Relativity, EM Forces: Historical Introduction

Alex Bogacz

Jefferson Lab

Relativistic Beams - 'Big Picture'

In a typical storage ring particles are accelerated and stored for **∼ 12 − 15 hours**

The distance traveled by particles moving at nearly the speed of light for 12 hours is:

$$\approx \mathbf{1.3 \times 10^{10}\ km}$$

This is about the distance from the Sun to Pluto and back!

Challenge: How to maintain them in a few millimeter wide beam-pipe?

Energy Units

1 eV, or 1 electron volt, is the energy acquired by 1 electron falling through a one volt potential difference

$$\boxed{\Delta E = q\Delta\Phi = q\left(\Phi_a - \Phi_b\right)}$$

$$1\ \mathrm{eV} = 1.6\times10^{-19}\ \mathrm{C}\ \times\ 1\ \mathrm{V}\ = 1.6\times10^{-19}\ \mathrm{J}$$

$$1\ \mathrm{MeV} = 10^{6}\ \mathrm{eV} = 1.6\times10^{-13}\ \mathrm{J}$$

To convert rest mass to eV use Einstein relation

$$E_0 = mc^2$$

where *m* is the rest mass. For electrons

$$E_{electron,0} = 9.1\times10^{-31}\ \mathrm{kg}\ \left(3\times10^{8}\ \mathrm{m/sec}\right)^2 = 81.9\times10^{-15}\ \mathrm{J}$$

$$= 0.512\ \mathrm{MeV}$$

Convenient Units

$$1\ \mathrm{eV} = (1.602 \times 10^{-19}\ \mathrm{C})(1\ \mathrm{V}) = 1.602 \times 10^{-19}\ \mathrm{J}$$

$$1\ \mathrm{MeV} = 1.602 \times 10^{-13}\ \mathrm{J}$$

$$1\ \mathrm{GeV} = 1.602 \times 10^{-10}\ \mathrm{J}$$

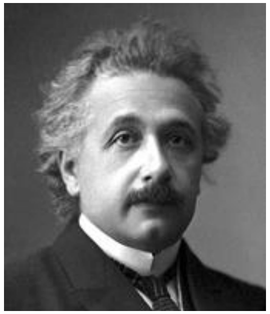
- How much is a TeV?
  - Energy to raise 1 g about 16 mm against gravity
  - Energy to power 100 W light bulb 1.6 ns
- But many accelerators have $10^{10\text{-}12}$ particles
  - Single bunch “instantaneous power” of tens of Terawatts
- Highest energy cosmic ray
  - ~300 EeV ($3\text{x}10^{20}$ eV or $3\text{x}10^{8}$ TeV!)

Reference Frames and Lorentz Transformation

- Lorentz transformation - The lab frame vs moving frame
  - Time/space coordinates in z velocity boost

$$\begin{pmatrix} ct' \\ x' \\ y' \\ z' \end{pmatrix} = \begin{pmatrix} \gamma & 0 & 0 & -\beta\gamma \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -\beta\gamma & 0 & 0 & \gamma \end{pmatrix} \begin{pmatrix} ct \\ x \\ y \\ z \end{pmatrix}$$

$$\beta = \frac{|\mathrm{v}|}{c} \qquad \gamma = \frac{1}{\sqrt{1-\beta^2}}$$



Albert Einstein (1879-1955)

Hendrik Lorentz (1853-1928)

- Invariance of space-time interval (Minkowski)

$$(ct')^2 - x'^2 - y'^2 - z'^2 = (ct)^2 - x^2 - y^2 - z^2$$

Hermann Minkowski (1864-1909)

Four-Velocity and Four-Momentum

- The proper time interval $d\tau = dt/\gamma$ is Lorentz invariant
- So, one can define a velocity 4-vector

$$cu^\alpha \equiv \left(\frac{dct}{d\tau}, \frac{dx}{d\tau}, \frac{dy}{d\tau}, \frac{dz}{d\tau}\right) = c\gamma(1, \beta_x, \beta_y, \beta_z)$$

- Minkowski Metric: $g^{\mu\nu} = g_{\mu\nu} = \mathrm{diag}(1, -1, -1, -1)$

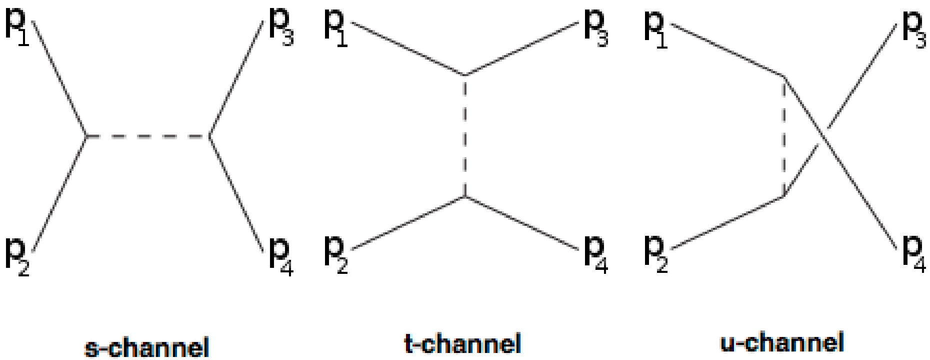
$$u^\alpha u_\alpha = u^\alpha g_{\alpha\beta} u^\beta = \gamma^2(1 - \beta^2) = 1$$

$$\gamma = \frac{1}{\sqrt{1-\beta^2}}$$

- One can also make a 4-momentum, etc

$$p^\alpha \equiv mcu^\alpha = mc\gamma(1, \beta_x, \beta_y, \beta_z)$$

Mandelstam Variables



where p_1 and p_2 are the four-momenta of the incoming particles and p_3 and p_4 are the four-momenta of the outgoing particles

$$s = (p_1 + p_2)^2 = (p_3 + p_4)^2$$
$$t = (p_1 - p_3)^2 = (p_2 - p_4)^2$$
$$u = (p_1 - p_4)^2 = (p_2 - p_3)^2$$

$$s + t + u = (m_1^2 + m_2^2 + m_3^2 + m_4^2)c^2$$

- Lorentz-invariant two-body kinematic variables
  - $p_{1\text{-}4}$ are four-momenta
- $\sqrt{s}$ is the total available center of mass energy
  - Often quoted for colliders
- Used in calculations of other two-body scattering processes
  - Moller scattering (e-e), Compton scattering (e-γ)

Stanley Mandelstam
1928 – present

Relativistic Newton Equation

- One can define a four-vector force in terms of four-momenta and proper time:

$$F^{\alpha} \equiv \frac{dp^{\alpha}}{d\tau}$$

$$\vec{F} = m\vec{a} = \frac{d\vec{p}}{dt}$$

$$\boxed{\vec{F} = q\left(\vec{E} + \vec{v} \times \vec{B}\right)}$$

- We are primarily concerned with electrodynamics, so now we must make the classical electromagnetic Lorentz force that obeys Lorentz transformations

Relativistic Electromagnetism

- The relativistic electromagnetic force equation becomes

$$\frac{dp^\alpha}{d\tau} = m\frac{du^\alpha}{d\tau} = \frac{q}{c}F^{\alpha\beta}u_\beta$$

$$F^{\alpha\beta} = \begin{pmatrix} 0 & E_x & E_y & E_z \\ -E_x & 0 & -B_z & B_y \\ -E_y & B_z & 0 & -B_x \\ -E_z & -B_y & B_x & 0 \end{pmatrix}$$

- It turns out, one can re-write this in somewhat simpler form

$$\frac{d(\gamma m\vec{v})}{dt} = q\left(\vec{E} + \vec{v}\times\vec{B}\right)$$

Relativistic Electromagnetism

- Relativistic Newton Equation with electromagnetic Lorentz force

$$\frac{d(\gamma m\vec{v})}{dt} = q\left(\vec{E} + \vec{v} \times \vec{B}\right)$$

- Can be treated as 'classical' Lorentz force equation with the momentum treated as 'relativistic'

$$\boxed{\vec{p} = \gamma m\vec{v} = \gamma\vec{\beta}mc}$$

Relativistic Electromagnetism

- Thankfully, we can re-write the relativistic force equation in somewhat simpler form:

$$\frac{d(\gamma m\vec{v})}{dt} = q\left(\vec{E} + \vec{v}\times\vec{B}\right) \;\Big/\, \vec{v}$$

- That is, "classical" E&M force equations hold, if we treat the momentum as relativistic:

$$\vec{p} = \gamma m\vec{v} = \gamma\vec{\beta}mc$$

- Multiplying both sides by the velocity, one gets energy transfer

$$\frac{d\gamma}{dt} = \frac{q\vec{E}\cdot\vec{v}}{mc^2} \qquad \frac{d}{dt}\Big/ (\beta\gamma)^2 = \gamma^2 - 1$$

- As expected, we can only get energy changes from electric fields, not from magnetic fields (static)

Special Relativity Review

- Accelerators: Applied Special Relativity
- Relativistic parameters:

$$\beta \equiv \frac{v}{c} \qquad \gamma \equiv \frac{1}{\sqrt{1-\beta^2}} \qquad \beta = \sqrt{1-1/\gamma^2}$$

  - Later β and γ will also be used for other quantities, but the context should usually make them clear
  - γ = 1 (classical mechanics) to ~ 2.05 x10^5

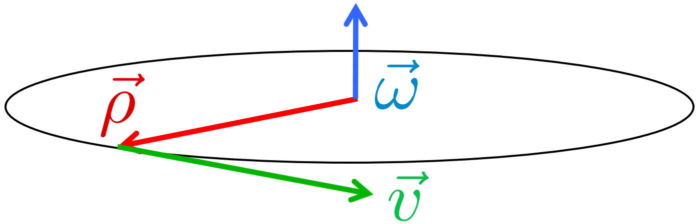
- Total energy U, momentum p, and kinetic energy W

$$U = \gamma m c^2 \qquad p = (\beta\gamma) m c = \beta \left(\frac{U}{c}\right) \qquad W = (\gamma - 1)\, m c^2$$

Particle Motion in Constant Magnetic Field (E = 0)

- In a constant magnetic field, charged particles move in circular arcs of radius ρ with constant angular velocity ω:

$$\vec{F} = \frac{d}{dt}(\gamma m \vec{v}) = \gamma m \frac{d\vec{v}}{dt} = q\vec{v} \times \vec{B}$$



$$\vec{v} = \vec{\omega} \times \vec{\rho} \qquad \Rightarrow \qquad q\vec{v} \times \vec{B} = \gamma m \vec{\omega} \times \frac{d\vec{\rho}}{dt} = \gamma m \vec{\omega} \times \vec{v}$$

- For $\vec{B} \perp \vec{v}$ we then have:

$$qvB = \frac{\gamma m v^2}{\rho} \qquad \text{or} \qquad p = \gamma m(\beta c) = q(B\rho)$$

$$\boxed{\omega = \frac{v}{\rho} = \frac{qB}{\gamma m}} \qquad \boxed{\frac{p}{q} = (B\rho)}$$

Magnetic Rigidity: Bending Radius vs Momentum

Particle beam

$$\boxed{\frac{p}{q} = (B\rho)}$$

Accelerator (magnets, geometry)

- This is such a useful expression in accelerator physics that it has its own name: 'magnetic rigidity'
- Ratio of momentum to charge
  - How hard (or easy) is a particle to deflect?
  - Often expressed in [T-m] (easy to calculate B)
- A very useful expression:

$$\boxed{\frac{p[\mathrm{GeV/c}]}{q[e]} \approx 0.3\, B[\mathrm{T}]\, \rho[\mathrm{m}]}$$

Cyclotron Frequency

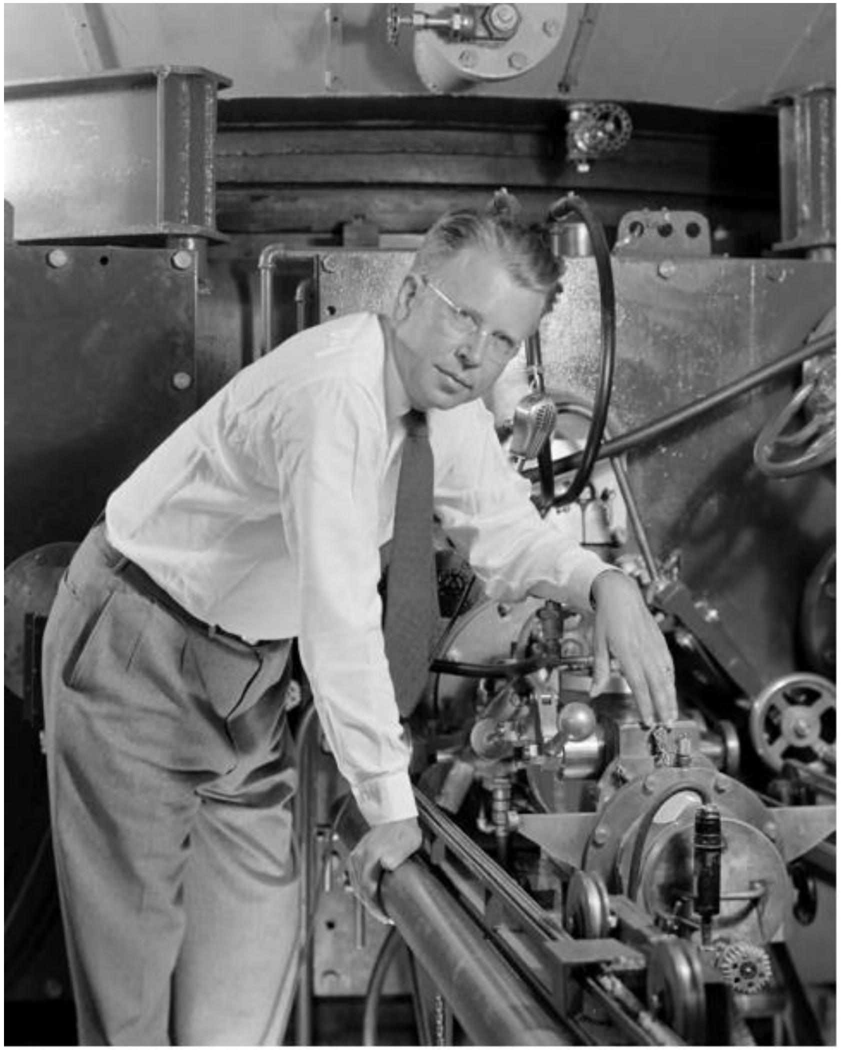
$$\boxed{\omega = \frac{v}{\rho} = \frac{qB}{\gamma m}}$$

- Another very useful expression for particle angular frequency in a constant field: cyclotron frequency

- In the nonrelativistic approximation: $\omega_{\text{nonrelativistic}} \approx \dfrac{qB}{m}$

- Revolution frequency is independent of radius or energy!

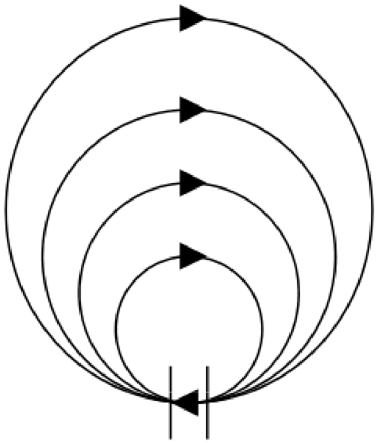
Lawrence and the Cyclotron



Ernest Orlando Lawrence

- Can one repeatedly spiral and accelerate particles through the same potential gap?

$\vec{B}$ $\odot$



Accelerating gap $\Delta\Phi$

Cyclotron Frequency Again

- Recall that for a constant B field

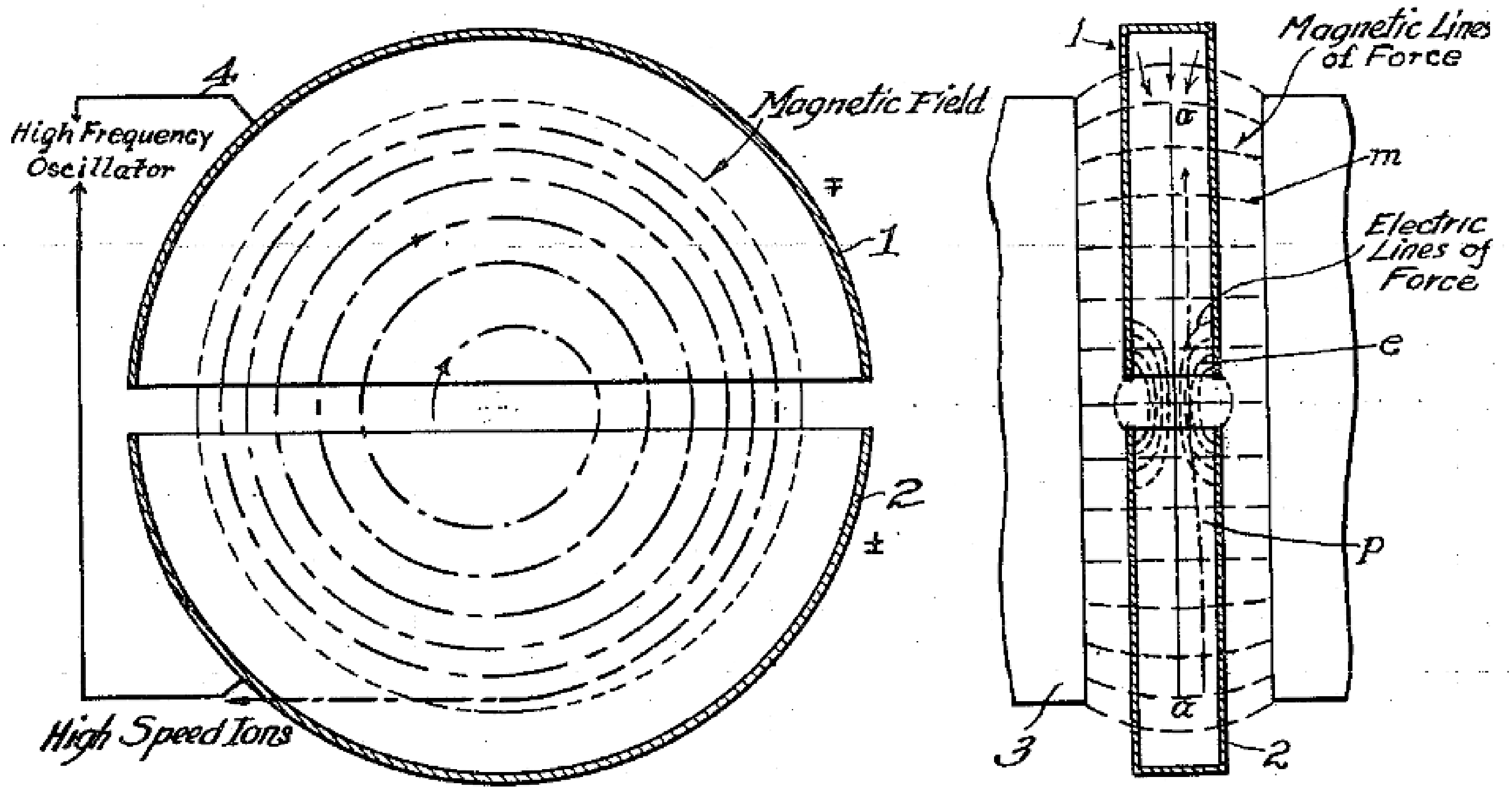
$$p = \gamma m v = q(B\rho) \quad \Rightarrow \quad \rho = \left(\frac{\gamma m}{qB}\right) v$$

  - Radius/circumference of orbit scale with velocity
    - Circulation time (and frequency) are independent of v
  - Apply AC electric field in the gap at frequency f_{rf}
    - Particles accelerate until they drop out of resonance

$$\omega = \frac{v}{\rho} = \frac{qB}{\gamma m} \qquad f_{\mathrm{rf}} = \frac{\omega}{2\pi} = \frac{qB}{2\pi\gamma m}$$

    - Works best with heavy particles (hadrons, not electrons)

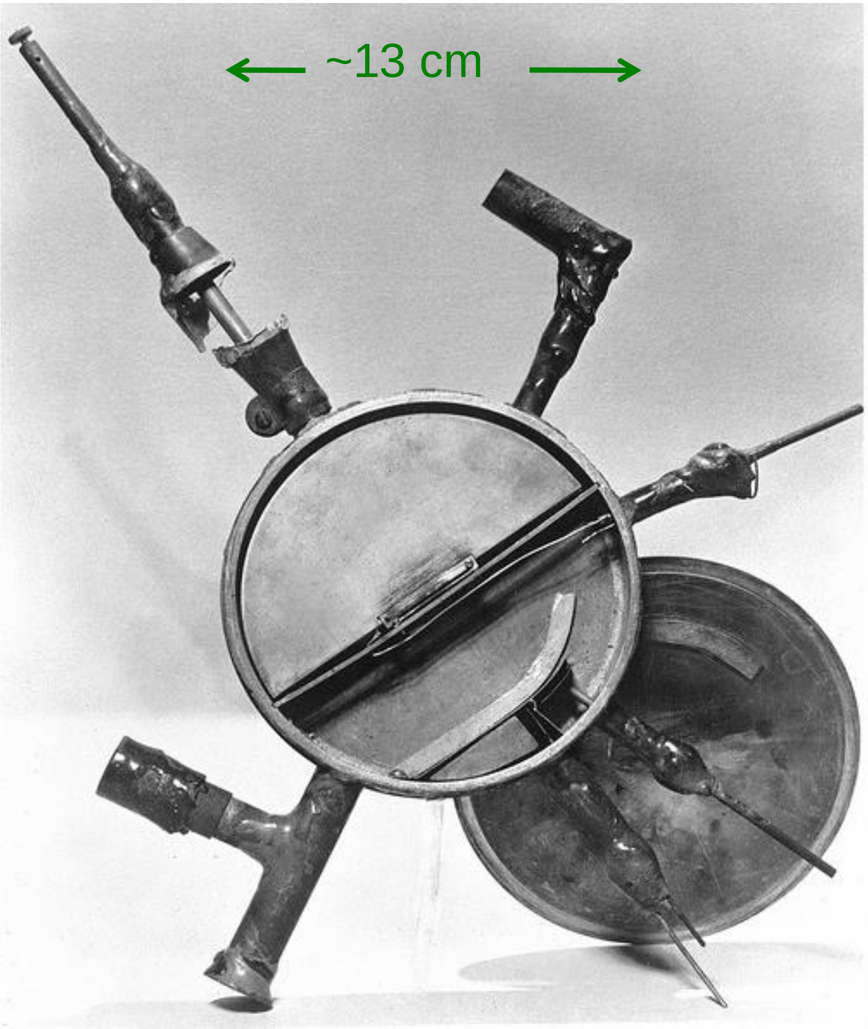
A Patentable Idea



- 1934 patent 1948384
  - Two accelerating gaps per turn!

All the Fundamentals of an Accelerator

- Large static magnetic fields for guiding (~1T)
  - But no vertical focusing
- HV RF electric fields for accelerating
  - (No phase focusing)
  - (Precise f control)
- p/H source, injection, extraction, vacuum
- 13 cm: 80 keV
- 28 cm: 1 MeV
- 69 cm: ~5 MeV
- … 223 cm: ~55 MeV



(Berkeley)

Livingston, Lawrence, 27”/69 cm Cyclotron

M.S. Livingston and E.O. Lawrence, 1934

The Joy of Physics

- Describing the events of January 9, 1932, Livingston is quoted saying:

> *"I recall the day when I had adjusted the oscillator to a new high frequency, and, with Lawrence looking over my shoulder, tuned the magnet through resonance. As the galvanometer spot swung across the scale, indicating that protons of 1-MeV energy were reaching the collector, Lawrence literally danced around the room with glee. The news quickly spread through the Berkeley laboratory, and we were busy all that day demonstrating million-volt protons to eager viewers."*

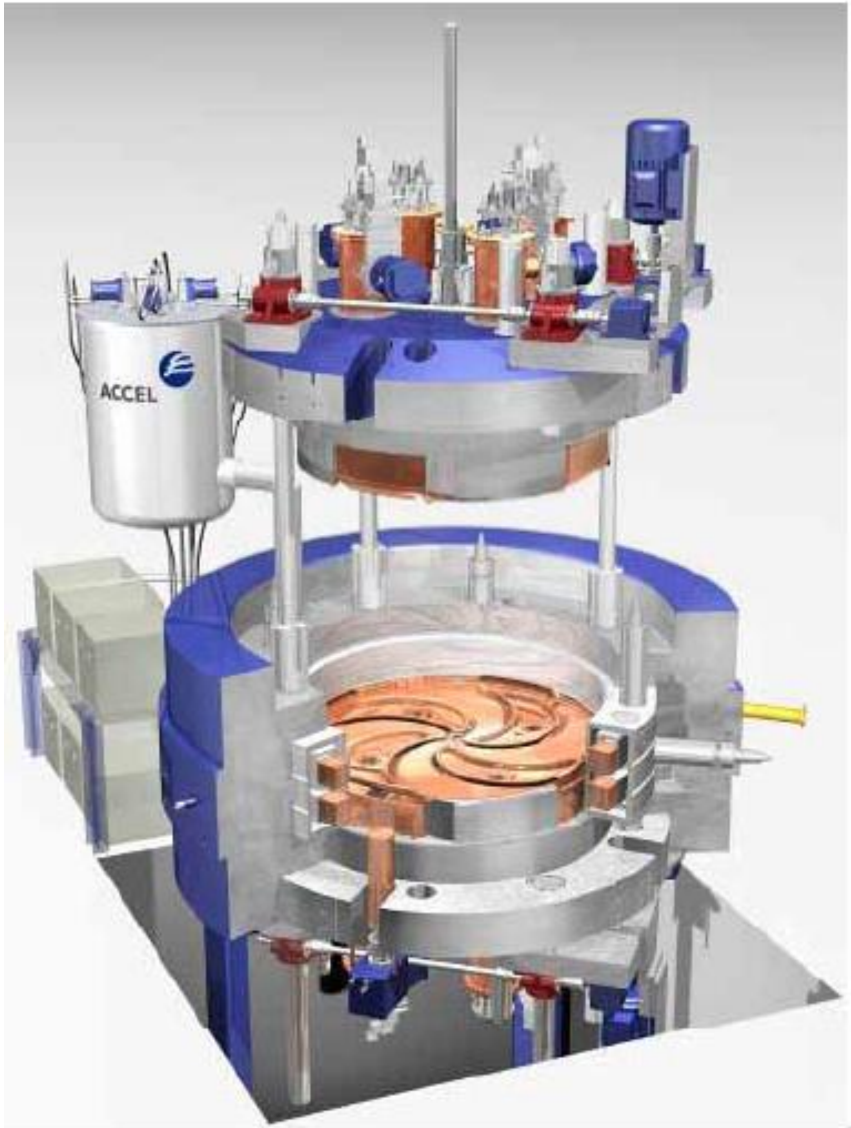
APS Physics History, Ernest Lawrence and M. Stanley Livingston

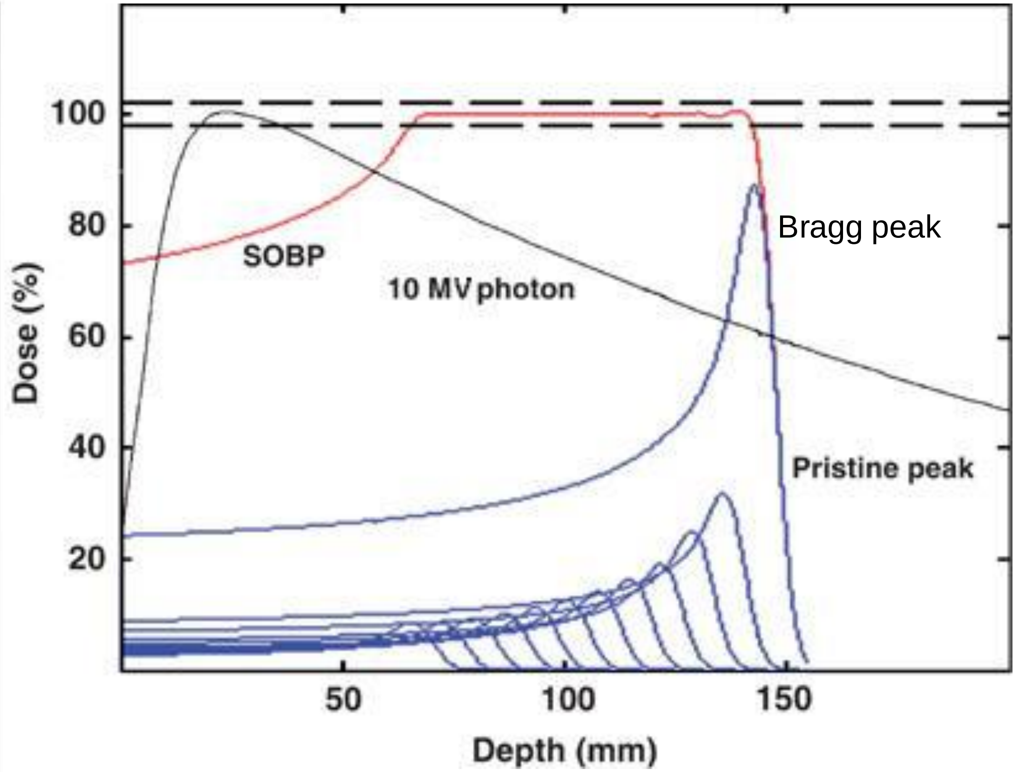
Cyclotrons Today

- Cyclotrons continue to evolve
  - Many contemporary developments
    - Superconducting cyclotrons
    - Synchrocyclotrons (FM modulated RF)
    - Isochronous/Alternating Vertical Focusing (AVF)
    - FFAs (Fixed Field Alternating Gradient)
  - Versatile with many applications even below ~ 500 MeV
    - High power (>1MW) neutron production
    - Reliable (medical isotope production, ion radiotherapy)
    - Power + reliability: ~5 MW p beam for ADSR (accelerator driven subcritical reactors, e.g. Thorium reactors)

Modern Medical Cyclotron (IBA)

ACCEL Radiotherapy Cyclotron





Distinct dose localization advantage for hadrons over X-rays

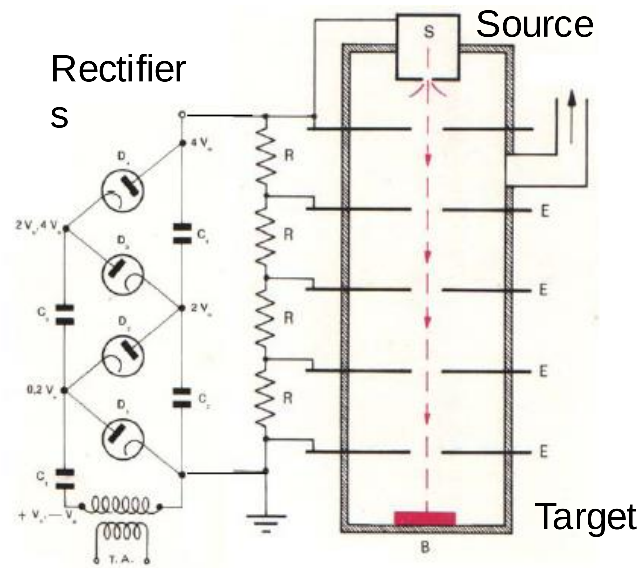
Also present work on proton and carbon radiotherapy fast-cycling synchrotrons

Brief Survey of Accelerator Concepts

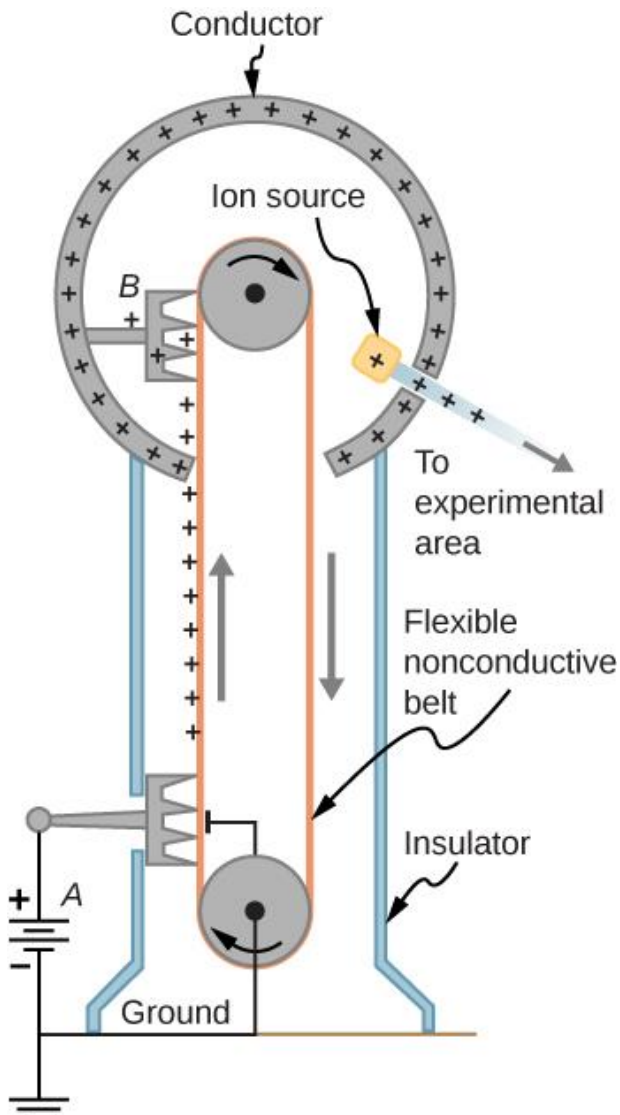
- Producing accelerating gaps and fields (DC/AC)
- Microtrons and their descendants
- Betatrons (and betatron motion)
- Synchrotrons
  - Fixed Target Experiments
  - Colliders and Luminosity (Livingston Plots)
  - Light Sources (FELs, Compton Sources)
- Others include
  - Medical Applications (radiotherapy, isotope production)
  - Spallation Sources (SNS, ESS)
  - Power Production (ADSR)

DC Accelerating Gaps: Cockcroft-Walton

- Accelerates ions through successive electrostatic voltages
  - First to get protons to > MeV
  - Continuous HV applied through intermediate electrodes
  - Rectifier-multipliers (voltage dividers)
  - Limited by HV sparking/breakdown
  - FNAL still uses a 750 kV C-W
- Also example of early ion source
  - H gas ionized with HV current
  - Provides high current DC beam



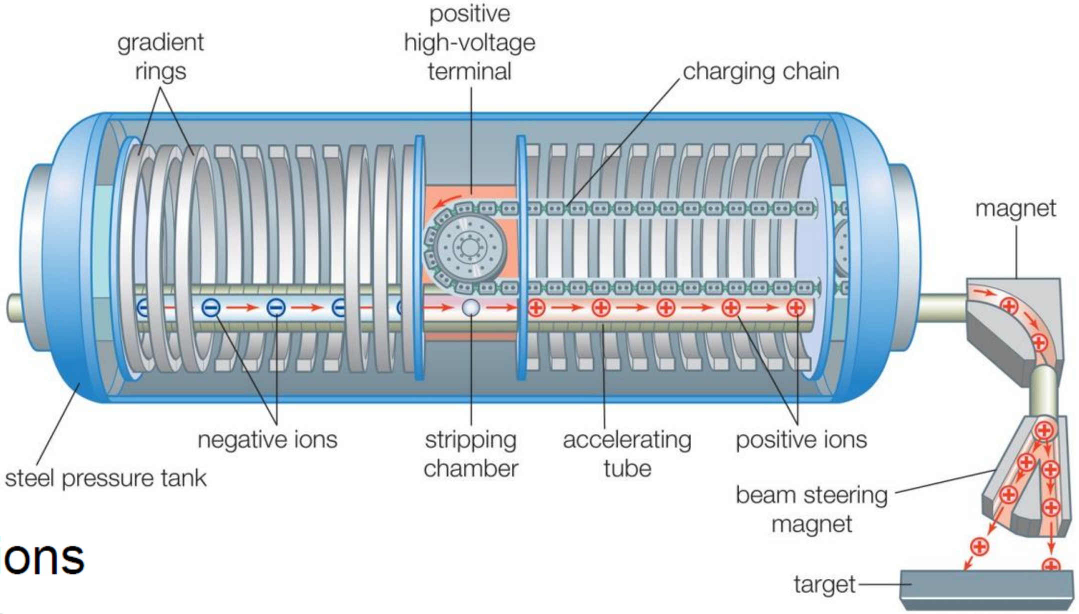
DC Accelerating Gaps: Van de Graaff



- How to increase voltage?
  - R.J. Van de Graaff: charge transport
  - First electrode sprays HV charge onto insulated conveyer belt
  - Carried up to spherical Faraday cage
  - Removed by second electrode and distributed over sphere
- Limited by discharge breakdown
  - ~ 2 MV in air
  - Up to 20+ MV in SF_6!
  - Ancestors of Pelletrons (chains)/Laddertrons (stripes)

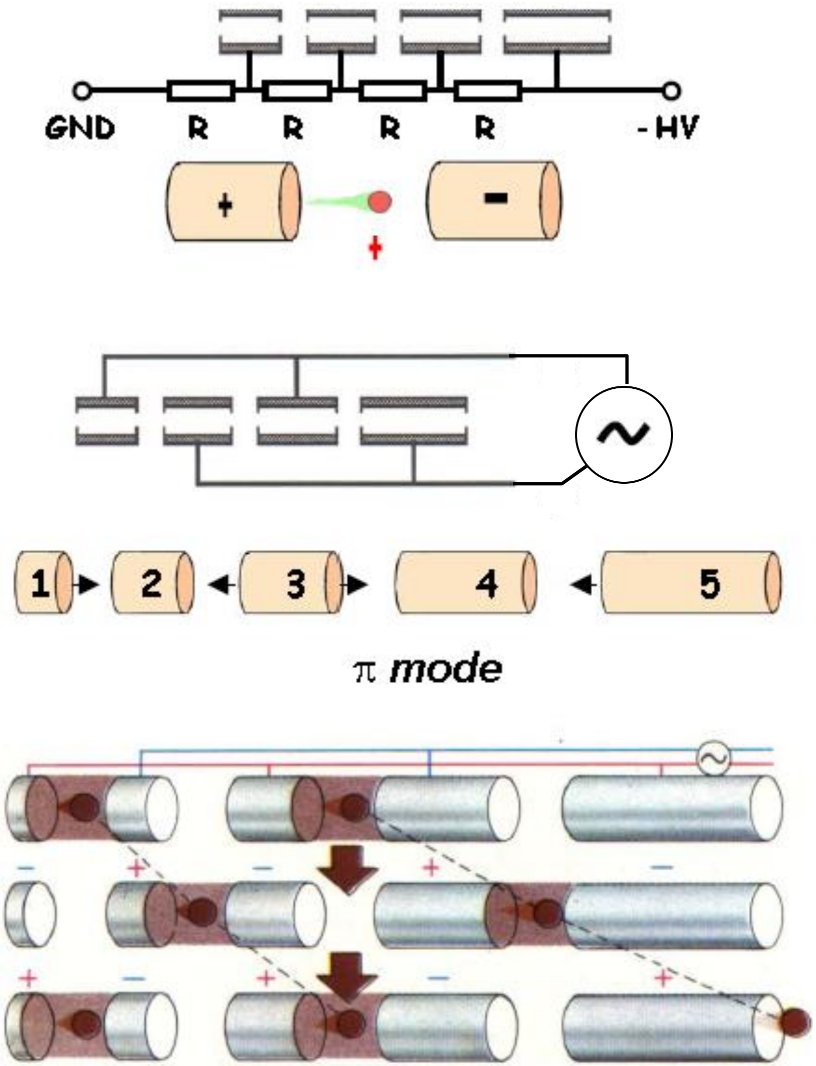
DC Accel Gaps: Tandem Van de Graaff

- Reverse ion charge state in middle of Van de Graaff allows over twice the energy gain
  - Source is at ground



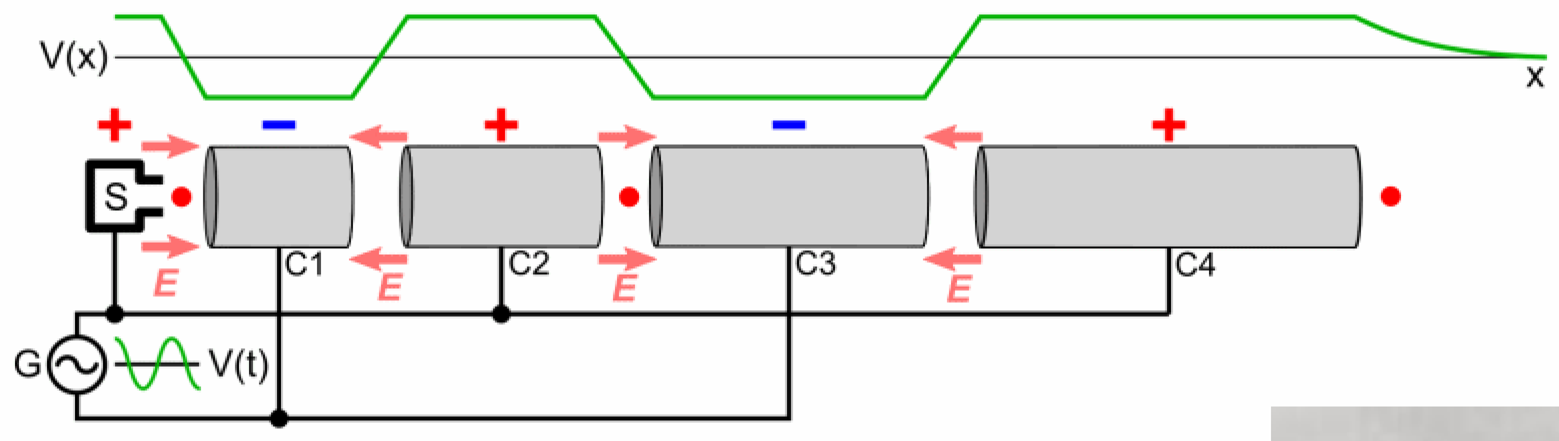
- This only works for negative ions
- However, stripping need not be symmetric
  - Second stage accelerates more efficiently
- BNL: two Tandems (1970, 14 MV, 24m)
  - Au^{-1} to $Au^{+10}/Au^{+11}/Au^{+12}$ to Au^{+32} for RHIC
  - About a total of 0.85 MeV/nucleon total energy

From Electrostatic to RF Acceleration



- Cockcroft-Waltons and Van de Graaffs have DC voltages, E fields
- What about putting on AC voltage?
  - Attach consecutive electrodes to opposite polarities of ACV generator
  - Electric fields between successive electrodes vary sinusoidally
  - Consecutive electrodes are 180 degrees out of phase (π mode)
- At the right drive frequency, particles are **accelerated in each gap**
  - While polarity change occurs, **particles are shielded in drift tubes**
  - To stay in phase with the RF, **drift tube length or RF frequency must increase** at higher energies

Widerøe Drift-tube Linac



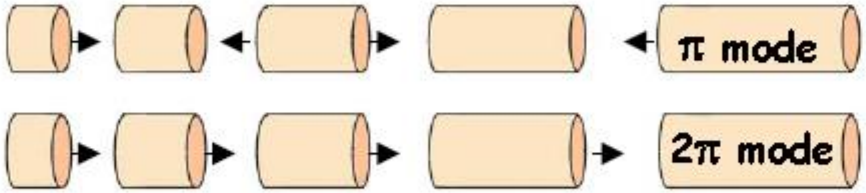
This experiment was successful and published in 1928, and became the progenitor of all high-energy particle accelerators. Widerøe's article was studied by Ernest Lawrence and used as the basis for his creation of the cyclotron in 1929.

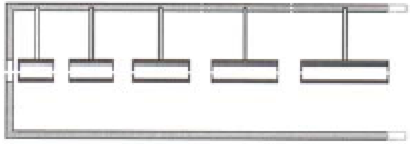
Rolf Widerøe

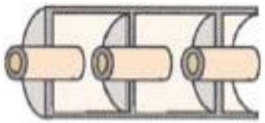
1902 -1996

Resonant Linac Structures

- Wideroe linac: π mode
- Alvarez linac: 2π mode
- Need to minimize excess RF power (heating)
  - Make drift tubes/gaps resonant to RF frequency
  - In 2π mode, currents in walls separating two subsequent cavities cancel; tubes are passive
  - We'll cover RF and longitudinal motion later this week…







Wideroe linac

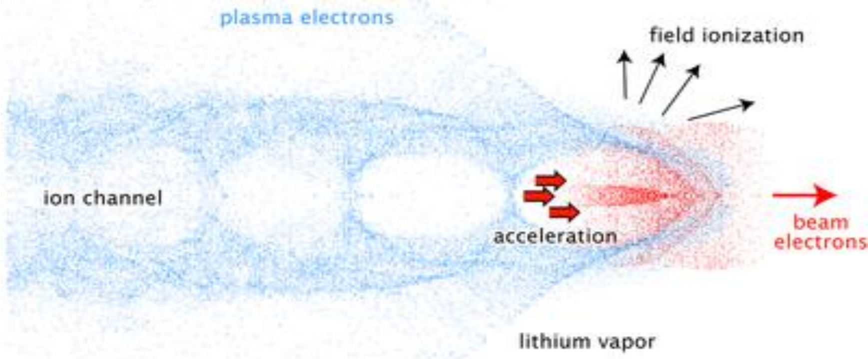
ALICE HI injector, IPN Orsay

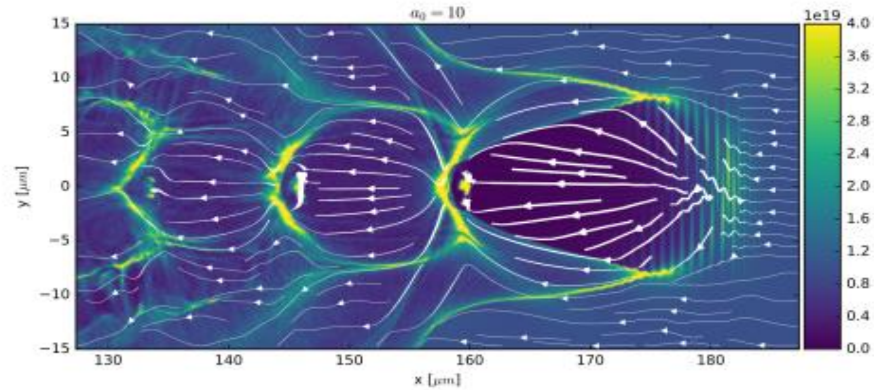
Drift tube linac

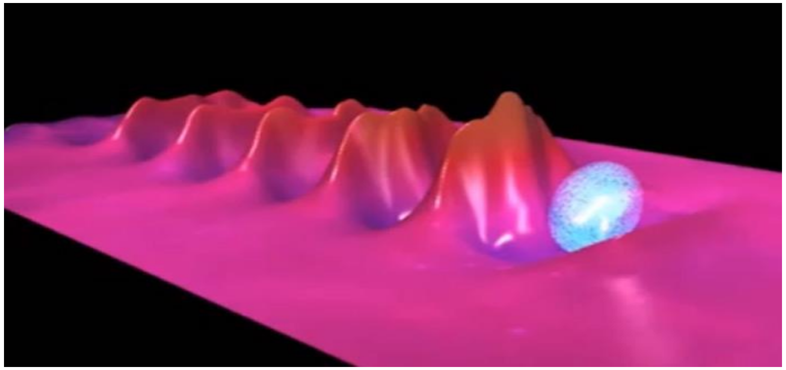
Saturne, Saclay

Advanced Acceleration Methods

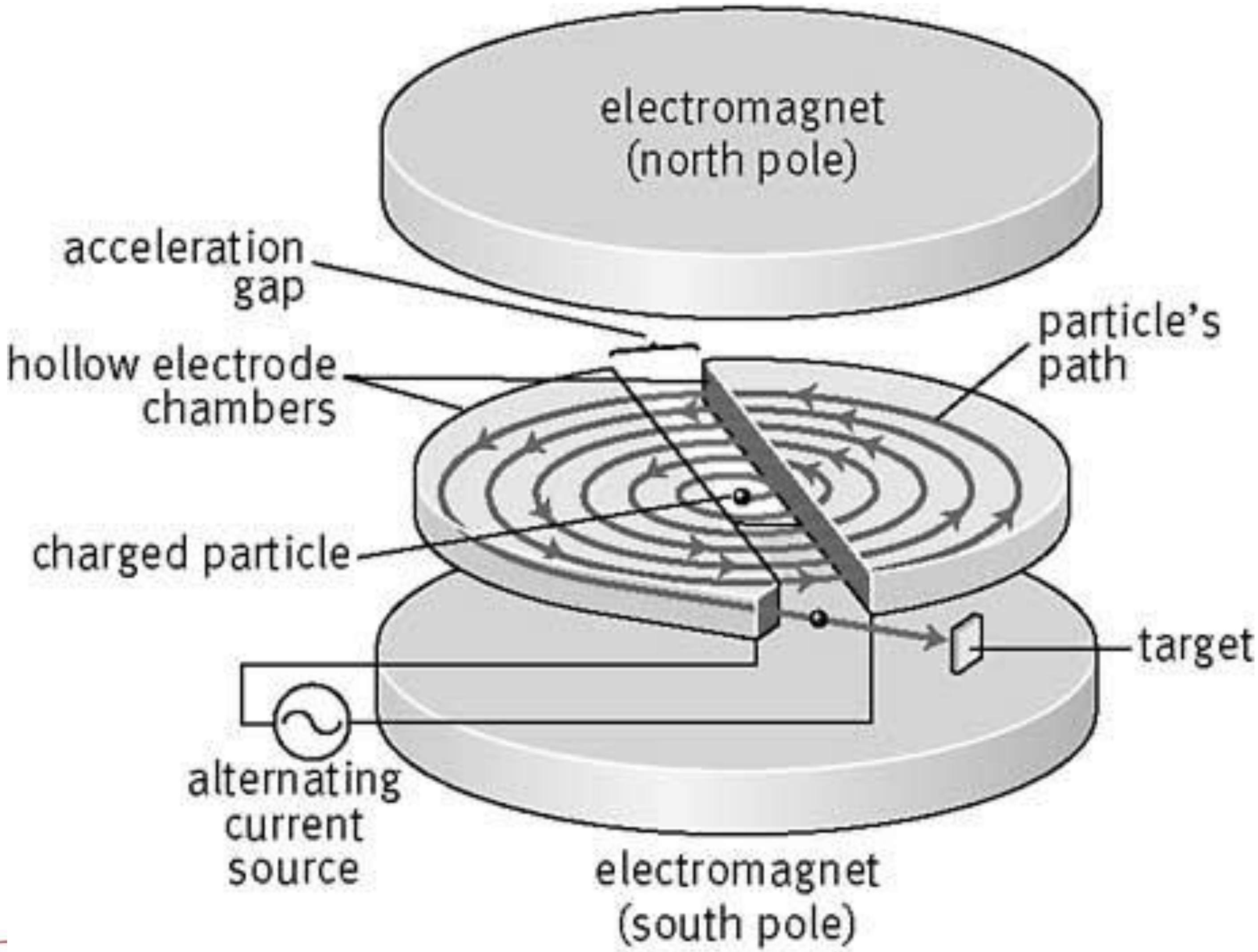
- How far do accelerating gradients go?
  - Superconducting RF acceleration: ~40 MV/m
  - CLIC: ~100 MV/m
    - Two-beam accelerator: drive beam couples to main beam
  - Dielectric wall acceleration: ~100 MV/m
    - Induction accelerator, very high gradient insulators
  - Dielectric wakefield acceleration: ~GV/m
  - Laser plasma acceleration: ~30 GV/m
    - electrons to 1 GeV in 3.3 cm
    - particles ride in wake of plasma charge separation wave



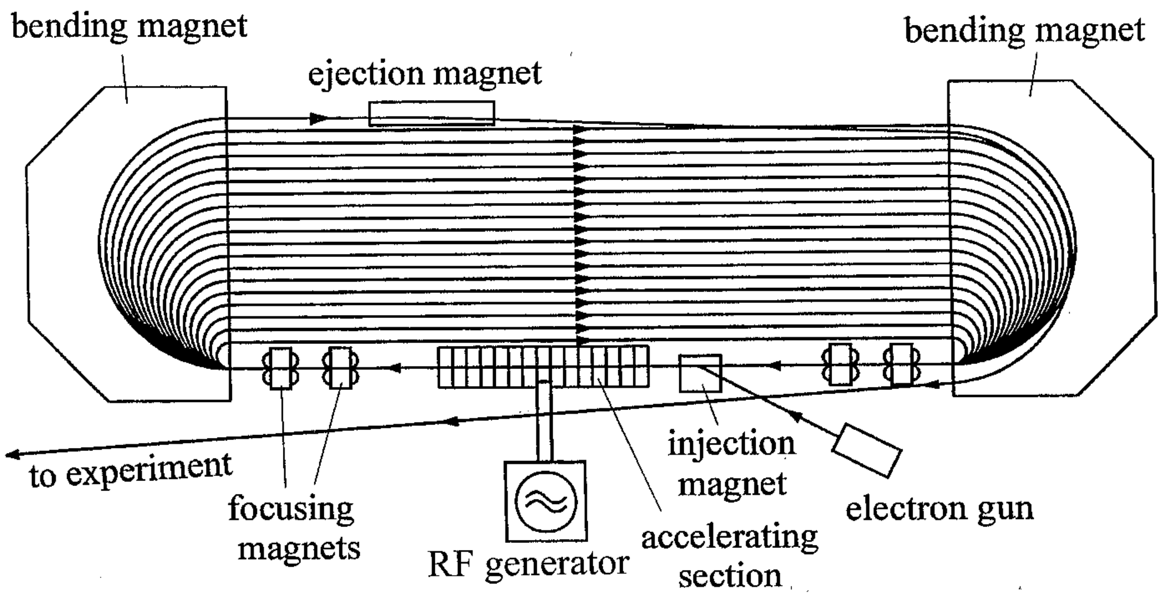




Cyclotrons (Again)



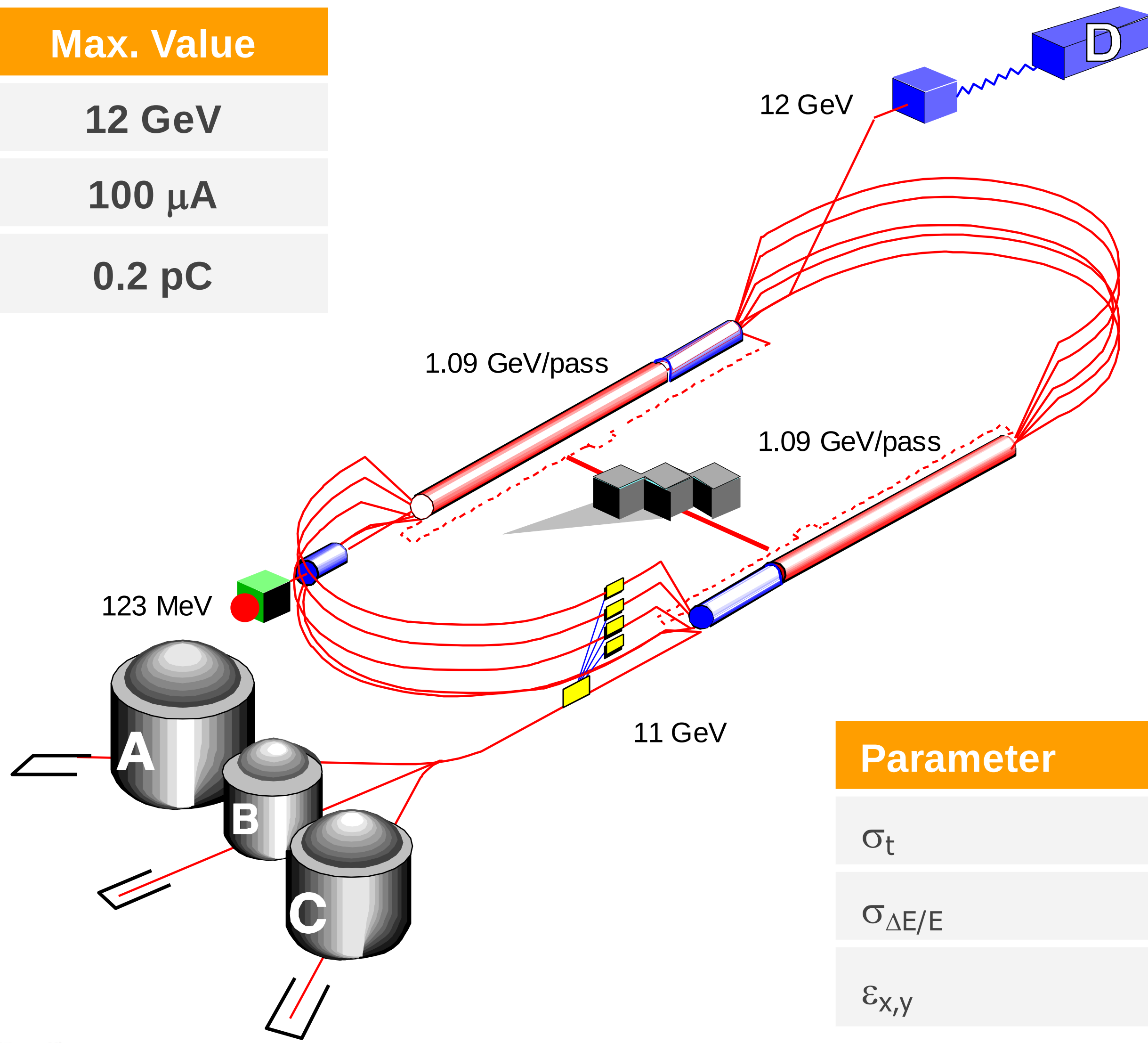
Microtrons



- What about electrons? Microtrons are like cyclotrons
  - but each revolution electrons "slip" by integer # of RF cycles
  - Trades off large # of revs for minimal RF generation cost
  - Bends must have large momentum aperture
  - Used for medical applications today (20 MeV, 1 big magnet)
  - Mainz MAMI: 855 MeV, used for nuclear physics

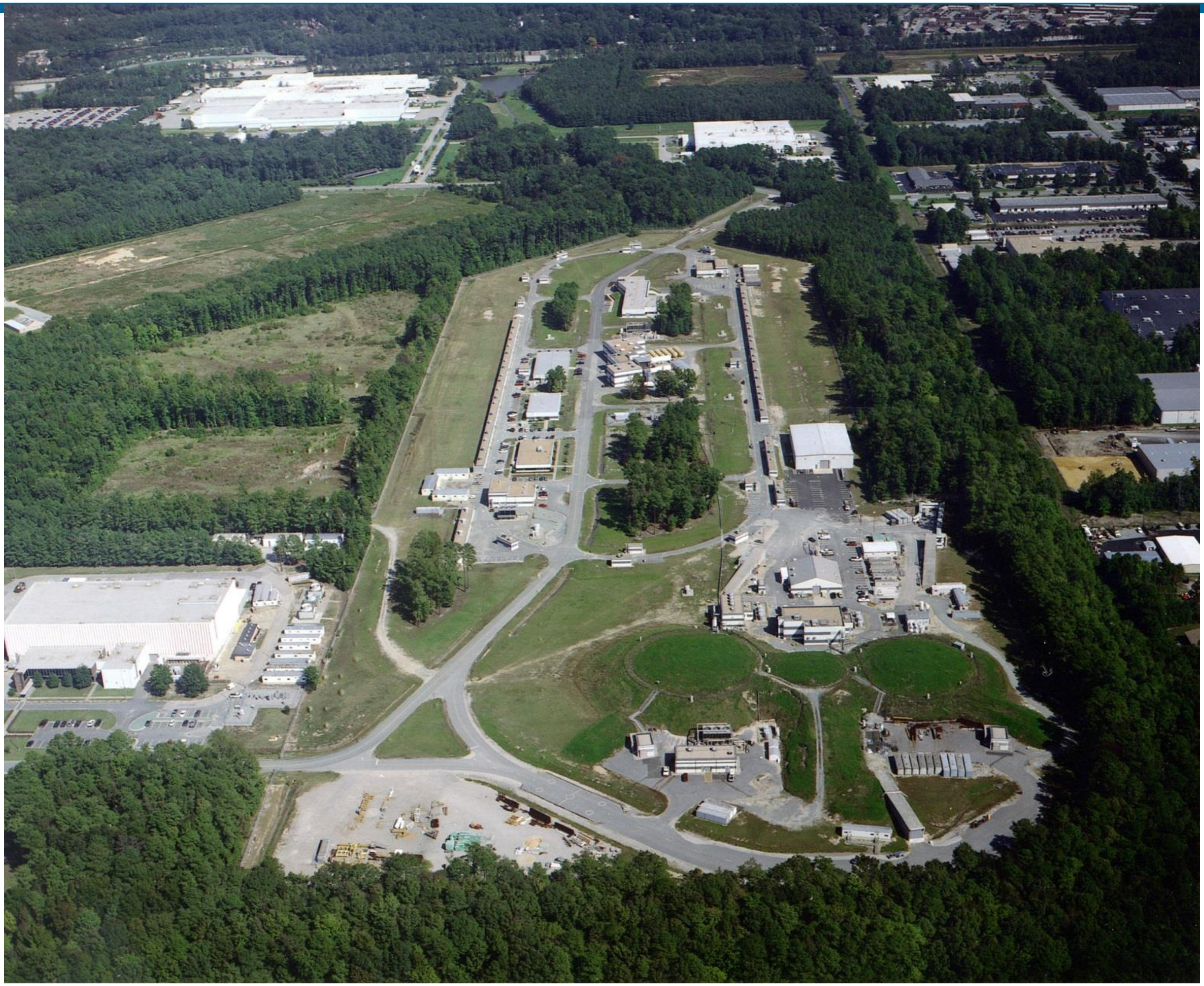
Recirculating Linac - CEBAF

Parameter	Max. Value
Energy	12 GeV
Current	100 μA
Charge	0.2 pC



Parameter	RMS
σ_t	0.7 ps
$\sigma_{\Delta E/E}$	0.01 %
$\varepsilon_{x,y}$	1 mm-mrad

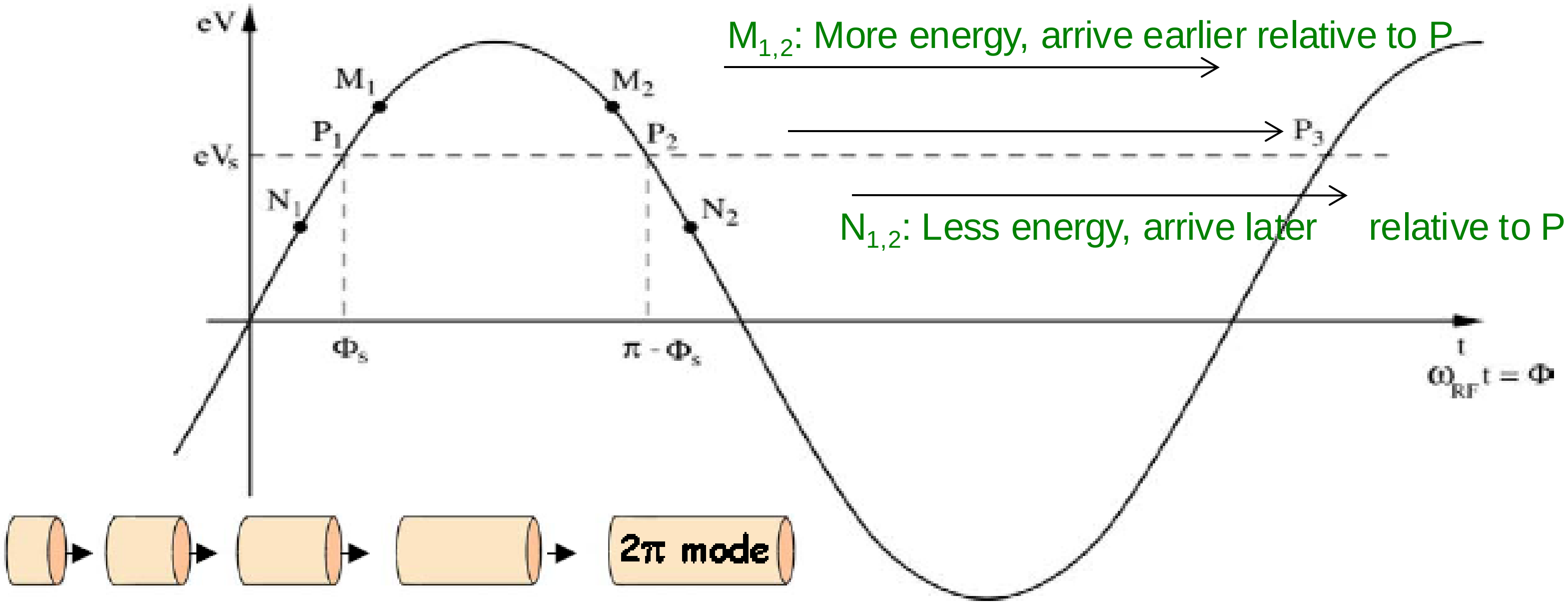
CEBAF at Jefferson Lab



New 12 GeV Linac

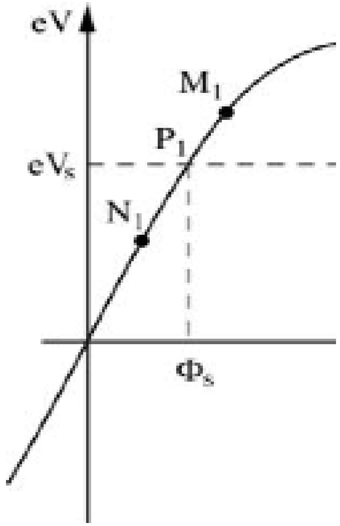
CEBAF Arcs

Phase Stability



- Consider a series of accelerating gaps (or a ring with one gap)
  - **By design** there is a synchronous phase Φ_s that gains just enough energy to hit phase Φ_s in the next gap
  - $P_{1,2}$ are fixed points: they "ride the wave" exactly in phase
- If increased energy means increased velocity ("below transition")
  - M_1,N_1 will move towards P_1 (local stability) => **phase stability**
  - M_2,N_2 will move away from P_2 (local instability)

Phase Stability Implies Transverse Instability



$$\frac{\partial V}{\partial t} > 0 \quad \Rightarrow \quad \frac{\partial E_z}{\partial z} < 0$$

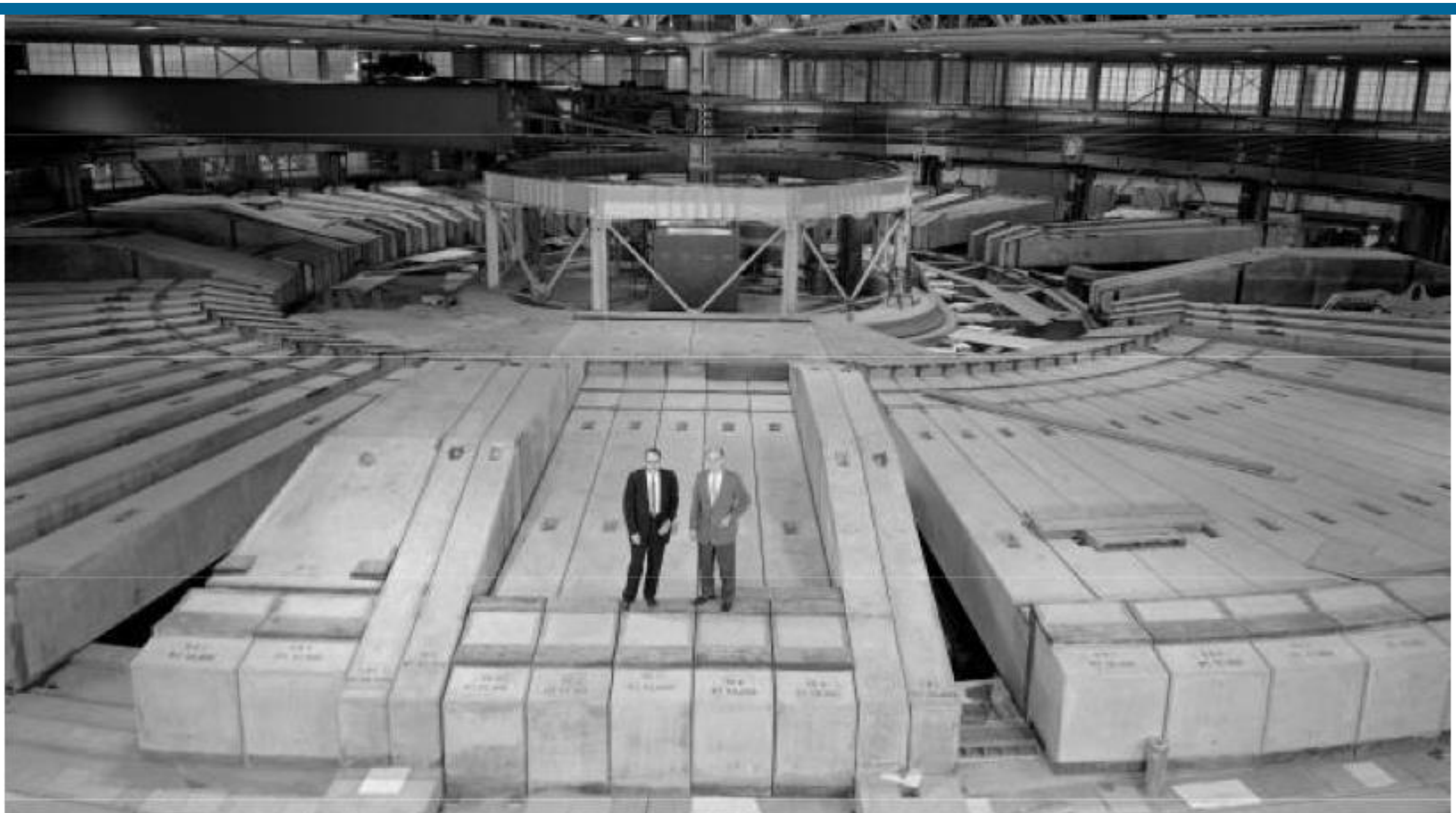
- For phase stability, longitudinal electric field must have a negative gradient. But then (source-free) Maxwell says

$$\vec{\nabla} \cdot \vec{E} = 0 \quad \Rightarrow \quad \frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z} = 0 \quad \Rightarrow \quad \frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} > 0$$

There must be some transverse defocusing/diverging force!

Any accelerator with RF phase stability (longitudinal focusing) needs transverse focusing! (solenoids, quads…)

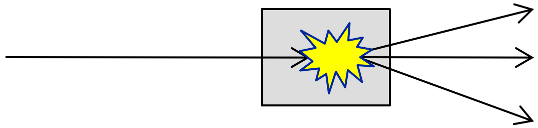
LBL Bevatron


Ed McMillan and Ed Lofgren

- Last and largest weak-focusing proton synchrotron
- 1954, Beam aperture about 4' square!, beam energy to 6.2 GeV
- Discovered antiproton 1955, 1959 Nobel for Segre/Chamberlain
  (Became Bevelac, decommissioned 1993, demolished recently)

Fixed Target Experiments

- Anti-proton production: Why did the Bevatron need 6.5 GeV protons?
  - Antiprotons are "only" 938 MeV/c² (times 2…)
  - Bevatron used Cu target, p + n → p + n + p + pbar
  - Mandelstam variables give:



$$\frac{E_{\rm cm}^2}{c^2} = 2\left(\frac{E_1 E_2}{c^2} - p_{z1}p_{z2}\right) + (m_{01}c)^2 + (m_{02}c)^2$$

  - Fixed Target experiment: $p_{z2} = 0$

$$(4m_{p0}c)^2 < \frac{E_{\rm cm}^2}{c^2} = 2\frac{E_1 m_{p0}\,c^2}{c^2} + 2(m_{p0}c)^2 \quad \Rightarrow \quad E_1 > 7m_{p0}c^2$$

  - Available CM energy scales with root of beam energy
    - Main issue: forward momentum conservation 'steals' energy

$$\boxed{E_{\rm cm} = \sqrt{2E_1(m_{02}\,c^2)}}$$

Two Serious Problems

- These machines were getting way too big
  - Bevatron magnet was 10,000 tons
  - Apertures scale linearly with machine size, energy

    (Length/circumference scales linearly with energy at fixed field strength too…)
- Fixed target energy scaling is painful
  - Available CM energy only scales with $\sqrt{E_{beam}}$
- Accelerator size grew with the square of desired CM energy
  - Something had to be done…..

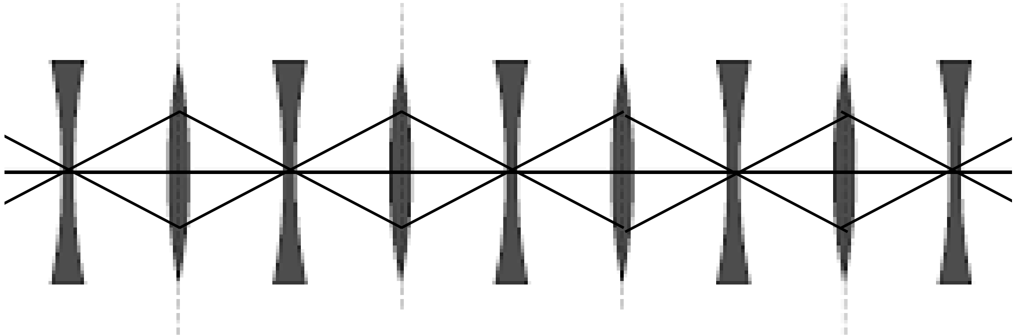
Strong Focusing (1952) and Colliders (1958-62)

Alternating Gradient Synchrotron (AGS)

First strong-focusing proton synchrotron

Strong Focusing – FODO Lattice

- In a quadrupole magnet field that focuses in one transverse direction, defocuses in the other
- Question: is it possible to develop a system that focuses in both directions simultaneously?
- Strong focusing: alternate the signs of focusing and defocusing: achieve net focusing!



FODO Lattice

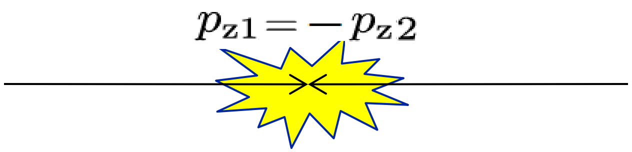
Ernest Courant
(1920 - 2020)

Hartland Snyder
(1913 – 1962)

Collider Experiments

- What if the Bevatron was a collider?
  - Antiprotons are "only" 938 MeV/c^2 (times 2…)
  - Two-body system (Mandelstam variables) gives (again):

$$(4m_{\mathrm{p}0}c)^2 < \frac{E_{\mathrm{cm}}^2}{c^2} = 2\left(\frac{E_1E_2}{c^2} - p_{\mathrm{z}1}p_{\mathrm{z}2}\right) + (m_{01}c)^2 + (m_{02}c)^2$$



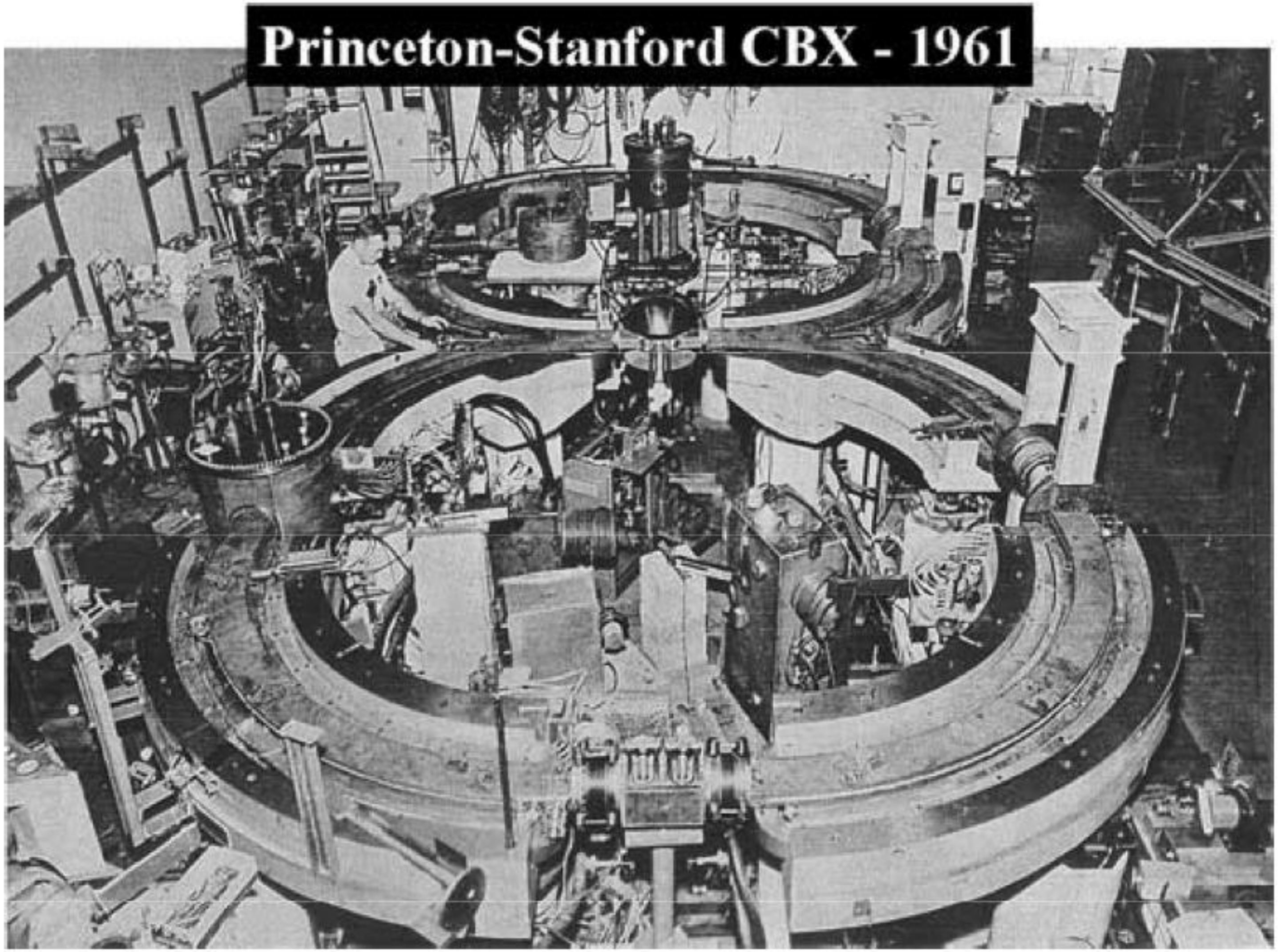
  - Collider's case

$$E_1 \gg m_{01}c^2 \qquad E_2 \gg m_{02}c^2 \qquad E_1 = c\,p_{\mathrm{z}1} \qquad E_2 = c\,p_{\mathrm{z}2}$$

$$\boxed{E_{\mathrm{cm}} = 2\sqrt{E_1E_2} = 2E} \text{ if } E_1 = E_2$$

  - Linear scaling with beam energy! $\Rightarrow \quad E_1 > 2m_{\mathrm{p}0}c^2$

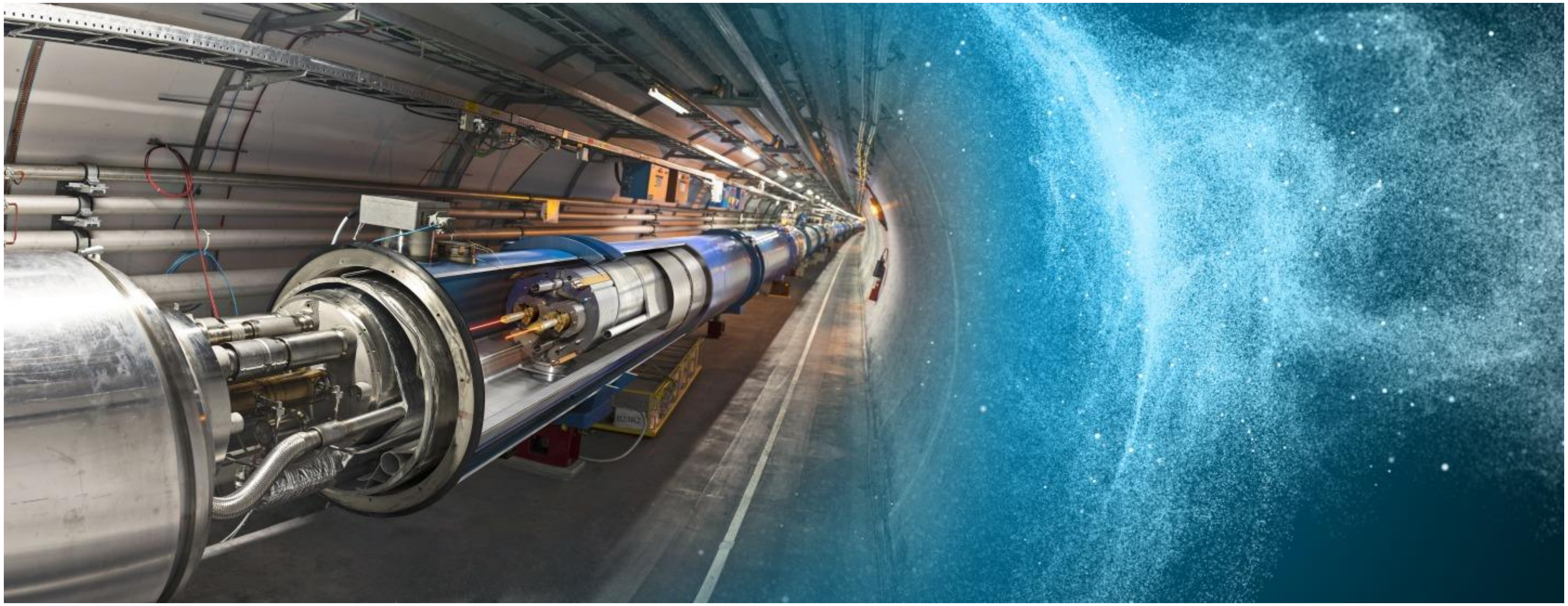
First Electron Collider



Tevatron/Main Injector Fermilab

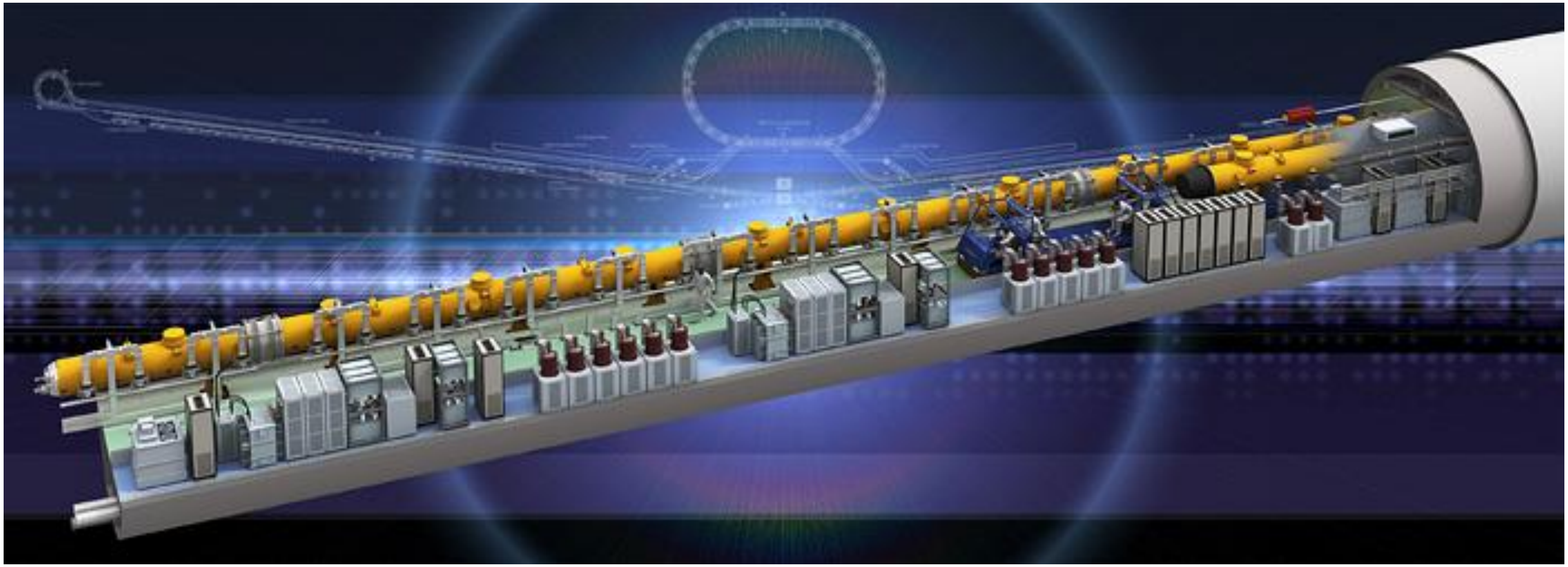
Tevatron: First TeV-scale accelerator; Large Superconducting Bends

Large Hadron Collider (former LEP tunnel)



The Large Hadron Collider (**LHC**) is the world's largest and most powerful particle collider ever built, and the largest single machine in the world. It was built CERN between 1998 and 2008. It lies in a tunnel 27 kilometres in circumference, as deep as 175 metres . On May 20 2015, the LHC reached center of mass energy of 13 TeV (the current world record).

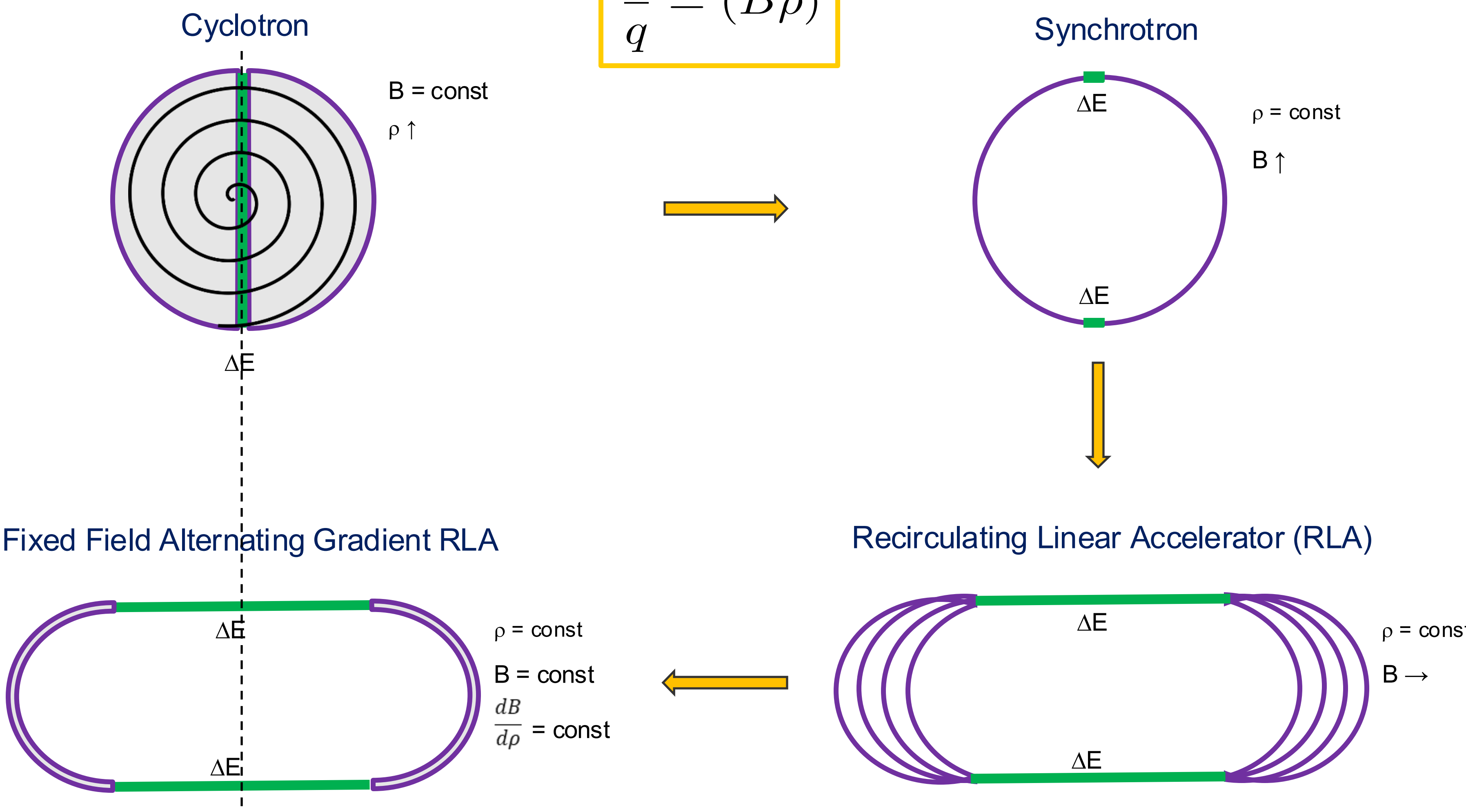
ILC - Energy Frontier Lepton Collider



The International Linear Collider (ILC) is a proposed particle accelerator that would be built in a 31km long underground tunnel. It would collide electrons and positrons head on in the middle of the tunnel reaching multi-TeV center of mass energies.

‘Evolutionary Path’ of Particle Accelerators

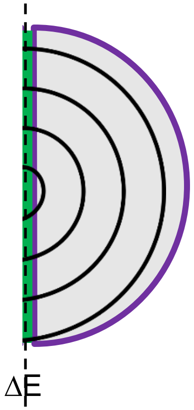
$$\frac{p}{q} = (B\rho)$$



‘Evolutionary Path’ of Particle Accelerators

$$\frac{p}{q} = (B\rho)$$

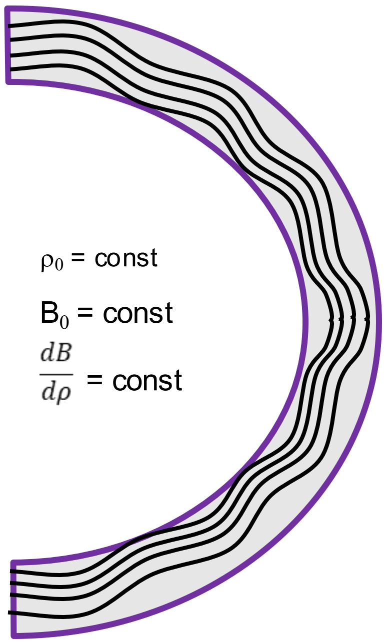
Cyclotron



B = const

$\rho \uparrow$

FFA Arc



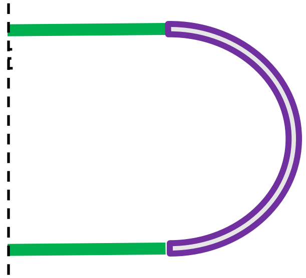
ρ_0 = const

B_0 = const

$\frac{dB}{d\rho}$ = const

$$\left(B_0 + \frac{dB}{d\rho}\Delta\rho\right) \times (\rho + \Delta\rho) = \frac{p}{q}$$

Fixed Field Alternating Gradient RLA

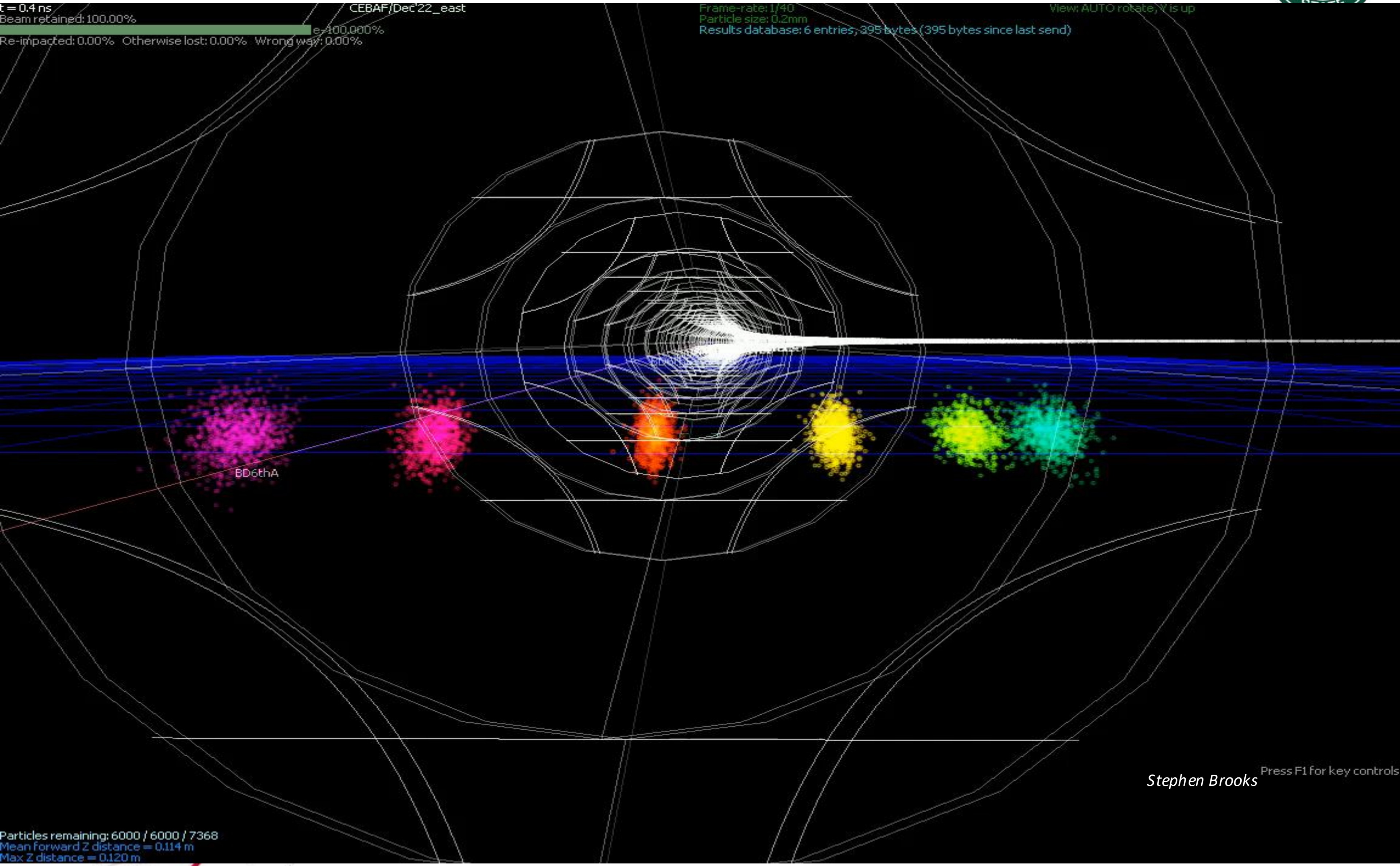


ρ = const

B = const

$\frac{dB}{d\rho}$ = const

Six beams with energies from 10 to 22 GeV

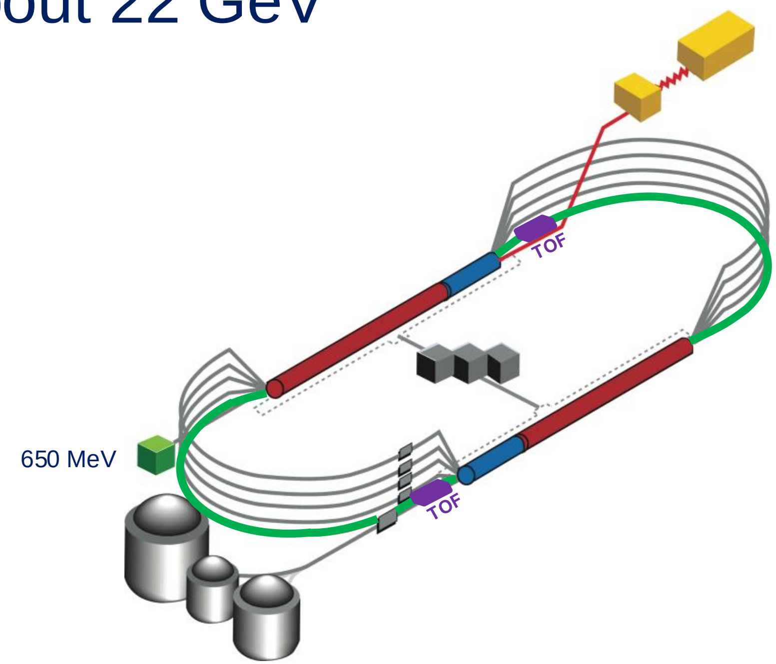


CEBAF Energy Upgrade Path via FFA* Arcs

Additional FFA* racetrack in the existing CEBAF tunnel to reach the top energy of about 22 GeV

*FFA (Fixed Field Alternating-Gradient)

- Remove the highest recirculation pass (Arc 9 & A) and replace them with two FFA arcs including time-of-flight chicanes
- Recirculate 4 + **6** times to get to **22 GeV**



Pass Arithmetic: **5** -1 + **6** = **10**

CEBAF Arcs

Luminosity

- **Luminosity** L is a measure of how many interactions of **cross section** σ can be created per unit time

$$L\sigma = \frac{dN}{dt} \qquad N = \sigma \int L\, dt = \sigma\, L_{\text{int}}$$

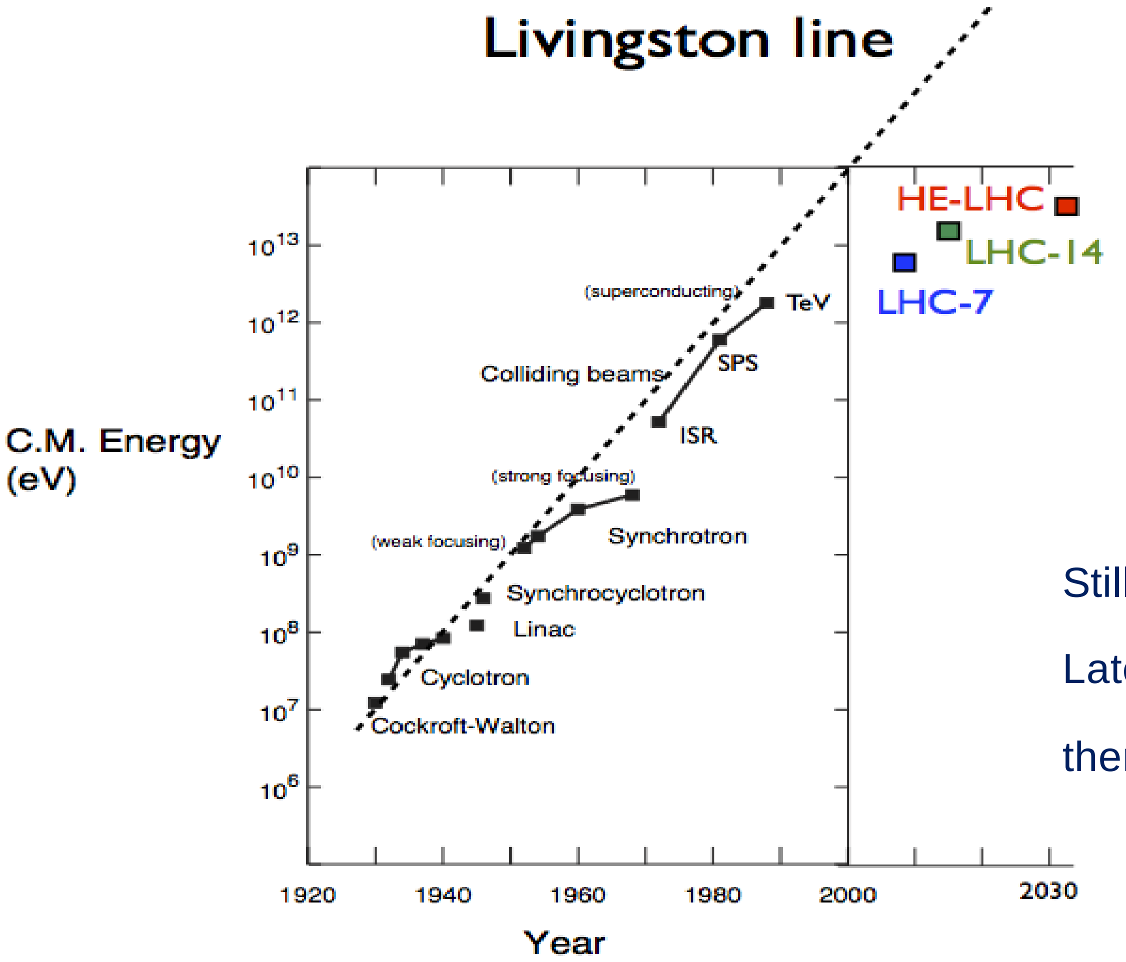
  - L_{int} is integrated luminosity, an important factor of production for colliders
  - $[L] = \text{cm}^{-2}\ \text{s}^{-1}$, $[L_{int}] = \text{cm}^{-2}$ (1 ba=10^{-24} cm; 1 $\text{pb}^{-1}=10^{36}\ \text{cm}^{-2}$)
- For equal-sized head-on Gaussian beams in a collider

$$L = \frac{f_{\text{rev}}\, h\, N_1\, N_2}{4\pi\sigma_x\sigma_y}$$

  - $\sigma_{x,y}$ are rms beam sizes, h is number of bunches
    - Colliding 100 μm 7.5×10^9 p bunches at 100 kHz for 1 year gives about 1 pb^{-1} of integrated luminosity

Livingston Plot

Stan Livingston observed that accelerator energy was growing exponentially (in 1950)



M. Stanley Livingston
(1905-1986)

Still holds true over 60 years (!)

Later technologies tend to saturate

then new technologies are introduced...