Project 3

Compact
Laser-plasma light source

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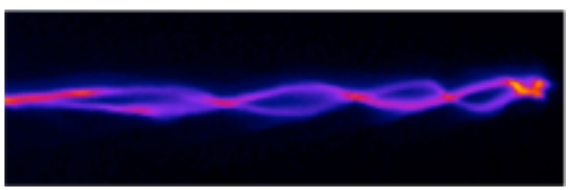

Outline

Laser parameter design

- Laser energy / power / length characteristics
- Radial spot - Gaussian Laser Rayleigh length
- Self-guiding distance of laser - **critical power**

Plasma parameter design

- Gas jet - super-sonic gas flow – Gas density profile
- Plasma density - **depletion length**
- Beam offset from acc. phase - **de-phasing length**

Laser-plasma interaction design

- Plasma density choice – field / depletion length / de-phasing length
- Plasma field × acceleration length – beam energy
- Beam properties – divergence / bunch length – emittance / beta-func

Parameter Suggestions

Laser parameter design

Laser pulse design

Laser pulse parameters :

Pulse-length

Power / Energy

Focal-spot

Laser pulse design

Design e-beam energy → 1 GeV

- choice of **beam charge** – depends upon **laser-energy**
- **assume** **laser-energy** on target – **10 J**
- **assume** **laser coupling eff** into plasma ~ **50%** **(FWHM)**
- plasma wavelength : $\lambda_{pe} \simeq 2\pi \frac{c}{\omega_{pe}} a_0$ - **pulse-length**
- plasma electric field : $E_{pe} = \frac{m_e c \omega_{pe}}{e} a_0^{3/2}$ - **energy-loss**
- **nearly consistent** acc. field / structure over acc. length :

  a_0 (exit) ~ 0.5 a_0 (input)
- charge → 3.75 J × 0.3 (beam-loading) / (10^9 eV) → ~ **1 nC**
- **realistic charge** → **10 pC**

Laser pulse – Radial focal spot

Radial dynamics of the laser

- Gaussian Laser Rayleigh length – Radial spot

$$\mathbf{E}(r,z) = E_0\,\hat{x}\,\frac{w_0}{w(z)}\exp\left(\frac{-r^2}{w(z)^2}\right)\exp\left(-i\left(kz + k\frac{r^2}{2R(z)} - \psi(z)\right)\right)$$

$$I(r,z) = \frac{|\Re(\mathbf{E}\times\mathbf{H})|}{2} = \frac{|E(r,z)|^2}{2\eta} = I_0\left(\frac{w_0}{w(z)}\right)^2\exp\left(\frac{-2r^2}{w^2(z)}\right)$$

$$w(z) = w_0\,\sqrt{1+\left(\frac{z}{z_R}\right)^2} \qquad z_R = \frac{\pi w_0^2}{\lambda}$$

w_0 = 20 um **z_R = 1.5 mm** **λ = 0.8 um**

- Self-focusing / guiding of laser – **prolong high-a_0 interaction > few mm**

Guiding the Laser pulse

Self-guiding of the laser by the plasma

- Self-guiding in a Gas (homogeneous) – **self-focusing**
- Physical Optics equation :

$$\frac{d}{dt}\tilde{\boldsymbol{r}}_\perp = \frac{\partial \omega}{\partial \boldsymbol{k}_\perp}$$

$$\frac{d}{dt}\boldsymbol{k}_\perp = -\frac{\partial \omega}{\partial \tilde{\boldsymbol{r}}_\perp}$$

- Ray transverse equation :

$$\frac{d^2\tilde{x}}{dt^2} + \Omega^2(\tilde{r}, \tilde{z})\tilde{x} = 0$$

$$\frac{d^2\tilde{y}}{dt^2} + \Omega^2(\tilde{r}, \tilde{z})\tilde{y} = 0$$

$$\Omega^2(r, z) = \frac{1}{2k^2 r}\frac{\partial \omega_p^2(r, z)}{\partial r}$$

- Radial variation of ω_{pe} :

$$\omega_p^2(r, z) = \frac{n(r, z)}{n_0}\frac{\omega_{p0}^2}{\gamma_\perp(r, z)}$$

relativistic

channel-based

Laser power required

Relativistic self-guiding critical laser power

- Self-guiding – Critical laser power :

$$P_{\text{crit}} = 2c\left(\frac{m_e c^2}{e}\right)^2 \frac{\omega^2}{\omega_{p0}^2} \simeq 17.4 \times 10^9 \frac{\omega^2}{\omega_{p0}^2}\ \text{W}$$

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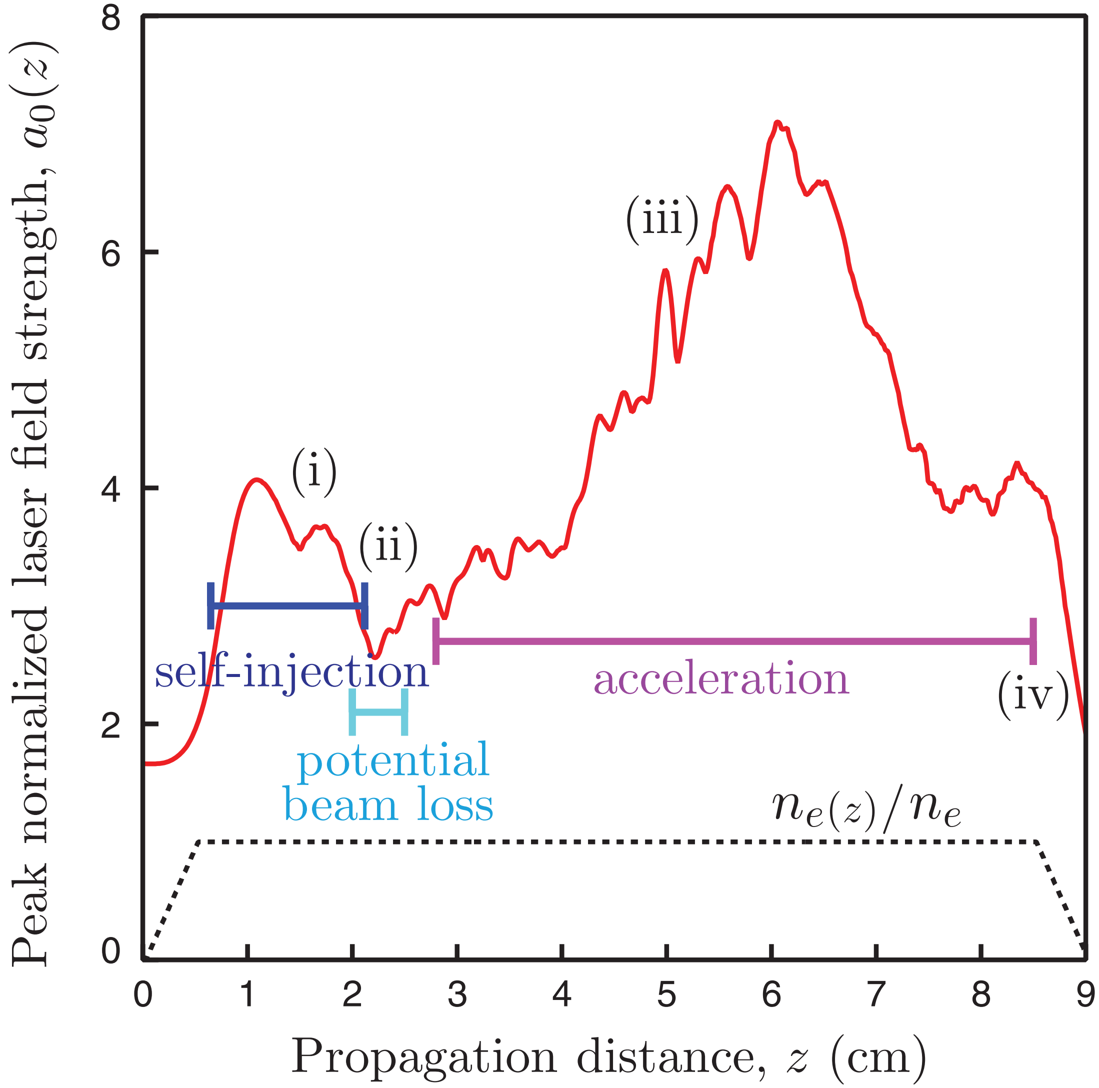
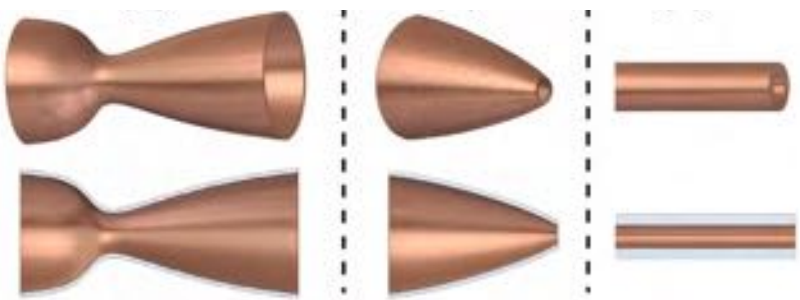
Numerical solution – Laser guiding



DOI: 10.1103/PhysRevLett.113.245002

Plasma parameter design

Gas jet – design

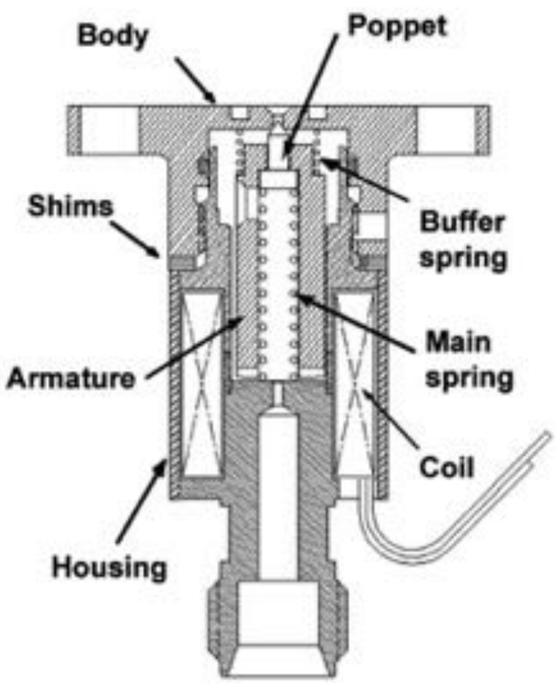


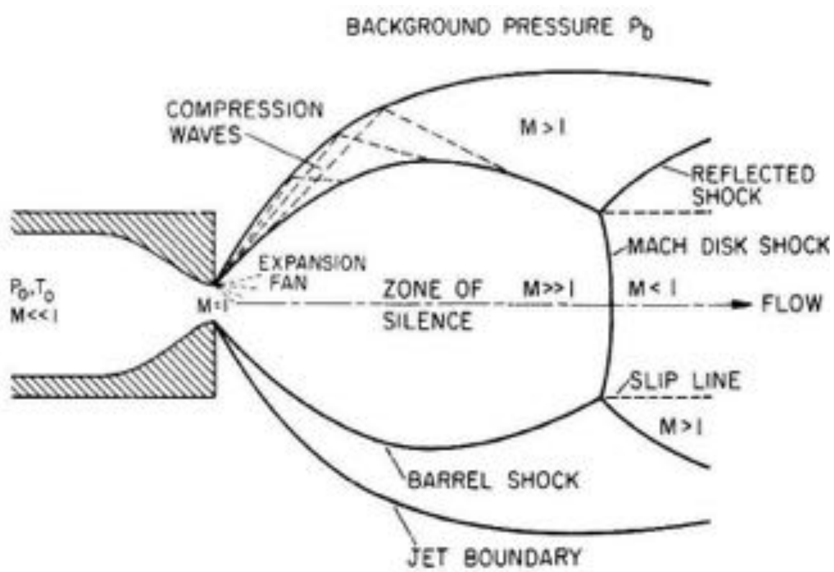
$$v_{Gas} = \sqrt{\frac{\gamma_a (N_A k_B)\ n_{Gas} T_{Gas}}{(N_A m_{Gas})}}$$

$$M \equiv v_{Gas}/c_s$$

$$R = P_0/P_a \quad G = \left(\frac{\gamma+1}{2}\right)^{\frac{\gamma}{\gamma-1}}$$

supersonic flow : $R > G$





Plasma – de-phasing length

Laser group velocity
Bubble velocity

$$v_g\ (laser) = c\left[1-\left(\frac{\omega_{pe}}{\omega_0}\right)^2\right]^{1/2}$$

$$\beta_\phi^{pe} \simeq \beta_g^{laser}(x) = \left(1-\frac{n_e(x)}{n_{crit}}\right)^{1/2}$$

time for e-beam @ c
to slip from “good”
Phase of bubble field

$$\frac{\lambda_{pe}}{4(c-v_\phi^{pe})}$$

c t_{slip} : de-phasing length

$$L_d = \frac{\lambda_{pe}}{4}\frac{1}{1-\beta_\phi^{pe}} = \frac{\lambda_{pe}}{2}\gamma_\phi^{pe\ 2}\Big|_{\beta_\phi\to 1}$$

$$\gamma_\phi^{pe\ 2} = \frac{\omega_0^2}{\omega_{pe}^2} = \frac{\lambda_{pe}^2}{\lambda_0^2}$$

$$L_d = \frac{1}{2}\frac{\lambda_{pe}^3}{\lambda_0^2} = \frac{1}{2\lambda_0^2}\left(\frac{c^2 m_e}{2e^2}\right)^{3/2} n_e^{-3/2}(x)$$

Plasma – depletion length

Laser energy : $$\mathcal{E}_L = \frac{E_0^2}{8\pi} \, c\tau_p \, \pi r_0^2$$

Plasma wave energy : $$\mathcal{E}_{wake} = \frac{E_{pe}^2}{8\pi} \, L_p \, \pi r_0^2$$

Total energy exchange : $$E_0^2 \; c\tau_p = E_{pe}^2 \; L_p$$

$$L_p = \frac{1}{2a_0} \frac{\lambda_{pe}^3}{\lambda_0^2} = \frac{1}{2\lambda_0^2} \left(\frac{c^2 m_e}{2e^2} \right)^{3/2} n_e^{-3/2}(x) a_0^{-1}$$

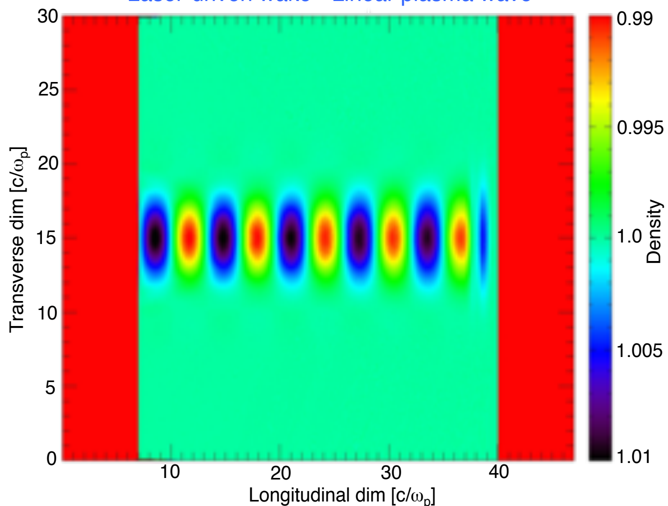
Driven electron density waves

Linear electron density waves

FWHM radius $4\frac{c}{\omega_{pe}}$ $\delta n \propto a_0^2 = 0.01$

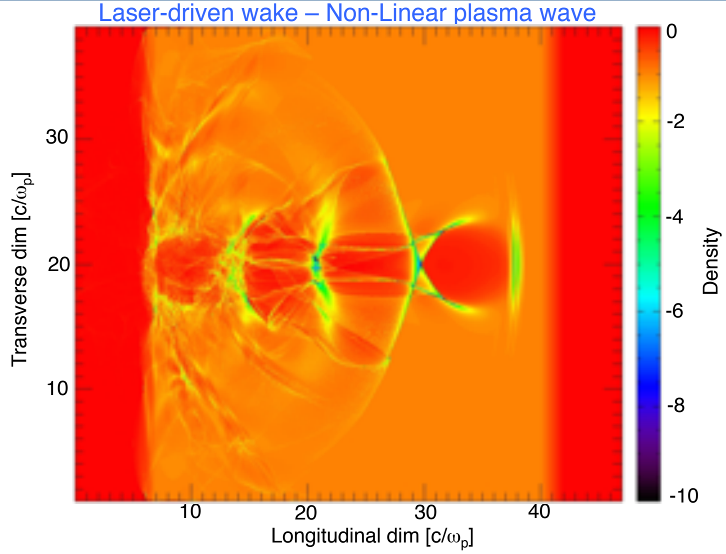
FWHM pulse length $2\frac{c}{\omega_{pe}}$ $\frac{\omega_0}{\omega_{pe}} = 10$

Laser-driven wake - Linear plasma wave



Non-linear electron density waves

Laser-driven wake – Non-Linear plasma wave



Laser-driven linear electron density wave :

$$\beta_\phi^{pe} \simeq v_g^{laser}/c$$

$$\left(\frac{1}{c^2}\frac{\partial^2}{\partial t^2} + k_{pe}^2\right)\frac{n^{(2)}}{n_0} = \nabla^2\frac{a_\perp^2}{2}$$

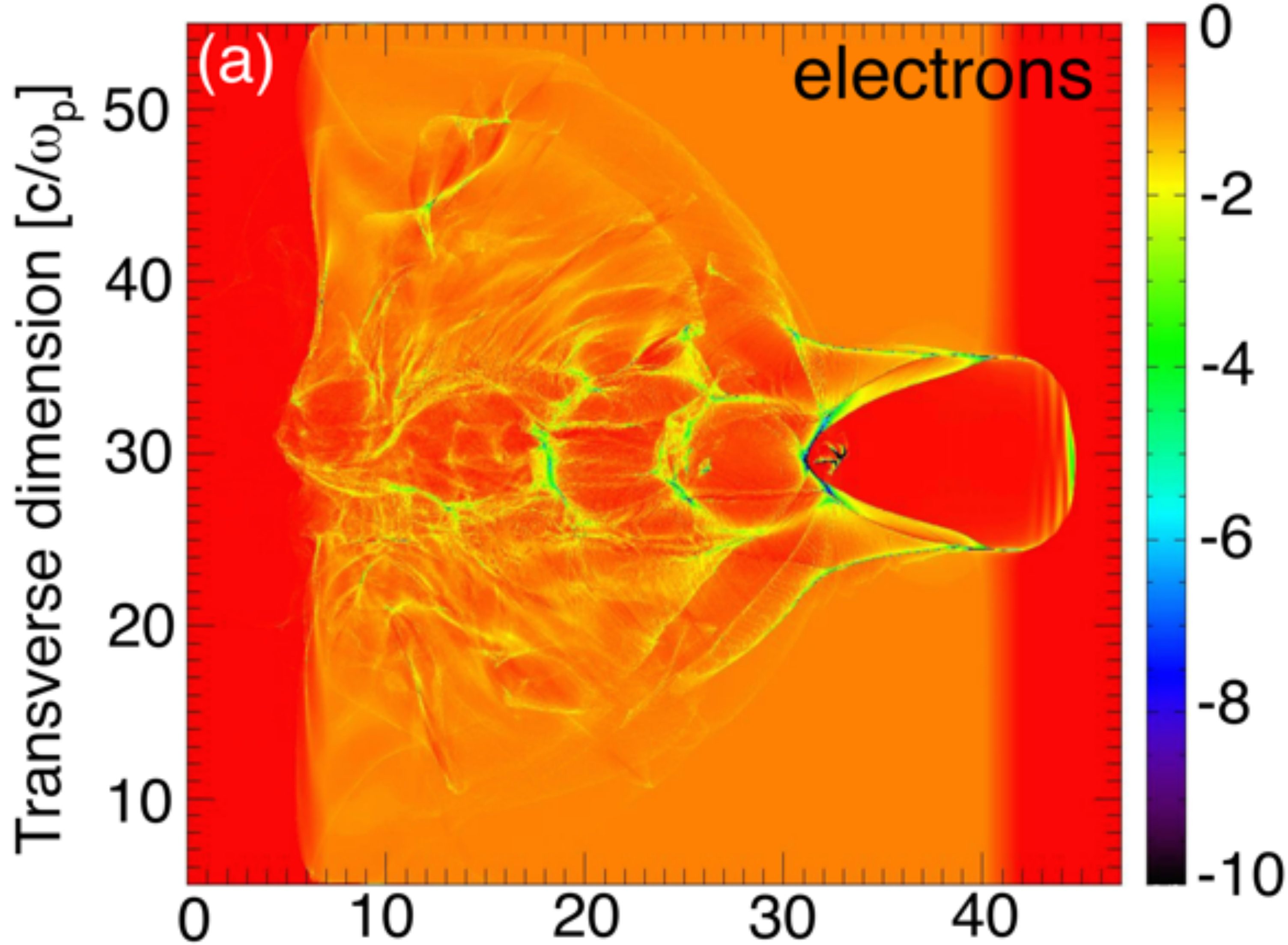
$$\left(\frac{1}{c^2}\frac{\partial^2}{\partial t^2} + k_{pe}^2\right)\phi = k_{pe}^2\frac{a_\perp^2}{2}$$

$$\left(\frac{1}{c^2}\frac{\partial^2}{\partial t^2} + k_{pe}^2\right)[\gamma\beta_{\|}] = -\frac{1}{c}\frac{\partial}{\partial t}\nabla\frac{a_\perp^2}{2}$$

Non-linearity & Relativistic effects:

- density compression >> n_0
- Wave steepening
- Phase-mixing / trajectory crossing
- Lorentz factor >> 1

FWHM radius $= 2\frac{c}{\omega_{pe}}$ $a_0 = 1.0$ $\frac{\omega_0}{\omega_{pe}} = 10$


(a)
electrons
Transverse dimension [c/ωp]
50
40
30
20
10
0
10
20
30
40
0
-2
-4
-6
-8
-10