

Proton and Ion Linear Accelerators

Week 2, part 2

Yuri Batygin and Sergey Kurennoy

LANL

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Proton and Ion Linear Accelerators – Week 2, Part 2

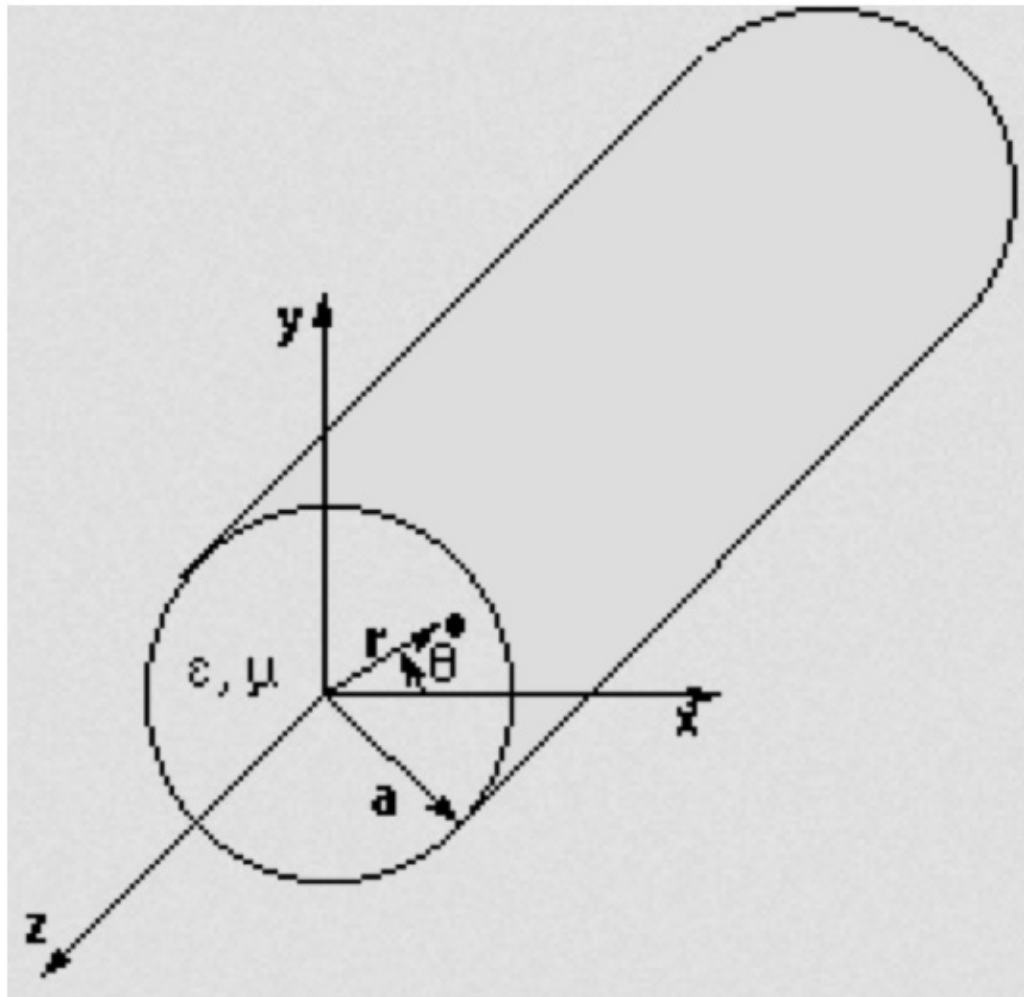
- Why linacs & RF together?
- Reminder: basics of linacs
- RF cavities + Superfish code & exercises
- Accelerating structures: RFQ, DTL, CCL, etc.
- Electromagnetic (EM) design of accelerating structures
- Linac components

Sources:

T.P. Wangler. *RF linear accelerators*, Wiley-VCH, 2nd Ed., 2008.

Handbook of Accelerator Physics and Engineering. Eds. A. Chao *et al.* World Scientific, 2013.

Uniform cylindrical waveguide



Circular waveguide.

Vacuum inside; for
 $\mu \neq 1, \epsilon \neq 1$

$$c \rightarrow c / \sqrt{\epsilon\mu}$$

Wave equations

$$\Delta \vec{E} - \frac{1}{c^2} \frac{\partial^2 \vec{E}}{\partial t^2} = 0$$

$$\Delta \vec{B} - \frac{1}{c^2} \frac{\partial^2 \vec{B}}{\partial t^2} = 0$$

Waveguide cross section
can have another shape

Waves in uniform circular cylindrical waveguide

Wave equation for electrical field

$$\Delta \vec{E} - \frac{1}{c^2} \frac{\partial^2 \vec{E}}{\partial t^2} = 0$$

Wave equation for E_z component in cylindrical coordinates

$$\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial E_z}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 E_z}{\partial \theta^2} + \frac{\partial^2 E_z}{\partial z^2} - \frac{1}{c^2} \frac{\partial^2 E_z}{\partial t^2} = 0$$

The solution is for TM wave:

$$E_z = R(r)\Theta(\theta)Z(z)T(t)$$

$$T(t) = T_o e^{-i\omega t} \quad \Theta(\theta) = \Theta_o e^{-in\theta} \quad Z(z) = Z_o e^{-ik_z z}$$

$$\frac{d^2 R}{dr^2} + \frac{1}{r} \frac{dR}{dr} + \left(\frac{\omega^2}{c^2} - k_z^2 - \frac{n^2}{r^2} \right) R = 0$$

Transverse wave number

$$k_r^2 = \frac{\omega^2}{c^2} - k_z^2$$

Wave equation can be rewritten as

$$\frac{d^2 R}{d(k_r r)^2} + \frac{1}{(k_r r)} \frac{dR}{d(k_r r)} + \left(1 - \frac{n^2}{(k_r r)^2} \right) R = 0$$

Its solution that is finite at $r=0$ is the Bessel function of 1st kind

$$R = A J_n(k_r r)$$

Waves in uniform circular cylindrical waveguide – 2

Longitudinal component vanishes at the boundary of cavity

Transverse wave number is determined as

v_{nm} is the root of equation $J_{nm}(x)=0$

Traveling wave in uniform waveguide

Wave number $k_z = \frac{2\pi}{\lambda}$ and wavelength

Cut-off frequency $k_z = 0$:

Phase of the wave

Phase velocity of the wave in uniform homogeneous single-connected waveguide is always above the speed of light in media

If $\omega < \omega_c$ in a uniform homogeneous single-connected waveguide, it is an evanescent wave: its amplitude exponentially decreases.

$$E_z(a) = 0 \quad J_n(k_r a) = 0$$

$$k_r a = v_{nm}$$

$$k_r = \frac{v_{nm}}{a}$$

$$E_z = E_0 J_n\left(v_{nm} \frac{r}{a}\right) \cos n\theta e^{-i(\omega t - k_z z)}$$

$$k_z^2 = \frac{\omega^2}{c^2} - \frac{v_{nm}^2}{a^2} \quad \lambda = \frac{2\pi}{k_z}$$

$$\omega_c = c \frac{v_{nm}}{a}$$

$$\varphi = \omega t - k_z z$$

$$v_{ph} = \frac{\omega}{k_z} = \frac{c}{\sqrt{1 - \left(\frac{\omega_c}{\omega}\right)^2}} > c$$

Dispersion diagram in uniform cylindrical waveguide

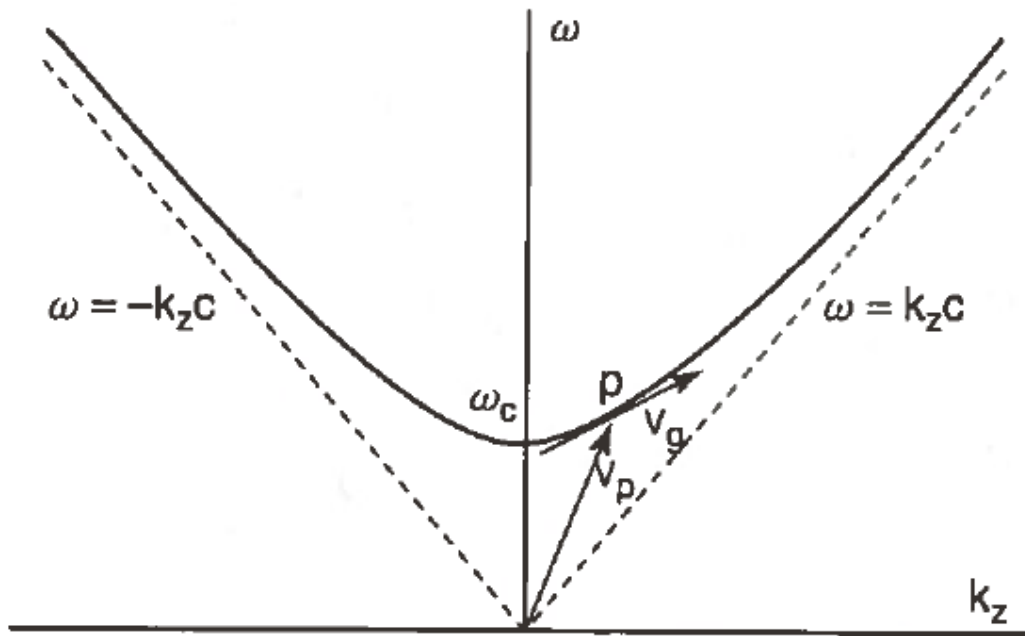


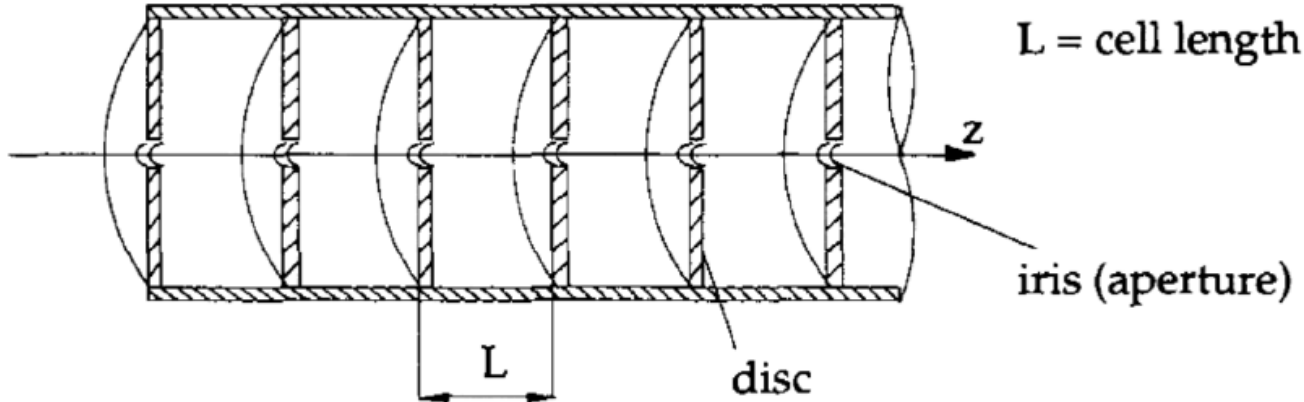
Figure 1.13 Example of dispersion curve for uniform waveguide, $\omega^2 = \omega_c^2 + (k_z c)^2$, showing graphically the meaning of phase and group velocity at the point p on the curve. The group velocity at point p is the tangent to the curve at that point. The phase velocity is the slope of the line from the origin to the point p .

Dispersion (Brillouin) diagram

The slope a of the line $\omega = a(k_z c)$ line determines the **wave phase velocity**:
 $a > 1$ means $v_{ph} > c$.

The **wave group velocity** gives the speed of energy propagation along the waveguide: $v_g = d\omega/dk_z < c$.

Disk-loaded waveguide (= traveling wave structure)



Longitudinal electric field with periodic conditions

$$E_z(r, z, t) = F(r, z)e^{j(\omega t - k_0 z)} \quad F(r, z + L) = F(r, z)$$

Expansion in Fourier series

$$F(r, z) = \sum_n a_n(r) e^{-j(2\pi n/L)z}$$

Substitution into wave Equation

$$e^{j\omega t} \sum_n e^{-j(k_0 + 2\pi n/L)z} \left[\frac{d^2 a_n(r)}{dr^2} + \frac{1}{r} \frac{da_n(r)}{dr} + K_r^2 a_n(r) \right] = 0$$

Transverse wave number

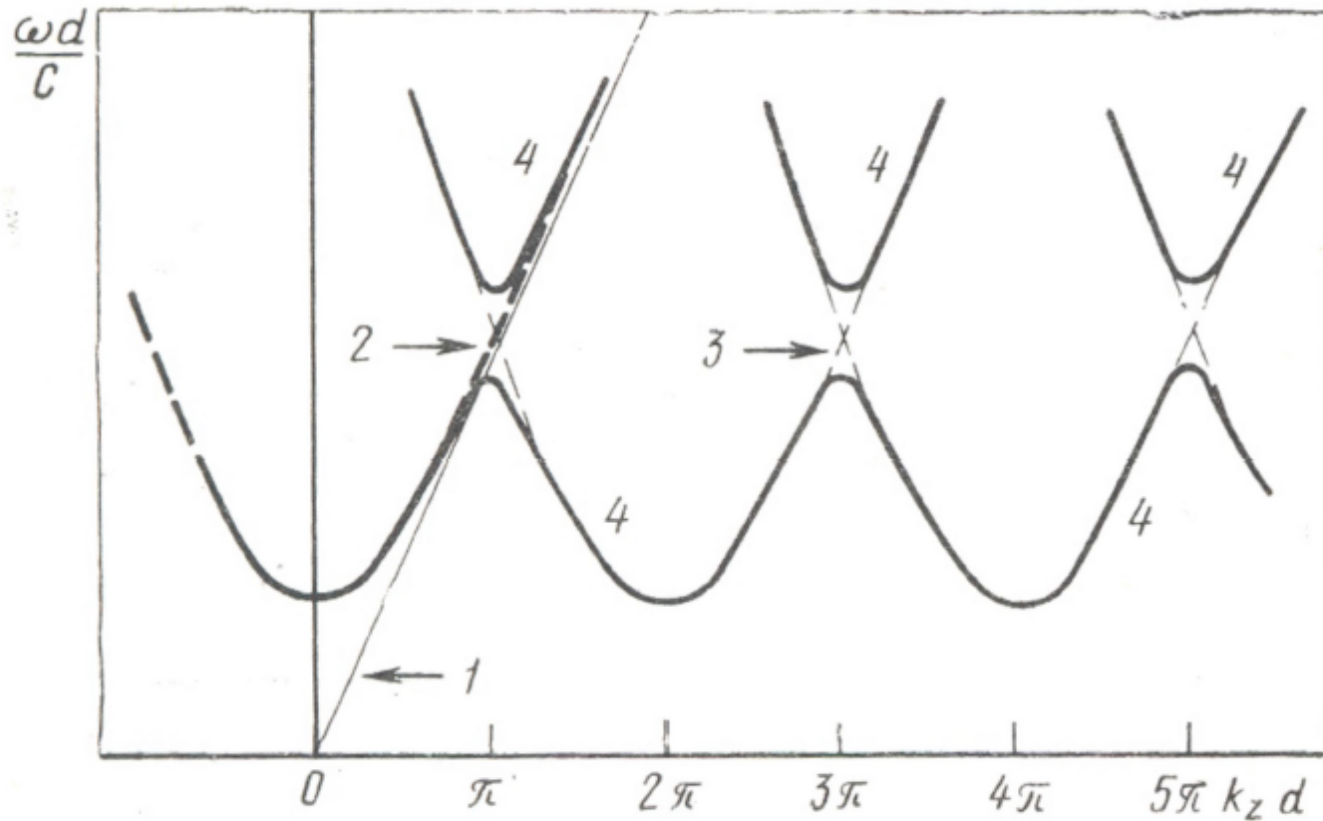
$$K_r^2 = \left(\frac{\omega}{c} \right)^2 - \left[k_0 + \frac{2\pi n}{L} \right]^2$$

Phase velocity

$$v_{ph} = \frac{\omega}{k_0 + 2\pi n/L} = \frac{\omega}{k_n}$$

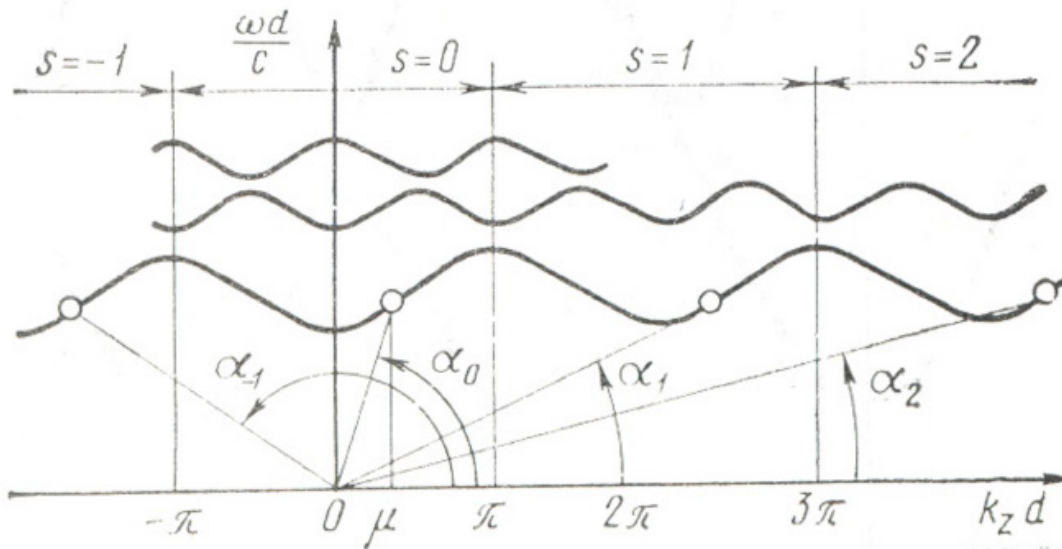
Different space harmonics have different phase velocities

Dispersion diagram of periodic waveguide

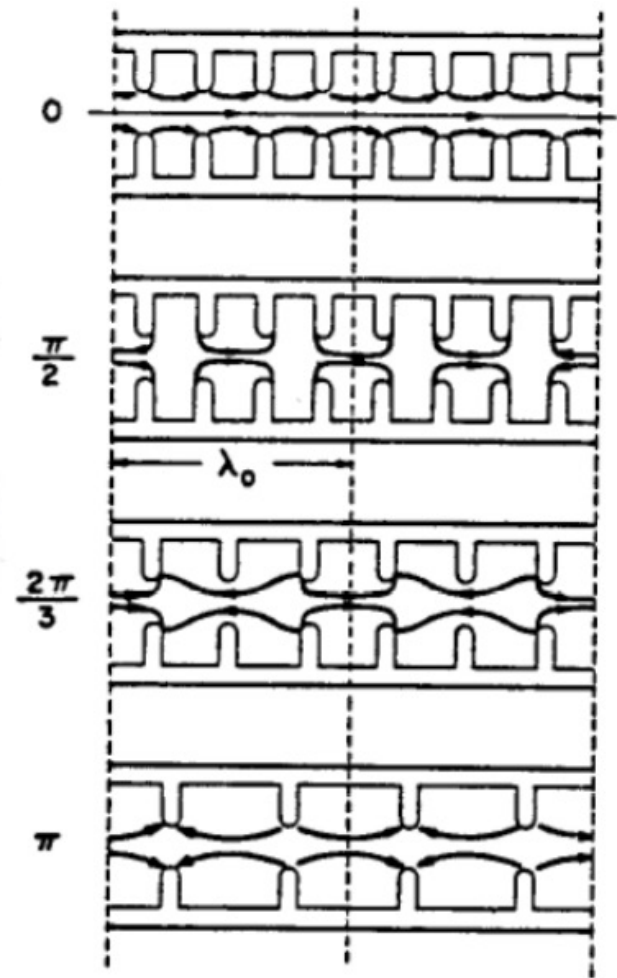


Dispersion diagram of periodic structure is a combination of diagrams for uniform waveguide periodically repeated after one period of the structure.

Traveling wave structures

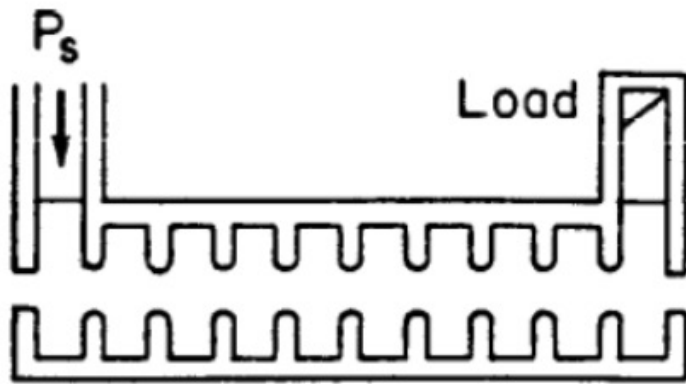


Brillouin diagram for disk-loaded waveguide. Angles α_1 , α_2 , ... correspond to phase velocities of various space harmonics.



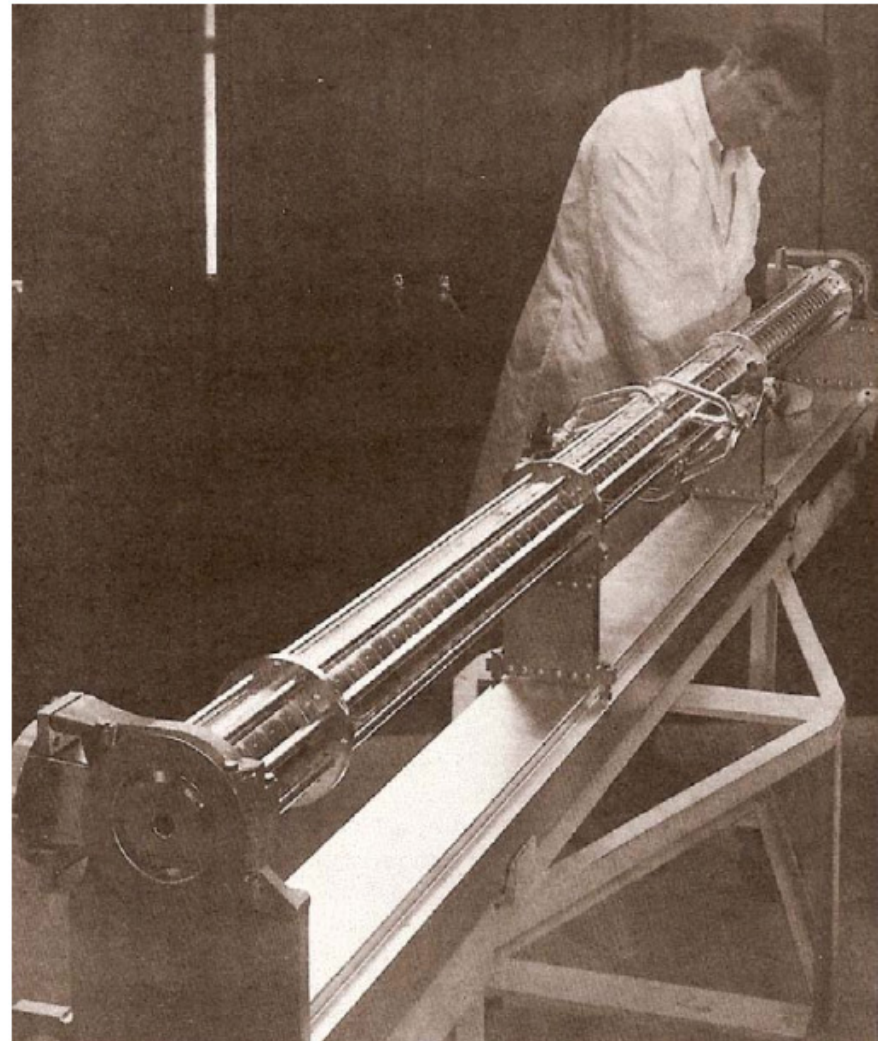
Snapshots of electric field configurations for disk-loaded structures with various phase shifts per period.

Traveling wave accelerator structures

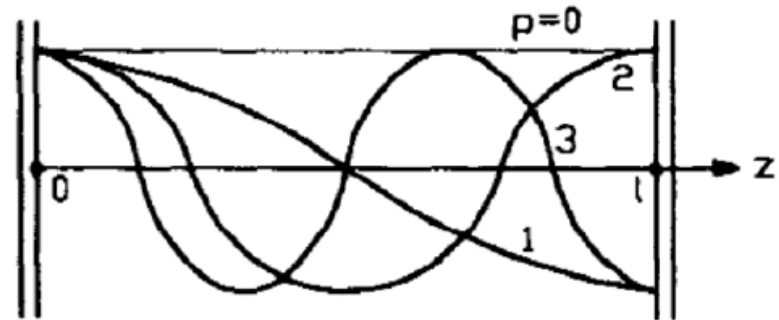
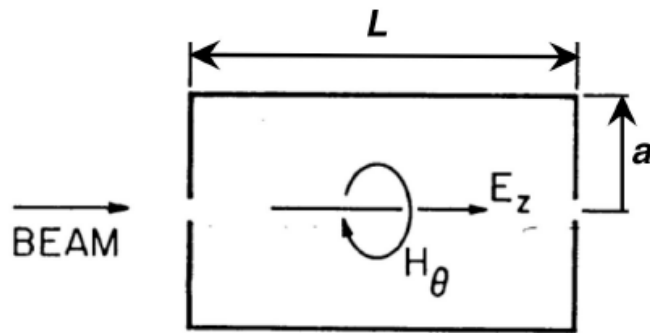


Linac with traveling wave. Primarily used for electrons.

SLAC accelerating structure: 10-foot disk-loaded, 2856 MHz, 86 cells per structure, 960 structures make up the SLAC 3-km linac.



Cylindrical resonator (pillbox)



Longitudinally integer number of half-variations can be excited

Transverse boundary condition:

$$k_z = \frac{\pi p}{L}$$

$$E_z(a) = 0 \quad J_n(k_r a) = 0 \quad k_r = \frac{v_{nm}}{a}$$

$$\frac{\omega_o^2}{c^2} - k_z^2 = \frac{v_{nm}^2}{a^2}$$

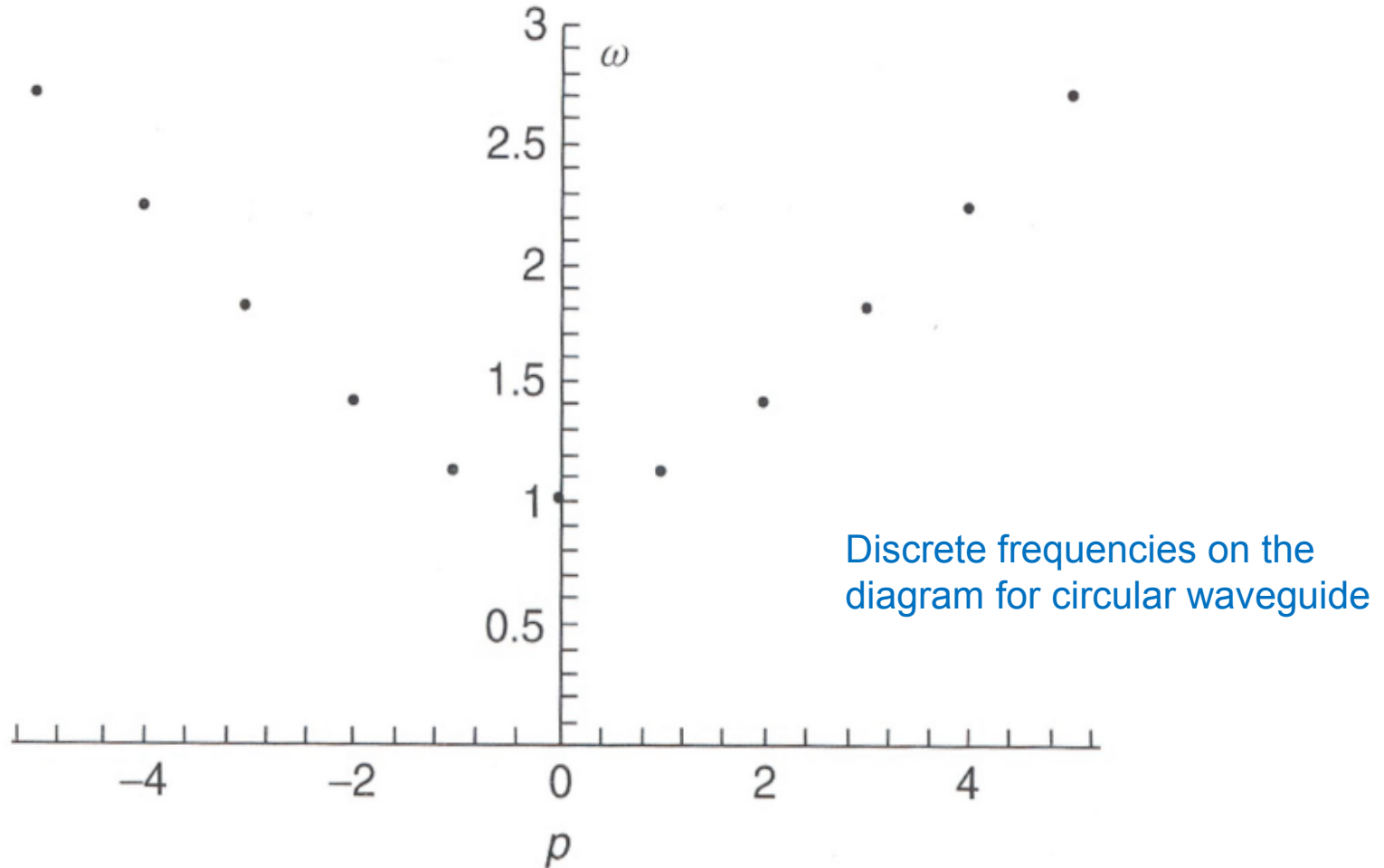
Frequency of oscillation mode is

$$\omega_o = c \sqrt{\frac{v_{nm}^2}{a^2} + \left(\frac{\pi p}{L}\right)^2}$$

Longitudinal component

$$E_z = E_o J_n\left(v_{nm} \frac{r}{a}\right) \cos n\theta \cos \frac{\pi p z}{L}$$

Dispersion diagram for cylindrical cavity



Dispersion curve for the TM_{01p} family of modes of a cylindrical circular cavity.

TM_{nmp} modes in cylindrical cavity (E modes)

Field components of TM_{nmp} modes in cylindrical cavity

$$\rightarrow E_z = E_o J_n(\chi r) \cos n\theta \cos \chi_z z$$

$$E_r = -E_o \frac{\chi_z}{\chi} J'_n(\chi r) \cos n\theta \sin \chi_z z$$

$$E_\theta = E_o \frac{n\chi_z}{\chi^2 r} J_n(\chi r) \sin n\theta \sin \chi_z z$$

$$H_r = -iE_o \frac{n\omega_o \epsilon_o}{\chi^2 r} J_n(\chi r) \sin n\theta \cos \chi_z z$$

$$H_\theta = -iE_o \frac{\omega_o \epsilon_o}{\chi} J'_n(\chi r) \cos n\theta \cos \chi_z z$$

$$\rightarrow H_z = 0$$

n – number of variation in azimuthal angle θ

m – number of variation in radius r

P – number of variation in longitudinal direction z

$$\chi = \frac{v_{nm}}{a} \quad \chi_z = \frac{\pi p}{L}$$

Example: TM₀₁₀ mode

Field components

$$E_z = E_o J_o(v_{01} \frac{r}{a}) \cos \omega_o t$$

$$B_\theta = -\frac{E_o}{c} J_1(v_{01} \frac{r}{a}) \sin \omega_o t$$

Boundary condition

$$E_z(a) = 0$$

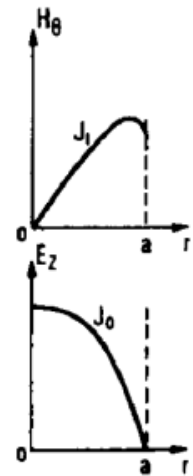
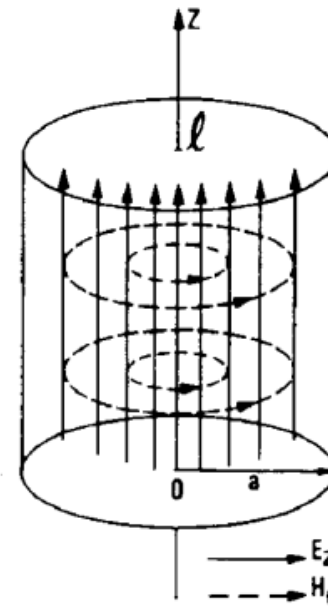
$$v_{01} = 2.405$$

Frequency of resonator

$$k_z = 0$$

$$\omega_o = 2\pi f = \frac{c v_{01}}{a}$$

$$f = \frac{2.405 c}{2\pi a}$$

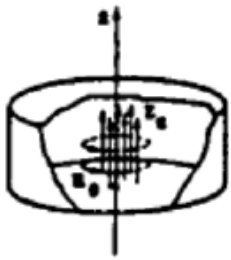


Example: radius of resonator for $f = 201.25$ MHz:

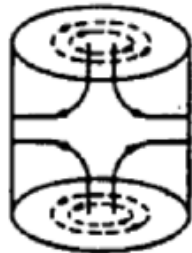
$$a = \frac{2.405 c}{2\pi f} = 0.57m$$

TM₀₁₀ mode in a pill-box cavity.

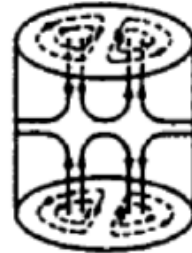
TM modes in cylindrical resonator



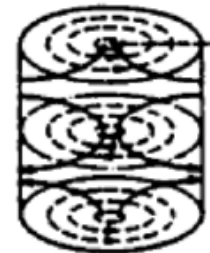
TM₀₁₀ mode



TM₀₁₁ mode

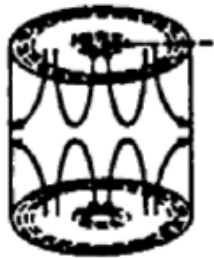


TM₁₁₁ mode

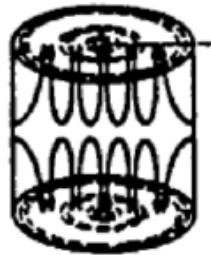


TM₀₁₂ mode

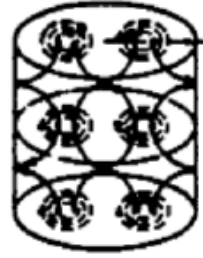
E ———
H - - - -



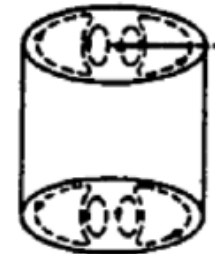
TM₀₂₁ mode



TM₀₃₁ mode



TM₁₁₂ mode



TM₁₂₁ mode

TM-mode field patterns in cylindrical resonator (T.Wangler, LA-UR-93-805).

TE_{nmp} modes in cylindrical cavity (H modes)

Field components of TE_{nmp} modes in cylindrical cavity

$$\rightarrow H_z = H_o J_n(\chi r) \cos n\theta \sin \chi_z z$$

$$H_r = H_o \frac{\chi_z}{\chi} J_n'(\chi r) \cos n\theta \cos \chi_z z$$

$$H_\theta = -H_o \frac{n\chi_z}{\chi^2 r} J_n(\chi r) \sin n\theta \cos \chi_z z$$

$$\rightarrow E_z = 0$$

$$E_r = iH_o \frac{n\omega_o \mu_o}{\chi^2 r} J_n(\chi r) \sin n\theta \sin \chi_z z$$

$$E_\theta = iH_o \frac{\omega_o \mu_o}{\chi} J_n'(\chi r) \cos n\theta \sin \chi_z z$$

$$\chi_z = \frac{\pi p}{L}$$

Boundary condition:

$$E_\theta(a) = 0$$

$$J_n'(\chi a) = 0 \quad \chi = \frac{v_{nm}'}{a}$$

v_{nm}' is the root of equation $J_n'(x) = 0$

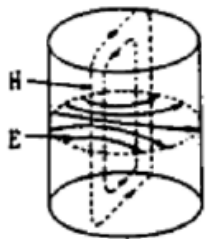
Frequency of TE_{nmp} oscillations

$$\omega_o = c \sqrt{\frac{v_{nm}'^2}{a^2} + \left(\frac{\pi p}{L}\right)^2}$$

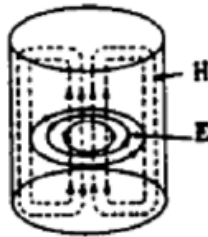
Zeros v_{nm}' of equation $J_n'(x) = 0$

	$m = 1$	$m = 2$	$m = 3$	$m = 4$
$n = 0$	3.832	7.016	10.173	13.324
$n = 1$	1.841	5.331	8.536	11.706
$n = 2$	3.054	6.706	9.969	13.170
$n = 3$	4.201	8.015	11.346	

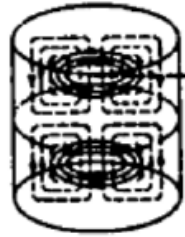
TE modes in cylindrical resonator



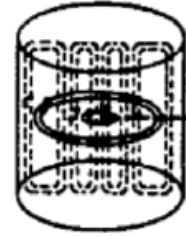
TE_{111} mode



TE_{011} mode



TE_{012} mode



TE_{021} mode



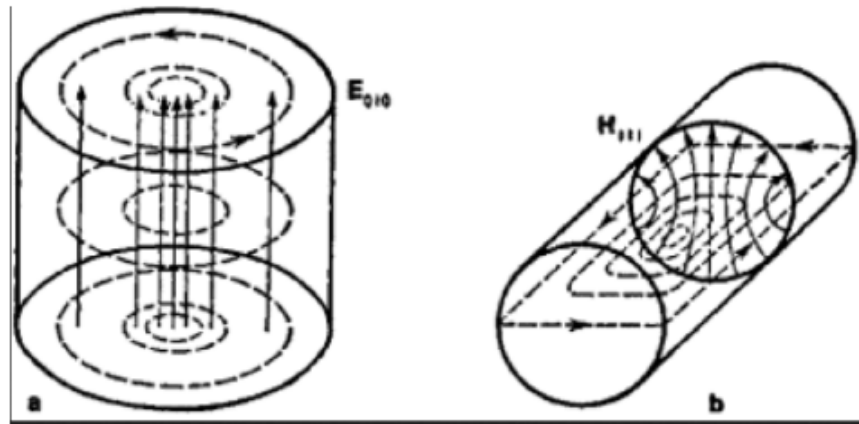
TE_{112} mode



TE_{121} mode

TE-mode field patterns in cylindrical resonator (T.Wangler, LA-UR-93-805).

Fundamental modes of cylindrical resonator



Oscillations TE₁₁₁ and TM₀₁₀ are fundamental modes which frequencies coincide if

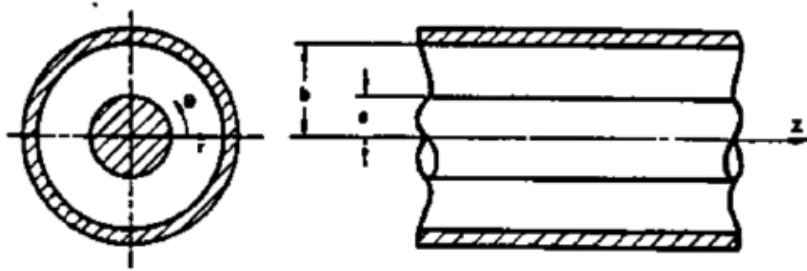
$$\frac{v_{01}^2}{a^2} = \frac{v_{11}^2}{a^2} + \left(\frac{\pi}{L}\right)^2$$

In this case of ratio of length of resonator to radius L/a is

$$\frac{L}{a} = \frac{\pi}{\sqrt{v_{01}^2 - v_{11}^2}} = 2.03$$

For long cylinder $L/a > 2.03$ the fundamental mode is TE₁₁₁ while for “flat” resonator $L/a < 2.03$ the fundamental mode is TM₀₁₀.

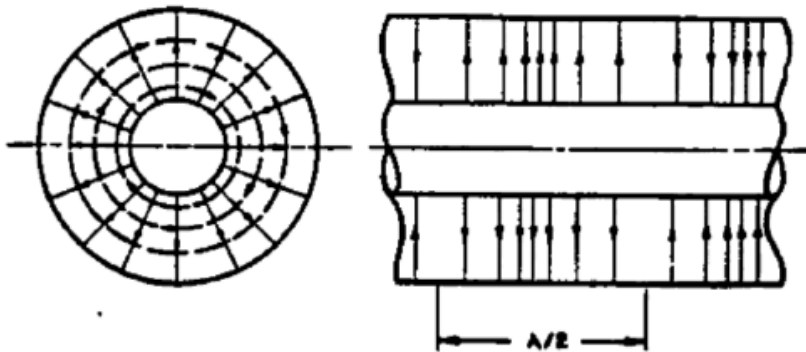
Coaxial line



A section of coaxial transmission line

Field components of TEM wave propagating in coaxial transmission line

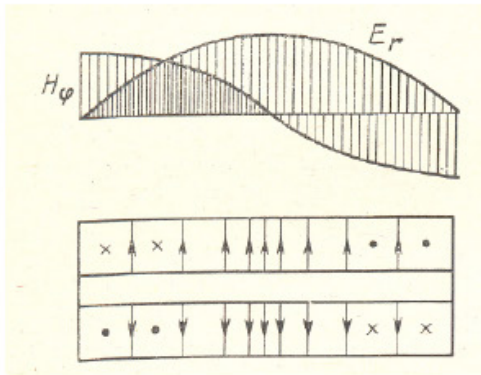
$$B_{\theta} = \frac{\mu_o I}{2\pi r} \exp[i(\omega t - k_z z)]$$



$$E_r = \sqrt{\frac{\mu_o}{\epsilon_o}} \frac{I}{2\pi r} \exp[i(\omega t - k_z z)]$$

Note that unlike single-connected waveguides there is no cut-off frequency in coaxial lines. They can transmit waves even at very low frequencies, and the phase velocity $v_{ph}=c$.

Coaxial resonators: half-wave & quarter-wave

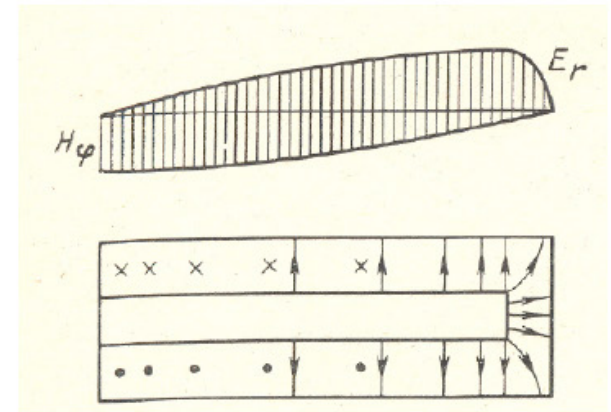
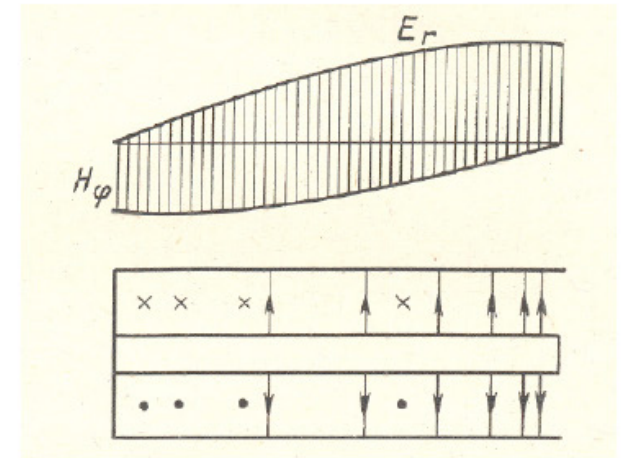
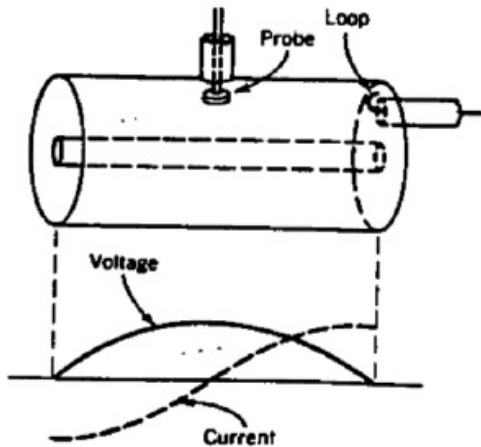


Resonance condition: $L = \frac{p\lambda}{2}$

Component of RF field

$$B_{\theta} = \frac{\mu_o I}{2\pi r} \cos\left(\frac{\pi p z}{L}\right) \cos \omega t$$

$$E_r = \sqrt{\frac{\mu_o}{\epsilon_o}} \frac{I}{2\pi r} \sin\left(\frac{\pi p z}{L}\right) \sin \omega t$$



Coaxial resonator with voltage and current standing waves.

Quarter wave resonator

The wavelength (and frequency) of coaxial resonators is defined mainly by their length. Note: RF coupling – magnetic loop here; ferrite loading – lowering and tuning frequency.

Field energy in RF cavities

EM energy in cavity

$$W = \frac{1}{2} \int_V (\mu_o H^2 + \epsilon_o E^2)$$

Energy balance

$$\oint_S [\vec{E}, \vec{H}] d\vec{S} = -\frac{d}{dt} \int_V \left(\frac{\mu_o H^2}{2} + \frac{\epsilon_o E^2}{2} \right) dV - \int_V \vec{j} \vec{E} dV$$

Poynting's theorem in integral form: the rate of energy change in cavity equals the rate of work done on a charge distribution plus the energy flux through its surface.

Energy dissipation in resonator and Q factor

Dissipated power is a combination of power losses inside cavity and outside cavity

Energy stored in cavity

Quality factor
(= 2π * stored energy / energy loss per period)

Q-factor is a combination of unloaded quality factor of cavity and external quality (loaded Q factor)

External quality factor

Losses in metal with surface resistance R_s [Ohm]

$$R_s = \frac{1}{\sigma\delta} = \sqrt{\frac{\mu_0\omega}{2\sigma}}, \text{ where } \sigma \text{ is the surface conductivity,}$$

and δ is the skin depth $\delta = \sqrt{2 / (\mu_0\sigma\omega)}$.

Unloaded quality factor

$$Q_o = \frac{\omega_o W_o}{P_o}$$

Physical meaning: $Q = G \frac{V}{S\delta}$

$$P = P_o + P_{ext}$$

$$W_o = \frac{1}{2} \int_{V_o} \mu H_m^2 dV = \frac{1}{2} \int_{V_o} \epsilon E_m^2 dV$$

$$Q = \frac{\omega_o W_o}{P}$$

$$\frac{1}{Q} = \frac{1}{Q_o} + \frac{1}{Q_{ext}}$$

$$Q_{ext} = \frac{\omega_o W_o}{P_{ext}}$$

$$P_o = \frac{R_s}{2} \int_s H_m^2 dS$$

$$Q_o = \frac{\omega_o \int_{V_o} H_m^2 dV}{R_s \int_s H_m^2 dS}$$

Quality factor of TM₀₁₀ cavity

Magnetic field

$$H_{m\theta} = -E_o \sqrt{\frac{\epsilon_o}{\mu_o}} J_1\left(\nu_{01} \frac{r}{a}\right)$$

Energy stored in cavity

$$W_o = \frac{1}{2} \int_{V_o} \mu_o H_{m\theta}^2 dV = \frac{\pi \epsilon_o E_o^2 L a^2 J_1^2(\nu_{01})}{2} = 0.135 \pi \epsilon_o L a^2 E_o^2$$

Loss power in cavity

$$P_o = \frac{R_s}{2} \int_S H_{m\theta}^2 dS = \pi a R_s E_o^2 \frac{\epsilon_o}{\mu_o} J_1^2(\nu_{01}) (L + a)$$

Unloaded quality factor

$$Q_o = \frac{\omega_o W_o}{P} = \frac{\nu_{01}}{2R_s} \sqrt{\frac{\mu_o}{\epsilon_o}} \frac{1}{\left(1 + \frac{a}{L}\right)} = 1.2025 \frac{376.7[\text{Ohm}]}{R_s} \frac{1}{\left(1 + \frac{a}{L}\right)}$$

For ideal copper surface $\sigma = 5.8 \cdot 10^7$ Sm/m, so that $R_s = 2.6 \cdot 10^{-4} \sqrt{f(\text{MHz})}$ Ω . At 201.25 MHz, $R_s = 3.7$ m Ω , and $Q_o = 66500$ for $a/L = 1$. In practice, typically 10%-20% less.

Superconducting RF cavities

For RF cavities the power loss depends on the surface resistance: for normal-conducting

$$R_s = \frac{1}{\sigma\delta} = \sqrt{\frac{\mu_0\omega}{2\sigma}} \text{ scales with RF frequency as } \sqrt{f}.$$

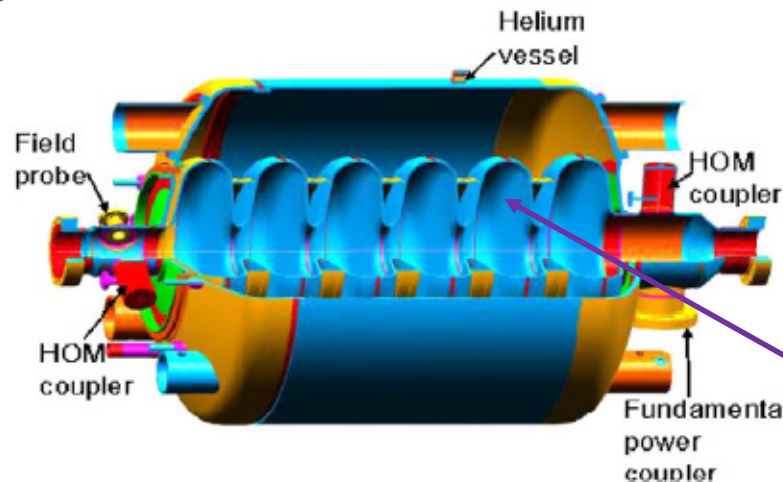
In superconducting (SC) RF cavities the surface resistance is much lower; e.g., for Nb

$$R_s (\Omega) = 9 \cdot 10^{-5} \frac{f^2 (\text{GHz})}{T (\text{K})} \exp\left(-\alpha \frac{T_c}{T}\right) + R_{res},$$

where R_{res} is the residual resistance ($\sim 1\text{-}10$ n Ω), $\alpha = 1.83$, and $T_c = 9.2$ K is the critical temperature.

SC R_s is $\sim 10^{-5}$ of that in copper, and so are the cavity surface losses!

For ideal copper surface, typical $Q_0 = 10^4\text{-}10^5$. In SC cavities, typical $Q_0 = 10^8\text{-}10^{10}$, so it is especially advantageous to use SC RF cavities in CW machines (operating at 100% RF duty factor).



SC cavity assembly
for SNS linac

6-cell Nb 805-MHz
elliptical RF cavity

Quality factor of coaxial resonator

Azimuthal magnetic field

$$H_{m\theta} = \frac{I_m}{2\pi r} \cos \frac{p\pi z}{L}$$

Integral over volume

$$\int_{V_o} H_m^2 dV = \pi L \left(\frac{I}{2\pi}\right)^2 \ln \frac{R_2}{R_1}$$

Integral over surface

$$\int_{V_o} H_m^2 dS = \pi \left(\frac{I}{2\pi}\right)^2 \left[4 \ln \frac{R_2}{R_1} + L \left(\frac{1}{R_1} + \frac{1}{R_2}\right)\right]$$

Unloaded quality factor

$$Q_o = \frac{p\pi}{R_s} \sqrt{\frac{\mu_o}{\epsilon_o}} \frac{\ln \frac{R_2}{R_1}}{4 \ln \frac{R_2}{R_1} + L \left(\frac{1}{R_1} + \frac{1}{R_2}\right)}$$

Filling time of resonator

Power losses is a rate of decrease of stored energy

$$P = -\frac{dW_o}{dt}$$

Substitution into equation $Q = \frac{\omega_o W_o}{P}$ gives equation for decrease of stored energy

$$\frac{dW_o}{dt} = -\frac{\omega_o W_o}{Q}$$

Solution

$$W_o = W_o(0)e^{-\frac{\omega_o t}{Q}}$$

Electrical field changes with two times smaller rate:

$$\alpha = \frac{\omega_o}{2Q}$$

Electric field

$$E = E_m e^{-i\omega_o t} e^{-\frac{\omega_o t}{2Q}}$$

Filing time of the cavity

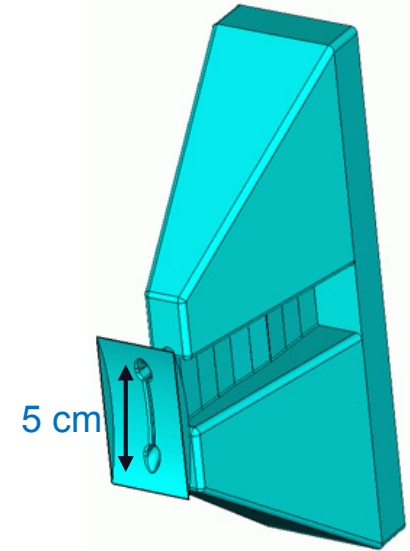
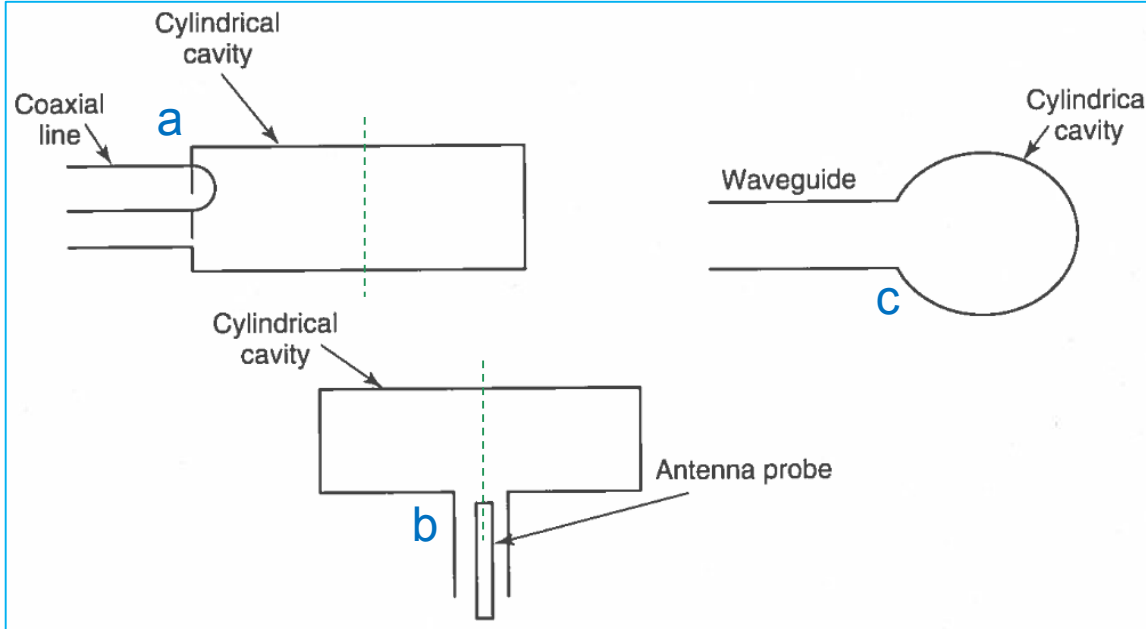
$$t_f = \frac{2Q}{\omega_o}$$

Note: filling time $t_f = QT_{RF}/\pi$

Sometimes it is convenient to use complex frequency of cavity

$$\tilde{\omega}_o = \omega_o \left(1 + i \frac{1}{2Q}\right)$$

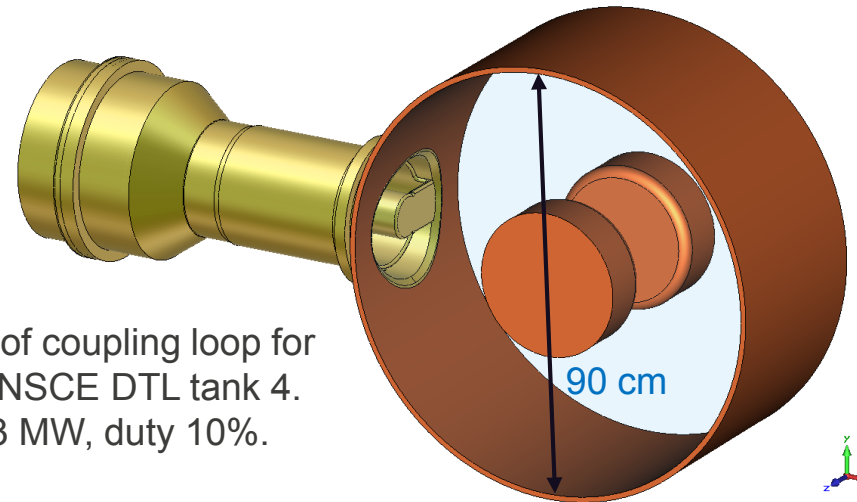
Coupling RF power to cavities



"Dog-bone" coupling iris for high-power FEL photoinjector. Up to 0.5 MW at 100% duty.

Methods of coupling to RF cavities:

- (a) magnetic – current loop;
- (b) electric – antenna;
- (c) magnetic – iris WG-cavity



Model of coupling loop for the LANSCE DTL tank 4. Up to 3 MW, duty 10%.



Filling time with external load

The cavity coupling coefficient is defined as $\beta \equiv \frac{P_{ext}}{P_0} = \frac{Q_0}{Q_{ext}}$.

When the power source is matched to the resonant structure through a coupling loop, such that no power is reflected toward the source, then the loaded Q

$$Q = \frac{Q_0}{1 + \beta}$$

where β is the coupling coefficient. For negligible beam current $\beta = 1$.

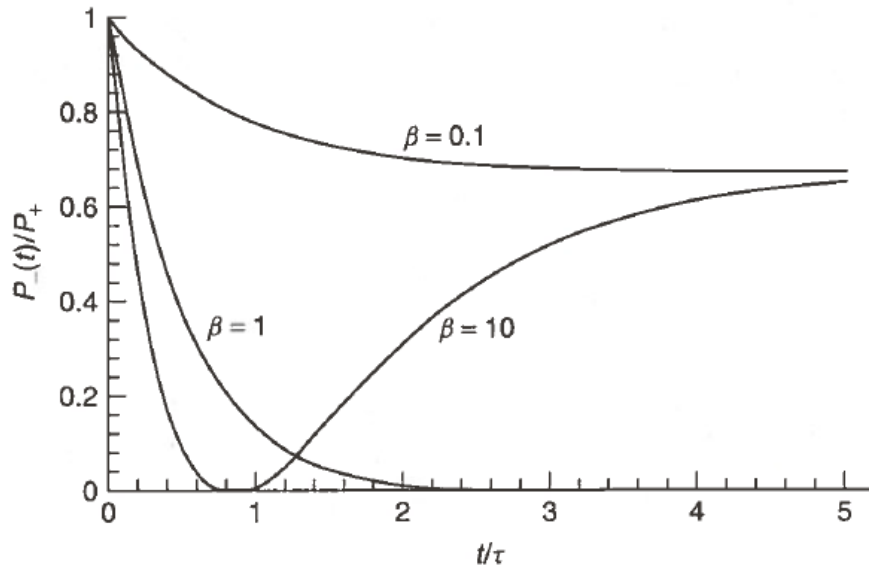
The filling time becomes

$$t_f = \frac{2Q}{\omega_o} = \frac{2Q_0}{\omega_o(1 + \beta)}$$

During the filling time, the transient effect exists when reflected power cannot be avoided.

RF coupling and reflected power

When coupling coefficient $\beta = 1$, the cavity is matched to RF feed (critically coupled); $\beta > 1$ – over-coupled, and $\beta < 1$ – under-coupled.



Backward (reflected) power vs time for different values of β . Here $\tau = t_f$ – cavity filling time.

The cavity RF coupling is usually designed such that

$$\beta = \frac{P_0 + P_{beam}}{P_0}.$$

In this case no RF power is reflected back to the RF source when the cavity is operating with the beam.

Figures of merit for accelerating structures

Quality factor (stored energy U , averaged power loss P)

$$Q = \frac{\omega U}{P}$$

Shunt impedance (total cavity voltage V_0)

$$R_{sh} = \frac{V_0^2}{P}$$

Effective shunt impedance (effective voltage $V_0 T$)

$$R_{eff} = R_{sh} T^2$$

Shunt impedance per unit length (voltage $V_0 = E_0 L$)

$$Z = \frac{E_0^2}{P / L}$$

Effective shunt impedance per unit length (Z_{eff})

$$Z T^2 = \frac{(E_0 T)^2}{P / L}$$

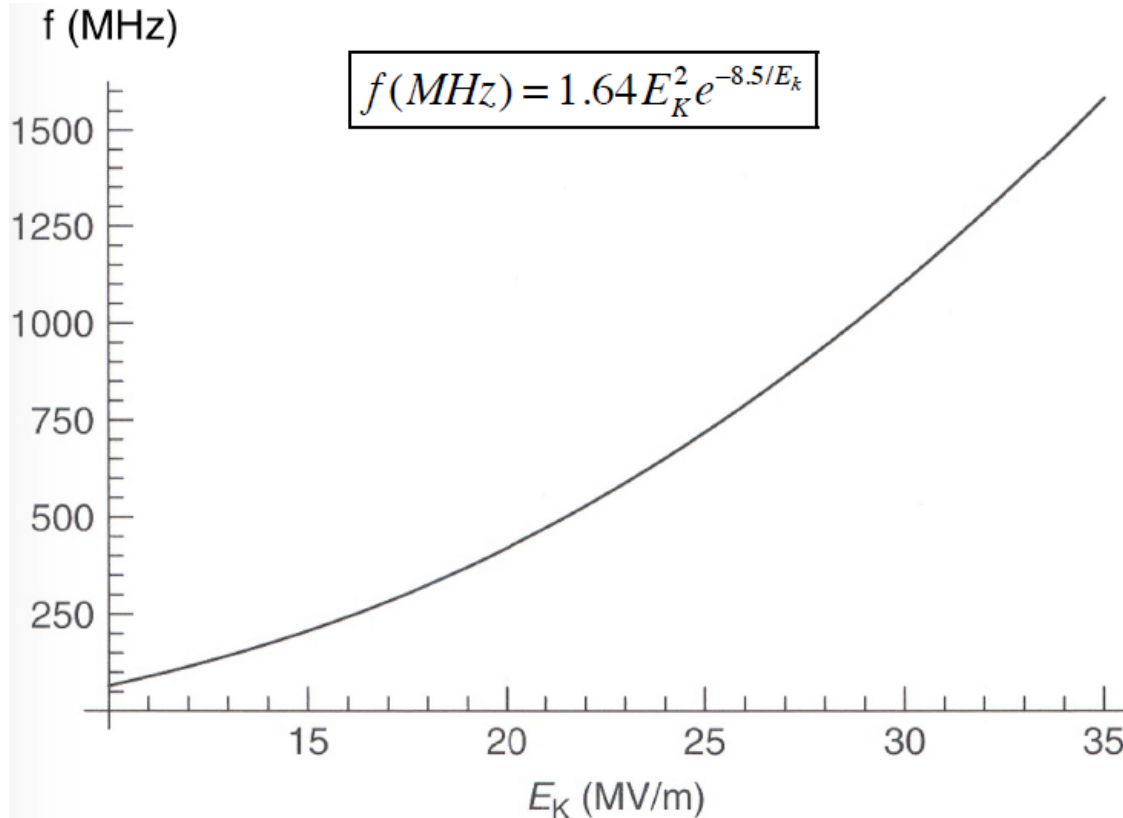
Ratio R_{eff}/Q is independent of surface losses and depends only on the cavity (structure) geometry

$$\frac{R_{eff}}{Q} = \frac{(V_0 T)^2}{\omega U}$$

Ratios E_{max}/E_{acc} ($E_{acc} = E_0 T$) and B_{max}/E_{acc} – lower is better. The latter is very important for SC cavities.

Cavity RF electric breakdowns: Kilpatrick criterion

RF breakdowns – uncontrolled discharges in cavities – limit the cavity max electric fields.



The maximal surface field also depends on the RF pulse length. The cavity design fields are typically $(1.3-2)E_K$ for pulses longer than 1 ms. For very short pulses, below 1 μs , they can be higher and scale as

$$E_s \propto f^{1/2} / t^{1/4}.$$

The critical value of the surface electric field (E_K – Kilpatrick field) versus RF frequency (empirical, conservative).

Other deleterious electron-related effects in RF cavities: [multipacting](#) (usually occurs at low RF field levels) and [electron RF loading](#). In SC RF cavities – quench (\rightarrow NC).

RF cavity design issues

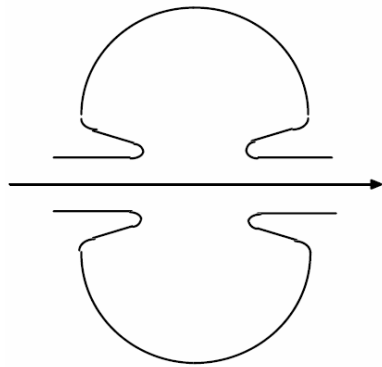
Cavity design goals depend on many factors including its application and the cavity type: maximize accelerating gradient, minimize losses (NC), minimize max surface fields, etc.

Frequency dependence of cavity parameters: $a \sim 1/f$,

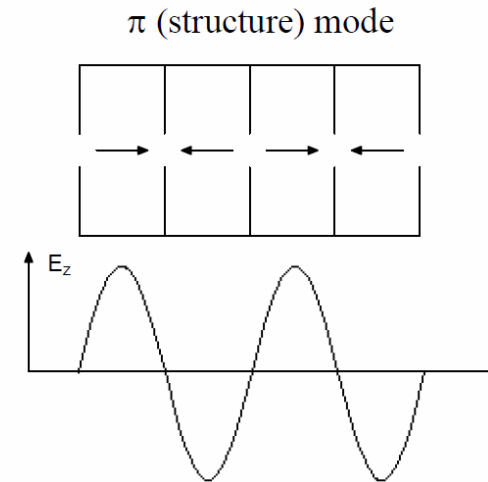
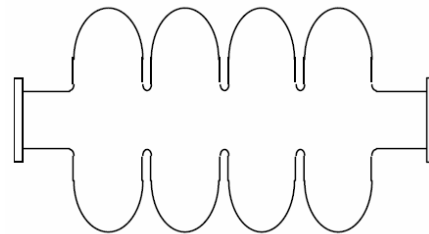
$$P \propto \begin{cases} f^{-1/2}, & \text{NC} \\ f, & \text{SC} \end{cases} \quad Q \propto \begin{cases} f^{-1/2}, & \text{NC} \\ f^{-2}, & \text{SC} \end{cases} \quad ZT^2 \propto \begin{cases} f^{1/2}, & \text{NC} \\ f^{-1}, & \text{SC} \end{cases}$$

Frequency choice also depends on available RF sources and beam parameters.

Changing cavity shape is the common way of achieving the design goals. Examples:



Nose-cone cavity



4-cell elliptical cavity (SC)

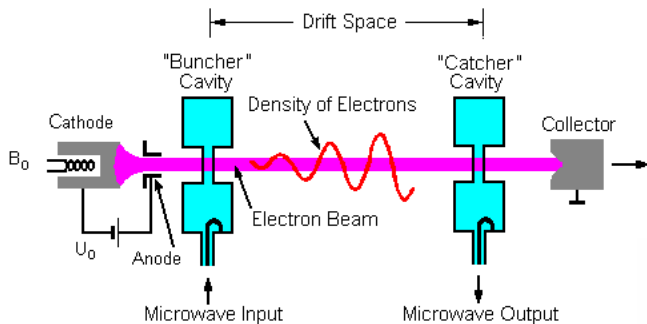
RF power sources

High-power vacuum-tube amplifiers: gridded tubes (triodes, tetrodes – below 300 MHz, pulsed) and klystrons (300 MHz – 10s GHz; from 1 μ s pulses to CW operation).

Klystrons were invented in 1937 (Varian). Peak power up to 60 MW, average 50 kW (SLAC).

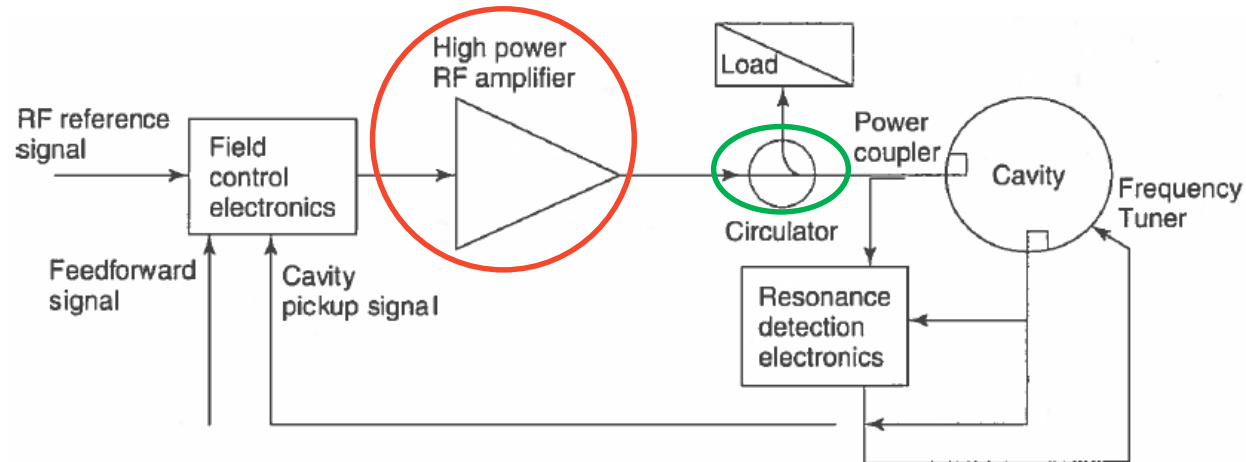
Power gain up to 50-60 dB, efficiency 40-60%.

Reliable: MTBF > 30000 h.



Klystron principle

Scheme of RF system for accelerator cavity



- The choice depends on application (pulsed or CW), frequency, peak and average power, efficiency, and cost. The RF cost is typically the largest part of linac cost.
- Magnetrons are rarely used (cheaper but lack phase stability).
- More recently – multi-beam klystrons, solid-state amplifiers, and inductive-output tubes (IOTs).

Summary of part 2

RF waveguide, cavity, and power source basics are reviewed.