HMW 2 USPAS LLRF

SOLUTIONS

1) $TF = \frac{V}{Ref} = \frac{G}{1+GH}$

1.1) $GH > 1 \rightarrow \sim \frac{G}{1+GH} \cong \frac{G}{GH} = \frac{1}{H}$

$GH < 1 \sim G$

1.2) $TF \cong G_0 + \Delta G$ when $GH << 1$ → we see the effect in closed loop

$TF \cong \frac{1}{H}$ when $GH >> 1$ → no effect due to disturbance

1.3) $V = V_0 + \Delta V = (G_0 + \Delta G)(R_{ref} + \Delta R_{ef})$

$$= \underbrace{G_0 R_{ref}}_{V_0} + G_0 \Delta R_{ref} + \Delta G R_{ref} + \cancel{\Delta G \Delta R}^{\,0}$$

$$\frac{\Delta V}{V_0} \cancel{+1} = \cancel{1+} \frac{\Delta R_{ref}}{\Delta R} + \frac{\Delta G}{\Delta G}$$

$$\left|\frac{\Delta V}{V_0}\right| < \left|\frac{\Delta R}{R_{ref}}\right| + \left|\frac{\Delta G}{G}\right| \simeq 0.1$$

$TF = \frac{KG}{1+KGH}$, $KGH >> 1 \Rightarrow \sim \frac{\cancel{KG}}{\cancel{KG}H} = \frac{1}{H}$

$\frac{V}{R_{ref}} = \frac{1}{H}$ $(V_0 + \Delta V) = (R_{ref} + \Delta R)\left(\frac{1}{H_0 + \Delta H}\right)$

$$H_0 \Delta V + \cancel{H_0 V_0} + \cancel{\Delta H \Delta V} + \Delta H V_0 = \cancel{R_{ref}} + \Delta R$$

$$H_0 \Delta V + \Delta H V_0 = \Delta R$$

$$\frac{\Delta V}{V_0} = \frac{\Delta R}{R_{ref}} + \frac{\Delta H}{H} \leq 2ppm$$

2)

$$\epsilon = R_{ref} - HV$$

$$V = \frac{R_L}{R_L + R_o} A\epsilon = \frac{R_L}{R_L + R_o} A\left(R_{ref} - HV\right)$$

$$V\left(1 + \frac{R_L A H}{R_L + R_o}\right) = \frac{A R_L}{R_L + R_o} R_{ref}$$

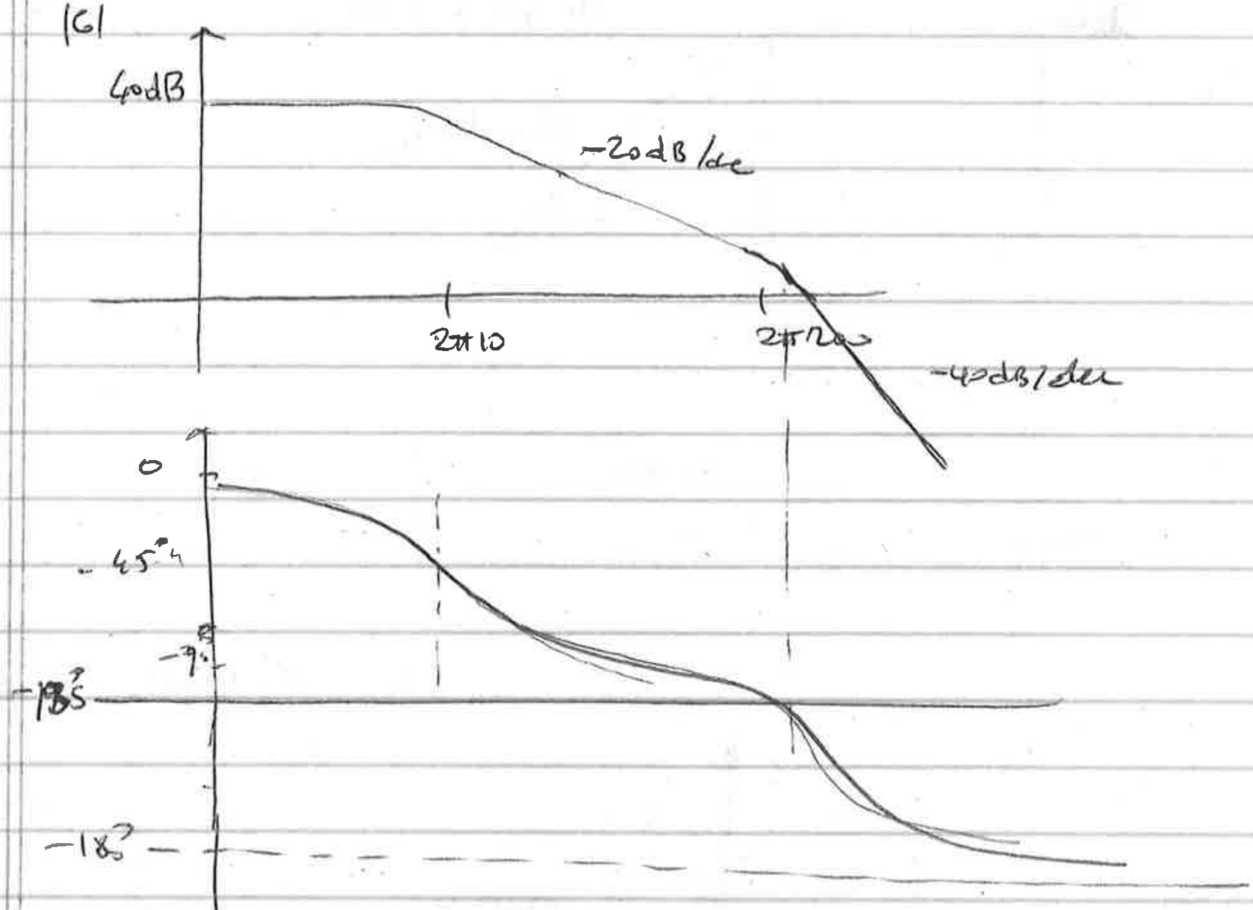
$$\frac{V}{R_{ref}} = \frac{\frac{A R_L}{R_L + R_o}}{1 + \frac{R_L}{R_L + R_o} A H} = \frac{A R_L}{R_L + R_o + R_L A H}$$

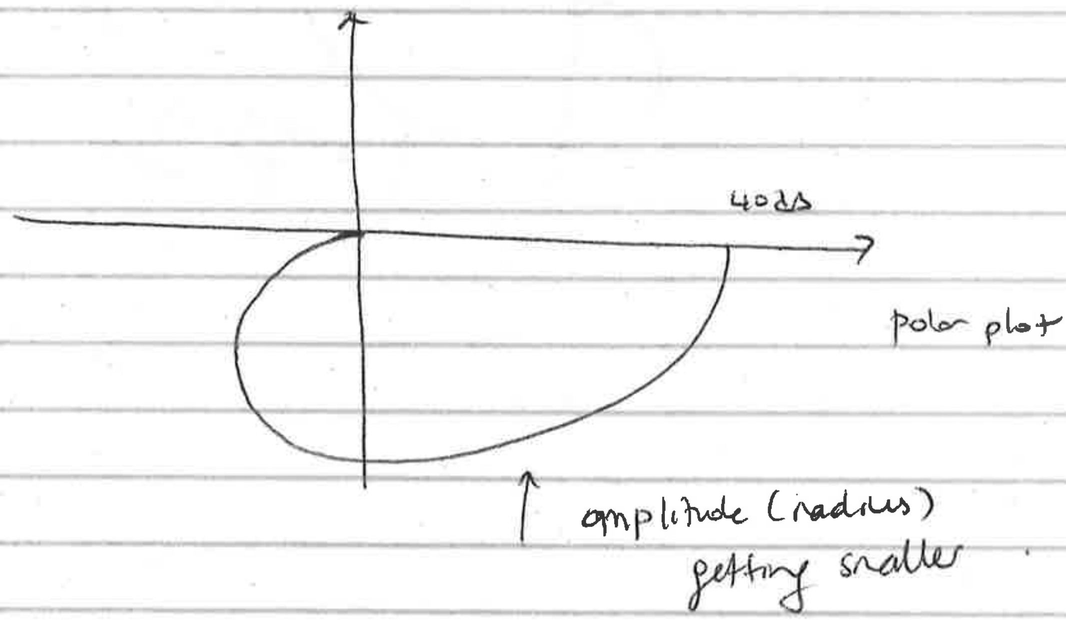
b)

$$I_{OUT} = \frac{V - A\epsilon}{R_o} = \frac{V + AHV}{R_o} =$$

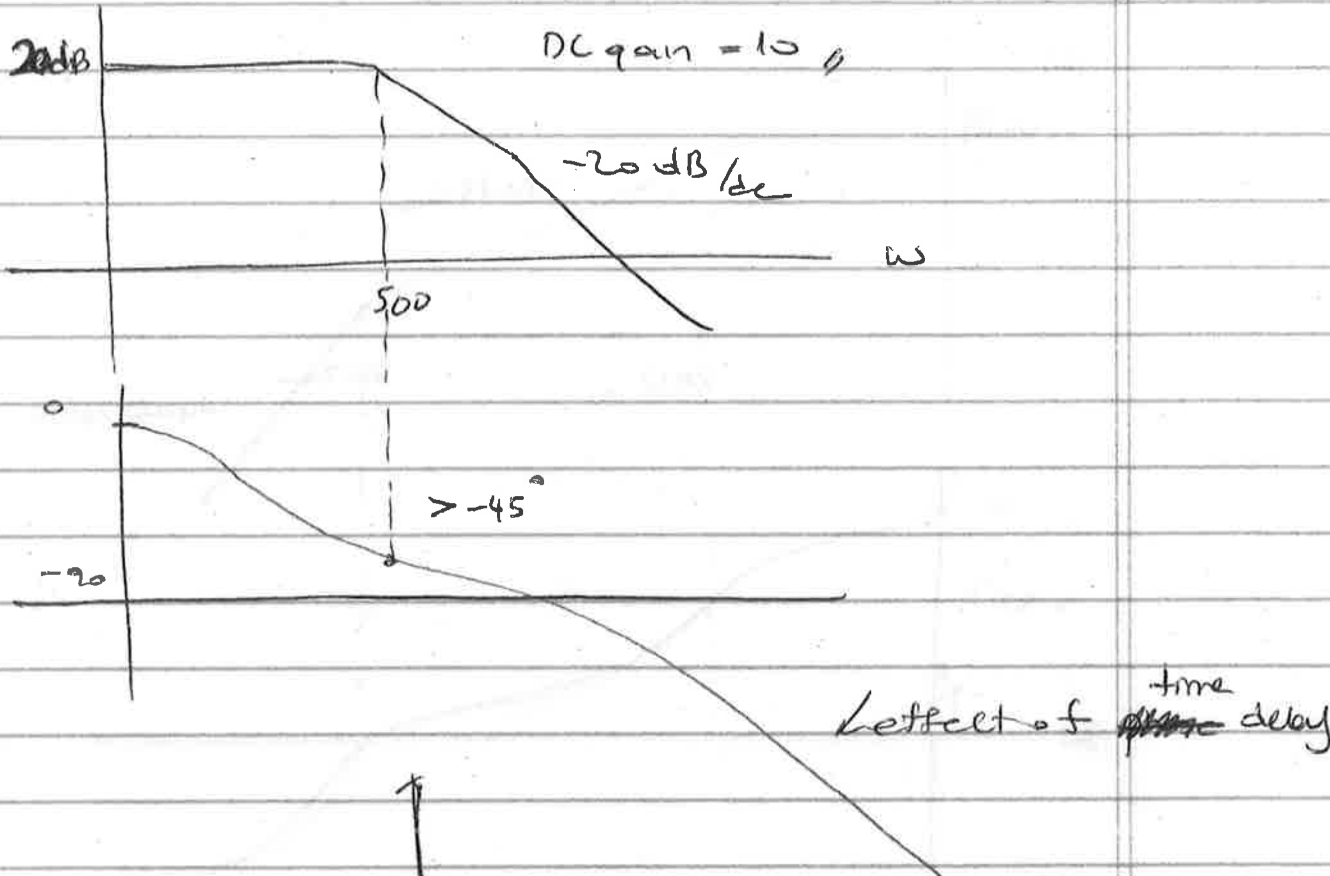
$$Z_{OUT} = V / I_{OUT} = \frac{R_o}{1 + AH} \quad //$$

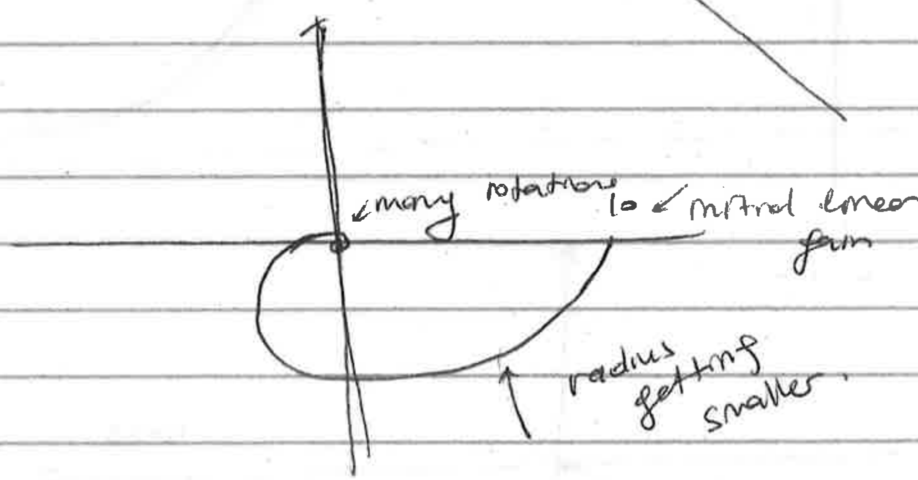
3) $$G(s) = \frac{100}{(1 + s/2\pi 10)(1 + s/2\pi 200)}$$





$$G(s) = \frac{5000\, e^{-1\mu s\, s}}{s+500}$$





a) $\dfrac{s+a}{(s^2+2\xi\omega_0 s+\omega_0^2)(s+b)}$ $b<a$ $b<\xi\omega_0<a$ $\xi<1$

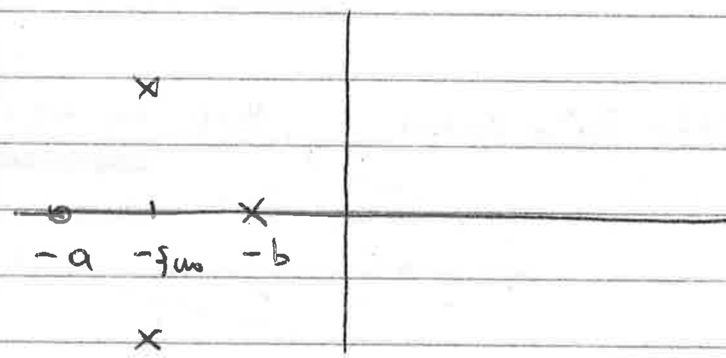
$s = \dfrac{-b \mp \sqrt{b^2-4ac}}{2a}$

(as^2+bs+c)

$\dfrac{-2\xi\omega_0 \mp \sqrt{4\xi^2\omega_0^2-4\omega_0^2}}{2} = -\xi\omega_0 \mp \sqrt{(\xi^2-1)\omega_0^2}$

$= -\xi\omega_0 \mp \omega_0\sqrt{\xi^2-1}$ $1>\xi$

$= -\xi\omega_0 \mp j\omega_0\sqrt{1-\xi^2}$



stable

b) $\dfrac{s-a}{s^2+bs+c} \rightarrow$ zero $= a$

poles $= \dfrac{-b \mp \sqrt{b^2-4ac}}{2a}$

if $b^2-4ac<0$ poles are $\dfrac{-b \mp j\sqrt{4ac-b^2}}{2a}$ ← complex

if $b>0$ stable

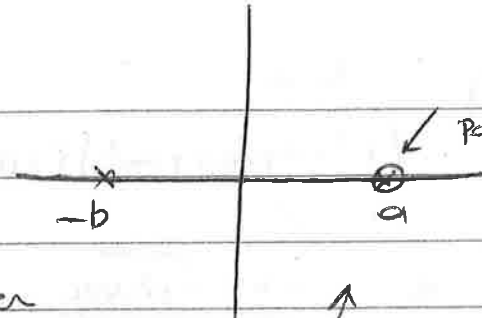
if $b^2-4ac>0$ $-b+\sqrt{b^2-4ac}$

and $b>0$ and $-b \mp \sqrt{b^2-4ac}$ $-b-\sqrt{b^2-4ac}$

can be negative or positive

stability depends on b, c

c) $\dfrac{s-a}{(s-a)(s+b)}$



↓ pole zero cancellation

this transfer function is stable

d) $\dfrac{K}{s^5+5s^3+4s^2+s}$ → Hurwitz unstable, missing term

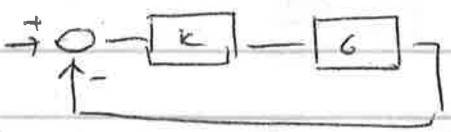
e) $\dfrac{K}{s^4+5s^3-3s^2+2s+1}$ Hurwitz unstable ~~[illegible]~~ coefficient is (−)

Important note : In general pole-zero cancellations are really dangerous. Mathematically it look the system is stable but in reality pure pole zero cancellations are very difficult so a system with pole zero cancellation on an unstable pole is practically <u>unstable</u>!!

4) $G(s) = \dfrac{1360}{s+1360}$.

$$|B(s)|_{s=\omega_B} = |G(s)|_{DC} \cdot \frac{1}{\sqrt{2}} \quad (-3dB)$$

$$|G(s)|_{j\omega_B} = \frac{\sqrt{1360^2}}{\sqrt{j\omega^2 + 1360^2}} = \frac{1}{\sqrt{2}} \rightarrow \cancel{\omega_{BW}} \cdot \omega_{BW} = 1360$$



$$TF_{CL} = \frac{GK}{1+GK}$$

$K=10$ $\quad TF_{CL} = \dfrac{13600/s+1360}{1+\dfrac{13600}{s+1360}} = \dfrac{13600}{s+1360+13600} = \dfrac{13600}{s+14960}$

$$|G(s)_{CL}|_{s=j\omega_{BW}} = |G(s)|_{DC} \cdot \frac{1}{\sqrt{2}} = \frac{13600}{14960} \cdot \frac{1}{\sqrt{2}}$$

$\omega_\beta = 14925$ //

$K=100$ $\quad TF_{CL} = \dfrac{136000}{s+137360}$ $\quad \omega_\beta \sim 137030$ //

$$C_L\,TF = \frac{GK}{1+GK} = \frac{\frac{1360K}{s+1360}}{1+\frac{1360K}{s+1360}} = \frac{1360k}{s+(K+1)1360}$$

$\Delta Ch = s + (k+1)1360 \rightarrow$

$\Delta_{ch.} = 0 \Rightarrow s = -(K+1)1360 > 0$

unstable

$\boxed{K > -1 \rightarrow \text{stable}}$

$$TF_{CL} = \frac{G\cancel{K}}{1+GK} \cdot e^{-T_d s} \quad \text{where} \quad T_d = 1\times10^{-6}\,s$$

$$\Delta\Theta_{needed} = -90^\circ$$

at what ω, $e^{-T_d s} = e^{-T_d j\omega}$ $\Rightarrow$ will introduce 90 phase shift.

~~dΦ~~ $\tau = \frac{d\phi}{d\omega}$

$$\Delta\Phi = \omega \cdot T_d \Rightarrow \frac{\pi}{2} = \omega \cdot 1\times10^{-6}\,s$$

T_d ↓ since this is constant

ω $(2\pi f)$ ↓ $\frac{1}{s}$

watch careful about units.

$$\omega = \frac{\pi}{2\times10^{-6}} = \frac{\pi}{2}\times10^{6}\ \text{rad/s}$$

what is the gain of the system at this frequency, it shouldn't be 1.

$$\left|TF_{CL}\right|_{\omega=\frac{\pi}{2}\times10^6} = \left|\frac{1360\,K_{critical}}{s + 1360 + 1360\,K_{critical}}\right| = 1$$

@ $j\omega$, $\omega = \frac{\pi}{2}\times10^{6}$

$$\frac{\sqrt{(1360\,K_{critical})^2}}{\sqrt{\cancel{(\frac{\pi}{2}\times10^6)^2 + (1360)^2} + (1360(K_{critical}+1))^2}} = 1$$

$$\left(j\,\frac{\pi}{2}\times10^{6}\right)^2 + \left((K_{critic}+1)\,1360\right)^2$$

$$K_{critical} \cong 1122$$

~~dB~~

$$TF_{CL} = \frac{136000}{s + 137360}$$
$K = 100$

$$\frac{\sqrt{(136000)^2}}{\sqrt{j\omega^2 + 137600^2}} = 1$$

$$\frac{136000^2}{1} = (j\omega)^2 + (137600)^2$$

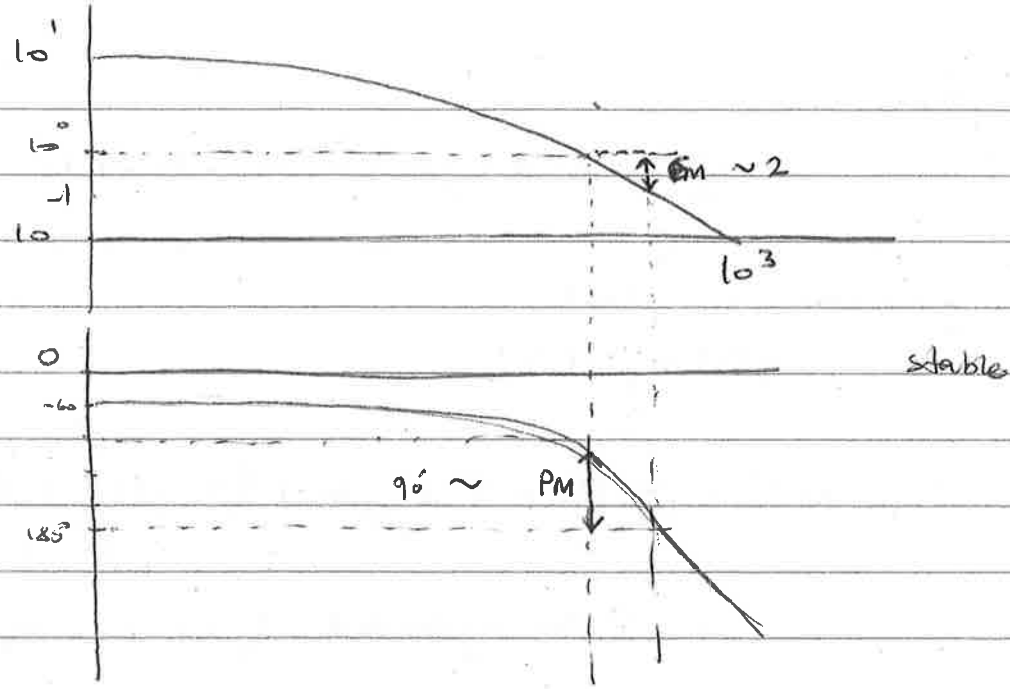
$$\omega = 19281 \text{ rad/s}$$

$$\theta = \omega \tau_{Delay}$$

$$\theta_{MAX} = 90°$$

$$\frac{\theta_{max}}{\omega} = T_D \Rightarrow \frac{\pi/2}{19281} = 8.1467e^{-5} \text{ second} = 0,81467 \mu s.$$

5)



b) → GM ~ 2

c) 3 > GM → unstable

Problem 6)

$$\underline{\dot{\tilde{V}}_c(t)} = (-\sigma - j\Delta\omega)\,\tilde{V}_c(t) + R_L\,\tilde{i}_T(t)$$

$$\tilde{V}_c(t) = V_{IN}(t) + j V_Q(t)$$
$$\tilde{i}_T(t) = I_{IN}(t) + j i_Q(t)$$

$$\dot{V}_{IN}(t) + j\dot{V}_Q(t) = (-\sigma - j\Delta\omega)\left(V_{IN}(t) + jV_Q(t)\right) + R_L\left(I_{IN}(t) + jI_Q(t)\right).$$
$$= -\sigma V_{IN} - j\Delta W V_{IN} - \sigma j V_Q + \Delta W V_Q + R_L I_{IN}(t) + R_L j I_Q$$

$$\dot{V}_{IN} = -\sigma V_{IN} + \Delta W \cancel{V_Q} + R_L I_{IN}$$
$$j\dot{V}_Q(t) = -j\Delta W V_{IN} - \sigma j V_Q + R_L j I_Q$$

$$\begin{bmatrix} \dot{V}_{IN} \\ \dot{V}_Q \end{bmatrix} = \begin{bmatrix} -\sigma & \Delta W \\ -\Delta W & -\sigma \end{bmatrix} \begin{bmatrix} V_{IN} \\ V_Q \end{bmatrix} + R_L \begin{bmatrix} I_{IN} \\ I_Q \end{bmatrix}$$

MONDAY

7) $Z_{co}(\omega) = \dfrac{R_o}{1+jR_o\left(\omega C - \frac{1}{\omega L}\right)}$ then $Z_c(\omega) = Z_{co}(\omega) \| Z_{cow}(\omega)$

$$= \frac{\dfrac{R_o}{1+jR_o(\omega C - \frac{1}{\omega L})} \cdot N^2 Z_o}{N^2 Z_o + \dfrac{R_o}{1+jR_o(\omega C - 1/\omega L)}} = \frac{N^2 Z_o R_o}{R_o + N^2 Z_o + j N^2 Z_o R_o\left(\omega C - \frac{1}{\omega L}\right)} =$$

$$= \frac{\cancel{N^2 Z_o} R_o}{\cancel{N^2 Z_o}\left[1+\beta + j R_o\left(\omega C - \frac{1}{\omega L}\right)\right]} = \frac{R_o/1+\beta}{1 + j\frac{R_o}{1+\beta}\left(\omega C - \frac{1}{\omega L}\right)} =$$

$$= \boxed{\frac{R}{1+jR\left(\omega C - \frac{1}{\omega L}\right)}}$$

2) $Q_L = \dfrac{Q_o}{1+\beta} = \dfrac{32 \times 10^3}{1+3.6} = \boxed{6956}$

$R_L = \left(\dfrac{\Gamma}{Q}\right) \dfrac{1}{2} \cdot Q_L = \dfrac{233.1\,\Omega}{2} \cdot 6956 = \boxed{8.1 \times 10^5\,\Omega}$

3) $\omega_{1/2} = \dfrac{\omega_o}{2Q_L} = \dfrac{476 \times 10^6 \times \cancel{2} \times \pi}{\cancel{2} \times 6956} = 2.15 \times 10^5$

$\omega_{1/2} = \dfrac{\omega_o}{2Q_o} = \dfrac{476 \times 10^6 \times 2 \times \pi}{2 \times 32 \times 10^3} = 4.67 \times 10^4$

WEDNESDAY #1

$$P_g\Big|_{Cav} = \frac{\hat{V}_c^{2}}{4\left(\frac{r}{Q}\right) Q_L}\left[\left(1 + \frac{\left(\frac{r}{Q}\right) Q_L I_{bo}}{\hat{V}_c}\cos\phi_b\right)^2 + \left(2 Q_L \frac{\Delta\omega}{\omega} + \frac{\left(\frac{r}{Q}\right) Q_L I_{bo}}{\hat{V}_c}\sin\phi_b\right)^2\right]$$

a) $\hat{V}_c = 16MV \quad \Delta\omega = 0,\ I_{bo} = 0$

$$P_g\Big|_c = \frac{(16e^6)^2}{4(1036)\ 4.12\times10^7} = 1500W \qquad \frac{1500}{0.9} = 1.67kW$$

$$P_g = P_{LWG} + P_g\Big|_c = 1.1 \cdot P_g\Big|_c = \boxed{1650W}$$

b) $$\frac{\left(\frac{r}{Q}\right) Q_L I_{bo}}{\hat{V}_c} = \frac{1036 \times 4.12\times10^7 \cdot 100e^{-6}A}{16e^6 V} = 0.2668.$$

$$P_g\Big|_c = \underbrace{1500W}_{\text{Part a)}}\left[\left(1 + 0.2668\cos 30^\circ\right)^2 + \left(0.2668\sin 30^\circ\right)^2\right] =$$

$$= 2300W$$

$$P_g = 1.1 \times 2300W = \boxed{2530W}$$

c) $$2 Q_L \frac{\Delta\omega}{\omega} = \frac{2 \cdot 4.12\times10^7 \cdot 20}{2\pi\ 1.3GHz} = \frac{2 \times 4.12\times10^7}{\cancel{2\pi} \cdot 1.3e^9\ Hz} \cdot \cancel{2\pi}\ 20Hz = 1.2831$$

$$P_g\Big|_c = 1500W\left[(1 + 0.2668)^2 + (1.2831)^2\right] = 4877W$$

at $\Delta\omega = \pm 20Hz$

$$\boxed{P_g = 1.1 \times 4877W = 5.364\ kW}$$