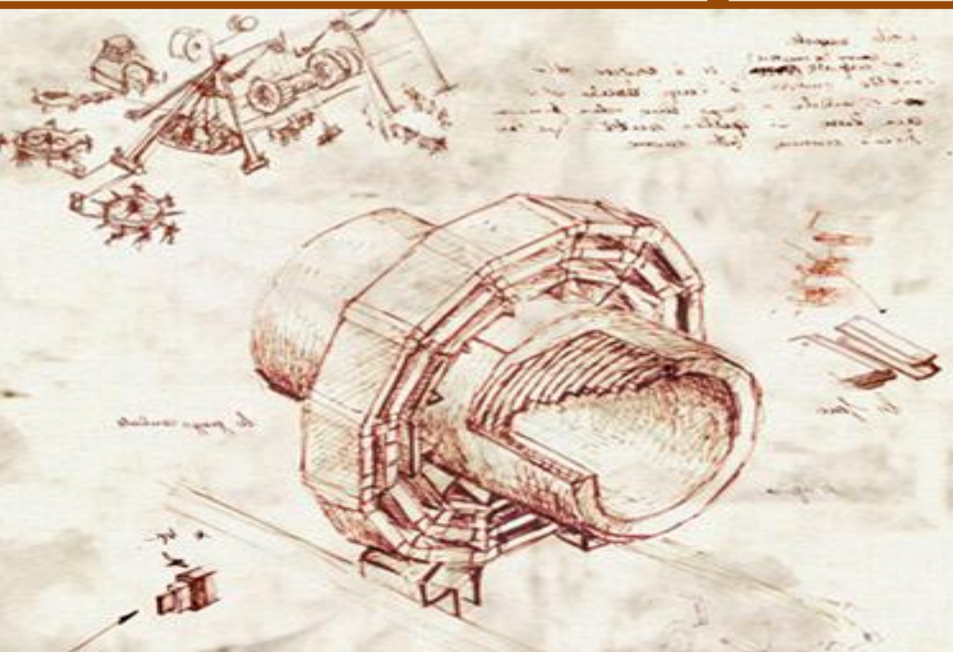
John Adams Institute for Accelerator Science

Unifying physics of accelerators, lasers and plasma



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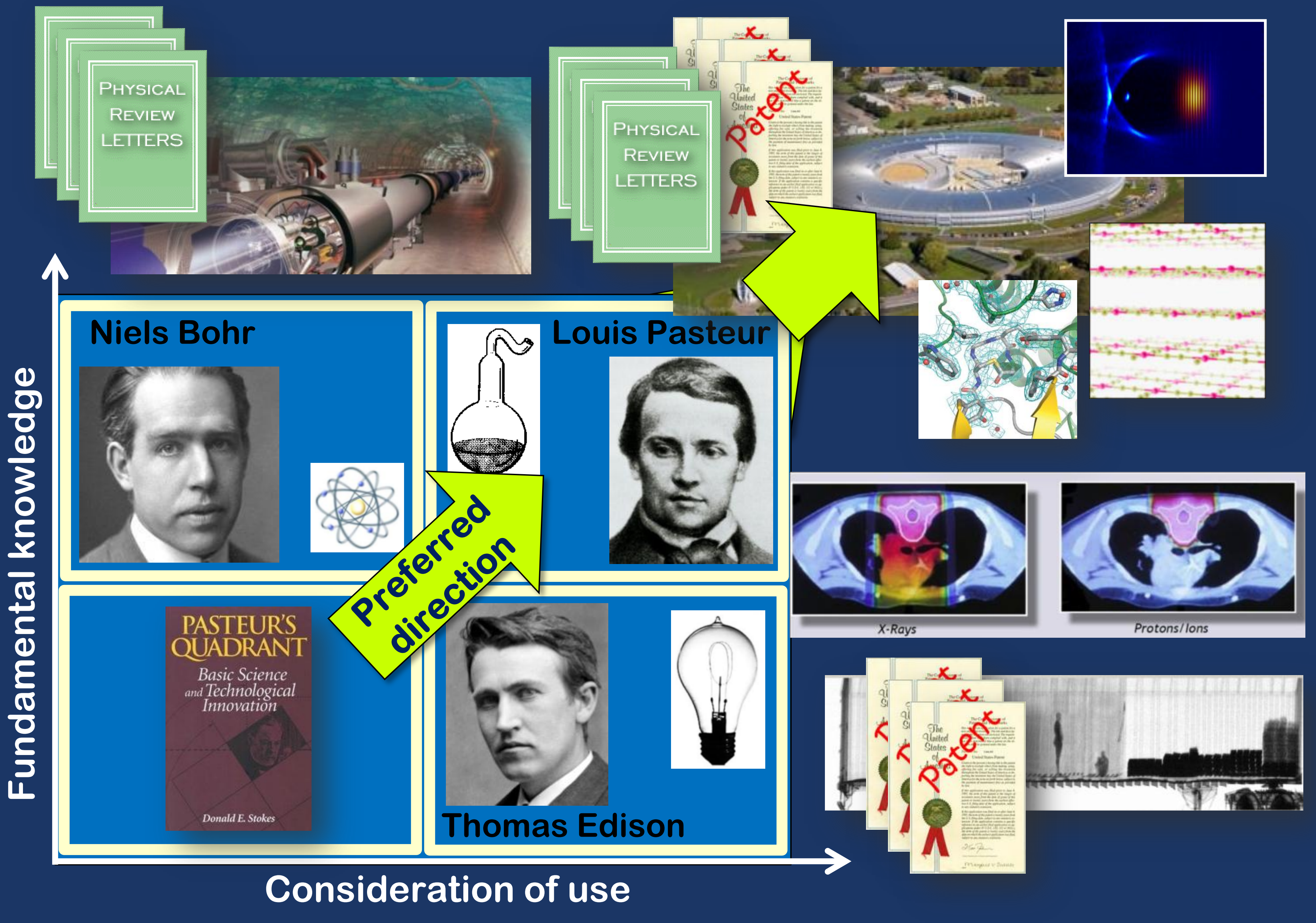
Lecture 3: Synchrotron radiation

USPAS 16

June 2016



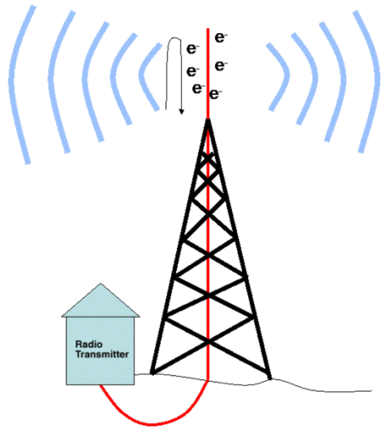
Physical Review Letters
Physical Review Letters
Patent
Patent
Niels Bohr
Louis Pasteur
Preferred direction
Pasteur's Quadrant
Basic Science and Technological Innovation
Donald E. Stokes
Thomas Edison
X-Rays
Protons/Ions
Fundamental knowledge
Consideration of use

Diamond light source

How does it work?

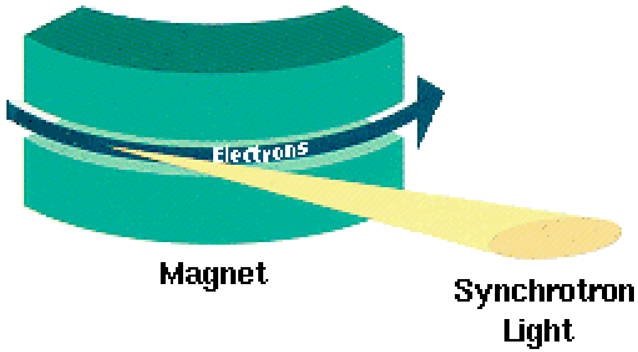
What is synchrotron radiation

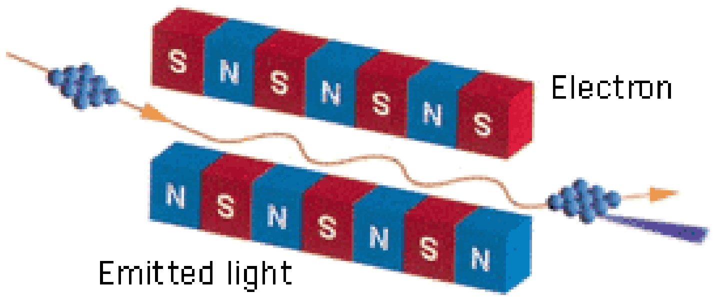
- **Electromagnetic radiation is emitted by charged particles when accelerated**



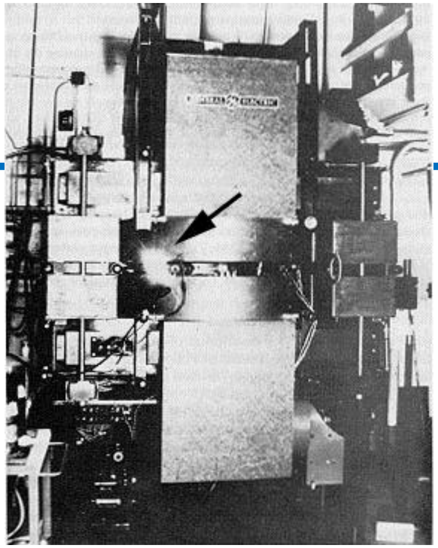
The electromagnetic radiation emitted when the charged particles are accelerated radially ($v \perp a$) is called synchrotron radiation

It is produced in the synchrotron radiation sources using bending magnets undulators and wigglers





SR brief history



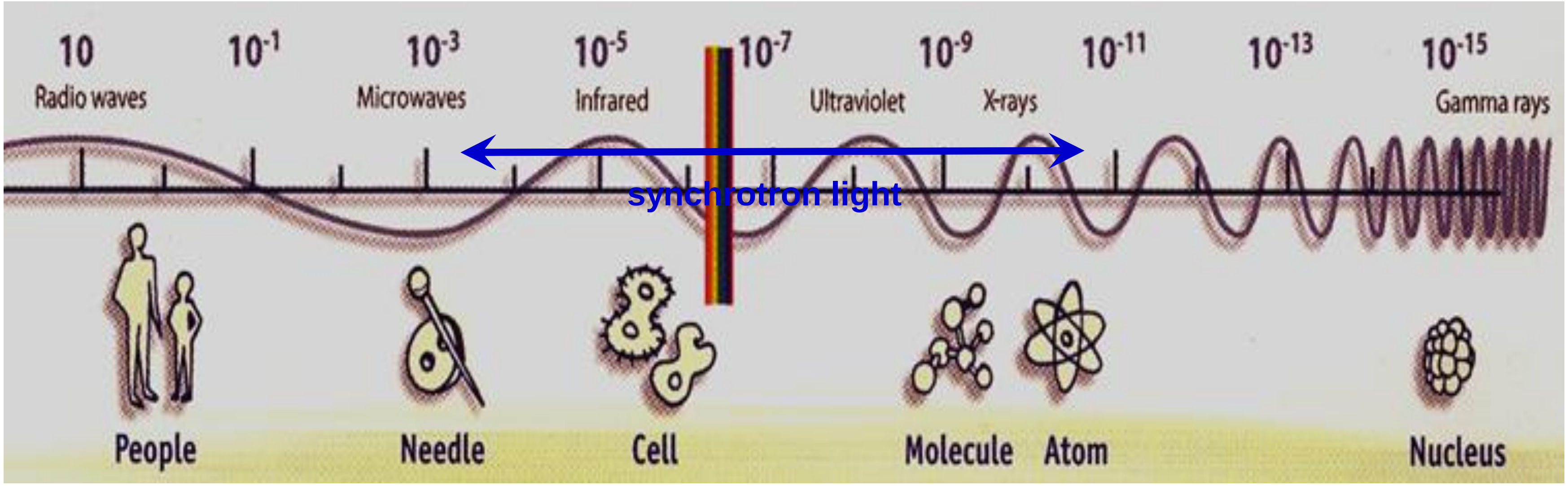
- In 1944, D. Ivanenko and I. Pomeranchuk predicted that the maximum energy of electrons in a betatron is limited due to energy losses caused by radiation of relativistic electrons
- This radiation was first observed around 1947, by accident, in a General Electric 70 MeV synchrotron (this gave the name *synchrotron* to the observed radiation), in the *visible* spectrum
  - It is interesting to note that earlier deliberate attempts to find this radiation in a betatron had failed, as researchers looked for the radiation in a microwave range where the betatron walls were opaque
- The first physics experiments with SR were conducted in 1956 at Cornell, on a 320~MeV synchrotron
  - In this run, D. Tomboulian and P. Hartman studied the spectral and angular properties of the radiation and also made the first soft X-ray spectroscopy experiments, investigating the transparency of beryllium and aluminum foils near the K and L edges
- The National Bureau of Standards (now National Institute of Standards and Technology) was the next to use SR properties to their advantage, modifying a section of a vacuum chamber of a 180~MeV electron synchrotron to enable access to SR
- Soon, it was apparent that the era of SR light sources had begun

Synchrotron radiation sources properties

Broad Spectrum which covers from microwaves to hard X-rays:

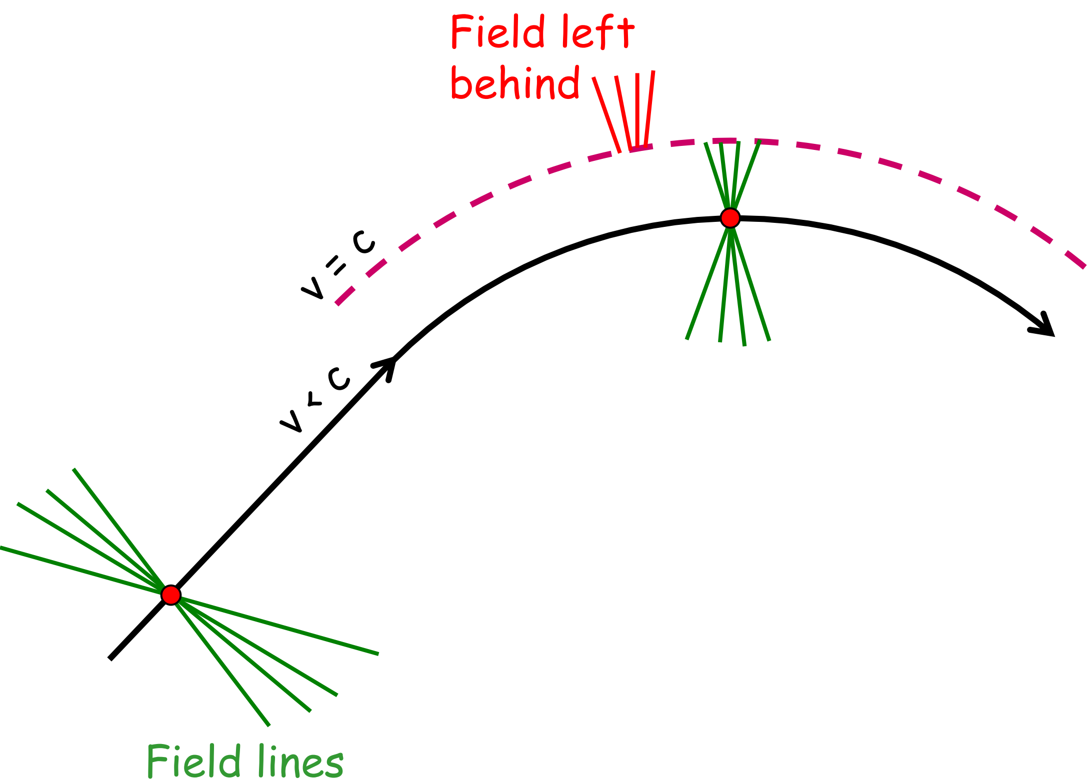
The user can select the wavelength required for experiment;

either with a monochromator
or adjusting the emission wavelength of insertion devices



Synchrotron Radiation (SR)

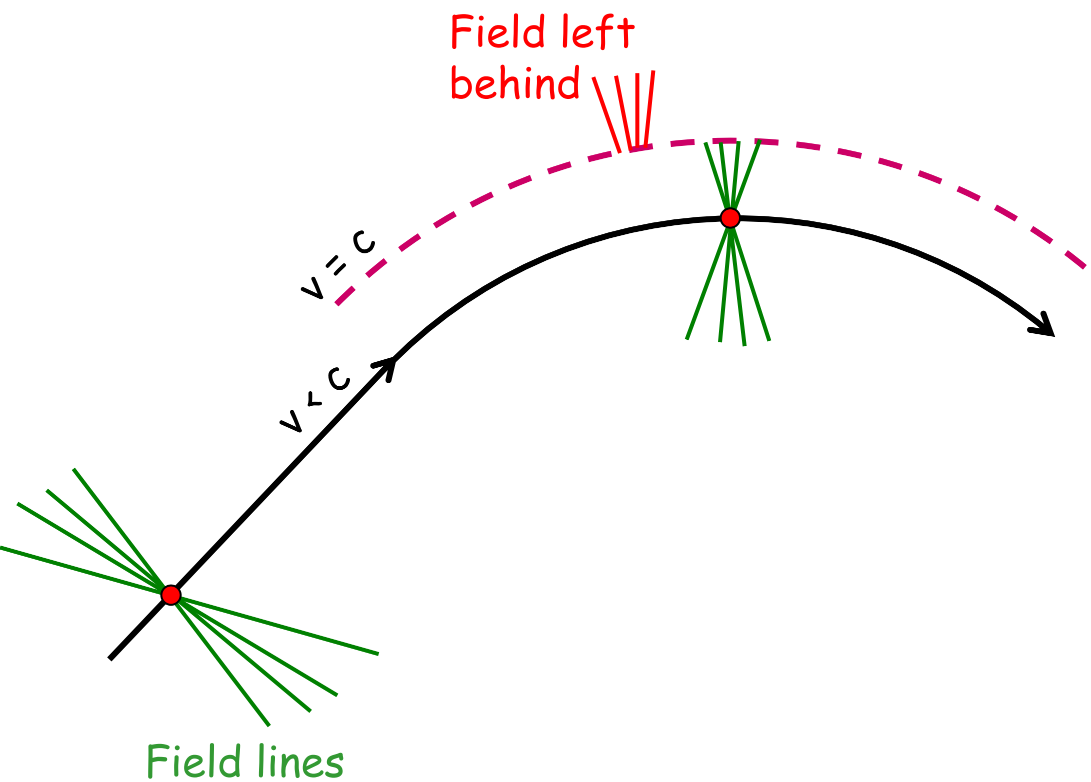
SR caused by leaving part of fields behind when the beam moves along the curve



Energy loss per meter is proportional to the energy in the 4th degree

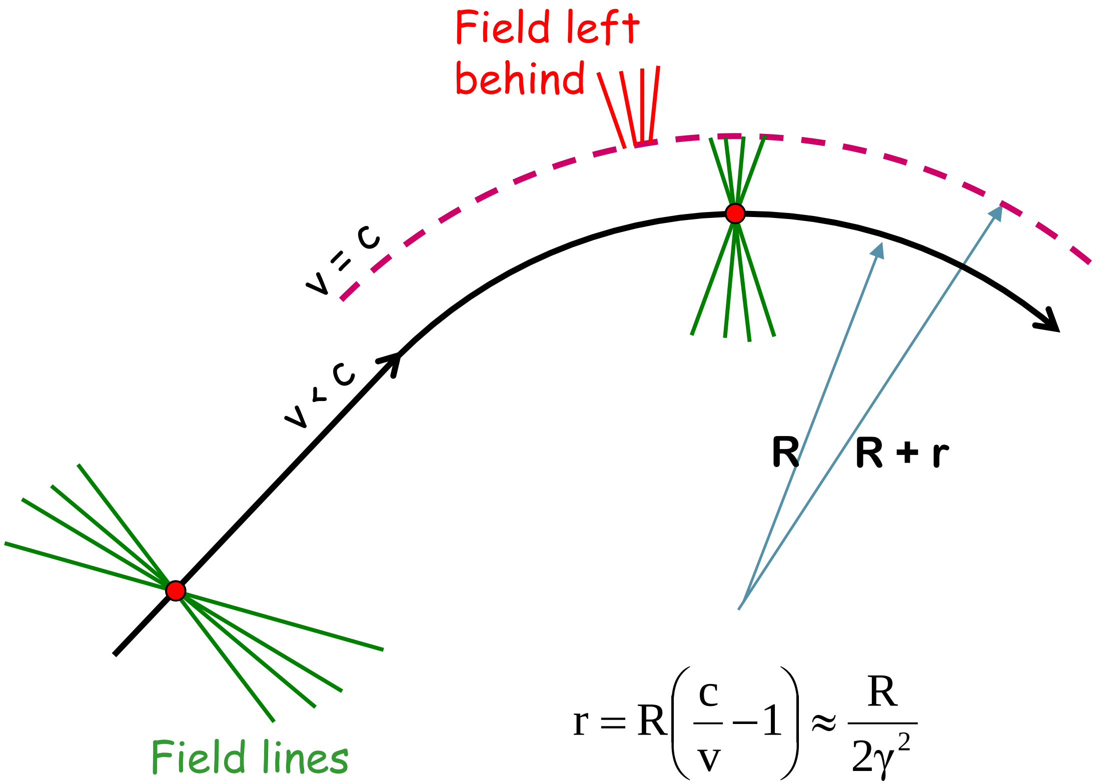
This radiation can be harmful and beneficial

SR effects - qualitatively



- Simplistic picture – SR caused by leaving part of the fields behind
- Can be useful as it
  - Creates high brightness radiation source
- Can be harmful as it
  - Creates additional energy spread
  - Creates additional emittance growth

Synchrotron radiation *on-the-back-of-the envelope* – power loss

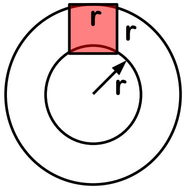


$$r = R\left(\frac{c}{v} - 1\right) \approx \frac{R}{2\gamma^2}$$

Energy in the field left behind (radiated !):

$$W \approx \int E^2 \, dV$$

The field $E \approx \dfrac{e}{r^2}$ **the volume** $V \approx r^2 \, dS$



Energy loss per unit length:

$$\frac{dW}{dS} \approx E^2 r^2 \approx \left(\frac{e}{r^2}\right)^2 r^2$$

Substitute $r \approx \dfrac{R}{2\gamma^2}$ **and get an estimate:**

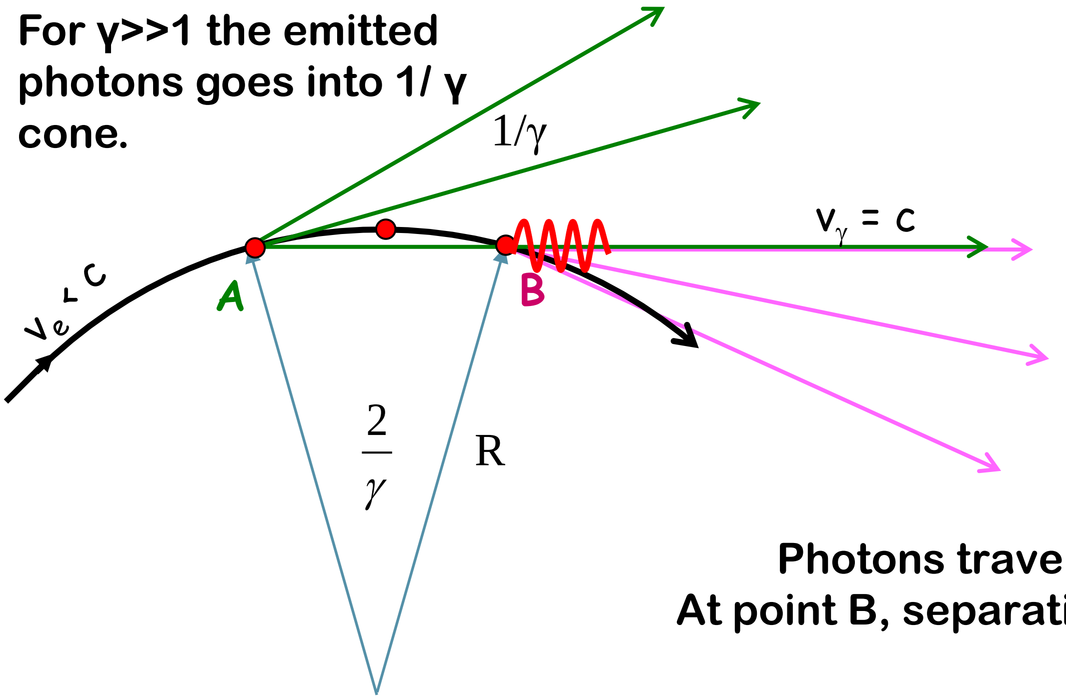
$$\boxed{\frac{dW}{dS} \approx \frac{e^2 \gamma^4}{R^2}}$$

Compare with exact formula: $\dfrac{dW}{dS} = \dfrac{2}{3} \dfrac{e^2 \gamma^4}{R^2}$

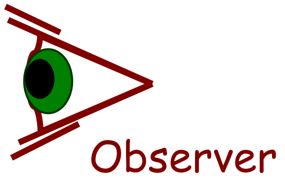
Gaussian units on this page!

Synchrotron radiation
on-the-back-of-the envelope – photon energy

For γ>>1 the emitted photons goes into 1/ γ cone.



During what time Δt the observer will see the photons?



Photons emitted during travel along the 2R/γ arc will be observed.

Photons travel with speed c, while particles with v. At point B, separation between photons and particles is

$$dS \approx \frac{2R}{\gamma}\left(1-\frac{v}{c}\right)$$

Therefore, observer will see photons during $\Delta t \approx \frac{dS}{c} \approx \frac{2R}{c\gamma}(1-\beta) \approx \frac{R}{c\gamma^3}$

Estimation of characteristic frequency

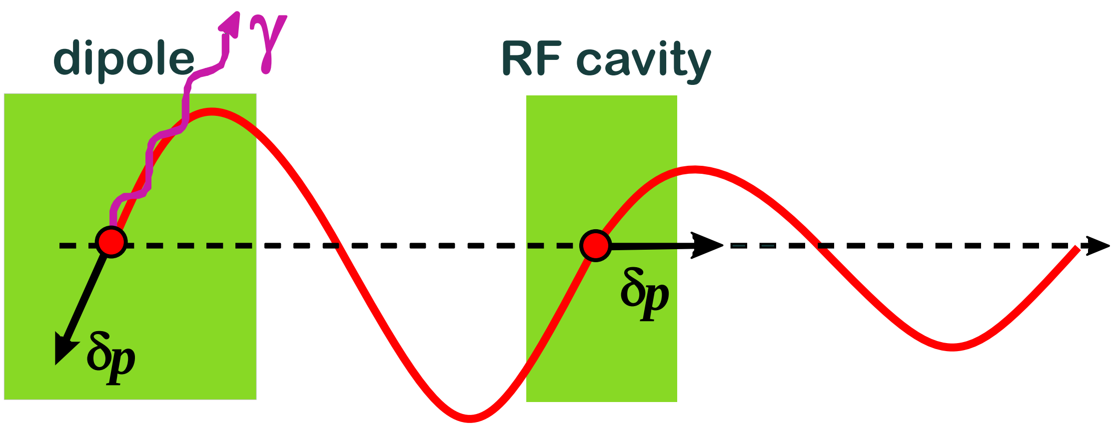
$$\boxed{\omega_c \approx \frac{1}{\Delta t} \approx \frac{c\gamma^3}{R}}$$

Compare with exact formula:

$$\omega_c = \frac{3}{2}\frac{c\gamma^3}{R}$$

Cooling time

- e- and particularly e+ from the source often have too high emittance ε
  $\Rightarrow$ we have to reduce the bunch size
- solution: use synchrotron radiation in a damping ring



- γ emission with transverse component
- acceleration only in longitudinal direction

} radiation damping!

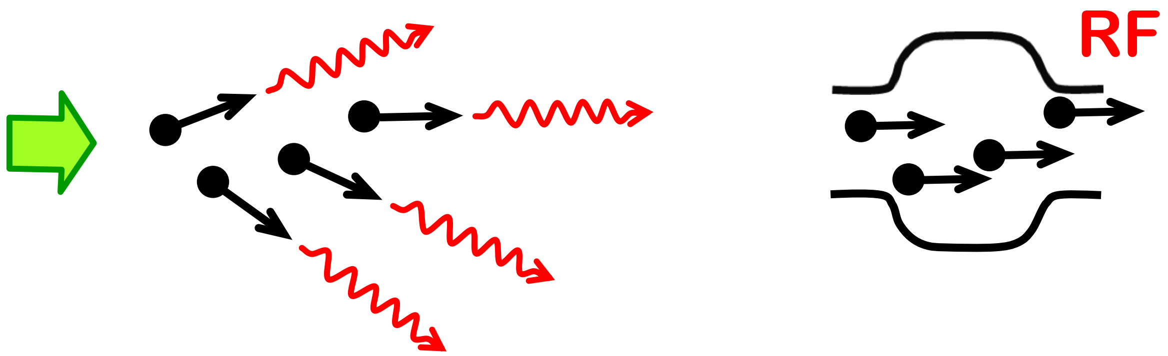
- Let's estimate cooling time

Let's estimate cooling time

We estimated that losses per unit length are: $\frac{dW}{dS} = \frac{2}{3}\frac{e^2\gamma^4}{R^2}$ **or** $\frac{dW}{dS} = \frac{2}{3}\frac{r_e\,\gamma^4}{R^2}mc^2$

Thus losses per turn are: $U_0 = \frac{4\pi}{3}\frac{r_e\,\gamma^4}{R}mc^2$

When electron radiate a photon, its momentum decrease



RF cavity restores only longitudinal momentum, thus other degrees of freedom cooled

Estimate cooling time τ as E_0T_0/U_0: $\tau \approx \frac{2\pi R}{c}\frac{\gamma mc^2}{U_0}$ **or** $\tau^{-1} \approx \frac{2}{3}\frac{c\,r_e\,\gamma^3}{R^2}$

Cooling time & partition

So, we estimated cooling time as $\tau^{-1} \approx \frac{2}{3}\frac{c\, r_e\, \gamma^3}{R^2}$

Usually, there is factor of 2 in the definition: $\tau = 2E_0T_0/U_0$ => $\tau^{-1} = \frac{1}{3}\frac{c\, r_e\, \gamma^3}{R^2}$

The evolution of emittance under SR damping is given by $\varepsilon(t) = \varepsilon_0 \exp(-2t/\tau)$

Both transverse planes and longitudinal motion in rings are usually coupled
Thus we can expect that the damping will be distributed between these degrees of freedom in some proportion depending on details of the optics

Distribution of cooling is defined by so called partition numbers J_x, J_y, J_E

Cooling time of a degree of freedom is $\tau_i = \frac{\tau}{J_i}$

Total radiated power fixed => $\sum \tau_i^{-1} = const$

Mentioned to you without derivation

Usually $J_x \approx 1, \quad J_y \approx 1, \quad J_E \approx 2$

Partition theorem $\sum J_i = 4$

Equilibrium emittance

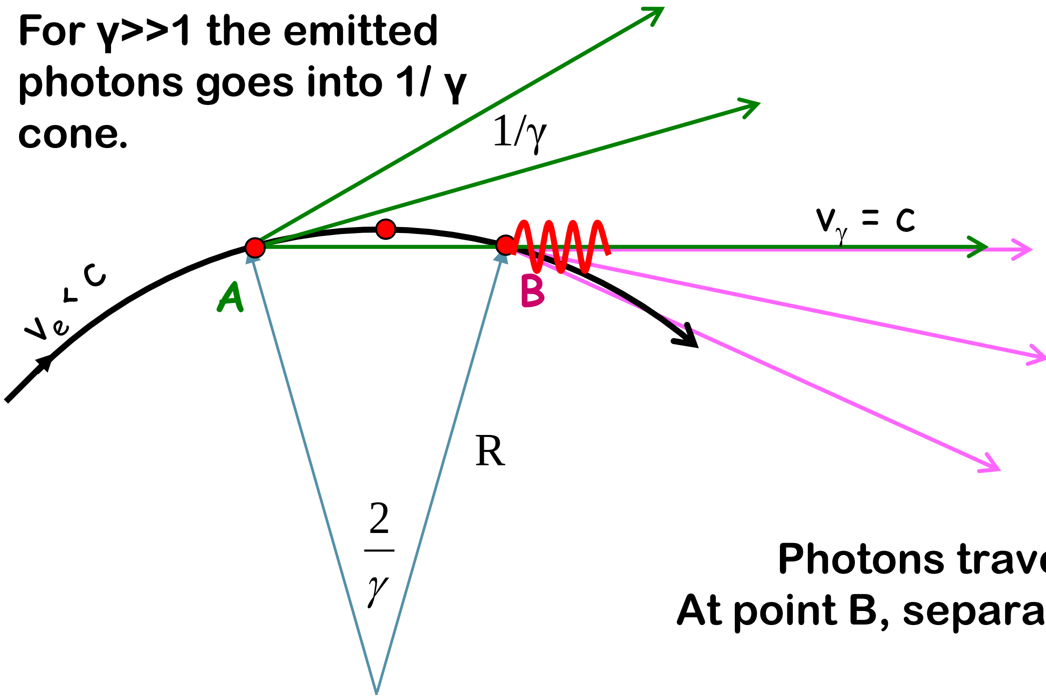
– Have SR cooling – would beam emittance reduce to zero?

– No, as there are quantum fluctuations

– Let's make simple estimations of the effects

SR – photon energy (recall)

For γ>>1 the emitted photons goes into 1/ γ cone.



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Photons travel with speed c, while particles with v. At point B, separation between photons and particles is

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Estimation of characteristic frequency

$$\boxed{\omega_c \approx \frac{1}{\Delta t} \approx \frac{c\gamma^3}{R}}$$

Compare with exact formula:

$$\omega_c = \frac{3}{2}\frac{c\gamma^3}{R}$$

Synchrotron radiation *on-the-back-of-the envelope* – number of photons

We estimated the rate of energy loss: $\frac{dW}{dS} \approx \frac{e^2 \gamma^4}{R^2}$ **And the characteristic frequency:** $\omega_c \approx \frac{c\gamma^3}{R}$

&

The photon energy $\varepsilon_c = \hbar\omega_c \approx \frac{\gamma^3 \hbar c}{R} = \frac{\gamma^3}{R} \lambda_e mc^2$ **where** $r_e = \frac{e^2}{mc^2}$ $\alpha = \frac{e^2}{\hbar c}$ $\lambda_e = \frac{r_e}{\alpha}$

=>

Number of photons emitted per unit length $\frac{dN}{dS} \approx \frac{1}{\varepsilon_c} \frac{dW}{dS} \approx \frac{\alpha\gamma}{R}$ **(per angle** θ: $N \approx \alpha\gamma\theta$ **)**

Gaussian units on this page!

Let's estimate energy spread growth due to SR

We estimated the rate of energy loss: $\frac{dW}{dS} \approx \frac{e^2 \gamma^4}{R^2}$ And the characteristic frequency: $\omega_c \approx \frac{c\gamma^3}{R}$

The photon energy $\varepsilon_c = \hbar\omega_c \approx \frac{\gamma^3 \hbar c}{R} = \frac{\gamma^3}{R} \lambda_e mc^2$ where $r_e = \frac{e^2}{mc^2}$ $\alpha = \frac{e^2}{\hbar c}$ $\lambda_e = \frac{r_e}{\alpha}$

Number of photons emitted per unit length $\frac{dN}{dS} \approx \frac{1}{\varepsilon_c} \frac{dW}{dS} \approx \frac{\alpha\gamma}{R}$ (per angle θ: $N \approx \alpha\gamma\theta$)

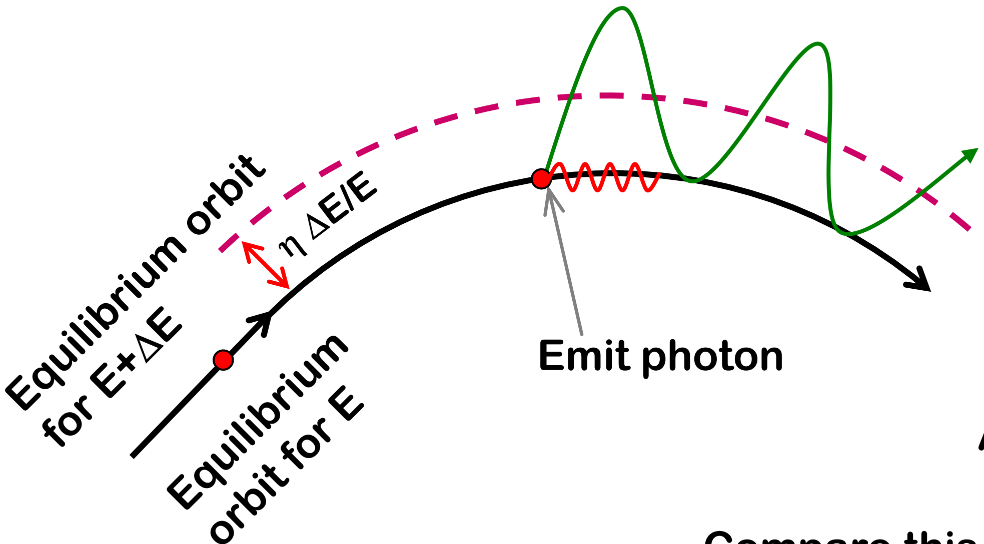
The energy spread ΔE/E will grow due to statistical fluctuations ($\sqrt{N}$) of the number of emitted photons :

$$\frac{d\left((\Delta E/E)^2\right)}{dS} \approx \varepsilon_c^2 \frac{dN}{dS} \frac{1}{\left(\gamma mc^2\right)^2}$$

Which gives: $\boxed{\frac{d\left((\Delta E/E)^2\right)}{dS} \approx \frac{r_e \lambda_e \gamma^5}{R^3}}$

Compare with exact formula: $\frac{d\left((\Delta E/E)^2\right)}{dS} = \frac{55}{24\sqrt{3}} \frac{r_e \lambda_e \gamma^5}{R^3}$

Let's estimate emittance growth rate due to SR



Dispersion function η shows how equilibrium orbit shifts when energy changes

When a photon is emitted, the particle starts to oscillate around new equilibrium orbit

Amplitude of oscillation is $\Delta x \approx \eta\, \Delta E/E$

Compare this with betatron beam size: $\sigma_x = (\varepsilon_x\, \beta_x)^{1/2}$

And write emittance growth: $\Delta\varepsilon_x \approx \dfrac{\Delta x^2}{\beta}$

Resulting estimation for emittance growth:

$$\frac{d\varepsilon_x}{dS} \approx \frac{\eta^2}{\beta_x}\,\frac{d\left((\Delta E/E)^2\right)}{dS} \approx \boxed{\frac{\eta^2}{\beta_x}\,\frac{r_e\,\lambda_e\,\gamma^5}{R^3}}$$

Compare with exact formula (which also takes into account the derivatives):

$$\frac{d\varepsilon_x}{dS} = \underbrace{\boxed{\frac{\left(\eta^2 + (\beta_x \eta' - \beta_x' \eta/2)^2\right)}{\beta_x}}}_{=\mathcal{H}}\,\frac{55}{24\sqrt{3}}\,\frac{r_e\,\lambda_e\,\gamma^5}{R^3}$$

Equilibrium emittance

We estimated the rate of emittance growth: $$\frac{d\varepsilon_x}{dS} \approx \frac{\eta^2}{\beta_x} \frac{d\left(\left(\Delta E/E\right)^2\right)}{dS} \approx \frac{\eta^2}{\beta_x} \frac{r_e \lambda_e \gamma^5}{R^3}$$

SR cooling gives $$\frac{d\varepsilon}{ds} = -\frac{2}{c\tau}\varepsilon$$ with $$\tau^{-1} = \frac{1}{3}\frac{c\, r_e \gamma^3}{R^2}$$

The equilibrium emittance is thus: $$\varepsilon_{x0} \approx \frac{c\tau}{2} \frac{\eta^2}{\beta_x} \frac{r_e \lambda_e \gamma^5}{R^3}$$ or $$\varepsilon_{x0} \approx \frac{3}{2} \frac{\eta^2}{\beta_x} \frac{\lambda_e \gamma^2}{R}$$

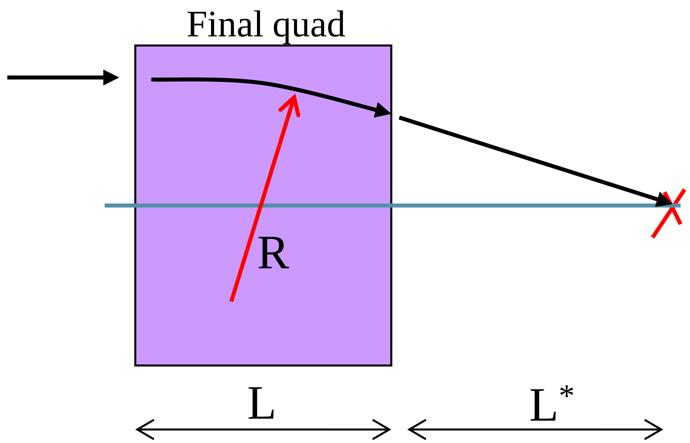
(these are estimations –> for accurate formulas need to take into account average values <1/R^2> and <1/R^3> over the orbit period)

In vertical plane, SR contribution to emittance is only due to 1/γ angles of photons, and this effect is usually very small

Vertical emittance usually defined by coupling coefficient k (<<1) of x-y planes:

$$\varepsilon_{y0} \approx k\, \varepsilon_{x0}$$

Oide effect (SR in FD)



IP divergence:

$$\theta^* = \sqrt{\varepsilon/\beta^*}$$

IP size:

$$\sigma^* = \sqrt{\varepsilon\,\beta^*}$$

Energy spread obtained in the quad:

$$\left(\frac{\Delta E}{E}\right)^2 \approx \frac{r_e\,\lambda_e\,\gamma^5\,L}{R^3}$$

Radius of curvature of the trajectory: $R = L / \theta^*$

Growth of the IP beam size: $\sigma^2 \approx \sigma_0^2 + \left(L^*\,\theta^*\right)^2 \left(\frac{\Delta E}{E}\right)^2$

Which gives $\sigma^2 \approx \varepsilon\beta^* + C_1 \left(\frac{L^*}{L}\right)^2 r_e\,\lambda_e\,\gamma^5 \left(\frac{\varepsilon}{\beta^*}\right)^{5/2}$ (where C_1 is ~ 7 (depend on FD params.))

This achieve minimum possible value:

$$\sigma_{min} \approx 1.35\, C_1^{1/7} \left(\frac{L^*}{L}\right)^{2/7} \left(r_e\,\lambda_e\right)^{1/7} \left(\gamma\varepsilon\right)^{5/7}$$

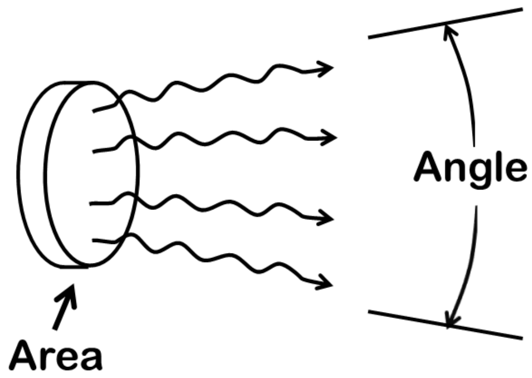
When beta* is:

$$\beta_{optimal} \approx 1.29\, C_1^{2/7} \left(\frac{L^*}{L}\right)^{4/7} \left(r_e\,\lambda_e\right)^{2/7}\,\gamma \left(\gamma\varepsilon\right)^{3/7}$$

Note that beam distribution at IP will be non-Gaussian. Usually need to use tracking to estimate impact on luminosity. Note also that optimal β may be smaller than the σ_z (i.e cannot be used).

Brightness

- Now we have almost everything to estimate brightness of synchrotron light sources



Brightness
photons / (s m^2 rad^2 (%bandwidth))

- And know how we can increase brightness
  - Smaller size of emitting area
  - Smaller angular divergence

To a certain limit,
as single photon has "emittance" –
diffraction limited sources

- We will discuss it in future lectures

SR spectral characteristics

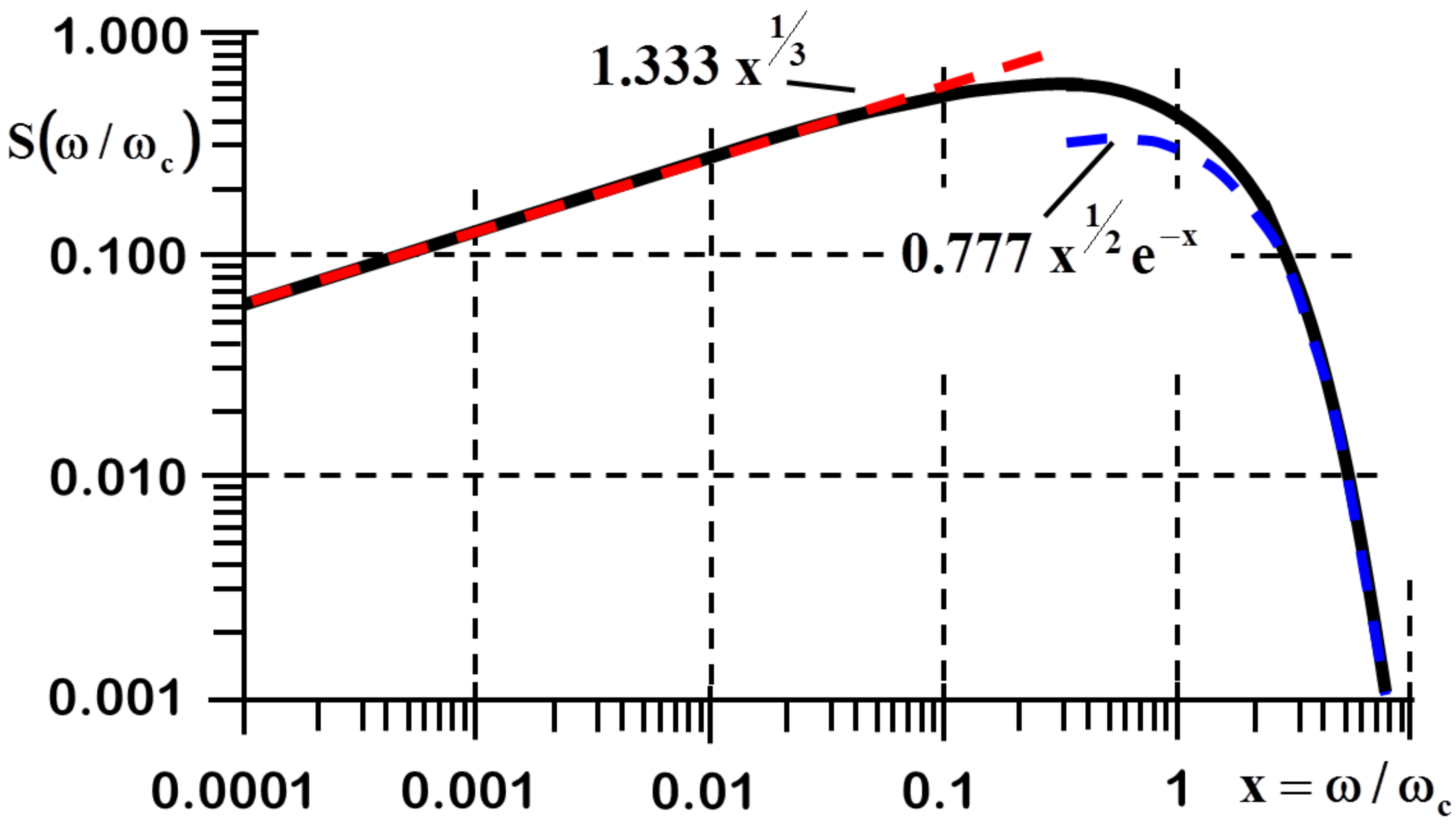
– Classical case

– Quantum case

– Also, let’s touch on SR from “insertion devices”

SR spectrum

Accurate math, which we do not show here, predicts that SR spectrum looks like this:

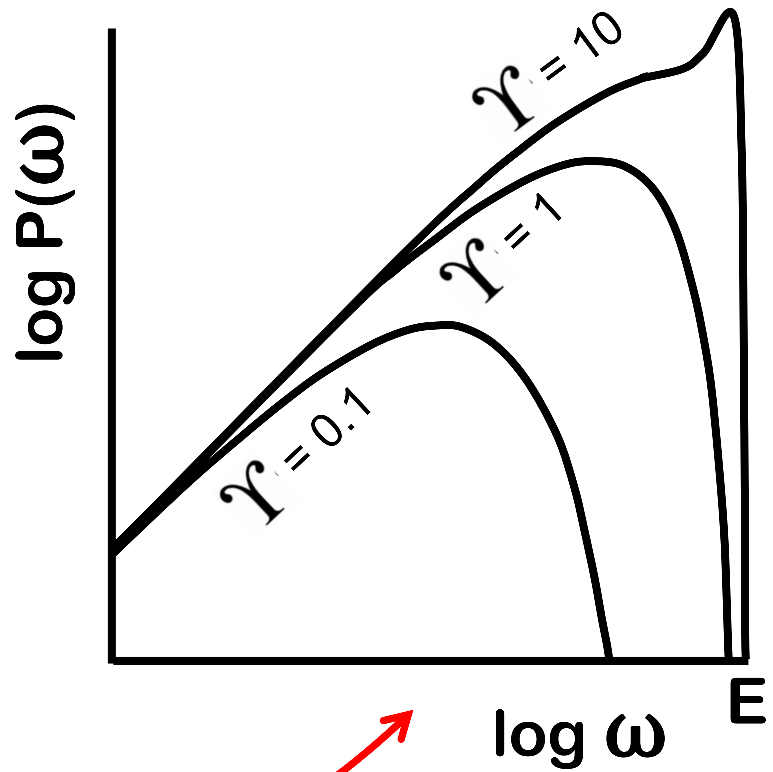


Classical and quantum SR regime

- **Let's define parameter "Upsilon" as**

$$\Upsilon = \hbar\omega_c/E$$

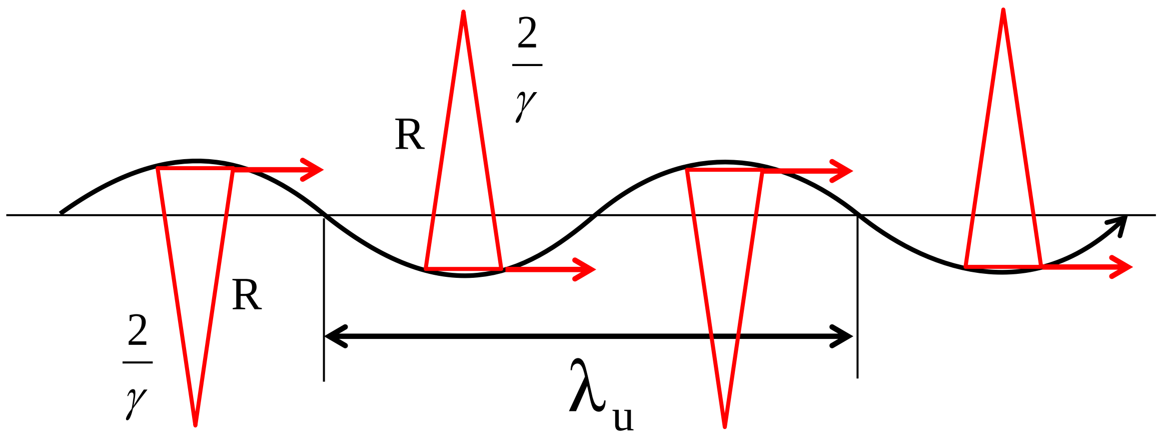
- **Regimes and meaning**
  - When "upsilon" <<1, it has the meaning of ratio of photon energy to beam energy
  - When "upsilon"~1 and larger, the classical regime of synchrotron radiation is not applicable, and quantum SR formulas of Sokolov-Ternov should be used



- **In quantum regime the spectrum of SR change …**

Radiation from sequence of bends

Assume that bends are arranged in sequence with +-+-+- polarity with period λ_u, so that trajectory wiggles:



Observer will see photons emitted during travel along the arc $2R/\gamma$

If $2R/\gamma << \lambda_u/2$, then radiation emitted at each wiggle is independent

If $2R/\gamma >> \lambda_u/2$ => regime where entire wiggling trajectory contribute to radiation

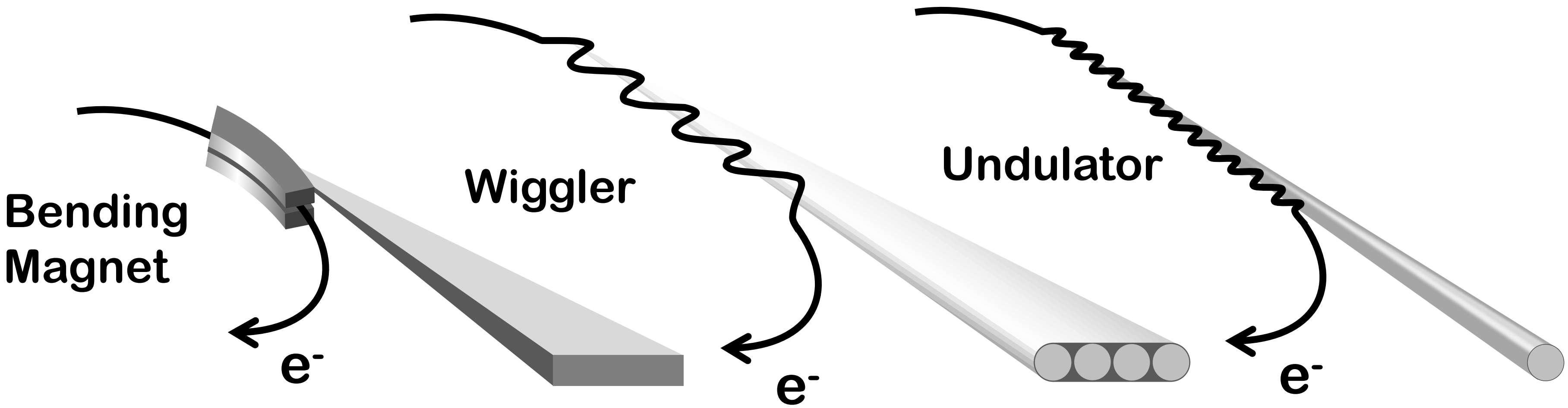
Define $K \sim \gamma\, \lambda_u/R$

K >> 1 – wiggler regime

K << 1 – undulator regime

Wiggler and undulator radiation

Parameter $K \sim \gamma \lambda_u / R$ defines different regimes of synchrotron radiation



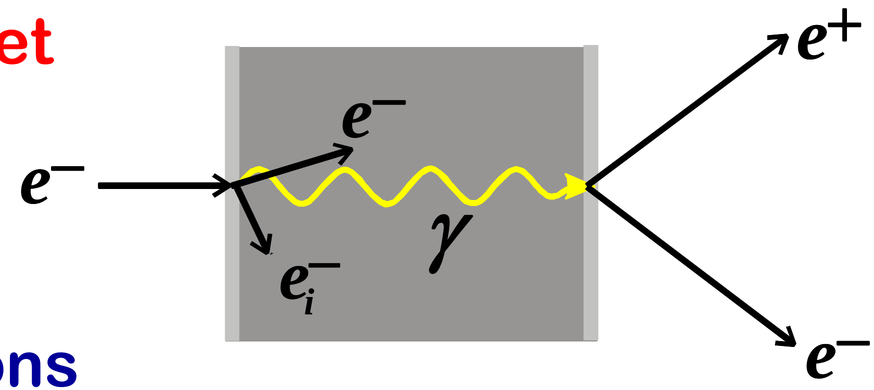
K >> 1
wiggler
regime

K << 1
undulator
regime

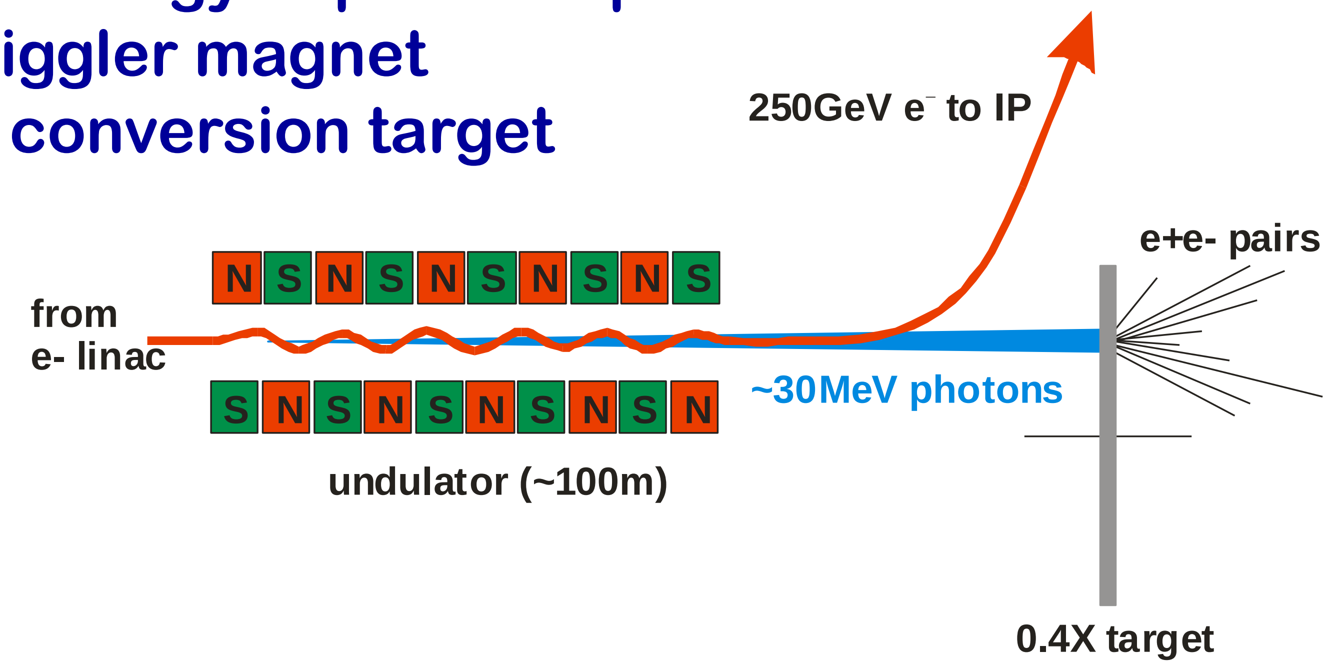
We will consider this in more details in the lecture about light sources

SR and e+ source

- basic mechanism: pair production in target material
- standard method: 'thick' target
  primary e- generate photons
  these converts into pairs



- undulator source:
  high energy e- produce photons
  in wiggler magnet
  thin conversion target



Summary of the lecture

- We have discussed
  - Basics of Synchrotron Radiation
    - Without complicated derivations
  - Some examples of its use