

GEOMETRICAL OPTIMIZATION of THE PLANE WAVE TRANSFORMER

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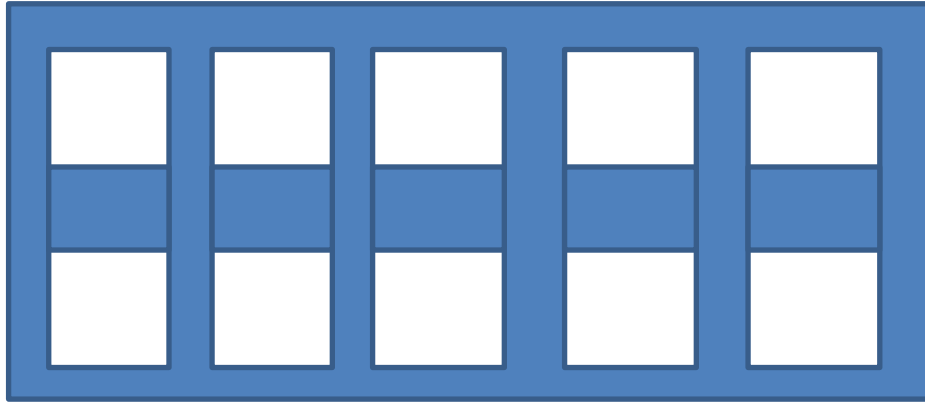
Standard simulation tools were used to characterize the Plane Wave Transformer (PWT) Accelerator Structure and determine the dependence of frequency and  $r$  over  $Q$  on the geometrical dimensions of the structure. For PWTs with flat washers, sets of dimensions maximizing  $r$  over  $Q$  at 157 ohms per cell for 2856 MHz and maximizing  $r$  over  $Q$  at 103 ohms per cell for 11424 MHz were determined.

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*Figure 1:Plane Wave Transformer Structure*

## I.INTRODUCTION

In a particle accelerator, charged particles interact with electric fields via the force,  $F=QE$ , gaining kinetic energy. The output of the accelerator is thus a beam of particles, moving through an evacuated enclosure. For lower energies and smaller beam currents, an electrostatic accelerator suffices, using a static field between two conductors, similar to a capacitor. The energy gained by the particles is thus equal to the charge times the voltage across the accelerator,  $W=QV$ . To accelerate larger numbers of particles to higher energy values, a radio-frequency (RF) accelerator is necessary.

In an RF accelerator, particles travel through an accelerating structure or cavity in which electromagnetic waves oscillate. The mode of the structure that is energized has a strong electric field parallel to the beam path on the beam path. The beam is usually bunched, so its charge distribution is periodic along its length. As the position of the bunch changes with time and the electric field varies with both time and position, the work done by the electric field is a transit-time integral. Synchronization of the bunches with the phase of the oscillating field is crucial to maximize the energy transfer.

In a linear accelerator, the accelerating structure is very long compared to the free-space wavelength of the accelerating fields. The bunches ride along at the phase velocity of the waves. In a regular waveguide mode, the phase velocity is faster than the speed of light, making it impossible for particles with nonzero mass to keep up. Therefore, the accelerating structure must be a slow-wave structure, with periodic loading that leads to a mode whose phase velocity can be matched to the motion of the particles.

Microwave power is fed into the slow-wave structure via a coupler from a waveguide. The coupler needs to provide impedance matching between the energized mode of the structure and a transmission mode of the waveguide to maximize transmission of energy to the beam and minimize reflection back into the waveguide. Designing a coupler to achieve this goal with very little ohmic loss can be a challenge, especially when coupler design comes after design of the slow-



wave structure. One proposed type of structure for which the coupler interface is natural is the Plane Wave Transformer.

The plane wave transformer (PWT)[1,2] consists of a cylindrical metal pipe containing metal washers arranged periodically along the axis. The axis of the pipe is normal to the plane of each washer. The washers do not touch the walls of the pipe, making the PWT different from many other linac designs.

The name “plane wave transformer” stems from the fact that the peripheral region between the washers and the walls is similar to coaxial cable, while the region near the axis between two washers is similar to a cylindrical pillbox cavity. In the mode selected to energize, the fields in the peripheral region are similar to the TEM (plane wave) mode of a coaxial cable, while the fields near the axis resemble those of the lowest-order transverse magnetic (TM ) mode of a pillbox cavity, having maximum electric field on axis to maximize coupling to the particle beam. This structure thus transforms power from the plane wave mode of coaxial cable into the TM mode for particle acceleration. Coupling the structure to a coaxial cable for power input should be straight forward.

Efficient coupling to a coaxial waveguide should reduce power reflected from the plane wave transformer back toward the microwave source. The washers are supported by dielectric rods running parallel to the axis. These rods could in principal be made hollow and used to transport deionized water or another coolant to and from hollow washers. If the washers are thick, hollow, and cooled, it might

be possible to lace permanent magnets inside the washers for periodic focusing of the beam.

The goal of this thesis is to find combinations of dimensions including pipe radius, structure wavelength, and thickness, inner radius, and outer radius for the washers such as to maximize performance and efficiency of the plane wave transformer.

## I.1 PHYSICS of ACCELERATING STRUCTURES

The goal of an accelerating structure is to impart maximum kinetic energy into the beam of charged particles. The continuing cost of operating such a structure is the microwave power fed into it.

For a normal conducting structure, most of the input microwave power is lost as heat via the skin effect in the walls of the cavity. In addition to the cost of energy, the operation of a cooling system to remove this heat adds to the cost of operation. If fields oscillating at a frequency  $\omega$  store energy  $U$  in a mode of the structure, then the power dissipated is

$$P_{dis} = \frac{\omega U}{Q}. \quad (1)$$

Here  $Q$  is the quality factor of the mode. Comparing a cavity mode with a parallel RLC circuit, wall losses can be represented by the shunt resistance in the circuit, whose quality factor is .

$$Q = R \sqrt{\frac{C}{L}}.$$

The gain in energy by a charged particle passing through a cavity is usually normalized by the charge of the particle to define the voltage of the cavity. The effective shunt impedance of the cavity is then that resistance which-when subjected to the cavity voltage- would dissipate power equal to that dissipated by the cavity:[4] .

$$R_{sh} = \frac{V^2}{P_{dis}} = \frac{QV^2}{\omega U}. \quad (2)$$

Thus a high value of shunt impedance means more beam energy and/or less power.

The quality factor of a cavity mode depends on both geometry and the material from which the cavity is built. A useful figure of merit for geometry alone is the ratio of shunt impedance to quality factor,  $r/Q$ . So “r over Q” compares the beam accelerating voltage to the energy stored in the cavity mode. Optimizing the geometry of a cavity means maximizing  $r/Q$ .

## I.2 LONGITUDINAL PHYSICS of CHARGED PARTICLE BEAMS

From the perspective of the charge moving in a beam, a particle accelerator is an environment with which to interact electromagnetically. In a

metal beam pipe, the charge distribution in the beam implies an image charge in the pipe, while the beam current implies an image current in the walls. As charge passes through a section of larger radius, such as an accelerating structure, its radial electric field and azimuthal magnetic field deposit energy in the normal modes of the structure, which ring at their natural frequencies. These wakefields represent radiation of energy from the beam to the structure.

For an antenna, the radiation of power to the environment is expressed in terms of the radiation resistance of the antenna. Radiation resistance is a single number that summarizes the integrated effect of the radiation pattern of the antenna. Analogously, the radiation of power from a particle beam is expressed in terms of longitudinal impedance per unit length. The frequency-dependent longitudinal impedance summarizes the integrated effect of electromagnetic interactions between the beam and whatever it sees.

Lorenz Reciprocity implies that an antenna functions equally well as either a transmitter or as a receiver. Similarly, the exchange of energy between a particle beam and a cavity mode occurs equally in both directions. The  $r$  over  $Q$  value for a cavity mode measures how strongly it transfers energy from the mode to the beam. Longitudinal impedance measures how strongly energy is transferred from the beam to the mode, so Lorenz Reciprocity requires a relationship between the two quantities.

### I.3 THE PLANE WAVE TRANSFORMER

The outer edges of the washers are close to the pipe wall, and the coax-TEM-like electric field in that region is radial, so that region behaves like a capacitor, with surface charge proportional to the radial electric field. As the radial field oscillates, so does the charge on the edge. This implies a radial current on the faces of the washer and an oscillating charge distribution near the hole. Charge concentration near the aperture leads to a strong accelerating field between the aperture of one washer and the aperture of the next.

One might imagine a current loop centered on the space between two adjacent washers. Conduction current flows radially inward on the face of the right washer, strong, oscillating electric field carries displacement in the axial direction near the axis, conduction current flows radially outward on the face of the left washer, oscillating radial electric field carries displacement from the left washer to the pipe, conduction current flows in the axial direction on the inner surface of the pipe, and oscillating radial electric field carries displacement radially inward back to the right washer. From this perspective, beam current replaces part of the axial displacement current, reducing the electric field as it loads the system. Nose-cones added to narrow the gap near the axis added localized capacitance near the axis.

#### I.4 SIMULATION SOFTWARE for ACCELERATING STRUCTURES

For cylindrically-symmetric structures, POISSON/SUPERFISH finds natural frequencies and normal modes via a finite-difference technique. The results are used to calculate transit-time integrals for particles having specified charge, rest mass, and initial velocity. A picture of lines of force of the electric field provides graphical output of the geometry of the mode. Quality factor,  $r$  over  $Q$ , natural frequency, and many other results are tabulated.

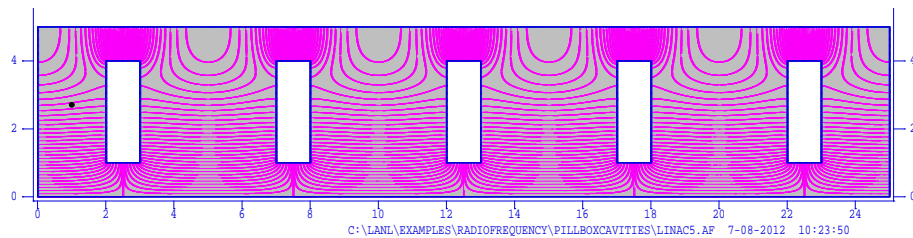
The Helmholtz equation for the H field is written in finite-difference form on a triangular mesh, yielding a huge matrix containing frequency as a parameter.

Superfish searches for zeroes of the determinant near a user-specified value. A null-space basis vector for the resulting matrix gives the azimuthal H field for the normal mode associated with the frequency. Numerically integrating Faraday's Law yields the radial and longitudinal electric field components.

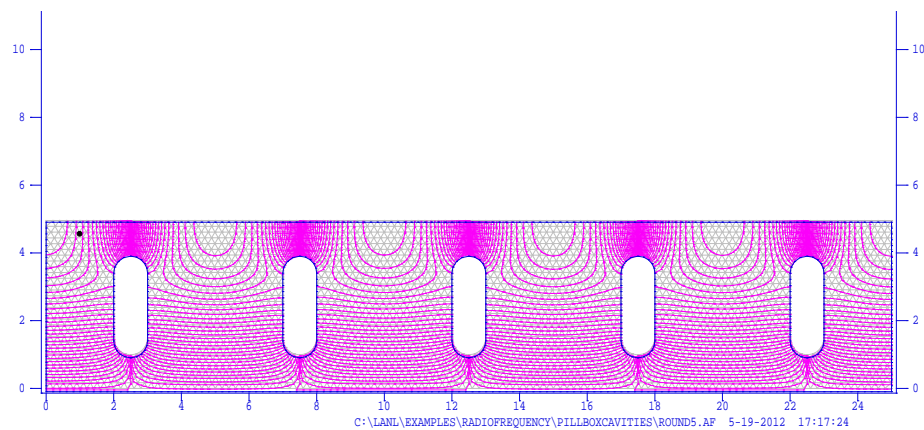
For cylindrically-symmetric structures, the program ABCI investigates Azimuthal Beam Cavity Interactions. A beam of bunches with Gaussian or other charge distribution is assumed to travel through the structure, and Maxwell's Equations are integrated numerically for the transient fields in the structure. Fast Fourier Transforms (FFT's) enable calculation of wakefunctions, longitudinal impedance, and other results.

## II. PHYSICS of THE PLANE WAVE TRANSFORMER

Superfish was used to simulate structures having one, two, three, four, and five half-wavelength cells. For all these structures, the pipe radius is five centimeters, the structure wavelength is ten centimeters, and the washers have one centimeter inner radius, four centimeter outer radius, and one centimeter thickness. For each number of cells, washers with square edges and washers with drastically rounded edges (radius of curvature equals half of washer thickness) were compared. Structure geometries, field patterns and tabulated results of calculation are shown in figure 2.



*Figure 2A: Fields with Square Washers*



*Figure 2B: Fields with Round Washers*



INITIAL	STUDY	of		PLANE	WAVE	TRANSFORMER	
		Number	of			Half	Wave
		1	2		3	4	5
Freq with	Square Edge	3034.22	3013.71		3013.7	3013.69	3013.69
Freq with	Round Edge	3100.43	3100.4		3100.39	3100.39	3100.39
Quality with	Square Edge	29495	29167		29167	29167	29167
Quality with	Round Edge	31522	31520		31520	31520	31520
r/Q with	Square Edge	113.54	269.15		402.01	533.54	661.23
r/Q with	Round Edge	106.09	208.07		301.94	384.2	451.92
Coupling with	Square Edge	0.01347343	Coupling				
Coupling with	Round Edge	1.9352E-05	Coupling				

*Figure 2C: Summary of Initial Study of PWT*

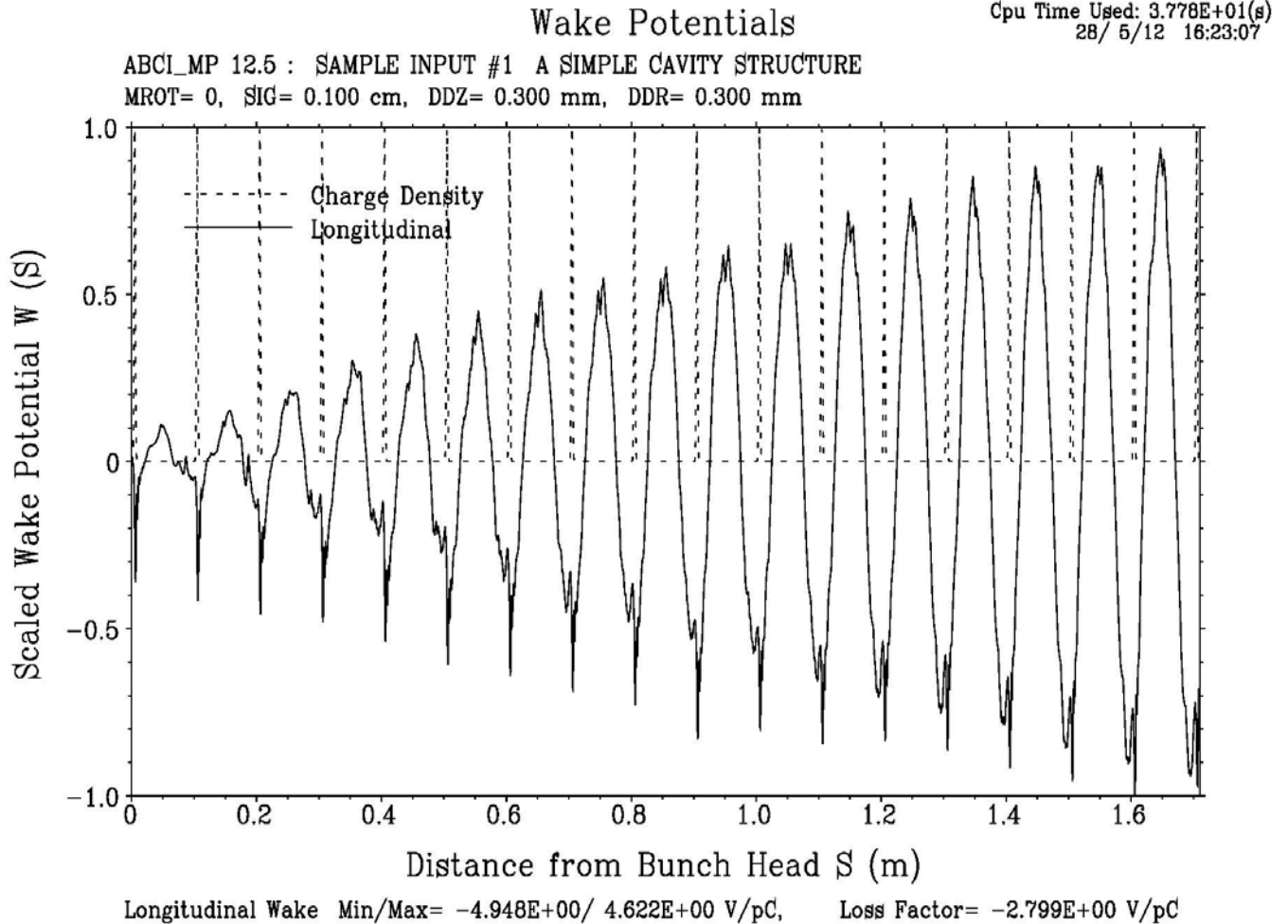
Initial results in which the particle velocity appeared to grow by the speed of light at each cell taught the importance of inserting lines “beta=0.999,” and “kmethod=1” in the input file for Poisson/Superfish..

If a normal mode of a single cell has a resonant frequency,  $\omega_o$ , the resonant frequency for a chain of any number of identical cells in the same mode will be  $\omega$ , and  $\cos\varphi = \frac{\omega_o^2 - \omega^2}{k\omega_o^2}$  [3], where  $\varphi$  is the phase shift from one cavity to the next and  $k$  is the coupling factor. This is independent of the number of cells in the chain[3]. This is well demonstrated by the data in figure 2. Assuming 180 degree cell-to-cell phase shift, the coupling factor for a given geometry can be calculated by comparing the frequency of a multicell chain to the frequency of a single cell using

$$k = \frac{\omega_o^2 - \omega^2}{k\omega_o^2 \cos\varphi}. \quad (3)$$

This value can be useful for calculating wave propagation velocities when the geometry is used for traveling-wave structures.

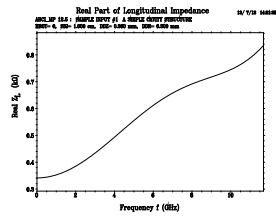
Drastically rounding the edges of the washers increases the frequency and quality factor and reduces the  $r/Q$ , but the shunt impedance per unit length stays fairly constant around 113 Megohms per meter. Rounding the outer edges of the washers would appear to reduce the effective capacitance between them and the pipe, which would increase the frequency but reduce the quality factor. In practice, sharp corners would be ground to a small radius, yielding results intermediate between the geometries used here. Note that the difference in quality factor, frequency and  $r/Q$  between sharp corners and drastically rounded edges is only less than five percent but rounding makes coupling much weaker.



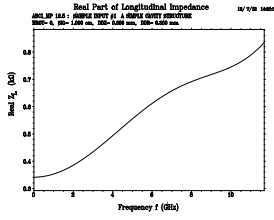
*Figure 3: Wakefields in Plane Wave Transformer*

The Azimuthal Beam-Cavity Interaction program ABCI was used to generate plots of wakefields and of frequency-dependent longitudinal impedance. Figure 3 shows the wakefields of 20 bunches in five-cell structures with (A) square-edged washers and (B) round-edged washers. Figure 4 shows plots of real impedance, and Figure 5 shows imaginary longitudinal impedance (reactance) versus frequency for the same structures. Nine runs were made with the bunch

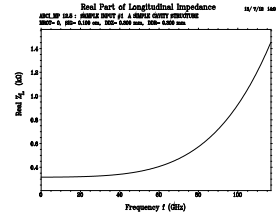
length one-tenth (short bunch), one-third (medium bunch) and the whole washer thickness of one cm. Beam radii varied as 0.2 (narrow beam), 0.6 (medium beam) and 1.0 (whole beam) of the aperture size. Consistent with Heisenberg Uncertainty, the spectral width of the Fourier Transform of a Gaussian is inversely proportional to the width of the original Gaussian. The energy in a shorter beam bunch is distributed over a wider spectrum while that in a longer bunch is concentrated at lower frequencies. Consequently, the wakefield of a shorter bunch illuminates a wider range of higher frequency modes while a longer bunch illuminates more low frequency modes. The FFT plots show negative reactance at low frequencies and extending to higher frequencies when the beam fills more of the aperture; this suggests capacitive coupling between the beam and washers. Reactance becomes inductive at high frequencies, probably due to the inertial mass of the electrons. Real impedance shows a dip at high frequency



Long Bunch

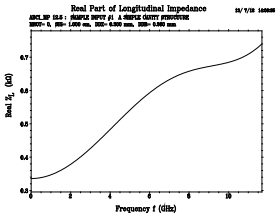


Medium Bunch

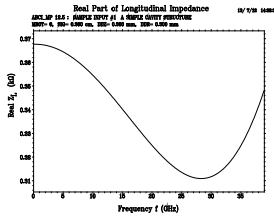


Short Bunch

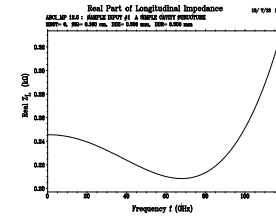
Narrow Beam Width



Long Bunch

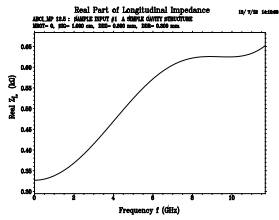


Medium Bunch Length

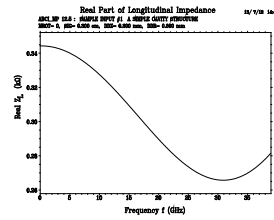


Short Bunch

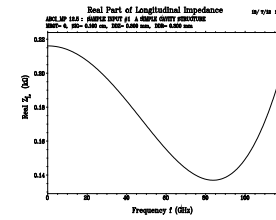
Medium Beam Width



Long Bunch



Medium Bunch



Short Bunch

Full Beam Width

Figure 4: Frequency Dependence of Real Longitudinal Beam Impedance

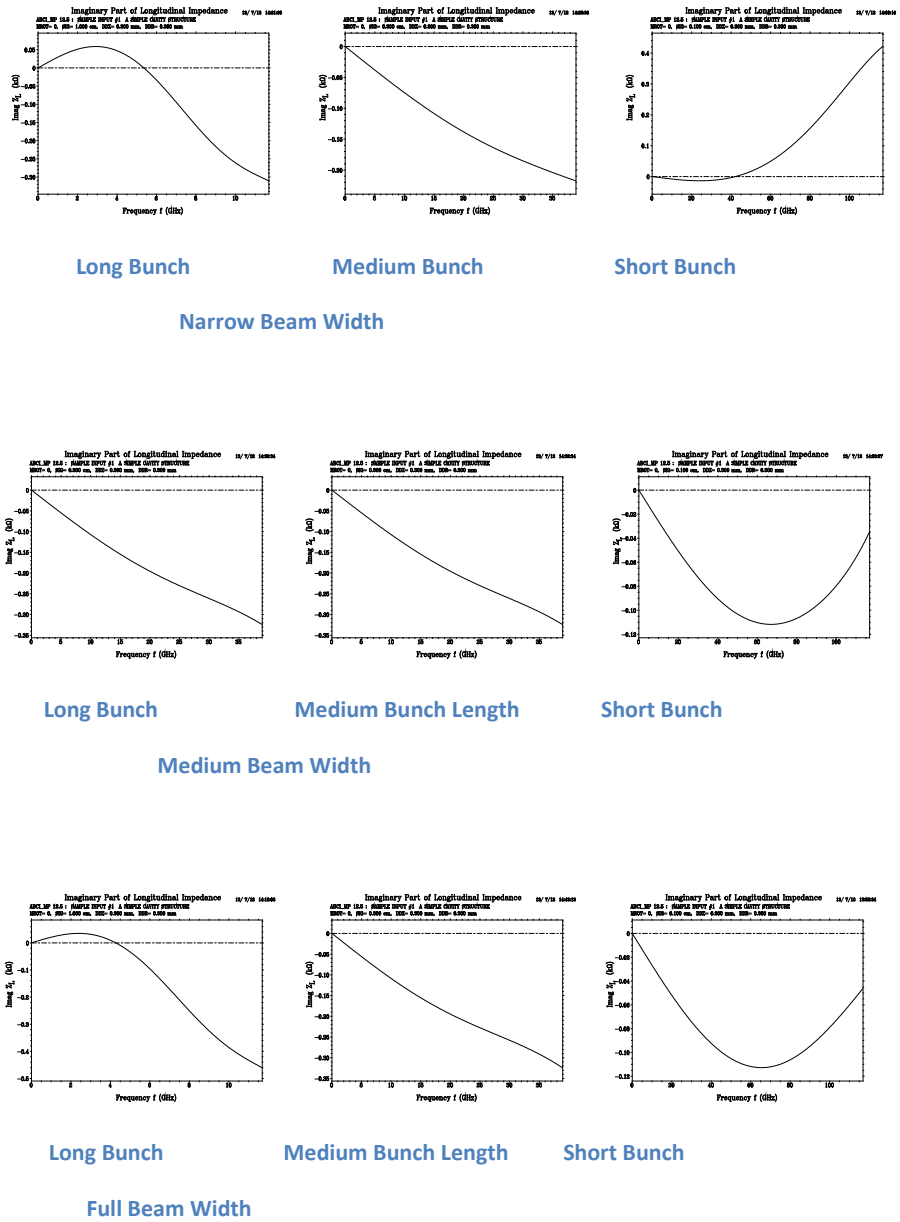


Figure 5: Frequency Dependence of Reactive Beam Impedance

### III. BROAD SURVEY of PARAMETER SPACE

Superfish was used to generate data to illuminate the variation of  $r$  over  $Q$  and frequency with all four dimensions. Appendix A contains a large spreadsheet table of the results. For each combination of dimensions, simulations were run for

both a half-wavelength single cell and a five-cell structure. This enables the cell-to-cell coupling coefficient to be calculated from (2). The  $r/Q$  values shown for five-cell structures is the value generated by Superfish divided by five for a per-cell value. For the first half of the table, three values of the ratio of pipe radius to washer outer radius were chosen; these are the ratios of radii for coax cable having impedances of 75, 50, and 10 ohms. Two values of structure wavelength, 10 cm. and 2.5 cm. were chosen, corresponding to free-space frequencies of 3 and 12 GHz. Washer inner radii and washer thickness values of 0.1, 0.3, and 1.0 cm. should give a broad idea of how frequency and  $r/Q$  vary with those dimensions. A point in parameter space corresponds to a row in the table. For each of these combinations of parameters, a frequency was found for which the electric field geometry plotted by Superfish resembles the PWT mode shown in Figure 1.

It is seen that structures may have lower-frequency modes similar to PWT modes, but with the planar nodes not aligned with the washers. Structures whose radius is larger than half the structure wavelength have higher-frequency PWT-like modes with cylindrical nodes and bands of strong axial field off axis. Both of these mode geometries tend to have low values of  $r/Q$ .

Initial data shows the value of  $r/Q$  increasing as the washer outer radius approaches the pipe radius. A more thorough investigation shows  $r/Q$  growing with the washer radius for a pipe radius of 5 cm but decreasing for 2.5 cm. pipe,.

Values of  $r/Q$  are a sensitive function of washer inner radius (aperture radius), being much larger for one millimeter than for one centimeter. This is

consistent with the model that the oscillating TEM electric field in the peripheral region drives surface charge on the outer and inner edges of the washers, and the fields of the surface charge near the aperture accelerate the beam. The Superfish program calculates  $r/Q$  for single particles travelling on axis. For large-radius beams, this may imply considerable potential depression, with particles near the axis gaining significantly less energy than particles closer to the washers.

#### IV. CONVERGING TOWARD OPTIMAL DIMENSIONS

The broad scan of parameter space shows values of  $r/Q$  larger than 120 ohms per cell for the 10 cm.wavelength with washer thickness=1 cm, washer outer radius=4 cm, aperture radius=0.3 cm, and pipe radius=5 cm , for which  $r/Q=130$  ohms per cell and frequency is 3004 MHZ. A two-cell structure was simulated to facilitate the adjustment of dimensions. Investigating that vicinity of parameter space and searching for maximum  $r/Q$  near the SLAC frequency of 2856 MHZ generated Table 1. Fixing the pipe radius at 5 cm, increasing the outer radius of the washers increases the  $r/Q$  but decreases the frequency.



	OPTIMIZING		for LOW		Frequency	
Outer Cyl Radius	Structure Wavelength	Washer Thickness	Washer Rin	Washer Rout	freq for 2/ wavelength	r/Q for 2/ wavelength
5	10	1	0.3	4	3004.13	282.538
5	10.8	1	0.3	4	2940.07	289.312
5	12	1	0.3	4	2854.05	283.168
5	12	1	0.3	4.5	2640.47	358.255
5	12	1	0.3	4.3	2730.98	333.805
5	12	1	0.3	4.7	2537.45	371.045
5	12	1.4	0.3	4.7	2511.69	376.026
5.5	12	1.4	0.3	4.7	2527.22	322.371
5.5	11.5	1.4	0.3	4.7	wont	run
5	11.5	1	0.3	4.7	2579.64	298.797
5	11	1.4	0.3	4.7	2580.59	267.849
5	11	1	0.3	4.7	2565.28	304.009
5	12	1	0.1	4.7	2564.87	309.28
5	10	1	0.1	4.7	2598.87	214.54
5	10	1	0.1	4.5	2729.15	242.579
5	10	1	0.1	4.3	2844.5	267.821
5	12	1	0.1	4.1	2813.95	314.928

Table 1: Optimizing a Two-Cell PWT structure Near the SLAC Frequency

Making the structure wavelength 12 cm, the pipe diameter 5 cm, washer thickness one cm, inner and outer washer radii 0.1 and 4.1 cm makes the  $r/Q$  157 ohms per cell and the frequency about two percent below the SLAC frequency. Shrinking the gap between washers and pipe to 3 mm and changing the washer thickness to 14 mm raises  $r/Q$  to 188 ohms per cell but drops the frequency to 2512 MHz.

For the 2.5 cm. wavelength, the broad scan shows maximum  $r/Q$  with a pipe radius of 1.25 cm, washer thickness and inner radius of 0.25 cm, and washer outer radius of 1.0581 cm. Searching parameter space nearby generated Table 2.

	OPTIMIZING		for High		Frequency		
Outer Cyl Radius	Structure Wavelength	Washer Thickness	Washer Rin	Washer Rout	freq for 2/ wavelength	r/Q for 2/2	
1.25	2.5	0.25	0.25	1.05	11688.76	220.96	
1.25	2.5	0.3	0.25	1.05	11631.44	220.925	
1.25	2.5	0.25	0.25	0.9	12896.95	213.338	
1.25	2.5	0.25	0.25	0.8	1359.92	174.867	
1.25	2.5	0.25	0.25	1.1	11241.49	206.769	
1.25	2.5	0.25	0.25	1.08	11424.27	213.465	
1.25	2.5	0.25	0.1	1.08	11346.7	255.776	

Table 2: Optimizing a Two-Cell PWT Structure Near The Fourth Harmonic of The SLAC Frequency.

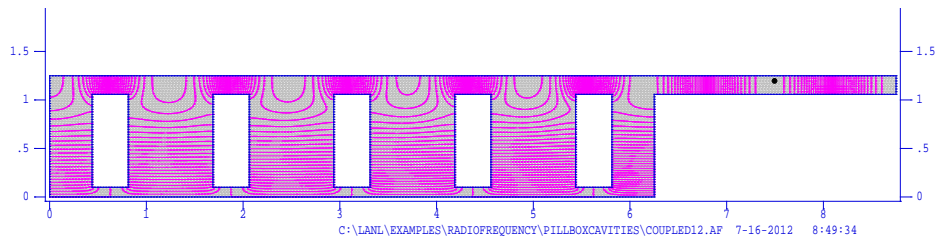
Table two shows that a multicell PWT whose resonant frequency is about a quarter megahertz above the fourth harmonic of the SLAC frequency can have  $r/Q$  of 103 ohms per cell. Reducing the aperture raises  $r/Q$  to 127 ohms per cell and drops the frequency by less than eight percent.

## V CONCLUSION: BEHAVIOR and COUPLING of THE OPTIMIZED PLANE WAVE TRANSFORMER

Dimensions were found for multicell Plane-Wave Transformer structures with maximal  $r/Q$  values resonant near the SLAC frequency and near its fourth harmonic. Near the SLAC frequency,  $r/Q$  values of 157 to 188 ohms per cell were obtained. Near the fourth harmonic,  $r/Q$  values of 103 to 127 ohms per cell were shown to be achievable depending how close the frequency must be to specified values.

Values of  $r/Q$  up to about 200 ohms per cell have been reported for PWT structures with nose cones on the washers to narrow the gap width. The current project studied only structures with flat-faced washers, however. The geometries reported had larger radius-to-wavelength ratios than those discussed here. With flat washers, structures tend to have more PWT-like modes with cylindrical nodes and lower  $r/Q$  values. Nose cones add localized capacitance near the axis, reducing the tendency toward those modes, in addition to narrowing the gap to increase transit time factors.

For the optimal geometry resonant near 11424 MHz, we consider the addition of a section of coaxial waveguide, one wavelength in length, having inner radius equal to the outer radius of the washers. The Superfish-generated field pattern is shown in Figure 5.



*Figure 6: Field Patterns in PWT with Coupler*

Without the coupler, the unloaded quality factor was  $Q_o = 11482$ . With the coupler, the loaded quality factor is  $Q_L = 7632$ . So the external Q associated with the coupler is  $Q_e = 22761$ . This

yields a coupling coefficient of  $\beta = \frac{Q_o}{Q_e} = \frac{11482}{22761} = 0.504$ .

Reasonably high  $r/Q$  values can be achieved with the flat-washer PWT geometry and the structure can indeed couple efficiently with a coaxial waveguide. Questions remain, however, concerning the mechanical stability of the structure and its ability to maintain precise alignments. The weight of the washers causes a large bending moment on the dielectric rods expected to support the rods. Considerable heat generated on the faces of the washers would lead to high temperatures and large temperature fluctuations during startup and powering down. For rigidity, the rods would need to be made of a rigid ceramic material and they would need a large cross section, filling much of the space between the washers. Then thermal expansion and contraction with temperature fluctuations would be expected to lead to more problems.



## APPENDIX B: Reference

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