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RF Geometries in Practice

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USPAS – Applied Electromagnetism Lecture 4

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Topological Morphing

- Both waveguide modes and coaxial modes can be topologically be manipulated to make an acceptable accelerating field.

- Most designs involve mapping the high electric field regions onto 'loading elements' where we can carefully control the geometry.

- For instance: noses in a pillbox cavity to focus the accelerating electric field, increasing the shunt impedance.

- This change in geometry doesn't fundamentally change the mode, but it does change the figures of merit we care about.

- We'll see this repeated over and over to turn these modes into cavities we can use.

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Topological Modifications of Single-Cell Cavities

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Topological Morphing of Modes
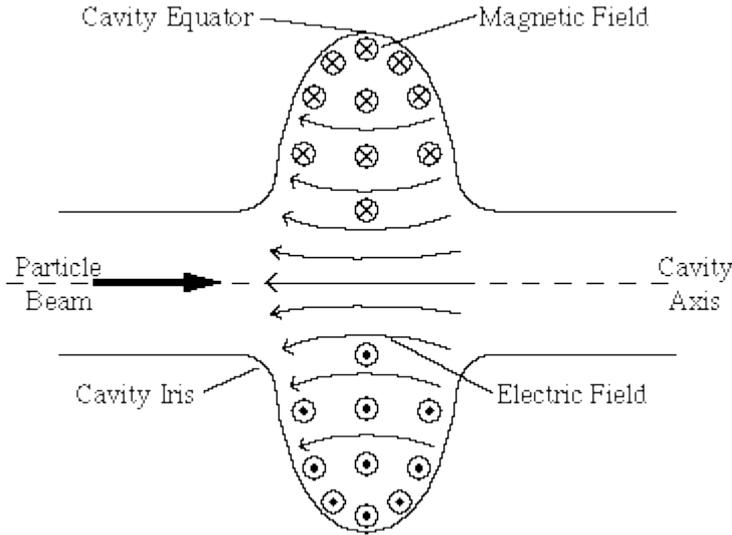
Actual Pillbox Cavity, same as we derived, but with beam pipes added and outer conductor modified to improve quality factor and other factors.
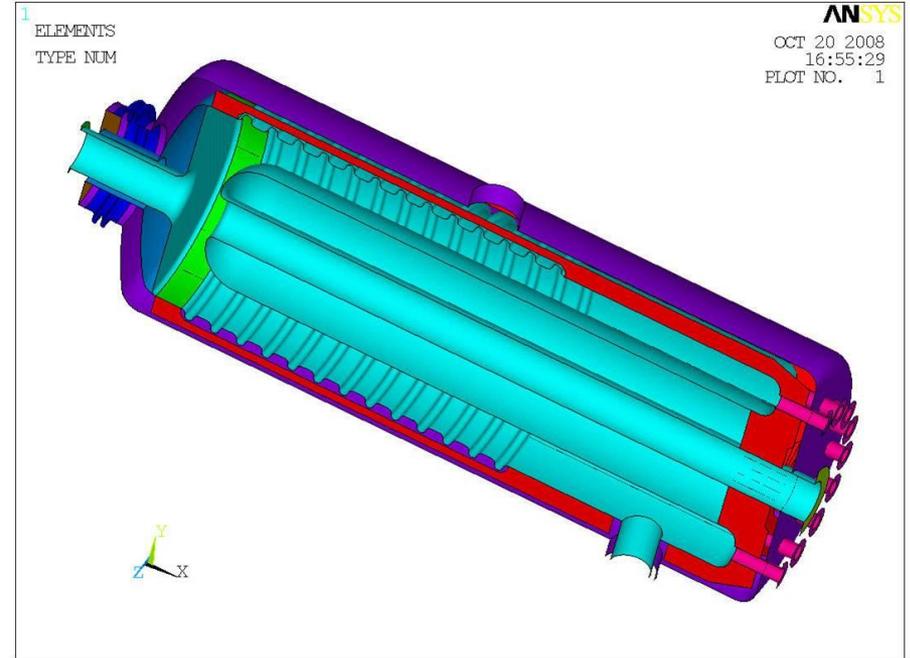
BNL Photo-Injector

"Quarter-Wave" Cavity

Not really. Modified for lower frequency in a compact shape.

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Split Ring Resonator with Loading Elements

Originally a TE mode cavity (not useful on it's own).

Adding the ring loading elements means that the magnetic flux moving through the rings 'loads' the electrodes in opposite directions, leading to a pi-mode like structure.

Unfortunately, the loading elements have significant low-frequency mechanical resonances, also hard to cool.
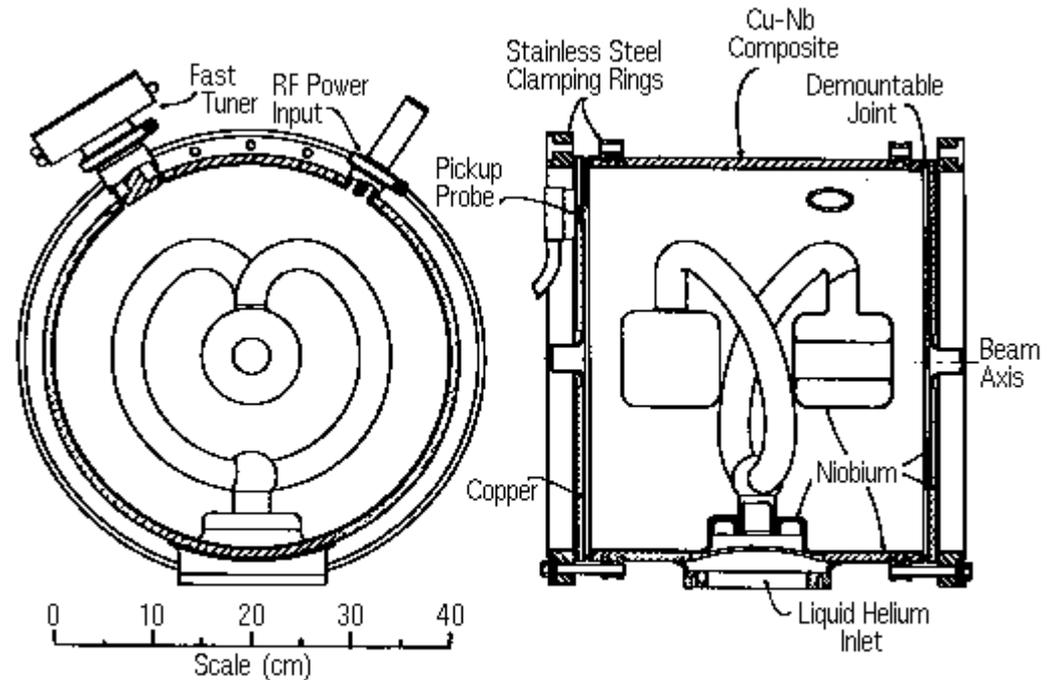
Superconducting split-ring resonator for β = 0.105.

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Increasing Efficiency

- Using many, independently driven and controlled cavities is very inefficient and expensive.

- While there are cases where this amount of flexibility is important, it's often very useful to try and increase the amount of acceleration you can get from every cavity.

- Using loading elements, like the split ring resonator, give multiple kicks for the same cavity.

- Let's explore different examples of this.

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Wideröe Linac – Sloan/Lawrence Structure

Dipole mode in a cylindrical waveguide cavity has opposite voltages at opposite edges of the cavity.

Putting loading elements bring this voltage to the beam axis, and oppositely loaded elements on the beam axis give a pi-mode like structure.

The spacing between the gaps has to be tuned to keep synchronization.
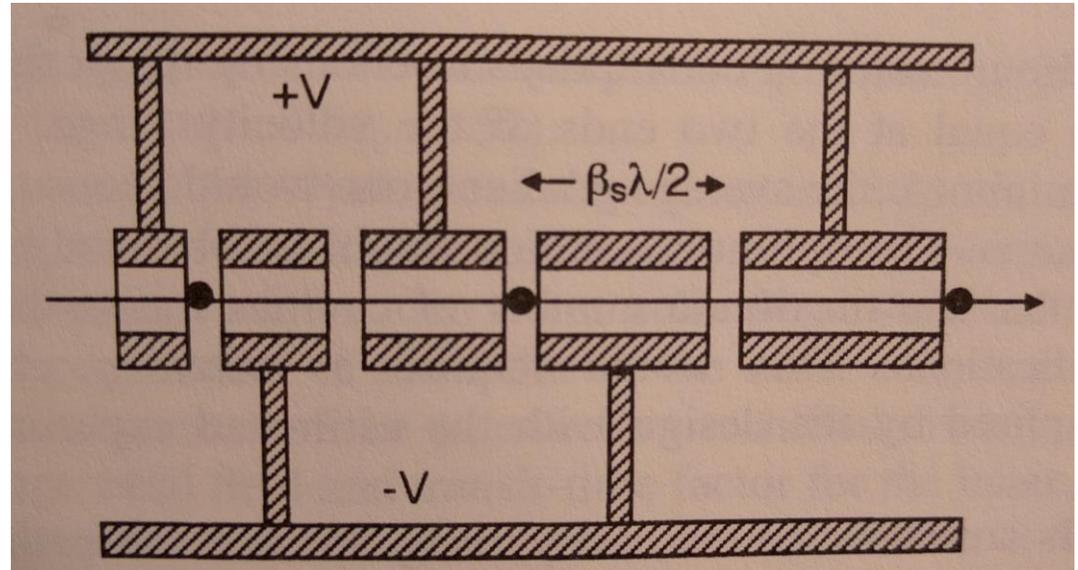
Wideröe (2)

- These tanks are designed for very low velocity acceleration, below about 3% of the speed of light, below 100 MHz.

- Above this frequency, the gap spacing gets too small to be practical.

- This low velocity means either heavy ions or protons.

- As the energy increases, the gap spacing must increase.

- The longer the gap spacing, the lower the shunt impedance (same voltage, further apart)

- Mainly, this tank benefits from being able to operate at very low frequencies with reasonable transverse size.

- Constant voltage per gap.

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Wideröe (3)

As the tank gets long, compared to the wavelength, it becomes better to drive the cavity in a quarter wave resonator mode.

In principle, this is the same, although the voltage on the central electrode goes like a QWR, highest at the end.

Electrodes can be made longer, keeping synchronism, to add focusing elements.
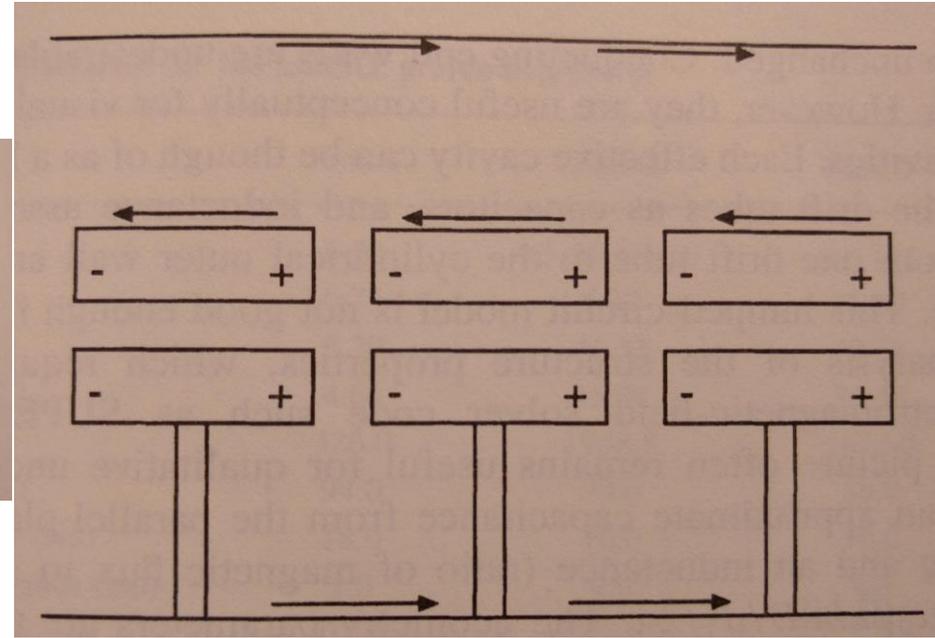
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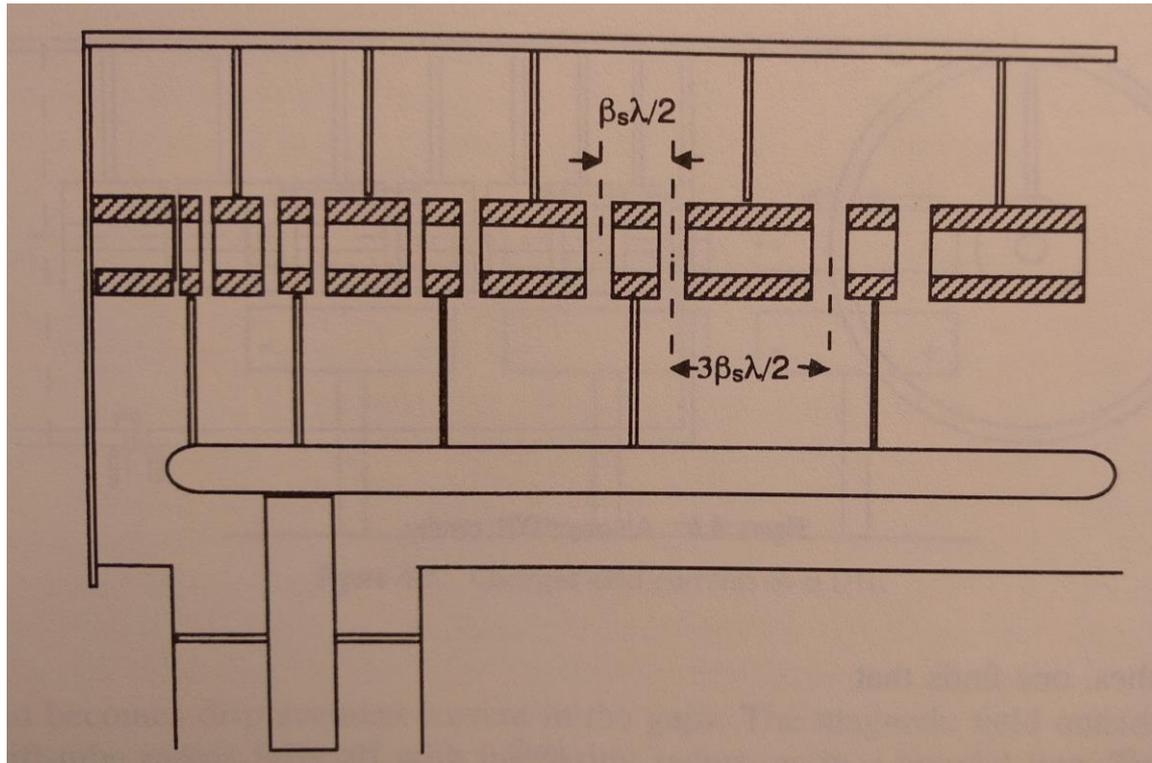
Alvarez Drift-Tube Linac
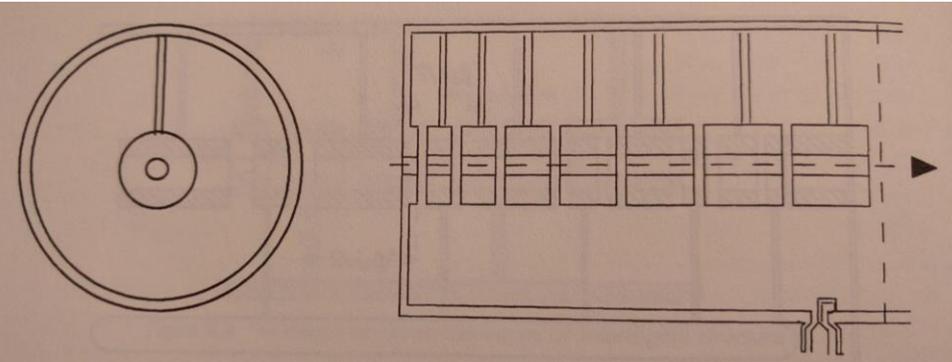
Using a pillbox cavity mode, but with drift tubes to shield the particle from deceleration fields.

Fields can be tuned to be uniform, giving uniform field per gap.

Each drift tube shields the fields, giving field-free regions. Synchronous acceleration requires $\beta\lambda$ gap separation.

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Drift Tube Linac Limitations

- Treating each cell like an individual cell is instructive.
- The effective circuit of each unit cell includes an inductance from the magnetic field region and a capacitance, which can be estimated as:

- $C_0 = \frac{\epsilon_0 \pi d^2}{4g}$ where d is the drift tube diameter, g is the gap

- $L_0 = \frac{\mu_0 \beta \lambda \ln\left(\frac{D}{d}\right)}{2\pi}$ remembering that $\beta\lambda$ is the gap spacing

- Also, $\omega_0^2 = \frac{1}{L_0 C_0}$

- You can see, to keep the frequency fixed as β increases, we must increase the gap (D, d can't change)
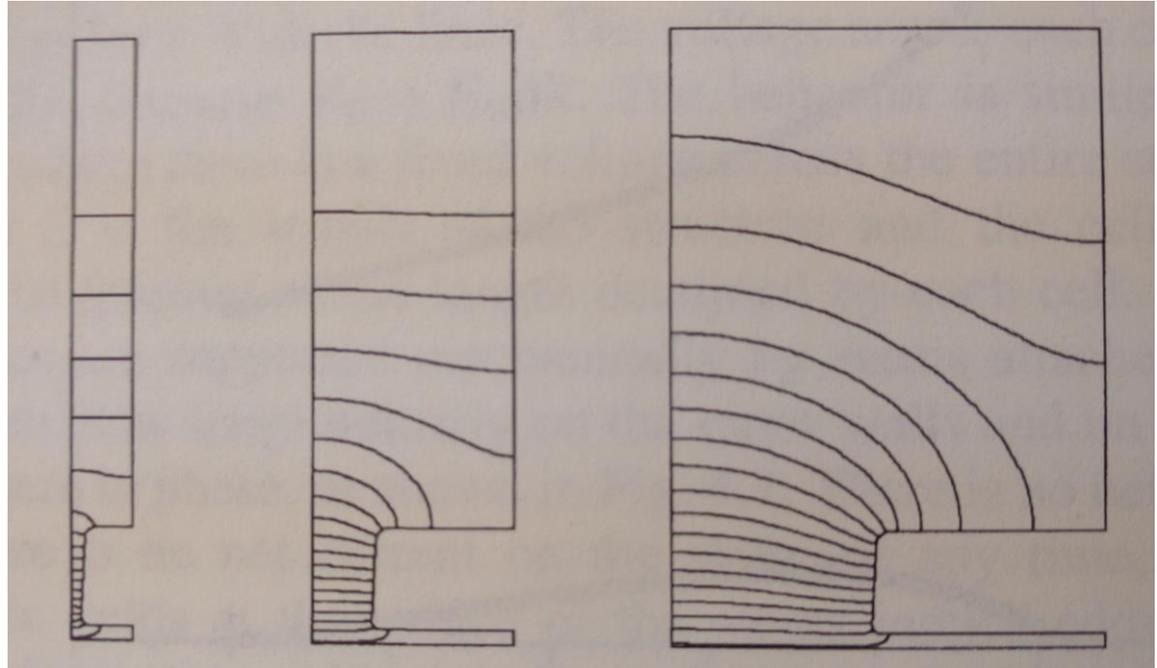
Drift Tube Fields – 2D simulation

The circuit model tells that to maintain the mode, the gap must increase at high β, but what happens to the field?
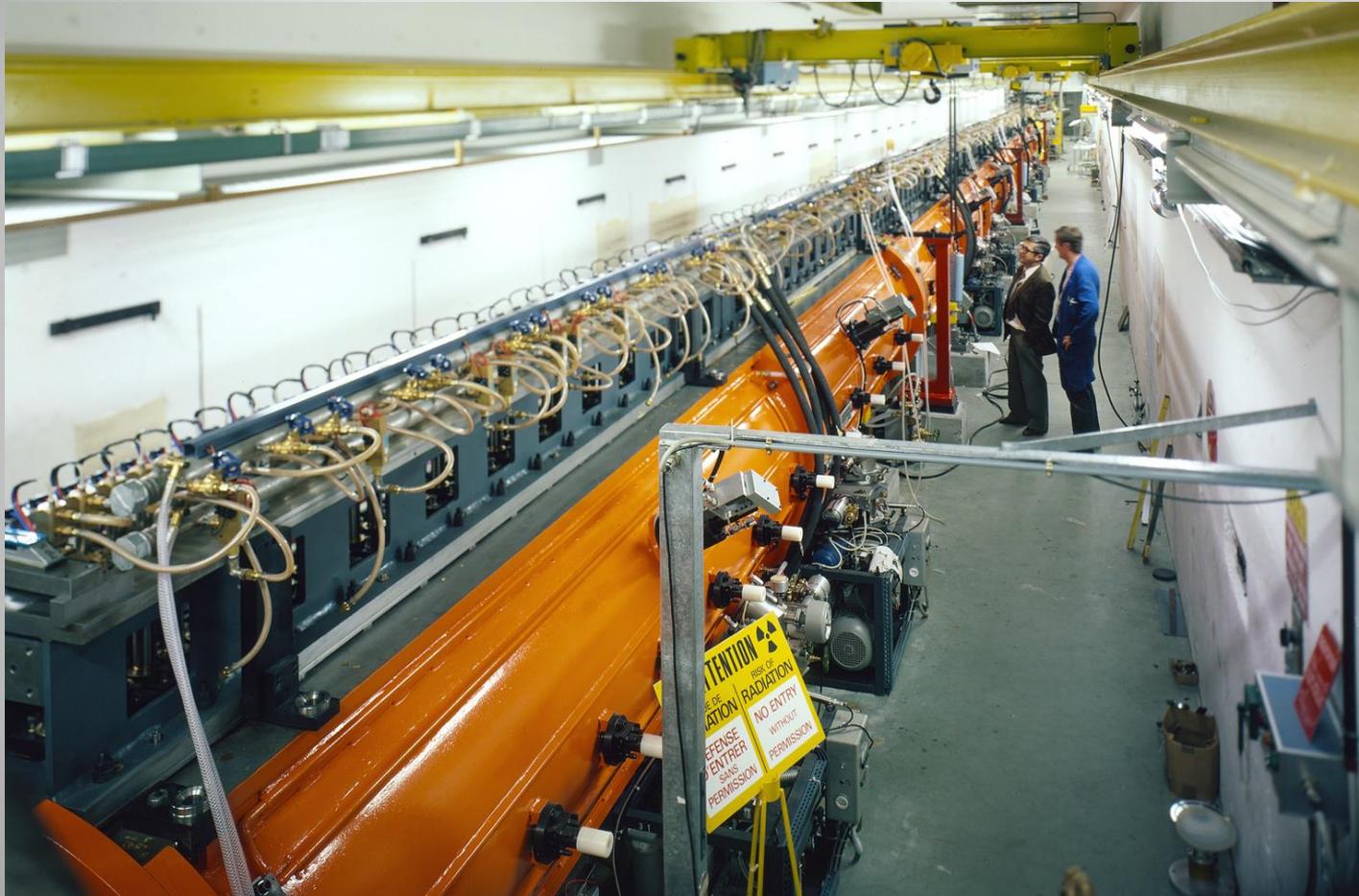
2D simulations give a good scaling:

Larger Gaps mean less field is focused on the beam axis, which is what we want!

DTLs lose efficiency at high β.

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Linac 2

- 3 RF tanks, 750 keV to 50 MeV, 34 m long.
- Frequency 200 MHz, about 30 MW.

# The drift tubes of Linac2

Erk Jensen, CERN

Linac4 DTL (3 ... 50 MeV)

Erk Jensen, CERN

- Three tanks – 39/42/30 cells
- Permanent magnetic quadrupoles
- Drift tube alignment relies on machining tolerances and not on alignment mechanism
- All PMQ centres aligned within ±0.1 mm!
- Conditioning time per tank: 1-2 weeks
- December 2015: Fully commissioned – if Linac2 fails, we have an emergency plan.
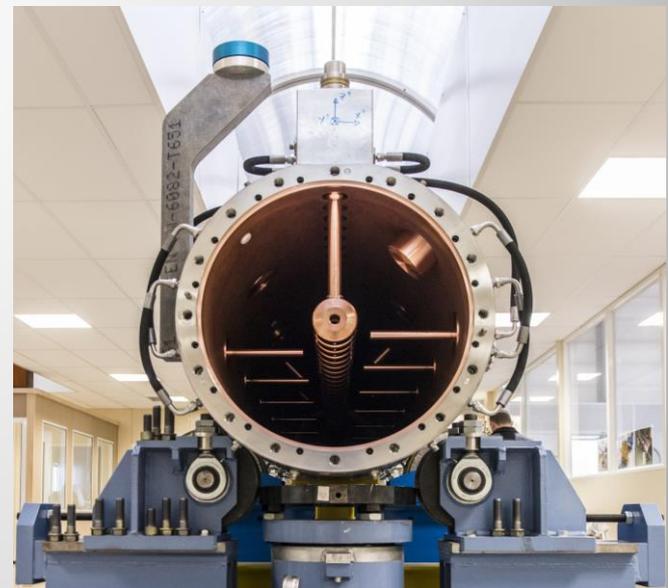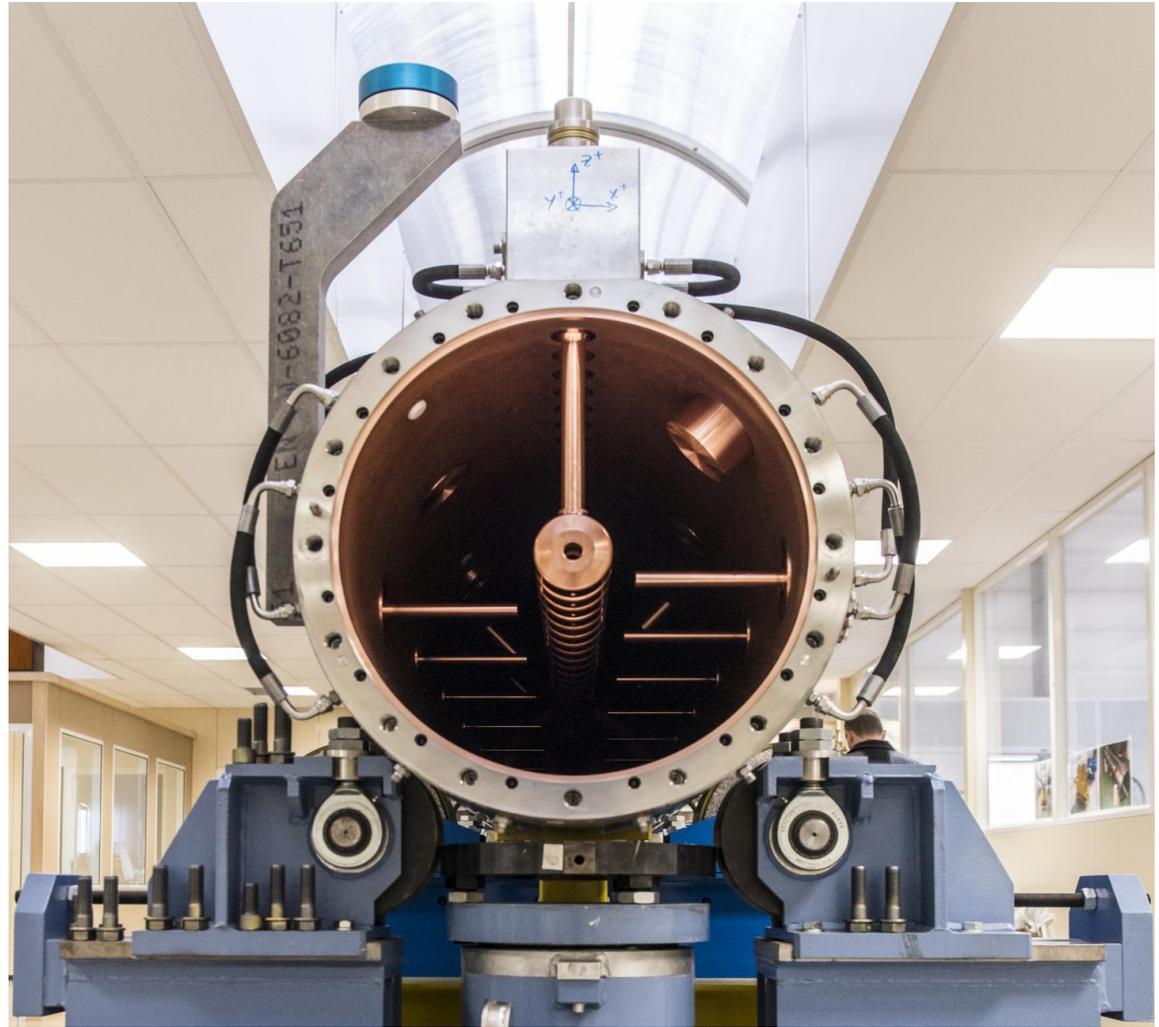
DTL Tuning

Large stubs are machined to tune inductance of tank.

Each cell has a stub tuner ($\lambda/4$) that can be trimmed to modify the capacitance of each drift tube.

Also! Nominally, each 'cell' is independent. In reality, this isn't true (errors, etc)

These tuning rods also perturb the fields, Coupling each 'cell' together.

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Design Workflow - DTL

- Go right to 3D and optimize altogether! (No, very silly).

- Design by parts.

- You know frequency, injection β, and particle type.

- This means you can design, cell by cell, to optimize for gap, shunt impendence, surface fields, size, and frequency.

- Make sure energy gain across the tank keeps the particles synchronous

- Only once you have a good estimate of each cell, then put it all together.

- Now insert non-symmetric elements, tuning bars, drift tube stems, etc.

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Coupled Cavities

- I've sort of danced around the topic, but now it's time to get into the topic of coupling.

- You can treat the a traveling wave structure as many individual pillbox cavities coupled together by the iris.

- For a standing wave structure, coupled pillbox cavities are useful for acceleration of particles up to $\beta = 1$, and with more efficiency than many single cell cavities.

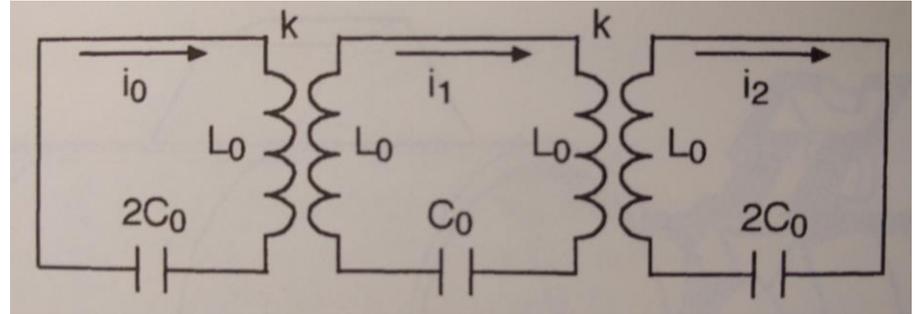- Let's take a simple circuit model and analyze it to get a feel.

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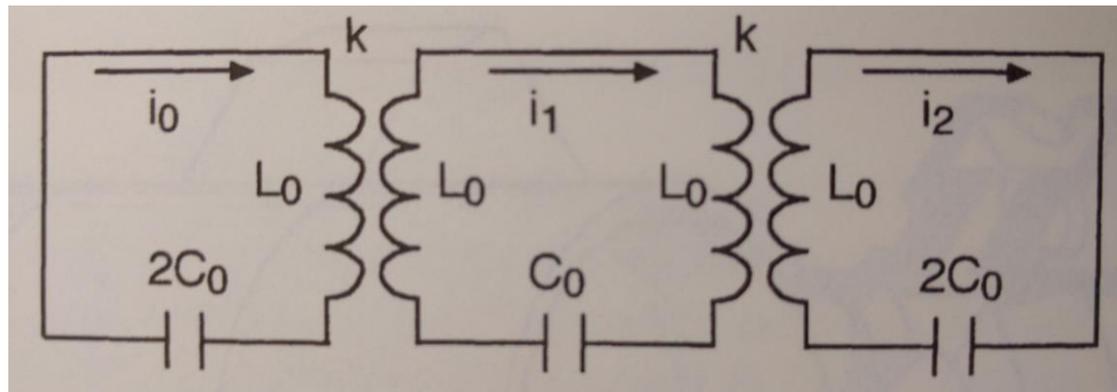
Ideal 3-Oscillator Model

- Note: End cells adjusted to have the same frequency due to lack of external coupling

- Summing voltages:

- $x_0 \left(1 - \frac{\omega_0^2}{\Omega^2}\right) + x_1 k = 0, \, n = 0$

- $x_1 \left(1 - \frac{\omega_0^2}{\Omega^2}\right) + \frac{(x_1 + x_2)k}{2} = 0,$
  $n = 1$

- $x_2 \left(1 - \frac{\omega_0^2}{\Omega^2}\right) + x_1 k = 0, \, n = 2$

Where:

$$x_n = i_n \sqrt{2L_0}, \, n = 0,1,2$$

$$k = \frac{M}{L_0}$$

$$\omega_0 = \frac{1}{\sqrt{2L_0 C_0}}$$

Ω is the normal mode frequency

Ideal 3-Oscillator Model (2)

- Rearranging these into the form:

- $LX_q = \frac{1}{\Omega_q^2} X_q$

- We get:

- $L = \frac{1}{\omega_0^2} \begin{bmatrix} 1 & k & 0 \\ k/2 & 1 & k/2 \\ 0 & k & 1 \end{bmatrix}$

- $X_q = \begin{pmatrix} x_0 \\ x_1 \\ x_2 \end{pmatrix}$

- With Ω_q as the eigen-frequencies.

- Solve!

- $\Omega_0 = \frac{\omega_0}{\sqrt{1+k}}$; $X_0 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

- $\Omega_1 = \omega_0$; $X_1 = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$

- $\Omega_2 = \frac{\omega_0}{\sqrt{1-k}}$; $X_2 = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$

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Ideal 3-Oscillator Model (3)

- Already a very interesting result! In general, for N coupled oscillators, we will get N eigen-modes.

- We will generally want our structures to have π phase advance, called π-mode structures

- We can also define the cell-to-cell coupling constant as such:

- $k = \frac{(\Omega_\pi - \Omega_0)}{\omega_0}$, the difference in frequency between the π and 0-modes normalized by individual cell frequency when $k \ll 1$.

- Now, what happens when the cells aren't made quite right?

Perturbed 3-Oscillator Model

- $\Delta X_q = \sum_{r \neq q} a_{qr} X_r$

- We're basically shifting modes by mixing them.

- The unperturbed modes are X_r, with

- $a_{qr} = \dfrac{X_q P X_r}{\frac{1}{\Omega_r^2} - \frac{1}{\Omega_q^2}}$

- Now, we have to be a little clever. All three frequencies are now different, so we have a little freedom. Let's say that the first and last cell have equal and opposite errors $\pm\delta\omega_0$, and the center cell has error $\delta\omega_1$. This makes things much easier.

MAAATTTTHHHHHH – Perturbative Solution to lowest order

- $\Omega_0 = \frac{\omega_0}{\sqrt{1+k}} \sqrt{1 - \frac{\delta\omega_1}{\omega_0}}$

- $\Omega_1 = \frac{\omega_0}{\sqrt{1 - 4\left(\frac{\delta\omega_0}{\omega_0}\right)^2}}$

- $X_1 = \begin{bmatrix} 1 + \frac{4}{k^2}\frac{\delta\omega_1}{\omega_0}\frac{\delta\omega_0}{\omega_0} - \frac{2}{k^2}\left(\frac{\delta\omega_0}{\omega_0}\right)^2 \\ -\frac{2}{k}\frac{\delta\omega_0}{\omega_0} \\ 1 + \frac{4}{k^2}\frac{\delta\omega_1}{\omega_0}\frac{\delta\omega_0}{\omega_0} - \frac{2}{k^2}\left(\frac{\delta\omega_0}{\omega_0}\right)^2 \end{bmatrix}$

- $\Omega_2 = \frac{\omega_0}{\sqrt{1-k}} \sqrt{1 - \frac{\delta\omega_1}{\omega_0}}$

- $X_2 = \begin{bmatrix} 1 - \frac{(1-k)}{2k}\left(\frac{\delta\omega_1}{\omega_0} - 4\frac{\delta\omega_0}{\omega_0}\right) \\ -1 - \frac{1-k}{2k}\frac{\delta\omega_1}{\omega_0} \\ 1 - \frac{(1-k)}{2k}\left(\frac{\delta\omega_1}{\omega_0} + 4\frac{\delta\omega_0}{\omega_0}\right) \end{bmatrix}$

- $X_0 = \begin{bmatrix} 1 + \frac{(1+k)}{2k}\left(\frac{\delta\omega_1}{\omega_0} - 4\frac{\delta\omega_0}{\omega_0}\right) \\ 1 - \frac{1+k}{2k}\frac{\delta\omega_1}{\omega_0} \\ 1 + \frac{(1+k)}{2k}\left(\frac{\delta\omega_1}{\omega_0} + 4\frac{\delta\omega_0}{\omega_0}\right) \end{bmatrix}$

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Perturbative Solutions

- 0 and π modes are sensitive to errors to 1st order.

- $\frac{\pi}{2}$ mode is sensitive to second order, except in normally unexcited cell.

- Including losses, driving from one end:

- $$X_0 = \begin{bmatrix} 1 \\ e^{-i\frac{3\sqrt{1+k}}{kQ}} \\ e^{-i\frac{4\sqrt{1+k}}{kQ}} \end{bmatrix} ; X_1 = \begin{bmatrix} 1 \\ \frac{1}{kQ}e^{i\frac{\pi}{2}} \\ -1 + \frac{2}{(kQ)^2} \end{bmatrix} ; X_2 = \begin{bmatrix} 1 \\ -e^{i\frac{3\sqrt{1-k}}{kQ}} \\ e^{i\frac{4\sqrt{1-k}}{kQ}} \end{bmatrix}$$

- Losses in 0 and π modes lead to phase shifts in later cells, while in the $\frac{\pi}{2}$ mode, it leads to a real amplitude 'droop'.

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# $\frac{\pi}{2}$ mode structures – "Side Coupled Linacs"

Operating a cavity in the $\frac{\pi}{2}$ mode means that the tuning and stability of the cavity is much easier.

However, you have to get the unexcited cavities out of the way, because they just take up space.

This is achieved with either small coupling cavities on axis of moving them off-axis.

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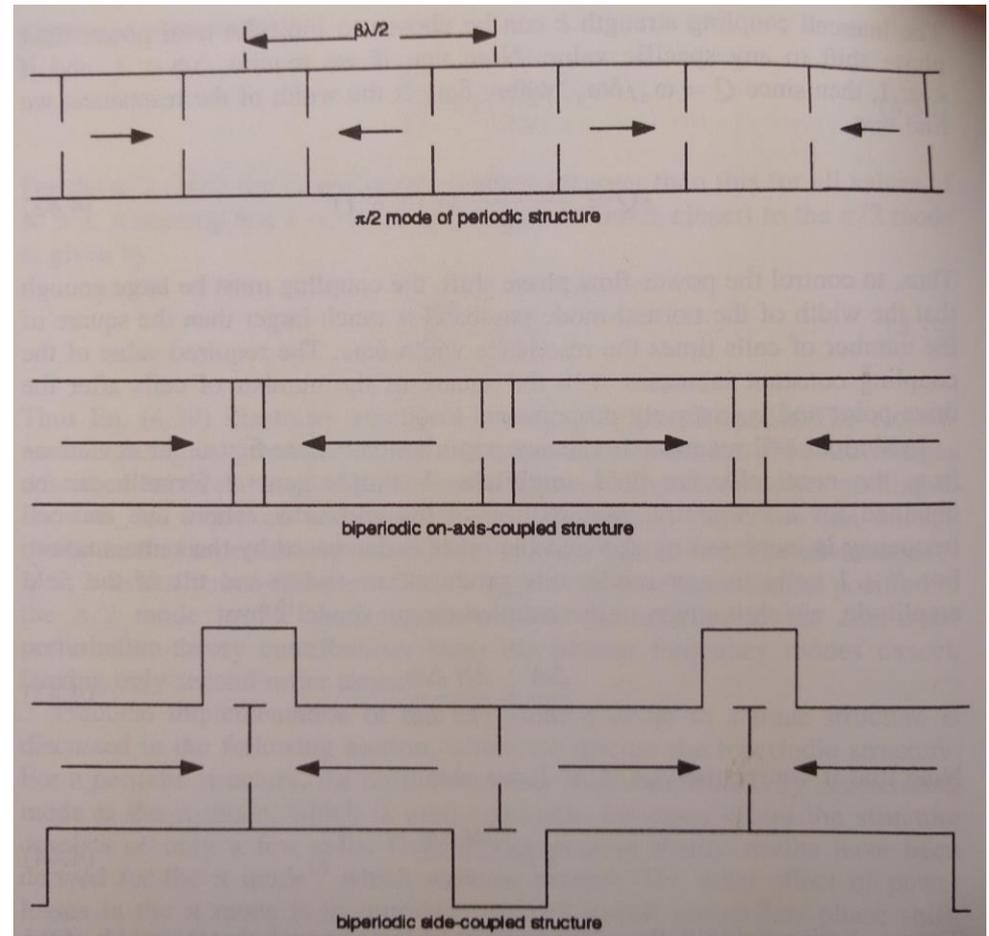
Side-Coupled Linacs in Practice

BEAM CHANNEL

ACCELERATING CAVITY
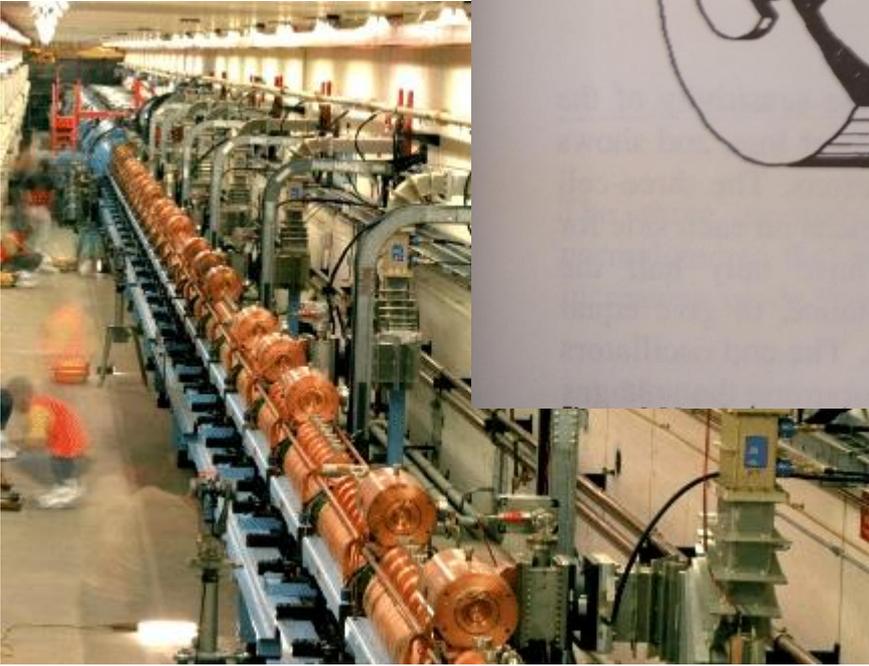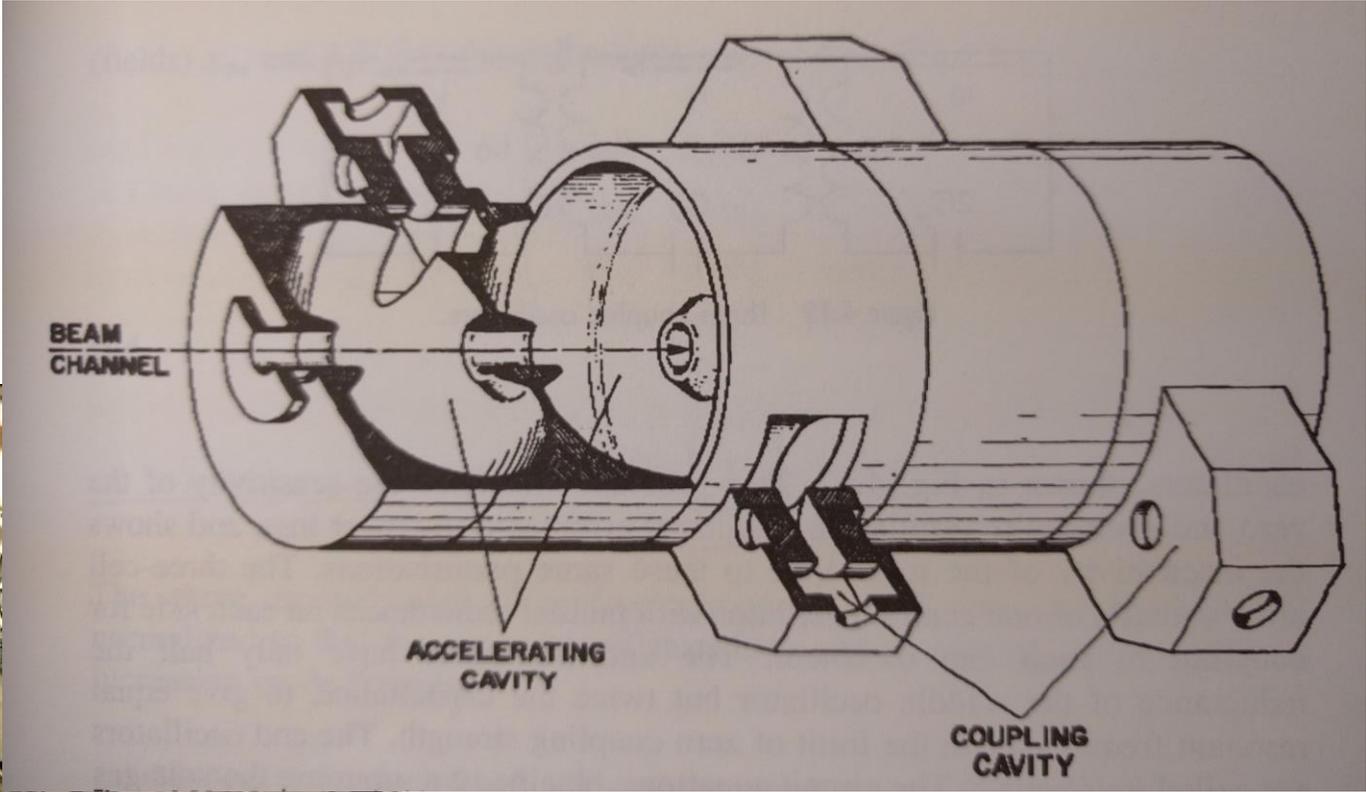
COUPLING CAVITY

Field droop means you excite in the middle, in practice.

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Superconducting Cavities

Superconducting cavities can be made with large apertures because the design focus is peak surface fields and Q rather than R/Q.

This means high coupling, and lower sensitivity to errors.

Also, you essentially never see superconducting cavities longer than 9-cells because of processing effects.

Still need to tune SRF cavities, though.

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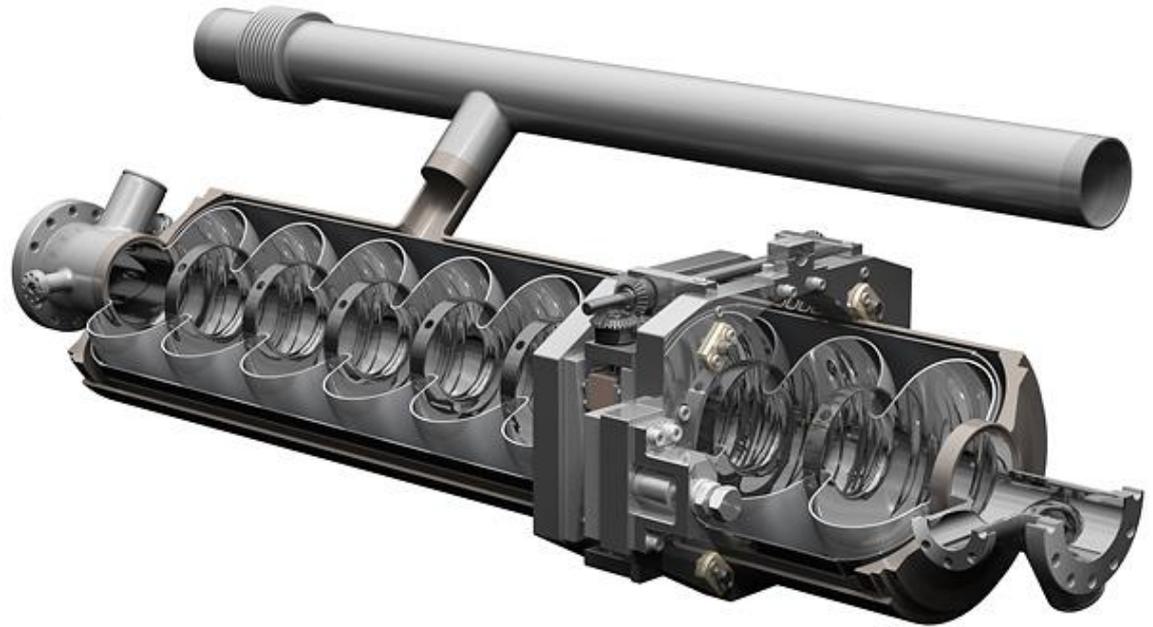
Bead Pull and Correction

Cavities are made field-flat by pulling a ceramic bead through the cavity.

This perturbs the resonant frequency of the cavity proportional to the field where the bead is.

This profile can be used to calculate the cell-by-cell errors, and each cell is tuned individually.

This process is repeated until the cavity gradient is even between all the cells.
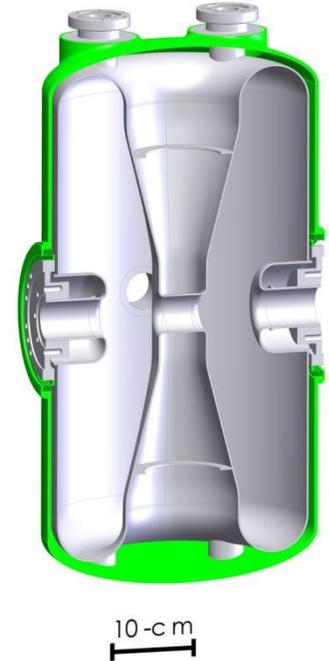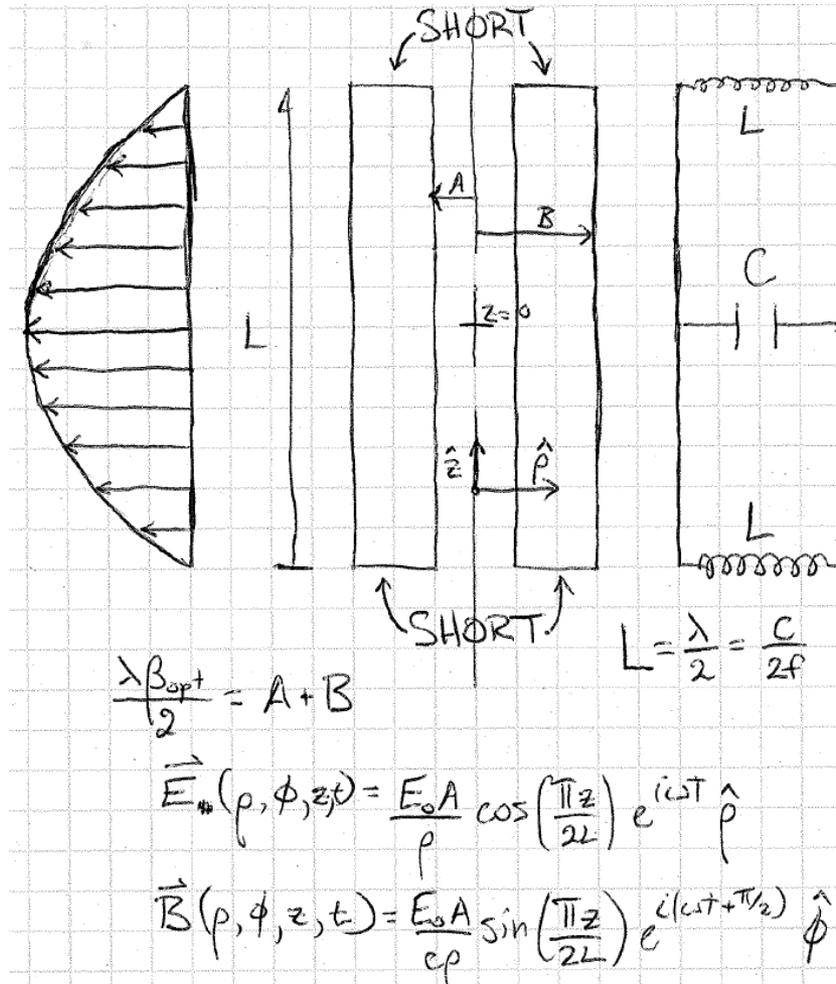
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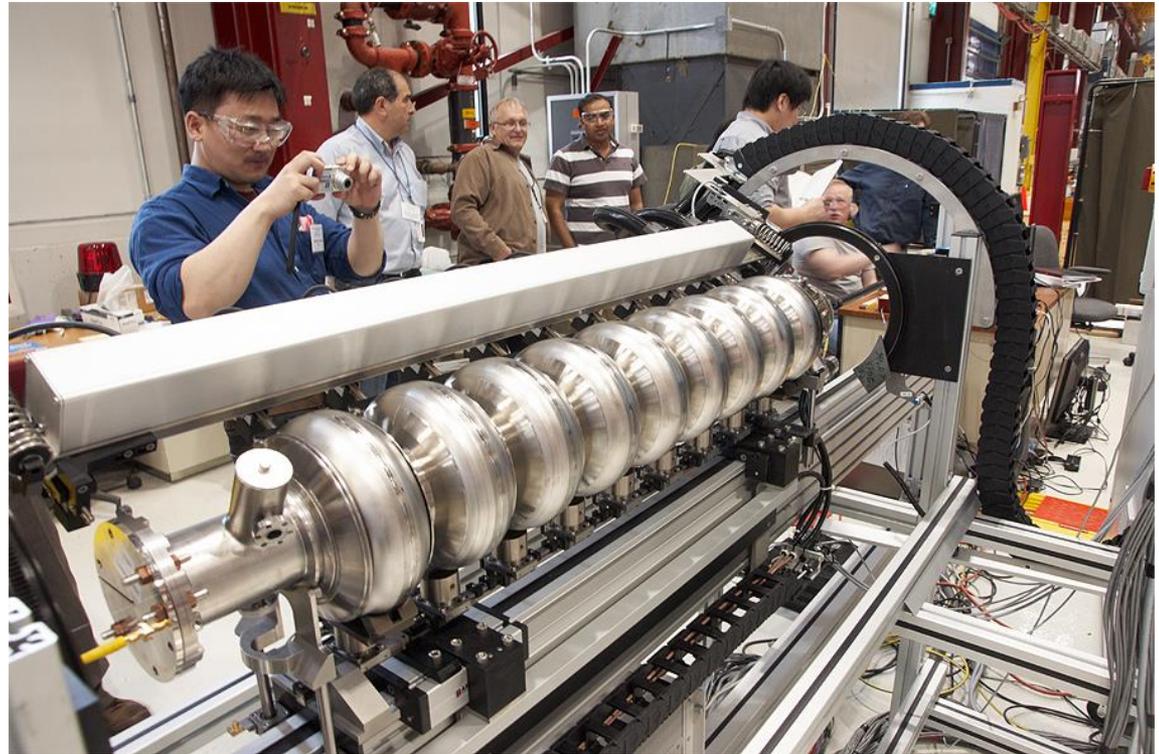
Quarter Wave Resonators

- **Coaxial Resonator**
  - Effective open and short termination

- **Low Frequency Structure**
  - Allows for efficient acceleration of low beta beams

- **Accelerating Field**
  - Two gap structure (Pi-Mode like)

- **Steering**
  - Asymmetric design leads to slight beam steering

- **Open end for access/processing**
 - Open end for cavity processing and inspection
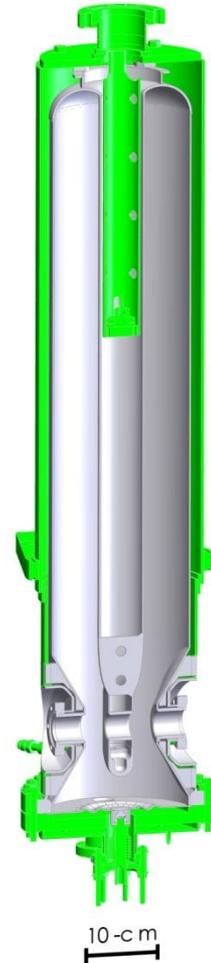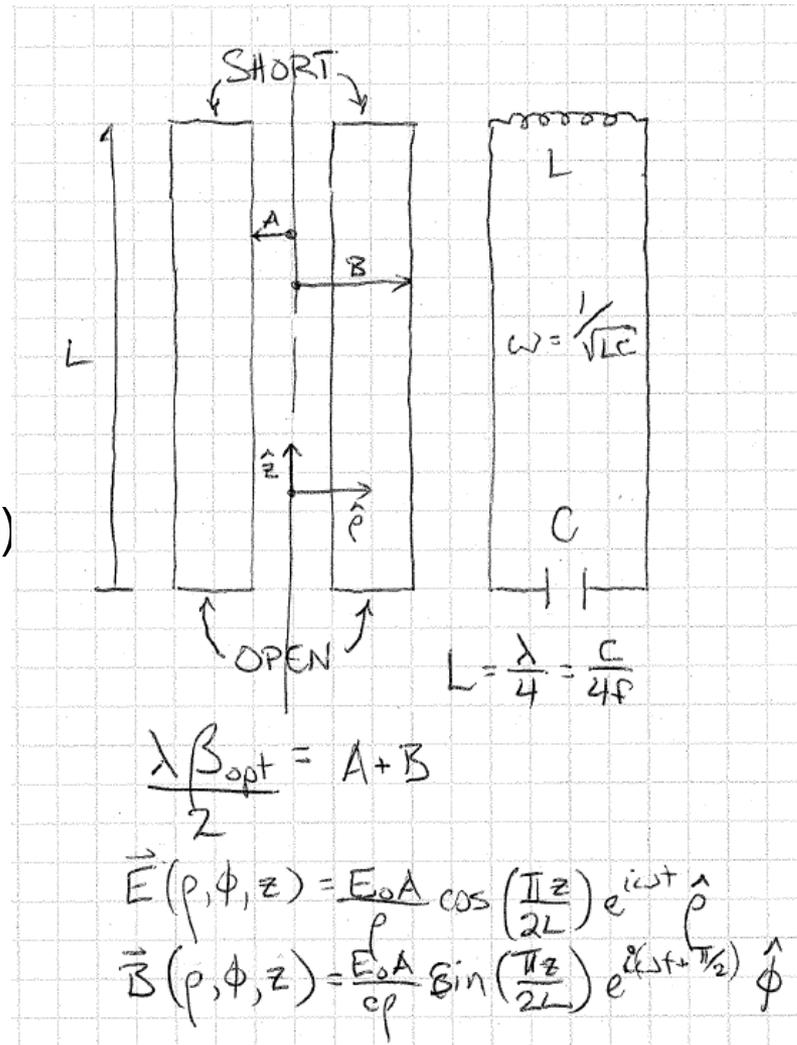
Half Wave Resonators

- **Coaxial Resonator**
  - Two effective short terminations

- **Higher Frequency Structure than QWR**

- **Accelerating Field**
  - Two gap structure (Pi-Mode like)

- **HWR v. QWR**
  - Higher optimum beta
  - No beam steering
  - Double the losses
  - No easy access

$$\frac{\lambda \beta_{opt}}{2} = A + B$$

$$L = \frac{\lambda}{2} = \frac{c}{2f}$$

$$\vec{E}_\phi (\rho, \phi, z, t) = \frac{E_0 A}{\rho} \cos\left(\frac{\pi z}{2L}\right) e^{i\omega T} \hat{\rho}$$

$$\vec{B}(\rho, \phi, z, t) = \frac{E_0 A}{c\rho} \sin\left(\frac{\pi z}{2L}\right) e^{i(\omega t + \pi/2)} \hat{\phi}$$

10-cm

Single Spoke Resonator

Topologically identical to
the Half Wave Resonator

Mechanically weaker in
the beam axis.

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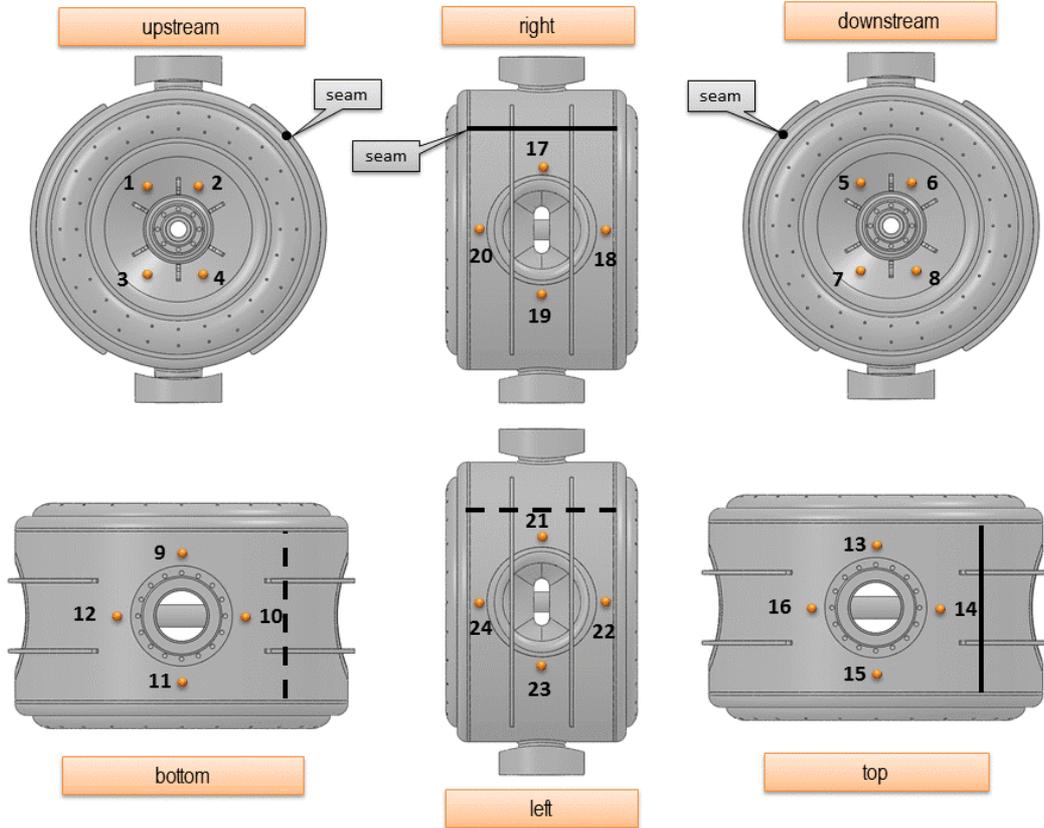
Multi-Spoke Resonator

Essentially a multi-cell version of the single spoke resonator/half wave resonator.

More compact than equivalent medium-beta geometries, but significantly more mechanically complex.

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