Engineering for Particle Accelerators, Tasks

Vyacheslav Yakovlev
U.S. Particle Accelerator School (USPAS)
Education in Beam Physics and Accelerator Technology

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Homework

Task 1. SRF 5-cell cavity designed for PIP II has the following parameters:

- Operating frequency is 650 MHz
- Acceleration voltage V is 20 MV
- R/Q is 620 Ohm
- O_0 at operation voltage is 3e10
- The beam current I is 2 mA

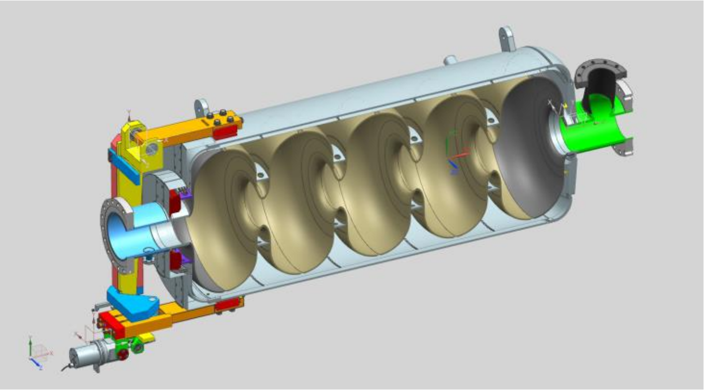
Estimate for CW operation:

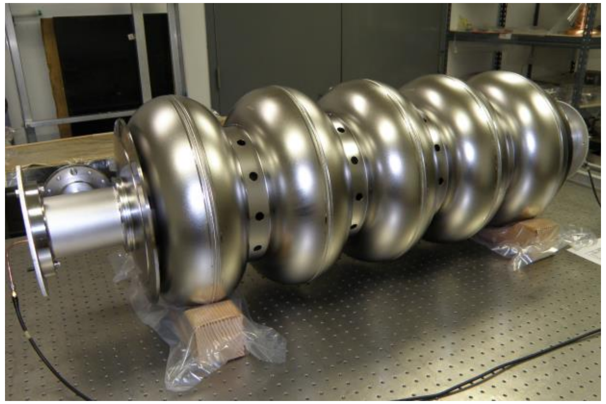
- The cavity loaded Q, $Q_L = V/(I \cdot (R/Q))$
- The cavity time constant, $\tau = 2Q_L/\omega$.
- The cavity bandwidth , $\delta f = f/Q_L$;
- Loss power in the cavity walls;
- The power transferred to the beam;
- Power required for refrigeration (conversion factor 1.e3 W/W,
  i.e., in order to remove 1 W from the cavity wall one needs wall plug power of 1 kW);
- Acceleration efficiency, the beam power over the sum of the beam power and power required for refrigeration.

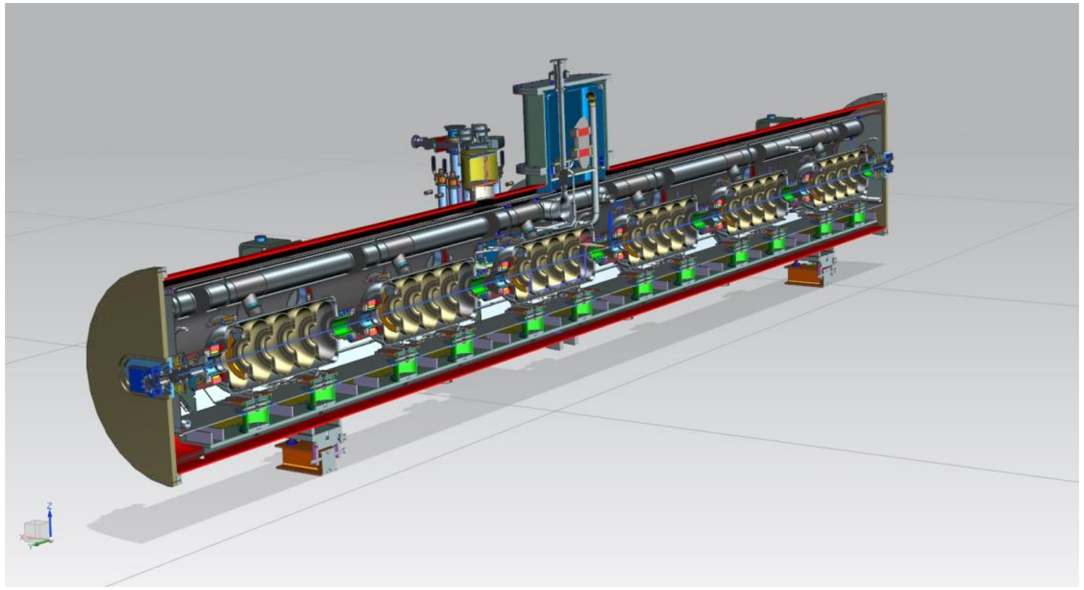
Solution

Task 1.

- The cavity loaded Q, $Q_L = V/(I\cdot(R/Q))$ = 20e6 V /(2e-3 A· 620 Ohm) = **1.6e7**
- The cavity time constant, $\tau = 2Q_L/\omega$ = 2·1.6e7/(2π·650e6 Hz)= 7.8e-3 sec=**7.8 msec** (compare to the beam pulse, 0.55 msec, see Task 2).
- The cavity bandwidth , $\delta f = f/Q_L$ = 650e6 Hz/1.6e7 = **40 Hz**.
- Loss power in the cavity walls, $P_{wall} = V^2/(R/Q\cdot Q_0)$= (20e6 V)²/(620 Ohm·3e10)=**21.5 W**;
- The power transferred to the beam, $P_{beam} = V\cdot I$ = 20e6 V·2e-3 A = **40 kW**;
- Power required for refrigeration (conversion factor 1.e3 W/W, i.e., in order to remove 1 W from the cavity wall one needs wall plug power of 1 kW); $P_{ref} = P_{wall}\cdot 1.e3$=**21.5 kW**.
- Acceleration efficiency η, the beam power over the sum of the beam power and power required for refrigeration. $\eta = P_{beam}/(P_{beam} + P_{ref})$=40e3/(40e3+21.5e3)=**65%**







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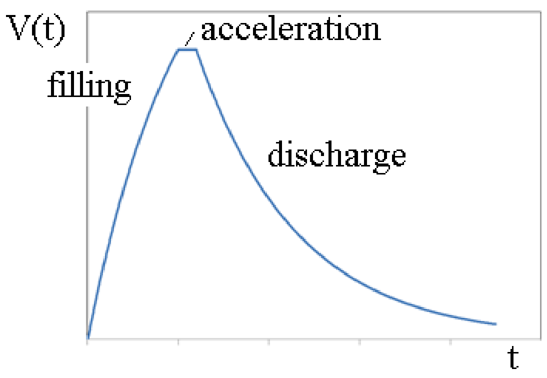
Task 2. PIP II SRF accelerator has CW capability, but will operate in the pulsed mode as an injector to the booster ring. The beam and cavity parameters are the same as for CW, Task 1. The beam pulse t_{beam} is 0.55 msec, repletion rate is 20 Hz. The beam appears when the cavity voltage reaches the operating value V, and backward wave (from the cavity to the RF source) is zero. Note that this wave is a sum of the reflection from the coupling element (which is equal to the incident wave), and the wave radiated from the cavity to the line. In the beginning of the cavity filling, the radiation is zero (the cavity is empty), and the backward wave is equal to reflection from the coupling element, and thus, to the incident wave. If there is no beam, the backward wave is again equal to the incident wave (no losses in the cavity) after the voltage reaches its maximal value, but it is again the sum of the wave reflected from the coupling element and radiated wave. It can be only if the radiated wave is two times larger than the wave reflected from the coupling element, and has opposite sign. It means that the beam appears when the cavity field reaches half of the maximal value (zero backward wave, the reflected wave is equal to the radiated wave, and they compensate each other). The cavity voltage, thus, increases during the filling as $V(t) = 2V(1-\exp(-t/\tau))$, τ is the time constant, $\tau = 2Q_L/\omega$. Filling is over when $V(t_{fill}) = V$, and therefore, the filling time t_{fill} is equal to $\tau \cdot \ln 2$. After the beam ends, the RF source is turned off, and cavity discharges as $V(t) = V\exp(-t/\tau)$. Thus, the cavity voltage has the following behavior:

Homework

- $V(t) = 2V(1-exp(-t/\tau))$, $t < t_{fill} = \tau \cdot ln2$ - filling; RF is on, no beam;
- $V(t) = V$, the beam acceleration; RF is on; $0<t<t_{beam}$, $t=0$ corresponds to the end of filling process
- $V(t) = Vexp(-t/\tau)$, cavity discharge; RF is off, no beam. $t=0$ corresponds to the end of acceleration

Estimate:

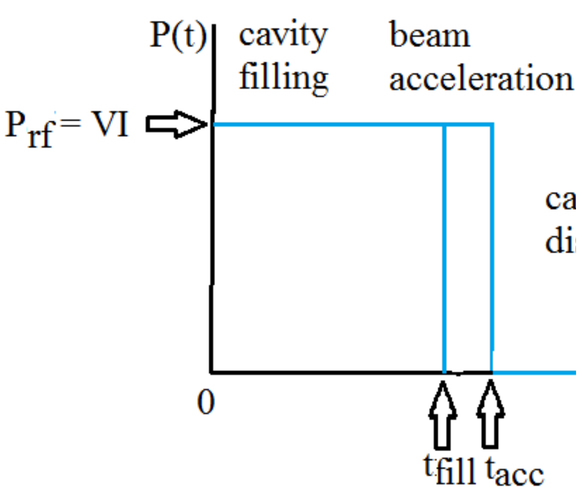
- Energy, delivered by the RF source to the beam during the pulse;
- Total energy, delivered by the RF source during the pulse;
- Total energy dissipated in the cavity wall during the pulse;
- Energy, required for refrigeration;
- Beam power /cavity (20 GHz repetition rate);
- Average RF power/cavity;
- Power necessary for refrigeration/cavity;
- Acceleration efficiency.

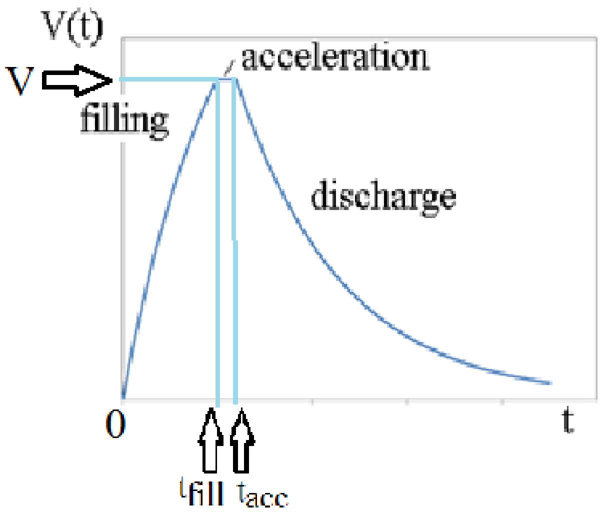


Homework

Task 2:

- Energy, delivered by the RF source to the beam during the pulse $w_{beam}=P_{beam}\cdot t_{beam}$ = 40 kW ·0.55e-3 sec= **22 J**;
- Total energy, delivered by the RF source during the pulse;





$w_{RF}=P_{beam}\cdot(t_{beam} + t_{fill}); t_{fill}=\tau ln2$;

t_{fill}= 7.8e-3 sec·ln2 = 5.4 msec;

w_{RF} = 40 kW · (5.4e-3 sec +0.55e-3 sec) = **238 J**.

The energy necessary from RF source for the cavity filling is

$w_{fill}=P_{beam}\cdot t_{fill} = w_{RF} - w_{beam}$= 40 kW · 5.4e-3 sec = **215 J**;

Note that all these 215 J go to the cavity; the energy stored in the cavity is $U=V^2/(R/Q\cdot\omega)$ = (20e6)2/(620·2π·650e6)=157 J.
The difference, *215 J-157 J= 58 J* is reflected during the filling process and will be dissipated in the RF load. The filling efficiency, $\eta_{fill} = U/w_{fill}$ = 157 J/215 J = 72%

- Total energy dissipated in the cavity wall during the pulse;

The energy dissipated in the cavity during the pulse w_{diss} is a sum of the energy dissipated during the cavity filling, w_{fill}; the energy dissipated during the beam acceleration, w_{acc}; and the energy dissipated during the cavity discharge, w_{dis}:

$w_{diss} = w_{fill} + w_{acc} + w_{dis}$

Homework

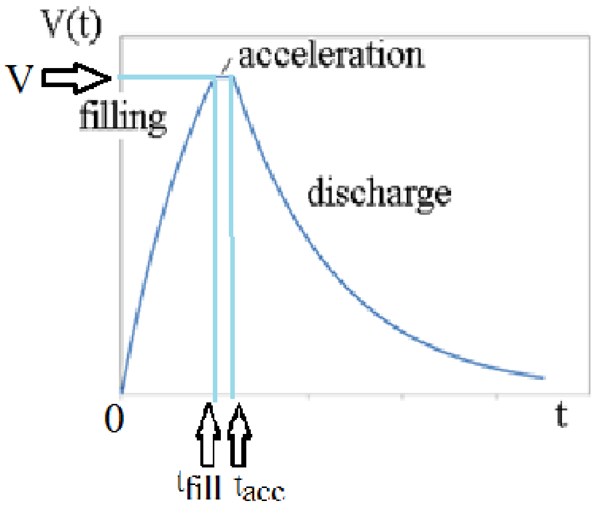
Task 2:

$$1.\ w_{fill} = \frac{\int_0^{t_{fill}} V(t)^2 dt}{\frac{R}{Q}Q_0} = \frac{4V^2 \int_0^{t_{fill}} \left(1-e^{-\frac{t}{\tau}}\right)^2 dt}{\frac{R}{Q}Q_0} =$$

$$= \frac{4V^2\tau \cdot \left(\ln 2 - \frac{5}{8}\right)}{\frac{R}{Q}Q_0} = \frac{0.27V^2\tau}{\frac{R}{Q}Q_0} =$$

$$= 0.27P_{wall}\tau = 0.27 \cdot 21.5\ \mathrm{W} \cdot 7.8\ \mathrm{msec} = \mathbf{45\ mJ};$$

(during the filling process $V(t) = 2V(1-\exp(-t/\tau))$, $t < t_f = \tau \cdot \ln 2$)



$$2.\ w_{acc} = \frac{\int_0^{t_{beam}} V(t)^2 dt}{\frac{R}{Q}Q_0} = \frac{V^2 t_{beam}}{\frac{R}{Q}Q_0} =$$

$$= P_{wall}t_{beam} = 21.5\ \mathrm{W} \cdot 0.55\ \mathrm{msec} = \mathbf{12\ mJ}$$

(during acceleration $V(t) = V$)

$$3.\ w_{dis} = \frac{\int_0^{\infty} V(t)^2 dt}{\frac{R}{Q}Q_0} = \frac{V^2 \int_0^{\infty} e^{-\frac{2t}{\tau}} dt}{\frac{R}{Q}Q_0} = \frac{0.5V^2\tau}{\frac{R}{Q}Q_0} =$$

(during the filling process $V(t) = V\exp(-t/\tau)$)

$$= 0.5P_{wall}\tau = 0.5 \cdot 21.5\ \mathrm{W} \cdot 7.8\ \mathrm{msec} = \mathbf{84\ mJ}$$

Therefore, $w_{diss} = w_{fill} + w_{acc} + w_{dis} = 45\ \mathrm{mJ} + 12\ \mathrm{mJ} + 84\ \mathrm{mJ} = \boldsymbol{141\ mJ}$

Homework

- Energy, required for refrigeration;
  $w_{ref} = w_{diss} \cdot 1000 = \mathbf{141\ J}$
- Beam power /cavity (20 GHz repetition rate);
  $P_{beam} = w_{beam} \cdot 20\ Hz = 22\text{J} \cdot 20\ Hz = \mathbf{440\ W}$
- Average RF power/cavity;
  $P_{RF} = w_{RF} \cdot 20\ Hz = 238\ \text{J} \cdot 20\ Hz = \mathbf{4.8\ kW}$
- Power necessary for refrigeration/cavity;
  $P_{ref} = w_{diss} \cdot 20\ Hz = \mathbf{2.8\ kW}$
- Acceleration efficiency
  $\eta = P_{beam}/(P_{RF} + P_{ref})$=440/(4.8e3+2.8e3)=**5.7%** (compared to **65%** in CW regime)

Homework

Thus, the pulse regime is not effective in this case, because we need additional high RF energy to charge the cavity, and we cannot use this energy after acceleration. Note, that if we define the “RF” efficiency η_{RF} , which is simply

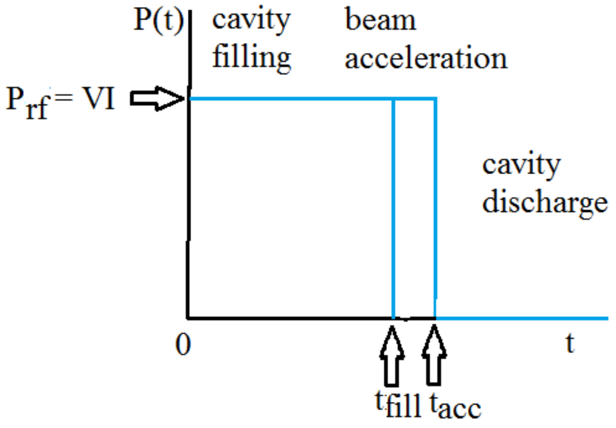
$$\eta_{RF}= w_{beam}/(w_{RF})=t_{beam}/(t_{fill}+t_{beam}) = 1/(t_{fill}/t_{beam}+1) = 1/[(2Q_L\cdot ln2/\omega)/t_{beam})+1] = 1/[2V\cdot ln2/(R/Q\cdot I\cdot\omega\cdot t_{beam})+1]=$$

$$t_{fill}=\tau ln2 = (2Q_L/\omega)\cdot ln2; \qquad Q_L = V/(R/Q\cdot I$$

$$= 1/[2V\cdot ln2/(R/Q\cdot\omega\cdot q)+1] \text{ ,}$$

i.e.,

$$\eta_{RF}= w_{beam}/(w_{RF})= 1/[2V\cdot ln2/(R/Q\cdot\omega\cdot q)+1]$$



where

- -V is the cavity voltage,
- -I is the beam current,
- -ω is cyclic frequency,
- -q is the total charge of the protons in the pulse, $q=I^* t_{beam}$ = 2 mA ·0.55 msec=1.1e-6 C

In the case considered in the Task 2 this RF efficiency is $t_{beam}/(t_{fill}+t_{beam})$ =0.55 msec/6.01 msec =9% (it is the fraction of the RF energy, which goes to the beam).

One can see that if q is large, the RF efficiency is high.

Note that for CW η_{RF}= =100%.

Homework

However, these estimation do not include two other sources of losses:

- Losses in the RF source;
- Decrease of the efficiency of a cryo-plant at lower load in pulse regime.

  Wall –plug efficiency $\eta_{\text{RF source}}$ of a typical RF source for conversion from AC → DC→RF is 50 % (or even lower);

  Relative efficiency η_{ref} of the cryo-plant at low load may be about 30% of maximal.

- Therefore, efficiency at CW will be:

$\eta = P_{beam}/(P_{beam}/0.5+P_{ref})$=40e3/(80e3+21.5e3)= **33% (instead of 65%);**

- and efficiency in pulse regime:

$\eta = P_{beam}/(P_{beam}/0.5+ P_{ref}/0.3)$=440/(9.6e3+5.6e3)= **2.9% (instead of 5.7%)**

Homework

For example, for SNS β=0.81 cavity one has:

- R/Q ~ 700 Ohms;
- f=805 MHz;
- I=~24 mA;
- t_{beam}=1 msec,
- V=10 MV

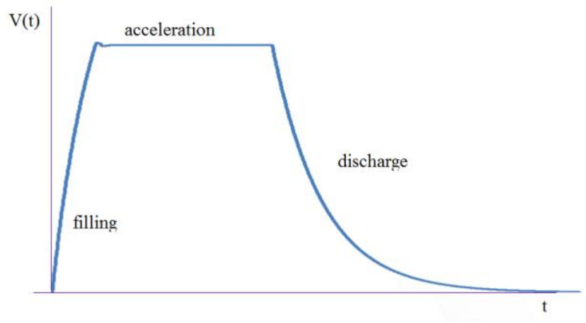
$$\eta_{RF} = 1/[2V\cdot \ln 2/(R/Q\cdot\omega\cdot q)+1]$$

Therefore,

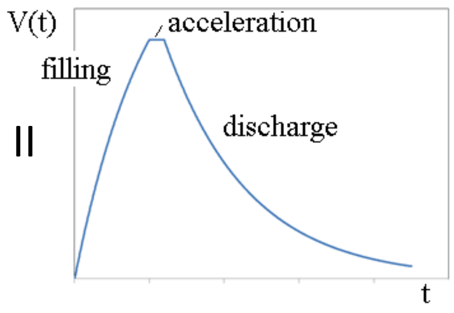
q = 24e-6 C (compared to 1.1e-6 C for the PIP II case we considered in the Task 2), and RF efficiency (not total acceleration efficiency) is 86%!

In this case the loaded quality factor is Q_L=V/(R/Q*I)=0.59e6, the time constant is $2Q_L$/omega = 0.24 msec, the filling time is 0.24msec*ln2=0.16 msec. Therefore, the RF sources works for 1.16 msec and RF efficiency (not total efficiency) is 1 msec/1.16 msec = 86% compared to 5.7% fr PIP II.

SNS



PIP II



Homework

We see that RF efficiency and therefore, overall plug efficiency, is determined by value of $1/2 \cdot R/Q \cdot \omega \cdot q$, which characterizes the beam loading.

What is it?

Perry B. Wilson
1927-2013

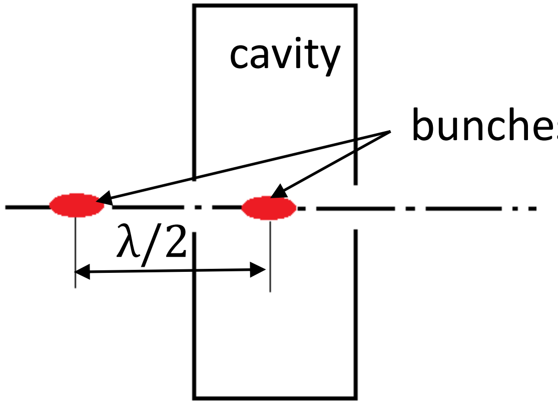
Wilson's theorems:

1. The bunch exiting the empty cavity, decelerates by $V_i/2$, where V_i is the voltage left in the cavity.

Two bunches with the distance between them of $\lambda/2$ excite total zero voltage.
If on bunches "sees" fraction α of V, one has: $q_b(V_i - \alpha V_i) = q_b \alpha V_i \rightarrow \alpha = 1/2$.

2. The voltage V exited by the bunch with the charge q_b is $V_i = 1/2 \cdot R/Q \cdot \omega \cdot q_b$

Energy conservation law: $1/2 \cdot V_i \cdot q_b = V_i^2/(R/Q \cdot \omega) \rightarrow V_i = 1/2 \cdot R/Q \cdot \omega \cdot q_b$



If the beam pulse is short compared to time constant τ (field decay time),
$V_I = 1/2 \cdot R/Q \cdot \omega \cdot q$ is a total voltage induced by the beam pulse in the cavity,
q is a total charge, $q = \Sigma q_b = I \cdot t_{beam}$.

Therefore

$$\eta_{RF} = w_{beam}/(w_{RF}) = 1/[\ln 2 \cdot V/V_I + 1]$$

For PIP II: V_I = 1.4 MV compared to V=20 MV; η_{RF}= 9%.; (q is low, $V_I \ll V$, ; η_{RF} is low)
For SNS: V_I = 42.5 MV compared to V=10 MV; η_{RF}= 86%; (q is high, $V_I \gg V$, ; η_{RF} is high)

Homework

Thus, overall efficiency:

$$\eta = P_{beam}/(P_{beam}/(\eta_{RF} \cdot \eta_{RF\ source}) + P_{ref}/\eta_{ref})$$

Large beam current, and thus, beam loading, always allows high efficiency – “RF” efficiency and total acceleration efficiency. But the beam current (and the beam pulse width) are determined by the linac applications, which finally determines the linac efficiency.