Unit 6
Flux Jumps and Motion in Superconductors

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Flux motion in superconductors

- Outline
  - Review of flux penetration in type II superconductors
  - Forces on fluxoids in type II superconductors
  - Pinning strength
    - Definition and theoretical concepts
    - Most common pinning sites
    - Influence of other parameters, e.g. temperature
  - Definition of stability
    - Adiabatic
    - Dynamic
  - HTS specific: Flux creep and irreversibility field

References

- This lecture relies heavily on:

  - Martin Wilson, "Superconducting Magnets"
  - Alex Gurevich, Lectures on Superconductivity
  - Ernst Helmut Brandt, "Electrodynamics of Superconductors exposed to high frequency fields"
  - Marc Dhallé, "IoP Handbook on Superconducting Materials" (preprint)
  - Arno Godeke, thesis: "Performance Boundaries in Nb3Sn Superconductors"
  - Feynman "Lectures on Physics"

Review

- A fluxoid is a "vortex" of normal material surrounded by circulating supercurrents generating a quantum of magnetic flux of amplitude:

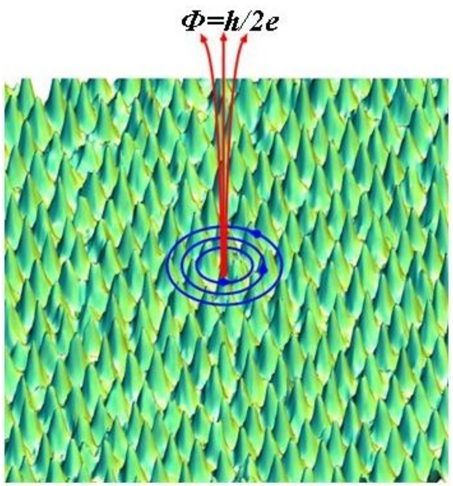
$$\phi_0 = \frac{\pi \hbar}{|e|} = 2.07 \times 10^{-15} \text{ Vs (=Weber=T-m}^2\text{)}$$

- The fluxoids penetrate the superconductor so as to minimize the total free Gibbs energy: whereas Type I superconductors are diamagnetic, the fluxoid field is paramagnetic, with fluxoid penetration when

$$G < 0$$

$$G = \varepsilon - H\phi_0;\ H_{c1} = \frac{\varepsilon}{\phi_0};\ \varepsilon \text{ is the vortex self-energy}$$

$$H_{c1} = \frac{\phi_0}{4\pi\lambda^2}\left(\ln\frac{\lambda}{\xi} + 0.5\right)$$

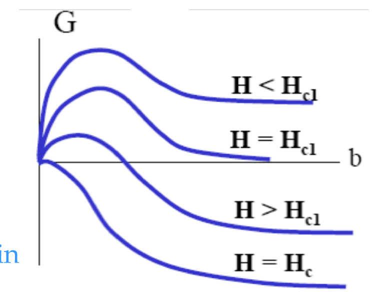


Vortex flux lattice in V_3Si STMFermi-level conductance image , H=3T, T=2.3K Center for Nanoscale Science, NIST

- The field in the sample is a function of the fluxoid

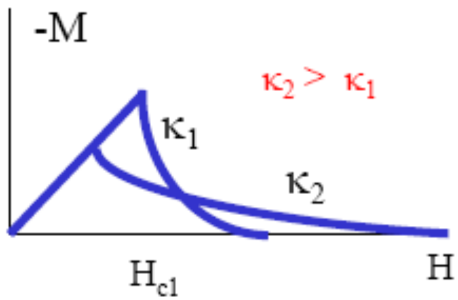
Cross-sectional density:

$$B = n\phi_0$$



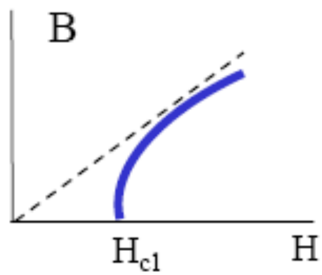
Review of fundamentals

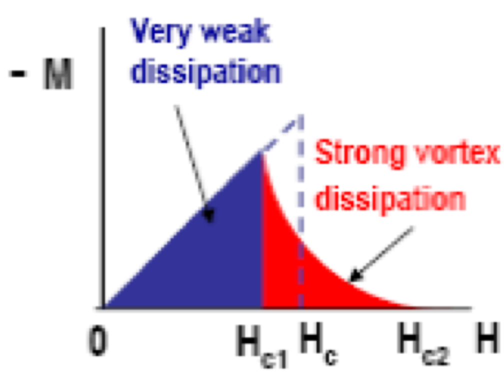
Gurevich lectures



The thermodynamic critical field H_c is defined as:

$$H_c = \frac{\phi_0}{2\sqrt{2}\pi\lambda\xi}$$





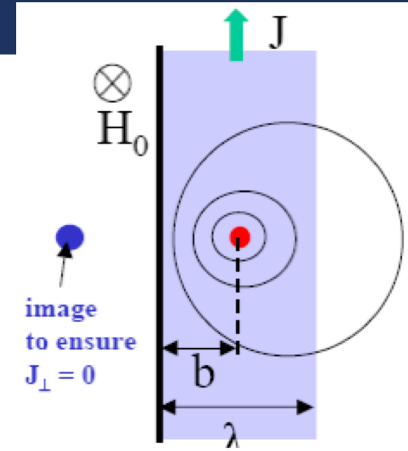
Material	T_c (K)	$H_c(0)$ [T]	$H_{c1}(0)$ [T]	$H_{c2}(0)$ [T]	$\lambda(0)$ [nm]
Pb	7.2	0.08	na	na	48
Nb	9.2	0.2	0.17	0.4	40
Nb_3Sn	18	0.54	0.05	30	85
NbN	16.2	0.23	0.02	15	200
MgB_2	40	0.43	0.03	3.5	140
YBCO	93	1.4	0.01	100	150

Material	T_c (K)	$\lambda(0)$, nm	$\xi(0)$, nm	H_{c2} (T)
Nb-Ti	9.5	240	4	13
Nb-N	16	200	5	15
Nb_3Sn	18	65	3	30
MgB_2 (dirty)	32-39	140	6	35
$YBa_2Cu_3O_7$	92	150	1.5	>100
Bi-2223	108	200	1.5	>100

Forces on the fluxoid

- The combination of Meissner current and interface forces results in a surface barrier energy that must be exceeded for flux to penetrate
    - The surface barrier energy decreases to zero at $H=H_c$, the thermodynamic critical

$$a_\Delta(H) = \left(\frac{4}{3}\right)^{1/4}\left(\frac{\phi_0}{\mu_0 H}\right)^{1/2}$$



- Fluxoids experience a variety of forces:
    - Oriented parallel to each other, fluxoids are mutually repulsive, leading to a natural equal hexagonal spacing in bulk material that maximizes their separation
    - In the vicinity of current, the fluxoid (oriented by n) sees a force

$$\vec{F}_v = \vec{J} \times \phi_0 \vec{n}$$

    - Near a superconductor – normal metal interface, a fluxoid experiences an attractive force towards the surface (from its attractive image)
    - The resulting motion of a fluxoid is countered by:
        - Viscous drag associated with an effective normal resistivity
        - Pinning of the fluxoids by energetically preferable sites

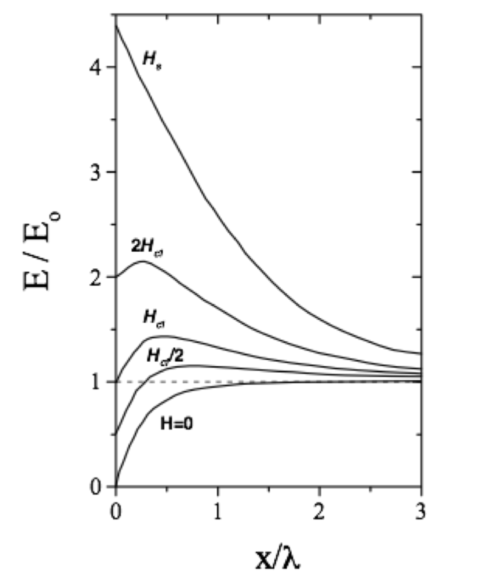


Figure 22 : calculated Bean-Livingston energy barrier for flux - entry and – exit. x is the distance to the surface and E_0 the energy per unit length of an flux line well inside the superconductor. Different curves correspond to different applied fields (from [49])

Energy perspective

- Pinning sites effectively reduce the energy state of the system; transport current has the effect of reducing the depth of the potential well

From Dhalle, IOP Handbook of Superconducting Materials

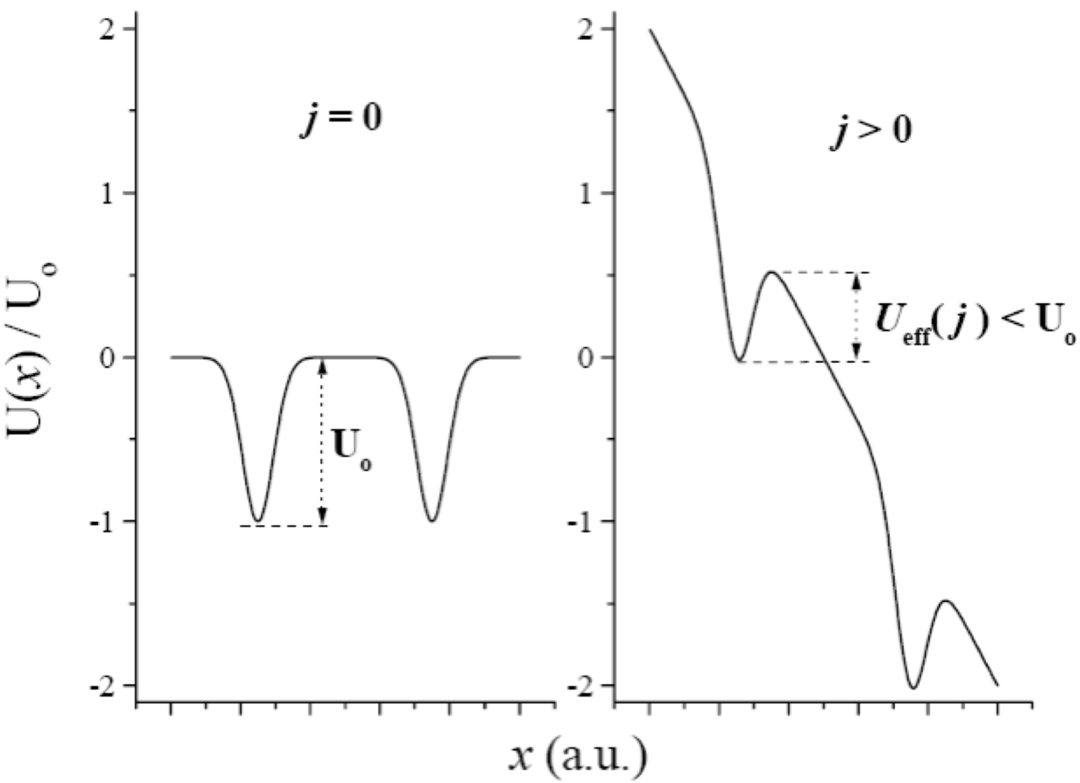


Figure 26 : schematic description of Anderson-Kim flux creep. The left panel shows the undisturbed pinning-potential landscape, to which in the right panel the free energy contribution of the 'Lorentz' force exerted by a current density is added. The effect of the current density is to lower the energy barrier for thermally activated hopping of flux lines (or flux bundles) from one pinning site to the next.

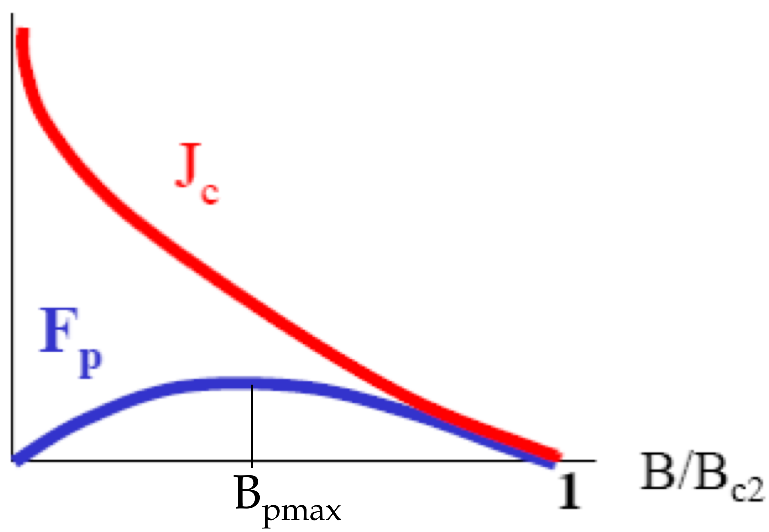
Pinning of fluxoids

- Useful type II superconductors require that no significant viscous flux-flow occur
  - Flux-flow results in heating, changing current with time, etc (see AC losses)
- The pinning must therefore balance the Lorentz force:

$$J_c(B,T)\times B = F_p(B,T)$$

- The pinning force must be zero at B=0 and at $B=B_{c2}$; it has a maximum at some intermediate field B_{pmax}.

Note: the pinning force defines J_c; whereas other critical values (H_{c1}, H_c, H_{c2}, T_c) are intrinsic to the material, the Jc can vary strongly with specific material characteristics (defects, etc)



Flux pinning – general comments

- First key question: what is the maximum current density that can be obtained?
  - "Unlimited" pinning force would result in $J_c = J_d$, the de-pairing current density (Dhalle) :

$$J_d(T) = \frac{4}{3\sqrt{6}} \frac{H_c(T)}{\lambda(T)}$$

  - Unfortunately (or fortunately!?), J_d is usually orders of magnitude larger than J_c
    - Real pinning sites differ in strength, distribution, etc
    - Key pinning sites:
      - Precipitates of non-superconducting materials
      - Dislocations
      - Grain boundaries and other planar defects
  - Details of pinning are complex:
    - Attraction to pinning sites competes with vortex-vortex repulsion

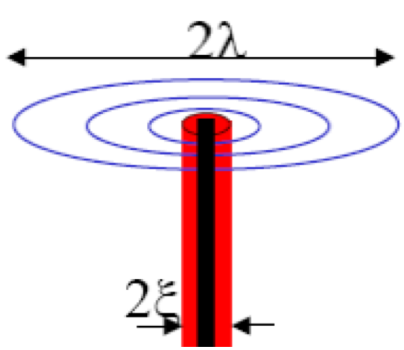
Pinning sites - mechanisms and strengths

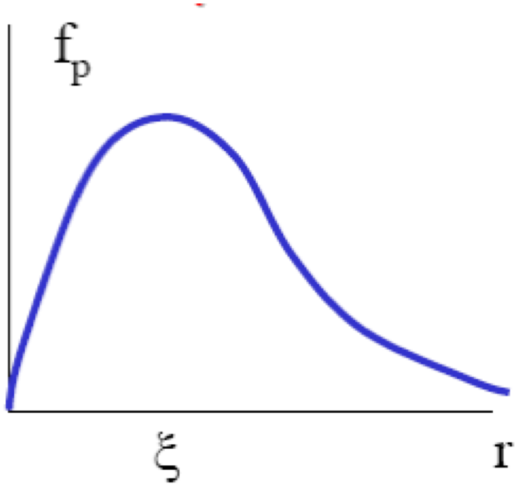
- Pinning a fluxoid "saves" a fraction of the vortex core energy:

$$\varepsilon_0 = \frac{1}{2}\pi\mu_0\xi^2 H_c^2$$

  - If the site has a characteristic dimension r<<ξ, only a small fraction of the core energy is saved, and the pinning strength is weak;

- The optimum pinning occurs for r~ ξ.





Pinning sites - mechanisms and strengths

- Futhermore, the pinning strength scales with the fraction of the fluxoid length subjected to pinning
  - Optimal sized pinning sites may not be all that effective if they are point sources
  - Interface sites (i.e. planar sites) often provide very strong pinning
  - Some of the strongest pinning sites are the result of image-vortex energies at insulating boundaries
  - Examples: α-Ti in NbTi, Nb_3Sn grain boundaries
- Note that large (insulating) planar defects inhibit normal transport current
  - ***The best superconductors are characterized by poor normal conductivity! (This is important from a stability point of view)***

Flux motion and Heat

- The movement of flux through a superconductor (in the absence of pinning) is accompanied by an electric field and hence a dissipative E-J relationship

Note: when pinned, the fluxoids generally exhibit no resistivity.
Exceptions:
- *flux jumps (coming soon)*
- *Thermally induced flux-flow (mainly seen in HTS materials)*
- *High-frequency applied fields: fluxoids are not infinitely rigid* =>*can vibrate, yielding losses despite pinning*

$$F_L = J \times \Phi_0$$

Faraday

Average vortex velocity

$$\begin{aligned} \vec{E} &= \vec{B} \times \vec{v}_f \\ \dot{w} &= nF_L \cdot \vec{v}_f = (\vec{J} \times \vec{B}) \cdot \vec{v}_L \\ &= \vec{J} \cdot \vec{E} = JE \end{aligned}$$

Schematic of a V-I transition

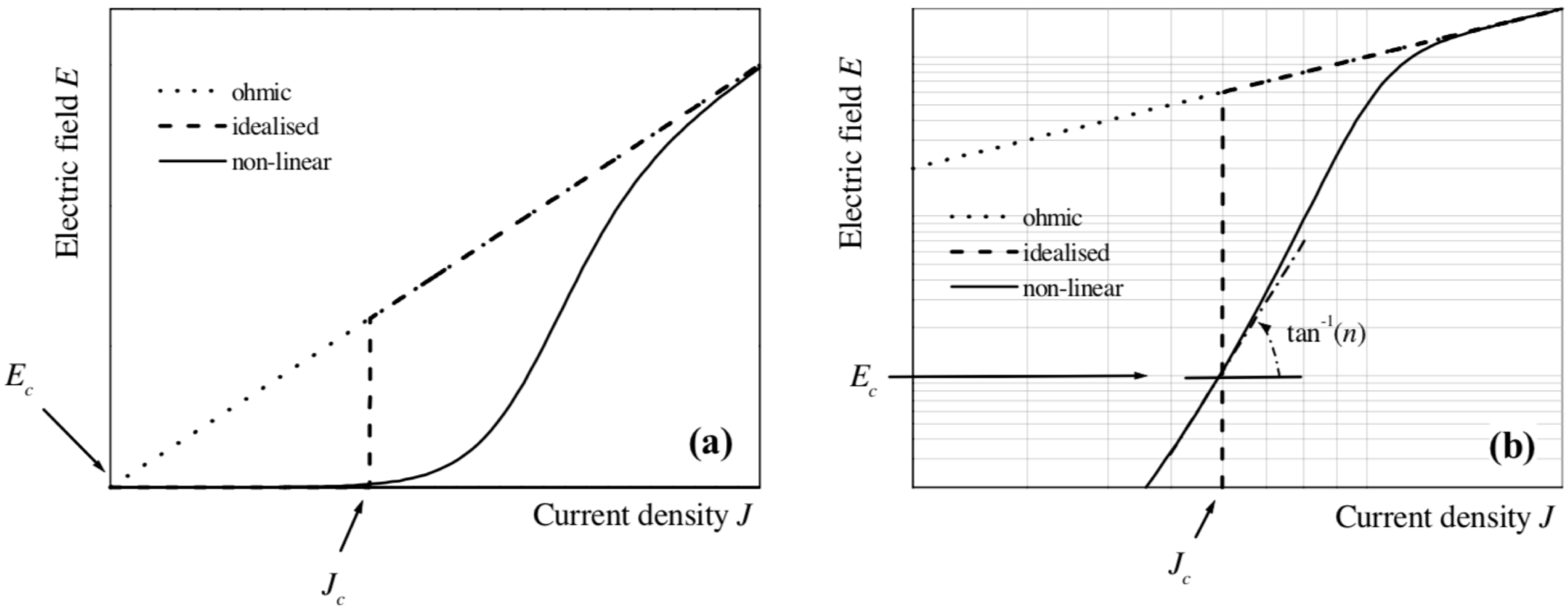


Figure 1 : Schematic electric field – current density curves for an ohmic and a superconducting material plotted on **(a)** a linear or **(b)** a double logarithmic scale.

Flux flow – critical current E-J relations

- Macroscopically, the nature of the E-J relation at the onset of flux-flow when transport current reaches critical current can be modeled using a power law or exponential relation:

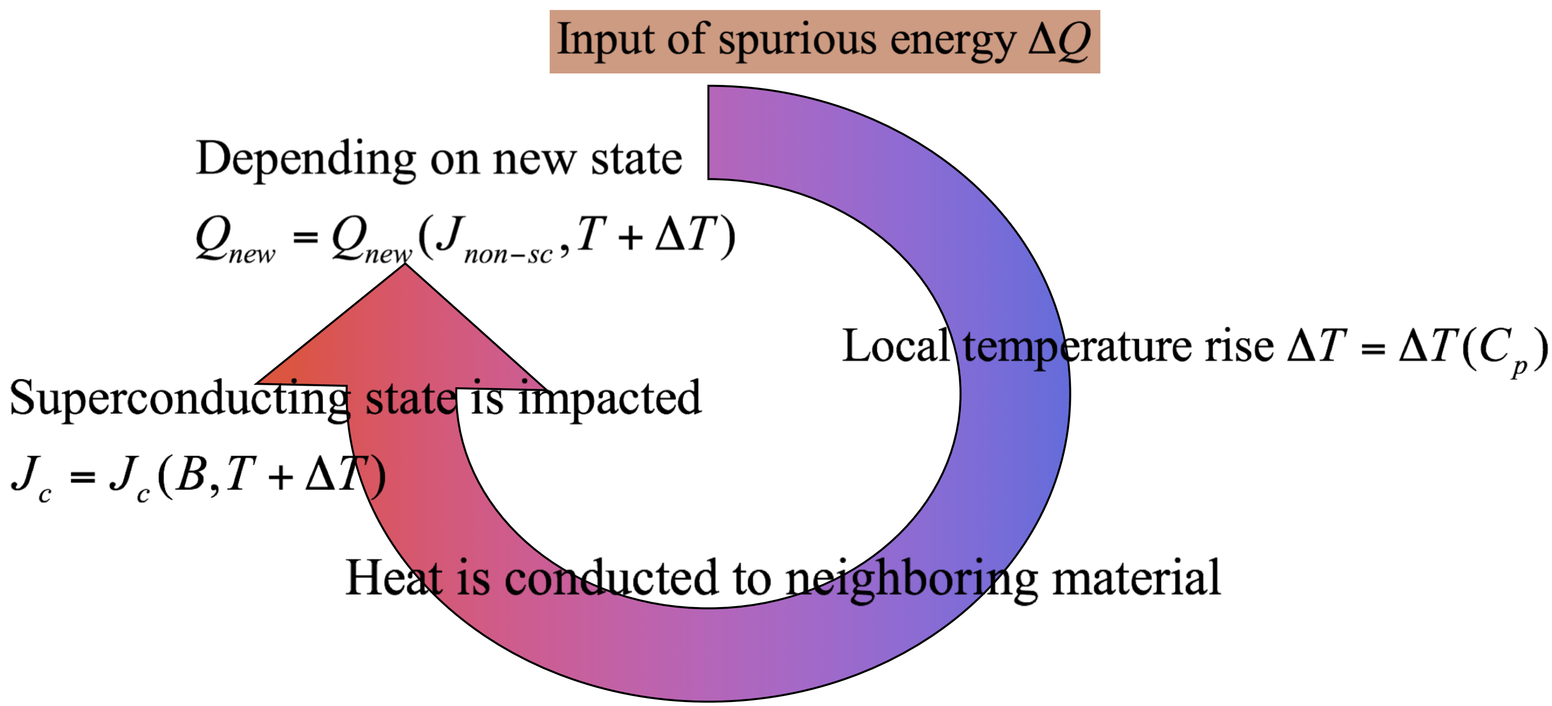
$$E = E_c \left[\frac{J}{J_c(H,T)} \right]^{n(H,T)}$$

$$E = E_c \exp\left[-n(H,T)\left(1 - \frac{J}{J_c(H,T)} \right) \right]$$

- Both formulations provide reasonable fits to data
- Both forms stem from a physical model of the depinning of fluxoids
  - Assumes flux depinning from thermal excitations
  - Assumes Maxwell-Boltzmann statistics for probability of depinning
  - Difference only in the *J*-dependence of the activation energy
- The parameter $n(H,T)$, characterizes the transition between normal and superconducting states; a higher n-value implies a more homogeneous pinning within the superconductor
  - The n-value is experimentally determined by measuring V-I transitions on short-samples and fitting the measured data (preferably in log-log plots!)

Concept of stability

- The concept of stability concerns the interplay between the following elements:
  - The addition of a (small) thermal fluctuation local in time and space
  - The heat capacities of the neighboring materials, determining the local temperature rise
  - The thermal conductivity of the materials, dictating the effective thermal response of the system
  - The critical current dependence on temperature, impacting the current flow path
  - The current path taken by the current and any additional resistive heating sources stemming from the initial disturbance



Source of initial disturbance: Flux jump

- We have seen that fluxoid motion induces heat
- For LTS materials:
  - The heat capacity at low temperatures is low
  - The critical current decreases strongly with increasing temperature

⇒ For sufficient flux-flow motion, the cycle described previously avalanches towards a cascade of local fluxoids (vortex bundle)

  ⇒ Depending on the ability of neighboring material to accommodate the resulting heat, the conductor may become normal, and possibly ***quench***.

Basics of stability analysis

- To analyze the stability of a conductor, we need to know:
  1. The amplitude and distribution (t,**x**) of the initiating heat source
  2. The heat capacity and thermal conductivity of the adjoining materials
  3. The relative volumes of the adjoining materials (or cross sections for typical conductors)
  4. The critical surface (J_c(B,t,…)) of the superconducting material

- In most cases the analyses can be done with a 1D model
  - Insulation between conductors in a coilpack results in

$$\kappa_{\perp} << \kappa_{||}$$

  - Material properties through the conductor cross section can be lumped together as appropriate (e.g. specific heat averaged, thermal and electrical conductivity modeled as parallel circuits, etc.)

Characteristic diffusion times

- In the case of a flux jump, do we need to include the temporal evolution of the flux motion in the stability analysis?
  - Consider characteristic times of
    1. thermal diffusion (time for thermal transport to neighboring materials)
    2. Magnetic diffusion (time for flux jump to proceed through avalanche)

Following Wilson:

$$\left.\begin{aligned} \frac{\partial \theta}{\partial t} &= D_\theta \nabla^2 \theta; \quad D_\theta = \frac{k}{\gamma C_p} \\ \frac{\partial B}{\partial t} &= D_m \nabla^2 B; \quad D_m = \frac{\rho}{\mu_0} \end{aligned}\right\} \quad \tau_\theta \sim \frac{L^2}{D_\theta}, \quad \tau_m \sim \frac{L^2}{D_m}$$

- Here L is the characteristic length of interest (the definitions can be made more precise for specific cases via direct solution of the PDE)

Typically $\tau_m \ll \tau_\theta$, so the flux jump can be viewed as a point source in time for stability purposes

Adiabatic stability

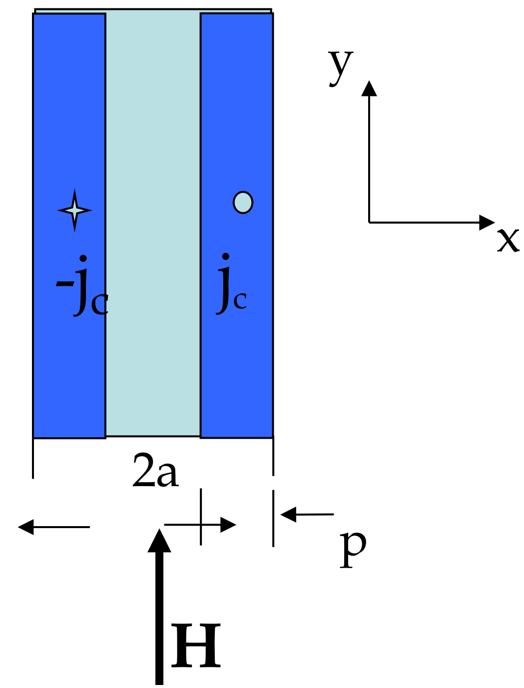
$$\phi = \mu_0 \int_{a-p}^{a} j_c x dx = \frac{\mu_0 j_c}{2}\left[2ap - p^2\right]$$

We have by definition:

$$\Delta q(x) = \int I(x)E(x)dt = J_c \Delta x \Delta\phi(x)$$

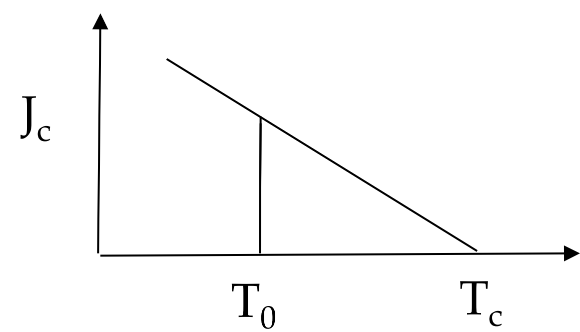
Hence the average heat/volume:

$$\Delta Q = \frac{1}{a}\int J_c \Delta\phi(x)dx$$



•Since Jc(T) decreases with temperature (e.g. linear), an incremental increase of temperature results in an additional decrease in available heat capacity – results in adiabatic stability criteria:

$$\frac{\mu_0 J_c^{\,2} a^2}{\rho C(T_c - T_0)} < 3$$



Energy deposited quenches
Point disturbances & dynamic stability

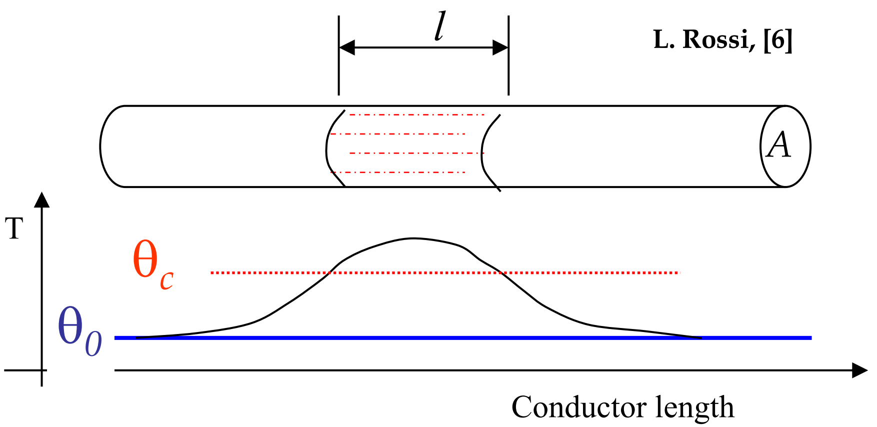
- We start considering a wire made purely of superconductor.
- Let's assume that a certain amount of energy *E* increased the temperature of the superconductor beyond θ_c over a length *l*. The segment *l* of superconductor is dissipating power given by $J_c^2 \rho A l$ [W].
- Part (or all) of the heat is conducted out of the segment because of the thermal gradient, which can be approximated as $(\theta_c - \theta_o)/(l/2)$. Therefore, <u>when the power dissipated equals the power conducted away</u>

$$\frac{2kA(\theta_c - \theta_0)}{l} = J_c^2 \rho A l$$

which results in

$$l = \sqrt{\frac{2k(\theta_c - \theta_0)}{J_c^2 \rho}}$$

L. Rossi, [6]



5. Energy deposited quenches Point disturbances

- The length l defines the MPZ (and MQE).
  - A normal zone longer that l will keep growing (quench). A normal zone shorter than l will collapse.
- An example [2]
  - A typical NbTi 6 T magnet has the following properties
    - $J_c = 2 \times 10^9$ A m^{-2}
    - $\rho = 6.5 \times 10^{-7}$ Ω m
    - $k = 0.1$ W m^{-1} K^{-1}
    - $\theta_c = 6.5$ K
    - $\theta_o = 4.2$ K

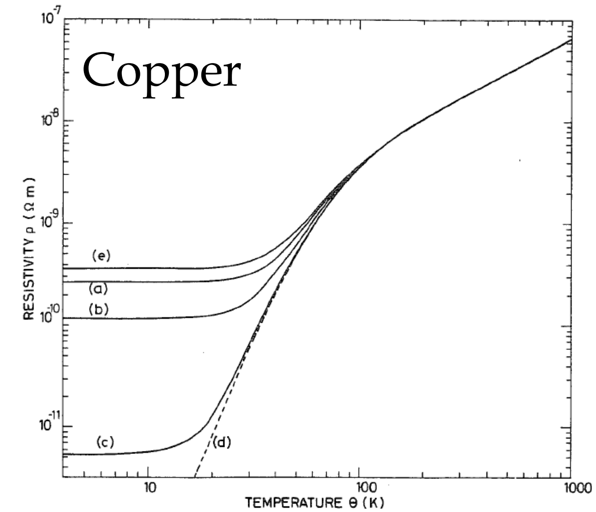
$$l = \sqrt{\frac{2k(\theta_c - \theta_0)}{J_c^2 \rho}}$$

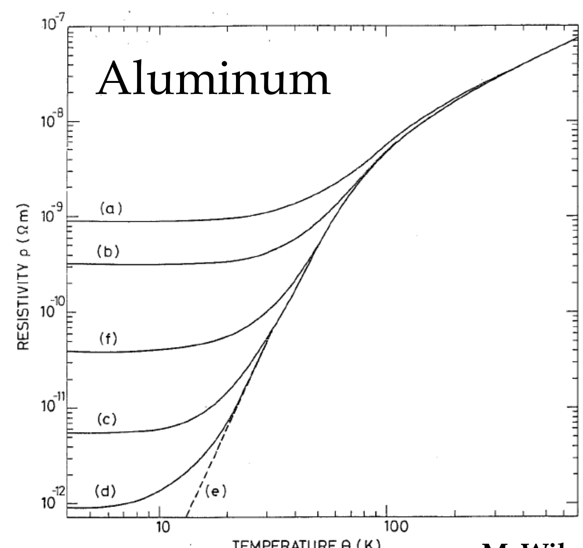
  - In this case, $l = 0.5$ µm and, assuming a 0.3 mm diameter, the required energy to bring to θ_c is 10^{-9} J.

- A wire made purely of superconductor, without any stabilizer (like copper) around, would quench with nJ of energy.
  - In order to increase l, since we do not want to reduce J_c, we have to increase k/ρ: we need a composite conductor!

5. Energy deposited quenches
Point disturbances

- We now consider the situation where the superconductor is surrounded by material with low resistivity and high conductivity.
- Copper can have at 4.2 K
  - resistivity $\rho = 3 \times 10^{-10}\ \Omega$ m (instead of $6.5 \times 10^{-7}\ \Omega$ m for NbTi)
  - $k = 350$ W m^{-1} K^{-1} (instead of 0.1 W m^{-1} K^{-1} for NbTi).
- We can therefore increase k/ρ by almost a factor 10^7.
- A significant improvement was achieved in the early years of superconducting magnet development after the introduction of composite conductor
  - Both for flux jump and stability viewpoint

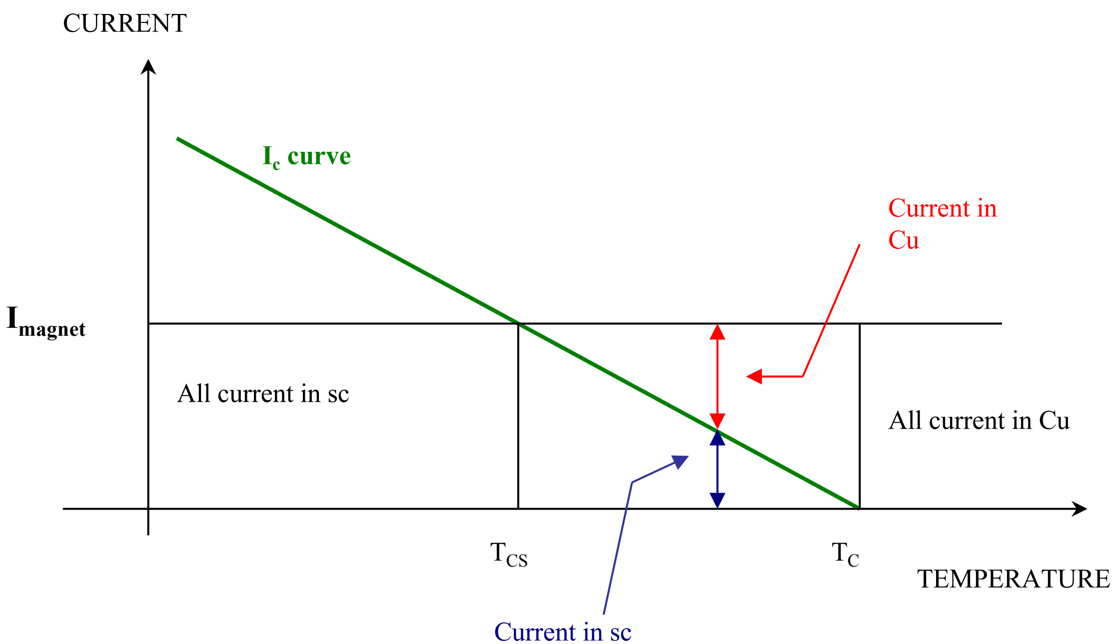


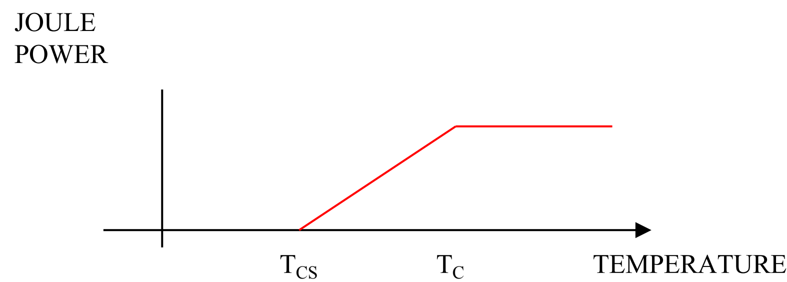


M. Wilson, [2]

5. Energy deposited quenches
Point disturbances

- In a composite superconductor, the heat dissipated when a transition from normal to superconducting state occurs can be subdivided in three parts:
  - All the current flows in the superconductor
  - The current is shared by the superconductor and the stabilizer
  - All the current flows in the stabilizer.



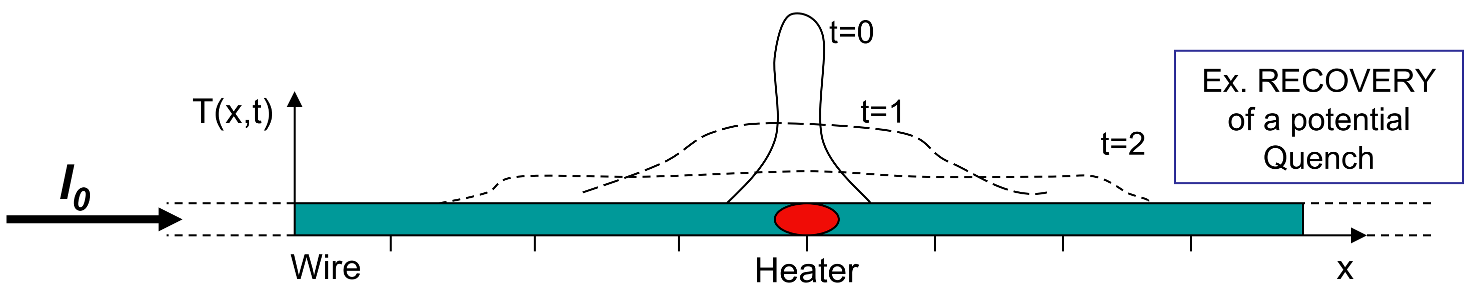


Calculation of the bifurcation point for superconductor instabilities

Thanks to Matteo Allesandrini, Texas Center for Superconductivity, for these calculations and slides

Heat Balance Equation in 1D, without coolant: *[W/m³]*

$$\underbrace{\frac{d}{dx}\left(k(T)\cdot\frac{dT}{dx}\right)}_{\text{Heat conduction}}+\underbrace{\rho(T)\cdot J^2}_{\text{Joule effect}}+\underbrace{Q_{initial\_pulse}}_{\text{Quench trigger}}-\underbrace{C(T)_{volume}\cdot\frac{dT}{dt}}_{\text{Heat stored in the material}}=0$$



Assumption on $\rho(T)\cdot J^2$:

CASE 1. $T < T_{sc\_crit}$

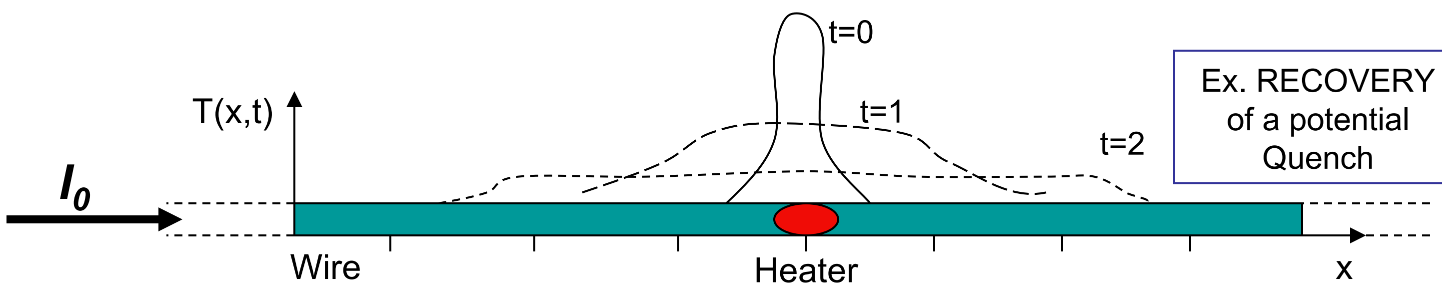
- 1.a) $T < T_{current\ sharing} \longrightarrow I_{sc}=I_o,\quad I_{metal}=0$
- 1.b) $T > T_{current\ sharing} \longrightarrow I_{sc}=I_{crit(T)},\quad I_{metal}=I_0-I_{sc}$

CASE 2. $T > T_{sc\_crit}$ -- $I_{sc}=0,\ I_{metal}=I_0$

Quench Propagation Rate

Heat Balance Equation in 1D, without coolant: *[W/m³]*

$$\underbrace{\frac{d}{dx}\left(k(T)\cdot\frac{dT}{dx}\right)}_{\text{Heat conduction}}+\underbrace{\rho(T)\cdot J^2}_{\text{Joule effect}}+\underbrace{Q_{initial\_pulse}}_{\text{Quench trigger}}-\underbrace{C(T)_{volume}\cdot\frac{dT}{dt}}_{\text{Heat stored in the material}}=0$$



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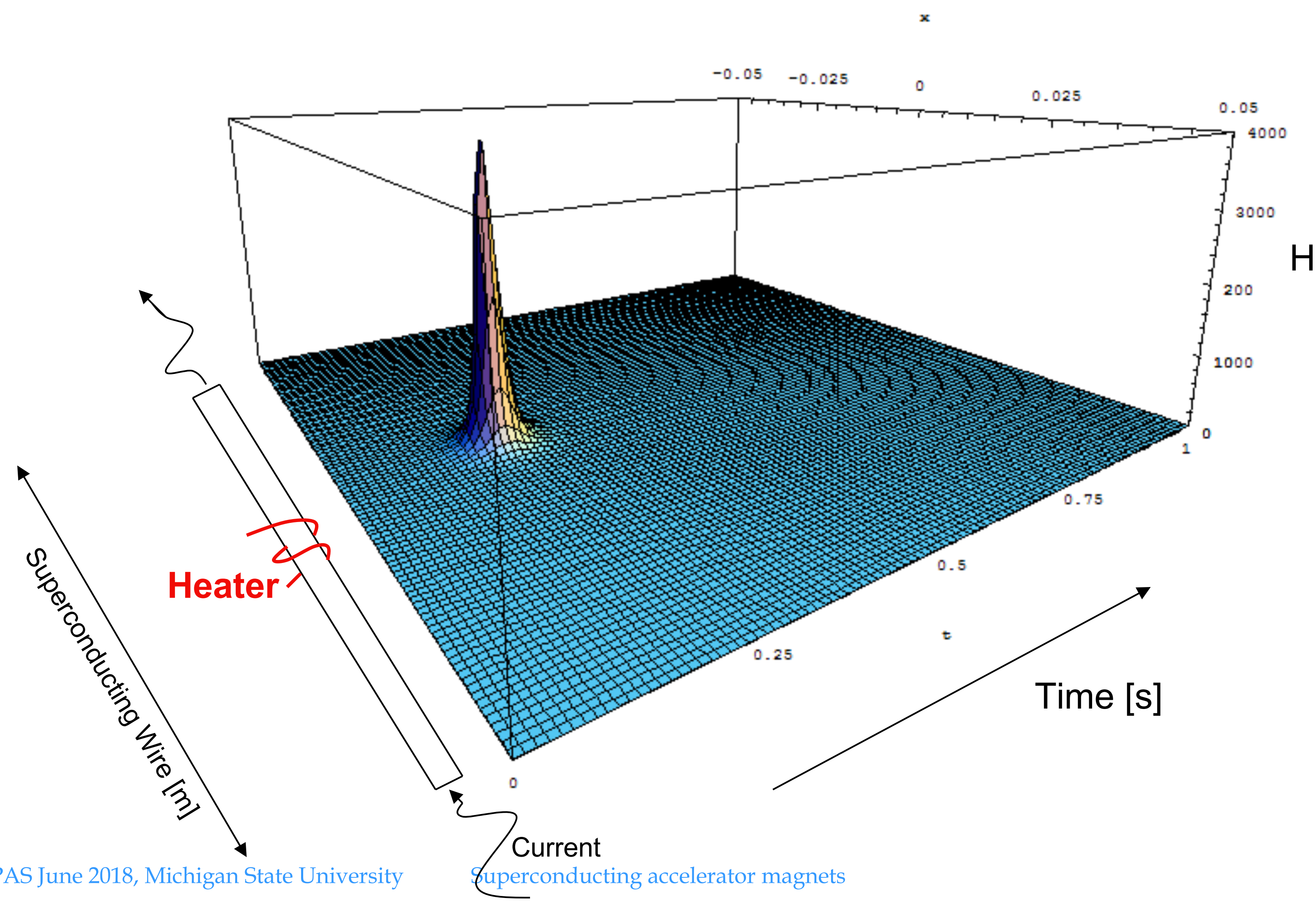
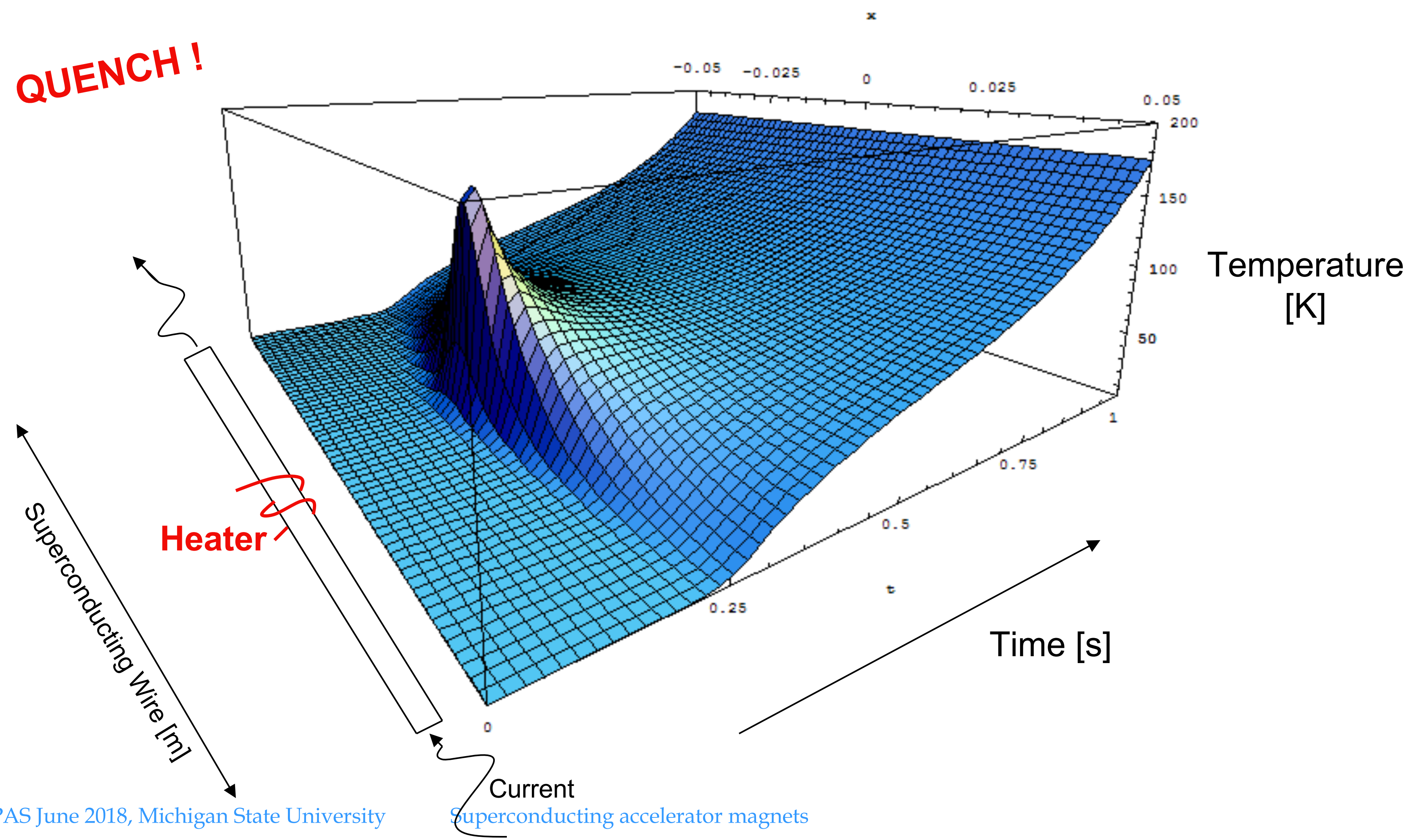
EXAMPLE: material properties are not realistic here.



Temperature profile

With T_{cr} = **39 K**

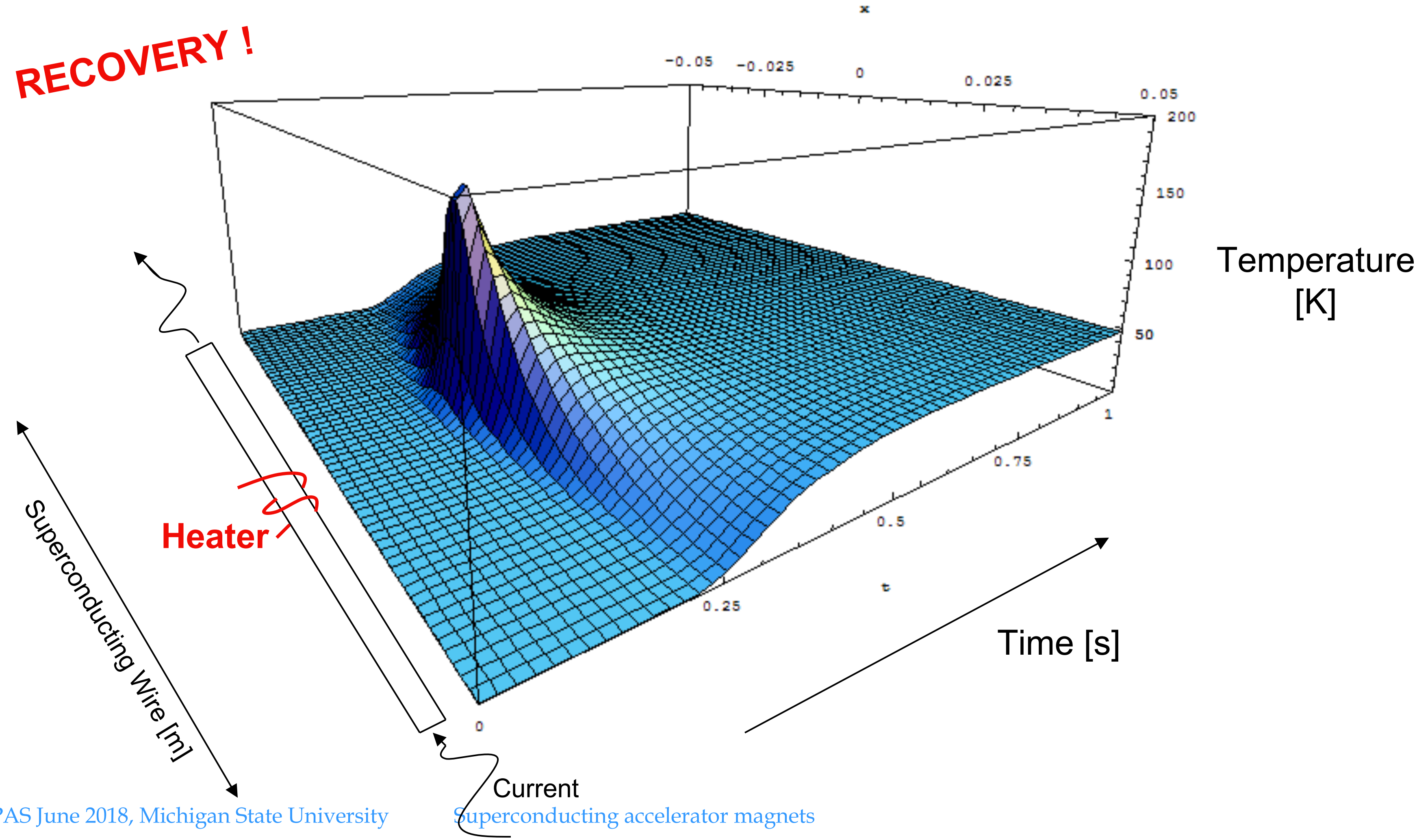
EXAMPLE: material properties are not realistic here.



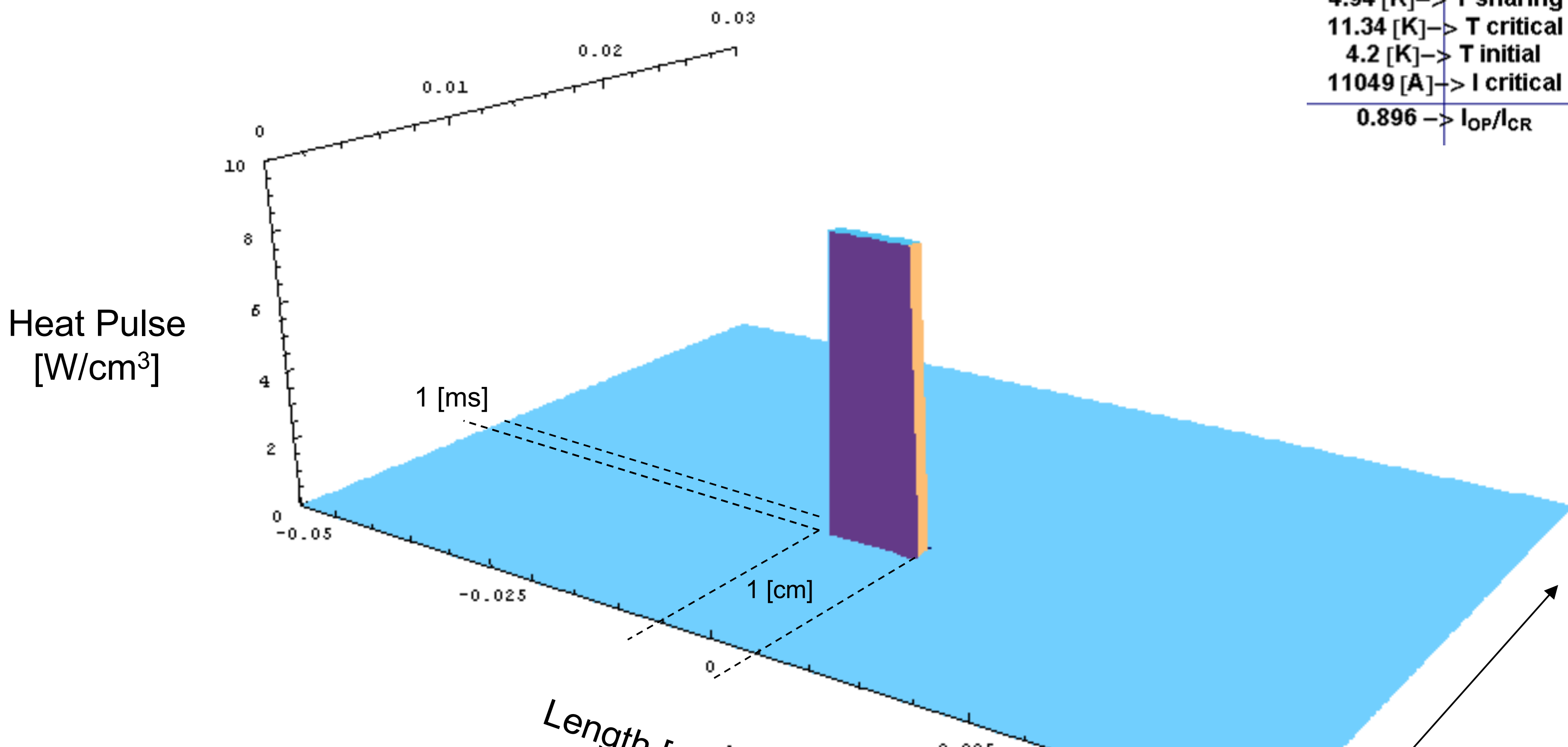
Temperature profile

With T_{cr} = **70 K**

EXAMPLE: material properties are not realistic here.



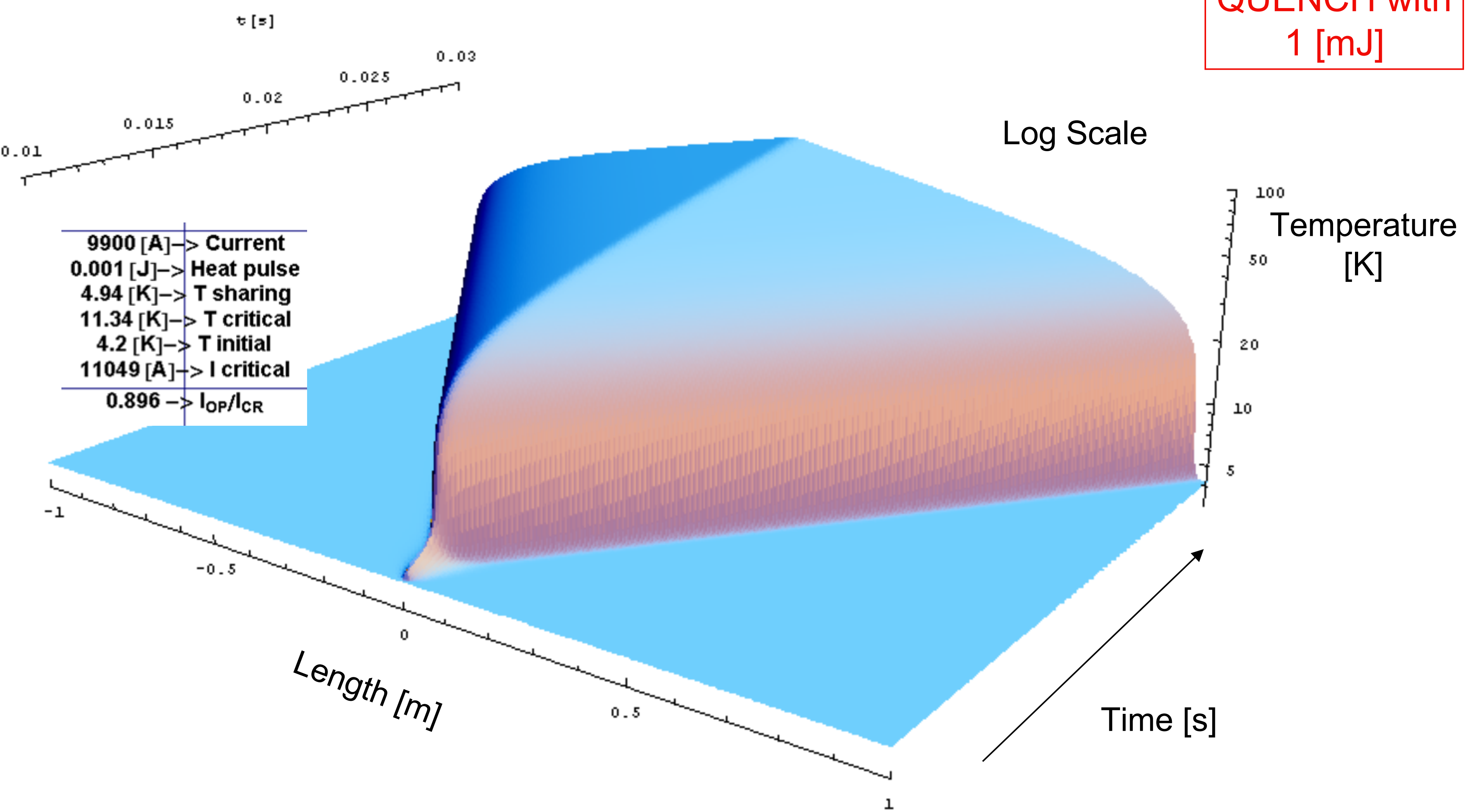
9900 [A]–>	Current
0.001 [J]–>	Heat pulse
4.94 [K]–>	T sharing
11.34 [K]–>	T critical
4.2 [K]–>	T initial
11049 [A]–>	I critical
0.896 –>	I_{OP}/I_{CR}



1 mJ is deposited in 1 ms over 1 cm of wire

Analysis of SQ02

QUENCH with 1 [mJ]



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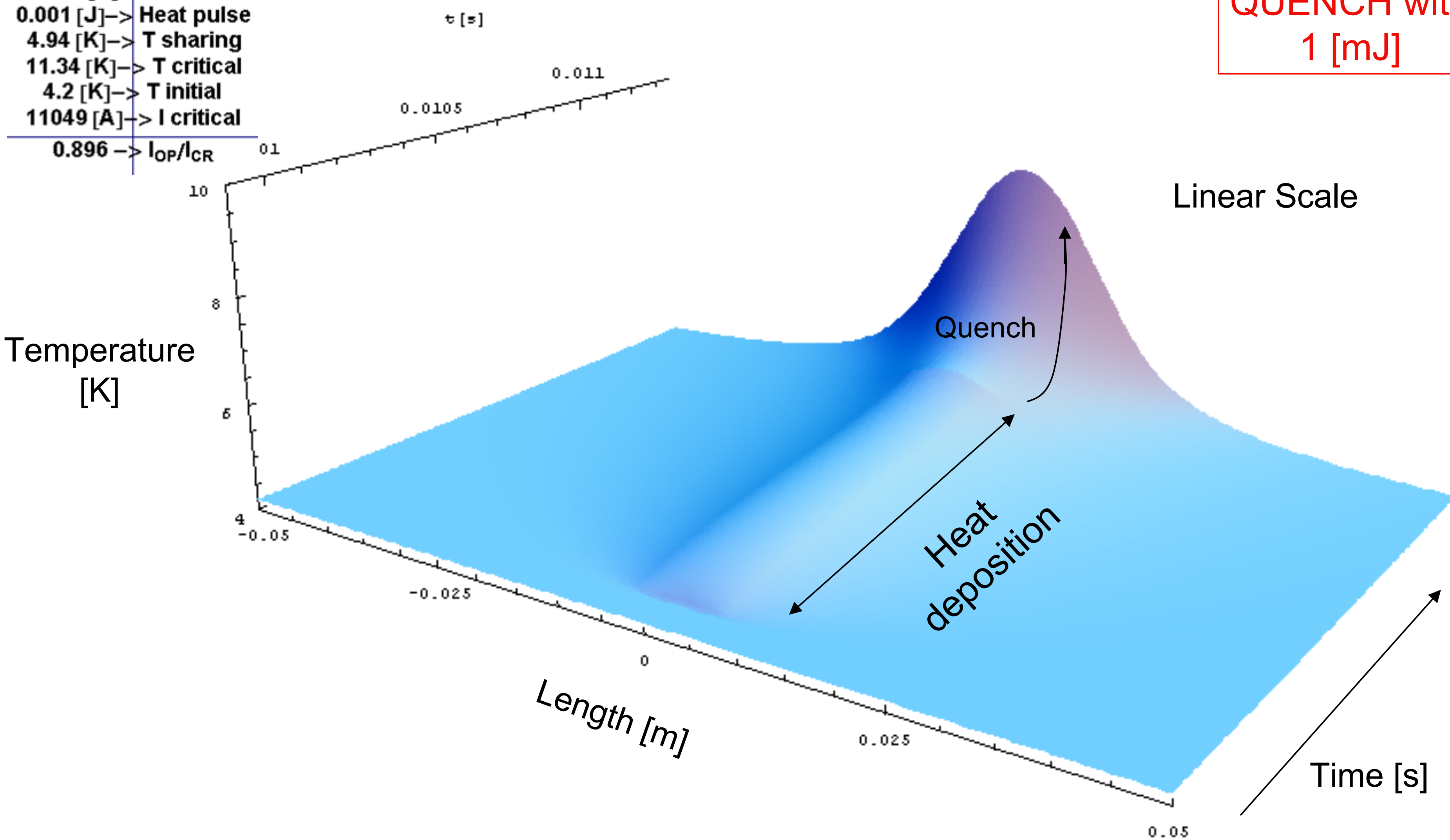

Analysis of SQ02

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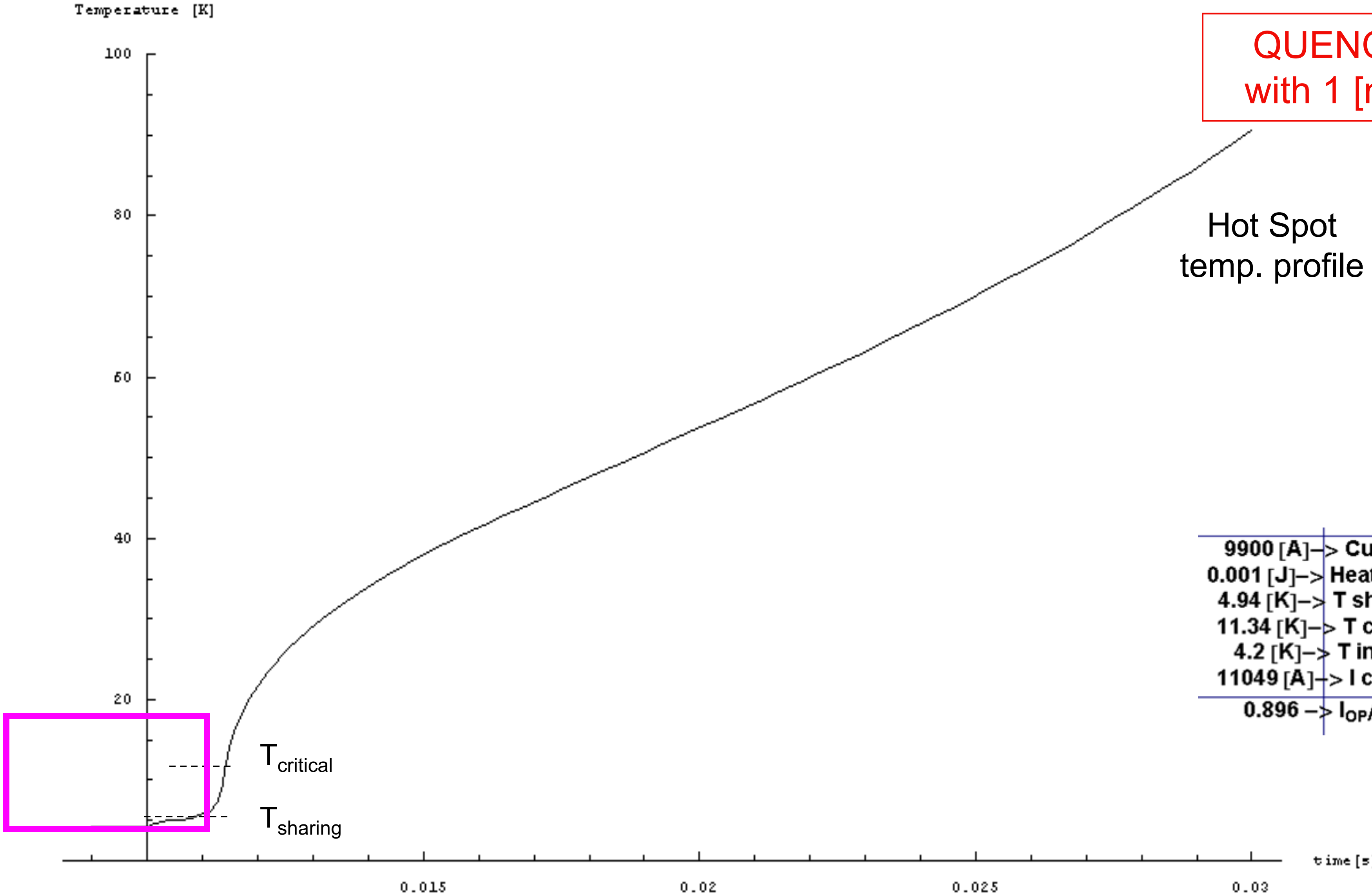
with 1 [mJ]



9900 [A]	-> Current
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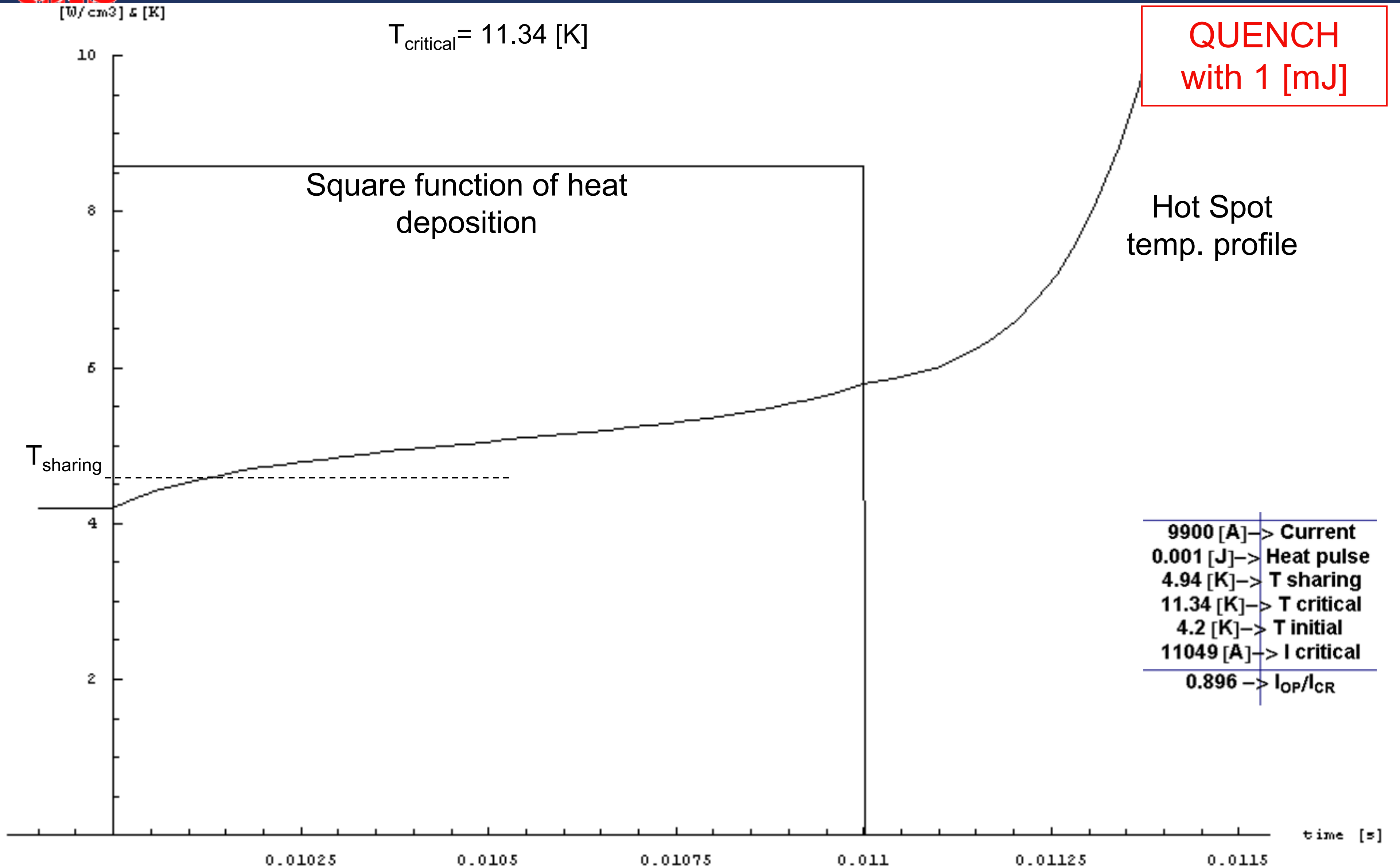
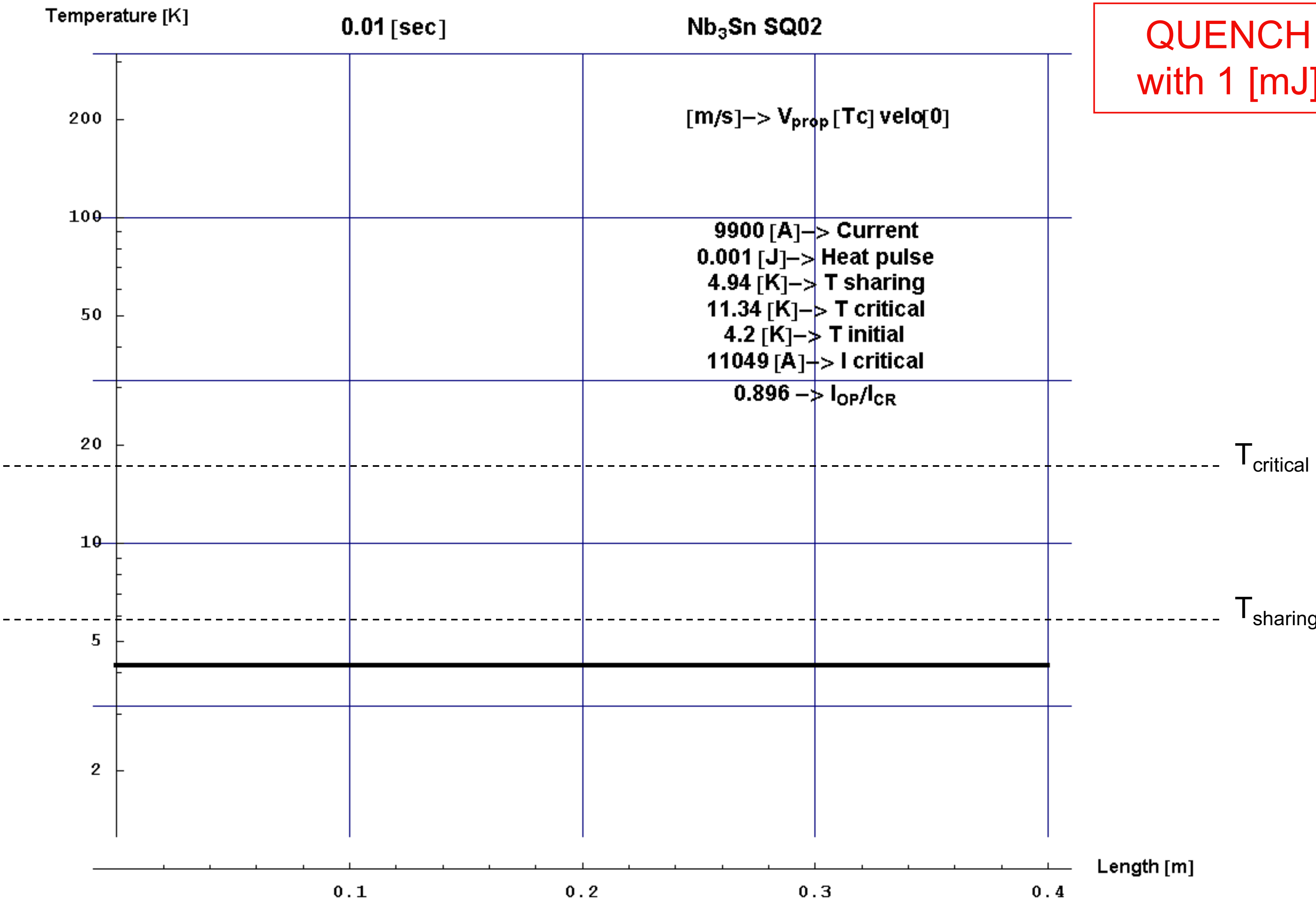
4.94 [K]	-> T sharing
11.34 [K]	-> T critical
4.2 [K]	-> T initial
11049 [A]	-> I critical
0.896	-> I_{OP}/I_{CR}

Analysis of SQ02



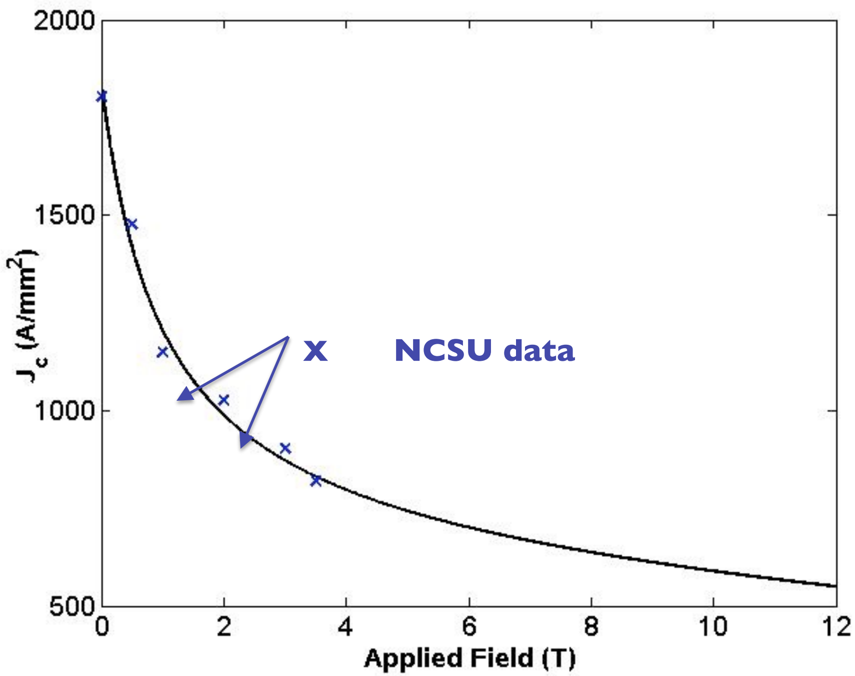
QUENCH
with 1 [mJ]

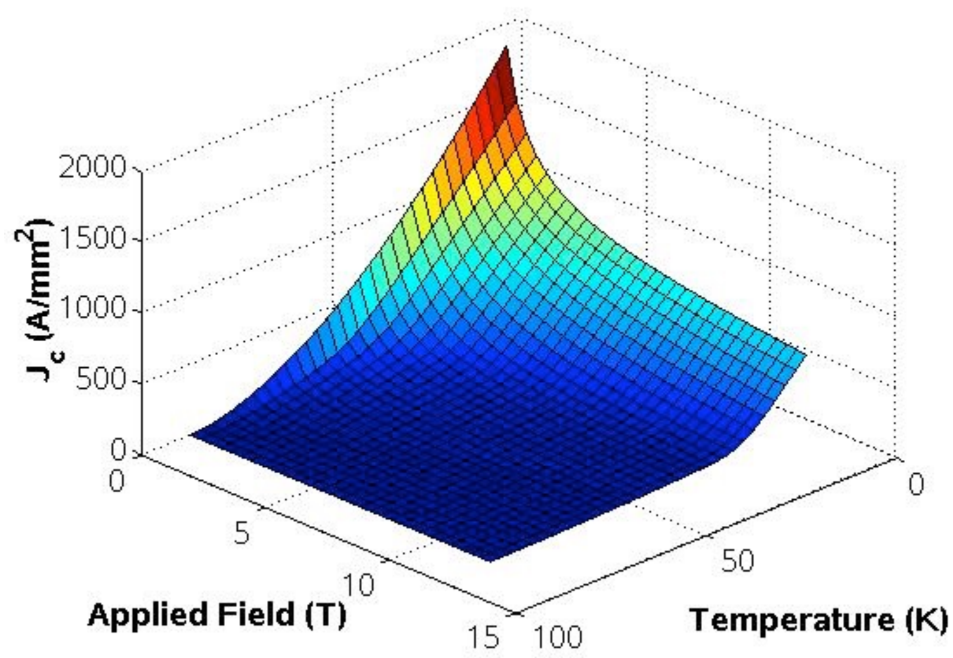


Bi2212 Model Properties

- $J_c(B,T)$ parameterization (Bottura, CryoSoft library) used to fit data from NCSU

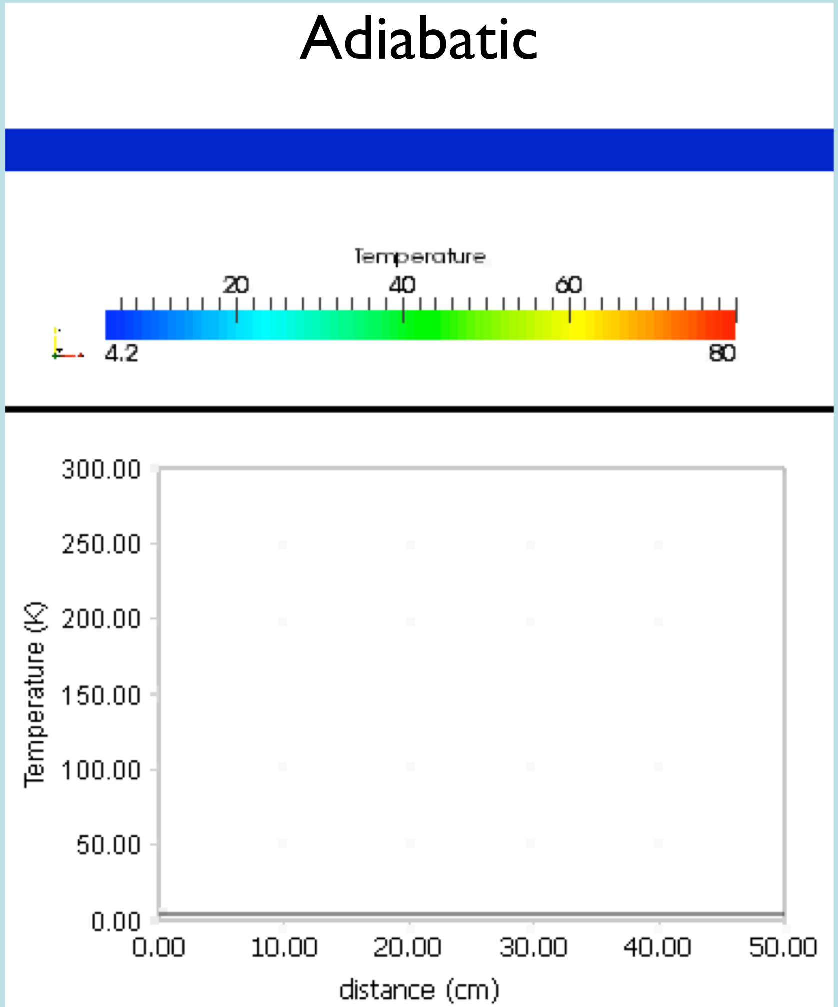
$$J_c(T,B) = J_0\left(1-\frac{T}{T_c}\right)^{\gamma}\left[(1-\chi)\frac{B_0}{B_0+B}+\chi\exp\left(-\frac{\beta B}{B_{c0}\exp\left(-\alpha T/T_c\right)}\right)\right]$$





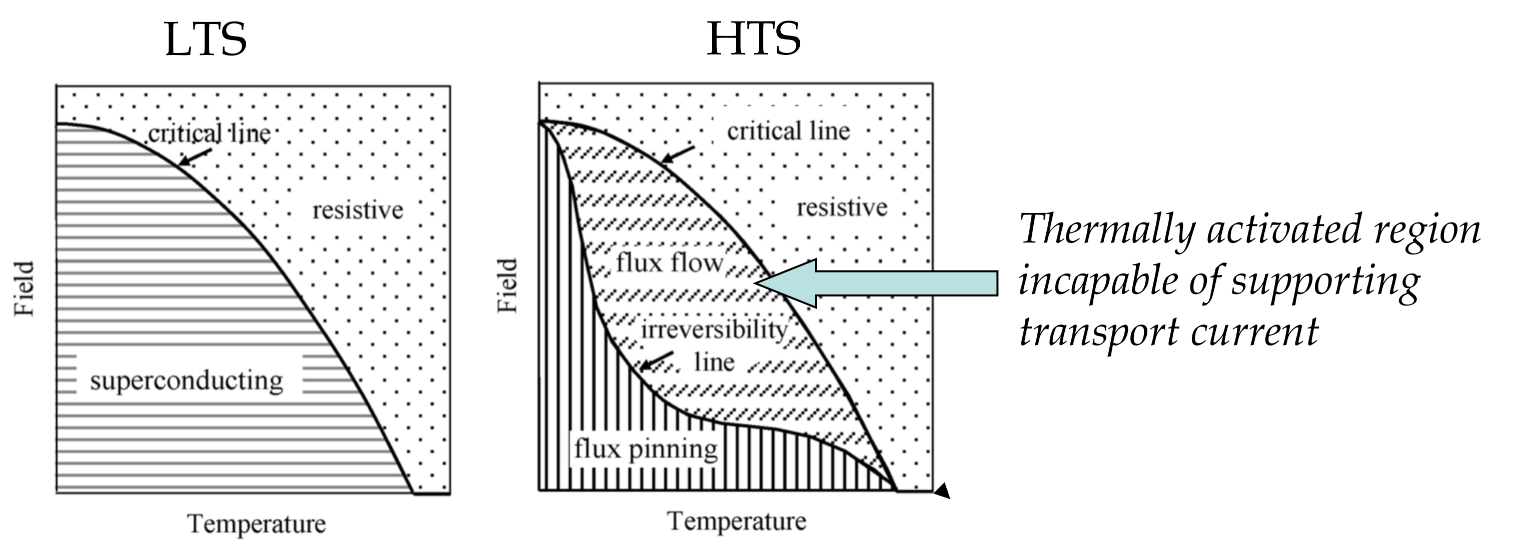
Note: need more measurements for material properties input

Quench Simulation Examples



HTS specific stability issues

- Whereas flux flow usually results in a flux jump condition in LTS materials, the far higher critical temperature of HTS materials provides significant heat capacity to mitigate the avalanche scenario
- The superconducting parameters for HTS materials and the (typically) higher operating temperature tend to increase the possibility of thermally induced flux motions: "melting" of the fluxoid lattice



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Summary

- Fluxoids in a type II superconductor carrying transport current are subjected to forces which, if not countered, will result in flux flow and associated heating.
- Useful conductors (i.e. capable of carrying transport current) have pinning sites that counter the forces on the fluxoid.
- In LTS materials, flux motion due to a breakdown of a pinning site will often initiate an avalanche of flux flow – a flux jump.
- Depending on the details of the conductor composition and the amplitude of the heat induced by a flux jump, the conductor will either recover or quench.
- The recovery or quench of a superconducting wire subjected to a thermal disturbance can be reasonably analyzed using simple 1D analytic models (with a little help from numerics to accommodate more accurate material property models).