Unit 9
Electromagnetic design
Episode II

Soren Prestemon and **Steve Gourlay**
Lawrence Berkeley National Laboratory (LBNL)

With significant re-use of material from the same unit lecture by Ezio Todesco, USPAS 2017

CONTENTS

1. Dipoles: short sample field versus material and lay-out

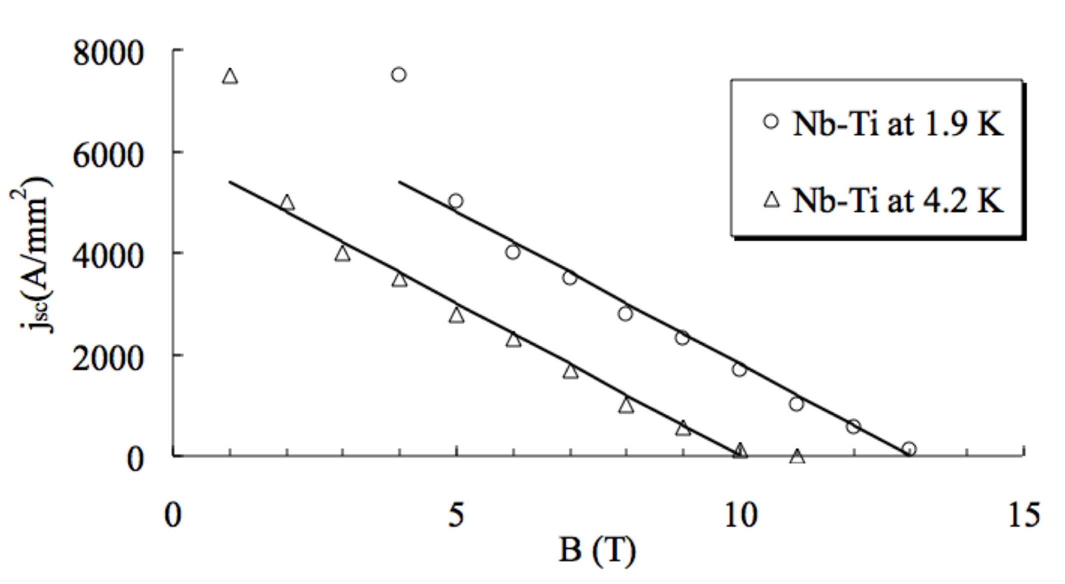
2. Quadrupoles: short sample gradient versus material and lay-out

3. A flowchart for magnet design

1. DIPOLES: FIELD VERSUS MATERIAL AND COIL THICKNESS

- We will use the following relation as a model for the $J_c(B)$ critical surface
  - Nb-Ti: linear approximation is reasonable: $J_c(B) = s(B^*_{c2} - B)$

    with s~6.0×10^8 [A/(T m^2)] and B^*_{c2}~10 T at 4.2 K or 13 T at 1.9 K
    - This is a typical mature and very good Nb-Ti strand
    - Tevatron superconductor had half of this J_c!



In reality in accelerator magnets our current density is a fraction of that shown in the previous model

- The current density in the coil is lower because…
  - Strand made of superconductor and normal conducting (~copper)
    - $\nu_{Cu\text{-}sc}$ is the ratio between the copper and the superconductor, usually ranging from 1 to 2 in most cases
  - If the strands are assembled in rectangular cables, there are voids:
    - $\kappa_{w\text{-}c}$ is the fraction of cable occupied by strands (usually ~85%)
  - The cables are insulated:
    - $\kappa_{c\text{-}i}$ is the fraction of insulated cable occupied by the bare cable (~85%)
- The current density flowing in the insulated cable is reduced by a factor κ (filling ratio)
  - The filling ratio ranges from 1/4 - 2/3
  - The critical surface for *j* (engineering current density) is

$$\kappa \equiv \kappa_{w-c}\kappa_{c-i}\frac{1}{1+\nu_{Cu/noCu}}$$

$$J_E(B) = \kappa J_c(B)$$

$$J_E(B) = \kappa s(B_{c2}^{*} - B)$$

The filling ratio is fairly consistent among accelerator magnets - with a few caveats

- Examples of filling ratio in dipoles (similar for quads)

$$J_E(B) = \kappa J_c(B)$$

$$\kappa \equiv \kappa_{w-c} \kappa_{c-i} \frac{1}{1+\nu_{Cu/noCu}}$$

Magnet	$\nu_{Cu/noCu}$	κ_{w-c}	κ_{c-i}	κ
Tevatron MB	1.85	0.82	0.81	0.23
HERA MB	1.88	0.89	0.85	0.26
SSC MB inner	1.5	0.84	0.89	0.30
RHIC MB	2.25	0.87	0.84	0.22
LHC MB inner	1.65	0.87	0.87	0.29
FRESCA	1.6	0.87	0.88	0.29
MSUT inner	1.25	0.85	0.88	0.33
D20 inner	0.43	0.83	0.84	0.49
FNAL HFDA	1.25	0.86	0.76	0.29

- Copper to superconductor ranging from 1.2 to 2.2
  - Extreme case of D20: 0.43
- Void fraction from 11% to 18%
- Insulation from 11% to 18%
  - Case of FNAL HFDA: 24% for insulation

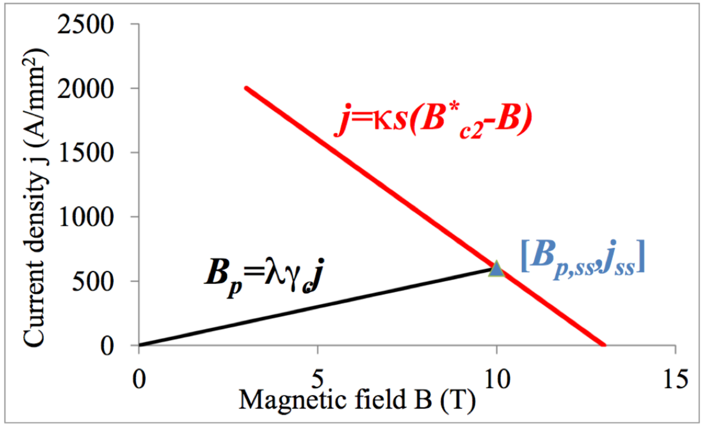
We can now relate field performance to conductor and coil size

- We characterize the coil by two parameters

$$B = \gamma_c J_E \qquad\qquad B_p = \lambda B = \lambda \gamma_c J_E$$

  - γ_c: how much field in the center is given per unit of current density
    - for a sector dipole or a cosθ , $\gamma_c \propto w$
  - λ: ratio between peak field and central field
    - for a cosθ dipole, λ=1
  - We can now compute what is the highest peak field that can be reached in the dipole in the case of a linear critical surface

$$B_p^{ss} = \lambda \gamma_c J_E \implies B_p^{ss} = \frac{\lambda \gamma_c \kappa s}{1 + \lambda \gamma_c \kappa s} B_{c2}^*$$



This model allows a closed-form estimate of short-sample current and field

- We can now compute the maximum current density that can be tolerated by the superconductor (short sample limit)

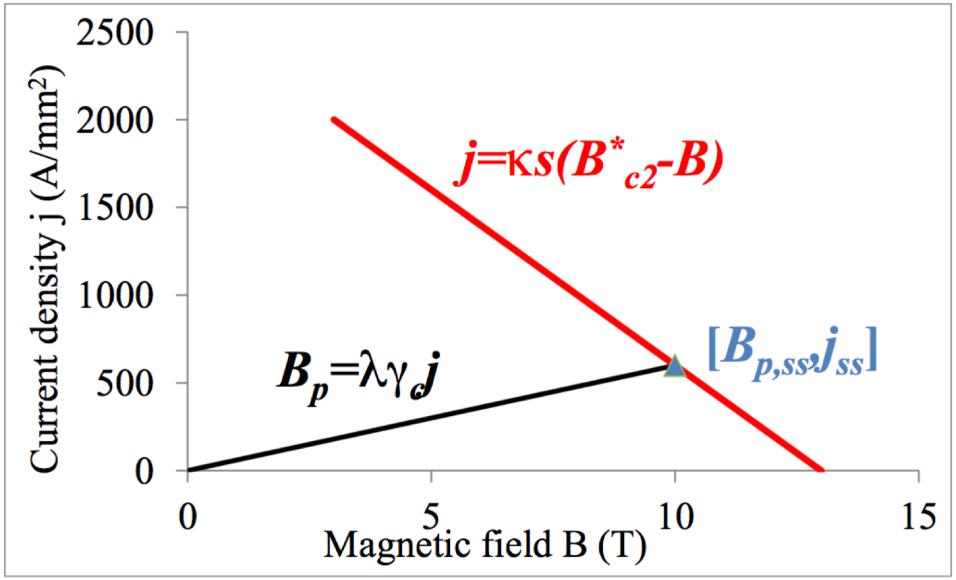
$$B_p^{ss} = \frac{\lambda \gamma_c \kappa s}{1 + \lambda \gamma_c \kappa s} B_{c2}^* \qquad B = \gamma_c J_E \qquad B_p = \lambda B = \lambda \gamma_c J_E$$

the short sample current is

$$J_E^{ss} = \frac{\kappa s}{1 + \lambda \gamma_c \kappa s} B_{c2}^*$$

and the bore short sample field is

$$B_{ss} = \frac{\gamma_c \kappa s}{1 + \lambda \gamma_c \kappa s} B_{c2}^*$$



We can now start to compare with magnets that have been built

- Limit of "large" coils:

$$B_{ss} = \frac{\gamma_c \kappa s}{1 + \lambda \gamma_c \kappa s} B_{c2}^* \xrightarrow{\gamma_c \kappa s} \frac{B_{c2}^*}{\lambda}$$

- Examples
  - The quantity $\lambda\gamma_c\kappa s$ is larger than 1 in the six analyzed dipoles, and is 4-5 for dipoles with large coil widths (SSC, LHC, Fresca)
    - This means that for SSC, LHC, Fresca we are rather close to the maximum field we can get with Nb-Ti

Magnet	κ (adim)	s (A/T/m^2)	λ (adim)	γ_c (T m^2/A)	$\lambda\gamma_c\kappa s$ (adim)
Tevatron MB	0.232	6.0E+08	1.13	1.23E-08	1.9
HERA MB	0.262	6.0E+08	1.08	1.64E-08	2.8
SSC MB	0.298	6.0E+08	1.05	2.14E-08	4.0
RHIC MB	0.226	6.0E+08	1.18	9.54E-09	1.5
LHC MB	0.286	6.0E+08	1.03	2.38E-08	4.2
FRESCA	0.293	6.0E+08	1.05	2.94E-08	5.4

Take a breather…

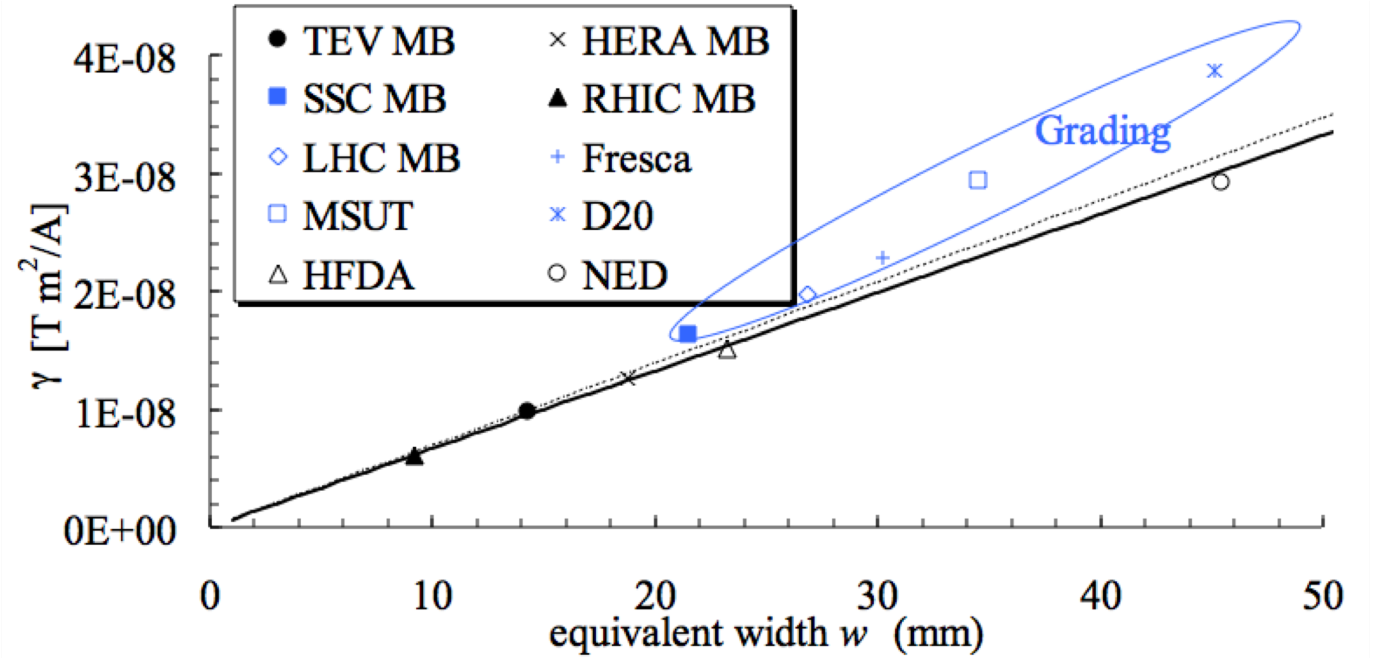
- We got an equation giving the field reachable for a dipole with a superconductor having a linear critical surface

$$B_{ss} = \frac{\gamma_c \kappa s}{1 + \lambda \gamma_c \kappa s} B^*_{c2}$$

- The plan: try to find an estimate for the two parameters γ_c and λ which characterize the lay-out
  - We want to have their dependence (even approximate) on the magnet aperture and on the thickness of the coil
  - This is what we are going to do in the next few slides

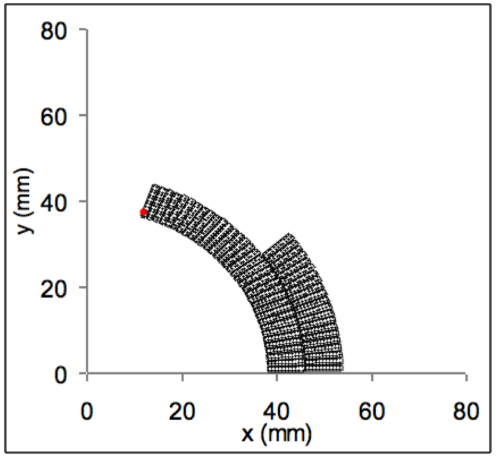
- What is γ_c (central field per unit of current density) ?
  - According to Biot Savart integration, central field per unit of current density is proportional to the coil thickness

$$\gamma_c = \gamma_{c0} w$$

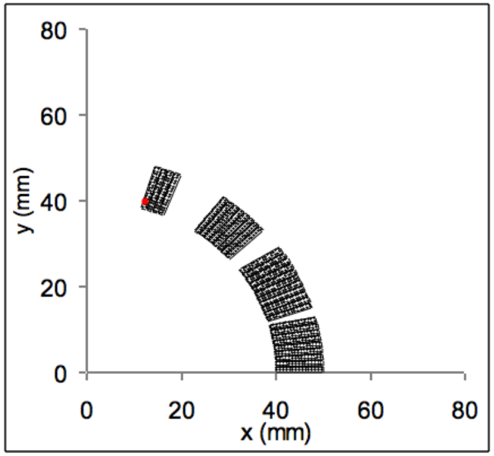


  - In most cases, magnet lay-out confirm this proportionality
  - The constant of the [0°-48°,60°-72°] (solid line) fits well the data
  - Some cases have 10-20% larger γ_c due to grading (see Unit 11)

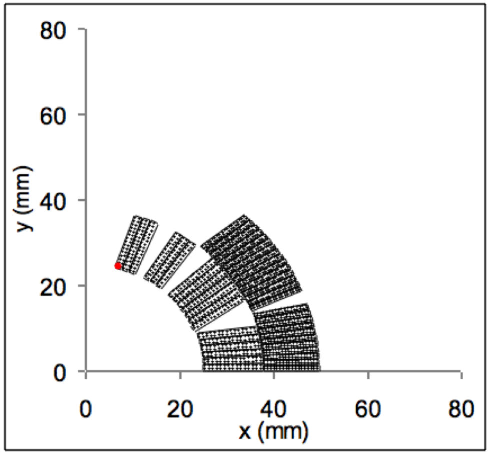
- What is λ (ratio between peak field and bore field)?
  - To compute the peak field one has to compute the field everywhere in the coil, and take the maximum
  - One can prove that if the current density is constant the maximum is always on the border of the coil – useful to reduce the computation time



Tevatron main dipole – location of the peak field



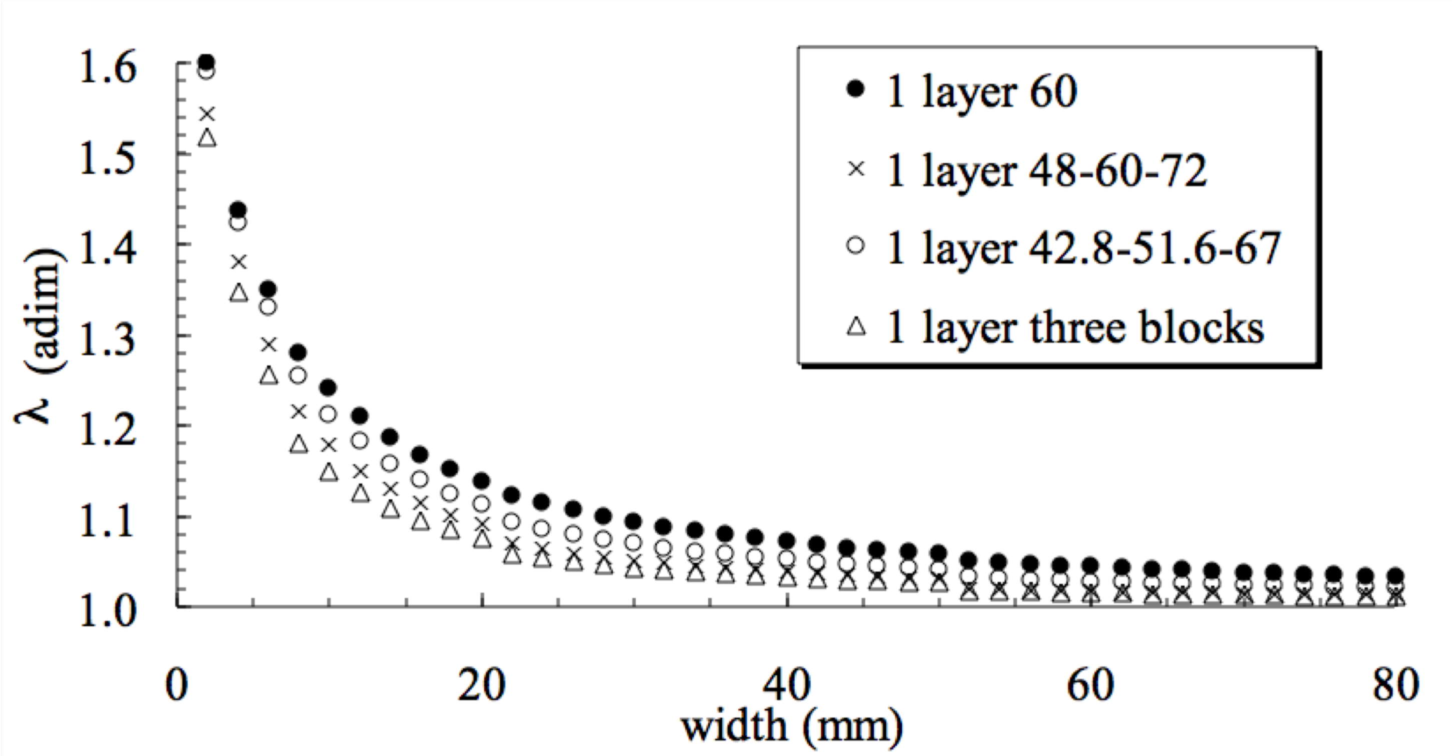
RHIC main dipole – location of the peak field



LHC main dipole – location of the peak field

As one might expect, more layers/sectors results in improved "efficiency"

- Numerical evaluation of λ for different sector coils

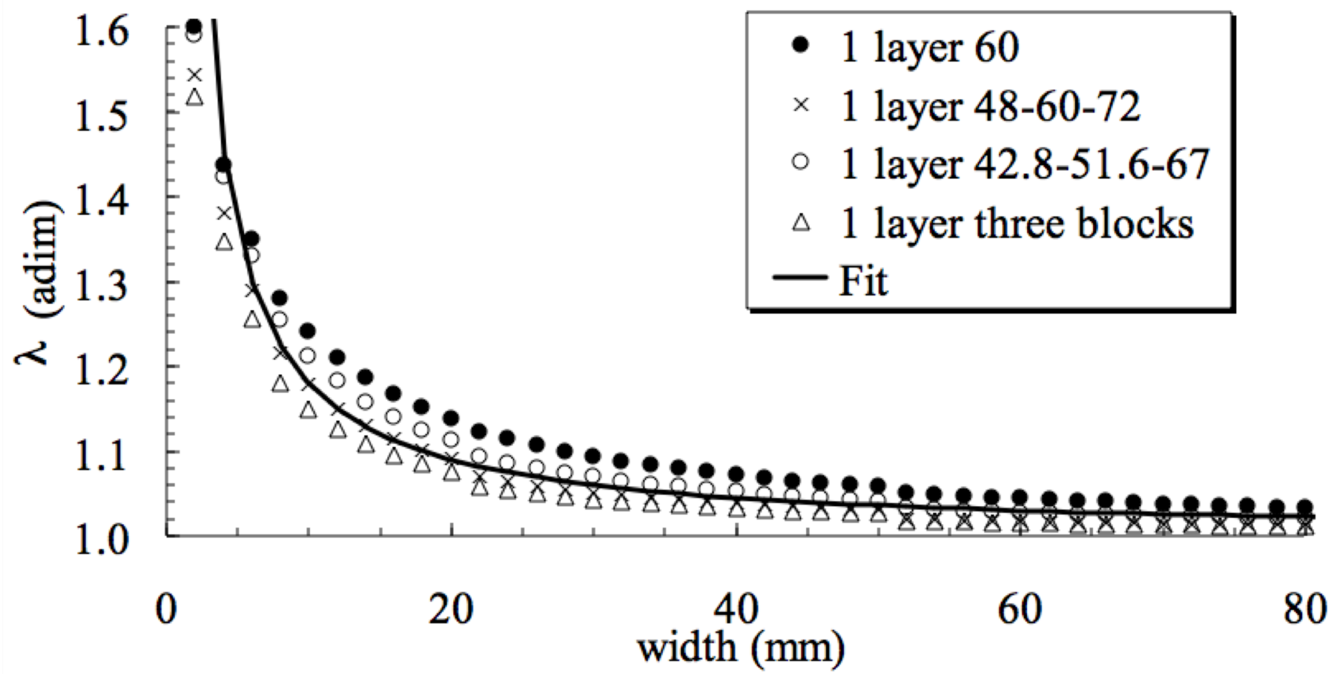


- For large widths, $\lambda \to 1$
- This means that for very large widths we can reach B^*_{c2}!

$$B_{ss} \approx \frac{B^*_{c2}}{\lambda}$$

But width plays a dominant role in reducing λ

- Numerical evaluation of λ for different sector coils



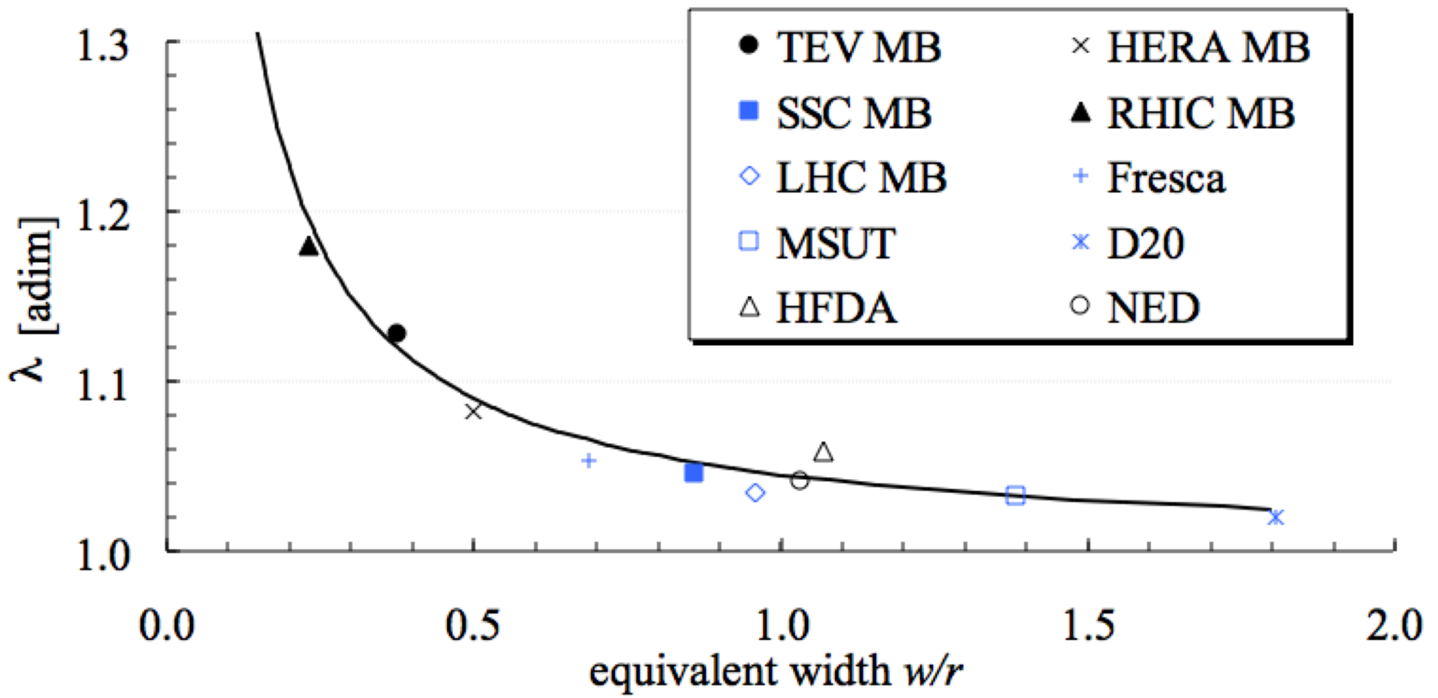
- For interesting widths (10 to 30 mm) λ is 1.05 - 1.15
  - This simply means that peak field 5-15% lager
- The cos(θ) approximately having λ=1 is not so bad for *w*>20 mm
- Typical hyperbolic fit with *a*~0.045

$$\lambda(w,r) \sim 1 + \frac{ar}{w}$$

And the model compares well with real examples

- Examples of λ (ratio between peak field and central field)
  - We now compute this parameter for built magnets
  - Agreement with the hyperbolic fit is very good (within 2% in the analyzed cases)

$$\lambda(w,r) \sim 1 + \frac{ar}{w}$$



Pulling it all together…
(and remembering this is for NbTi only!)

- We now can write the short sample field for a sector coil as a function of
  - Material parameters c, B^*_{c2}
  - Cable parameter κ
  - Aperture r and coil width w

$$B_{ss} = \frac{\gamma_c \kappa s}{1 + \lambda \gamma_c \kappa s} B^*_{c2} \qquad \lambda(w, r) \sim 1 + \frac{ar}{w}$$

$$\gamma_c \sim \gamma_{c0} w$$

a=0.045 γ_{c0}=6.63×10^{-7} [Tm/A]

for Nb-Ti s~6.0×10^8 [A/(T m^2)] and B^*_{c2}~10 T at 4.2 K or 13 T at 1.9 K

$$B_{ss} \sim \frac{\gamma_{c0} w \kappa s}{1 + \left(1 + \frac{ar}{w}\right) \gamma_{c0} w \kappa s} B^*_{c2}$$

- Cosθ model:

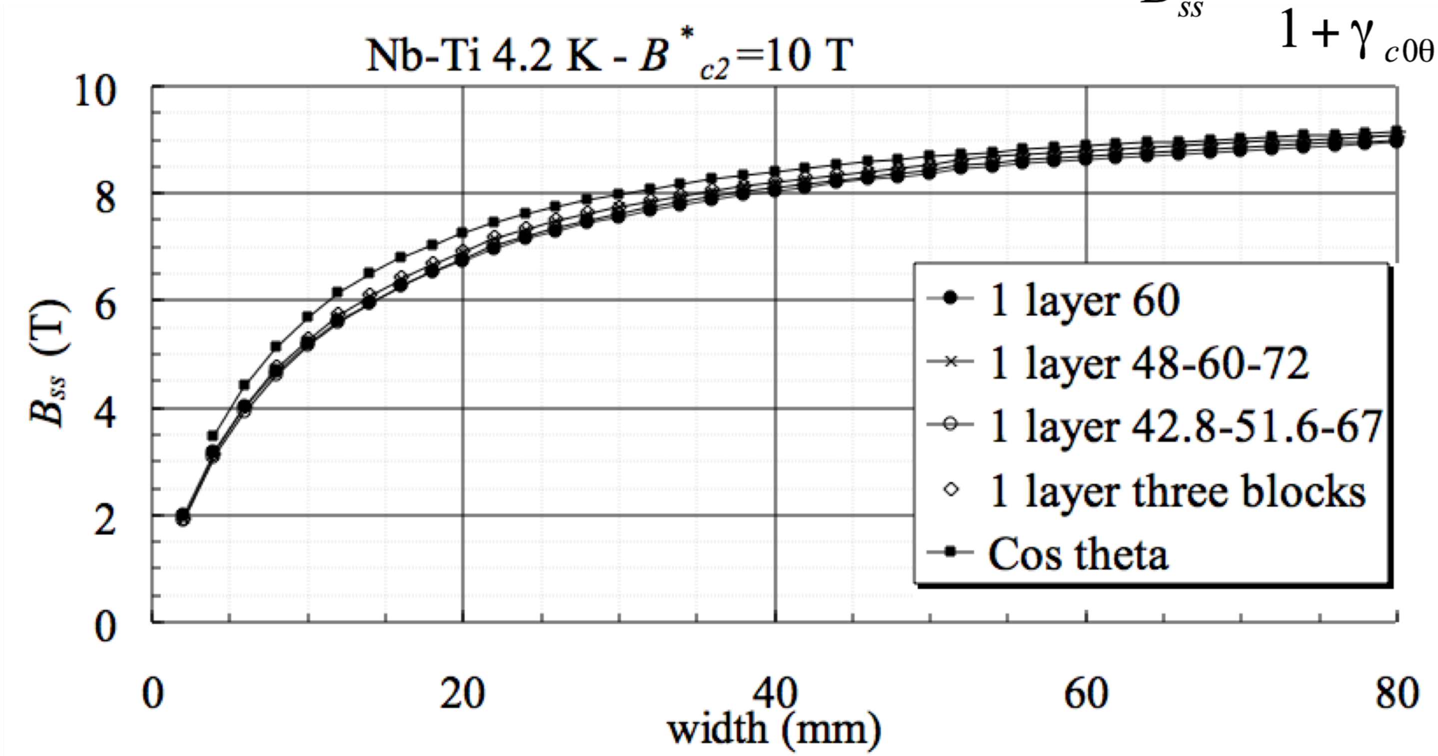
$\gamma_{c0\theta}$ =2π×10^{-7} [Tm/A]

$$B_{ss} \sim \frac{\gamma_{c0\theta} w \kappa s}{1 + \gamma_{c0\theta} w \kappa s} B^*_{c2}$$

The model shows the field vs coil width that can be obtained

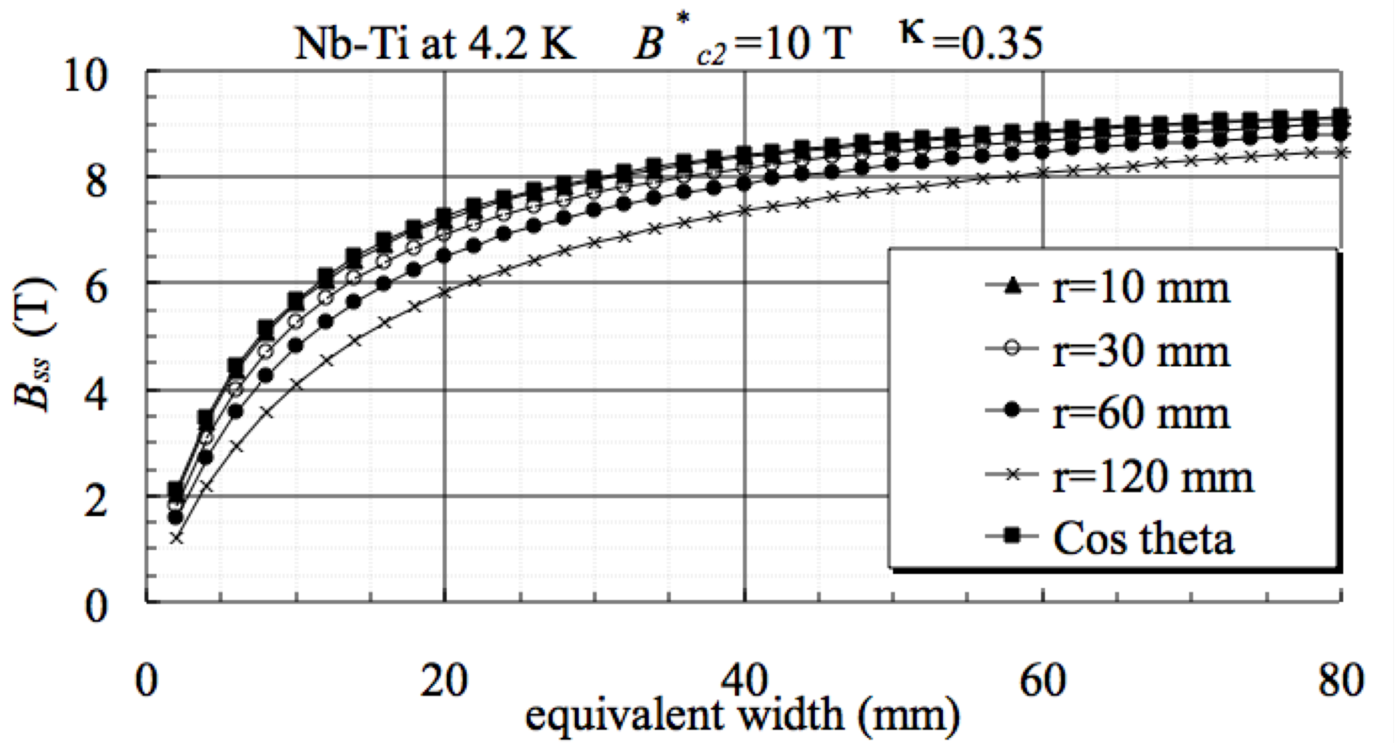
- Evaluation of short sample field in sector lay-outs and cosθ model for a given aperture (r=30 mm)
  - Tends asymptotically to B^*_{c2}, as $B^*_{c2}\, w/(1+w)$, for $w \to \infty$
  - Similar results for different position of wedges

$$B_{ss} \sim \frac{\gamma_{c0\theta}\, w\kappa s}{1+\gamma_{c0\theta}\, w\kappa s} B^*_{c2}$$



Probing the parameter space…

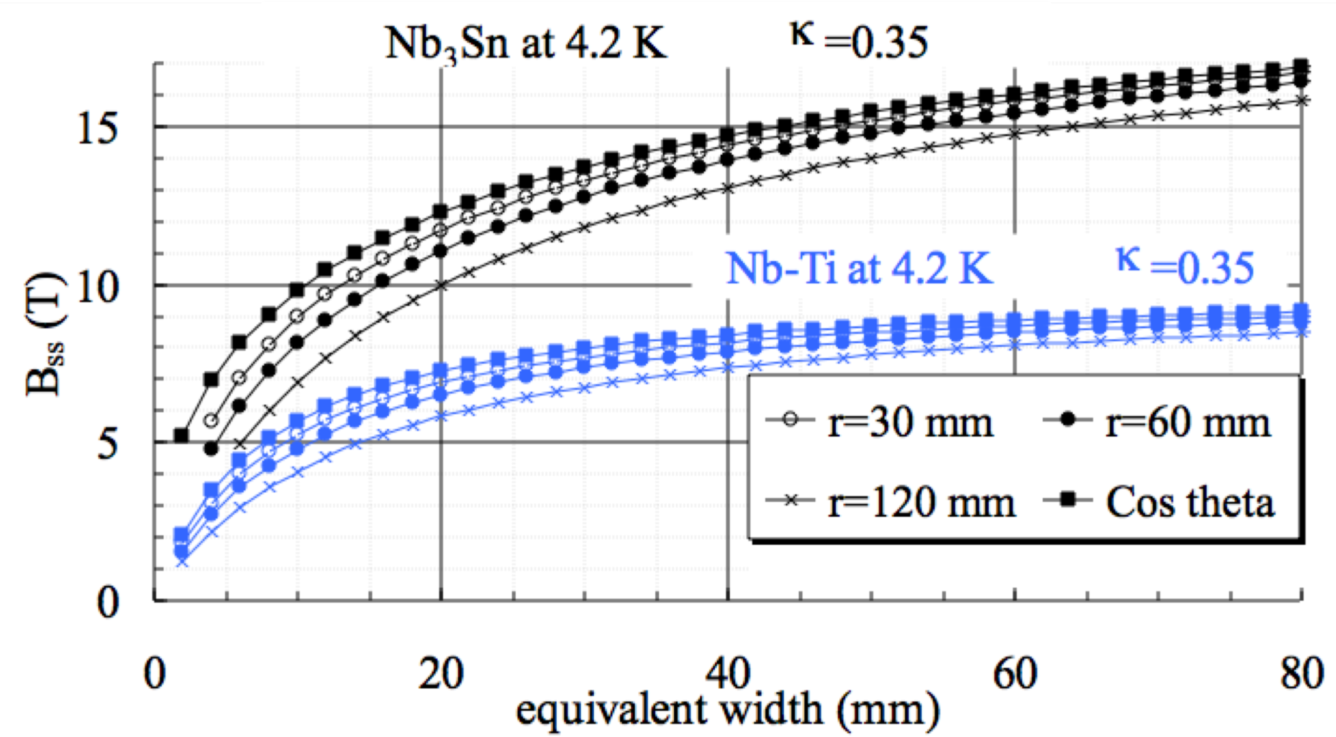
- Dependence on the aperture



  - For very large aperture magnets, one has less field for the same coil thickness
  - For small apertures it tends to the cosθ model

Application to Nb_3Sn can be done with some modifications to the model

- Case of Nb_3Sn



- The critical surface is not linear, but it can be solved with a similar approach
- The saturation for large widths is slower (due to different **s** and the shape of the surface)

Viewed another way…

(Caspi, ferracin, Gourlay 2005)

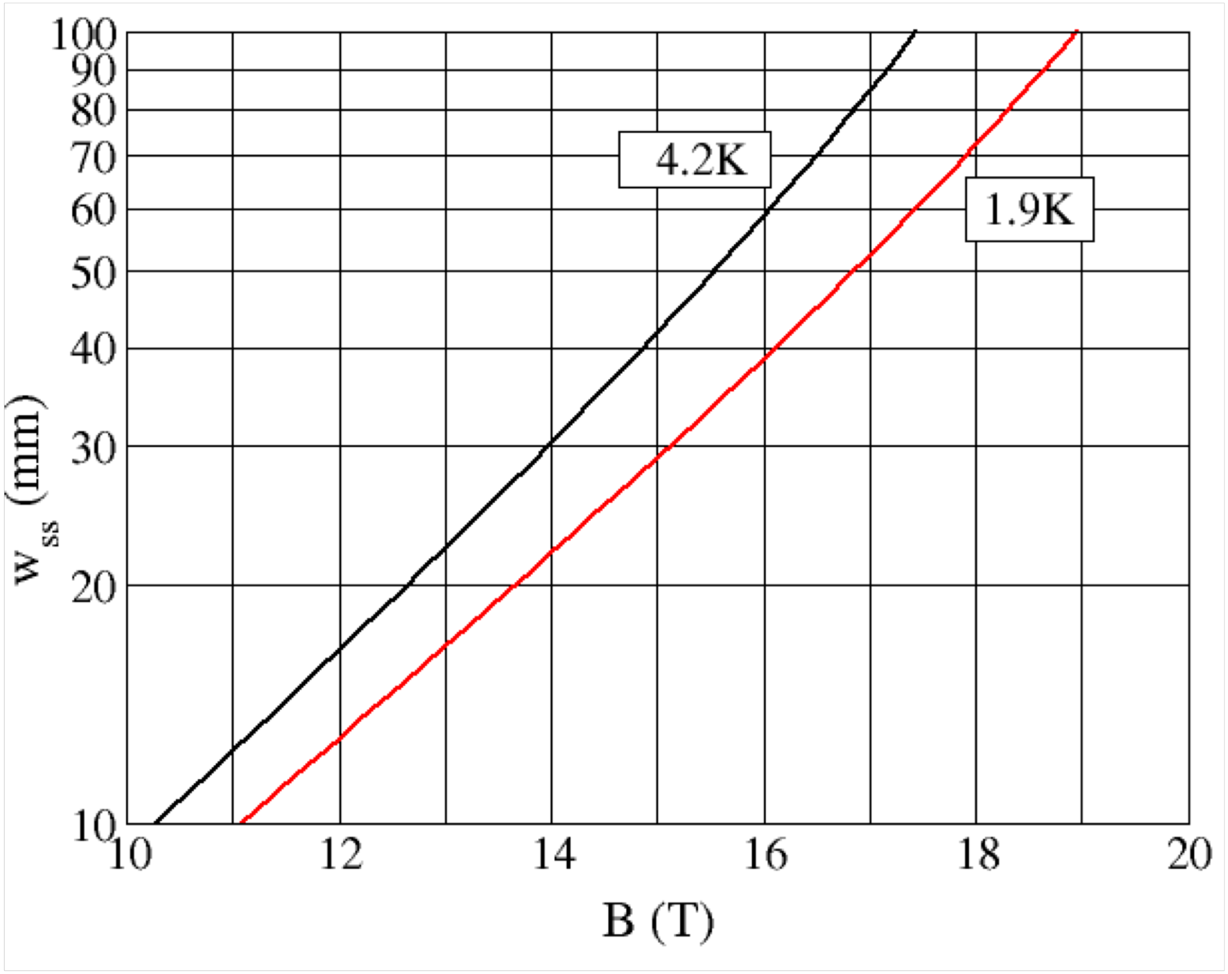
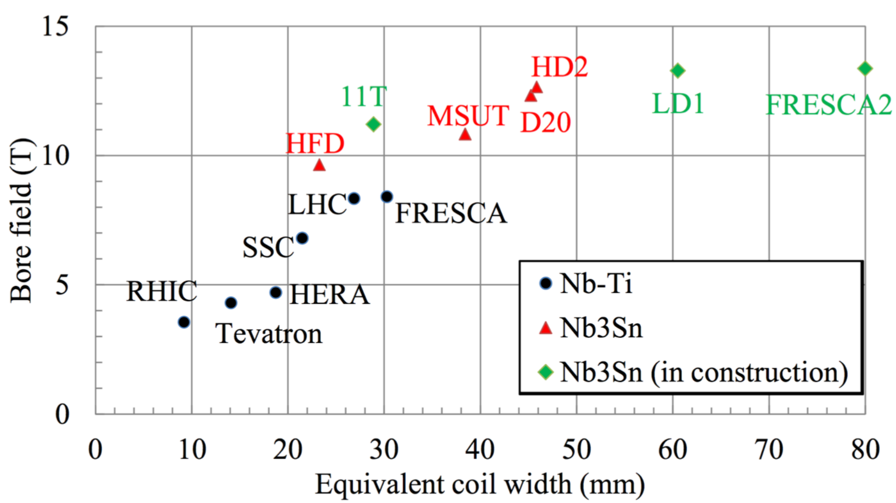


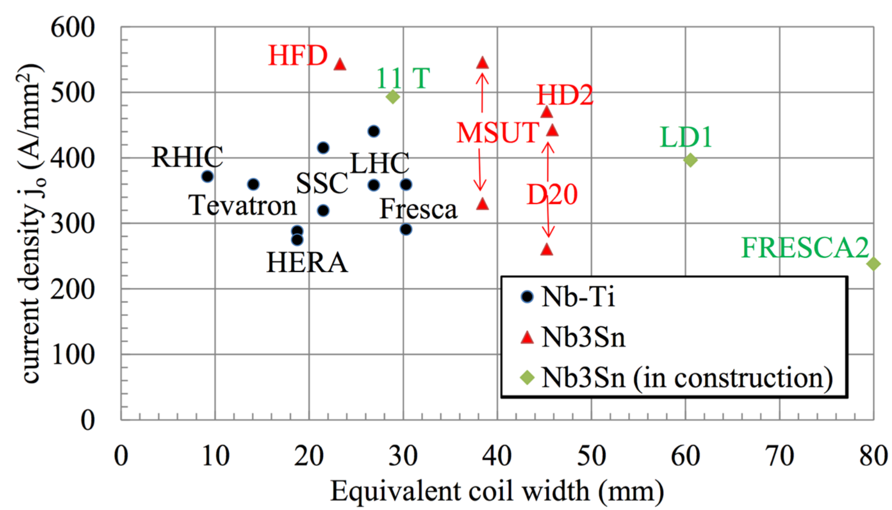
Fig. 3. Coil thickness of Nb_3Sn dipole magnets at short-sample.

Some actual coil performance values

- Approaching the limits of each material implies very large coil and lower current densities – not so effective
- Operational current densities are typically ranging between 300 and 600 A/mm^2



Operational bore field versus coil width
(80% of short sample at 1.9 K taken for models)



Operational overall current density versus coil width
(80% of short sample at 1.9 K taken for models)

We can apply the technique to quadrupoles too!

- The same approach can be used for a quadrupole
  - We define

$$\gamma_c \equiv \frac{G}{j} \qquad \lambda \equiv \frac{B_p}{rG}$$

the only difference is that now γ_c gives the gradient per unit of current density, and in B_p we multiply by r for having T and not T/m

  - We compute the quantities at the short sample limit for a material with a linear critical surface (as Nb-Ti)

$$B_{p,ss} = \frac{\lambda r \gamma_c \kappa s}{1+\lambda r \gamma_c \kappa s} B_{c2}^* \qquad j_{ss} = \frac{\kappa s}{1+\lambda r \gamma_c \kappa s} B_{c2}^* \qquad G_{ss} = \frac{\gamma_c \kappa s}{1+\lambda r \gamma_c \kappa s} B_{c2}^*$$

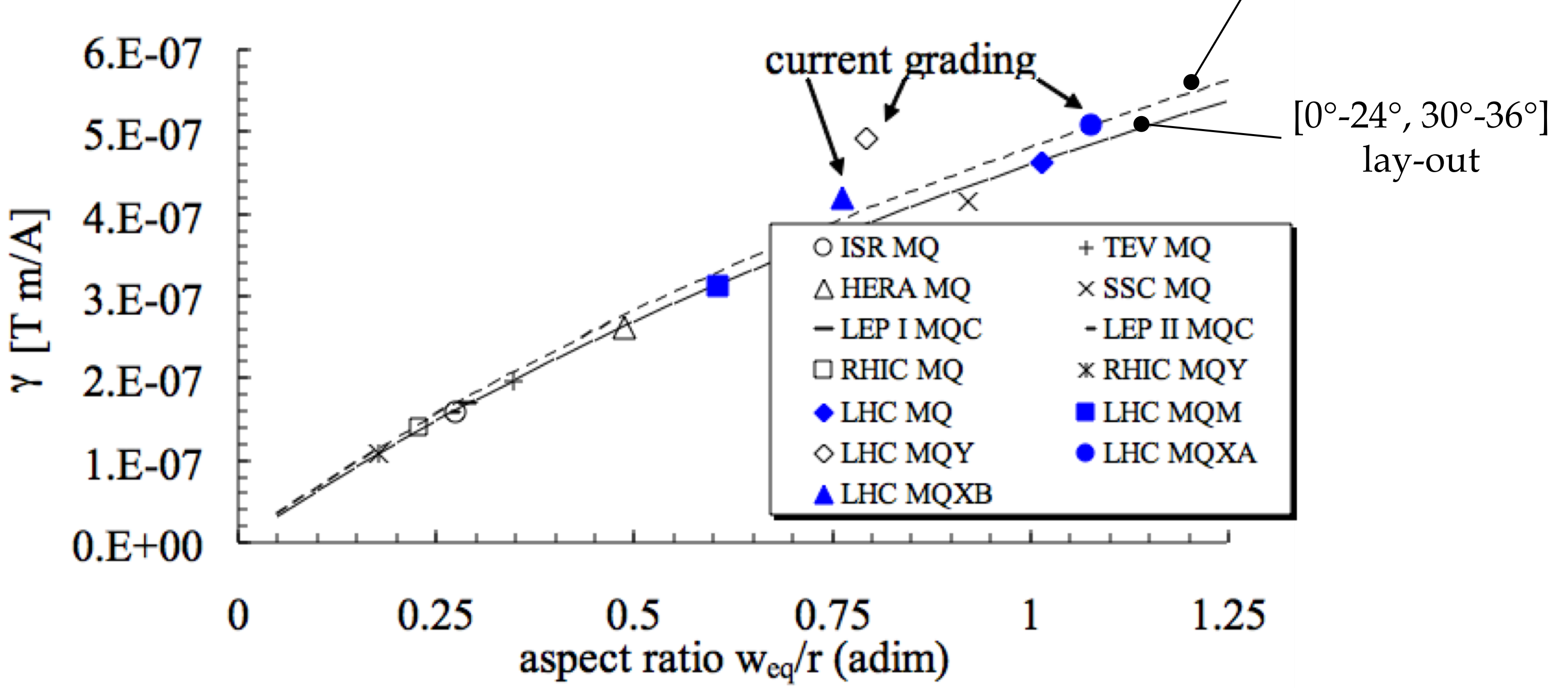
  - But note that γ is no longer proportional to w and no longer independent of r!

$$\gamma_c = \gamma_{c0} \ln\left(1+\frac{w}{r}\right)$$

We see good correlation with real magnets again

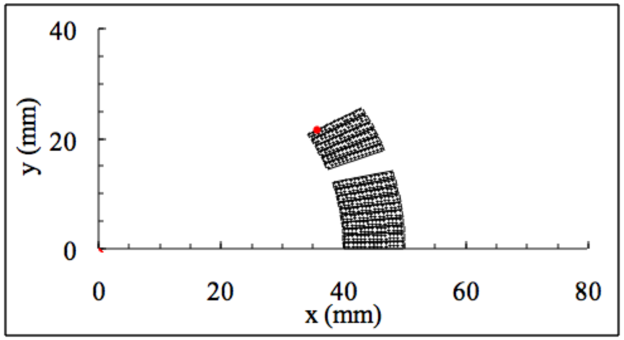
- Please note that γ is not any more proportional to w and independent of r !

$$\gamma_c = \gamma_{c0} \ln\left(1 + \frac{w}{r}\right)$$

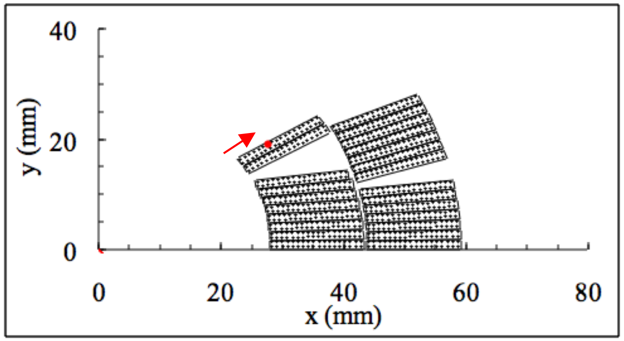


- The above equation fits very well the data relative to actual magnets built in the past years …

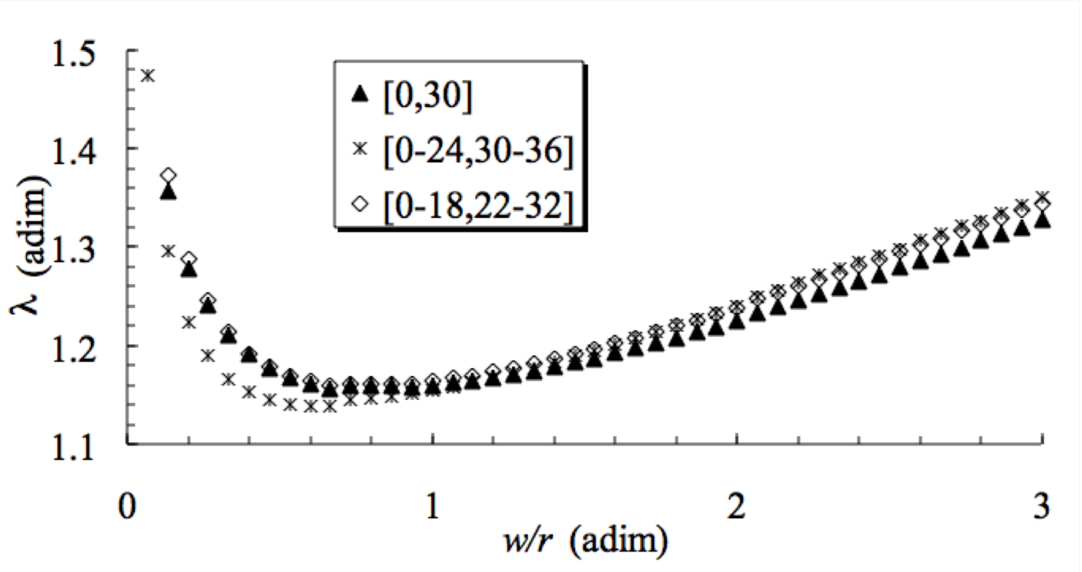
- The ratio λ is defined as ratio between peak field and gradient times aperture (central field is zero …)
  - Numerically, one finds that for large coils λ→∞
  - Peak field is "going outside" for large widths



RHIC main quadrupole

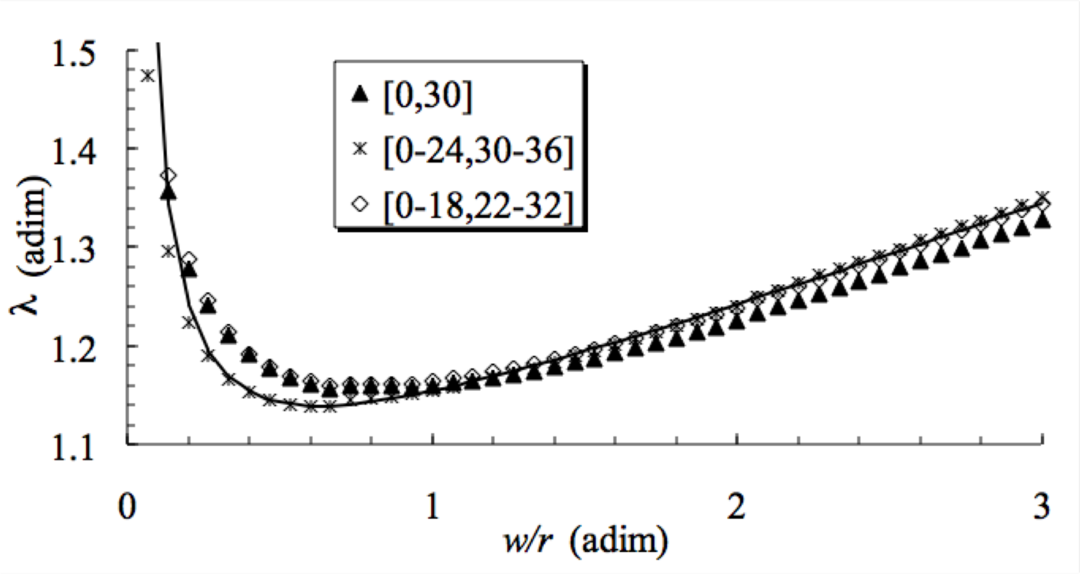


LHC main quadrupole



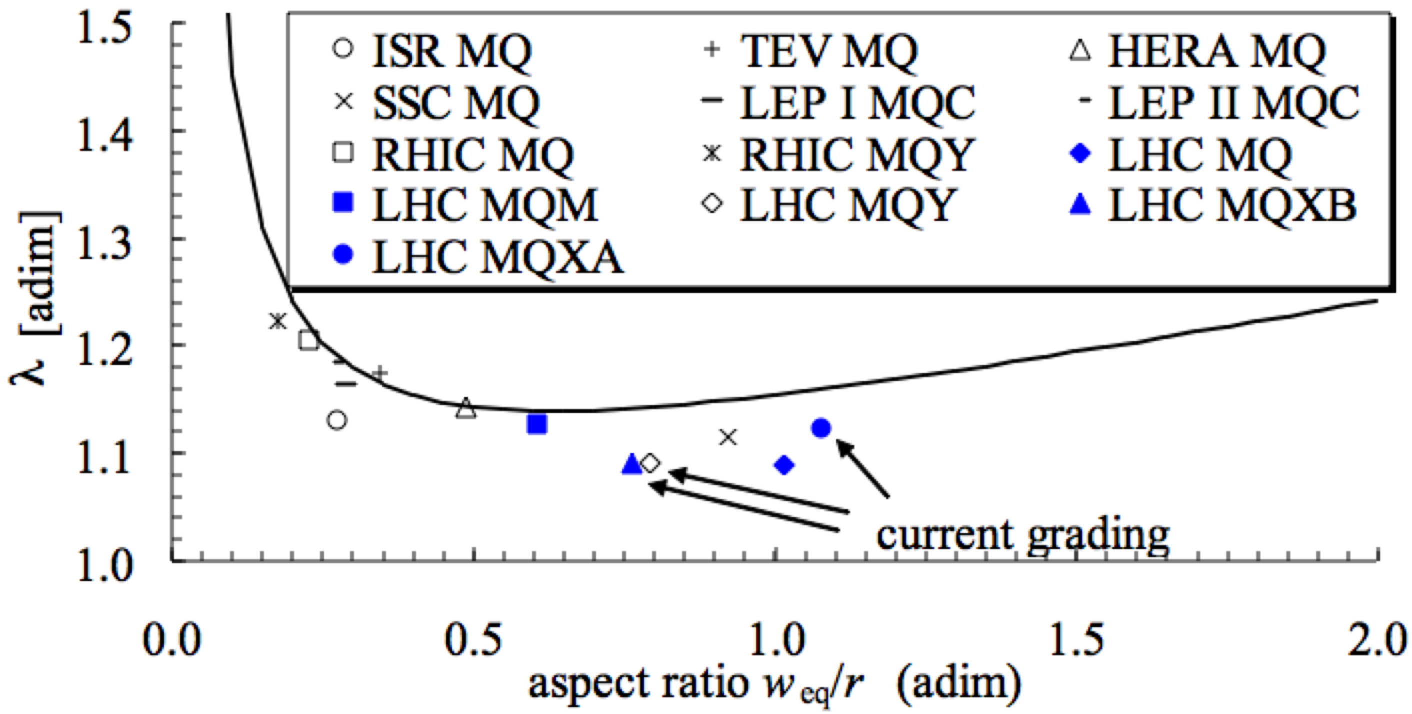
- The ratio λ is defined as ratio between peak field and gradient times aperture (central field is zero …)
  - The ratio depends on *w/r*
  - A good fit is $\lambda(w,r) = a_{-1}\frac{r}{w} + 1 + a_1\frac{w}{r}$

    a_{-1}~0.04 and a_1~0.11 for the [0°-24°,30°-36°] coil
  - A reasonable approximation is $\lambda \sim \lambda_0 = 1.15$ for ¼<*w/r*<1



The correlation is good, but some designs provided higher "efficiency"

- Comparison for the ratio λ between the fit for the [0°-24°,30°-36°] coil and actual values



We now pull the model together to enable prediction of gradient as a function of key parameters

- We now can write the short sample gradient for a sector coil as a function of
  - Material parameters s, B^*_{c2} (linear case as Nb-Ti)
  - Cable parameters κ
  - Aperture r and coil width w

$$G_{ss} = \frac{\gamma_c \kappa s}{1 + \lambda r \gamma_c \kappa s} B^*_{c2}$$

$$\lambda(w,r) \sim a_{-1}\frac{r}{w} + 1 + a_1\frac{w}{r}$$

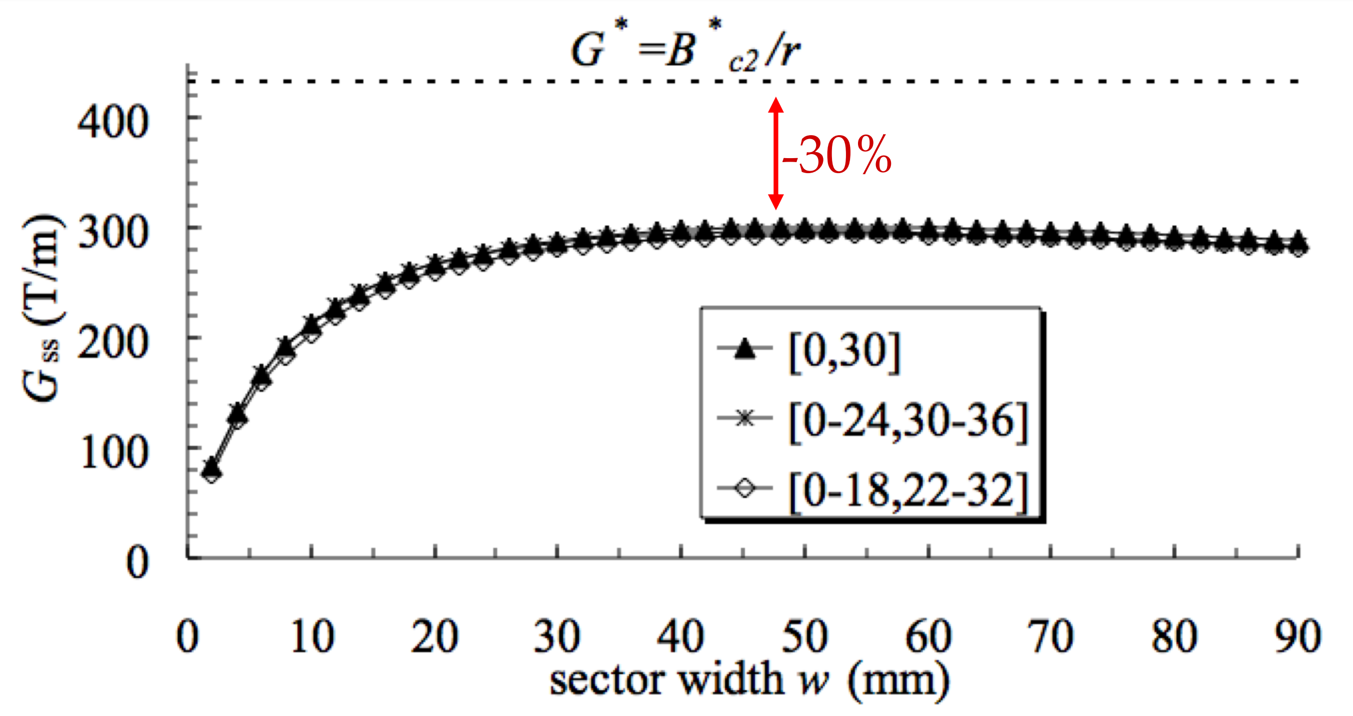
$$\gamma_c(w,r) = \gamma_{c0} \ln\left(1 + \frac{w}{r}\right)$$

$$G_{ss} = \frac{\gamma_c \kappa s}{1 + \lambda r \gamma_c \kappa s} B^*_{c2} = \frac{\gamma_{c0} \ln\left(1 + \frac{w}{r}\right)\kappa s}{1 + \left(a_{-1}\frac{r}{w} + 1 + a_1\frac{w}{r}\right) r \gamma_{c0} \ln\left(1 + \frac{w}{r}\right)\kappa s} B^*_{c2}$$

  - Relevant feature: for very large coil widths $w \to \infty$ the short sample gradient tends to zero !

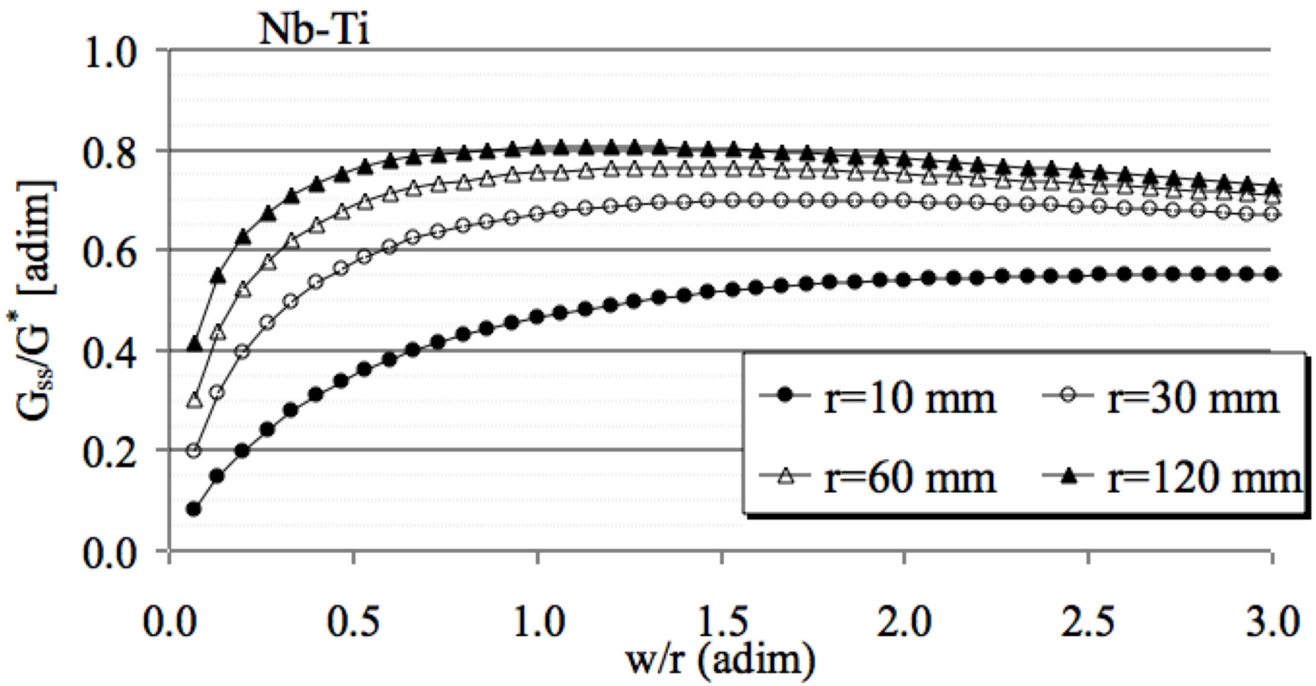
As one might expect, making coils bigger does not pay off for quadrupoles

- Evaluation of short sample gradient in several sector lay-outs for a given aperture (r=30 mm)



- No point in making coils larger than 30 mm!
- Max gradient is 300 T/m and not 13/0.03=433 T/m !! We lose 30% !!

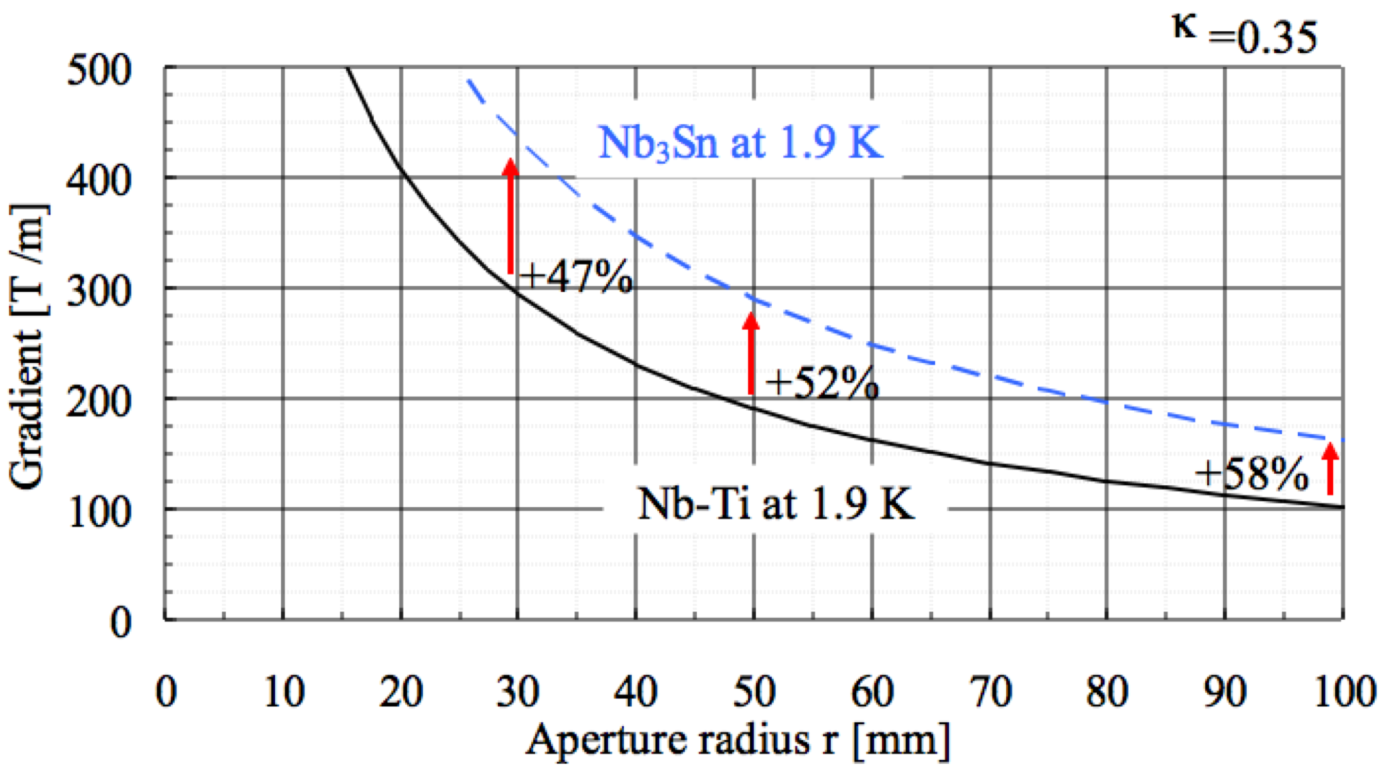
- Dependence of of short sample gradient on the aperture



  - Large aperture quadrupoles go closer to $G = B_{c2}/r$
  - Very small aperture quadrupoles do not exploit the sc !!
  - Large aperture need smaller ratio *w/r*
    - For r=30-100 mm, no need of having *w*>*r*

An estimate of the impact of switching from NbTi to Nb_3Sn for quadrupoles

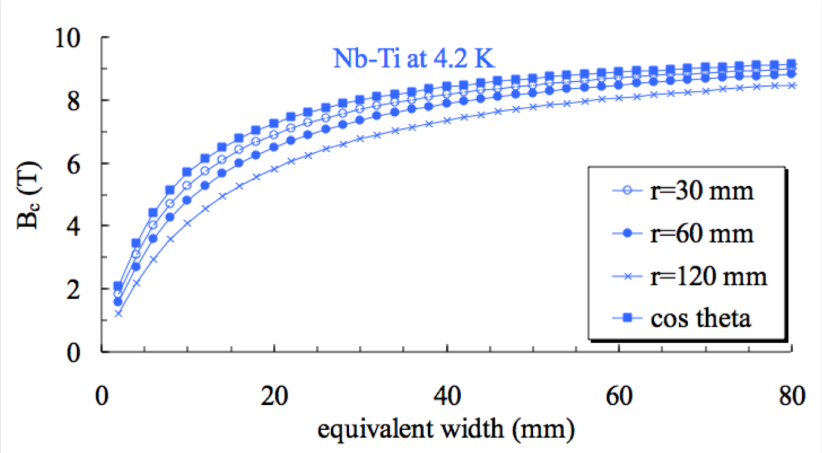
- Case of Nb_3Sn



  - Gain is ~50% in gradient for the same aperture (at 35 mm)
  - Gain is ~70% in aperture for the same gradient (at 200 T/m)

Summary comments

- Having an aperture
  - The technology gives the maximal field that can be reached
    - Nb-Ti: ~7-8 T at 4.2 K, ~10 T at 1.9 K (~80% of B^*_{c2})
    - Nb_3Sn: ~17-20 T ?
- Having an aperture and a field
  - One can evaluate the thickness of the coil needed to get the field using the equations for a sector coil
  - Cost optimization – higher fields costs more and more $$$ (or euro)



$$B_{ss} \sim \frac{\gamma_{c0} w \kappa s}{1 + \left(1 + \frac{a r}{w}\right) \gamma_{c0} w \kappa s} B^*_{c2}$$

REFERENCES

- Field quality constraints
  - M. N. Wilson, Ch. 1
  - P. Schmuser, Ch. 4
  - A. Asner, Ch. 9
  - Classes given by A. Devred at USPAS
- Electromagnetic design
  - S. Caspi, P. Ferracin, “Limits of Nb3Sn accelerator magnets“, *Particle Accelerator Conference* (2005) 107-11.
  - L. Rossi, E. Todesco, “Electromagnetic design of superconducting quadrupoles”, *Phys. Rev. ST Accel. Beams* **9** (2006) 102401.
  - Classes given by R. Gupta at USPAS 2006, Unit 3,4,5,6
  - S. Russenschuck, “Field computation for accelerator magnets”, J. Wiley & Sons (2010).

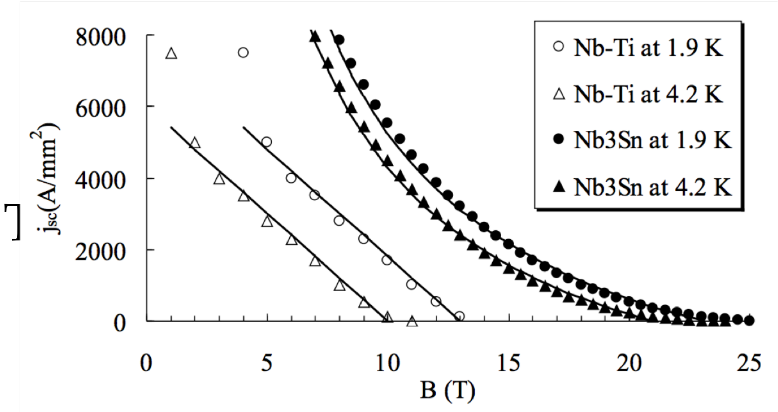
ACKNOWLEDGEMENTS

- B. Auchmann, L. Bottura, A. Den Ouden, A. Devred, P. Ferracin, V. Kashikin, A. McInturff, T. Nakamoto,, S. Russenschuck, T. Taylor, S. Zlobin, for kindly providing magnet designs … and perhaps others I forgot
- S. Caspi, L. Rossi for discussing magnet design, grading, and other interesting subjects …

Case of Nb_3Sn – an explicit expression

- An analytical expression can be found using a hyperbolic fit

$$j_c(B) = \kappa s\left(\frac{b}{B} - 1\right)$$



that agrees well between 11 and 17 T
with s~4.0×10^9 [A/(T m^2)]
and b~21 T at 4.2 K, b~23 T at 1.9 K

- Using this fit one can find explicit expression for the short sample field

$$B_{ss} = \frac{\kappa s \gamma_c}{2}\left(\sqrt{\frac{4b}{\lambda \kappa s \gamma_c} + 1} - 1\right)$$

and the constant γ_c λ are the same as before (they depend on the lay-out, not on the material)