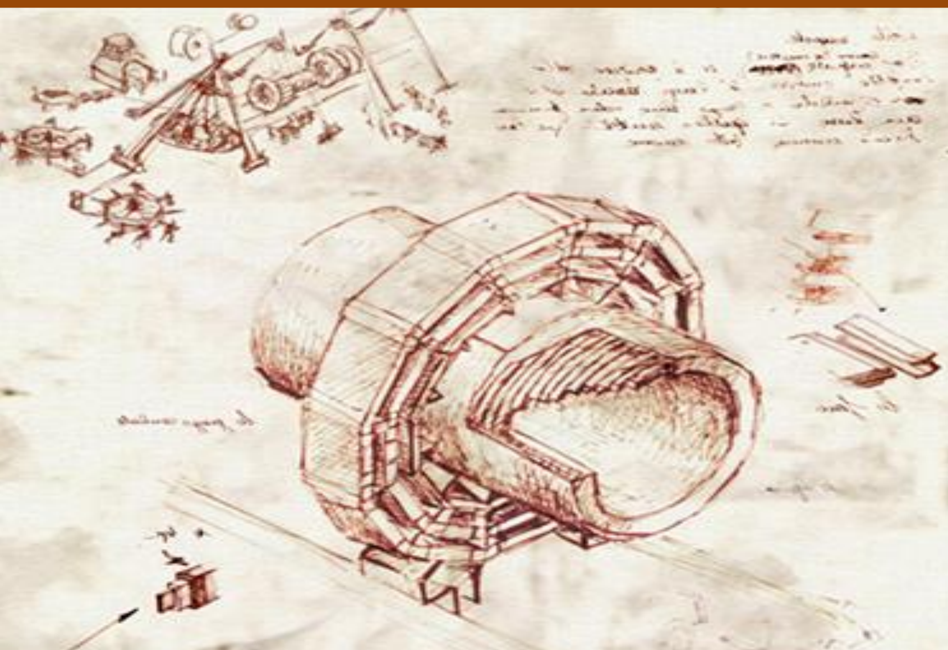
John Adams Institute for Accelerator Science

Imperial College London

Royal Holloway University of London

University of Oxford

Unifying physics of accelerators, lasers and plasma



LHC sketches by Sergio Cittolin (CERN) – used with permission

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John Adams Institute

Lecture 5: Conventional acceleration

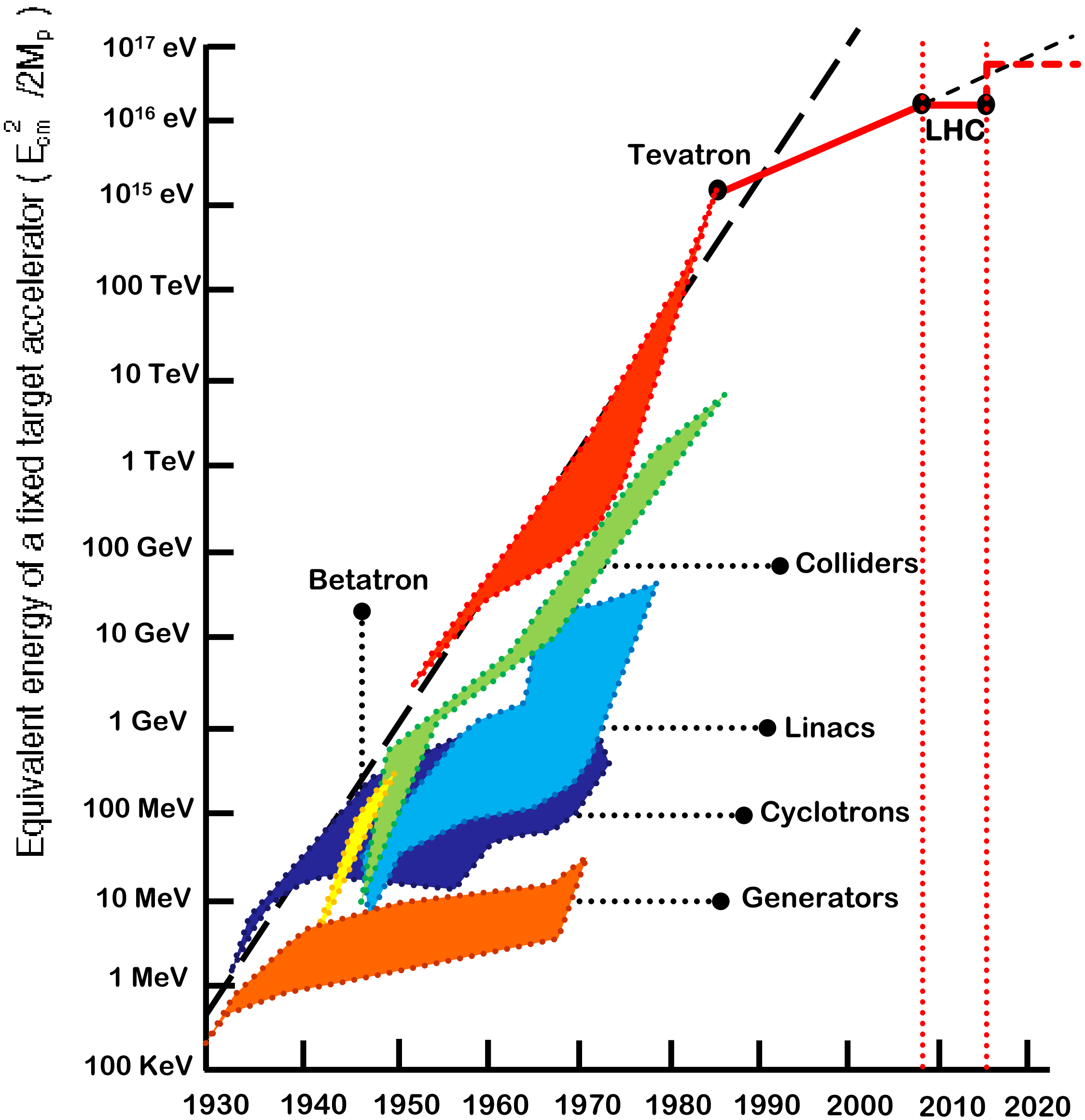
USPAS16

June 2016

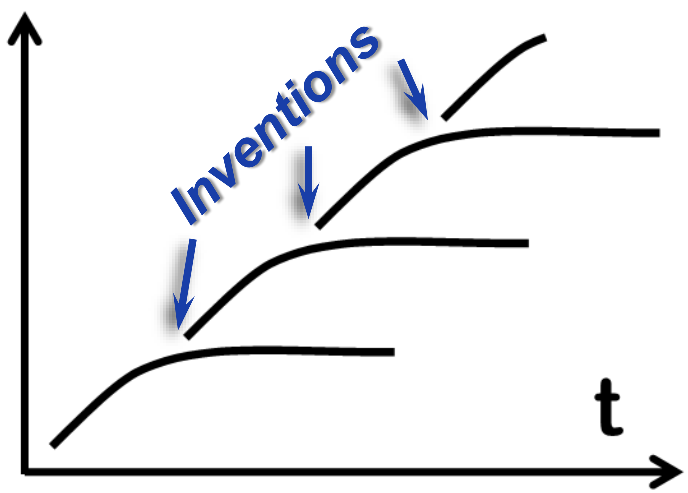
Conventional accelerations

- Historical Introduction
- Waveguides
- Resonant Cavities
- Power Sources
- Longitudinal dynamics

History



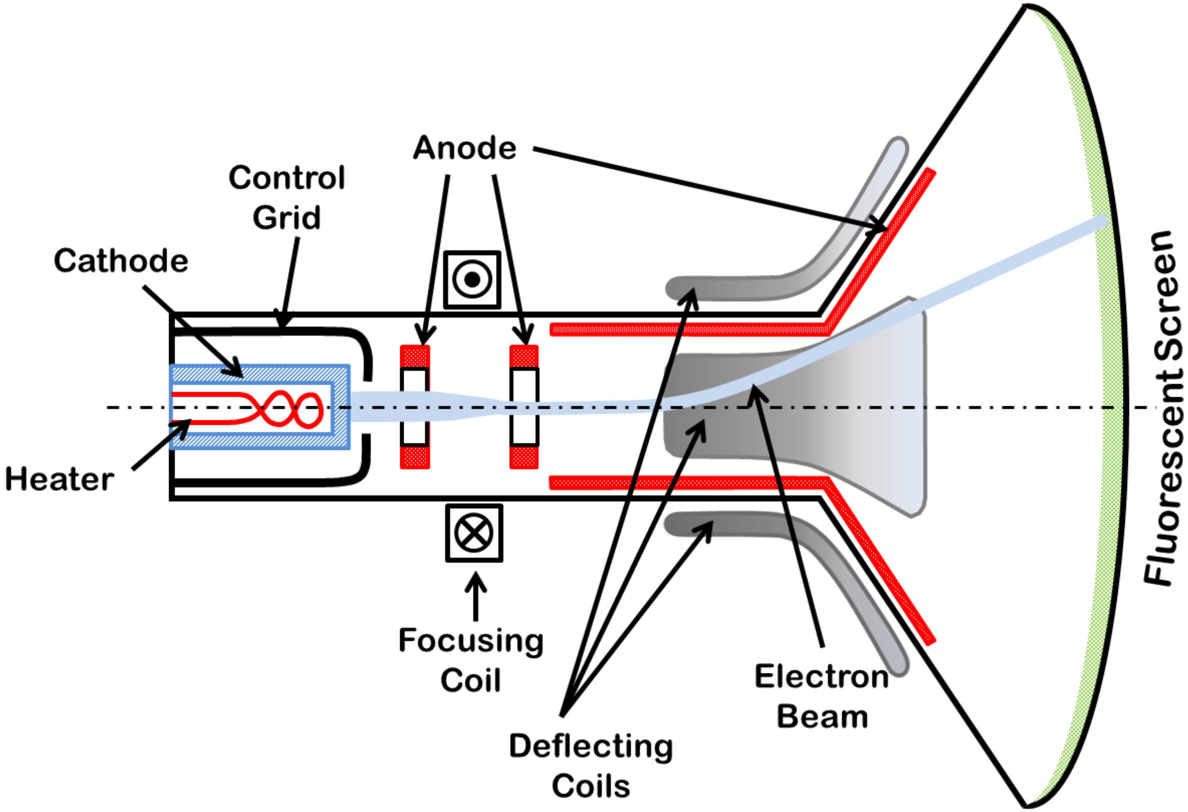
- "Livingston plot" shows great history of accelerators
- These are conventional accelerators as we call them now

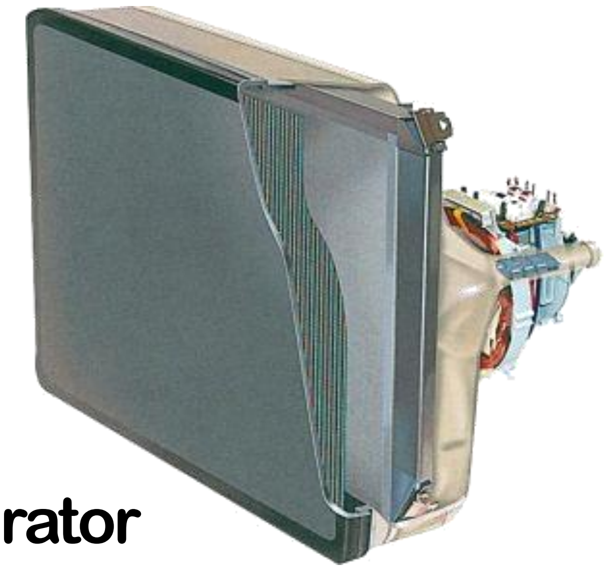


Natural accelerators

Natural accelerators

A TV as an Accelerator



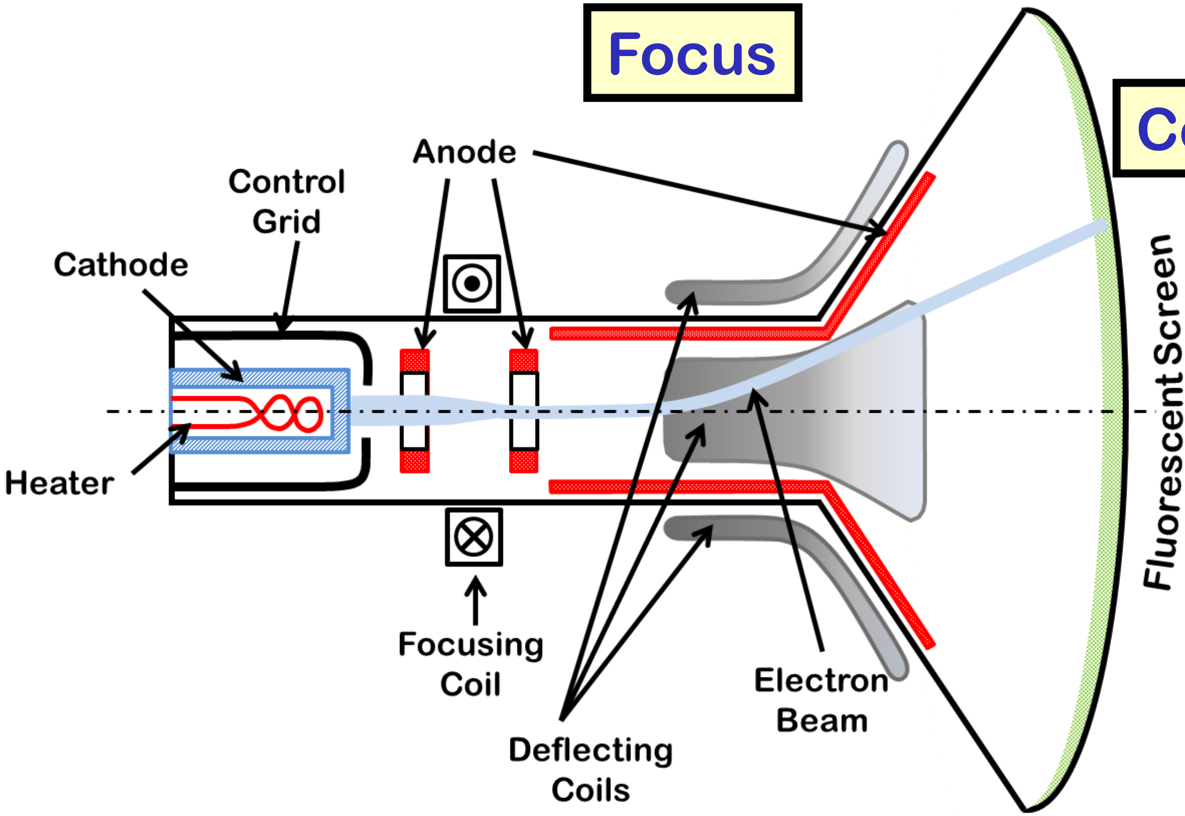


Your (old CRT) TV is an accelerator

A TV as an Accelerator

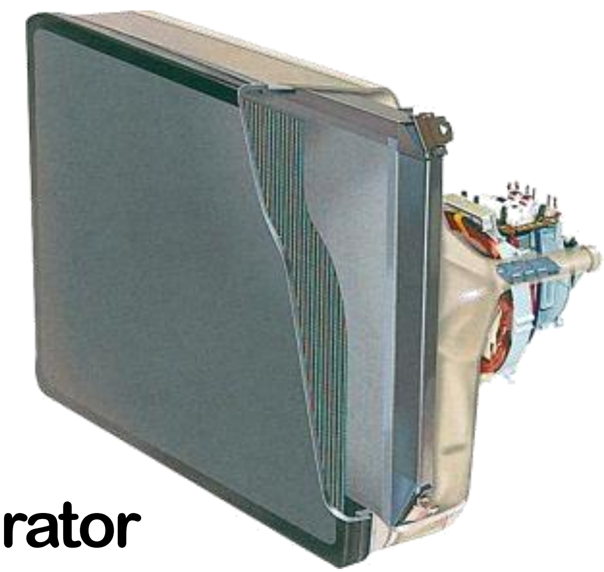
Focus

Collide with fixed target



Bend or steer

Accelerate

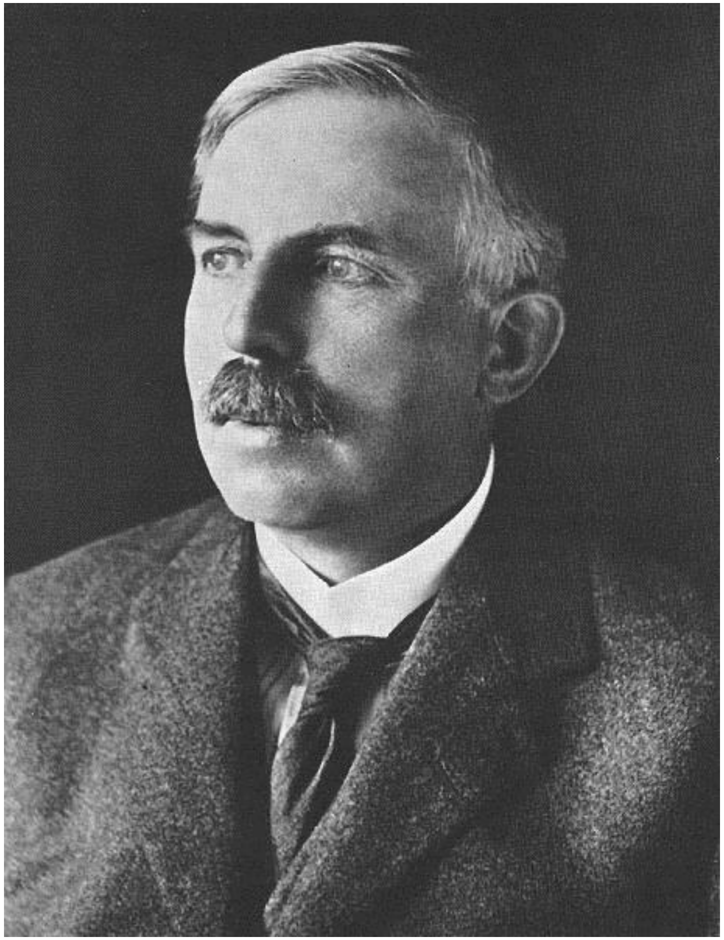


Your (old CRT) TV is an accelerator

Rutherford fired the starting pistol

At the Royal Society in 1928 he said:

"I have long hoped for a source of positive particles more energetic than those emitted from natural radioactive substances".

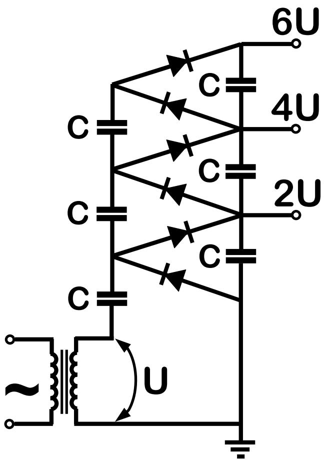


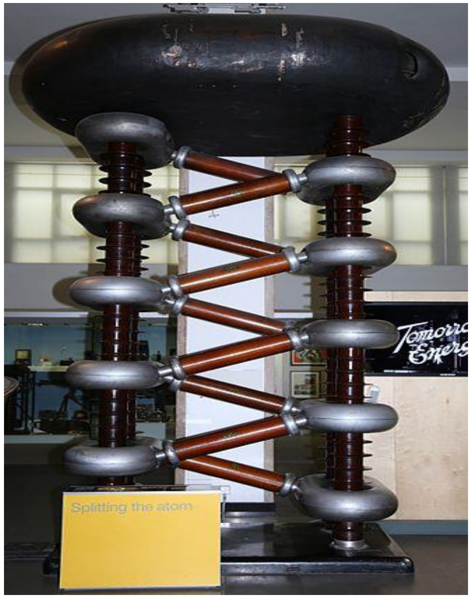
Electrostatic Accelerators
The Cockcroft-Walton

- **Based on system of multiple rectifiers**
- **Voltage generated by cascade circuit**

$$U_{tot} = 2Un - \frac{2\pi I}{\omega C}\left(\frac{2}{3}n^3 + \frac{1}{4}n^2 + \frac{1}{12}n\right)$$

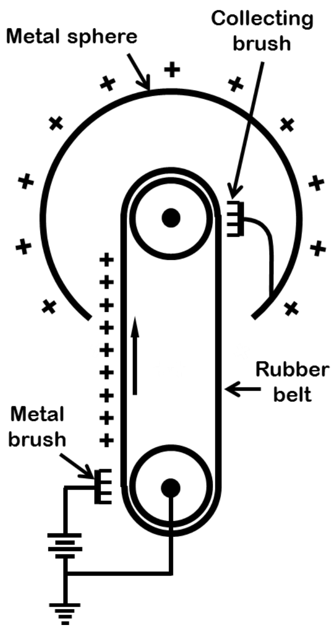
- **Voltages up to ~4 MV**
- **Beam currents of several hundred mA with pulsed particle beams of few μs pulse length**





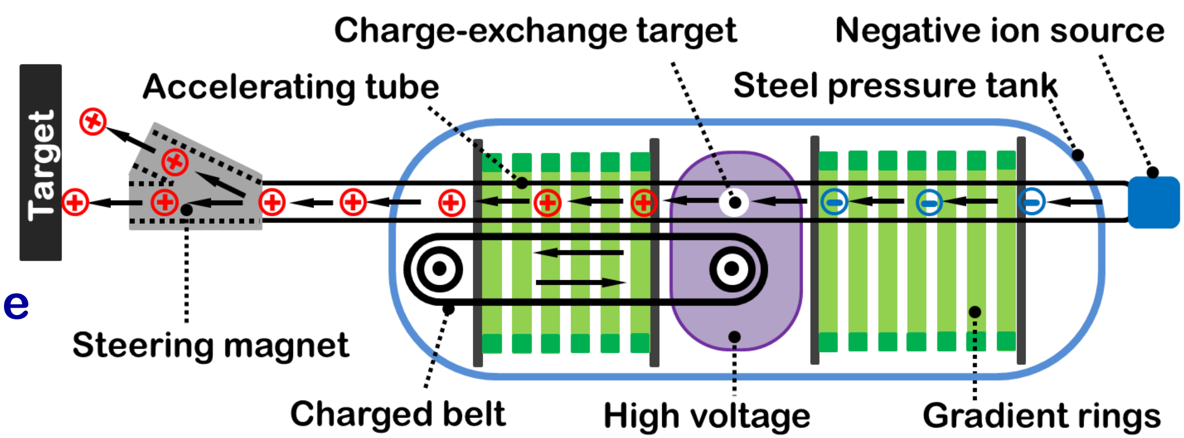
Electrostatic Accelerators
The Van der Graaf

- Example of electrostatic accelerators - "Van der Graaf"



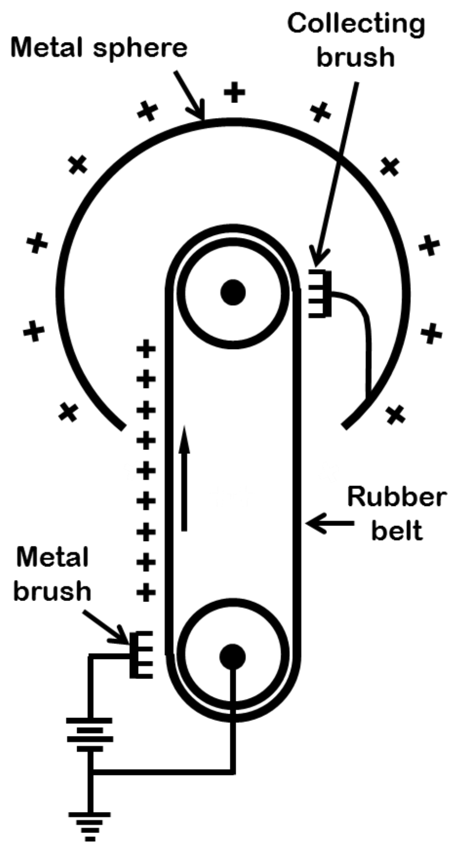
- With any electrostatic accelerator, it is difficult to achieve energy higher than ~20 MeV (e.g. due to practical limitations of the size of the vessels)

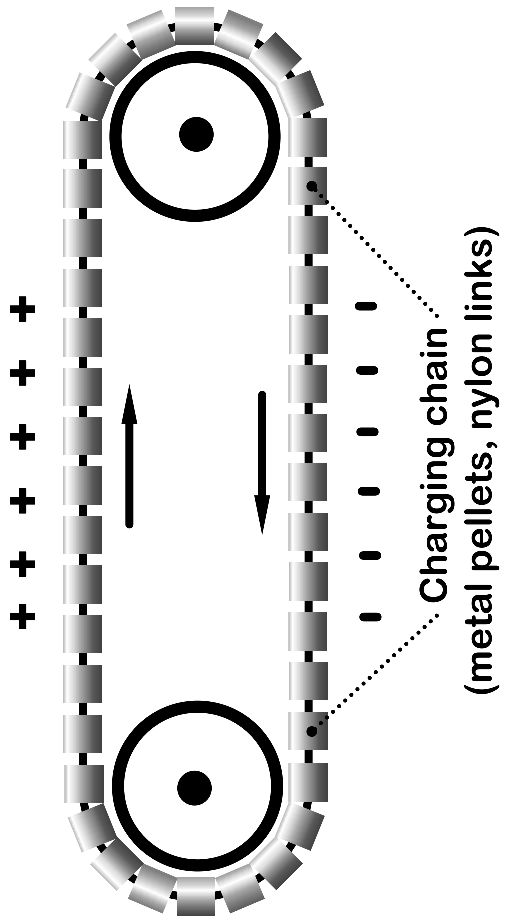
- Tandem is a version with charge exchange in the middle



Electrostatic Accelerators
The Van der Graaf and Pelletron

- Example of electrostatic accelerators - "Van der Graaf"





- In pelletron metal pellets are carrying the charge

Static => Time-varying Fields

- **Limitations to final beam energy achievable in electrostatic accelerators may be overcome by the use of high frequency voltages**

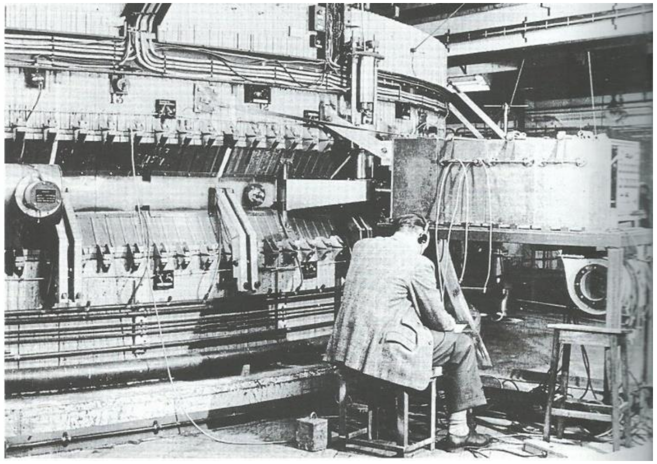
- **Virtually all modern accelerators use powerful radiofrequency (RF) systems to produce the required strong electric fields**
  - **Frequencies range from few MHz to several GHz**
  - **Much of the hardware derived from telecommunications industry**

Mark Oliphant & the Synchrotron

" Particles should be constrained to move in a circle of constant radius thus enabling the use of an annular ring of magnetic field...which would be varied in such a way that the radius of curvature remains constant as the particle gains energy through successive accelerations by an alternating electric field applied between coaxial hollow electrodes."

Mark Oliphant,
Oak Ridge, 1943

With Ernest Rutherford in 1932



1 GeV machine
at Birmingham University

Acceleration & Time-varying Fields

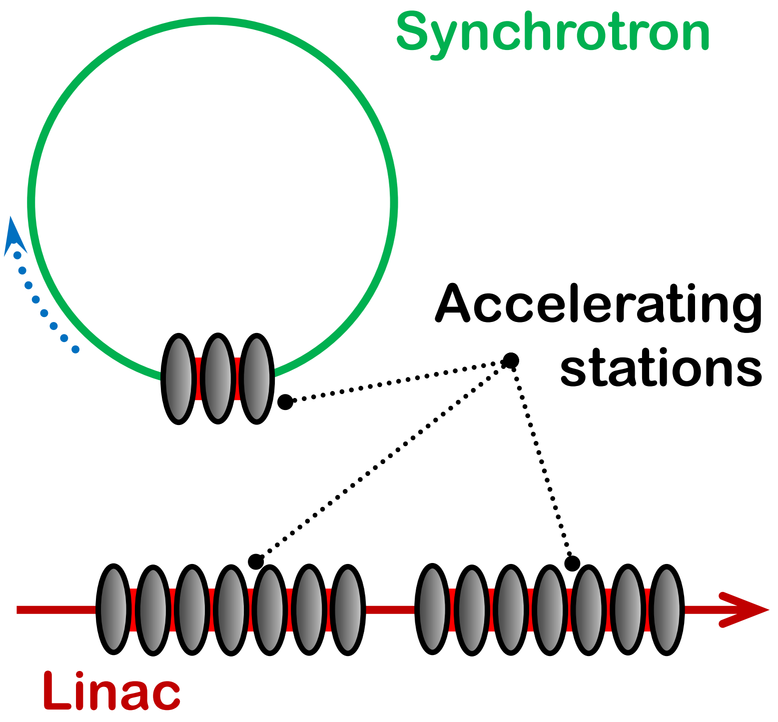
DC accelerator

DC versus RF

RF accelerator

Acceleration & Time-varying Fields

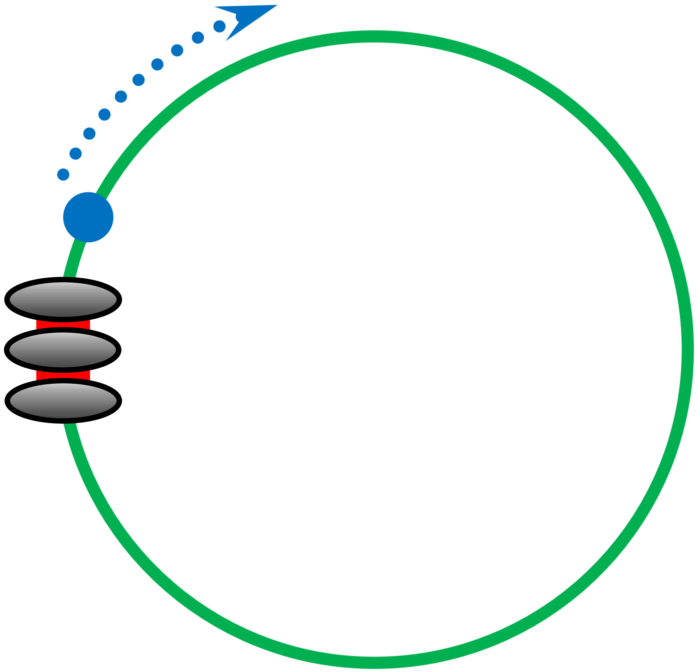
- Necessary conditions for acceleration
  - Both linear and circular accelerators use electromagnetic fields oscillating in resonant cavities to apply the accelerating force



- ➢ LINAC – particles follow straight path through series of cavities
- ➢ CIRCULAR ACCELERATORS – particles follow circular path in B-field and particles return to same accelerating cavity each time around

Synchrotrons

- Synchrotrons can accelerate to much higher energies
- LHC is synchrotron
- Limitation of synchrotrons (especially for electrons) is due to "synchrotron radiation"



Advantages- Time-varying Fields

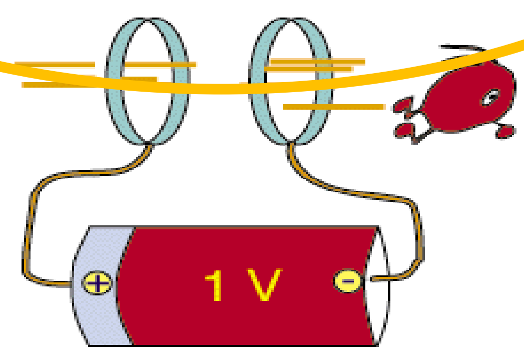
- In classical linac or synchrotron, EM field oscillates in resonant cavity and particles enter and leave by holes in end walls
- Energy is continuously exchanged between electric and magnetic fields within cavity volume
- The time-varying fields ensure finite energy increment at each passage through one or a chain of cavities
- There is no build-up of voltage to ground

Maxwell's Equations

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

Becomes in its integral form

$$\oint_{\partial\Sigma} \mathbf{E} \cdot d\boldsymbol{\ell} = -\frac{d}{dt} \iint_{\Sigma} \mathbf{B} \cdot d\mathbf{S}$$



There can be no acceleration without time-dependent magnetic field

Time-dependent flux may provide particle acceleration

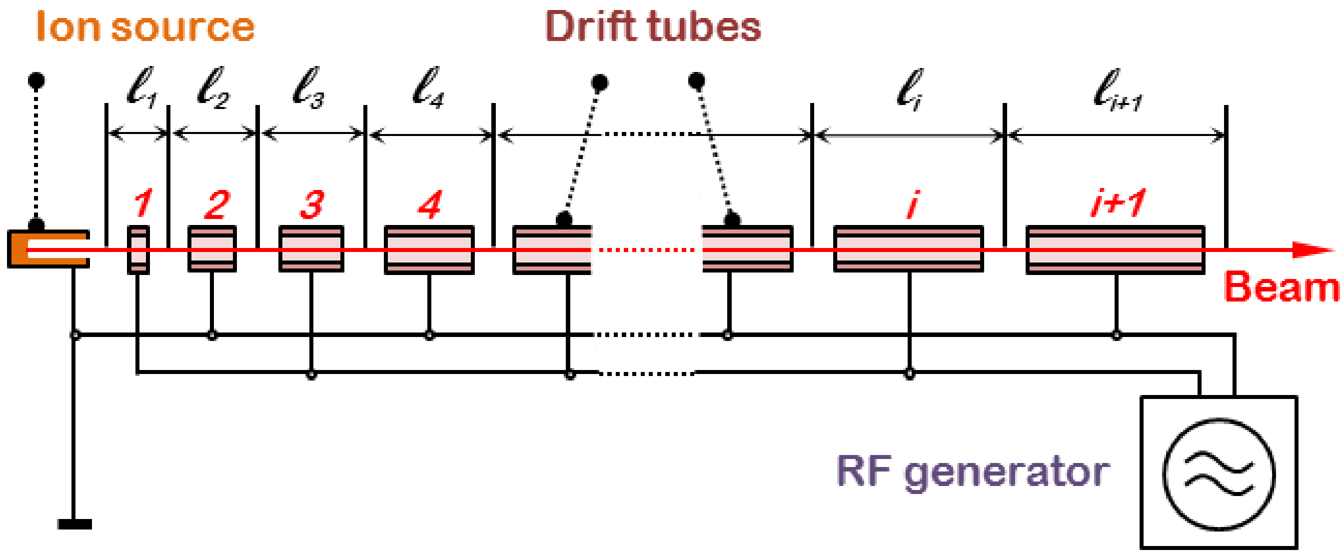
Wideröe Linear Accelerator

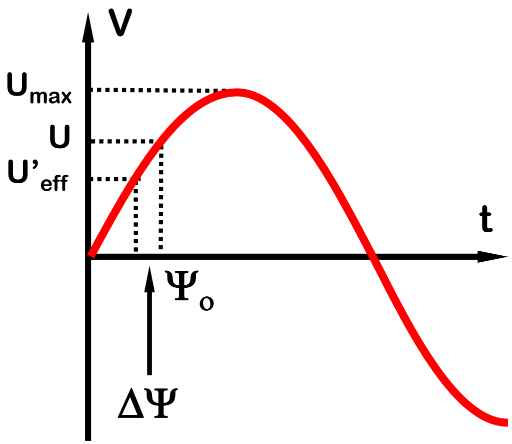
- In order to avoid limitations imposed by corona formation and discharge on electrostatic accelerators, in 1925 Ising suggested using rapidly changing high frequency voltages instead of direct voltages

- In 1928 Wideröe performed first successful test of linac based on this principle
- Series of drift tubes arranged along beam axis and connected with alternating polarity to RF supply
  - Supply delivers high frequency alternating voltage:

$$V(t) = V_{max} \sin(\omega t)$$

Wideröe Linear Accelerator





Wideröe Linear Accelerator

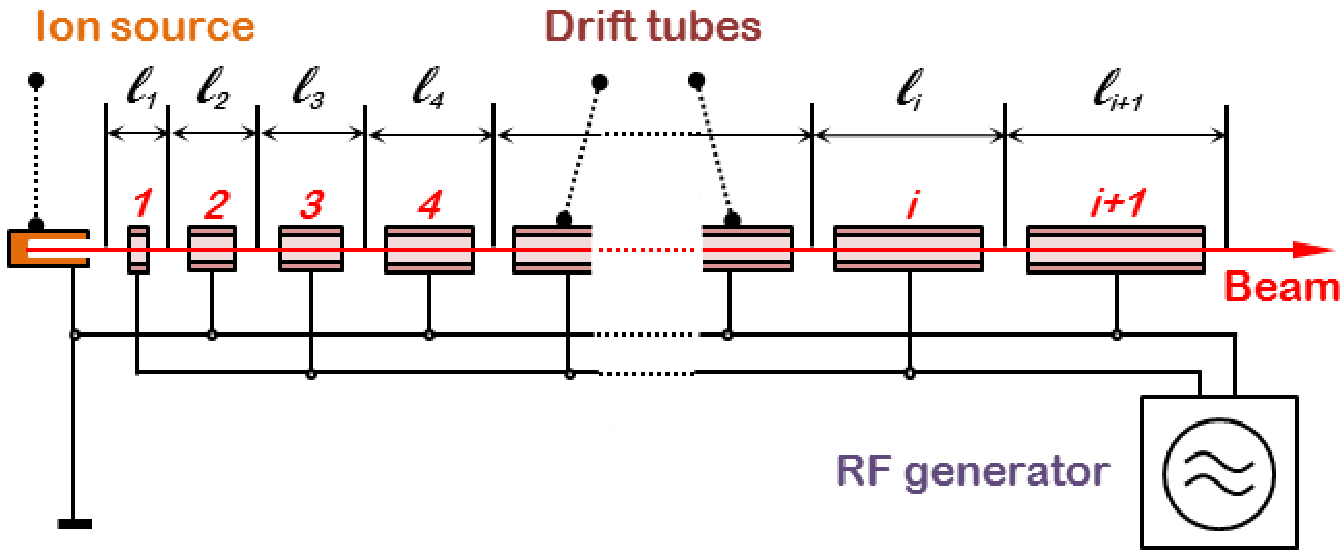
- Acceleration process
  - During first half period, voltage applied to first drift tube acts to accelerate particles leaving ion source.
    - Particles reach first drift tube with velocity v_1.
  - Particles then pass through first drift tube, which acts as a Faraday cage and shields them from external fields.
  - Direction of RF field is reversed without particles feeling any effect.
  - When they reach gap between first and second drift tubes, they again undergo an acceleration.

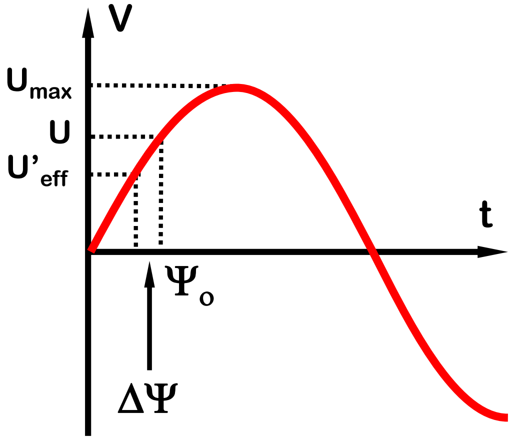
- Acceleration process
  - After the *i-th* drift tube the particles of charge *q* have reached energy

$$E_i = iqV_{max} \sin \Psi_0$$

where Ψ_0 is average phase of RF voltage that particles see as they cross gaps.

Wideröe Linear Accelerator





- **Observations**
  - Energy is proportional to number of stages *i* traversed by particle.
  - The largest voltage in entire system is never greater than V_{max}
    - Arbitrary high energies without voltage discharge

Wideröe Linear Accelerator

- Accelerating gaps
  - During acceleration particle velocity increases monotonically but alternating voltage remains constant in order to keep the costs of already expensive RF power supplies reasonable.
    - ➔ Gaps between drift tubes must increase.
- RF voltage moves through exactly half a period $\tau_{RF}/2$ as particle travels through one drift section.
- Fixes distance between *i-th* and *(i+1)-th* gaps

$$l_i = \frac{v_i \tau_{RF}}{2} = \frac{v_i}{2f_{RF}} = \frac{v_i \lambda_{RF}}{2c} = \beta_i \frac{\lambda_{RF}}{2} = \frac{1}{f_{RF}} \sqrt{\frac{iqV_{max} \sin \Psi_0}{2m}}$$

Modern Linear Accelerators

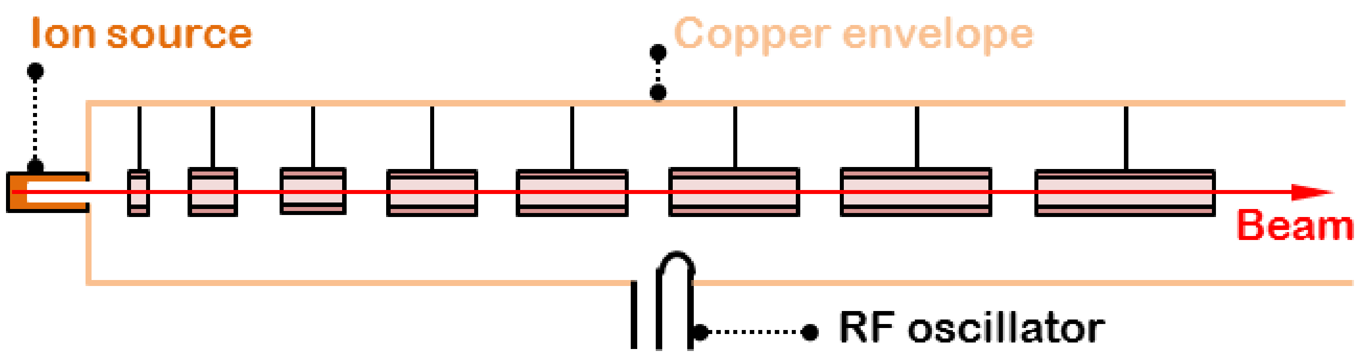
- Drift tubes typically no longer used and have been generally replaced by cavity structures
- Electron linacs
  - By energies of a few MeV, particles have already reached velocities close to light speed
  - As they are accelerated electron mass increases with velocity remaining almost constant
    - Allows cavity structures of same size to be situated along whole length of linac
      - Leading to relatively simple design

Modern Linear Accelerators

- **Hadrons**
  - Particles still have non-relativisitc velocities in first few stages and Wideröe-type structure is needed
  - Alvarez structure
    - Drift tubes are today arranged in a tank, made of good conductor (Cu), in which a cavity wave is induced
    - The drift tubes, which have no field inside them, also contain the magnets to focus the beam
    - Alternate tubes need not be earthed and each gap appears to the particle to have identical field gradient which accelerates particle from left to right

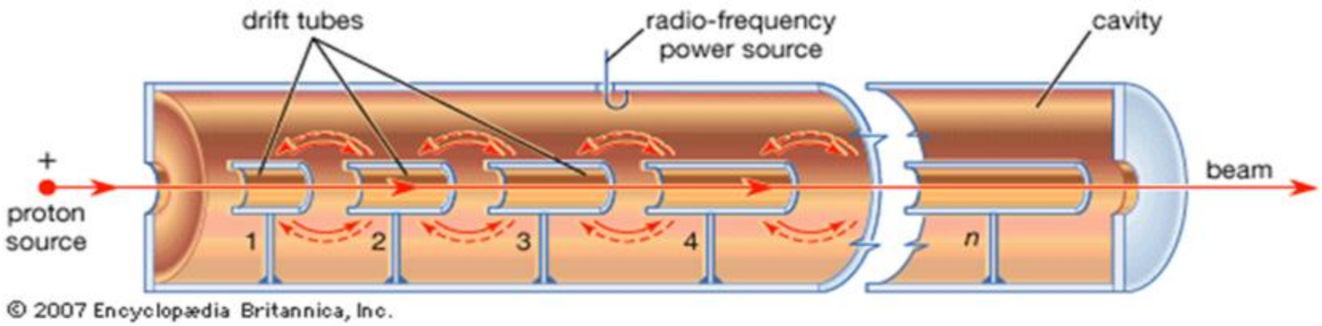
Drift tube linac - SuperHilac (Super Heavy Ion Linear Accelerator)LBNL

Alvarez Structure

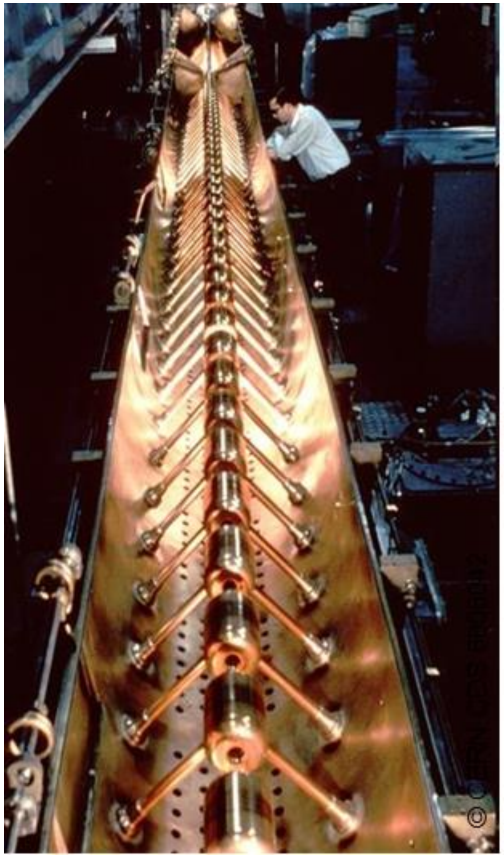


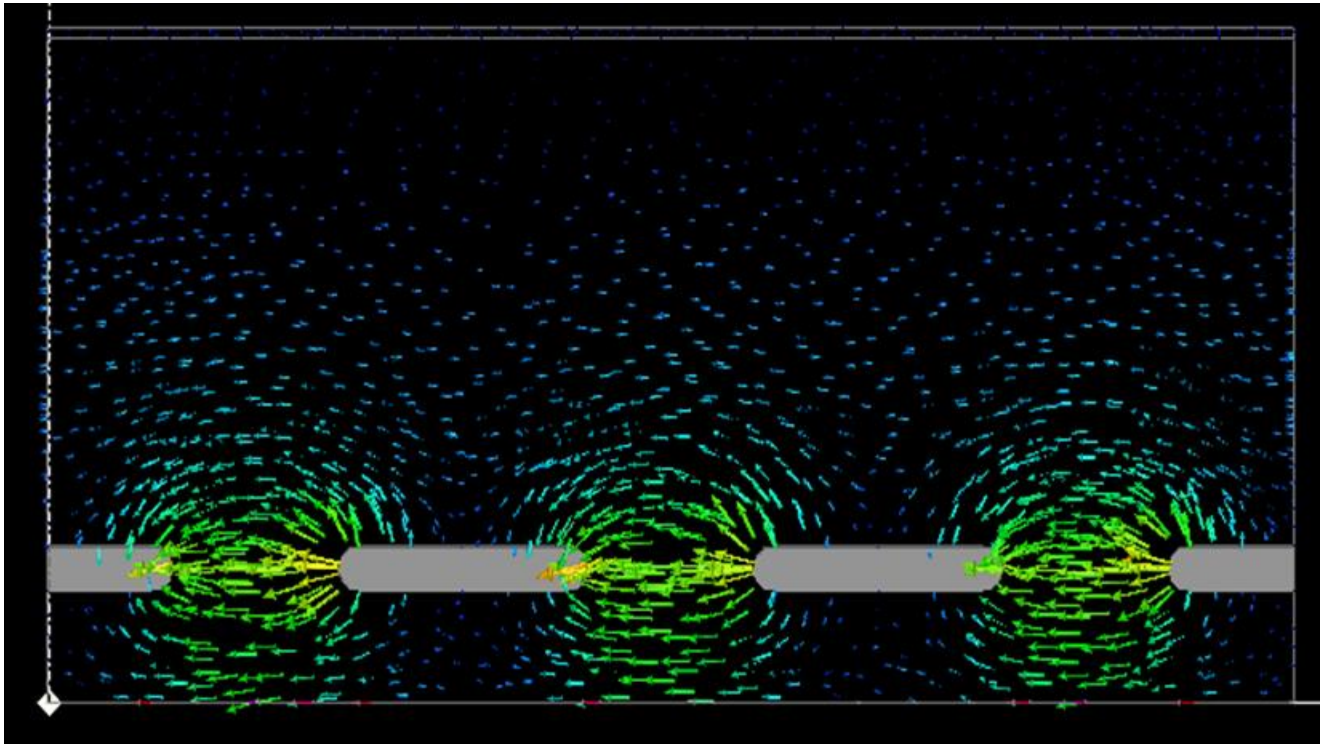
The concept of the Alvarez linear accelerator

Drift Tube Linac: Higher Integrated Field



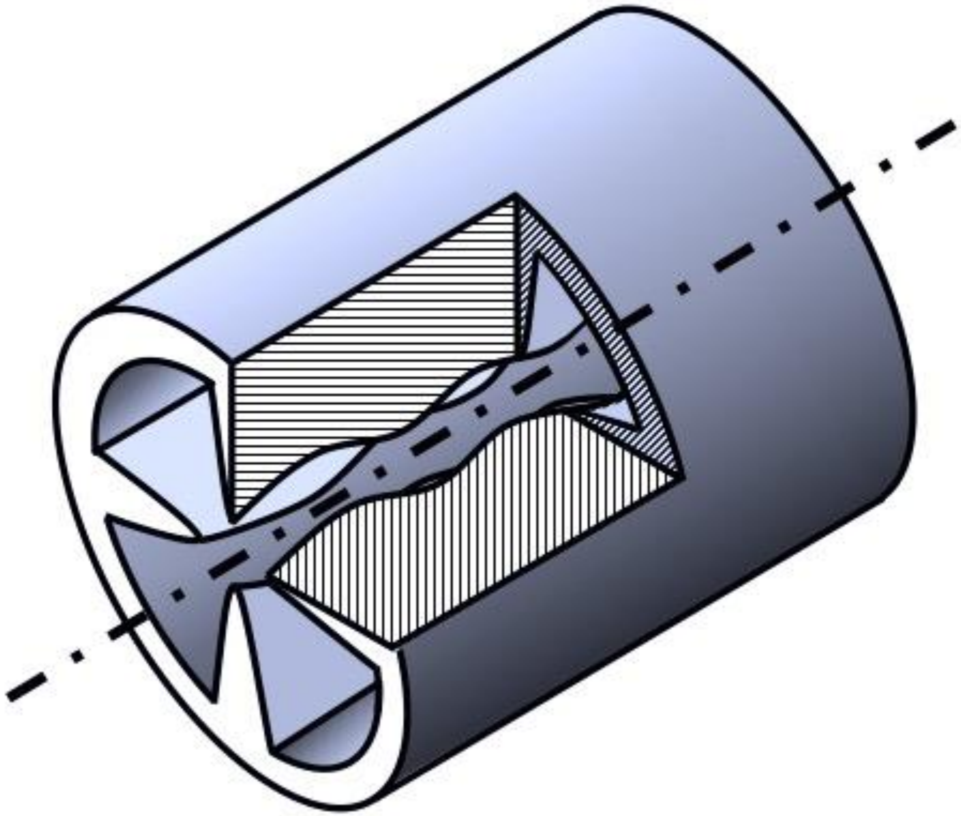
CERN LINAC1 1982-1992





Courtesy E. Jensen

RFQ accelerating structure



Drift tubes linacs in modern accelerators are now typically replaced by RFQ – radio frequency quadrupole structures, that combine acceleration and focusing

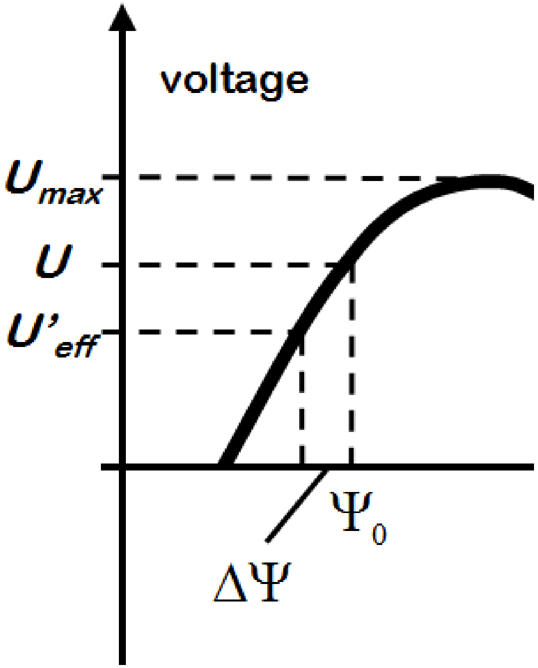
Phase Focusing

- **Energy transferred to particle depends on V_{max} and Ψ_0**
  - Small deviation from nominal voltage V_{max} results in particle velocity no longer matching design velocity fixed by length of drift sections.
    - Particles undergo a phase shift relative to RF voltage.
    - Synchronisation of particle motion and RF field is lost.

- **Solution based on using $\Psi_0 < \pi/2$ so that the effective accelerating voltage is $V_{eff} < V_{max}$**
  - Assume particle gained too much energy in preceding stage and travelling faster than ideal particle and hence arrives earlier.
  - Sees average RF phase $\Psi = \Psi_0 - \Delta\Psi$ and is accelerated by voltage

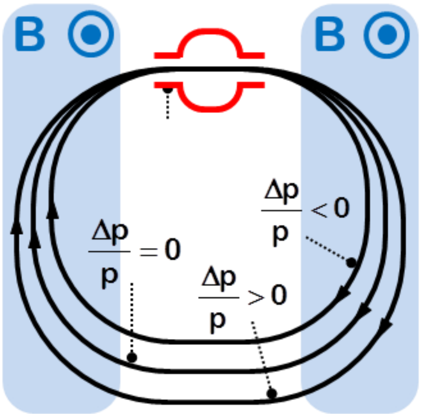
$$V'_{eff} = V_{max} \sin(\Psi_0 - \Delta\Psi) < V_{max} \sin \Psi_0$$

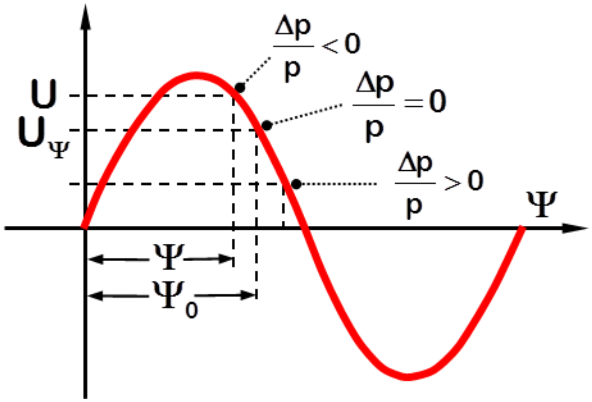
    which is below the ideal voltage.
  - Particle gains less energy & slows down again until it returns to nominal velocity.
  - All particles oscillate about nominal phase Ψ_0



Synchrotron Oscillations

- The periodic longitudinal particle motion about the nominal phase is called **synchrotron oscillation**





Phase focusing of relativistic particles in circular accelerators

- As the ideal particle encounters the RF voltage at exactly the nominal phase on each revolution, the RF frequency ω_{RF} must be an integer multiple of the revolution frequency ω_{rev}

$$h = \frac{\omega_{RF}}{\omega_{rev}}$$

where *h* is the harmonic number of the ring.

(We assumed above that the particle is relativistic. We will consider a general case later in the lecture)

Waves in Free Space

- **Wave parameters**

Velocity in vacuum $$v = c = \frac{1}{\sqrt{\varepsilon_0 \mu_0}}$$

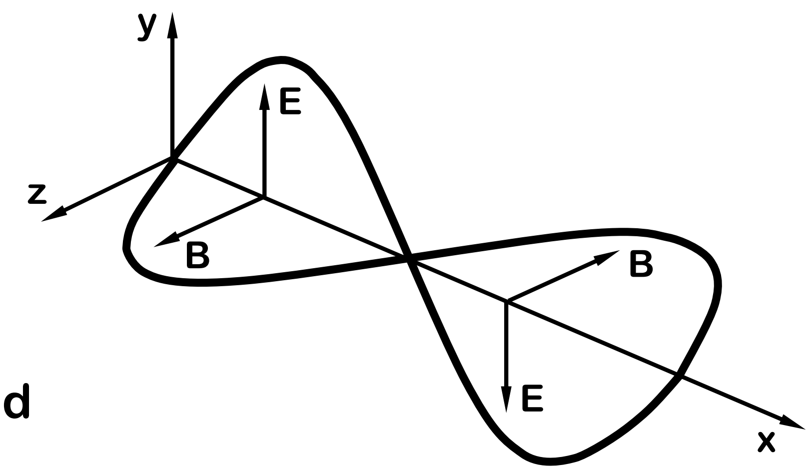
Velocity in medium $$v = \frac{1}{\sqrt{\varepsilon_0 \varepsilon_r \mu_0 \mu_r}}$$

With ε_r being the dielectric constant and the magnetic permeability is μ_r

The ratio between the electric and magnetic fields is

SI units $$\frac{E}{H} = 376.6\sqrt{\frac{\mu_r}{\varepsilon_r}} \quad (\Omega)$$

Plane transverse electric and magnetic wave (TEM) propagating in free space in x-direction.

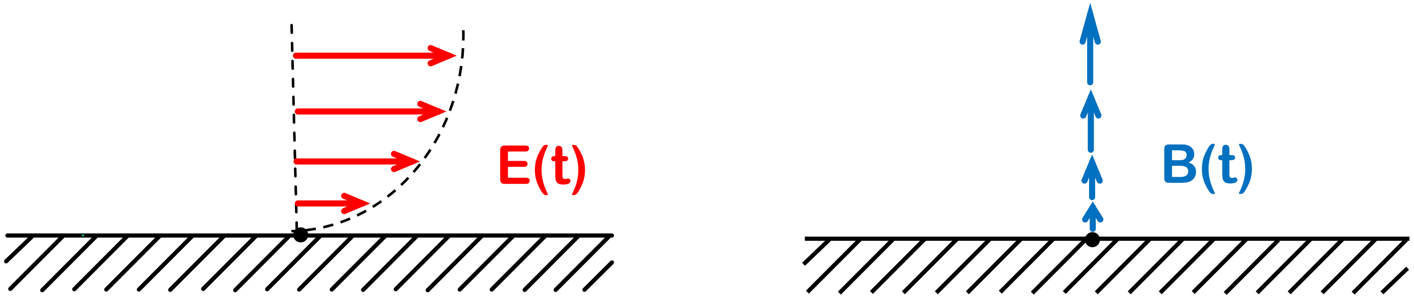


The Poynting flux (the local power flux) is

$$\mathbf{P} = (\mathbf{E} \times \mathbf{H}) \quad [\mathrm{W\ m^{-2}}]$$

Conducting Surfaces

- Consider waves in metal boxes, use boundary conditions of a wave at a perfectly conducting metallic surface
  - $E_{tangential}$ component and B_{normal} component to surface vanish



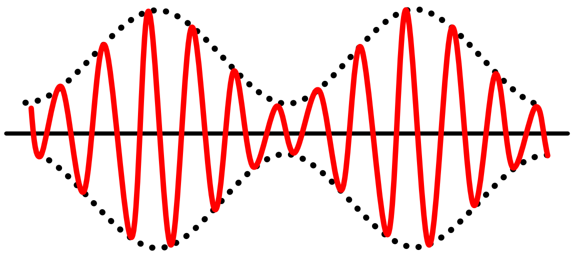
- Skin depth – EM wave entering a conductor is damped to 1/e of initial amplitude in distance

$$\delta_S = \frac{1}{\sqrt{\pi f \ \mu_0 \mu_r \sigma}}$$

- Surface resistance $R_S = \frac{1}{\sigma \delta_S}$

Group Velocity

- Energy (and information) travel with wave group velocity



- Interference of two continuous waves of slightly different frequencies described by:

$$E = E_0 \sin[(k+dk)x - (\omega + d\omega)t] + E_0 \sin[(k-dk)x - (\omega - d\omega)t]$$
$$= 2E_0 \sin[kx - \omega t]\cos[dk\, x - d\omega\, t]$$
$$= 2E_0 f_1(x,t) f_2(x,t)$$

Group Velocity

- **Mean wavenumber & frequency represented by continuous wave**

$$f_1(x,t) = \sin[kx - \omega t]$$

  - **Any given phase in this wave is propagated such that *$kx - \omega t$* remains constant => $v_p = \omega / k$**
  - **Or from zeroing the convective derivative:**

$$v_p = -\frac{\partial f_1(x,t)/\partial t}{\partial f_1(x,t)/\partial x} = \frac{\omega}{k}$$

- **Envelope of pattern described by**

$$f_2(x,t) = \cos[dk\,x - d\omega\,t]$$

-
  - **Any point in the envelope propagates such that *$x\,dk - t\,d\omega$* remains constant and its velocity, i.e. group velocity, is**

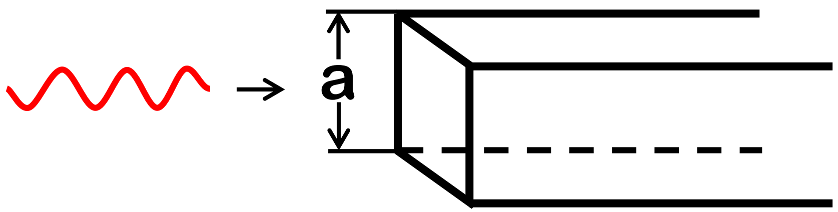
$$v_g = -\frac{\partial f_2(x,t)/\partial t}{\partial f_2(x,t)/\partial x} = \frac{d\omega}{dk}$$

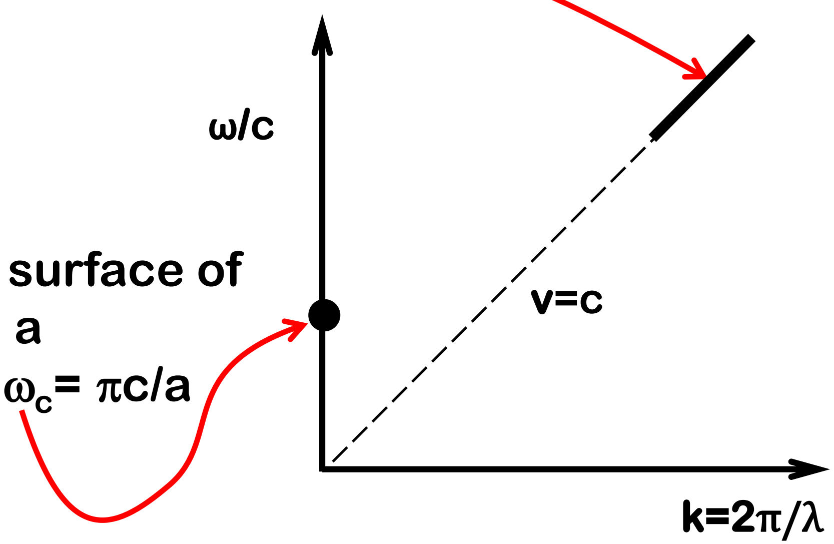
Dispersion Diagramme for Waveguide

Consider wave propagation down a waveguide and aim to plot graph of frequency, ω, against wavenumber, $k = 2\pi/\lambda$

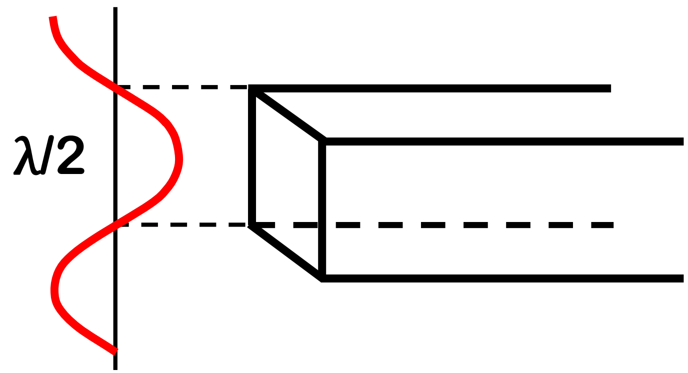
- Let's look at this from simple geometrical point of view

First of all, if λ in free space << a, waveguide does not matter, and we expect the dispersion at large ω to approach $\omega/c=k$





Boundary conditions at perfect conducting surface of waveguide can be satisfied only if $\lambda/2$ = or < a
This defines cut-off parameters $\lambda_c = 2a$ and $\omega_c = \pi c/a$

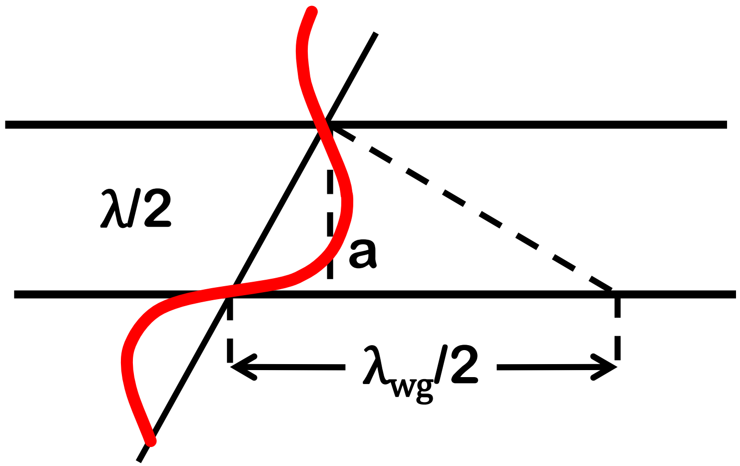


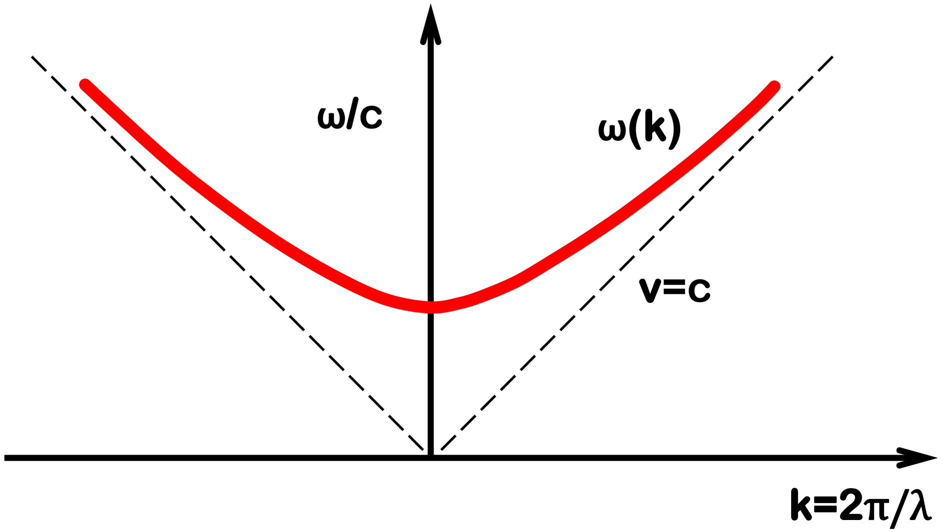
The case of $\omega = \omega_c$ corresponds to infinite wavelength in waveguide, or $k=0$

Dispersion Diagramme for Waveguide

From simple geometrical picture

$$\left(\frac{\omega}{c}\right)^2 = k^2 + \left(\frac{\omega_c}{c}\right)^2$$





- **Observations**
  - However small the *k*, the frequency is always greater than the cut-off frequency
  - The longer the wavelength or lower the frequency, the slower is the group velocity
  - At cut-off frequency, no energy flows along the waveguide
  - Also

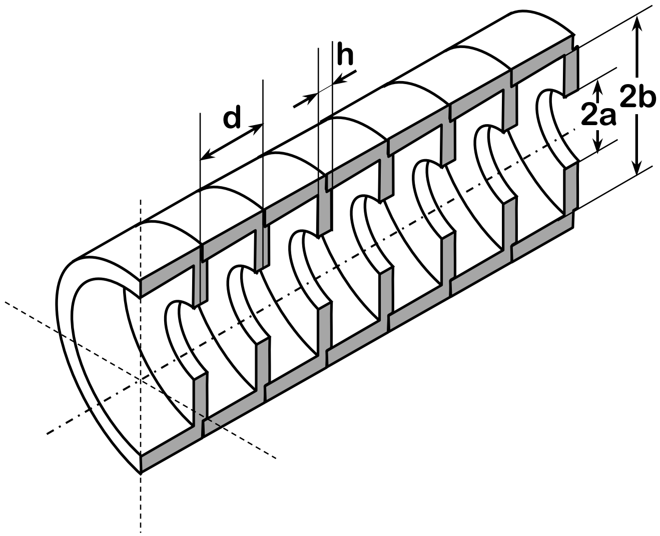
$$v_{ph} v_g = c^2$$

Iris-loaded Structures

- **Acceleration in a waveguide is not possible as the phase velocity of the wave exceeds that of light**
  - **Particles, which are travelling slower, undergo acceleration from the passing wave for half the period but then experience an equal deceleration**
  - **Averaged over long time interval results in no net transfer of energy to the particles**

•Need to modify waveguide to reduce phase velocity to match that of the particle (less than speed of light)

•Install iris-shaped screens with a constant separation in the waveguide

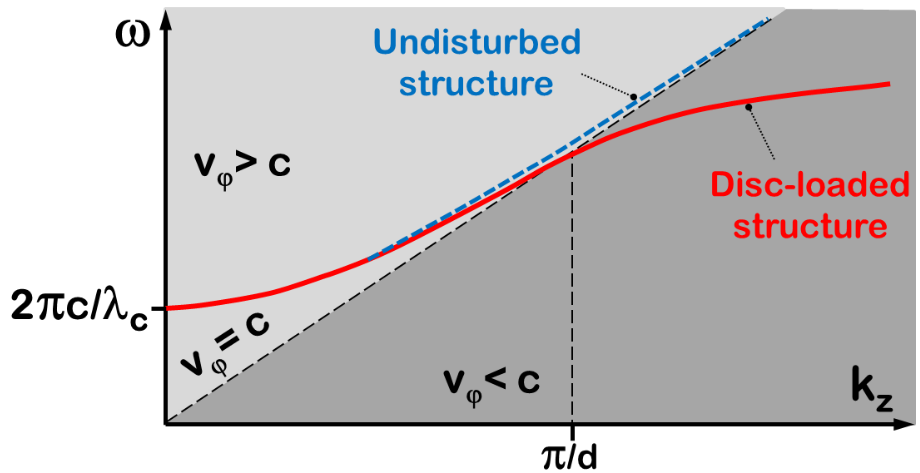


Iris-loaded Structures

- **Recall that the dispersion relation in a waveguide is**

$$\omega = c\sqrt{k_z^2 + \left(\frac{2\pi}{\lambda_c}\right)^2}$$

- **With the installation of irises, curve flattens off and crosses boundary at $v_\varphi = c$ at $k_z = \pi/d$**

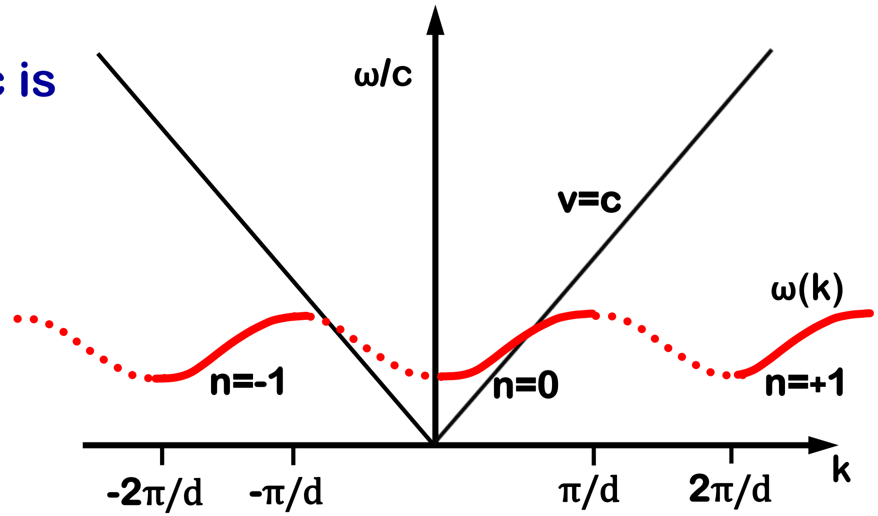


With suitable choice of iris separation *d* the phase velocity can be set to any value

Iris-loaded Structures

- The *k*-value for each space harmonic is

$$k_n = k_0 + \frac{2n\pi}{d}$$



By choosing any frequency in dispersion
diagramme it will intercept dispersion curve at k values spaced by 2nπ/d

First rising slope used for acceleration

Dispersion diagramme
for an iris-loaded structure

Resonant Cavities

- **General solution of wave equation**

$$W(r,t) = Ae^{i(\omega t + k\cdot r)} + Be^{i(\omega t - k\cdot r)}$$

  - Describes sum of two waves – one moving in one direction and another in opposite direction

- **If wave is totally reflected at surface then both amplitudes are the same, *A=B,* and**

$$W(r,t) = Ae^{i\omega t}(e^{ik\cdot r} + e^{-ik\cdot r})$$
$$= 2A\cos(k\cdot r)e^{i\omega t}$$

  - Describes field configuration which has a static amplitude *2Acos(k·r),* i.e. a standing wave

Resonant Cavities

- **Resonant Wavelengths**
  - Stable standing wave forms in fully-closed cavity if

$$l = q\frac{\lambda_z}{2} \quad \text{with} \quad q = 0, 1, 2, \ldots$$

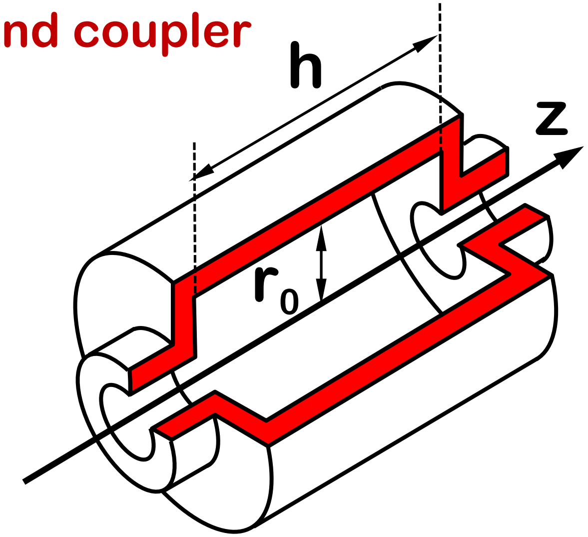
    - where *l* = distance between entrance and exit of waveguide after being closed off by two perpendicular sheets
    - → only certain well-defined wavelengths *λ* are present in the cavity

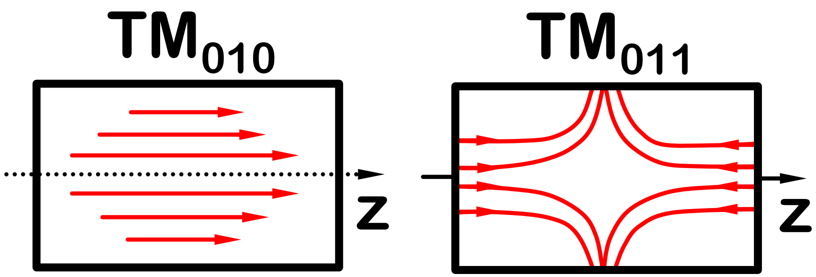
- **Near the resonant wavelength, resonant cavity behaves like electrical oscillator but with much higher Q-value and corresponding lower losses of resonators made of individual coils and capacitors**
  - Exploited to generate high-accelerating voltages

Pill-box Cylindrical Cavity

- The simplest RF cavity type
- The accelerating modes of this cavity are TM_{0lm}
- Indices refer to the polar co-ordinates φ, r and z

Cylindrical pill-box cavity with holes for beam and coupler





Lines of force for the electrical field

Quality Factor of Resonator, Q

- Ratio of stored energy to energy dissipated per cycle divided by 2π

$$Q = \frac{W_s}{W_d} = \omega \frac{W_s}{P_d}$$

W_s = stored energy in cavity
W_d = energy dissipated per cycle divided by 2π
P_d = power dissipated in cavity walls
ω = frequency

Quality Factor of Resonator, Q

- **Stored energy over cavity volume is**

$$W_s = \frac{\varepsilon_0}{2}\int |\mathbf{E}|^2 dv \qquad W_s = \frac{\mu_0}{2}\int |\mathbf{H}|^2 dv$$

where the first integral applies to the time the energy is stored in the *E*-field and the second integral as it oscillates back into the *H*-field

Quality Factor of Resonator, Q

- **Losses on cavity walls are introduced by taking into account the finite conductivity σ of the walls.**
- **Since, for a perfect conductor, the linear density of the current j along walls of structure is**

$$\mathbf{j} = \mathbf{n} \times \mathbf{H}$$

- **we can write**

$$P_d = \frac{R_{surf}}{2} \int_s |\mathbf{H}|^2 ds$$

with s = inner surface of conductor

Shunt Impedance - R_s

- Figure of merit for an accelerating cavity
  - Relates accelerating voltage to the power P_d to be provided to balance the dissipation in the walls.
- Voltage along path followed by beam in electric field E_z is

$$V = \int_{path} |E_z(x,y,z)|\, dl$$

  - from which (peak-to-peak)

$$R_s = \frac{V^2}{2P_d}$$

Energy Gain

- **Energy gain of particle as it travels a distance through linac structure depends only on potential difference crossed by particle:**

$$U = K\sqrt{P_{RF} l R_s}$$

where

$P_{RF} \equiv$ supplied RF power

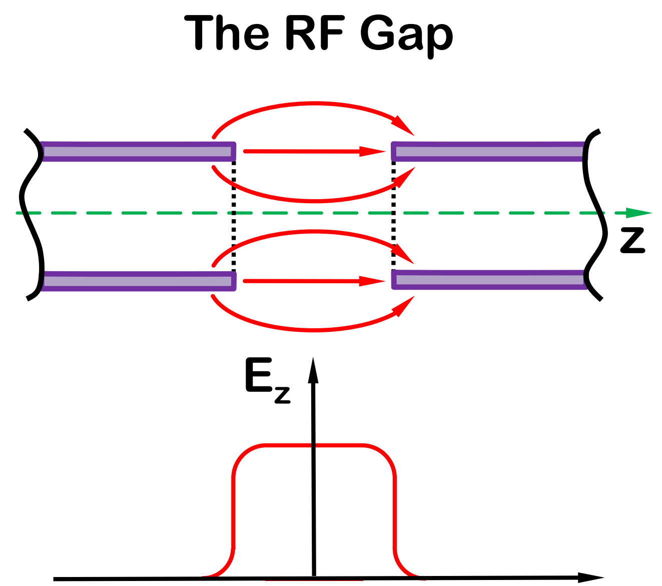
$l \equiv$ length of linac structure

$R_s \equiv$ shunt impedance

$K \equiv$ correction factor (≈ 0.8)

Transit-Time Factor

- Accelerating gap
  - Space between drift tubes in linac structure.
  - Space between entrance and exit irises of cavity resonator
- Field is varying as the particle traverses the gap
  - Makes cavity less efficient and resultant energy gain which is only a fraction of the peak voltage



$$E_z = E_0 \cos(\omega t + \varphi)$$

Field is uniform along gap axis and depends sinusoidaly on time

Phase φ refers to particle in middle of gap z=0 at t=0

Transit-Time Factor

- **Transit-Time Factor is ratio of energy actually given to a particle passing the cavity centre at peak field to the energy that would be received if the field were constant with time at its peak value**

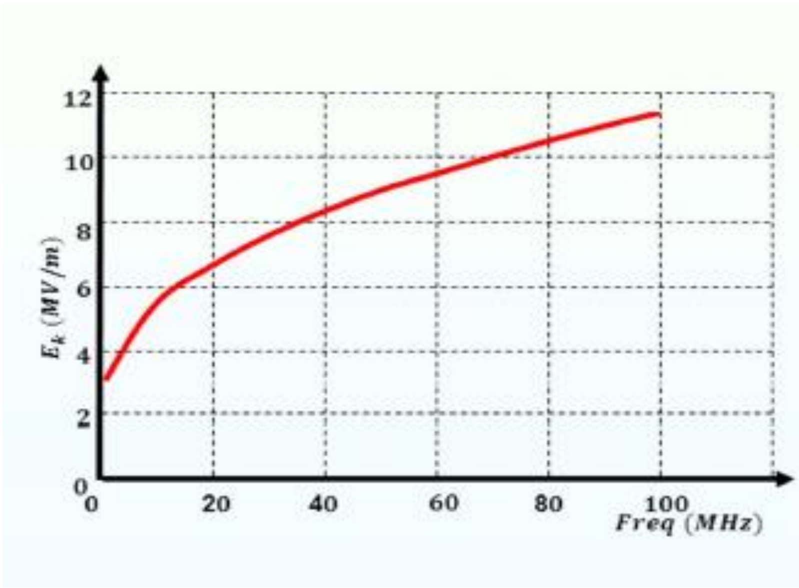
- **The energy gained over the gap G is:**

$$V = \int_{-G/2}^{+G/2} E_0 \cos(\omega t + \varphi) dz = \underbrace{\frac{\sin(\omega G / 2\beta c)}{\omega G / 2\beta c}}_{\text{Transit Gap Factor } \Gamma} (E_0 G \cos \varphi)$$

Transit Gap Factor Γ

Kilpatrick Limit

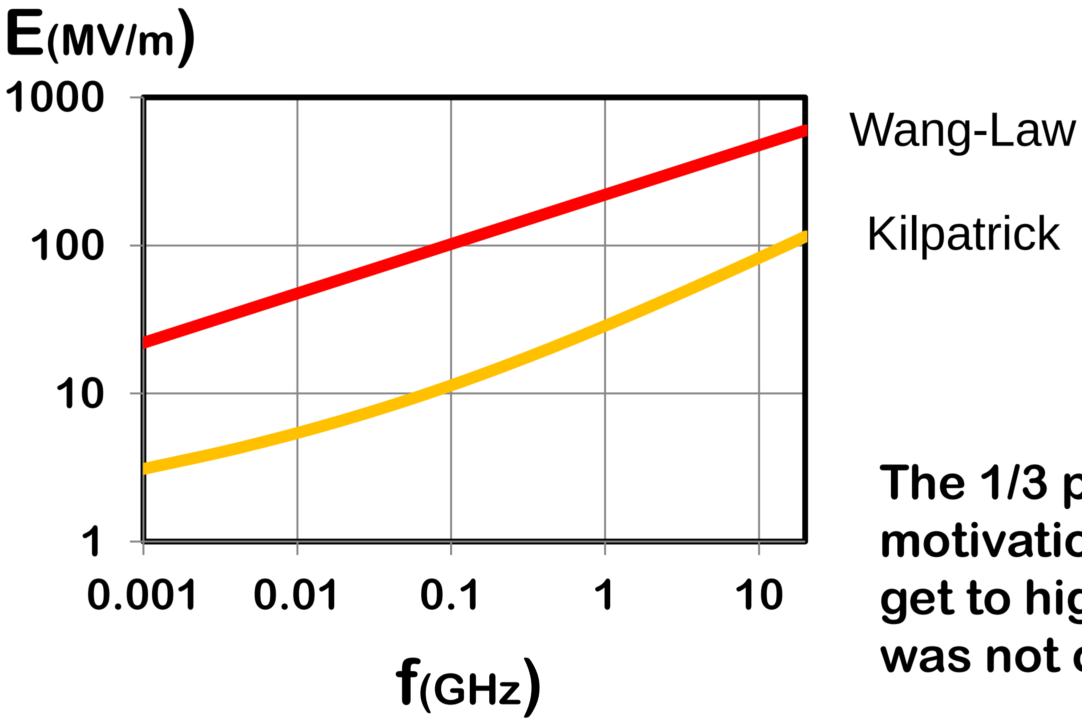
- **RF breakdown observed at very high fields**
- **Kilpatrick Limit (~1950) expresses empirical relation between accelerating frequency and E-field [in MV/m]:**
  - » $f\,[\mathrm{MHz}] = 1.64\, E_k^2\, e^{-8.5/E_k}$



Kilpatrick Limit & Wang-Law

- A lot of new knowledge on this came from linear collider high gradient studies
- Wang-Law formula:

  $E\,[MV/m] = 220\, f[GHz]^{1/3}$



The 1/3 power dependence, being a motivation for quest for higher f to get to higher gradients, however was not confirmed at the end

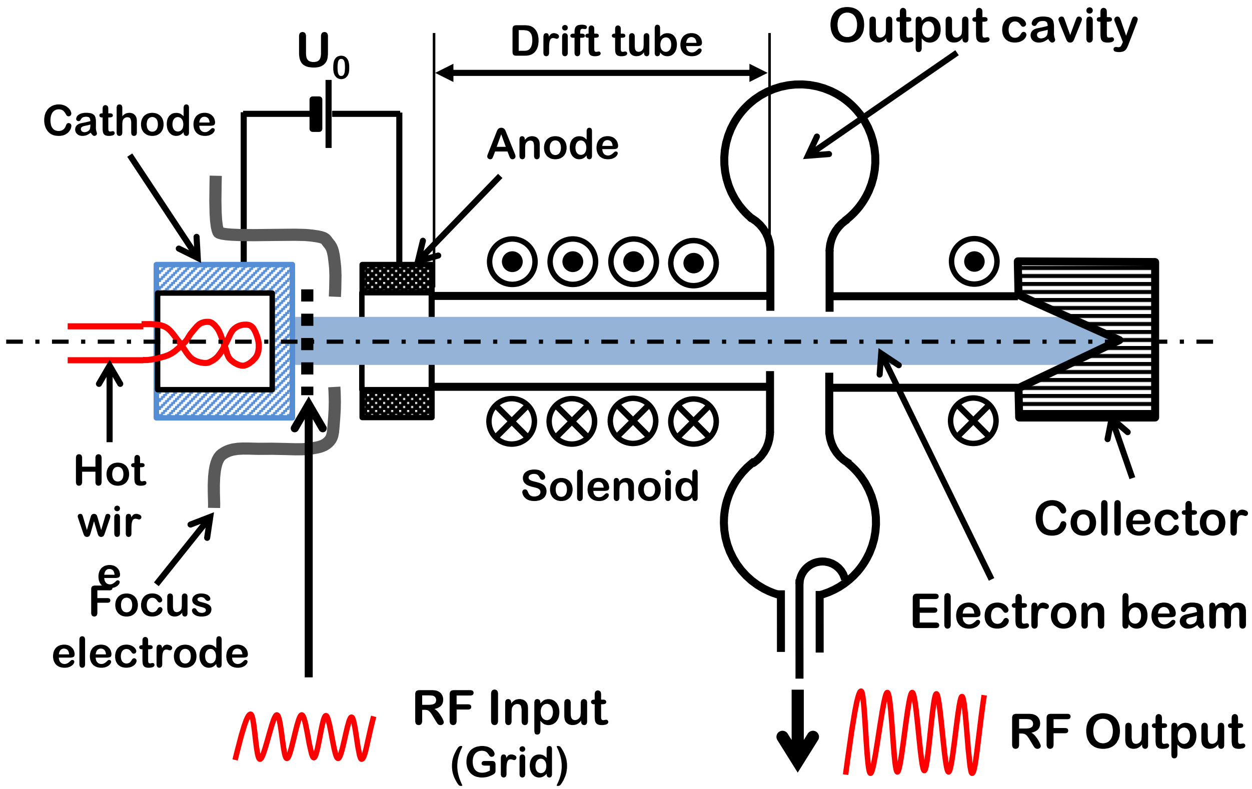
Power Generators for Accelerators

- The sinusoidal power needed to drive the accelerating structures ranges between a few kW to a few MW

- RF power amplifiers
  - Triodes & tetrodes: few MHz to few hundred MHz
  - Inductive Output Tubes
    - CW applications, tens of kW, high efficiency
  - Klystrons: above a few hundred MHz
    - Proven to be the most effective power generator for accelerator applications

IOT - Inductive Output Tubes

It was discovered (in 1939) that a toroidal cavity surrounding an electron beam of oscillating intensity could extract power from the beam without intercepting the beam itself

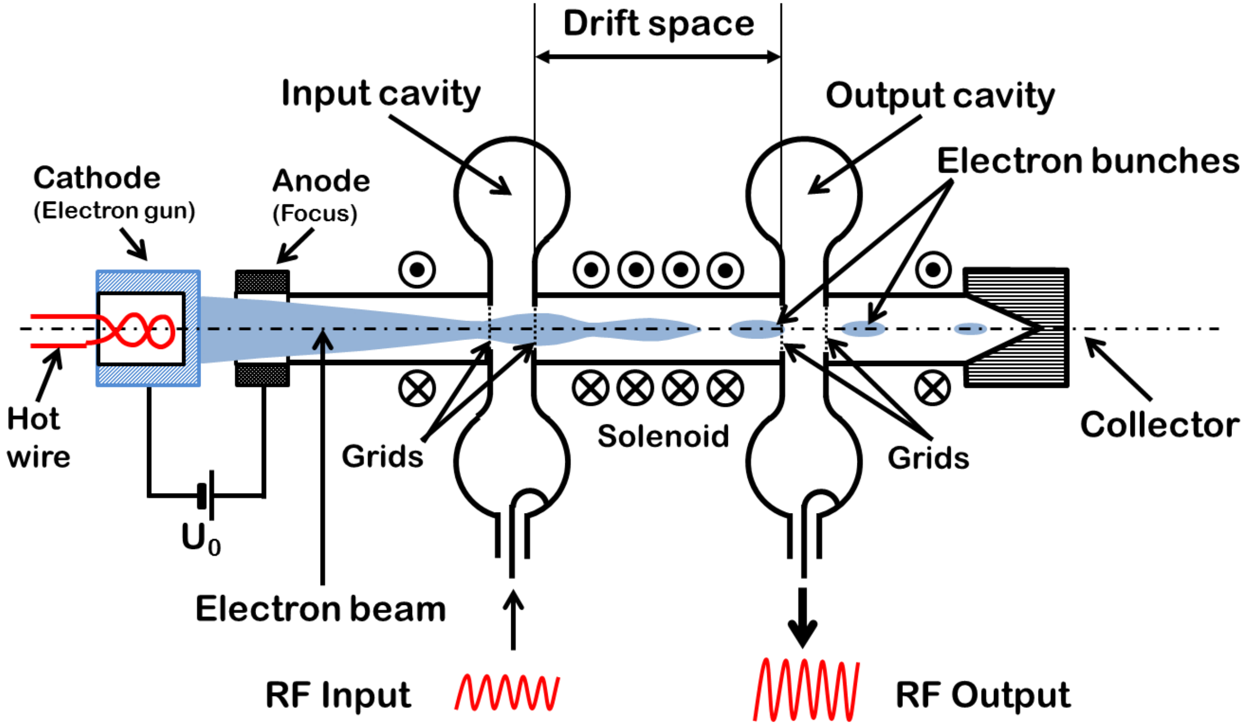


The oscillating EM fields associated with the beam "echoed" inside the cavity, allowing RF energy to be transferred from the beam to a waveguide or coaxial cable connected to the resonator with a coupling loop

This tube was called an ***inductive output tube***, or IOT

IOT => Klystron

Two researchers instrumental in development of IOT, Sigurd and Russell Varian, added a second cavity resonator for signal input to the inductive output tube This input resonator acted as a pair of inductive grids to alternately "bunch" and release packets of electrons down the drift space of the tube, so the electron beam would be composed of electrons traveling at different velocities

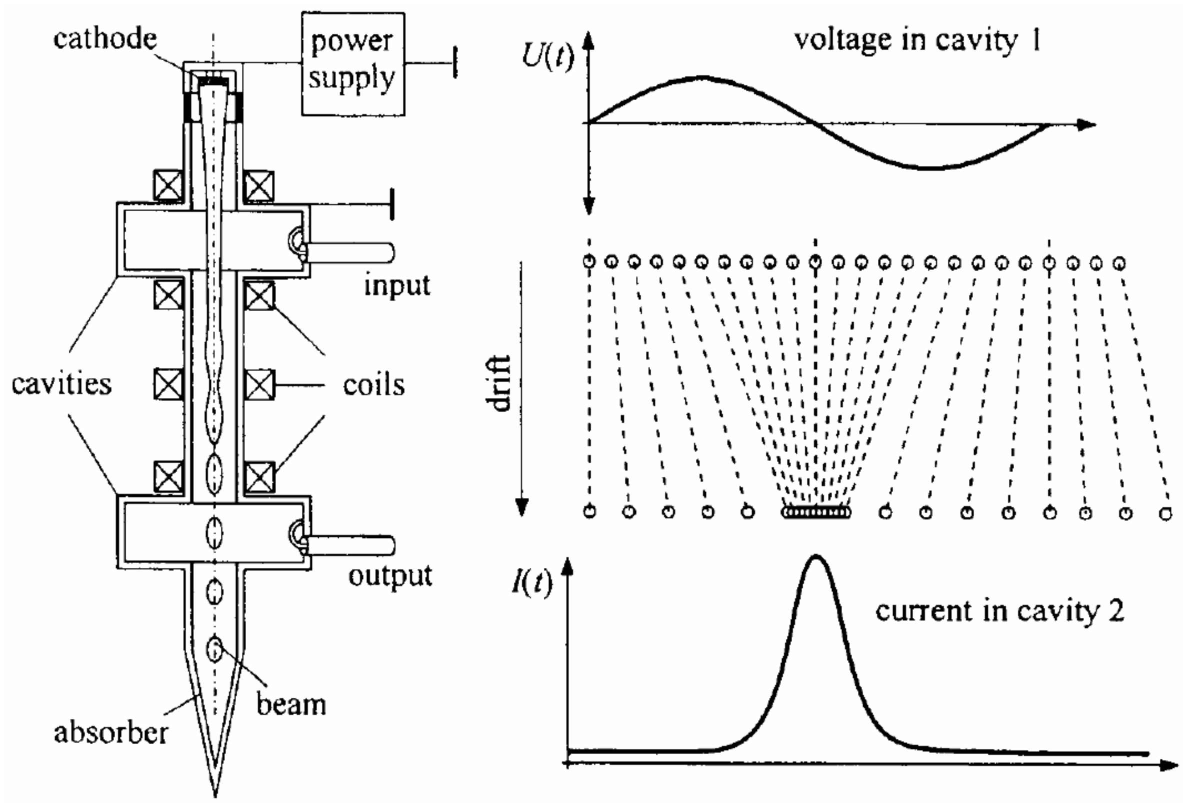


This "velocity modulation" of the beam translated into the same sort of amplitude variation at the output resonator, where energy was extracted from the beam

The Varian brothers called their invention a ***klystron***

Klystrons

- **Principle of operation**
  - Electrons emitted from round cathode with large surface area
  - Accelerated by voltage of a few tens of kV
  - Yields a round beam with a current of between a few amperes and tens of amperes
  - Electrodes close to the cathode focus the beam and solenoid along the tube ensure good beam collimation
  - Outgoing particles from cathode have a well-defined velocity and pass through cavities operated in TM_{011} mode
  - Wave excited in this resonator by external pre-amplifier



Klystrons are similar to a small linear accelerator

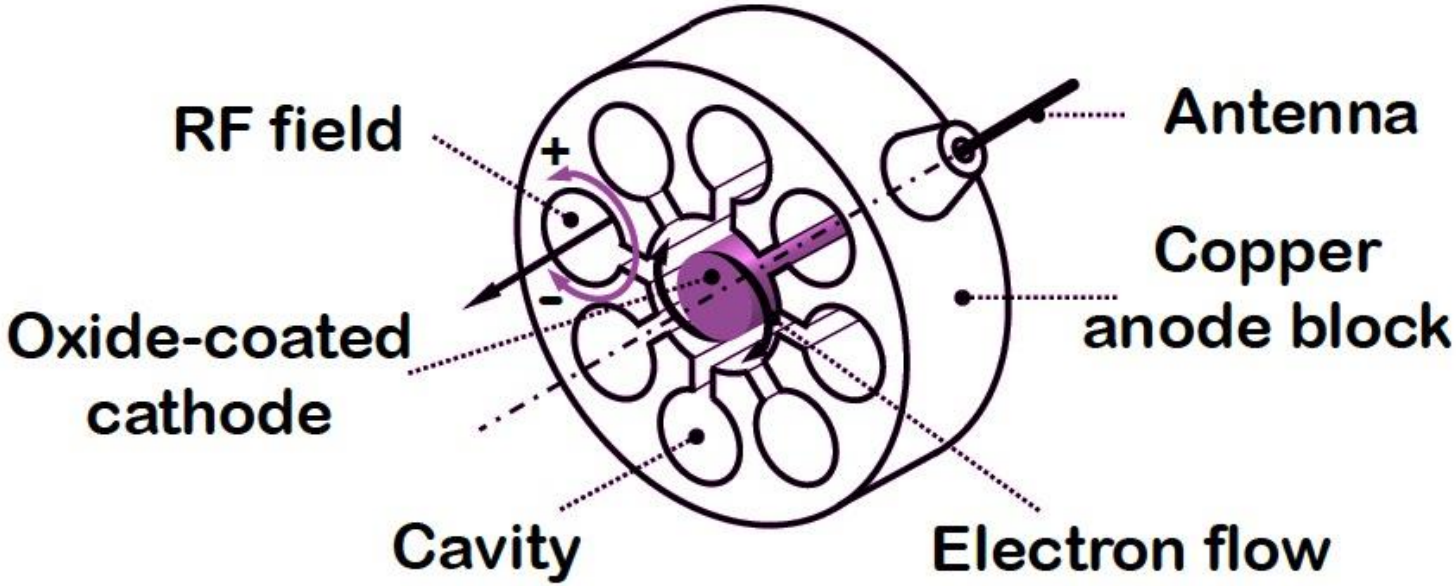
Klystrons

- **Klystron output power**

$$P_{klystron} = \eta U_0 I_{beam}$$

  - U_0 = klystron supply voltage (e.g. 45 kV)
  - I_{beam} = beam current (e.g. 12.5 A)
  - η = klystron efficiency (45% - 65%)

Magnetron

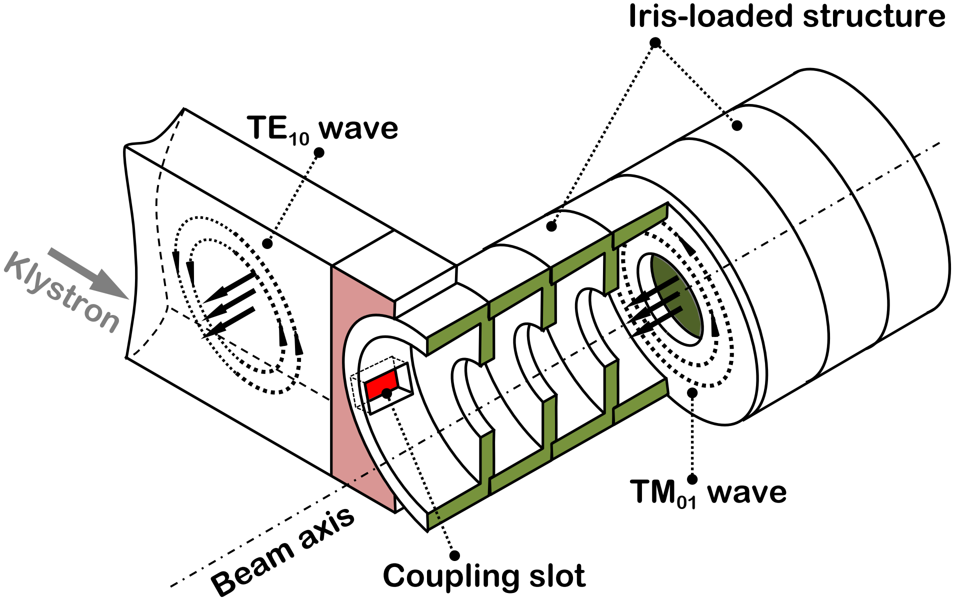


Magnetic field bends electrons, creating azimuthal variation of their trajectories and density. When it matches spacing of cavities, excited fields will further increase variation of density. Resonance. Amplitude grows until saturation.

Magnetron is a popular source for CW applications.
However, it is not an amplifier, but a generator starting from initial noise.
Combining many magnetrons and phasing them is a challenge.

Operation of LINAC Structure

- **Standard operation of linac structure is, e.g., in S-band**
  - λ=0.100m (f_{RF}=3 GHz)
- **As in radar technology, RF power supplied by pulsed power tubes – klystrons**
  - Power fed into linac structure by TE_{10} wave in rectangular waveguide which is connected perpendicular to cylindrical TM_{01} cavity



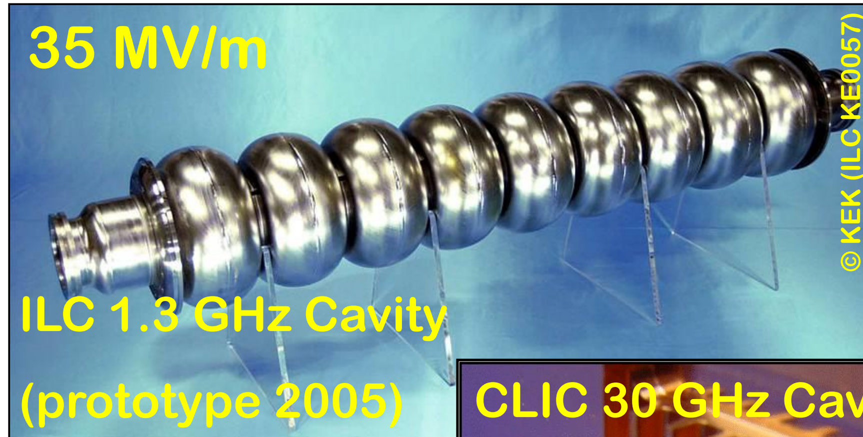
RF technology

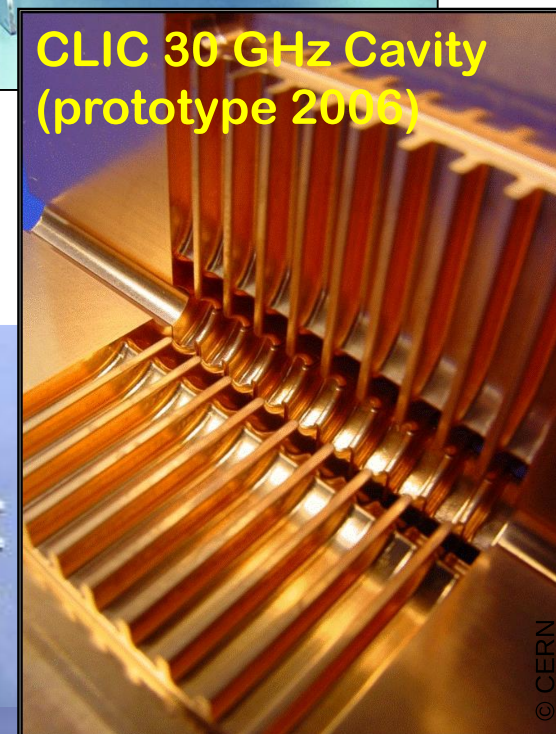
Usual operating frequencies for RF cavities for Linear accelerators are

Warm cavities	gradient	repetition rate
S-band (3GHz)	15-25 MV/m	50-300 Hz
C-band (5-6 GHz)	30-40 MV/m	<100 Hz
X-band (12 GHz)	100 MV/m	<100 Hz

Superconducting cavities		
L band (1.3 GHz)	< 35 MV/m	up to CW

Variety of Accelerating Cavities



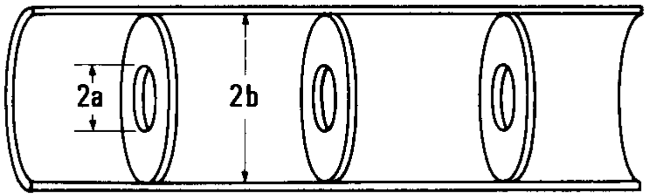


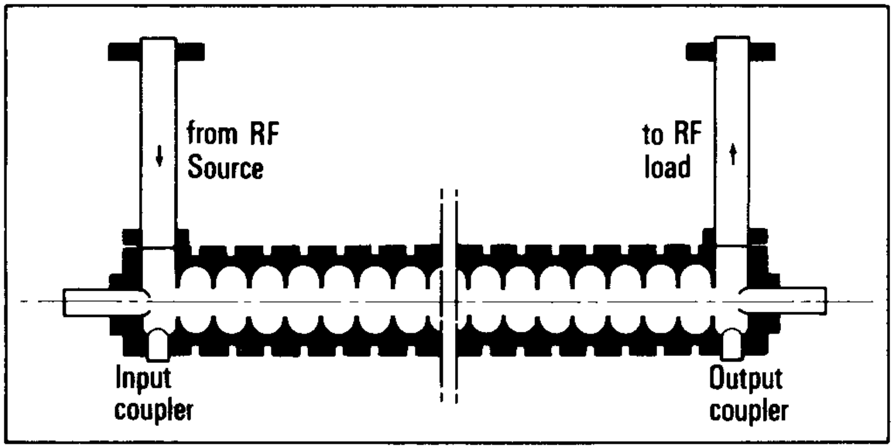
Travelling wave and standing wave structures

The wave velocity and the particle velocity have to be equal hence we need a disk loaded structure to slow down the phase velocity of the electric field

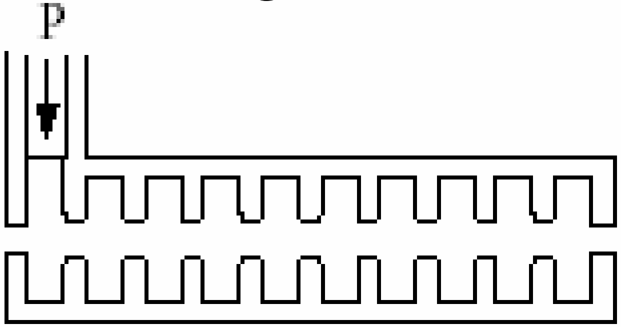
To achieve synchronism $v_p < c$

Slow down wave using irises





In a standing wave structure the electromagnetic field is the sum of two travelling wave structure running in opposite directions

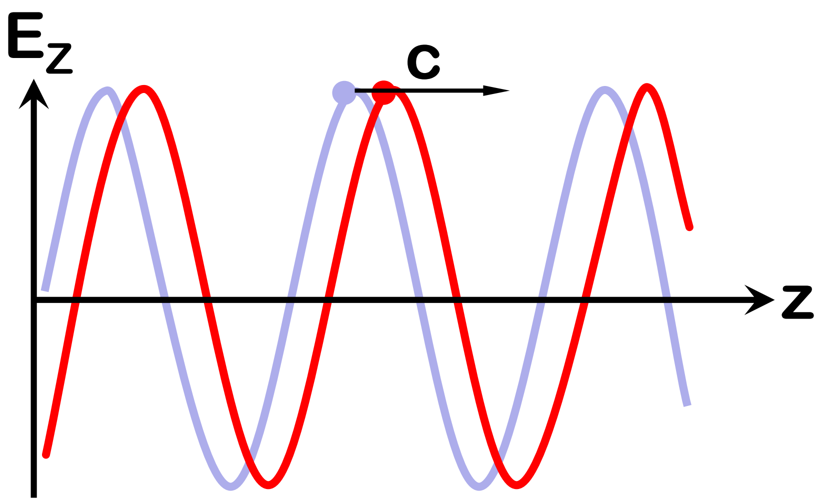


Only the forward travelling wave takes part in the acceleration process

Acceleration in RF structures

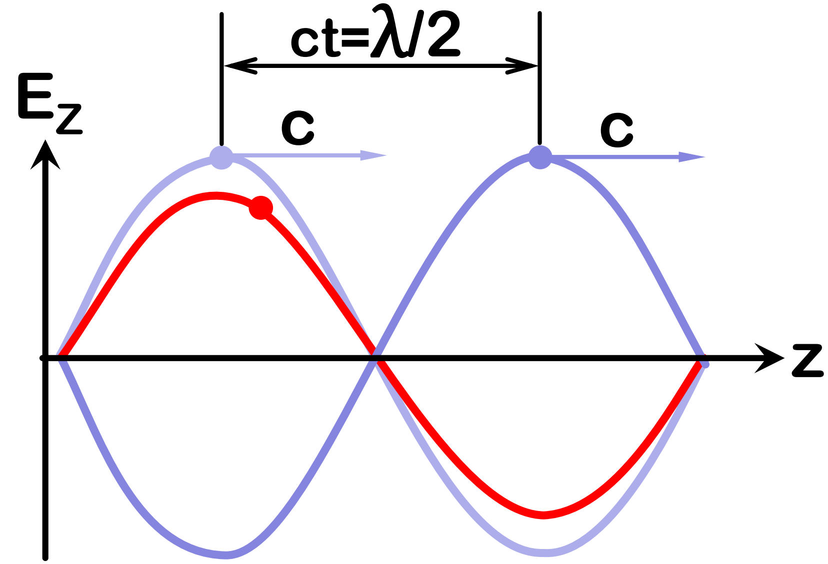
Acceleration is achieved with RF cavities, using EM modes with the electric field pointing in the longitudinal direction (direction of motion of the charged particle)

The RF electric field can be provided by travelling wave structure or standing wave structure



Travelling wave: the bunch sees a constant electric field

$E_z=E_0\cos(\phi)$



Standing wave: the bunch sees a varying electric field

$E_z=E_0\cos(\omega t+\phi)\sin(kz)$

Long. dynamics in travelling wave

Consider a particle moving in the electric field of a travelling wave

$$E_z = E_0 \cos(\omega t - kz)$$ **with a phase velocity** $$v_f = \frac{\omega}{k}$$

The equations used to describe the motion in the longitudinal plane are

$$\frac{dp_z}{dt} = eE_0 \cos(\omega t - kz) \qquad \frac{dE}{dt} = eE_0 \dot{z} \cos(\omega t - kz)$$

Define the synchronous particle as

$$\frac{dE_s}{dt} = eE_0 v_s \cos\varphi_s$$

For any other particle, we will use, as coordinates, the deviation from the energy and time from the synchronous particle

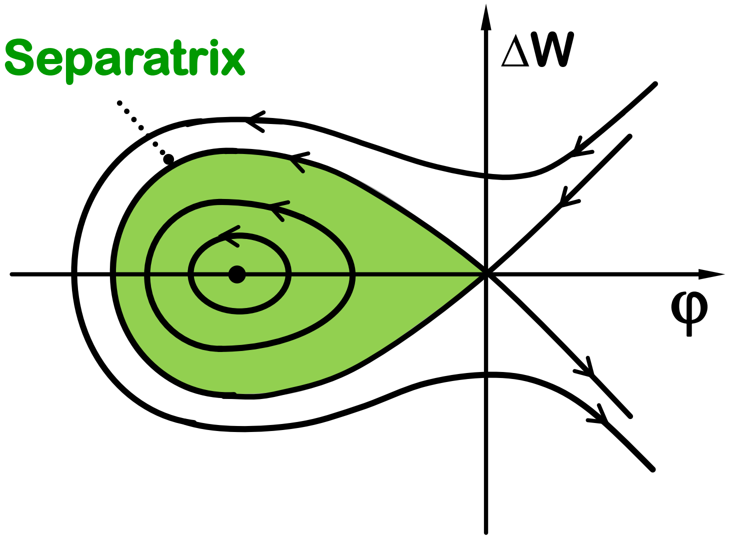
$$E = E_s + W \qquad z = z_s + u$$

After changing variables to $$\varphi = kz - \omega t = \varphi_s - \frac{\omega}{v_s} u$$

Long. dynamics in travelling wave

…we get the following system of equations for particle motion

$$\frac{dW}{ds} = eE_0\left[\cos\varphi - \cos\varphi_s\right] \qquad \frac{d\varphi}{ds} = -\frac{\omega}{\beta_s^3\gamma_s^3 c}\frac{W}{mc^2}$$



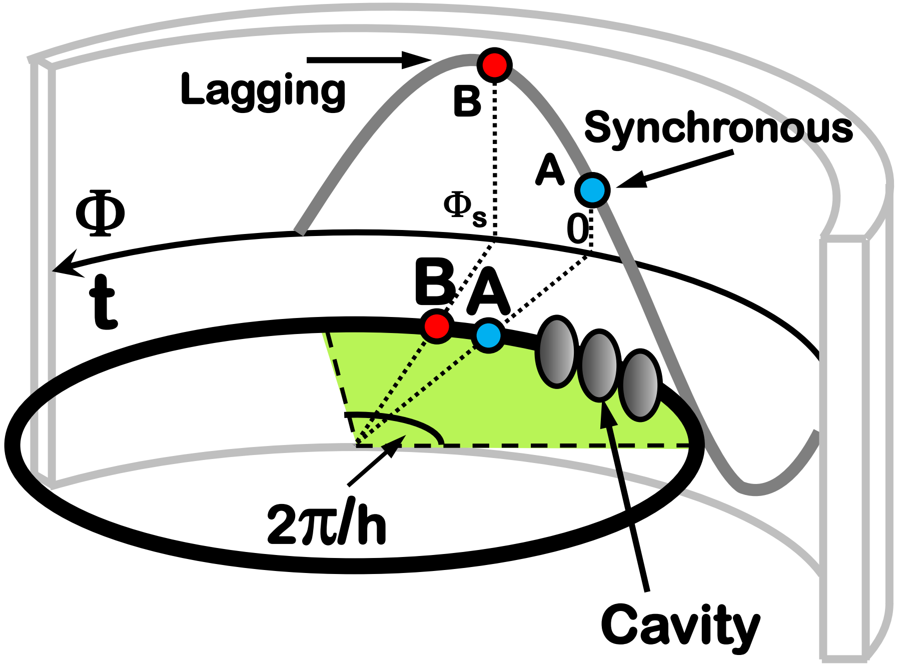
These describe the usual RF bucket in the longitudinal phase space (φ, W)

Longitudinal dynamics in a ring

$$\frac{d\bar{p}}{dt} = q(\bar{E} + \bar{v} \times \bar{B}) \qquad \bar{p} = m_0 \gamma \bar{v}$$

$$\frac{d\varepsilon}{dt} = \bar{F} \cdot \bar{v} \qquad \varepsilon = m_0 \gamma c^2$$

Acceleration is provided by longitudinal electric fields generate in RF cavities



A particle in an RF cavity changes energy according to the phase of the RF field found in the cavity

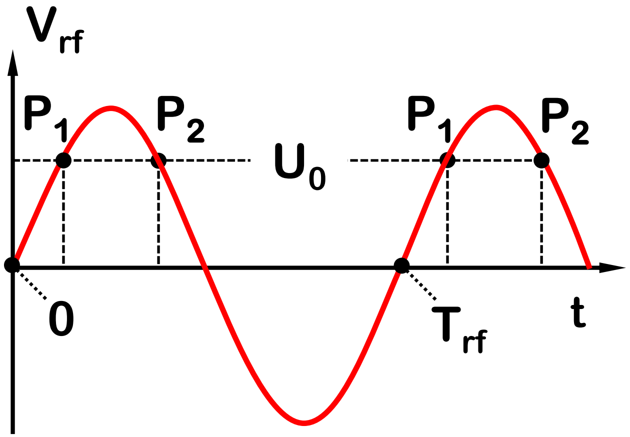
$$\Delta E = eV(t) = eV_o \sin(\omega_{RF} t + \varphi_s)$$

The synchronous particle is the particle that arrives at the RF cavity when the voltage is such that it compensate exactly the average energy losses U_0

$$\Delta E = U_0 = eV_0 \sin(\varphi_s)$$

Motion in the RF potential

There are two points in time where the particle will get the correct energy from the RF wave P1 and P2: one is stable one is unstable



$$\Delta E = eV(t) = eV_o \sin(\omega_{RF} t + \varphi_s)$$

What is the fate of the particle arriving earlier or later than the synchronous particles?

It depends on the time of flight of the particle all around the ring and how it behaves with energy

Synchronous particle has nominal energy E, velocity v and travels around the nominal circumference C in a time T **T = C/v**

Momentum compaction factor

Taking a log of T=C/v and differentiating this expression brings:

$$\frac{dT}{T} = \frac{dC}{C} - \frac{dv}{v}$$

The first term is expressed via α_c which is called momentum compaction factor and depends on the specific lattice. It can be positive (most common) or negative (higher energy particles travelling shorter paths)

$$\frac{dC}{C} = \alpha_c \frac{dp}{p}$$

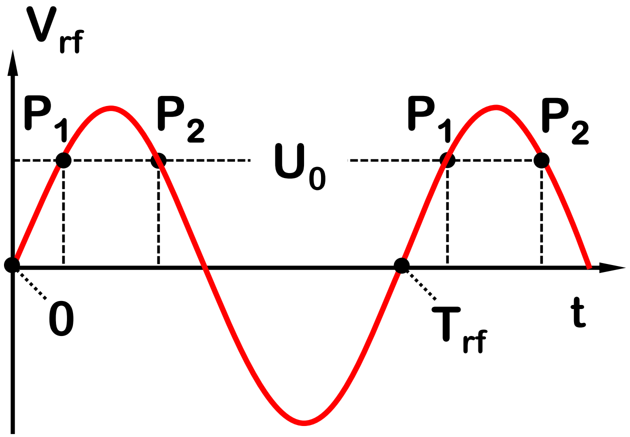
The second term can be expanded using $\frac{dv}{v} = \frac{d\beta}{\beta} = \frac{1}{\gamma^2}\frac{dp}{p}$

=> $$\frac{dT}{T} = \left(\alpha_c - \frac{1}{\gamma^2}\right)\frac{dp}{p} \sim \left(\alpha_c - \frac{1}{\gamma^2}\right)\frac{dE}{E}$$

We see that the time of flight depends on the energy deviation and on the momentum compaction factor

What is the fate of particle that arrives later or earlier than the synchronous particle?

There are two points in time where the particle will get the correct energy from the RF P1 and P2: one is stable one is unstable



$$\frac{dT}{T} \sim \left(\alpha_c - \frac{1}{\gamma^2}\right)\frac{dE}{E}$$

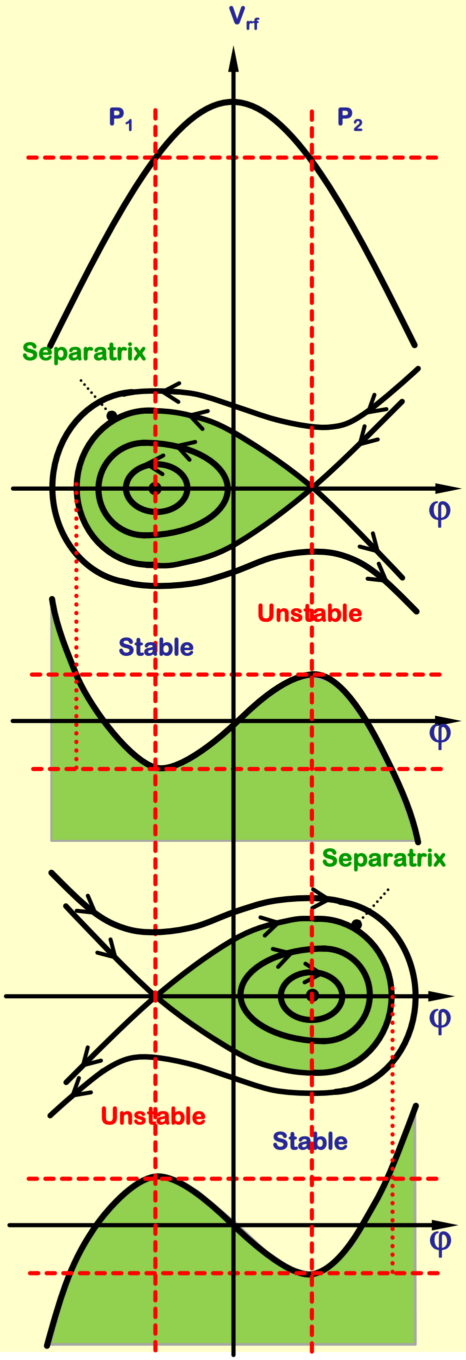
if $\alpha_c - \frac{1}{\gamma^2} > 0$ **Stable P_2**

A particle with higher energy → has a longer time of flight → it arrives later at the RF cavity → it gain less energy (sees a lower RF voltage) → stable

A particle with lower energy → has a shorter time of flight → it arrives earlier at the RF cavity → it gain more energy (sees a higher RF voltage) → stable

This mechanism is called "principle of phase stability"
(stable point – negative slope of the RF)
It ensures that we can capture particles in the RF potential

Transition energy



Following from

$$\frac{dT}{T} \sim \left(\alpha_c - \frac{1}{\gamma^2}\right)\frac{dE}{E}$$

We can introduce “transition energy”:

$$\gamma_t = \frac{1}{\alpha_c^{1/2}}$$

Stability point flips from P1 to P2 during acceleration through the transition energy

Long. dynamics in synchrotron

Start from particle moving in the electric field of a travelling wave

$$E_z = E_0 \cos(\omega t - kz) \quad \textbf{with a phase velocity} \quad v_f = \frac{\omega}{k}$$

The equations used to describe the motion in the longitudinal plane are

$$\frac{dp_z}{dt} = eE_0 \cos(\omega t - kz) \qquad \frac{dE}{dt} = eE_0 \dot{z} \cos(\omega t - kz)$$

Define again the synchronous particle as

$$\frac{dE_s}{dt} = eE_0 v_s \cos \varphi_s$$

For the arbitrary particle, use as coordinates the deviation from the energy and time from the synchronous particle:

$$E = E_s + \varepsilon \qquad t = t_s + \tau$$

and …

Long. dynamics in synchrotron

…we will get the first equation

$$\frac{d\varepsilon}{ds} = eE_0\left[\cos(\omega\tau + \varphi_s) - \cos\varphi_s\right]$$

Using the definition of momentum compaction factor, we have

$$\frac{dT}{T} \sim \left(\alpha_c - \frac{1}{\gamma^2}\right)\frac{dE}{E}$$

i.e. (at high energy $\gamma >> 1$)

$$\frac{d\tau}{dt} \sim \frac{\alpha_c}{E_s}\frac{d\varepsilon}{dt}$$

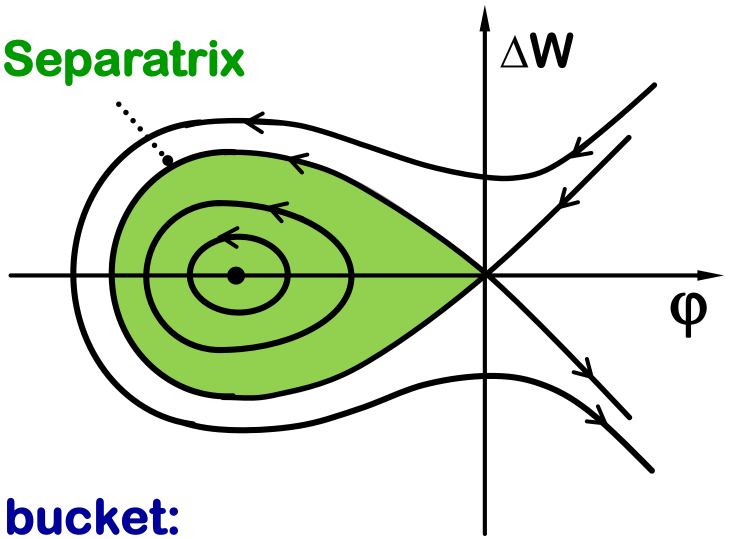
These two equations describe the RF bucket in the longitudinal phase space with coordinates (τ, ε)

RF bucket

Rewrite these equations for relat. case

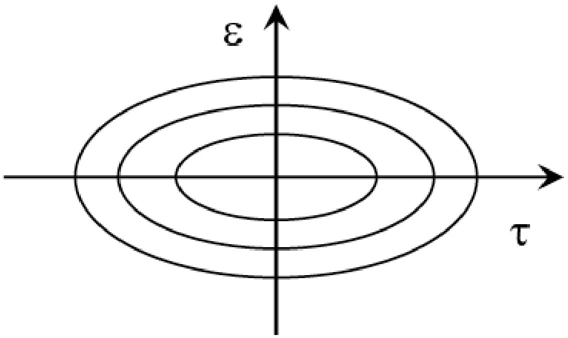
$$\varepsilon' = \frac{qV_0}{L}\left[\sin(\phi_s + \omega\tau) - \sin\phi_s\right]$$

$$\tau' = \frac{\alpha_c}{E_s}\varepsilon$$



Linearised equations for the motion in the RF bucket: the phase space trajectories become ellipses

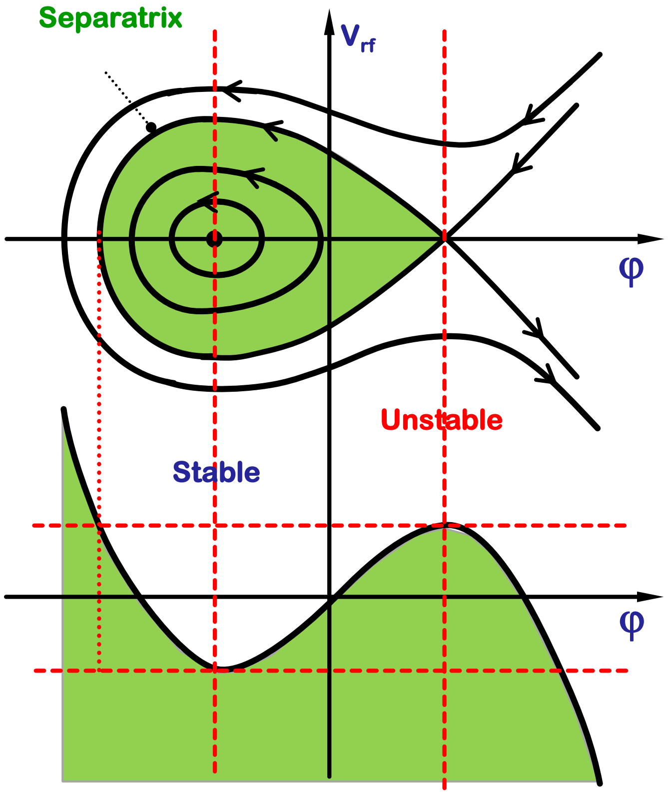
$$\varepsilon' = \frac{e}{T_0}\frac{dV}{d\tau}\tau \qquad \tau' = \frac{\alpha_c}{E_s}\varepsilon$$



$$\omega_s^2 = \frac{\alpha_c e\dot{V}}{T_0 E_0}$$ **angular synchrotron frequency**

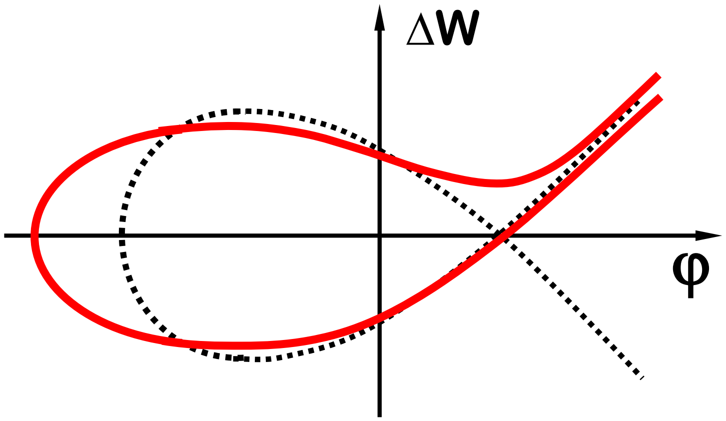
RF bucket – nonlinearity & adiabaticity

RF bucket potential is not linear



Passing near *saddle point* takes long time

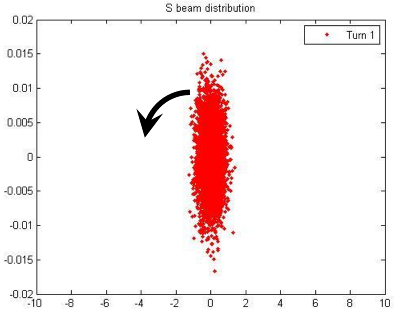
We assumed here that the acceleration is adiabatic i.e. $d\gamma_s/ds \sim 0$.
If this in not true, numerical integration shows that the RF bucket gets distorted into a "golf club"



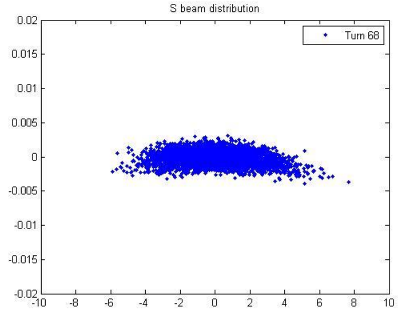
Tracking example: longitudinal plane

Consider a storage ring with a synchrotron tune of 0.0037 (273 turns); negligible radiation damping:

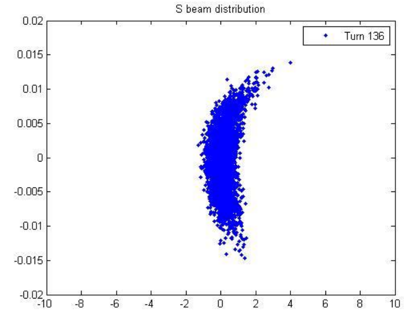
start



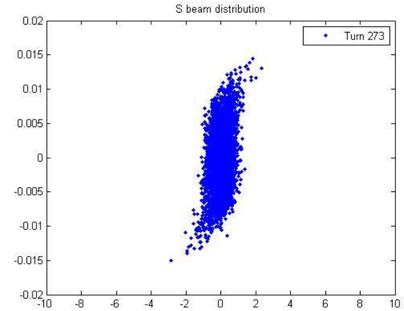
¼ of synch period



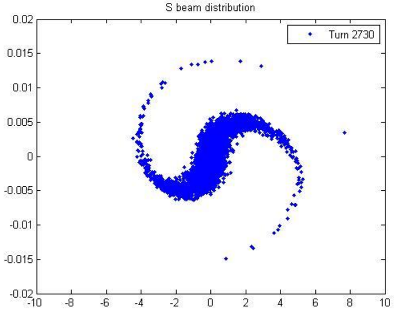
½ of synch period



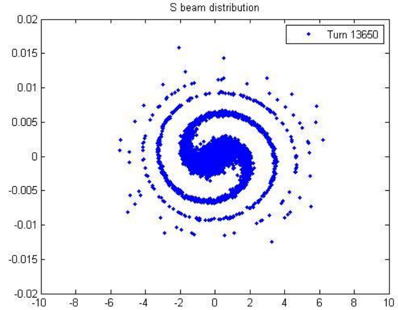
1 synch period



10 synch periods



50 synch periods



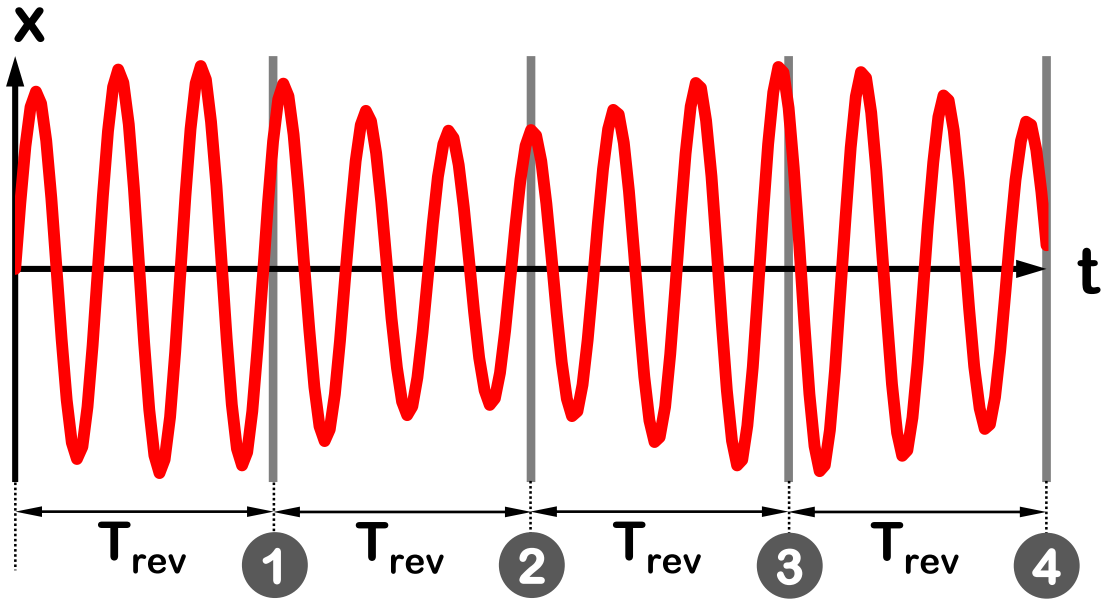
After 50 synchrotron periods the longitudinal phase space distribution is completely filamented (decoherence)) due to the nonlinearity of the RF potential

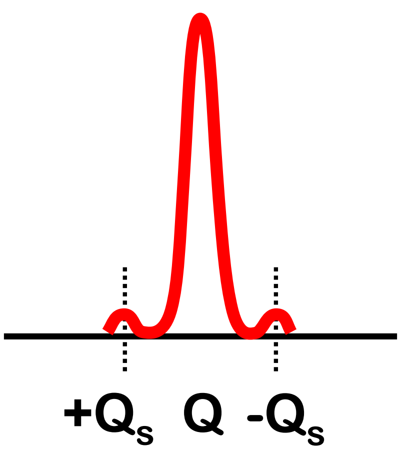
Betatron tune and synchrotron tune

Synchrotron motion is multi turn (slow). Synchrtotron tune: $Q_s = 2\pi T_{rev}/\omega_s$

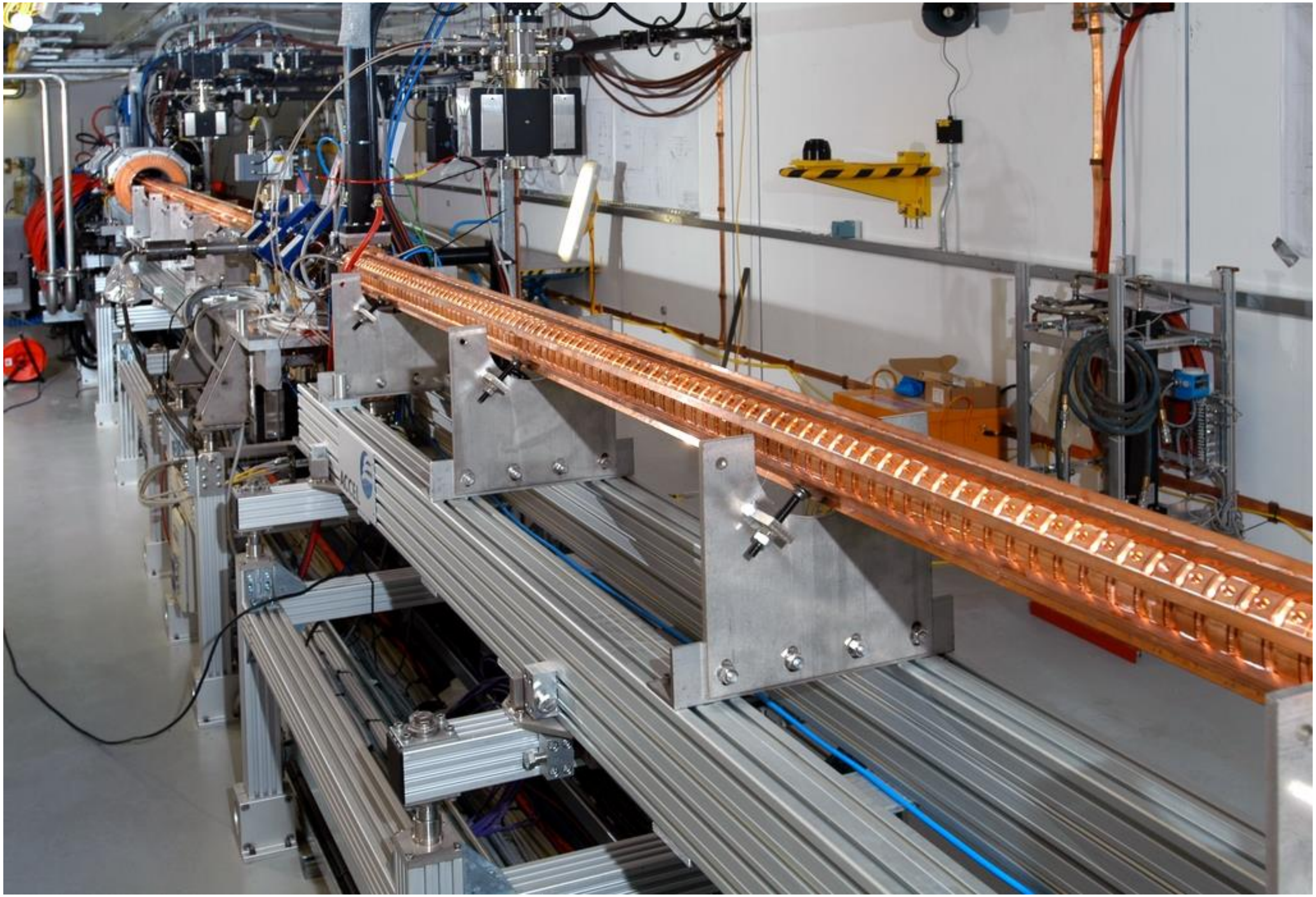
Betatron oscillations around the ring are fast.
Betatron tune: number of oscillations per revolution: $Q = \frac{\mu}{2\pi} = \frac{1}{2\pi}\int_0^L \frac{ds'}{\beta(s')}$

In case of coupling, betatron motion can be modulated by synchrotron





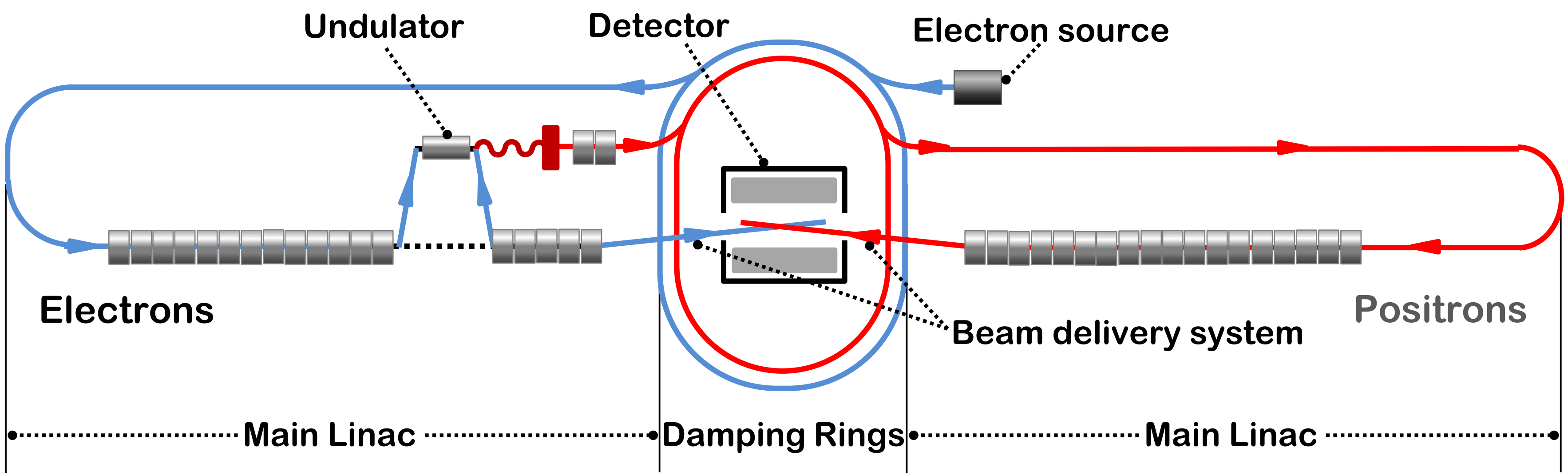
A 100 MeV LINAC in … Diamond Light Source



Linacs in … Linear Colliders

Linear accelerators are at the heart of the next generation of linear colliders

ILC (International Linear Collider)	CLIC (Compact Linear Collider)
L-band SC cavities	X-band NC cavities
30 MV/m	100 MV/m
500 GeV (36 km overall length)	3 TeV (48 km overall length)



Linacs in … Fourth generation light sources

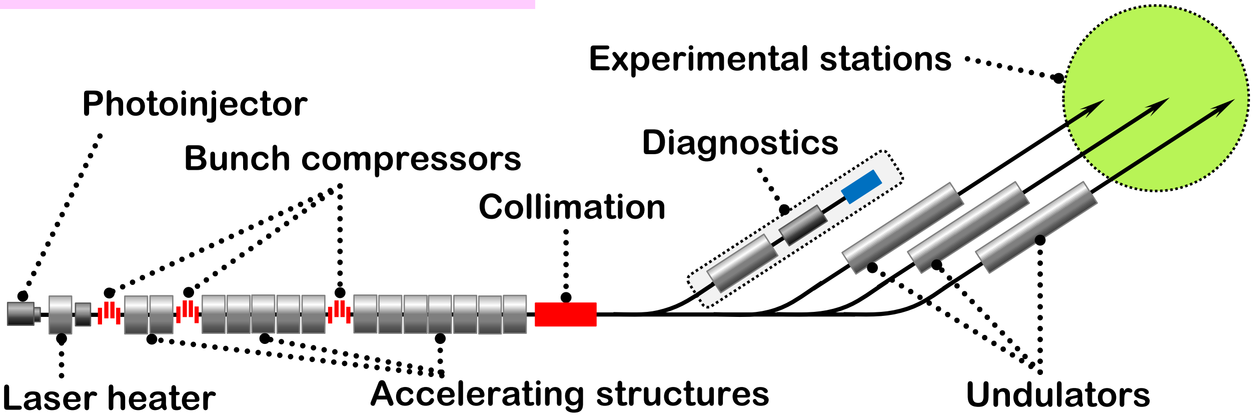
Linear accelerators are at the heart of the next generation of synchrotron radiation sources, e.g. the UK New Light Source project was based on

High brightness electron gun

SC CW linac L- band

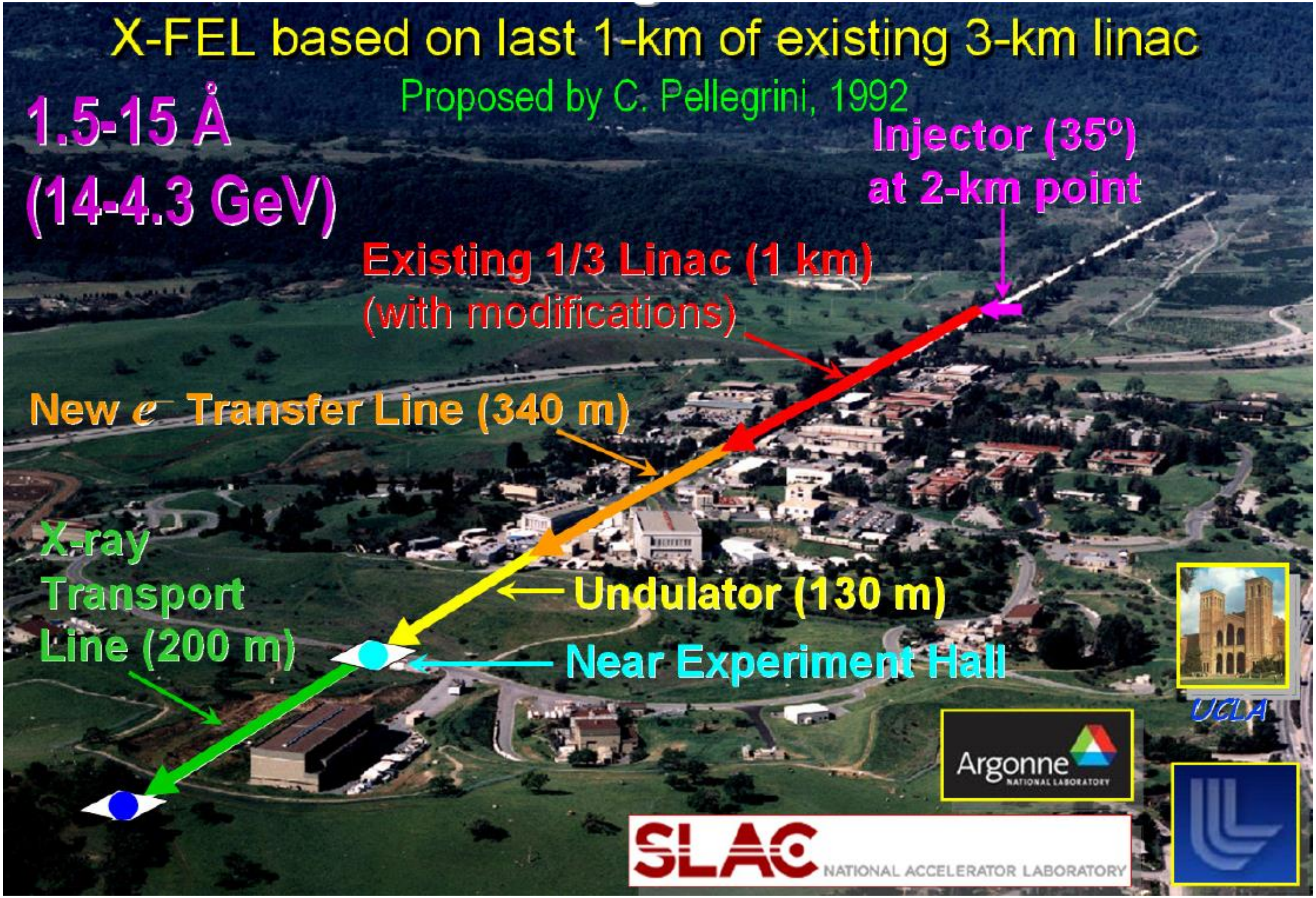
operating (initially) at 1 kHz

2.25 GeV



to feed 3 FELS covering the photon energy range 50 eV – 1 keV

Linac in … The first X-ray FEL at SLAC (LCLS)



Summary of the lecture

- We have discussed
  - Historical Introduction to acceleration methods
  - Waveguides
  - Resonant Cavities
  - Parameters of Cavities
  - Power Generation
  - Basics of linacs
  - Basics of longitudinal dynamics