U.S. Particle Accelerator School
Education in Beam Physics and Accelerator Technology

Fermilab

Part 1 Linear Accelerator Magnets

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January 13, 2020

Outline

- *Introduction to Magnetostatics*
- *Magnetic Field Equations*
- *Magnet Specifications*
- *Room Temperature Magnets*
- *Permanent Magnets*
- *Superconducting Magnets*
- *Magnetic Field Simulations*

Hans Christian Ørsted

Pictures in Public Domain

http://en.wikipedia.org/wiki/Hans_Christian_Oersted

In 1820, which Ørsted described as the happiest year of his life, Ørsted considered a lecture for his students focusing on electricity and magnetism that would involve a new electric battery. During a classroom demonstration, Ørsted saw that a compass needle deflected from magnetic north when the electric current from the battery was switched on or off. This deflection interestred Ørsted convincing him that magnetic fields might radiate from all sides of a live wire just as light and heat do. However, the initial reaction was so slight that Ørsted put off further research for three months until he began more intensive investigations. Shortly afterwards, Ørsted's findings were published, proving that an electric current produces a magnetic field as it flows through a wire.

This discovery revealed the fundamental connection between electricity and magnetism, which most scientists thought to be completely unrelated phenomena.

His findings resulted in intensive research throughout the scientific community in electrodynamics. The findings influenced French physicist André-Marie Ampère developments of a single mathematical form to represent the magnetic forces between current-carrying conductors. Ørsted's discovery also represented a major step toward a unified concept of energy.

Magnetostatics (Free Space With Currents & Conductors)

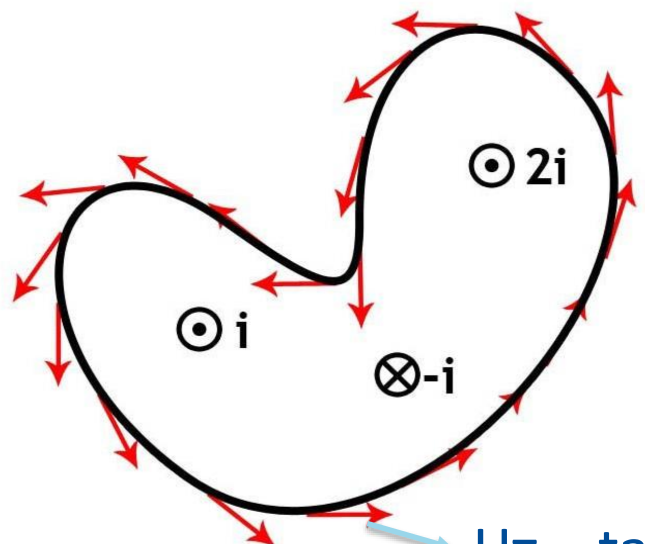
https://ocw.mit.edu/courses/electrical-engineering-and-computer-science

André-Marie Ampère,
1775-1836

portrait is in the Public Domain

Ampere's Low for Magnetostatics

$$\int_C \overline{H} \cdot d\bar{l} = \int_S \overline{J} \cdot d\overline{A} = I_{enclosed}$$

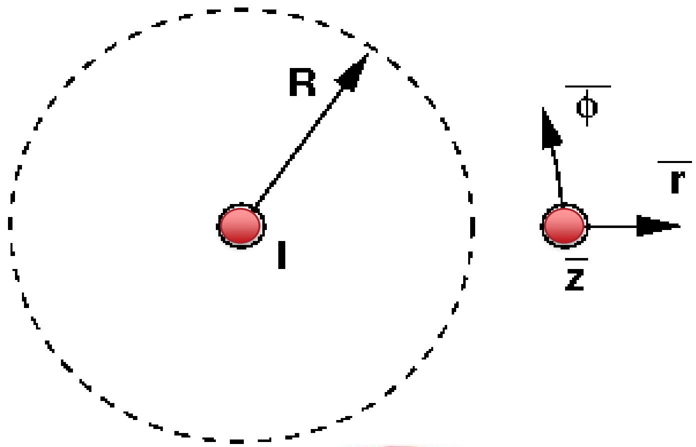


Hτ – tangential field component

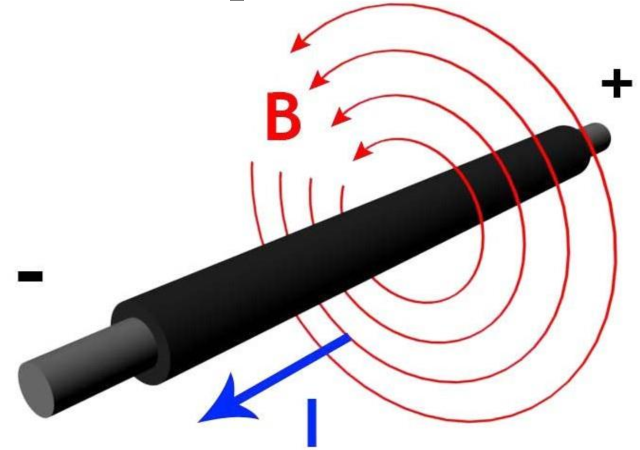
Andre-Marie Ampere, <u>Memoir on the Mathematical Theory of Electrodynamic Phenomena, Uniquely Deduced from Experience</u> (1826)

Magnetic Field Around a Very Long Wire Carrying Current

$$\int_C \overline{H} \cdot d\bar{l} = \int_S \overline{J} \cdot d\overline{A}$$



$$H_\phi 2\pi r = I \qquad \overline{H} = \frac{I}{2\pi r}\widehat{\phi}$$

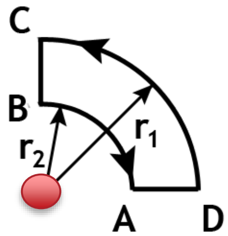


Ampere observe that:

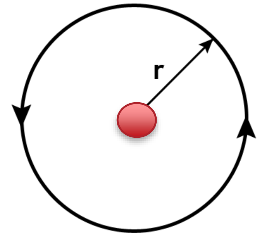
1) the H-field is rotationally symmetric around wire
2) the H-field falls off as 1/r
3) the H-field is proportional to the current in the wire

Ampere's Law Examples

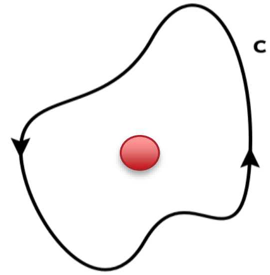
Current



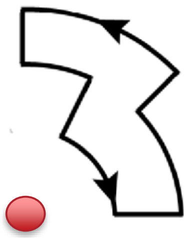
(a) Path lying in plane perpendicular to wire



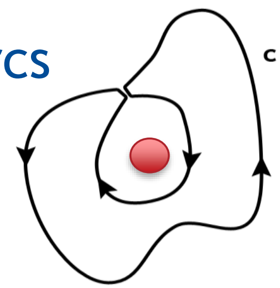
(d) Circular path enclosing wire



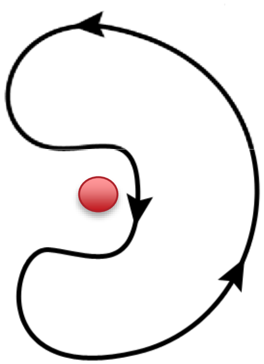
(d) Crooked path enclosing wire



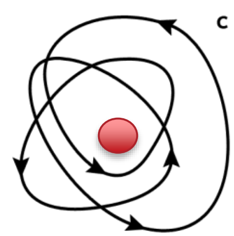
(b) Path constructed of Radial segments and arcs



(f) Circular and crooked path NOT enclosing wire



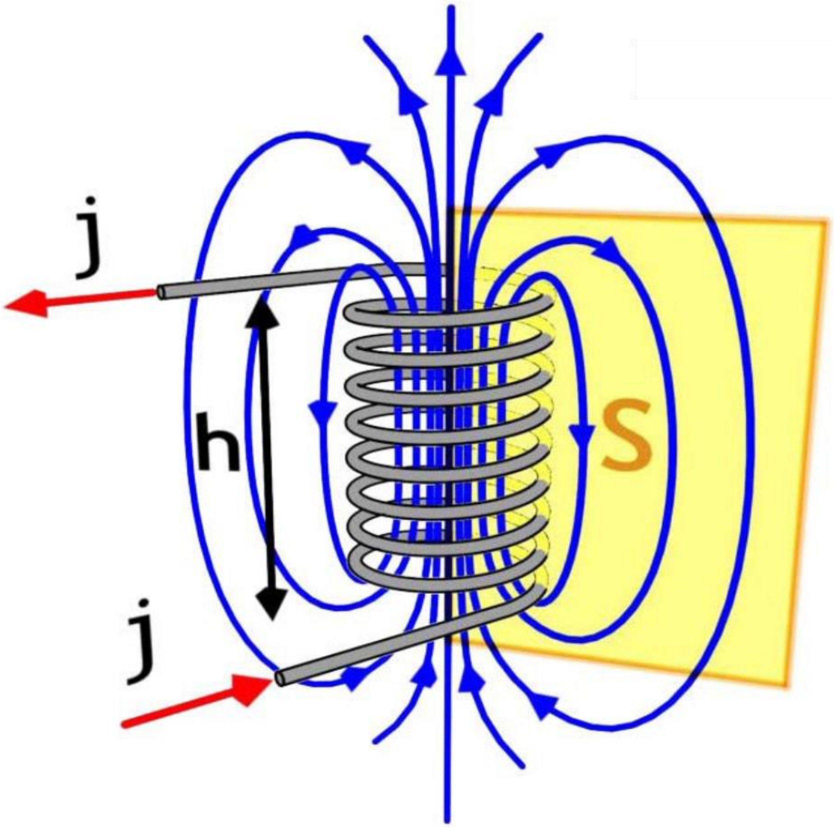
(c) Path which does not Enclose the wire

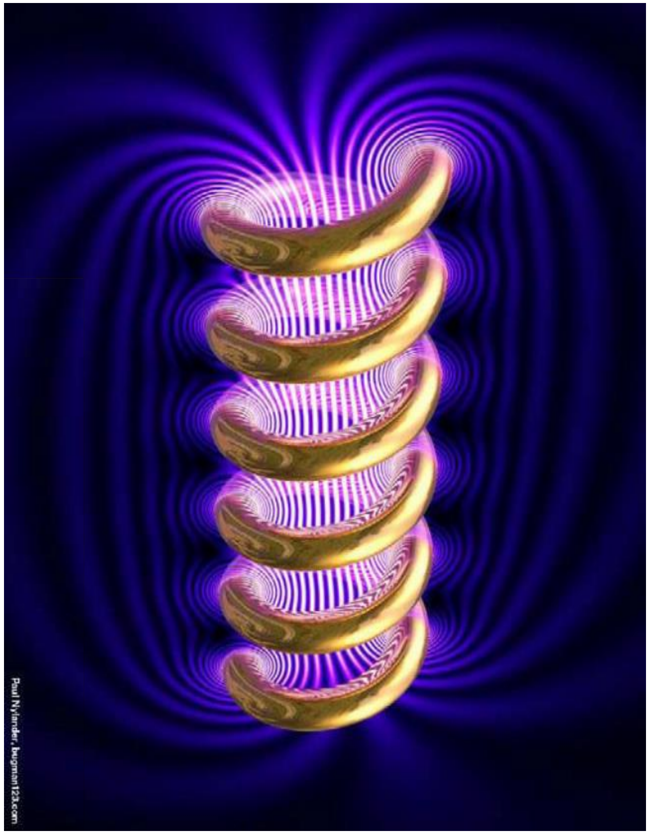


(f) Loop of N turns enclosing wire

Fields from a Solenoid

$$\int_C \overline{H} \cdot d\bar{l} = \int_S \overline{J} \cdot d\overline{A} = I_{enclosed}$$





Courtesy of Paul Nylander.

$$H_{inside} \approx \frac{N\,I}{h}$$

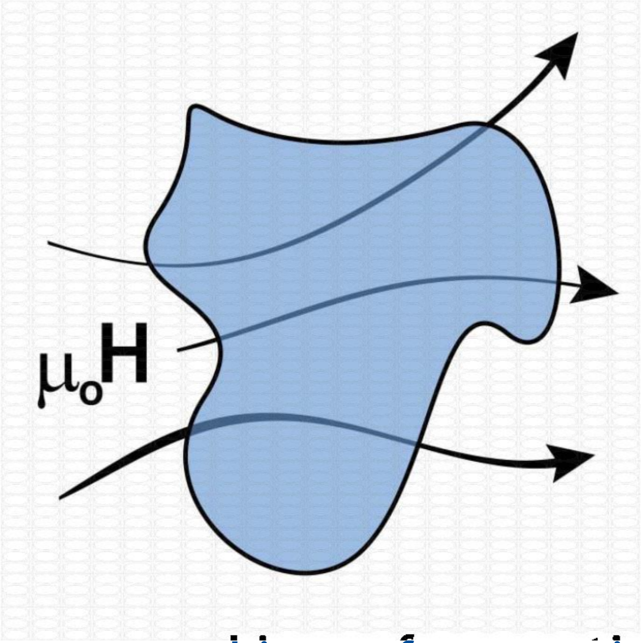
Gauss Law for Magnetic Fields

No Magnetic Monopoles

$$\nabla \cdot \mu_o \vec{H} = 0$$

$$\oiint_S \mu_o \vec{H} \cdot d\vec{S} = 0$$

$$\oiint_S \vec{B} \cdot d\vec{S} = 0$$



Magnetic flux conservation law:
No net magnetic flux enters of exits a closed surface.
What goes in must come out.

Lines of magnetic flux ($\mu_o \vec{H}$) never terminate.
Rather, they are solenoidal and close on themselves in loops.

Magnet Design Steps

1. *Magnet functional specification (physics requirement document).*
2. *Magnet engineering specification.*
3. *Conceptual magnetic and mechanical design.*
4. *Final magnetic and mechanical design.*
5. *Design verification by beam optics analysis.*
6. *Prototype fabrication.*
7. *Prototype magnetic measurements, and tests.*
8. *Correction if needed the magnet design.*
9. *Documentation for the serial production.*

Magnet Functional Specification

The functional specification usually prepared by physicists responsible for the beam optics analysis.

The specification includes:

- *Beam energy and type of particles: electrons, protons, muons…*
- *Magnet type: H-type dipole, C-type dipole, Shell-type dipole,*
- *Septum, Lambertson, Quadrupole, Sextupole, Octupole, Bump, Kicker, Solenoid, etc.*
- *Beam aperture dimensions;*
- *Field, or gradient strength in the magnet center;*
- *Magnet effective length;*
- *Good field area dimensions, and the field quality;*
- *Integrated field, or gradient along the beam path;*
- *Separation between beams for Septums, Lambertsons;*
- *Beam bending angle;*
- *Fringe field limitations.*

Magnet Engineering Specification

The engineering specification usually prepared by physicists and engineers responsible for the magnet design.

In general, the specification includes:

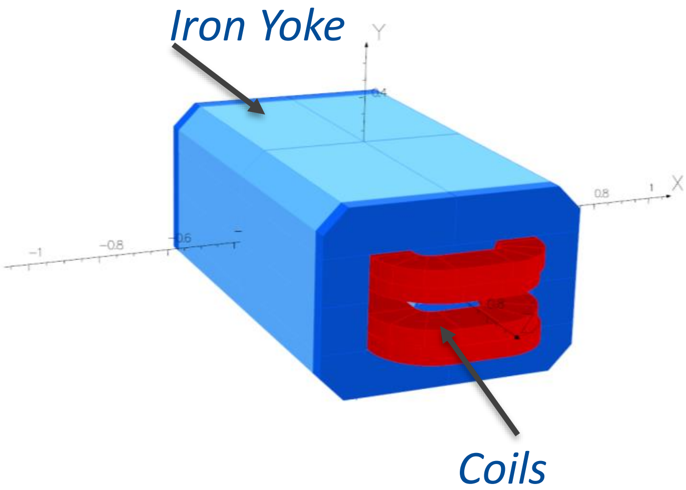
- *Magnet physical aperture dimensions;*
- *Beam pipe dimensions;*
- *Magnet total length and space slot available for the magnet;*
- *Space and weight limitations;*
- *Magnet peak field, or gradient;*
- *Type of cooling: air, water, liquid Helium (Lhe), conduction;*
- *Cooling system parameters;*
- *Power supply parameters: peak current and voltage, AC, pulsed.*
- *Magnet protection and instrumentation;*
- *Radiation level;*
- *Number of magnets.*

Examples of Specifications

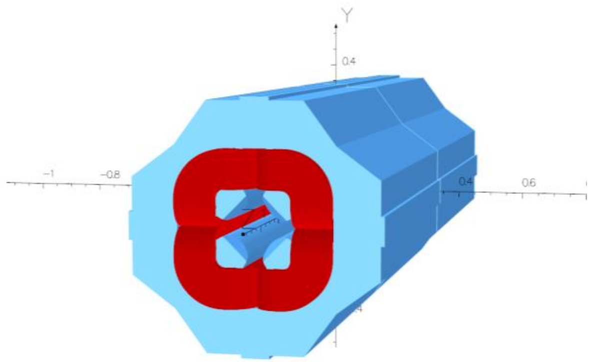
1. Physics Requirement Document. Magnets, LCLSII-2.4-PR-0081-R0 – describes functional specifications for all LCLS-II Linear Accelerator magnets (see Magnets_PRD_signed_012815.pdf).

2. Engineering Specifications Document. Cryomodule Magnet. – describes specifications for the superconducting magnets (see Cryomodule_Magnet_ESD_Signed_042015.pdf).

Conventional Iron Dominated Electromagnets



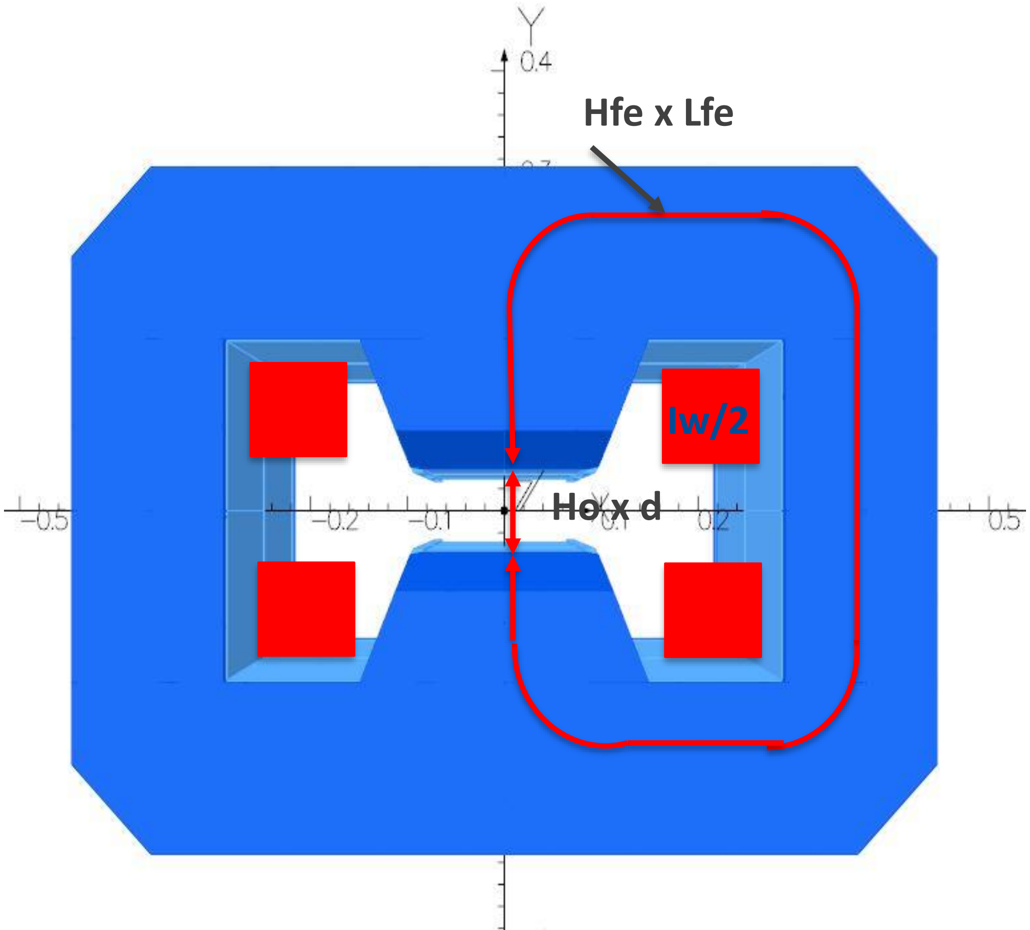
FNAL Main Injector Dipole



FNAL MI WQB quadrupole

Conventional iron dominated magnets still “a working horse” for many accelerator magnet systems having fields below 1.8 T.
They also could not be replaced by superconducting magnets for fast pulsed fields, and in very high radiation areas.
Some accelerators have hundreds, or even thousands dipole and quadrupole magnets connected in series.
In this case needed careful cost optimization to optimize capital and operational cost. This, in general, includes: cost of fabrication and cost of used electricity to power magnets. For most water cooled magnets the optimal current density in copper coils around 4 A/mm^2. Iron yoke made from solid, or laminated steel.

Ampere's Law for Electromagnets



d - magnet gap, m

Hd – field strength in the gap, A/m

Hfe – Field strength in the iron yoke, A/m

Lfe – the average flux path length in the iron yoke, m

Iw – total coils ampere-turns, A

μo= 4π*e-7 H/m

Bo – flux density in the gap, T

*Ho=(Iw-Bfe*Lfe)/d (SI)*

*Bo=(μo*Iw-Bfe*Lfe)/d (SI)*

*Bo ≈ Iw/0.8*d (CGS)*

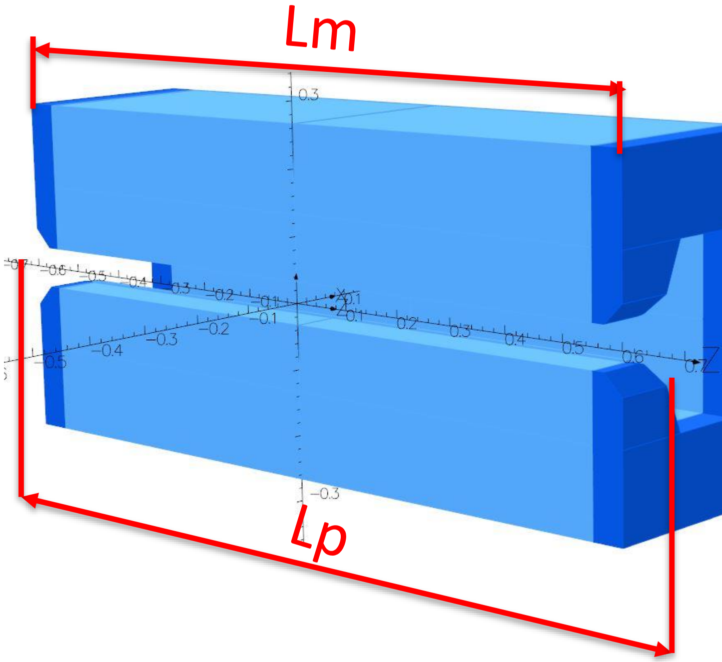
*Iw ≈ 0.8 *Bo*d (CGS)*

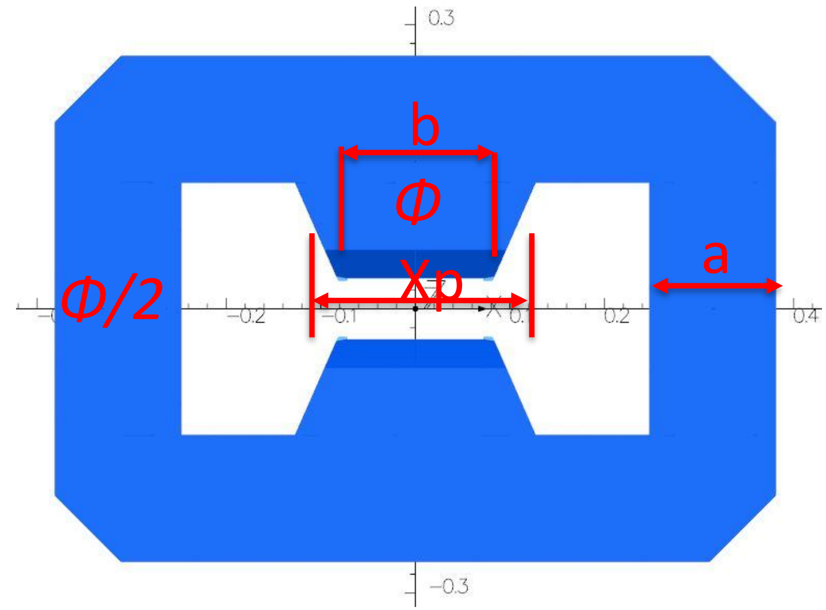
$$Ho * d + Hfe * Lfe = Iw$$

$$Bo*d/\mu o + \frac{Bfe}{\mu} * Lfe = Iw$$

Hfe = ?

Gauss Law for Electromagnets





Lp=Lp+d – pole effective length

Xp=b+d – pole effective width

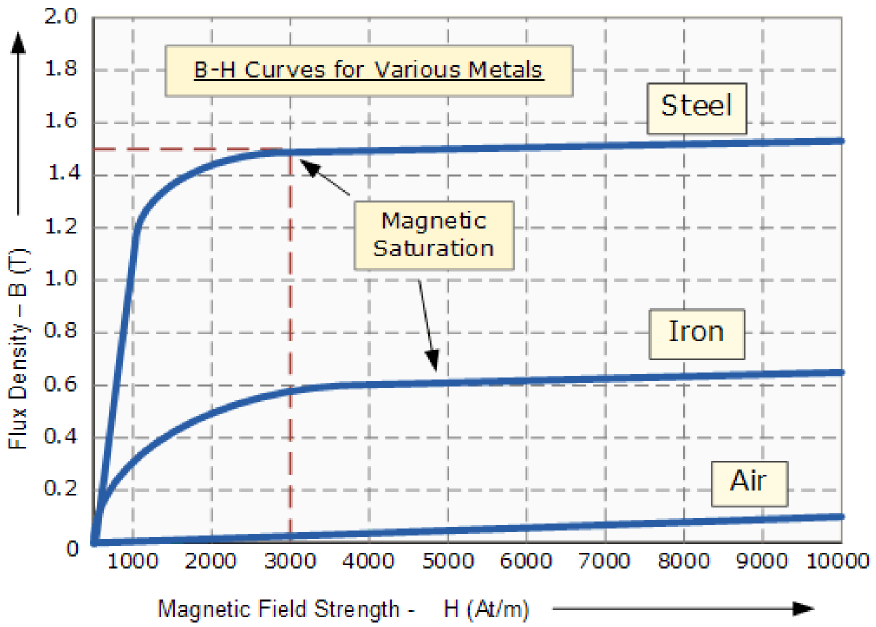
*Sp=Xp*Lp – pole effective area*

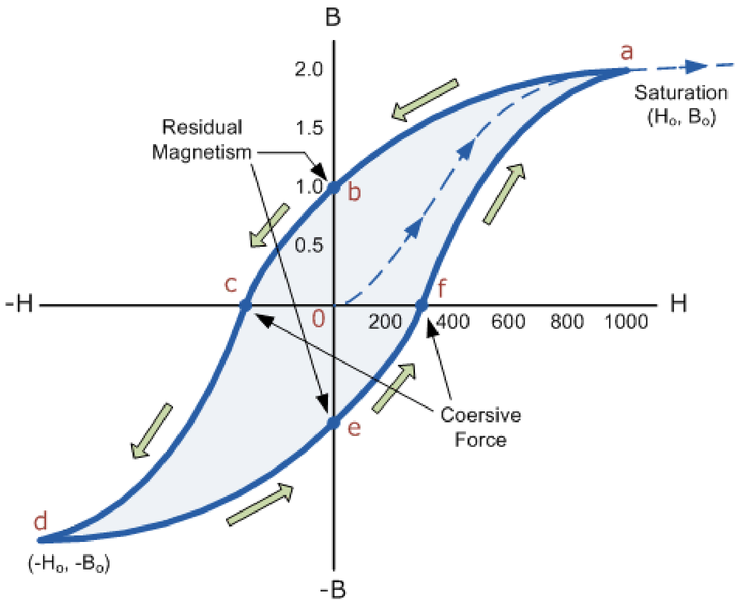
*Sy=a*Lm – yoke area*

*Φ = Bo*Xp*Lp=Bo*Sp – total flux*

Bfe= Φ/Sy

Ferromagnetic Material Properties





$$B = \mu o * H + M, \text{ or } B = \mu * H, \ \mu = B/H$$

For magnet design used *B=f(H)* measured magnetic properties of rings (from thick metal), or stacks of steel strips for thin steel forming closed magnetic circuit.

Soft Magnetic Materials

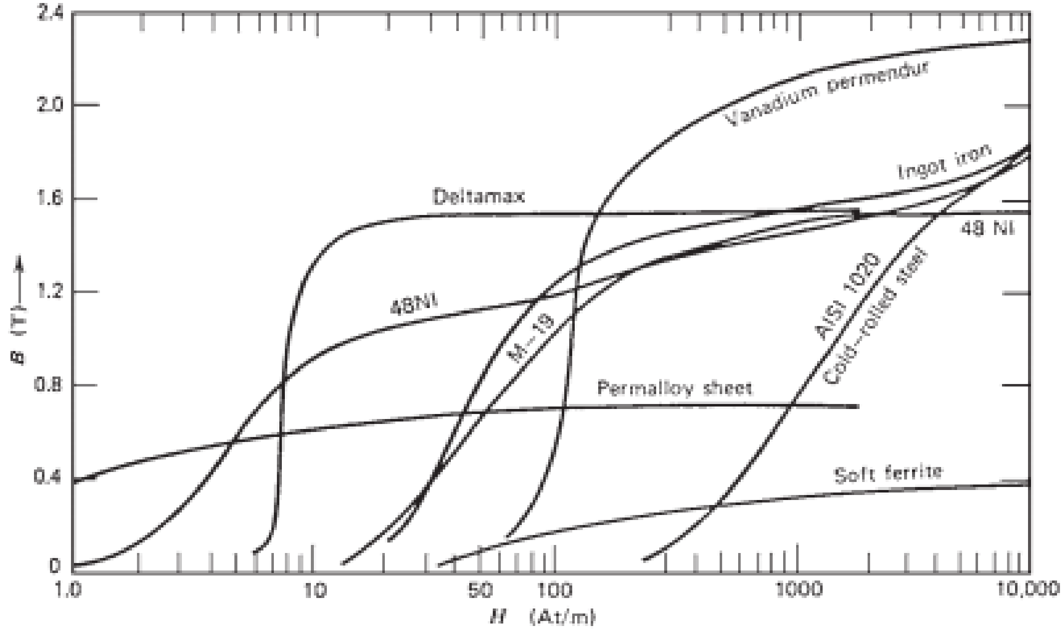


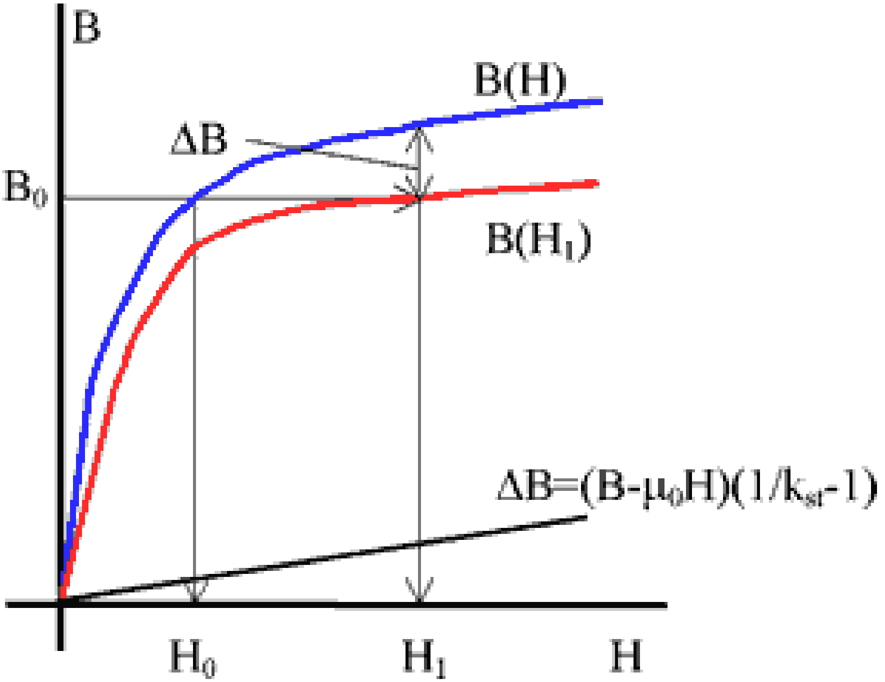
FIGURE 2.3 *B-H curves of selected soft magnetic materials.*

For accelerator magnets used low carbon steel: AISI 1006, AISI 1008, AISI 1010 with the low coercive force Hc < 2 Oe (160 A/m).
Sometimes Vanadium Permendur is used for fields close to 2 T.
Electrical type of steel used in AC magnets has up to 4% Si to reduce AC losses. It has the thickness of 0.35 mm – 0.5 mm for 50-60 Hz applications.

Ferromagnetic Material Properties

Fig. 17. Direct-current magnetization curves for various magnetic materials

Define Hfe in the Yoke



$$Ho * d + Hfe * Lfe = Iw$$

$$Bo*d/\mu o + \frac{Bfe}{\mu} * Lfe = Iw$$

If in the yoke the flux density is Bo then using B-H curve for solid material (blue) and for laminated (red) could be defined Hfe (Ho or H1) needed to finish the magnet parameters estimation.

The Kst is laminations stacking factor.

The Kst is around 0.96-0.98.

Total ampere-turns Iw includes gap and yoke used for coils design.

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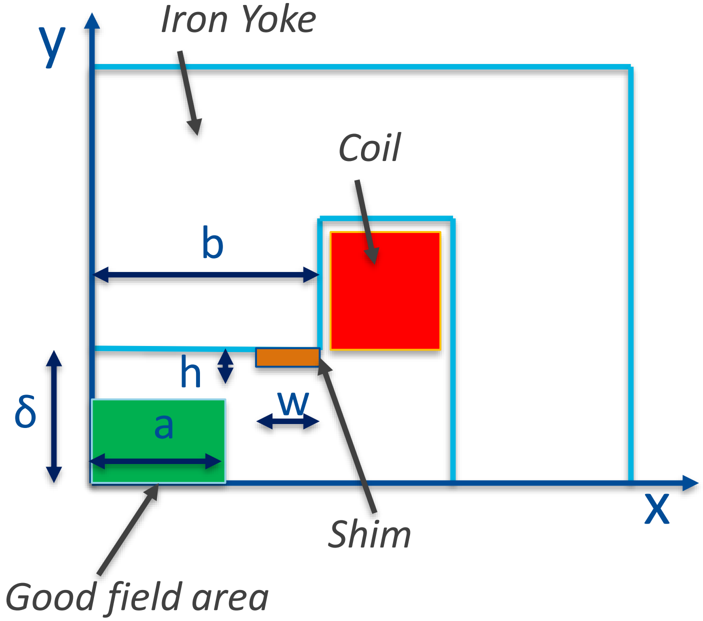
Dipole Magnet Field Quality



Shim area h*w : $S=0.021*\delta^2$

This relation is good for w/d in the range of 0.2 – 0.6.

Field in the magnet midplane:

$B=Bo(1+b1*x+b2*x^2+\ldots)$

Without shims the good field area width is:

- for 1% field homogeneity a=(b-d);

- for 0.1% field homogeneity a=(b-2d).

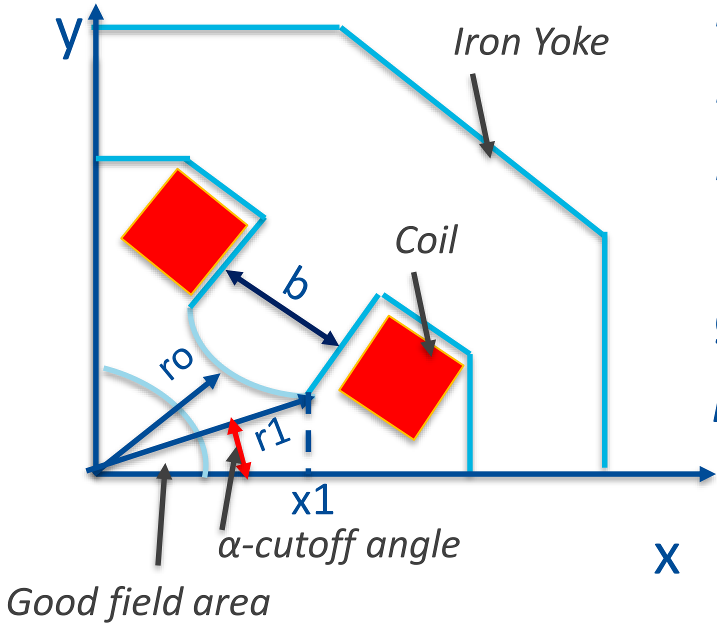
The good field area could be extended by adding shims:

- for 1% field homogeneity a=(b-d/2);

- for 0.1% field homogeneity a=(b-d).

For gap fields above 0.8 T used more smooth shims to reduce iron saturation effects in pole edges and shim areas.

Quadrupole Magnets



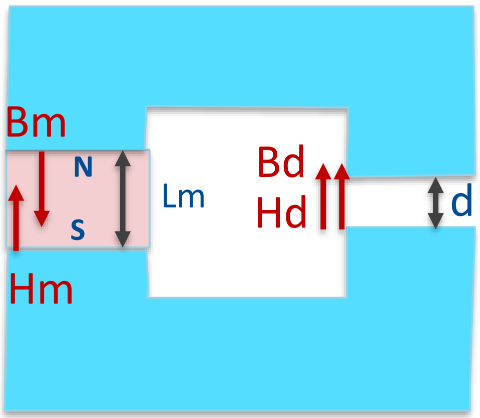
At $\alpha = 18^\circ$ the first undesired multipole b5 vanishes.
$r1=1.122*ro$, $x1=1.077*ro$
Field gradient at $\mu=\infty$:
$G=dBy/dx=b1=const$
$By=G*x$, $G=2\mu o*Iw/ro^2$

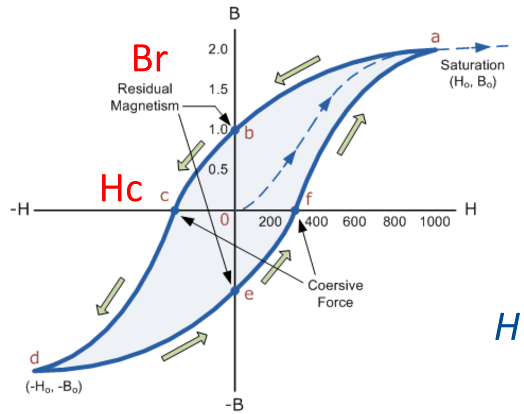
Field in the magnet midplane:
$B=Bo(1+b1*x+b2*x^2+\ldots)$
For the quadrupole Bo=0,
The ideal quadrupole field : $B=b1*x$ generated by a hyperbolic pole profile: $x*y=ro^2/2$
The quadrupole half gap ampere-turns: $(Hp+Ho)/2*ro=Iw$, or at Ho=0;
Quadrupole coil ampere-turns:

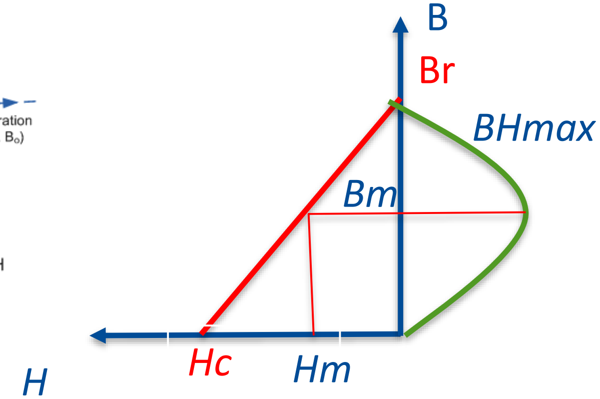
$$Hp*ro/2+Hfe*Lfe=Iw,$$
$$Bp*ro/2\mu o+Bfe/\mu*Lfe=Iw.$$

Hfe, Bfe –defined as for dipoles, but because of field gradient the flux through the yoke two times lower.

Permanent Magnets







$Hm*Lm - Hd*d = 0$ (Ampere's Law)

$Am*Bm = Ad*Bd$ (Gauss Law)

Where Bm, Hm permanent magnet working point on the demagnitization curve.
Bd,Hd – gap field.
Am, Ad – permanent magnet and gap areas.

$Lm=Hd*d/Hm=Bd*d/\mu o*Hm$

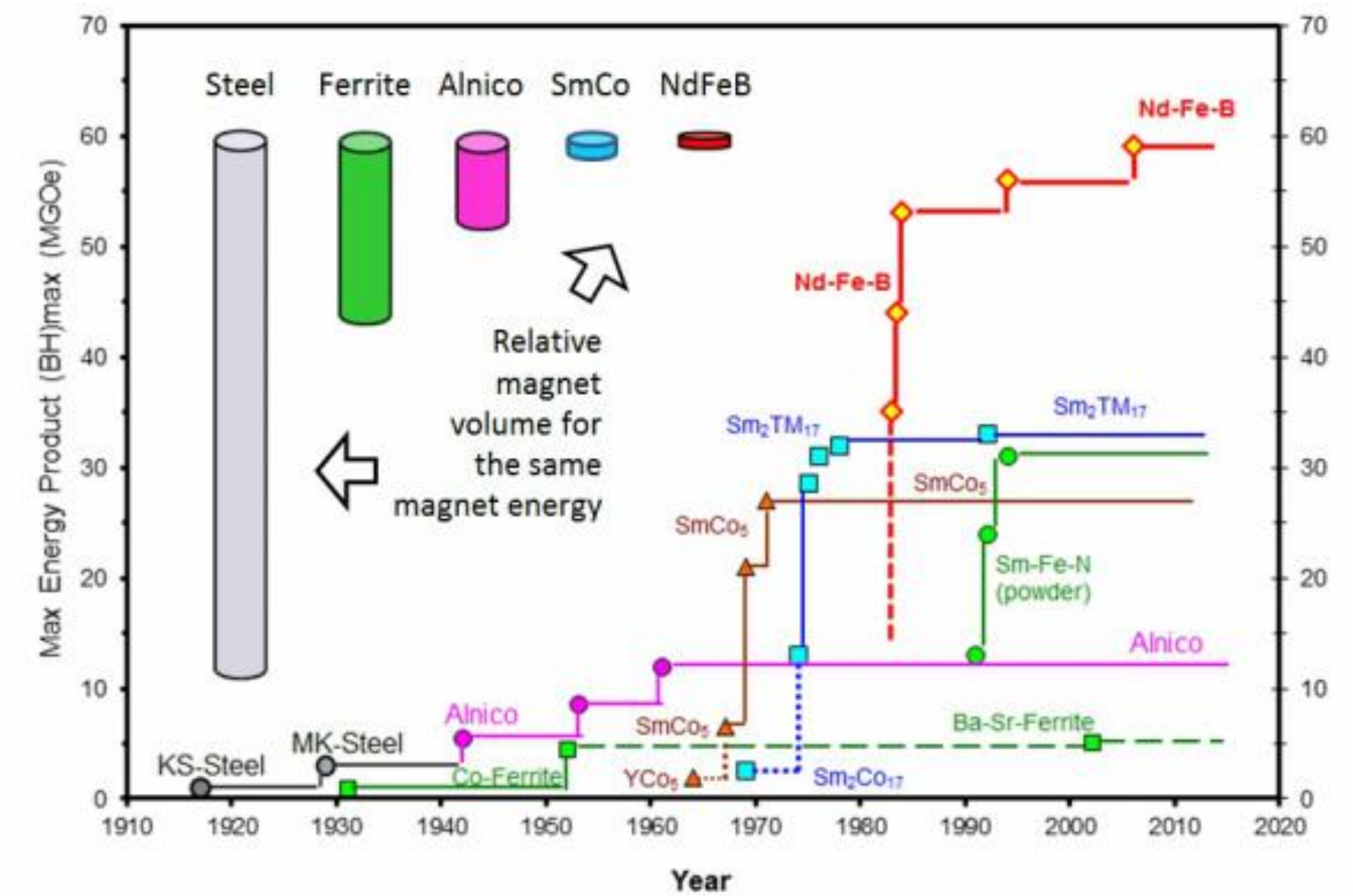
$Am=Ad*Bd/Bm$

*$Vm=Lm*Am=Bd^2*d*Ad/(\mu o*Bm*Hm)$ – the larger Bm*Hm the lower PM volume.*

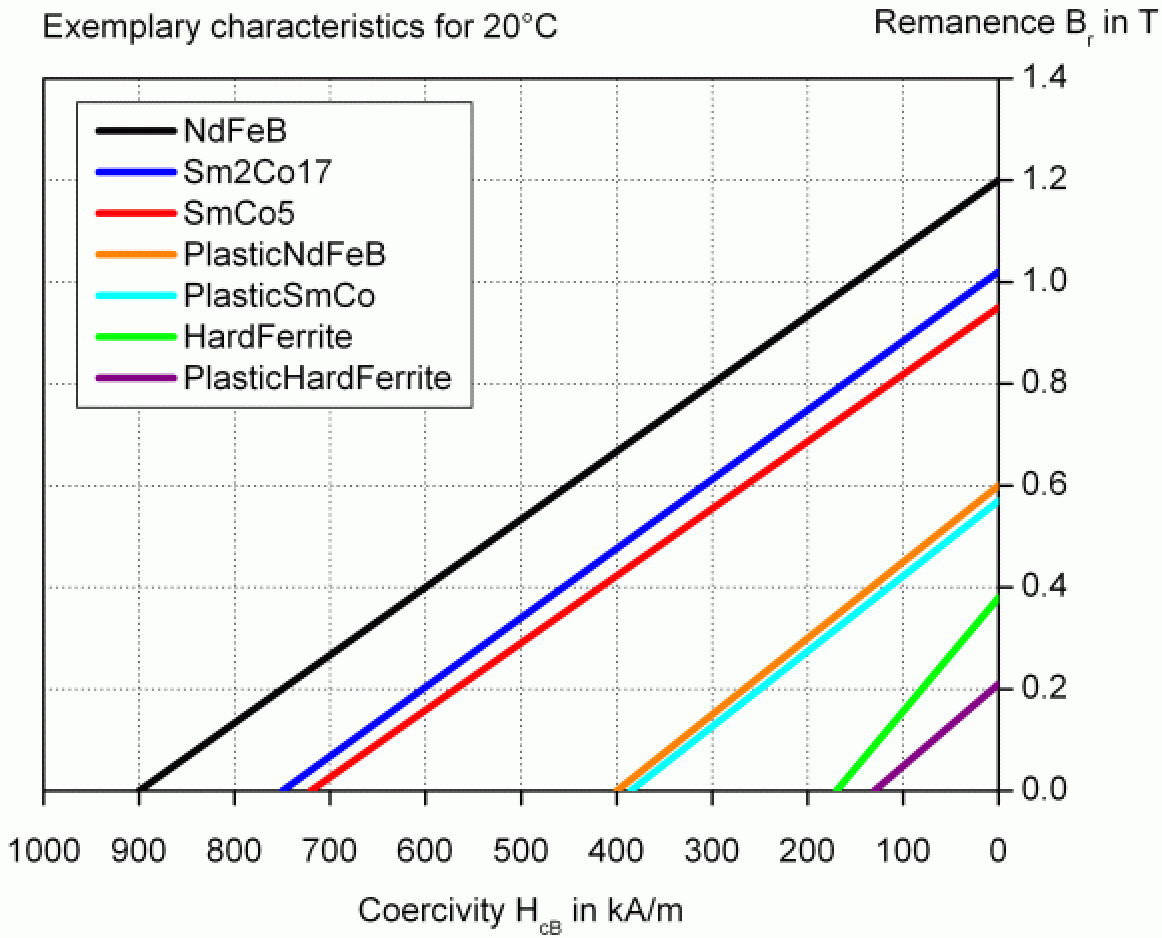
For SmCo5: $Br=-\mu oHc$, and $(BmHm)max$ at $Bm=Br/2$, $Hm=Hc/2$

Permanent magnets are the energy source which supply the maximum energy only at BHmax.

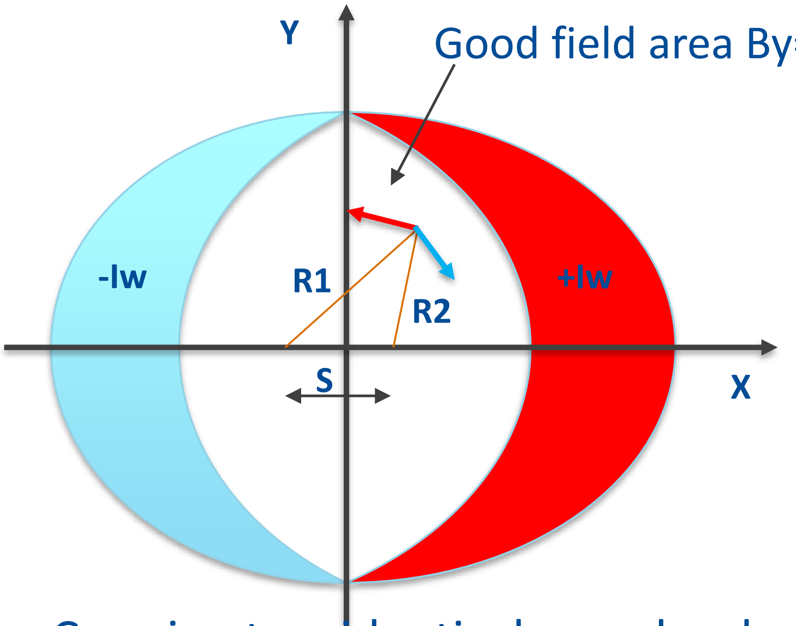
Permanent Magnets Energy Product

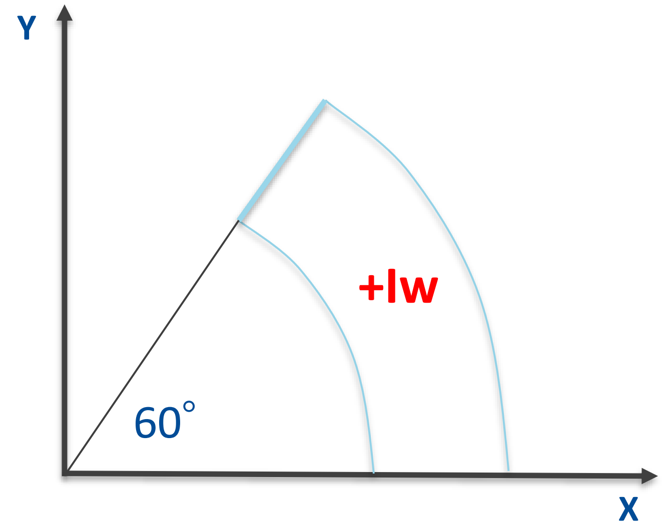


Permanent Magnets B(H)



Superconducting Magnets





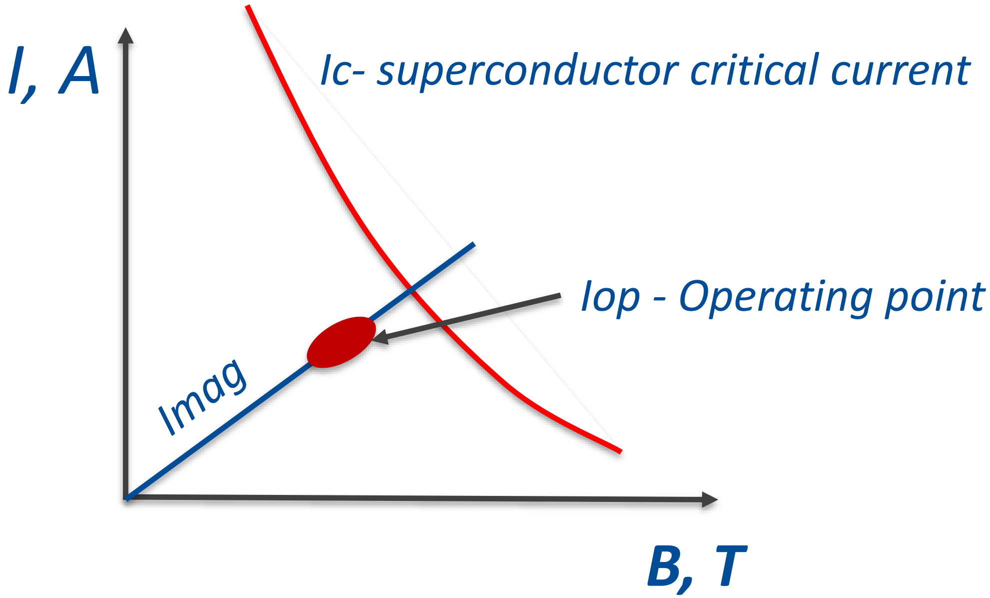
Crossing two identical round rods with opposite currents gives the homogeneous field.

By=μo*Iw/2*[-R1*cos(ϑ1)+ R2*cos(ϑ2)]=-μo*Iw*s/2

At 60° eliminated the first dipole field harmonic (sextupole).

The closer geometry approximation to the COS azimuthal current density distribution the better field homogeneity could be obtained.

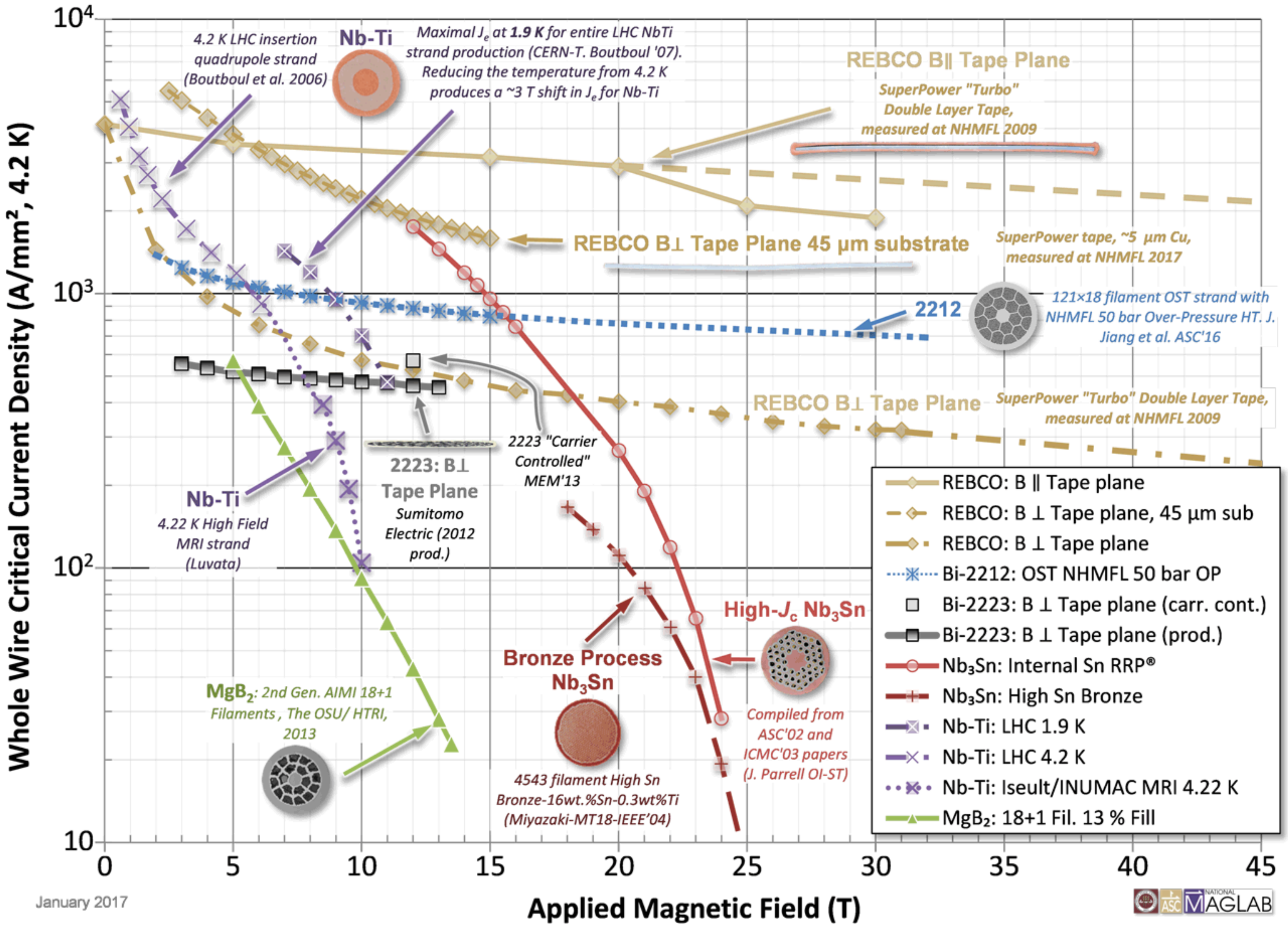
Superconducting Magnets



- ✓ *Choice of operating point (Iop) is the most critical decision in the superconducting magnet design.*
- ✓ *The closer Iop to the Ic the more risk for the magnet performance.*

- ➢ *For most accelerator magnets used NbTi superconductors: Tevatron –(4.5T, 4.2K), LHC (9 T, 2 K).*
- ➢ *Nb3Sn magnets have a good progress in recent years for LHC upgrade.*
- ➢ *HTS in an R&D phase for accelerator magnets with two main directions: low field with high temperature up to 77K, and very high field with low temperature LHe 4.5K. Hybrid magnets are a main stream.*

Superconductors (P. Lee)



Superconductor References

YBCO: Tape,∥Tape-plane, SuperPower "Turbo" Double layer (tested NHMFL 2009). Source: Aixia Xu and Jan Jaroszynski, June 2009. 20 T depression due to He bubble, dashed line estimates true performance.
YBCO: Tape, ⊥ Tape-plane, SuperPower "Turbo" Double layer (tested NHMFL 2009). Source: Aixia Xu and Jan Jaroszynski, June 2009.

YBCO: Tape, ⊥ Tape-plane, SuperPower 45 μm substrate with 5 μm Cu layer, sample courtesy of D. van der Laan (ACT), tested at NHMFL 2017 (D. Abraimov with A. Francis - Ic measurements, and N. Gibson - (IA)).

Bi-2223: B ⊥ Tape-plane "DI" BSCCO "Carrier Controlled" Sumitomo Electric Industries (MEM'13 presented by Kazuhiko Hayashi).
2212: OST NHMFL 50 bar overpressure HT, 18 x 121 filaments. J. Jiang et al., "Effects of Filament Size on Critical Current Density in Overpressure Processed Bi-2212 Round Wire," IEEE Transactions on Applied Superconductivity, vol. 27, no. 4, pp. 1–4, Jun. 2017. doi: 10.1109/TASC.2016.2627817

Nb-47Ti: 0-6 T - Boutboul et al. MT-19: Boutboul, T.; Le Naour, S.; Leroy, D.; Oberli, L.; Previtali, V.; , "Critical Current Density in Superconducting Nb-Ti Strands in the 100 mT to 11 T Applied Field Range," Applied Superconductivity, IEEE Transactions on , vol.16, no.2, pp.1184-1187, June 2006.
doi: 10.1109/TASC.2006.870777
Nb-47Ti 5-8 T Maximal: Nb-Ti: Max @4.2 K for whole LHC NbTi strand production (CERN-T. Boutboul '07)

Nb-47Ti 4.22 K for 11.75 T Iseult/INUMAC MRI: Kanithi H, Blasiak D, Lajewski J, Berriaud C, Vedrine P and Gilgrass G 2014 Production Results of 11.75 Tesla Iseult/INUMAC MRI Conductor at Luvata IEEE Transactions on Applied Superconductivity 24 1–4
http://dx.doi.org/10.1109/TASC.2013.2281417

Nb_3Sn (RRP®): Non-Cu J_c Internal Sn OI-ST RRP® 1.3 mm, Parrell, J.A.; Youzhu Zhang; Field, M.B.; Cisek, P.; Seung Hong; , "High field Nb_3Sn conductor development at Oxford Superconducting Technology," Applied Superconductivity, IEEE Transactions on , vol.13, no.2, pp. 3470- 3473, June 2003.
doi: 10.1109/TASC.2003.812360 and Nb_3Sn Conductor Development for Fusion and Particle Accelerator Applications J. A. Parrell, M. B. Field, Y. Zhang, and S. Hong, AIP Conf. Proc. 711, 369 (2004), DOI:10.1063/1.1774590.
Nb_3Sn (High Sn Bronze): T. Miyazaki et al. MT18 - fig3, Miyazaki, T.; Kato, H.; Hase, T.; Hamada, M.; Murakami, Y.; Itoh, K.; Kiyoshi, T.; Wada, H.; , "Development of high Sn content bronze processed Nb_3Sn superconducting wire for high field magnets," Applied Superconductivity, IEEE Transactions on , vol.14, no.2, pp. 975- 978, June 2004
doi: 10.1109/TASC.2004.830344
MgB_2: 18 Filament - The OSU/HTRI C 2 mol% AIMI ("Advanced Internal Mg Infiltration") 33.8 Filament to strand ratio, 39.1% MgB_2 in filament. G. Z. Li, M. D. Sumption, J. B. Zwayer, M. A. Susner, M. A. Rindfleisch, C. J. Thong, M. J. Tomsic, and E. W. Collings, "Effects of carbon concentration and filament number on advanced internal Mg infiltration-processed MgB 2 strands," Superconductor Science and Technology, vol. 26, no. 9, p. 095007, Sep. 2013. http://dx.doi.org/10.1088/0953-2048/25/11/115023

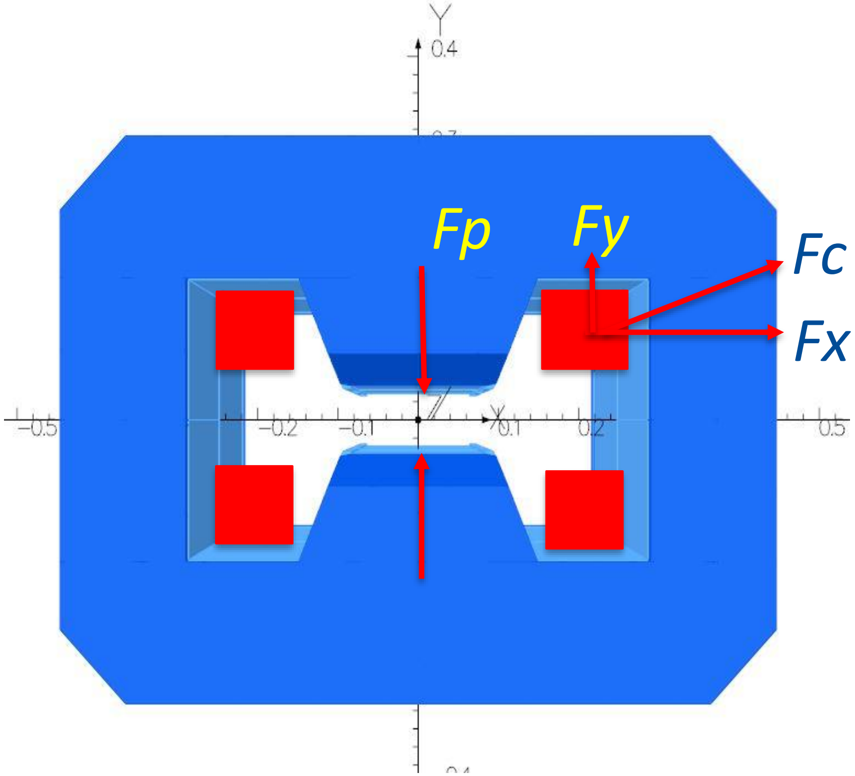
Links to ASC, MT and ICMC Proceedings can be found on the conferences page.

Superconducting Magnet Quenches and Protection

- ✓ Superconducting magnets quenched if there is generated enough heat to transfer the superconductor locally or globally in the normal condition.
- ✓ Quenches happened because of “flux jumps”, mechanical motion, epoxy cracking, not enough cooling, etc.
- ✓ Most magnets have quench detection and protection systems.
- ✓ Quench detected by monitoring the resistive voltage rise on the coil(s).
- ✓ Quench protection system initiate the stored energy extraction on dump resistor. Also used heaters to transfer the whole winding in the normal condition.

MRI Explosion:
https://www.youtube.com/watch?v=1R7KsfosV-o

Magnet Stored Energy and Lorentz Forces



Magnet stored energy:

$$W = \int_V \mathbf{B}*\mathbf{H}/2 *dV = \int_V \mathbf{B}^2/2\mu_o *dV$$

*More than 90% of magnet stored energy concentrated in the magnet air gap because **H** in the iron yoke is very small.*

Force between poles Fp:

Fy= -dW/dg = - W/g,

g- magnet gap

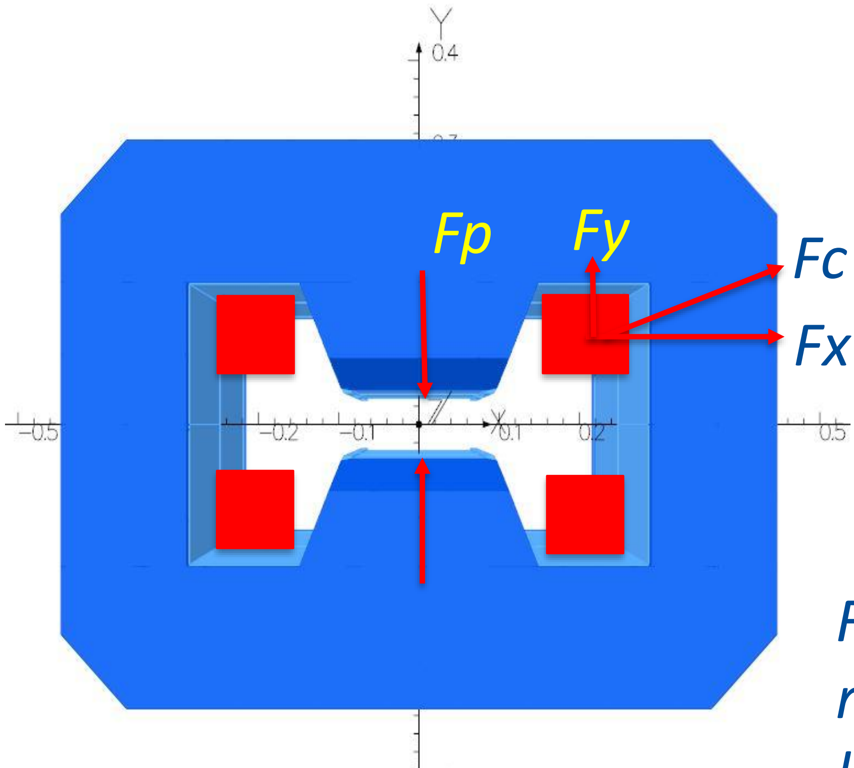
Lorentz forces on the coil Fc:

*Fx= By*Iw*Lc*

*Fy= Bx*Iw*Lc*

More accurate forces calculated by volumetric integration.

Magnet Resistance and Inductance



Magnet coil resistance at 20 C:

$R = (1+\alpha*dT)*\rho*Lc/q$

For Cu: $\alpha = 0.004\ [1/C^{\circ}]$

Lc – conductor length;

q – conductor cross-section;

α – temperature coefficient.

For superconducting magnets resistance defined by current leads and external cables.

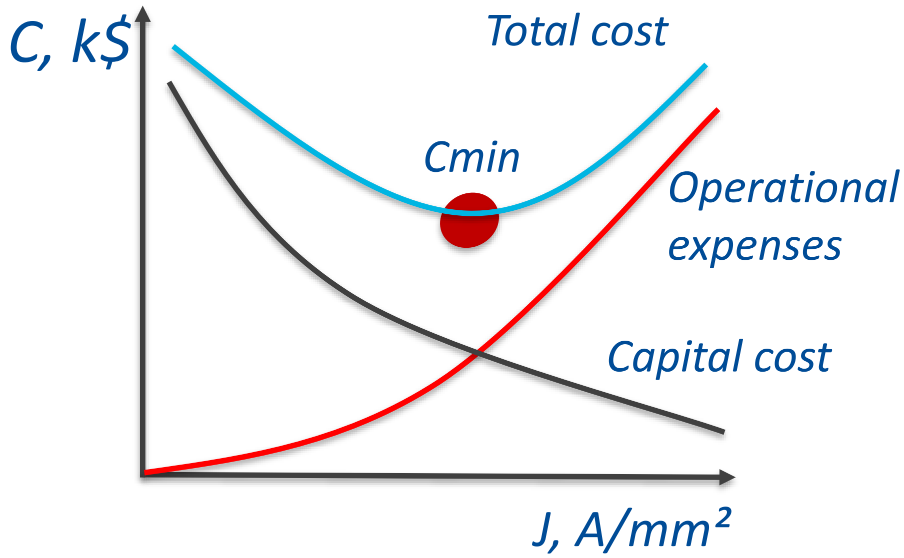
Magnet inductance:

$L = \psi/I = \omega\Phi/I$, *or*

$L = 2*W/I^2$

Magnet R, and L needed for the power supply design, and quench protection in the case of superconducting magnet.

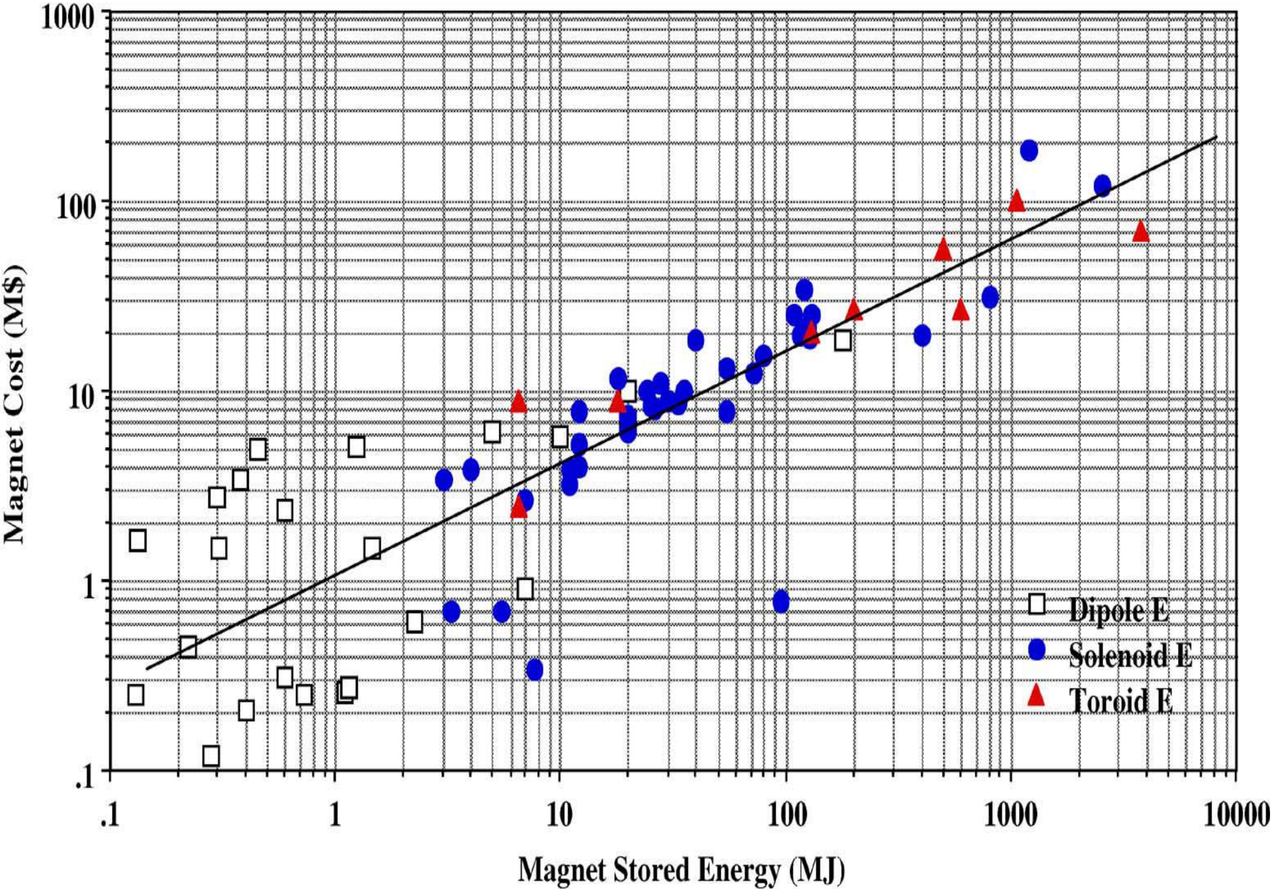
Magnet Cost



- ✓ *Cost of magnets:*
- *Small – 10k$- 50k$;*
- *Medium – 50k$ - 100k$;*
- *Large - > 100 k$.*
- *Cost of prototype is at least 3 times more than at a serial production.*

- ✓ *Capital cost includes cost of conductors, materials, tooling, and fabrication.*
- ✓ *Operational expenses include cost of electricity, water or LHe cooling for 10 years of operation at 5000-7000 hours/year.*
- ✓ *Optimal current density for air cooled magnets is 1.5 A/mm^2 - 2.0 A/mm^2.*
- ✓ *Optimal current density for water cooled magnets is 4.0 A/mm^2.*
- ✓ *For superconducting magnets the operational peak current density should be below critical, at least 20%, measured for the short sample.*

Magnet Stored energy vs. Cost



$$C(M\$) = 0.92\,[E(MJ)]^{0.60}$$

The Cost of Superconducting Magnets as a Function of Stored Energy and Design Magnetic Induction Times the Field Volume

Michael A. Green and Bruce P. Strauss

Magnetic Field Simulations

- Build or transfer from the CAD 2D or 3D model geometry;
- Input data for material properties and current sources;
- Specify type of field analysis: steady state, transient, current flow, motion, levitation, particles tracking;
- Specify multi physics combination: field-stress, field-thermal, quench for superconducting;
- Build the surface and volumetric meshes;
- Field calculation;
- Results analysis.

General 2D and 3D magnetic field simulation codes based on Finite Element Method. Commercial codes: OPERA, COMSOL, ANSYS, etc.