Unit 7
AC losses in Superconductors

Soren Prestemon and **Steve Gourlay**
Lawrence Berkeley National Laboratory (LBNL)

Scope of the Lesson

- AC losses – general classification
  1. Hysteresis losses
  2. Coupling and eddy current losses
  3. Self-field losses
  - Role of transport current in loss terms
  - Impact of AC losses on cryogenics
  - Specifying conductors based on the application

Following closely the presentation of Wilson "Superconducting magnets"

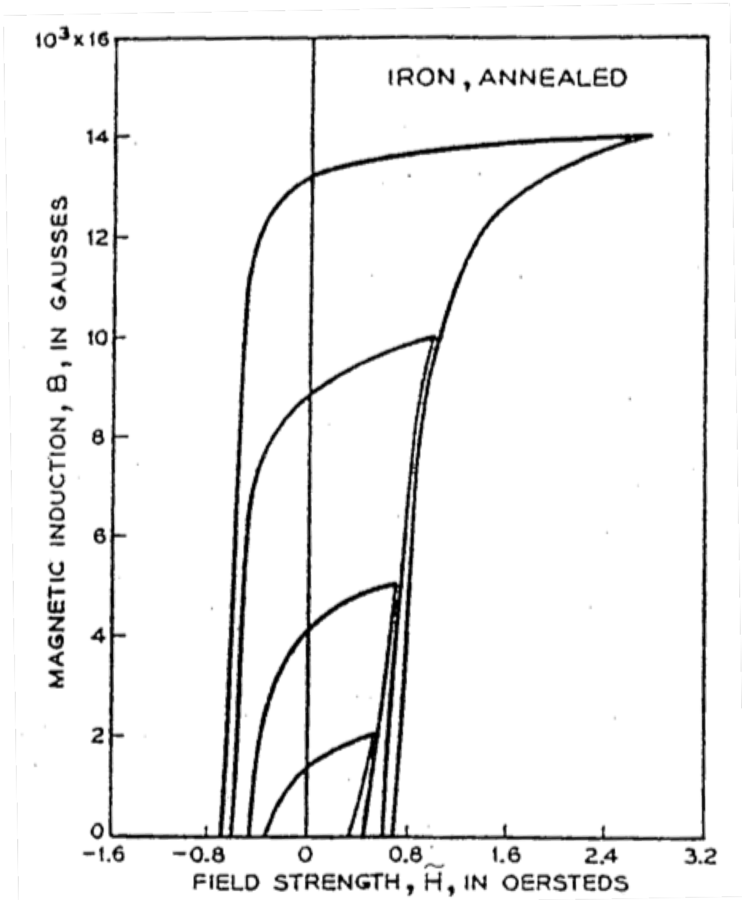
Also thanks to:
Mess, Schmueser, Wolff, "Superconducting Accelerator Magnets"
Marijn Oomen Thesis "AC Loss in Superconducting Tapes and Cables"
M.N. Wilson / Cryogenics 48 (2008) 381–395
T. M. Mower and Y. Iwasa, Cryogenics, vol. 26, no. 5, pp. 281–292, May 1986.

Introduction

- Superconductors subjected to varying magnetic fields see multiple heat sources that can impact conductor performance and stability
- All of the energy loss terms can be understood as emanating from the voltage induced in the conductor:
  - The hysteretic nature of magnetization in type II superconductors, i.e. flux flow combined with flux pinning, results in a net energy loss when subjected to a field cycle
  - The combination of individual superconducting filaments and a separating normal-metal matrix results in a coupling Joule loss
  - Similarly, the normal-metal stabilizer sees traditional eddy currents

Magnetization losses

- The superconductor B-H cycle defines losses associated with magnetization: the area enclosed in a loop is lost as heat



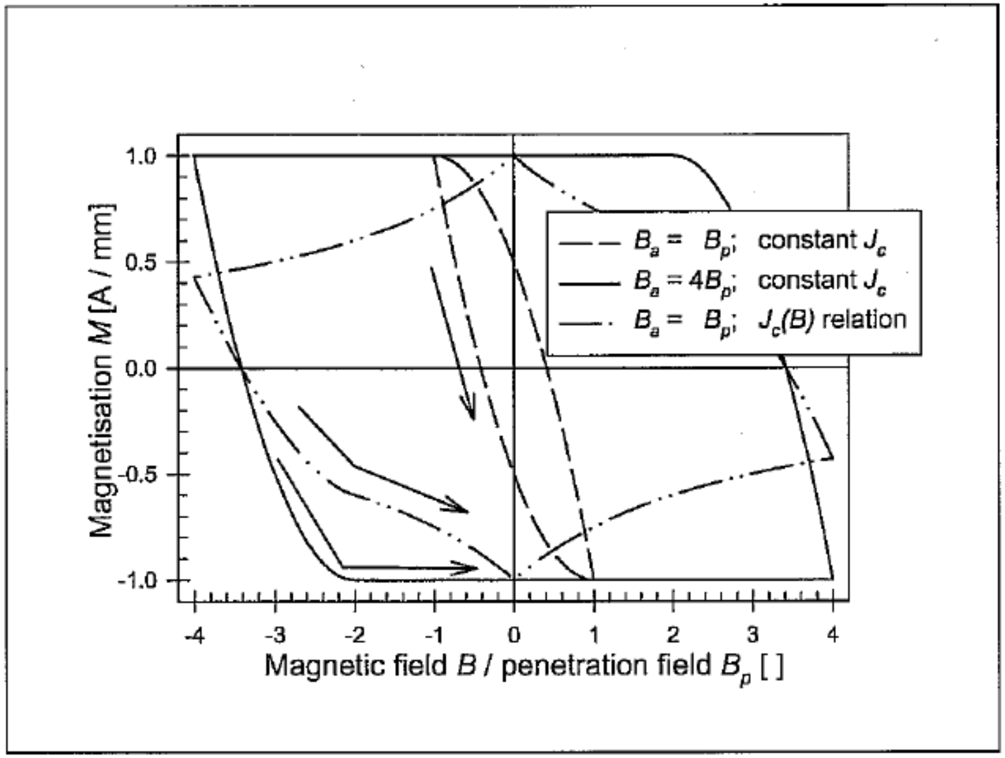


Figure 2.3 Magnetisation loops calculated for an infinite slab parallel to the magnetic field.

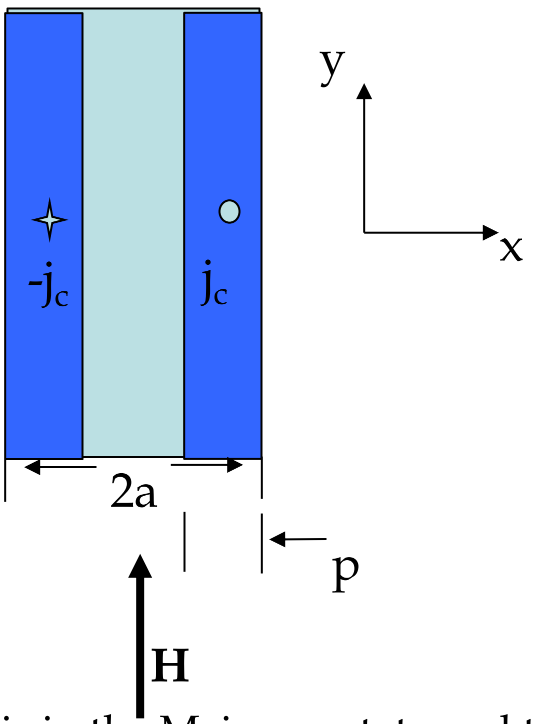
Hysteresis losses – basic model

Hysteresis loss is $Q = \int \vec{H} \cdot d\vec{M} = \int \vec{M} \cdot d\vec{H}$

Problem: how do we quantify this?
-Note that magnetic moment generated by a current loop *I* enclosing an area *A* is defined as

$$m = \mu_0 AI$$

$$\frac{\partial B_y}{\partial x} = \mu_0 J_c$$



The magnetization M is the sum of the magnetic moments/volume.
Assume $j=j_c$ in the region of flux penetration in the superconductor (Bean Model), then

$$\phi = \mu_0 \int_{a-p}^{a} j_c x dx$$

$$= \frac{\mu_0 j_c}{2}\left[2ap - p^2\right]$$

- Below H_{c1} the superconductor is in the Meissner state and the magnetization from dH/dt corresponds to pure energy storage, i.e. there is no energy lost in heat;
- Beyond H_{c1} flux pinning generates hysteretic B(H) behavior; the area enclosed by the B(H) curve through a dB/dt cycle represents thermal loss

Some basic definitions:

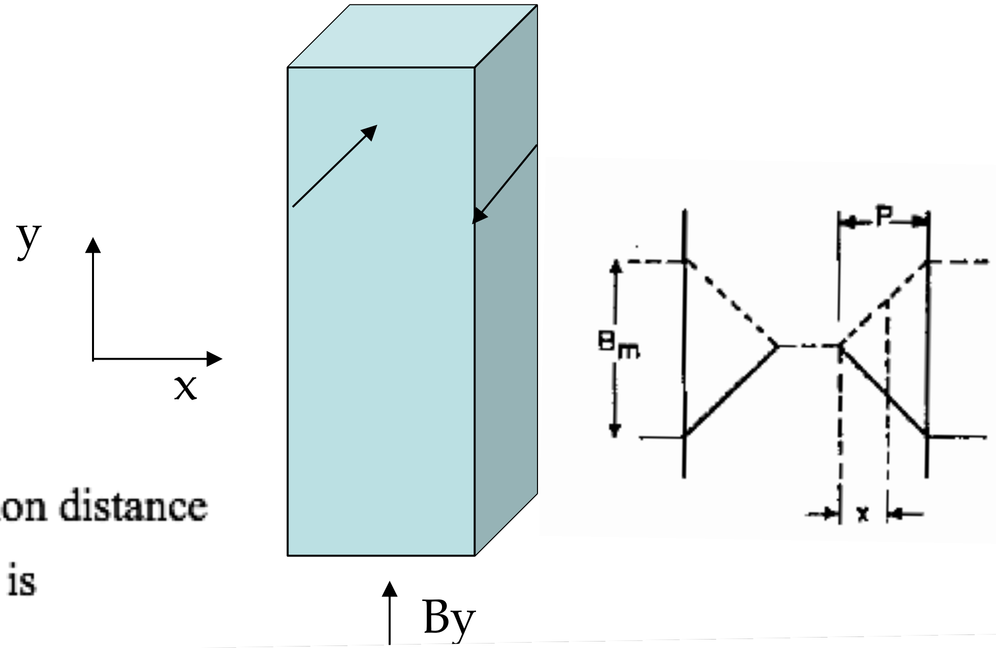
B_p = Penetration field (to center)

B_m = Field modulation

$B_m = 2\mu_0 J_c p$ for $p < a$, p is the field penetration distance

The power generated by the penetrating field is

$$P = E \cdot J_c = J_c \frac{\partial \phi}{\partial t}$$



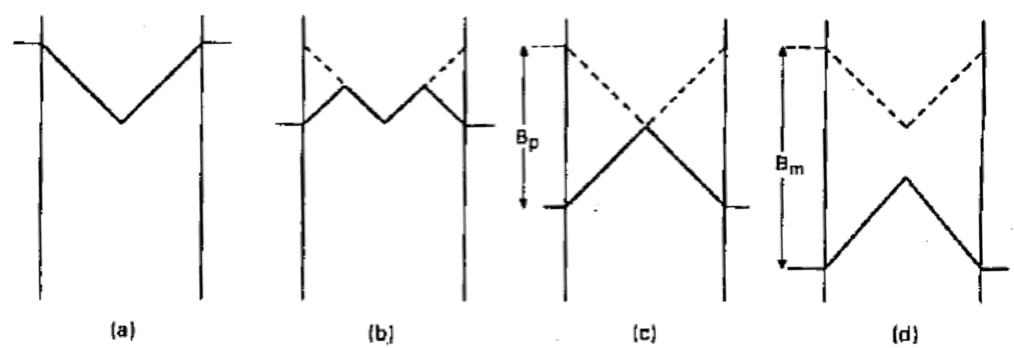


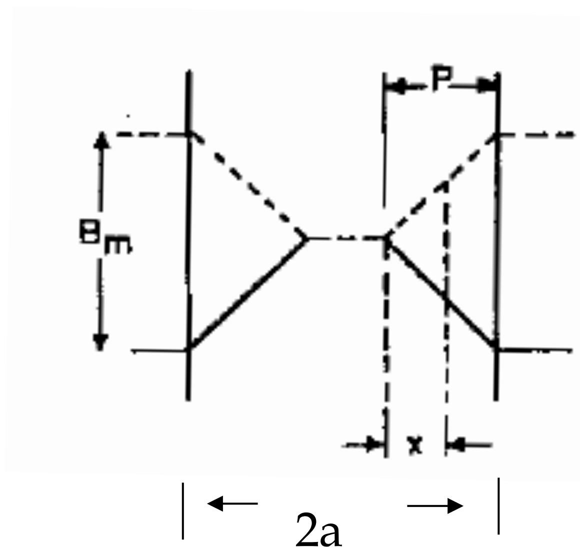
Fig. 8.4. (a) Field pattern within a superconducting slab subjected to large field change; (b) as the field is reduced; (c) when the field change penetrates to centre of slab; (d) when the field reaches a minimum value before rising again.

Calculating hysteresis losses

- The total heat generated for a half-cycle is then

$$\Delta\phi(x) = \int_0^x \Delta B(\varsigma)\, d\varsigma \simeq \int_0^x \mu_0 J_c \varsigma \, d\varsigma = \frac{\mu_0}{2} J_c x^2$$

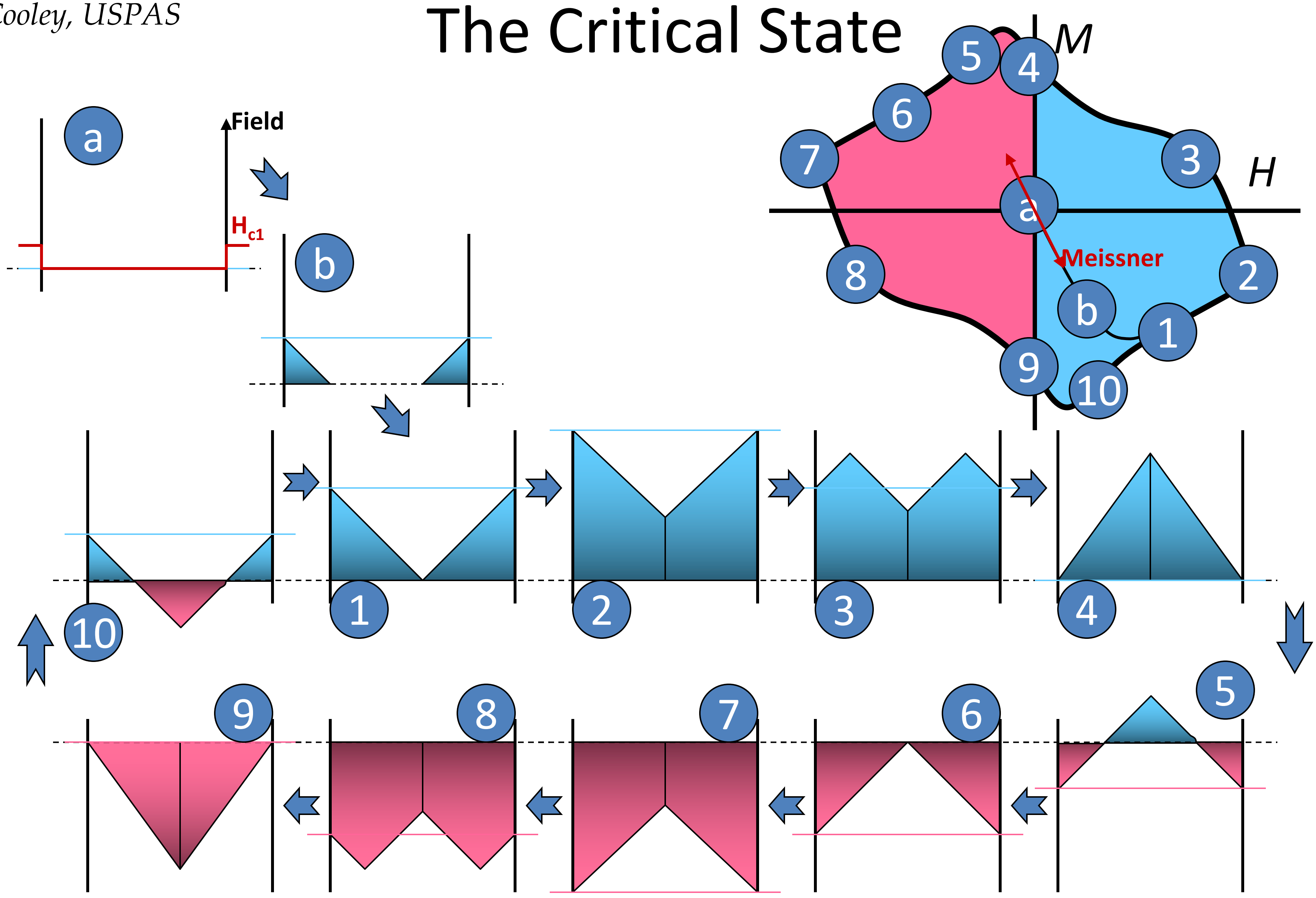
$$\Rightarrow q = \frac{1}{a}\int_0^p J_c\left(\mu_0 J_c x^2\right) dx = \frac{\mu_0 J_c^2 p^3}{3a}$$



- Note that this calculation assumed $p<a$; a similar analysis can be applied for the more generally case in which the sample is fully penetrated.

The Critical State

Slide taken from Lance Cooley, USPAS



Understanding AC losses via magnetization

- The screening currents are bound currents that correspond to sample magnetization.
    - Integration of the hysteresis loop quantifies the energy loss per cycle

=> Will result in the same loss as calculated using $E \cdot J_c$

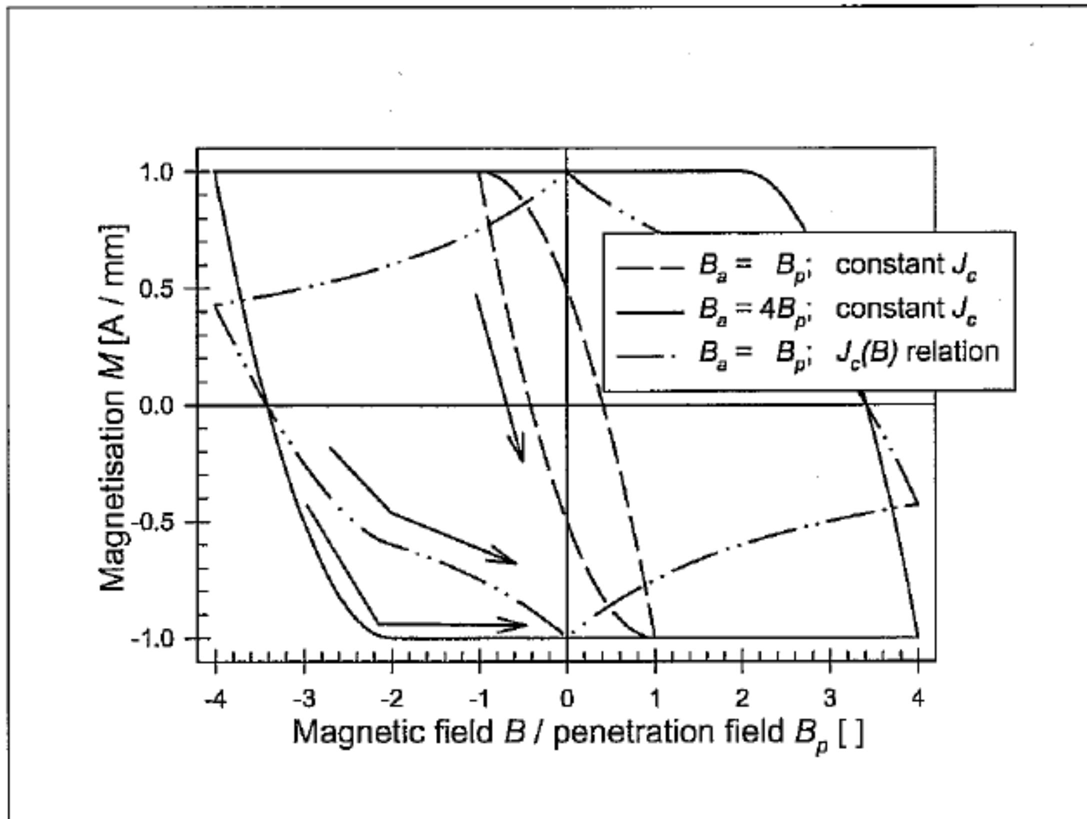


Figure 2.3 Magnetisation loops calculated for an infinite slab parallel to the magnetic field.

Hysteresis losses - general

- The hysteresis model can be developed in terms of:

$$\beta = \frac{B_m}{B_p} = \frac{B_m}{2a\mu_0 J_c}$$

The total cycle loss (for the whole slab) is then:

$$Q = \frac{B_m^{\ 2}}{2\mu_0}\Gamma(\beta);$$ The function Γ (geometry dependent) has a maximum near 1.

To reduce losses, we want β<<1 (little field penetration, so loss/volume is small) or β>>1 (full flux penetration, but little overall flux movement)

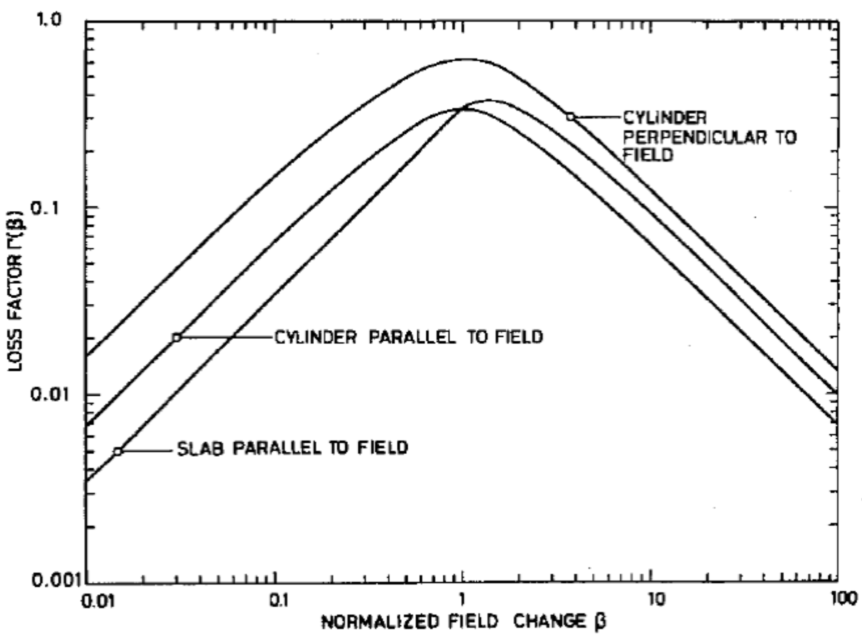


Fig. 8.5. Loss factor $\Gamma(\beta)$ for hysteresis loss per cycle in different shapes of superconductor.

Hysteresis losses

- The addition of transport current enhances the losses; this can be viewed as stemming from power supply voltage compensating the system inductance voltage generated by the varying background field.

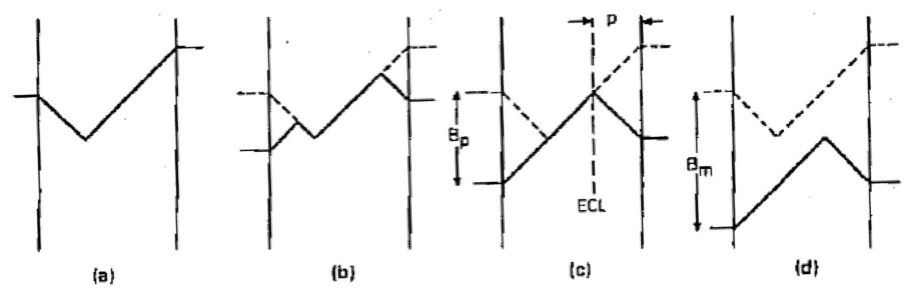


Fig. 8.11. (a) Slab carrying fixed transport current in external field; (b) as the field is reduced; (c) when the field change penetrates the entire slab; (d) when the field reaches minimum value before rising again.

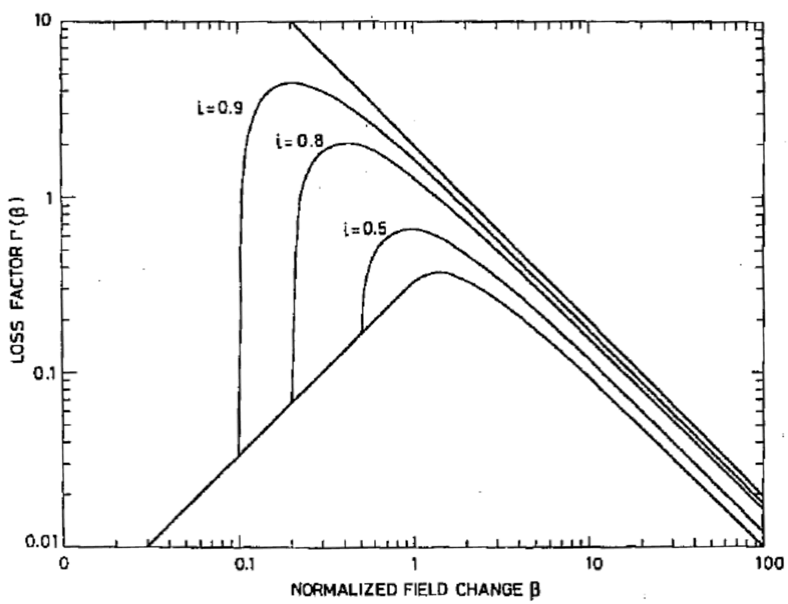
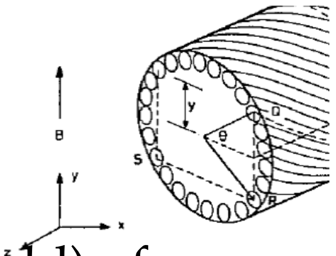


Fig. 8.12. Hysteresis loss in superconducting slab carrying fixed transport current $I_t = iI_c$ and subjected to a changing external field, calculated from eqns (8.28) and (8.7).

Coupling losses

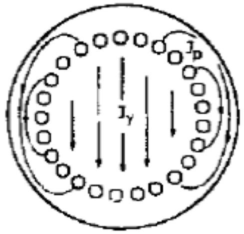
- A multifilamentary wire subjected to a transverse varying field will see an electric field generated between filaments of amplitude:

$$E = \frac{\dot{B}L}{2\pi};\ L \text{ is the twist-pitch of the filaments}$$



The metal matrix then sees a current (parallel to the applied field) of amplitude:

$$J = \frac{\dot{B}L}{2\pi\rho_t}$$



Similarly, the filaments couple via the periphery to yield a current:

$$J_p(\theta) = \frac{\dot{B}L\cos(\theta)}{2\pi\rho_m}$$

There are also eddy currents of amplitude:

$$J_e(\theta) = \frac{\dot{B}a\cos(\theta)}{\rho_m}$$

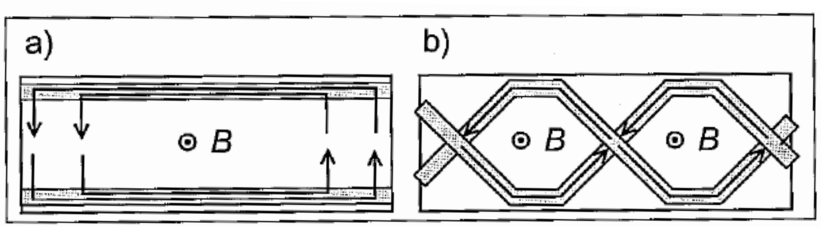


Figure 2.4 Schematic of coupling currents between two filaments in a wire or tape.

Coupling losses – time constant

- The combined Cos(θ) coupling current distribution leads to a natural time constant (coupling time constant):

$$\tau = \frac{\mu_0}{2\rho_{eff}}\left(\frac{L}{2\pi}\right)^2$$

- The time constant τ corresponds to the natural decay time of the eddy currents when the varying field becomes stationary.
- The losses associated with these currents (per unit volume) are:

$$Q_e = \frac{B_m^{\ 2}}{2\mu_0}\frac{8\tau}{T_m}, \text{ where } T_m \text{ is the half-time of a full cycle}$$

Here B_m is the maximum field during the cycle.

- Coupling currents also form between strands in cables

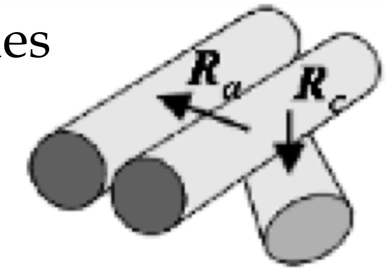


Fig. 18. Crossover resistance R_c and adjacent resistance R_a.

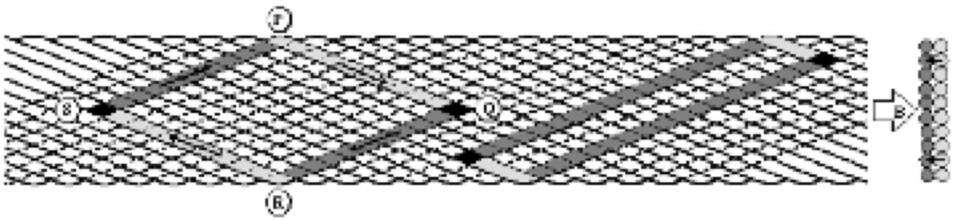
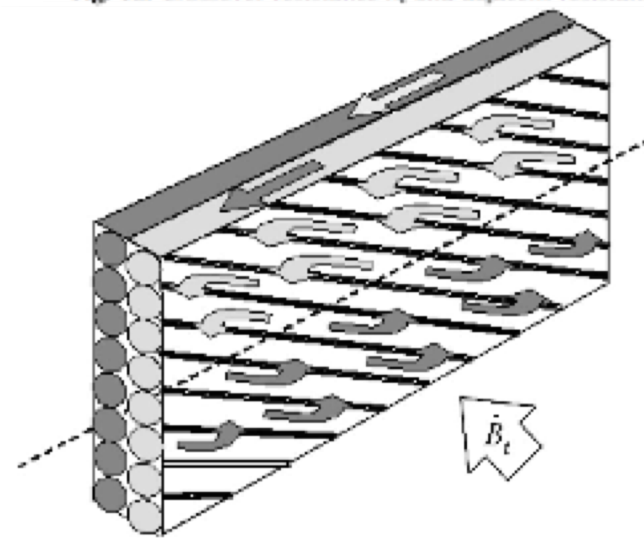
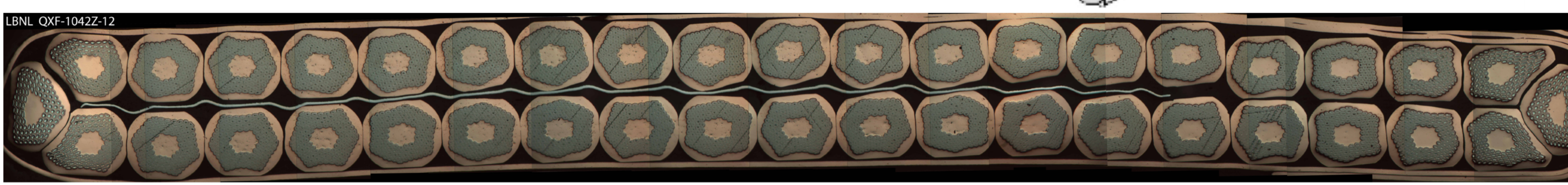


Fig. 19. Coupling currents flowing via crossover resistance R_c in transverse field (upper wires shown light grey).

$$\frac{Q_{tc}}{Q_{ta}} = \frac{R_a}{R_c}\,\frac{N(N-1)}{20}$$

Add core to dramatically reduce transverse coupling, while maintaining decent Ra for current sharing





Other loss terms

- In the previous analysis, we assumed the cos(θ) longitudinal current flowed on the outer filament shell of the conductor. Depending on dB/dt, ρ, and L, the outer filaments may saturate (i.e. reach Jc), resulting in a larger zone of field penetration. The field penetration results in an additional loss term:

$$Q_p = \frac{B_m^2}{2\mu_0} \frac{4\pi^2}{T_m^2} \Gamma(\beta')$$

$$\beta' = \frac{\pi B_m}{2\mu_0 \lambda J_c a} \frac{\tau}{T_m}$$

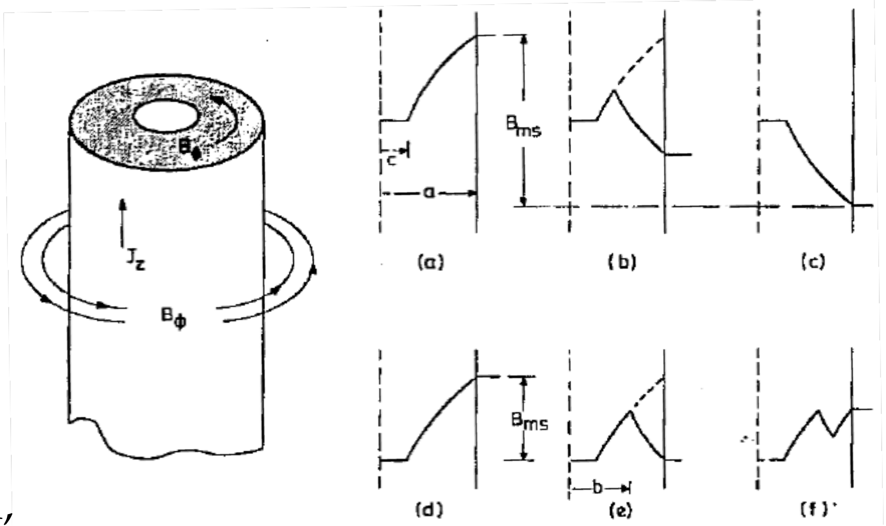


Fig. 8.24. Self-field in a superconducting cylinder or filamentary composite carrying transport current. (a), (b), and (c) show profiles of **B** within the cylinder when transport current is reversed; (d), (e), and (f) show effect of unidirectional current oscillations.

- Self-field losses: as the transport current is varied, the self-field lines change, penetrating and exiting the conductor surface. The effect is independent of frequency, yielding a hysteresis-like energy loss:

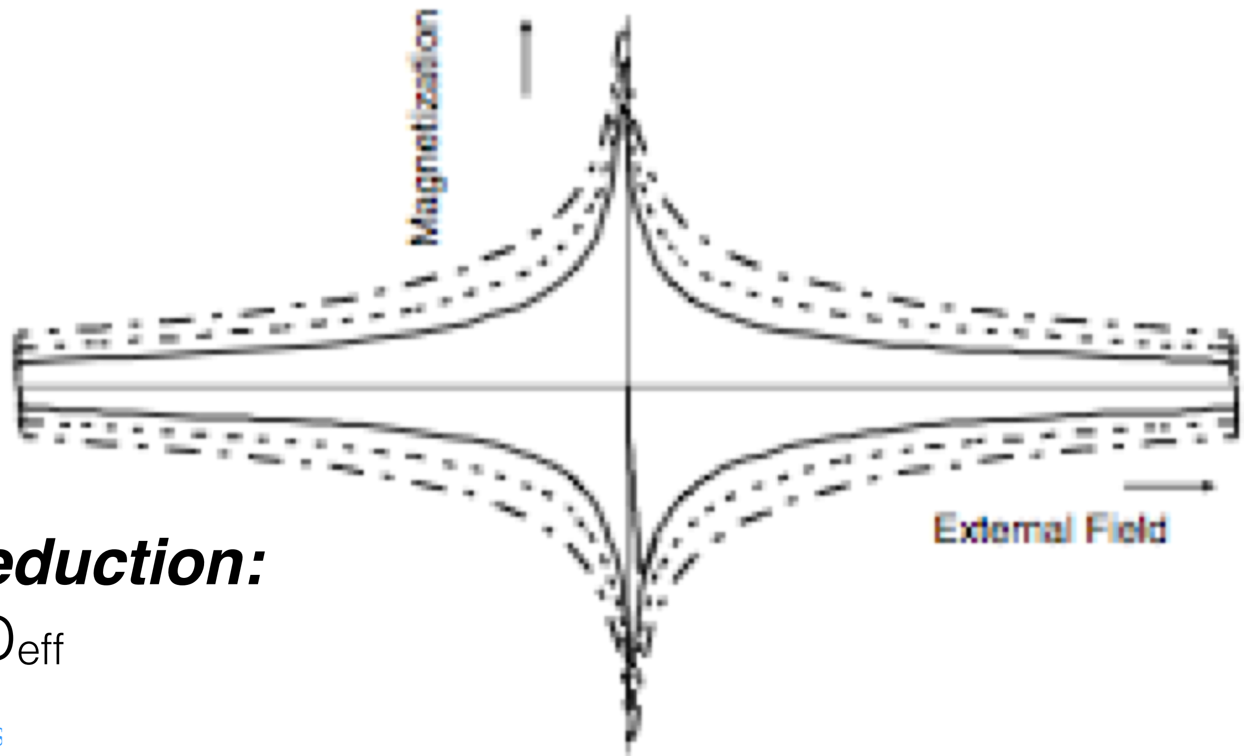
$$Q_{sf} = \frac{B_{ms}^2}{2\mu_0} \Gamma(\beta); \ \beta = \frac{B_{ms}}{B_p} = \frac{I}{I_c}$$

First estimate of AC losses: Hysteresis losses

$$Q_{cyc} = \int_0^{t_0} J_c(B) \frac{2D_{eff}}{3\pi} \frac{dB}{dt} dt$$ [J/m^3, per cycle]

$$Q_{hyst-tot} = Q_{cyc} * V_{sc}$$ [J, per cycle]

This has motivated the quest for fine filament wire!



Hysteresis loss reduction:

- minimize D_{eff}

First estimate of AC losses: Coupling losses

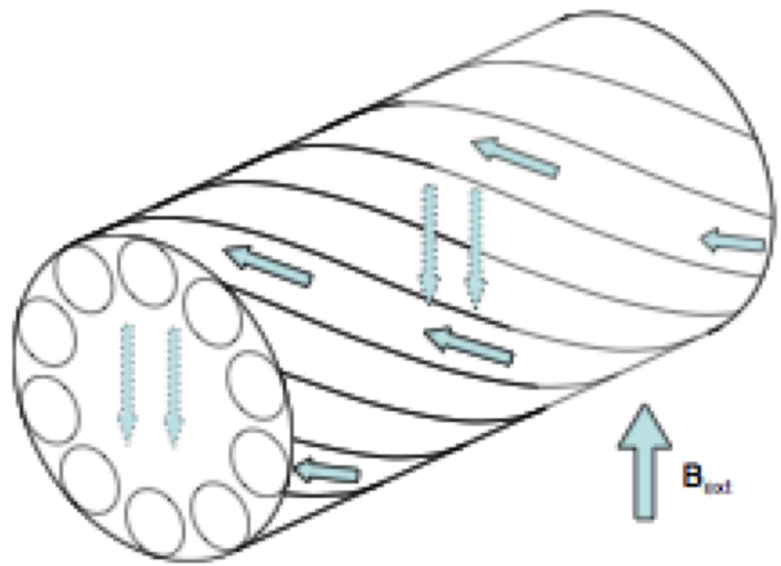
$$\tau = \frac{\mu_0}{2\rho_t}\left(\frac{p}{2\pi}\right)^2$$

$$q_{coupling} = \frac{(dB/dt)^2}{\mu_0} 2\tau \quad [\mathrm{W/m^3}]$$

$$Q_{coupling-tot} = Q_{coupling} * V_{cond}$$

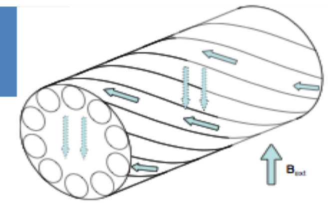
Coupling loss reduction:

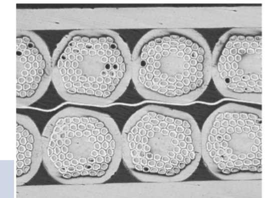
- minimize twist pitch

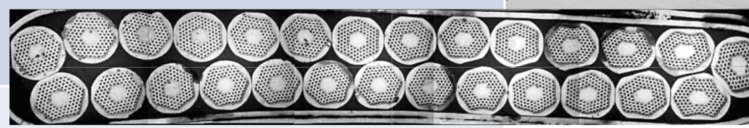


AC losses impact conductor, magnet and cryostat design

Loss term	Scaling	Notes
Conductor hysteresis	$P_{hys} \propto M\frac{dB}{dt} \sim J_c\frac{dB}{dt}d_{eff}$	$1+\left(\frac{I_{max}}{I_c}\right)^2$
Conductor filament coupling	$P_{coup} \propto \frac{2\tau}{\mu_0}\left(\frac{dB}{dt}\right)^2$	$\tau = \left(\frac{p}{2\pi}\right)^2\frac{\mu_0}{2\rho_t}$
Cable strand coupling	$P_{cable} \propto \left(\frac{dB}{dt}\right)^2\frac{p}{R_a}\frac{w}{t}$	
Eddy currents		







- Loss estimates are further complicated by field regime, operational current, etc.
- Final design is a balance between heat capacity, losses, heat transfer and duty cycle resulting in conductor temperature excursions and hence performance limitations

- Hysteresis:
  - Reduce d_{eff}
  - Increases with I/I_c
- Coupling:
  - Minimize twist pitch
  - Modify inter-filament resistance
- Eddy currents:
  - laminations

Use of the AC-loss models

- It is common (but not necessarily correct) to add the different AC loss terms together to determine the loss budget for an conductor design and operational mode.
- AC loss calculations are "imperfect":
  - Uncertainties in effective resistivities (e.g. matrix resistivity may vary locally, e.g. based on alloy properties associated with fabrication; contact resistances between metals may vary, etc)
  - Calculations invariably assume "ideal" behavior, e.g. Bean model, homogeneous external field, etc.
- For real applications, these models usually suffice to provide grounds for conductor specifications and/or cryogenic budgeting
  - For critical applications, AC-loss measurements (non-trivial!) should be undertaken to quantify key parameters

Special cases: HTS tapes

- HTS tapes have anisotropic Jc properties that impact AC losses.

- The same general AC loss analysis techniques apply, but typical operating conditions impact AC loss conclusions:
  - the increased specific heat at higher temperatures has significant ramifications - enhances stability
  - Cryogenic heat extraction increases with temperature, so higher losses may be tolerated

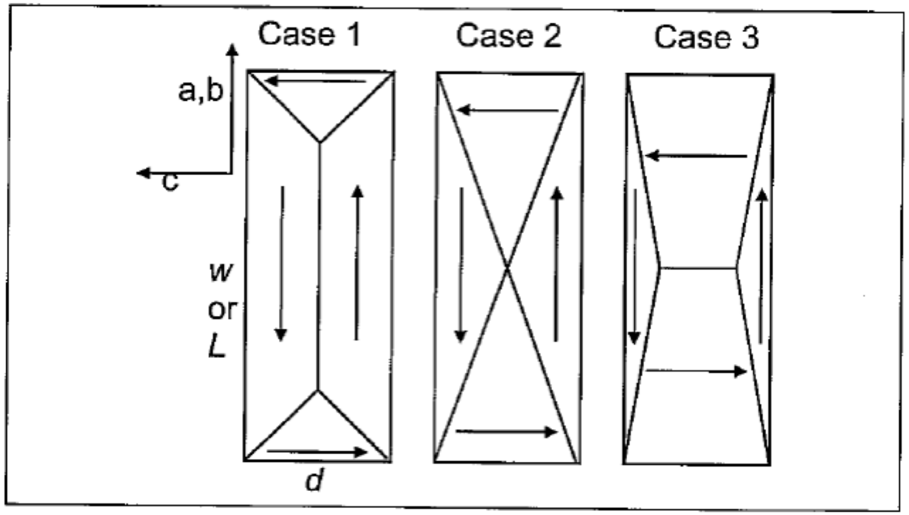


Figure 2.11 Screening currents in a slab with anisotropic critical-current density.

AC losses and cryogenics

- The AC loss budget must be accounted for in the cryogenic system
  - Design must account for thermal gradients – e.g. from strand to cable, through insulation, etc. and provide sufficient temperature margin for operation
  - Typically the temperature margin needed will also depend on the cycle frequency; the ratios of the characteristic cycle time (τ_w) and characteristic diffusion time (τ_d) separates two regimes:
    1. $\tau_w \ll \tau_d$: Margin determined by single cycle enthalpy
    2. $\tau_w \gg \tau_d$: Margin determined by thermal gradients

- The AC loss budget is critical for applications requiring controlled current rundown; if the AC losses are too large, the system may quench and the user loses control of the decay rate

Specifying conductors for AC losses

- As a designer, you have some control over the ac losses:
  - Control by conductor specification
    - Filament size
    - Contact resistances
    - Twist pitch
    - Sufficient temperature margin (e.g. material T_c, fraction of critical current, etc)
  - Control by cryogenics/cooling
    - Appropriate selection of materials for good thermal conductivity
    - Localization of cryogens near thermal loads to minimize ΔT
- Remember: loss calculations are imperfect! For critical applications, AC loss measurements may be required, and some margin provided in the thermal design to accommodate uncertainties