6
Magnetic Measurements

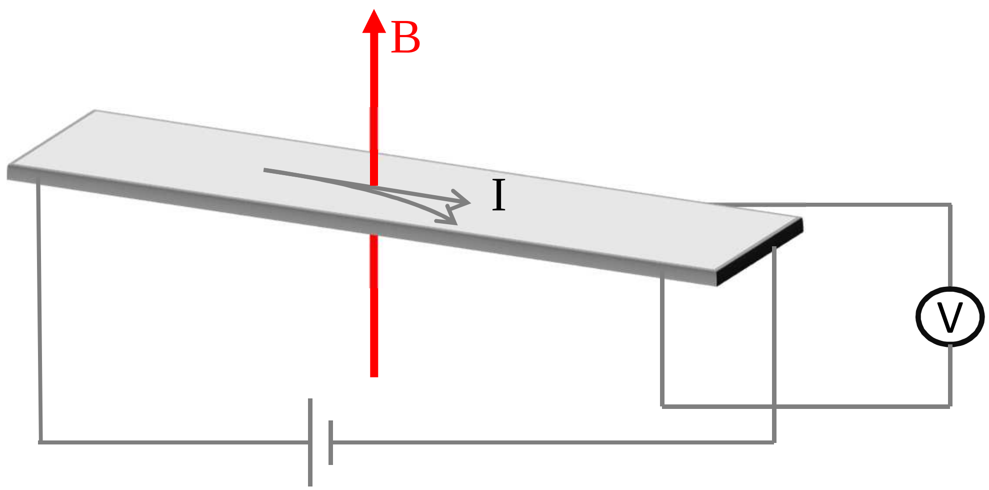
Mauricio Lopes – FNAL

Introduction

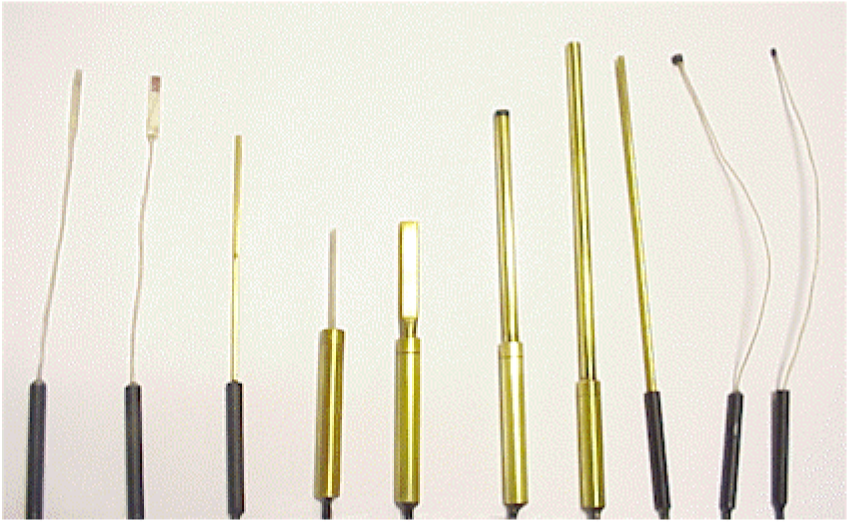
- Magnetic field is the “art” of measuring the magnetic field components, the field integral or the field errors of a particular magnet.
- There are many techniques. One may dedicate a career developing this field.
- A few techniques will be explored in this lecture.

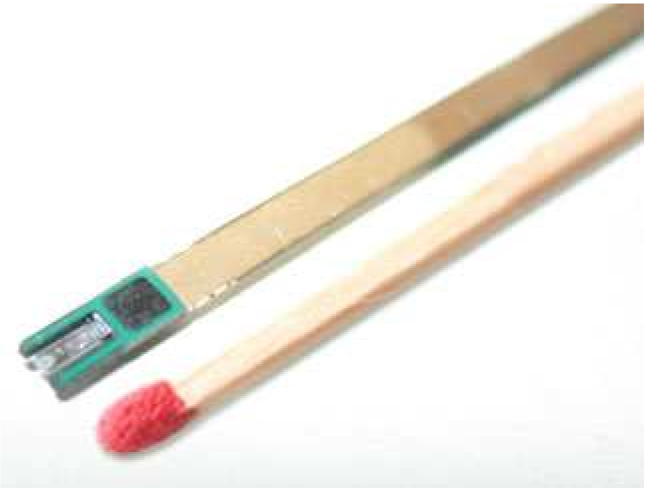
Hall Probes

$$\boldsymbol{F} = q\,\boldsymbol{v} \times \boldsymbol{B}$$



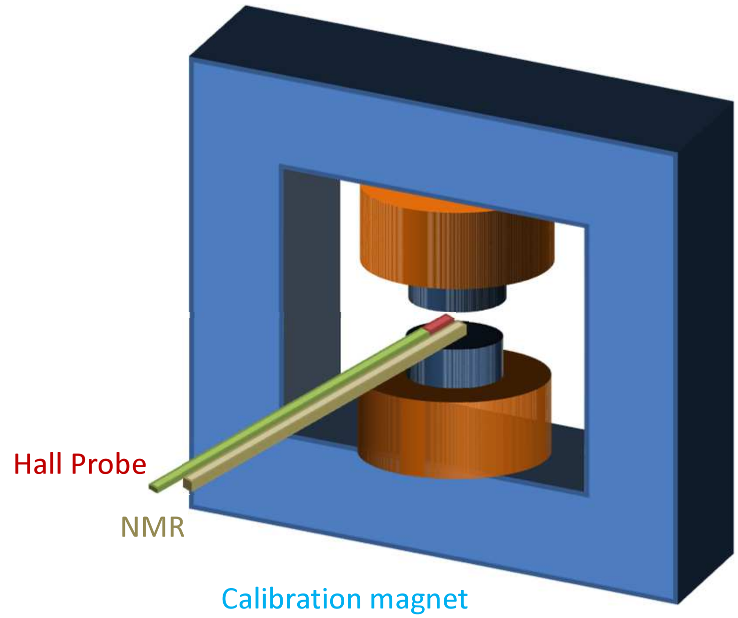
Hall Probes

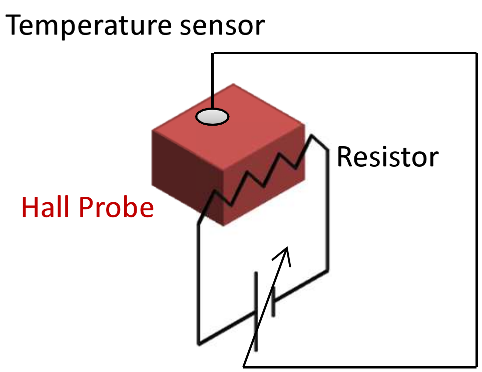


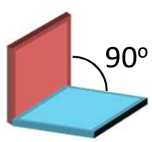


Things to keep in mind when working with hall probes…

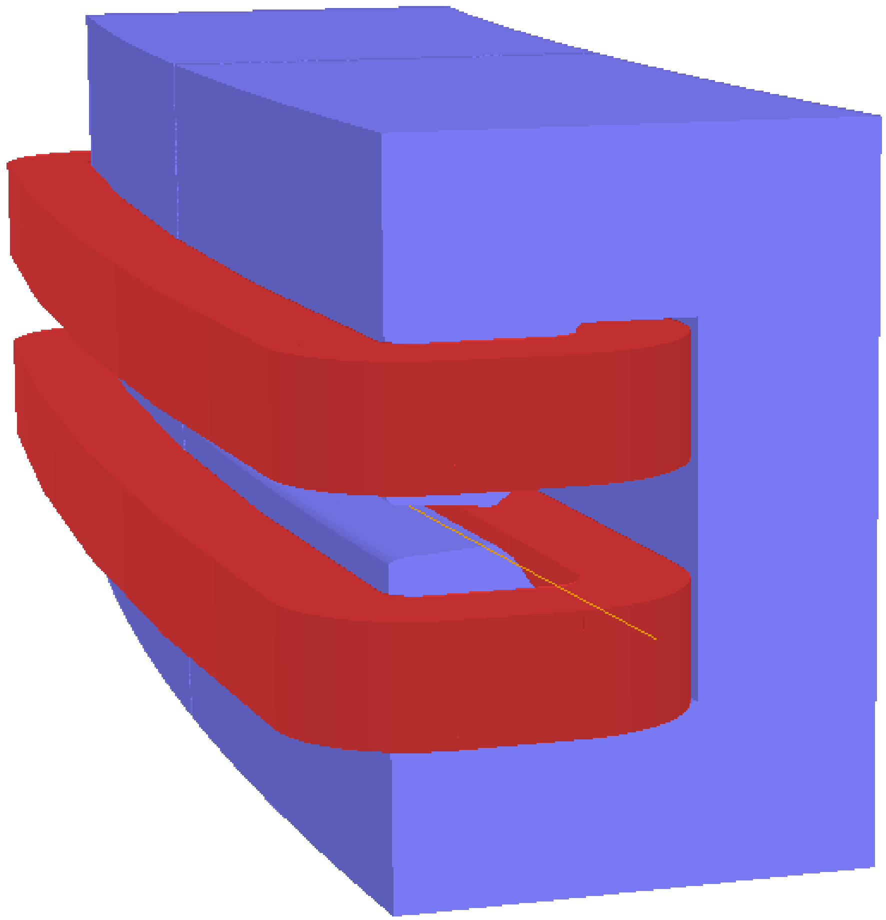
- Calibration/Linearity
- Temperature dependence
- Planar effects
- Positioning





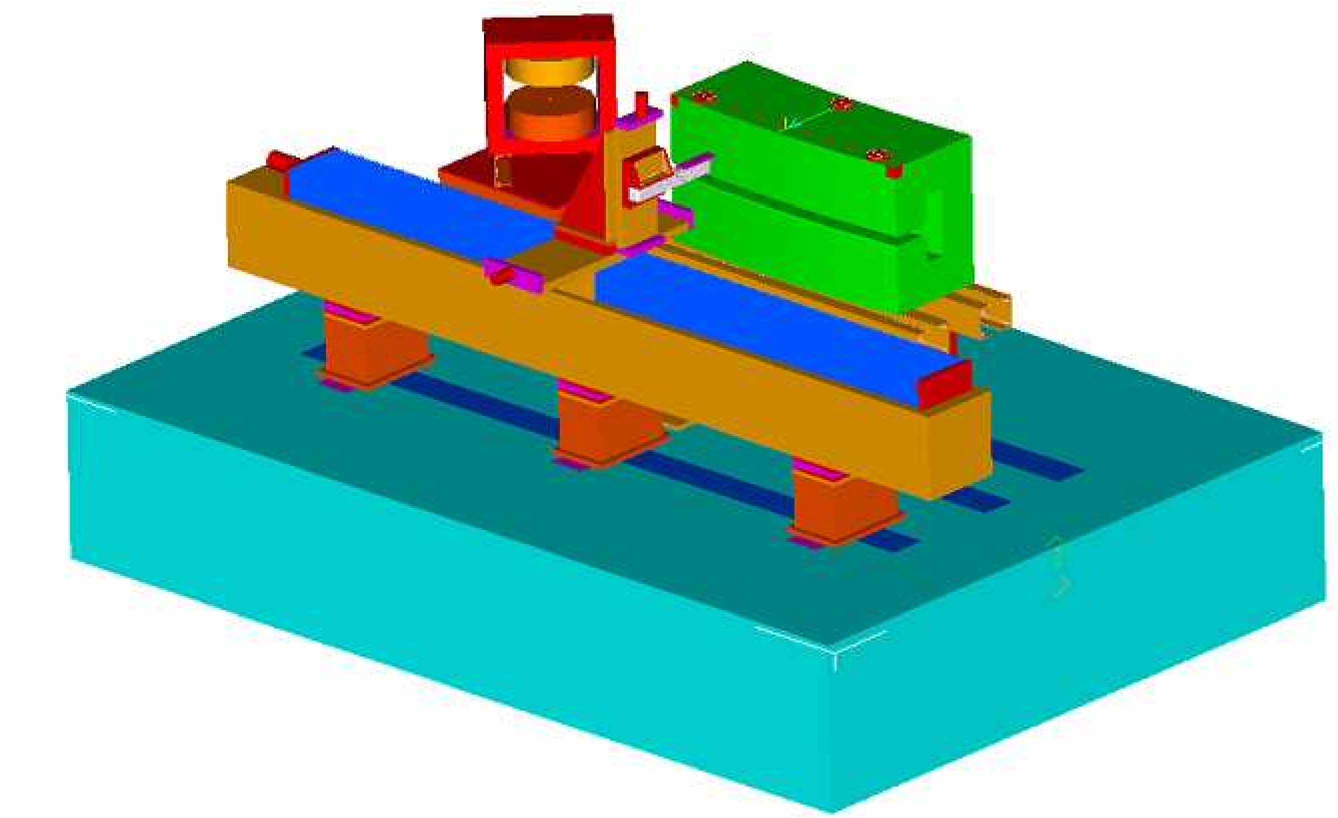


ALBA Combined Magnet

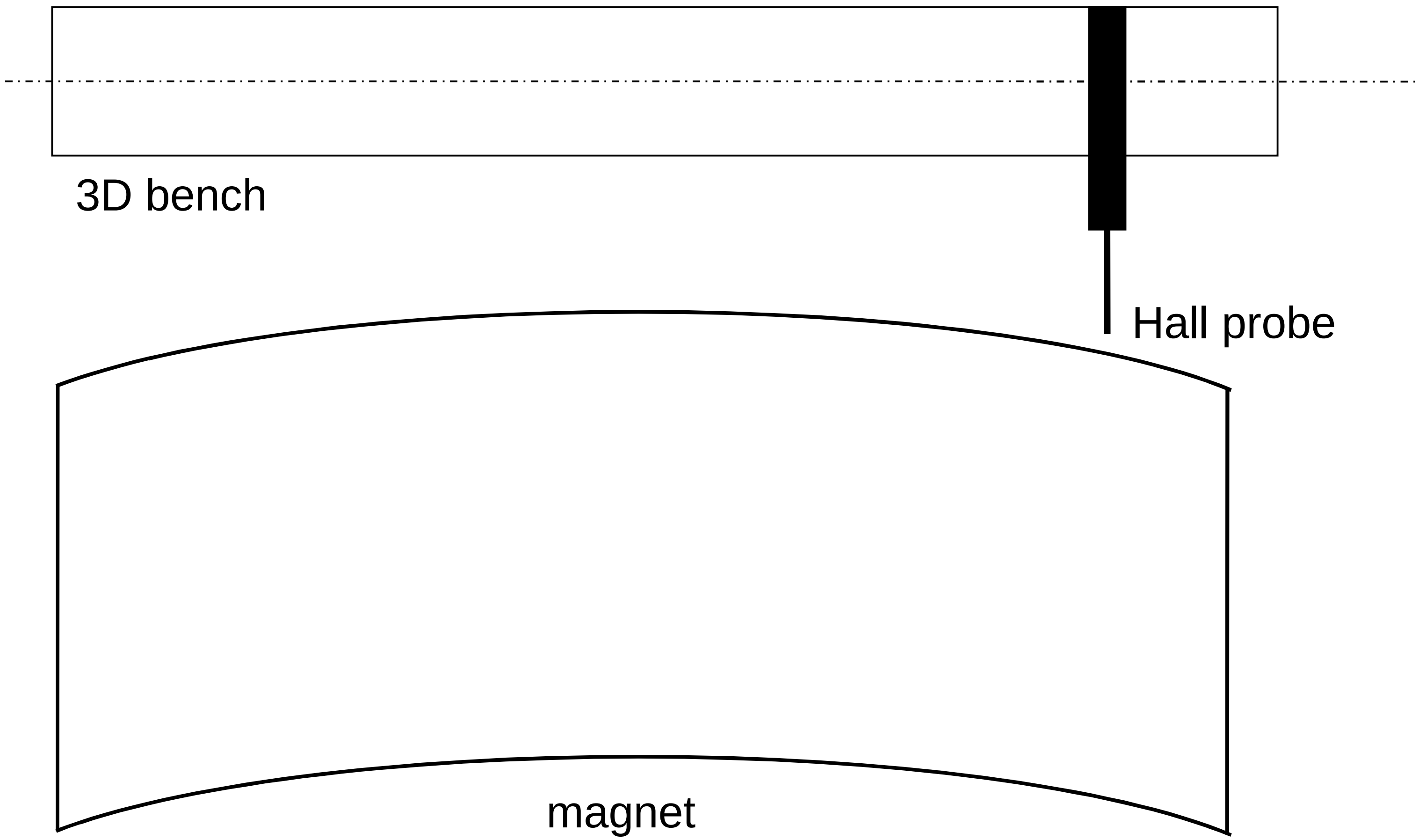


Curved
Dipole + Gradient
B=1.42 T
G=5.656 T/m

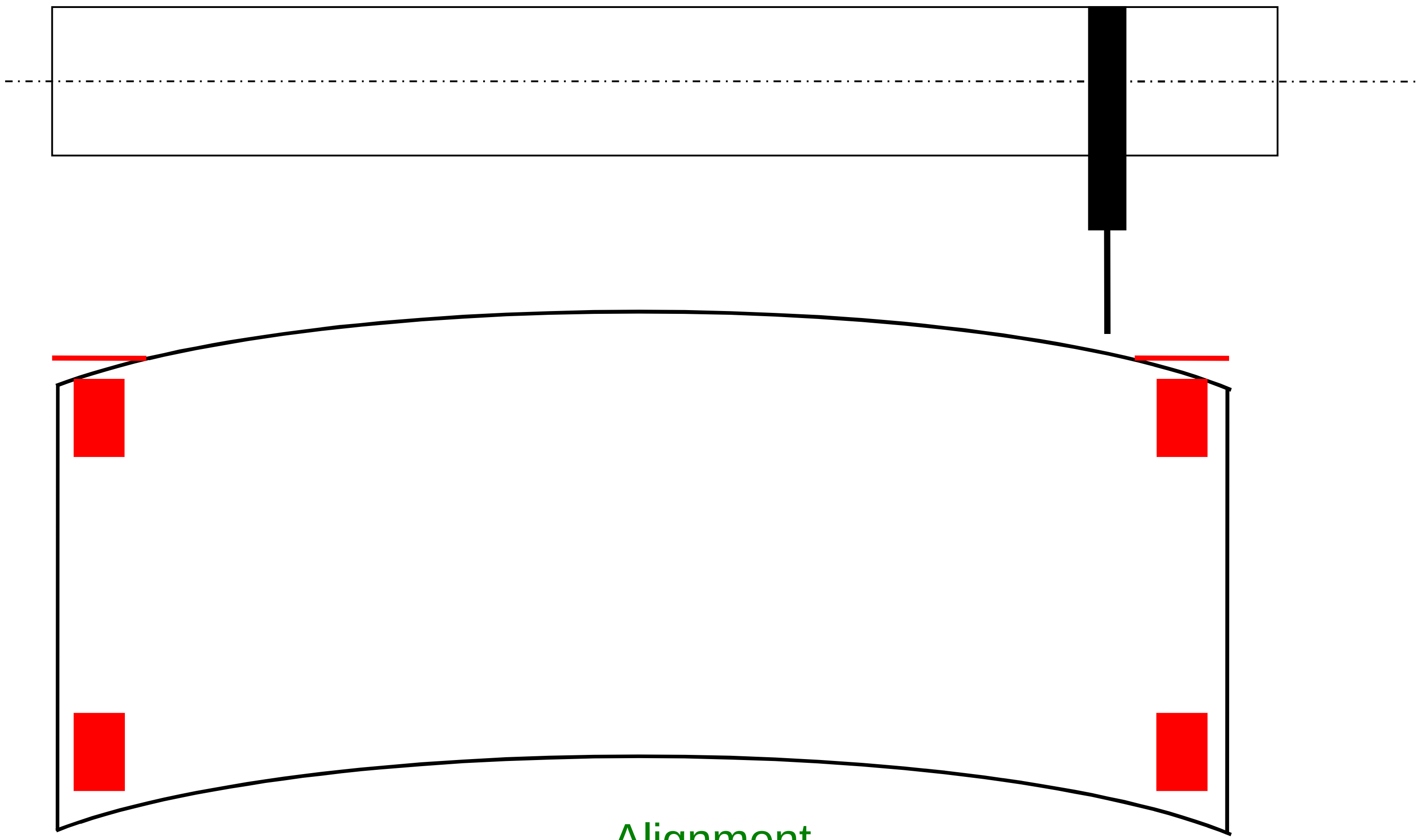
3D Hall Probe Bench



Positioning

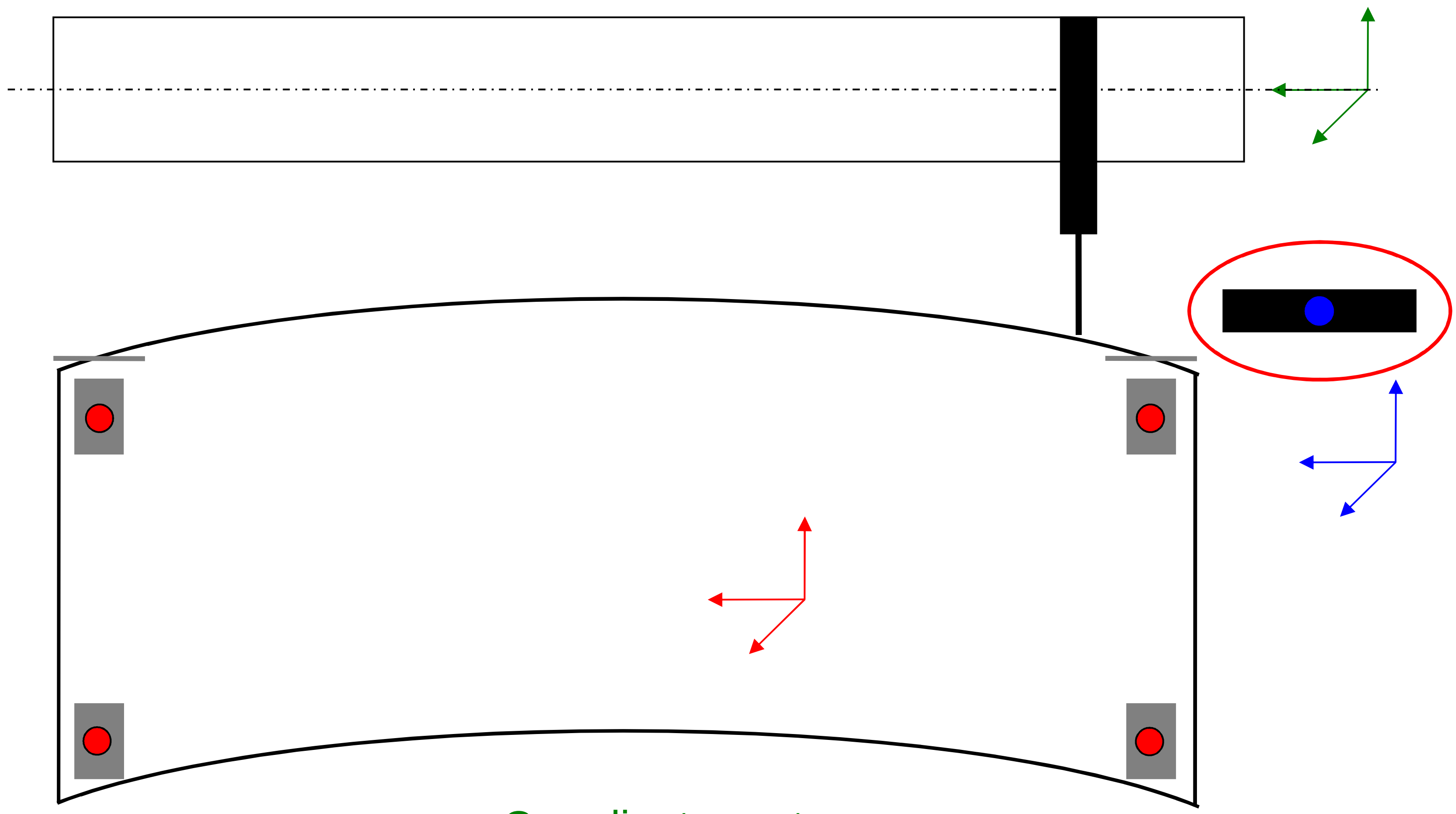


Positioning

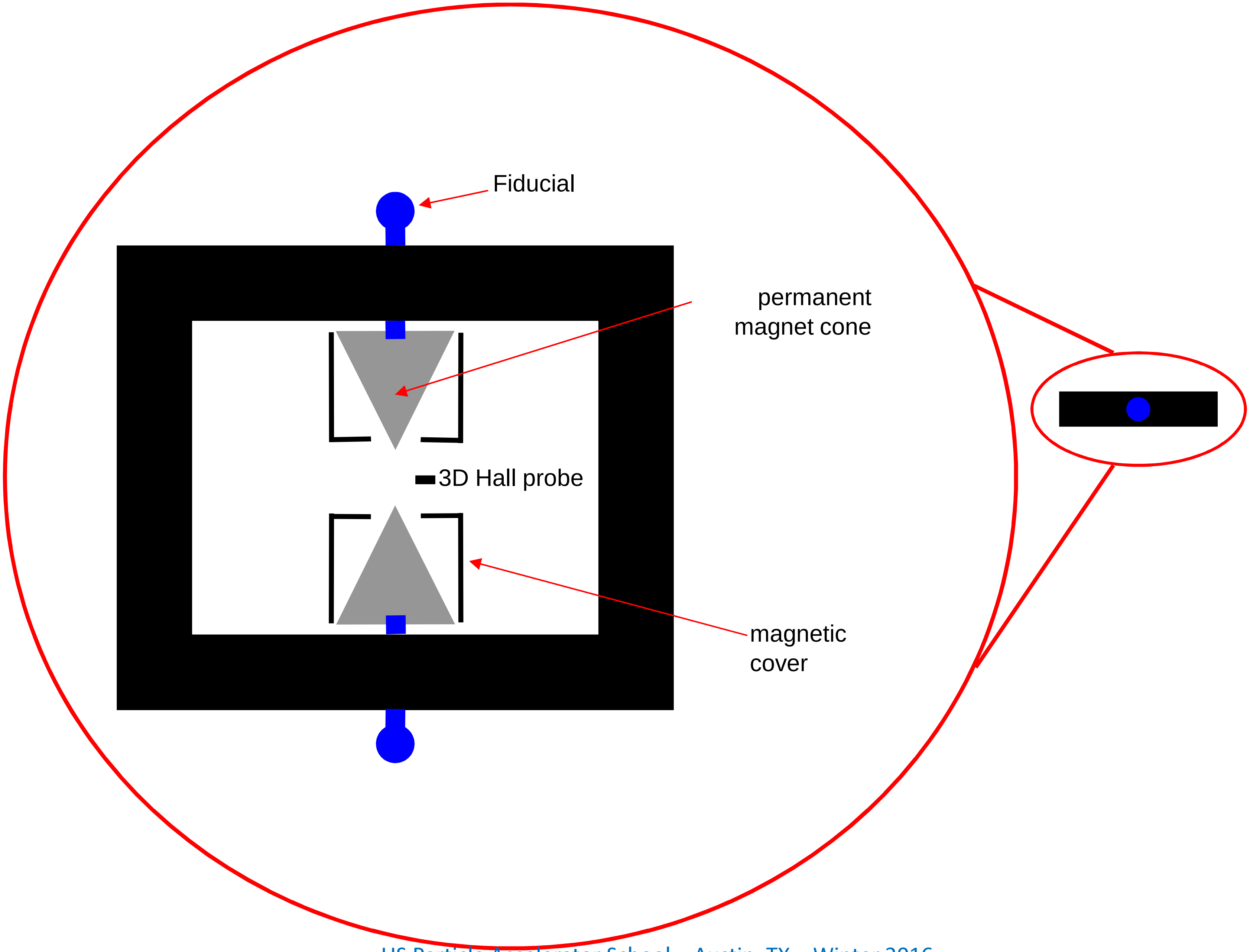


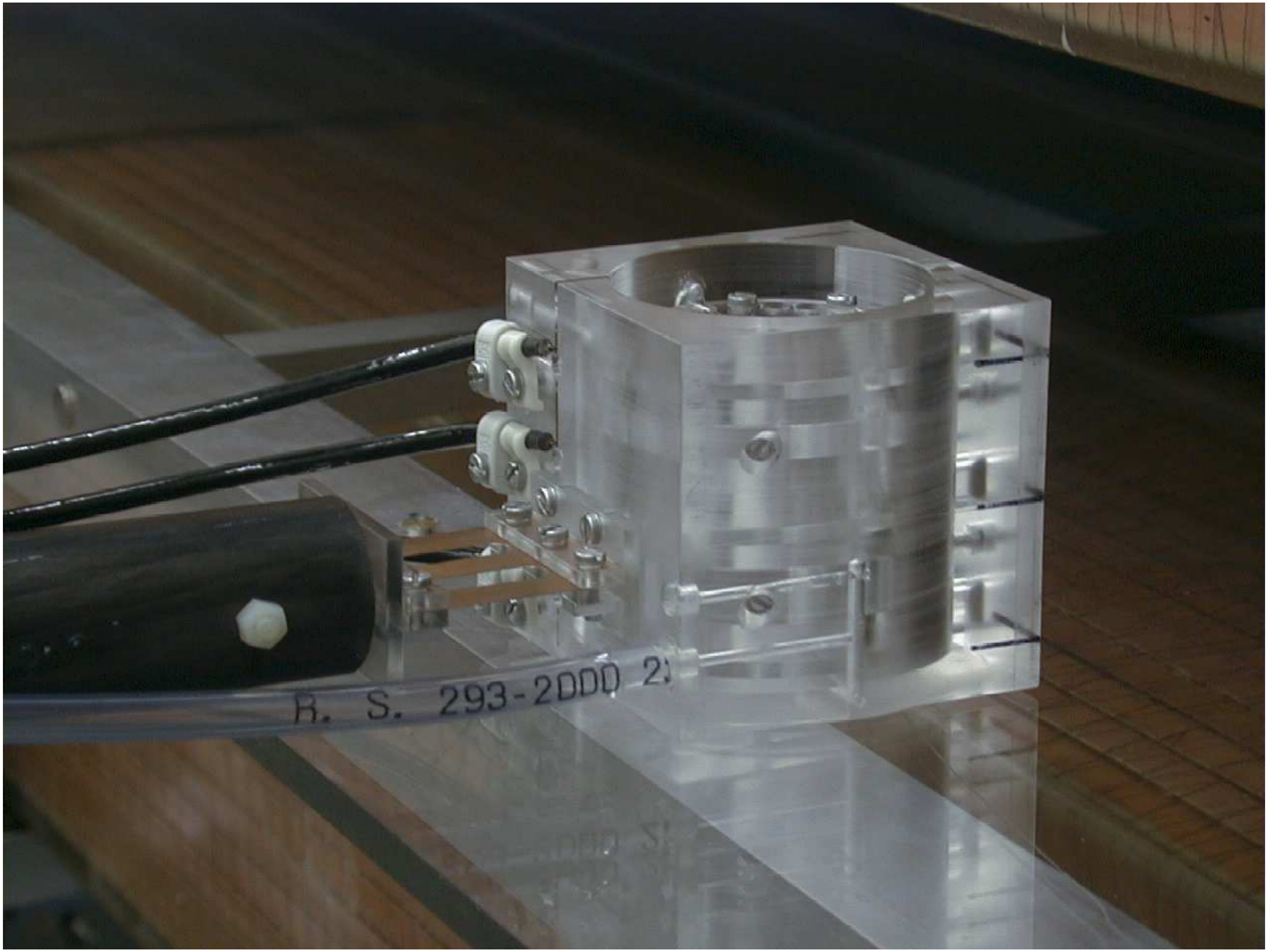
Alignment
Magnet vs. Bench

Positioning

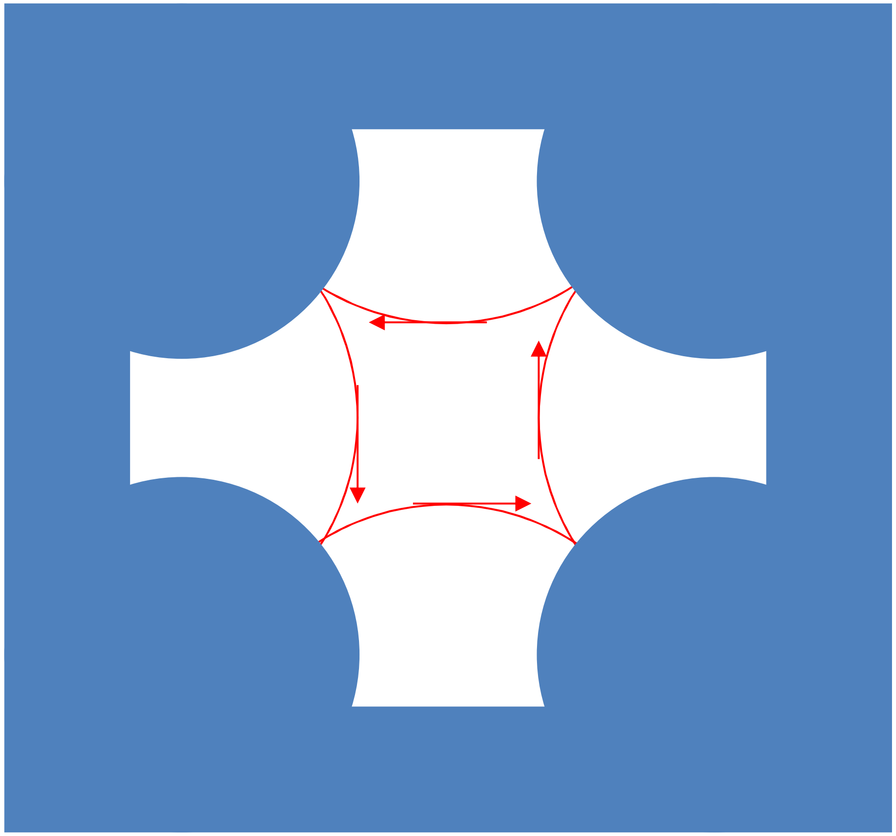


Coordinate systems

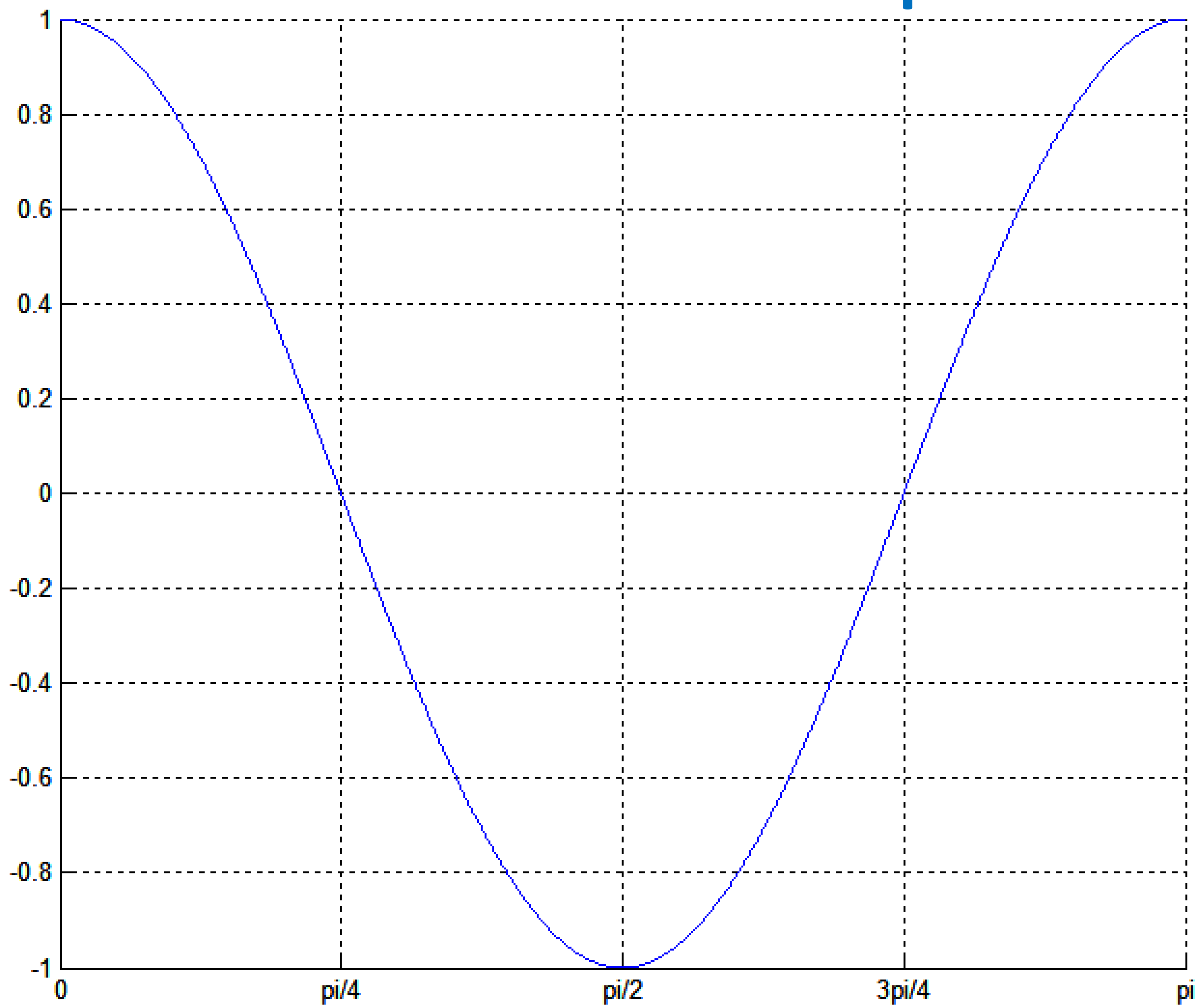

Fiducial
permanent magnet cone
3D Hall probe
magnetic cover


R. S. 293-2DDD 2

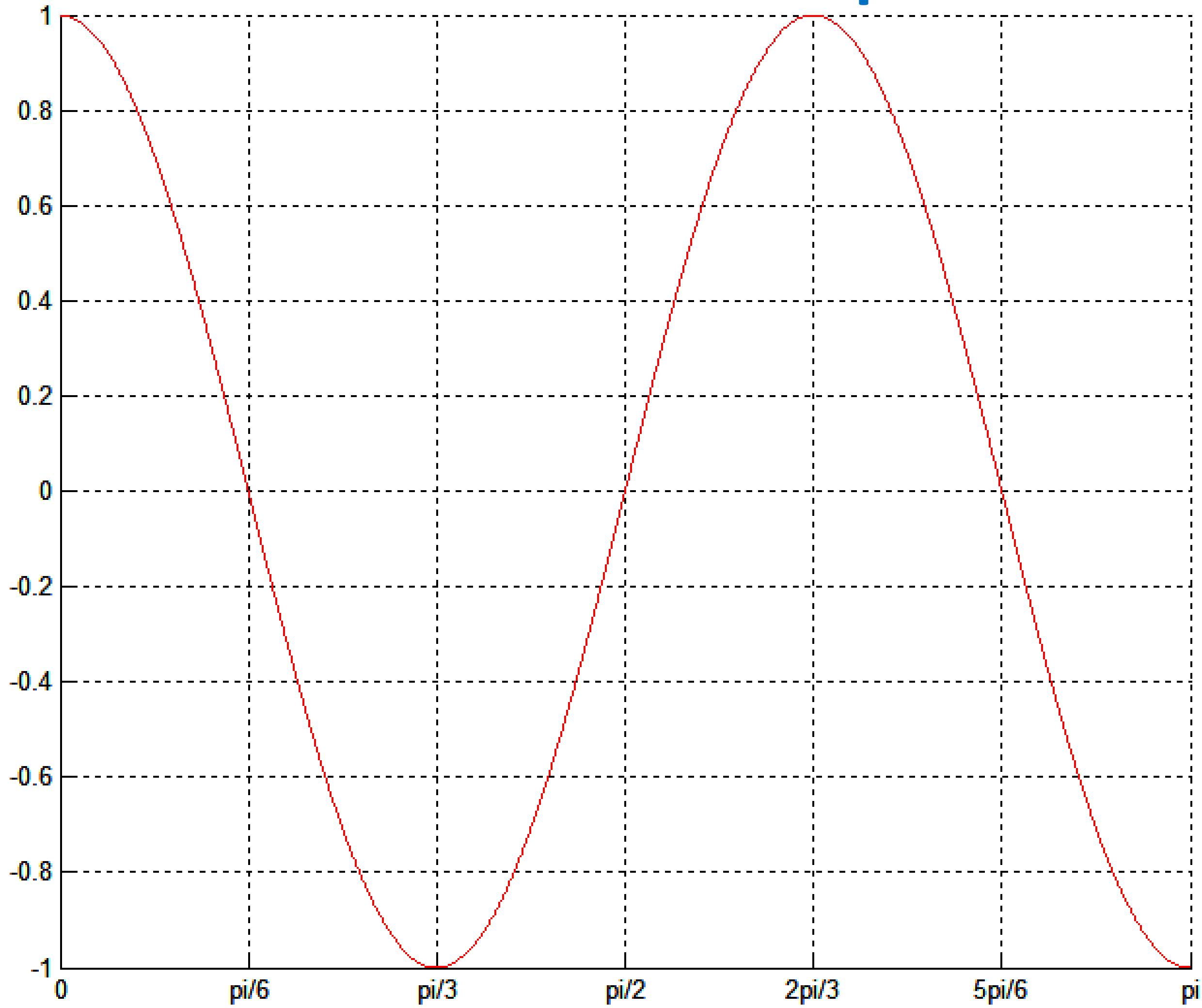
Field of a Quadrupole



Field of a Quadrupole



Field of a Sextupole



Fourier series

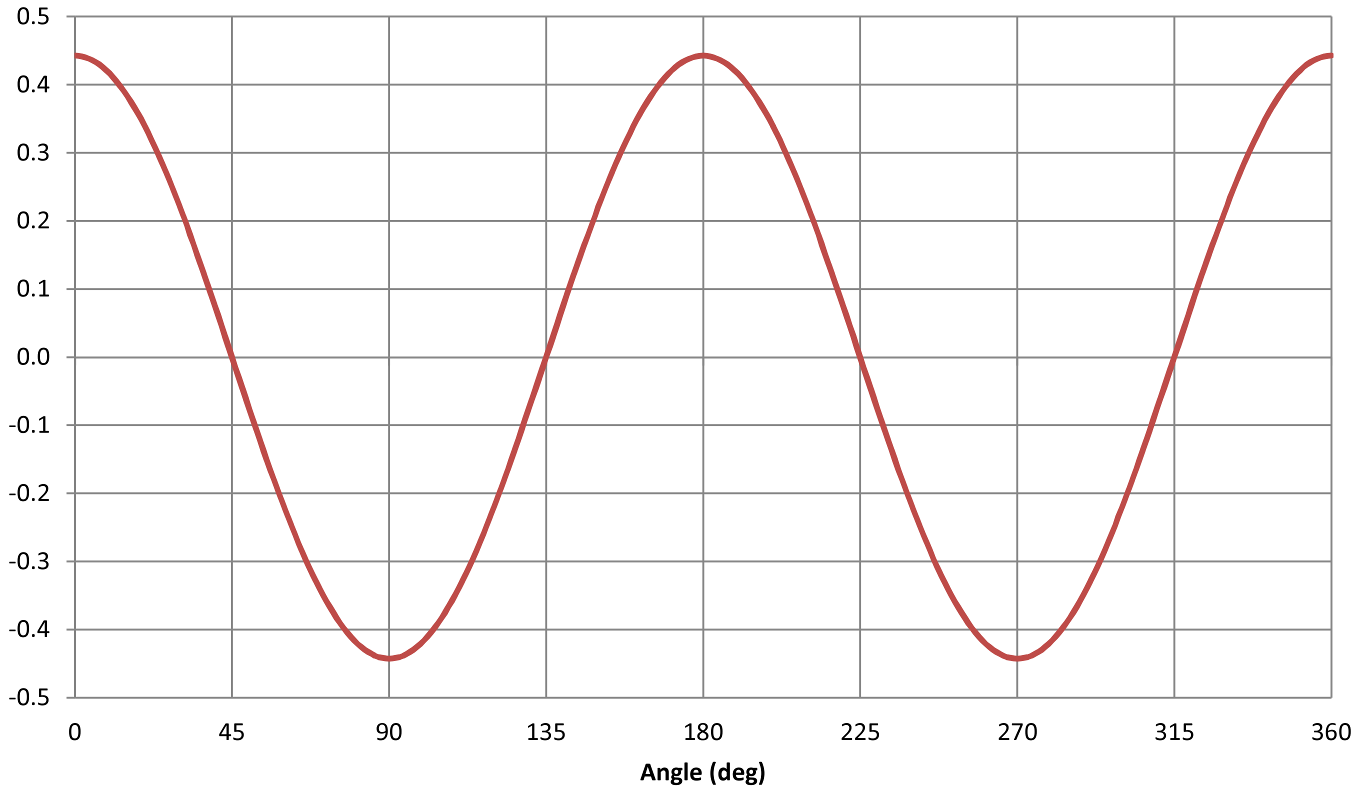
$$f(x) = \frac{b_0}{2} + \sum_{n=1}^{\infty} [a_n \sin(nx) + b_n \cos(nx)]$$

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin(nx)\, dx \qquad n \geq 1$$

$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos(nx)\, dx \qquad n \geq 0$$

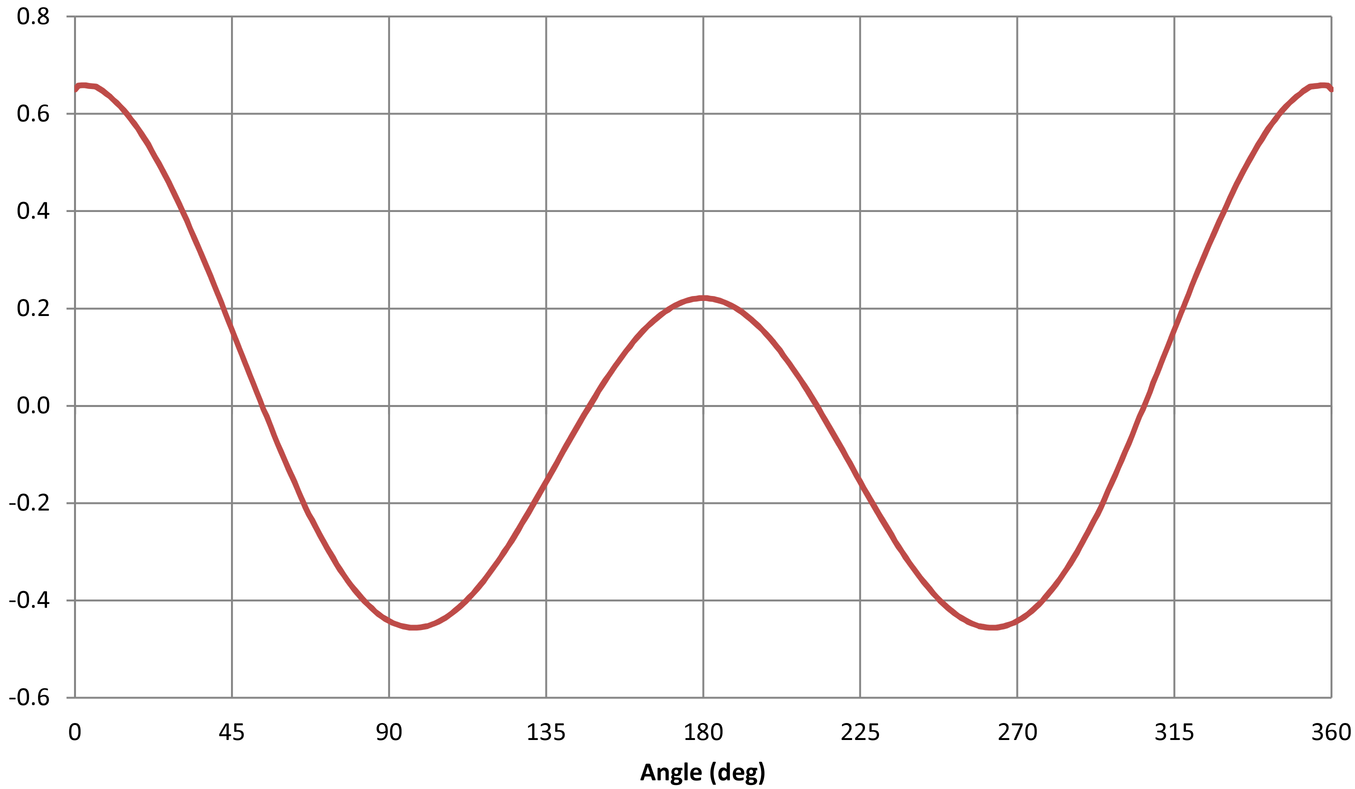
$$F(\mathbb{z}) = A + iV \quad = \sum_{n=1}^{\infty} C_n \mathbb{z}^n \quad = \sum_{n=1}^{\infty} C_n |\mathbb{z}|^n e^{in\theta} \quad = \sum_{n=1}^{\infty} C_n |\mathbb{z}|^n [\cos(n\theta) + i\sin(n\theta)]$$

Example 1



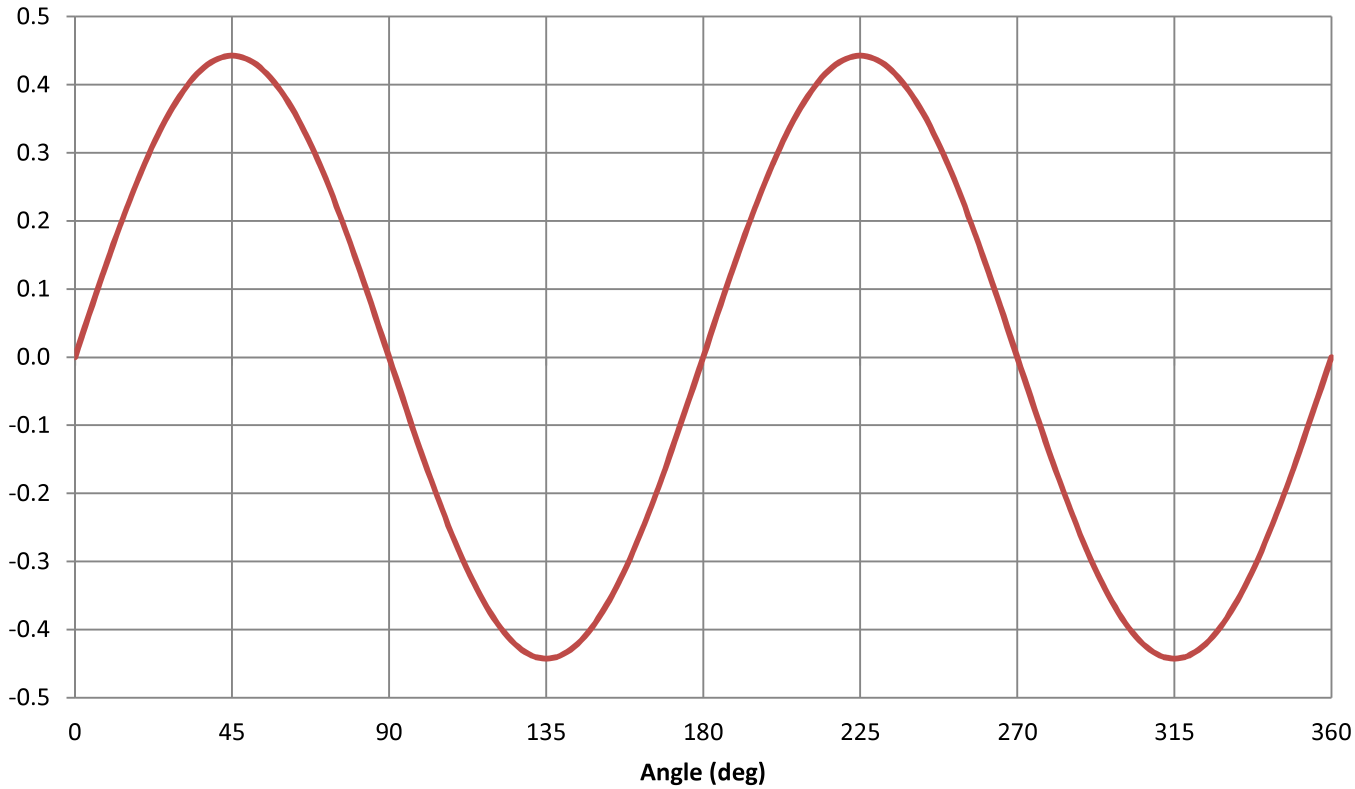
	an	bn
1	0.000000	0.000000
2	0.000000	0.442599
3	0.000000	0.000000
4	0.000000	0.000000
5	0.000000	0.000000
6	0.000000	0.000118
7	0.000000	0.000000
8	0.000000	0.000000
9	0.000000	0.000000
10	0.000000	-0.000006

Example 2



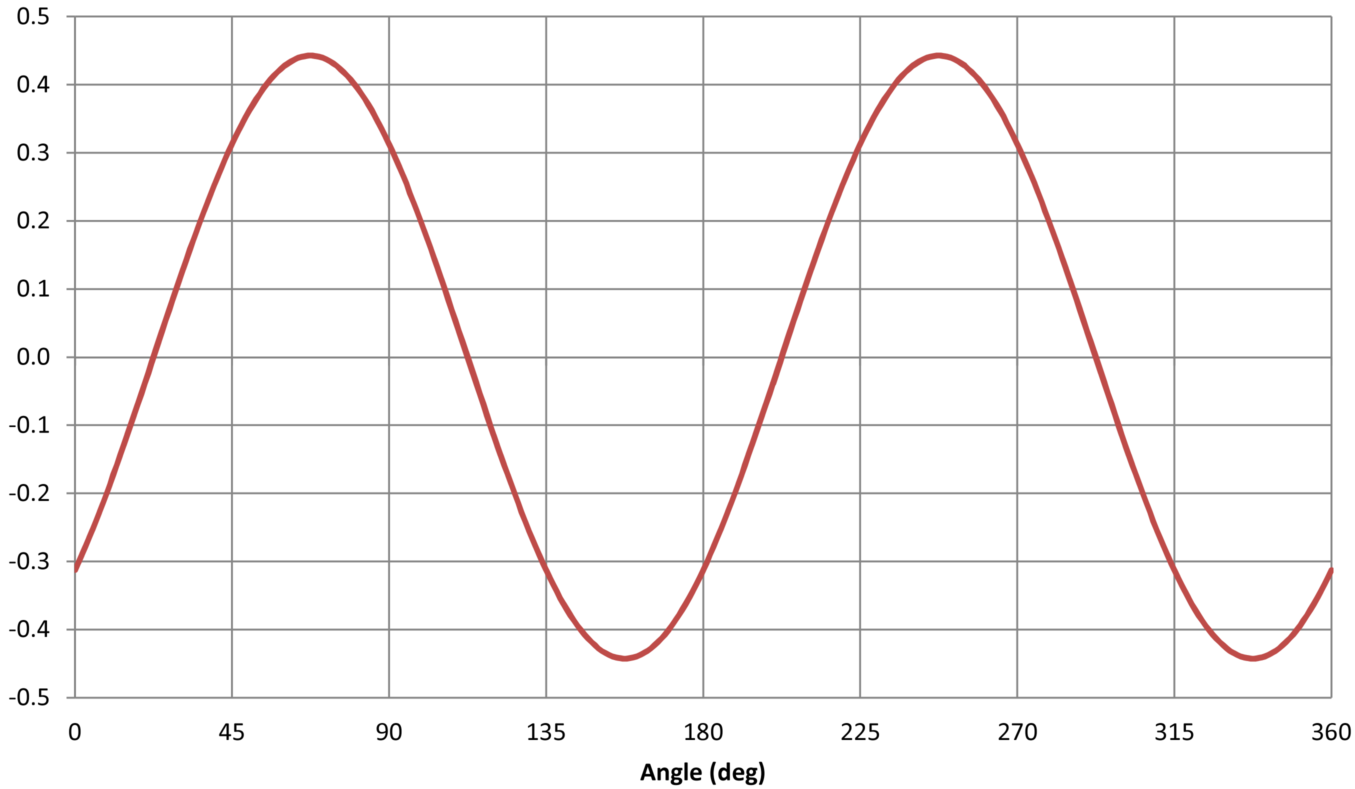
	an	bn
1	0.000000	0.110650
2	0.000000	0.442601
3	0.000000	0.000019
4	0.000000	0.000074
5	0.000000	0.000146
6	0.000000	0.000115
7	0.000000	-0.000009
8	0.000000	-0.000016
9	0.000000	-0.000021
10	0.000000	-0.000021

Example 3



	an	bn
1	0.000000	0.000000
2	0.442599	0.000000
3	0.000000	0.000000
4	0.000000	0.000000
5	0.000000	0.000000
6	-0.000118	0.000000
7	0.000000	0.000000
8	0.000000	0.000000
9	0.000000	0.000000
10	-0.000006	0.000000

Example 4



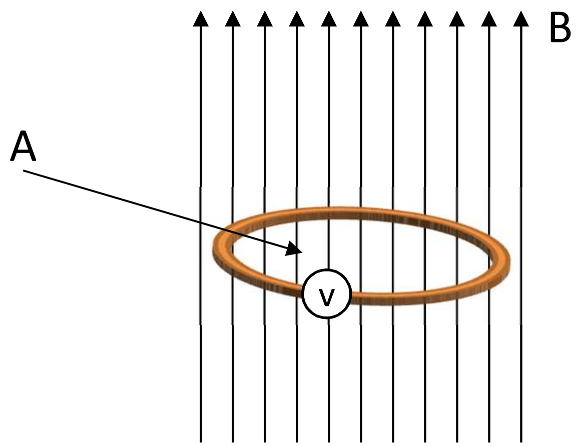
	an	bn
1	0.000000	0.000000
2	0.312965	-0.312965
3	0.000000	0.000000
4	0.000000	0.000000
5	0.000000	0.000000
6	0.000083	0.000083
7	0.000000	0.000000
8	0.000000	0.000000
9	0.000000	0.000000
10	0.000004	-0.000004

Maxwell's Equations

(in vacuum)

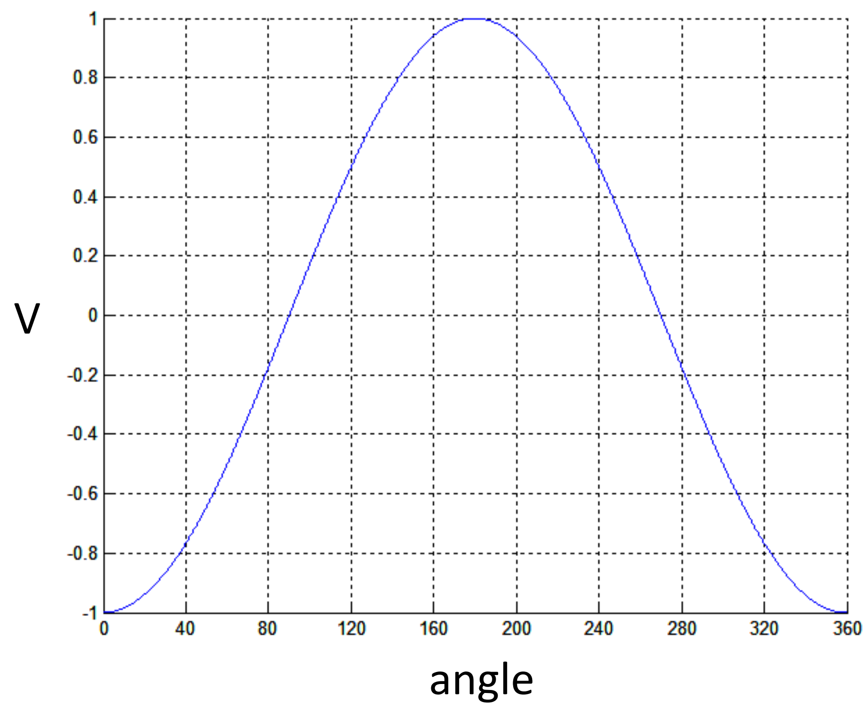
Gauss's law	$\nabla . \boldsymbol{E} = \dfrac{\rho}{\varepsilon_o}$	$\oiint \boldsymbol{E} . d\boldsymbol{A} = \dfrac{Q}{\varepsilon_o}$
	$\nabla . \boldsymbol{B} = 0$	$\oiint \boldsymbol{B} . d\boldsymbol{A} = 0$
Faraday's law	$\nabla \times \boldsymbol{E} = -\dfrac{\partial \boldsymbol{B}}{\partial t}$	$\oint \boldsymbol{E} . d\boldsymbol{l} = -\iint \dfrac{\partial \boldsymbol{B}}{\partial t} . d\boldsymbol{A}$
Ampere's law	$\nabla \times \boldsymbol{B} = \mu_o \boldsymbol{J} + \mu_o \varepsilon_o \dfrac{\partial \boldsymbol{E}}{\partial t}$	$\oint \boldsymbol{B} . d\boldsymbol{l} = \mu_o \boldsymbol{I} + \mu_o \varepsilon_o \iint \dfrac{\partial \boldsymbol{E}}{\partial t} . d\boldsymbol{A}$

Faraday's Law

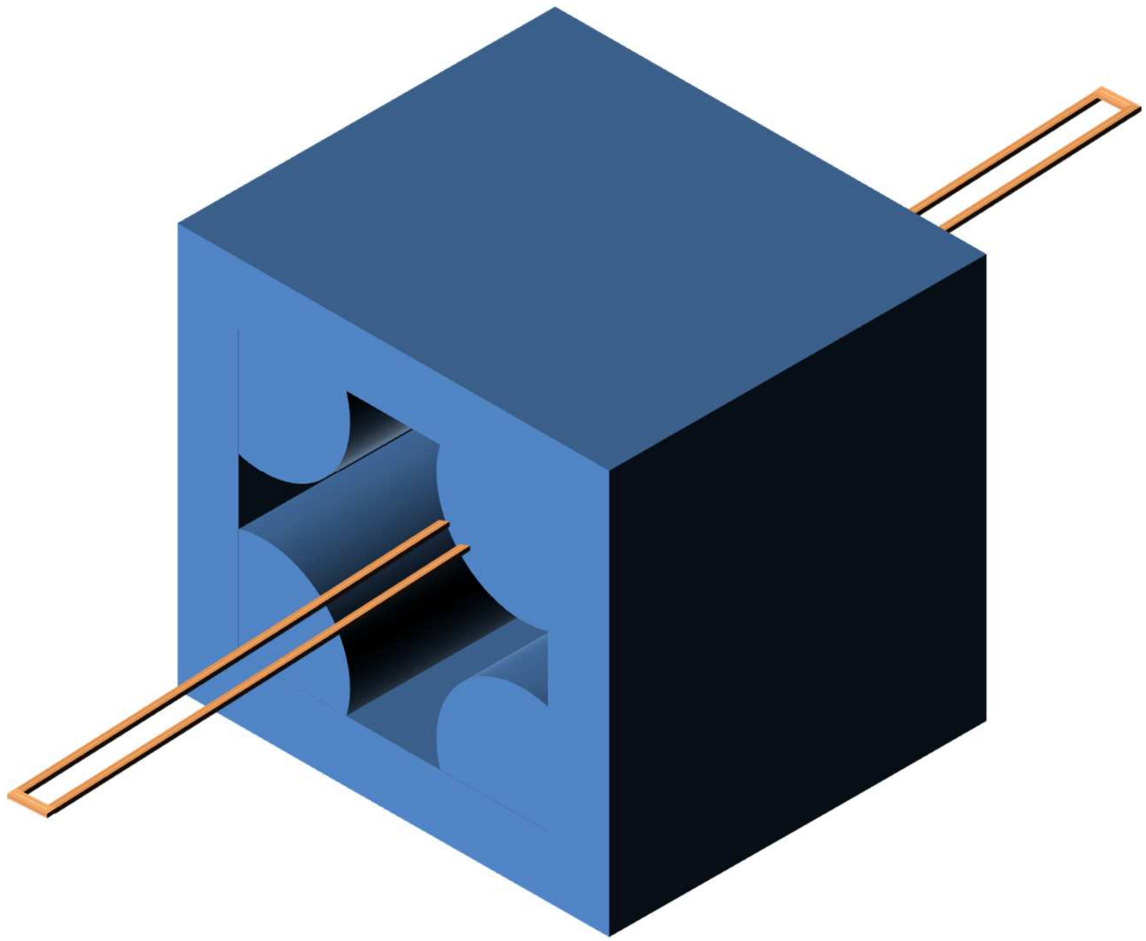


$$\oint \boldsymbol{E}.d\boldsymbol{l} = -\iint \frac{\partial \boldsymbol{B}}{\partial t}.d\boldsymbol{A}$$

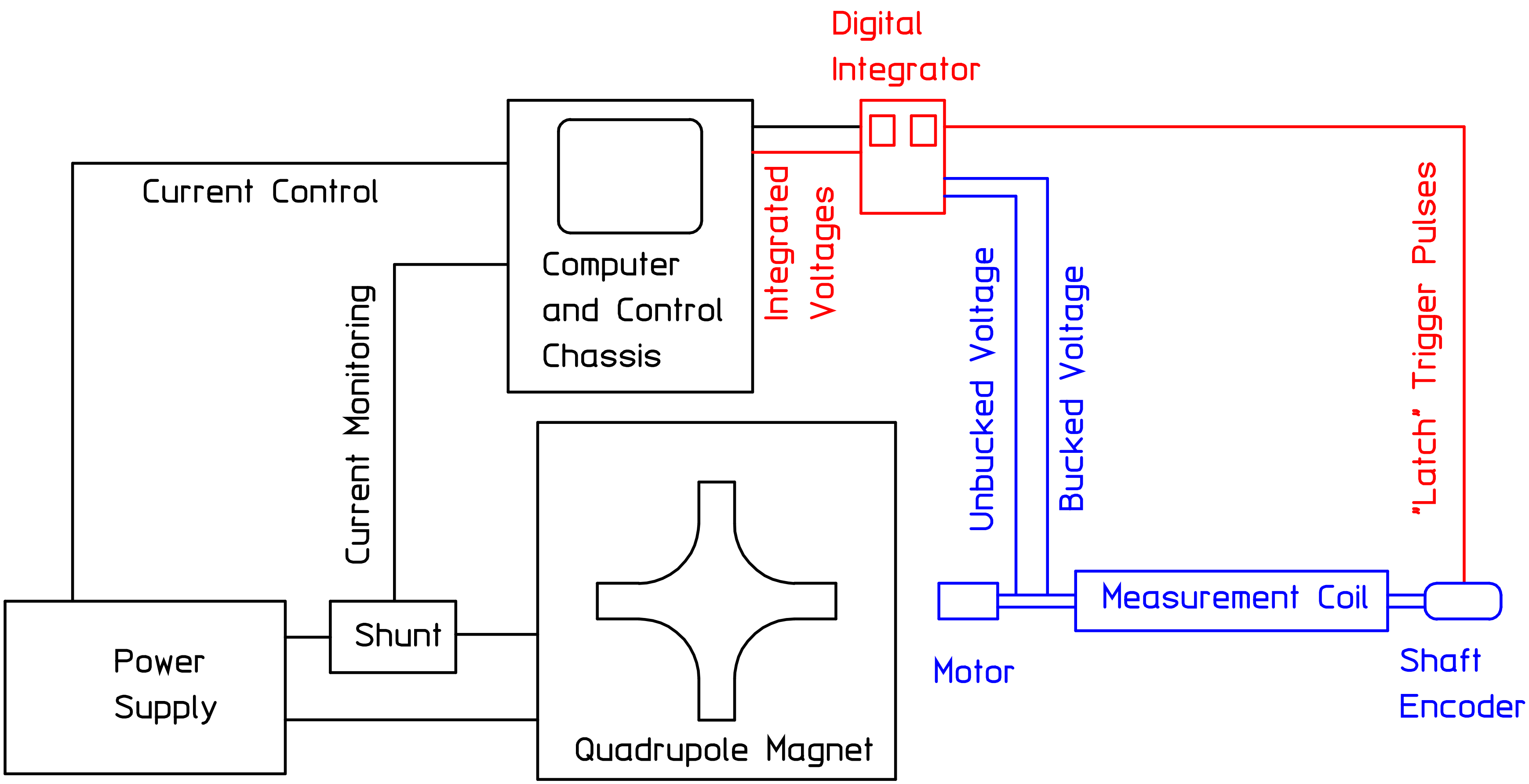
$$\boldsymbol{V} = -\frac{\partial \phi_{\boldsymbol{B}}}{\partial t} \qquad \phi_{\boldsymbol{B}} = \int \boldsymbol{B}.\boldsymbol{dA}$$



Rotating coil



Rotating Coil Schematics

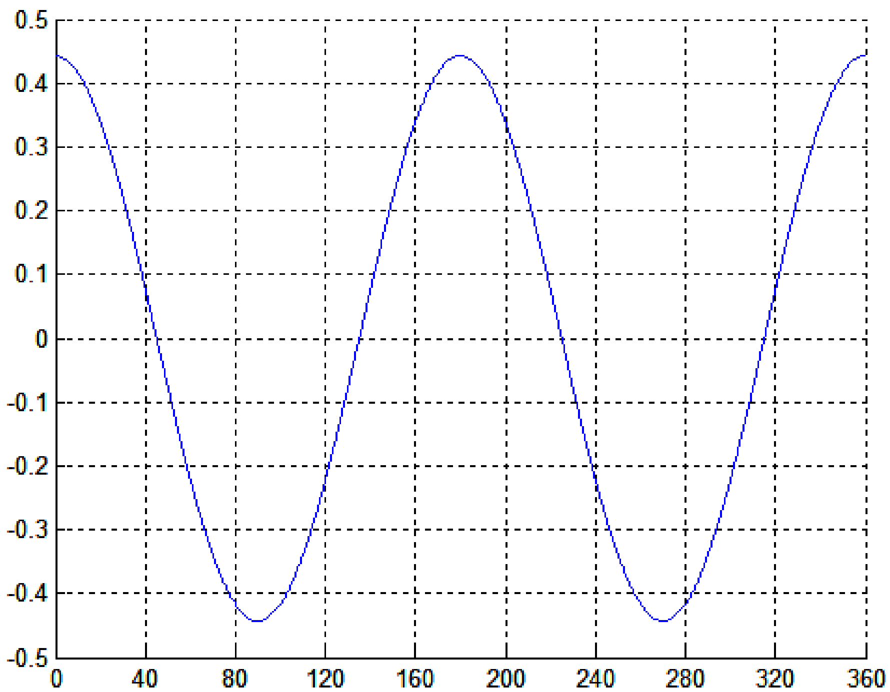


Compensated (Bucked) Coil

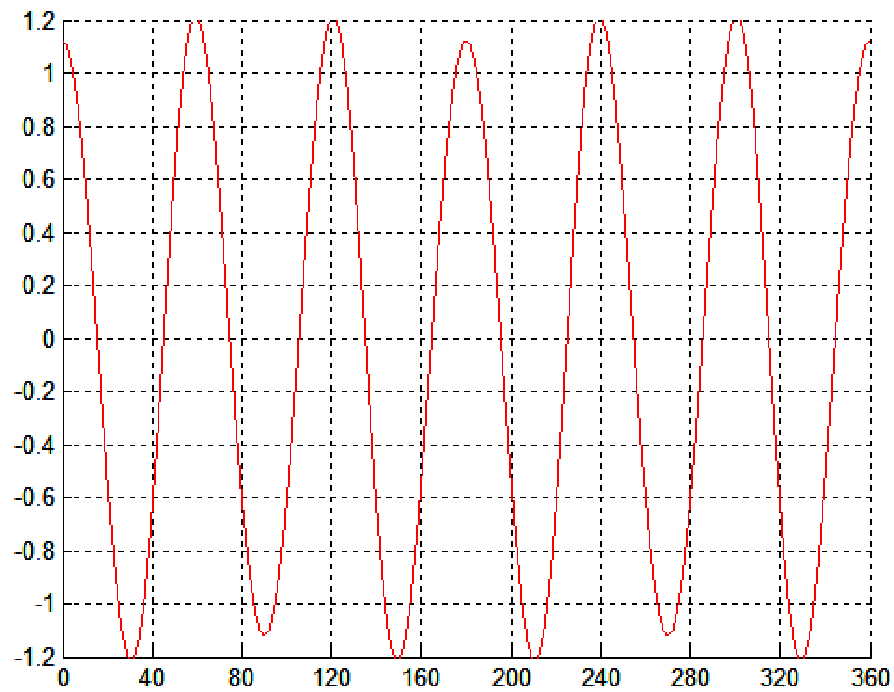
- The multipole errors are usually very small compared to the amplitude of the fundamental field. Typically they are $\leq 10^{-3}$ of the fundamental field at the measurement radius.
  - The accuracy of the measurement of the multipole errors is often limited by the resolution of the voltmeter or the voltage integrator.
- Therefore, a coil system has been devised to null the fundamental field, that is, to measure the error fields in the absence of the large fundamental signal.

Compensated (Bucked) Coil Example

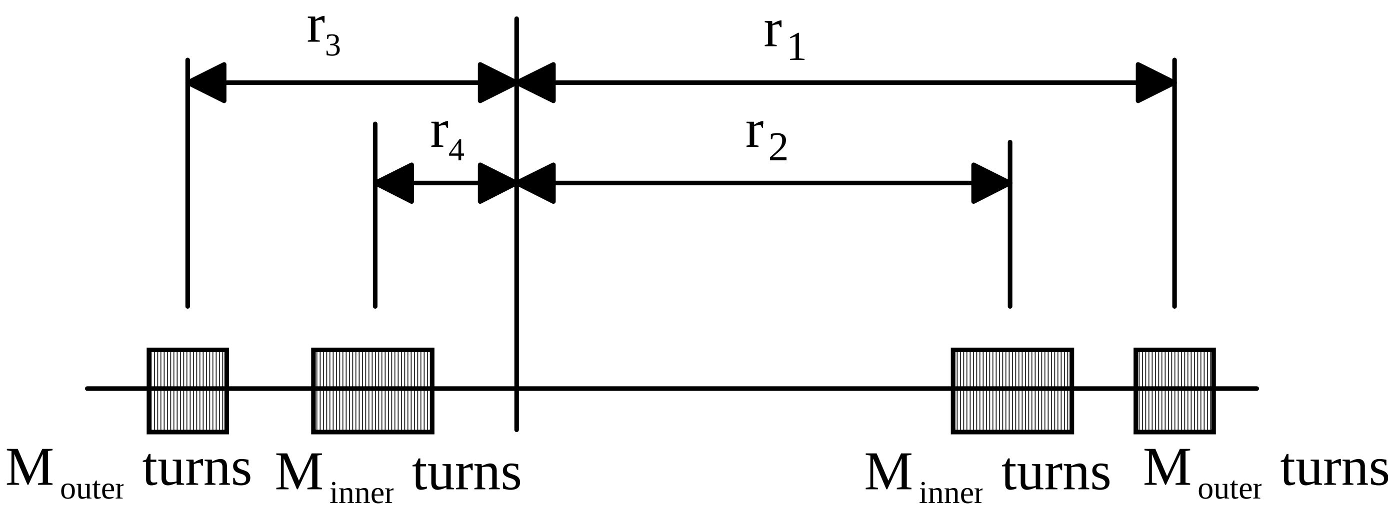
All terms



High Order Harmonics only



Compensated (Bucked) Coil Schematic



Two sets of nested coils with M_{outer} and M_{inner} number of turns to increase the output voltage for the outer and inner coils, respectively, are illustrated.

$$\left(\int V dt\right)_{outer} = L_{eff} M_{outer} \sum_{n} |C_n| \left(r_1^n - r_3^n\right) \cos\left(n\theta + \psi_n\right)$$

$$\left(\int V dt\right)_{inner} = L_{eff} M_{inner} \sum_{n} |C_n| \left(r_2^n - r_4^n\right) \cos\left(n\theta + \psi_n\right)$$

Compensated Connection

The two coils are connected in series opposition.

$$\left(\int V dt\right)_{compensated} = L\sum_n |C_n|\left[M_{outer}\left(r_1^n - r_3^n\right) - M_{inner}\left(r_2^n - r_4^n\right)\right]\cos\left(n\theta + \psi_n\right)$$

Define the following parameters:

$$\beta_1 \equiv \left|\frac{r_3}{r_1}\right| \qquad \beta_2 \equiv \left|\frac{r_4}{r_2}\right| \qquad \rho \equiv \frac{r_2}{r_1} \qquad \text{and} \qquad \mu \equiv \frac{M_{inner}}{M_{outer}}$$

$$\left(\int V dt\right)_{compensated} = L_{eff} M_{outer}\sum_n |C_n| r_1^n \left[\left(1-(-\beta_1)^n\right) - \mu\rho^n\left(1-(-\beta_2)^n\right)\right]\cos\left(n\theta + \psi_n\right)$$

We define the coil *sensitivities*; $\boxed{s_n \equiv \left(1-(-\beta_1)^n\right) - \mu\rho^n\left(1-(-\beta_2)^n\right)}$

then, $\left(\int V dt\right)_{compensated} = LM_{outer}\sum_n |C_n| r_1^n s_n \cos\left(n\theta + \psi_n\right)$

Compensation (Bucking)

The sensitivities for the fundamental (*n=N*) and the multipole *one under* the fundamental (*n=N-1*) are considered.

$$s_N = \left(1-(-\beta_1)^N\right) - \mu\rho^N\left(1-(-\beta_2)^N\right)$$

$$s_{N-1} = \left(1-(-\beta_1)^{N-1}\right) - \mu\rho^N\left(1-(-\beta_2)^{N-1}\right)$$

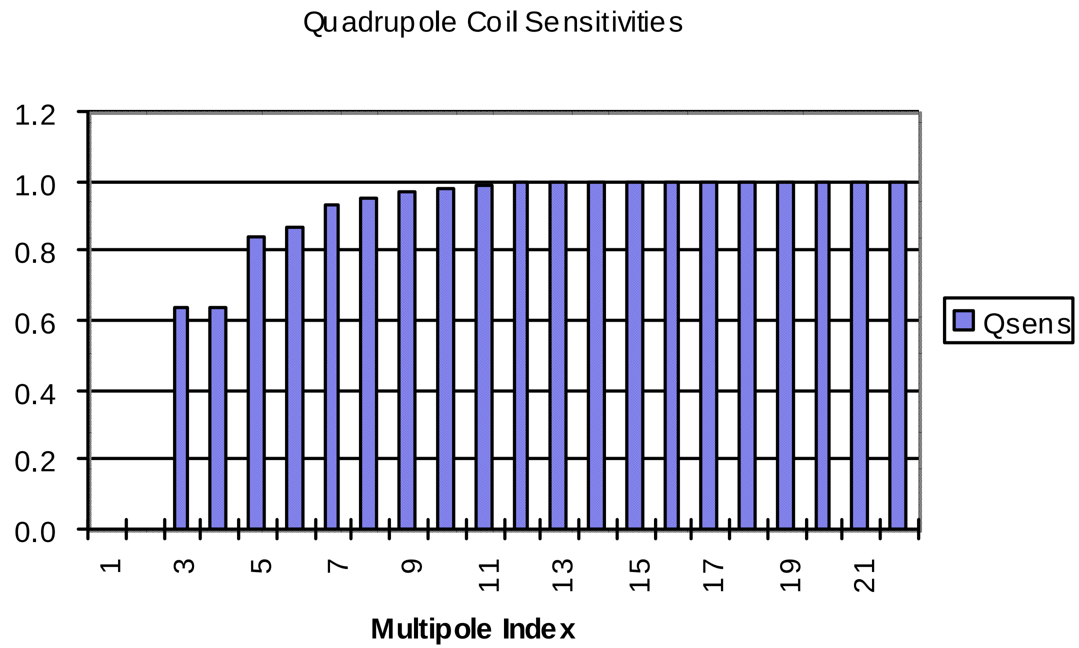
Consider the quadrupole, N=2

$$s_2 = \left(1-(-\beta_1)^2\right) - \mu\rho^2\left(1-(-\beta_2)^2\right)$$

$$s_1 = \left(1-(-\beta_1)\right) - \mu\rho\left(1-(-\beta_2)\right)$$

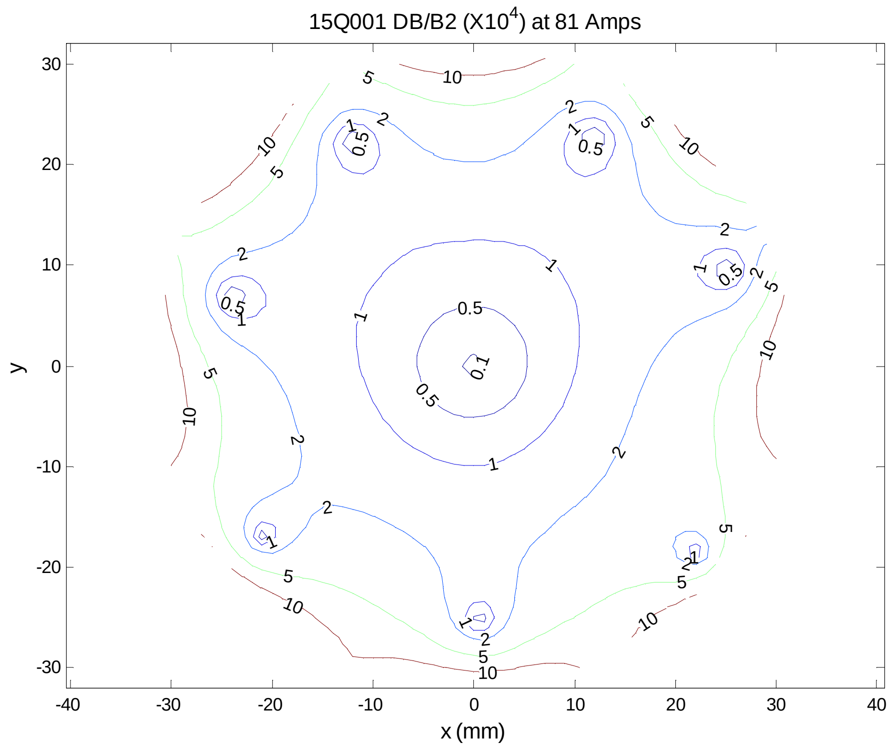
The *classical* geometry which satisfies the conditions for *nulling the N=2 and N=1 field components* in the compensated mode have the following geometry.

$$\beta_1 = 0.5, \quad \beta_2 = .2, \qquad \rho = 0.625, \qquad \mu = 2$$

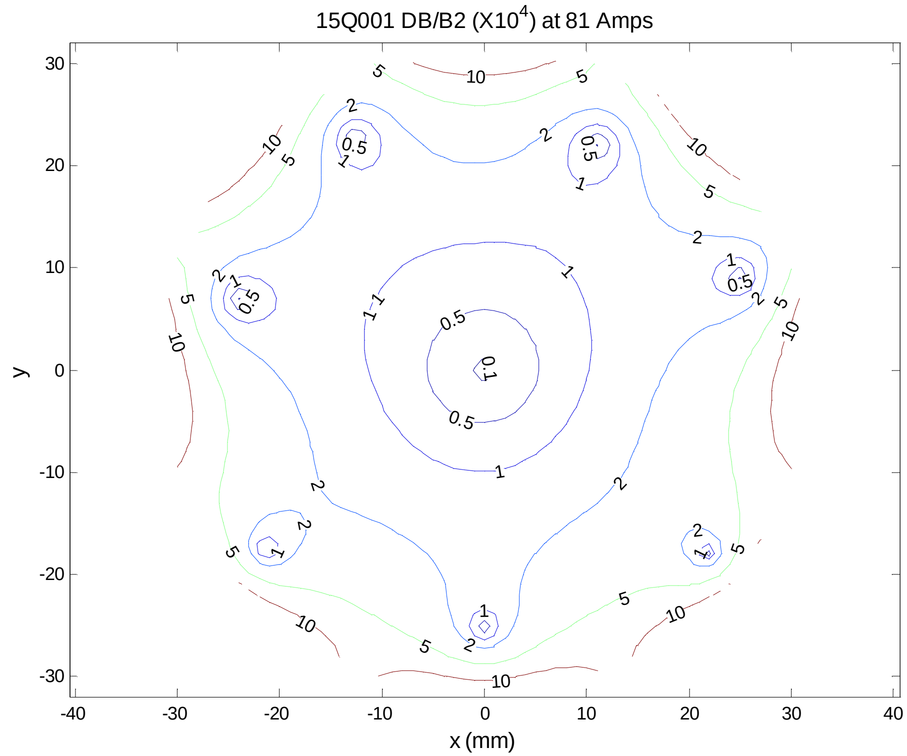


Iso-Errors

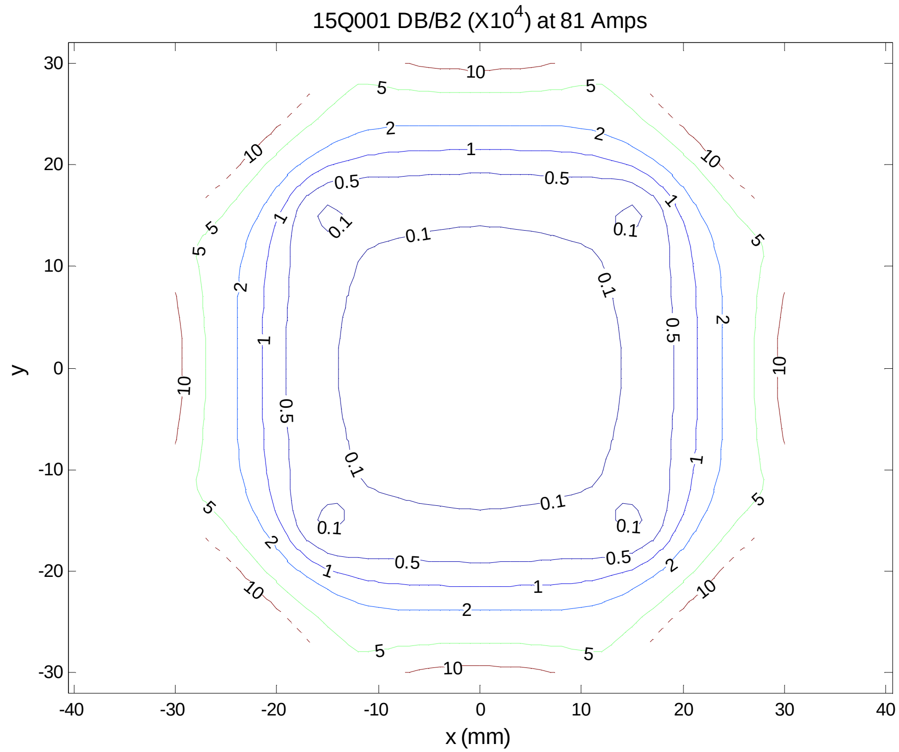
The normalized multipole errors and their phases provide information regarding the Fourier components of the error fields. Often, however, one wants to obtain a *map* of the field error distribution within the required beam aperture. This analog picture of the field distribution can be obtained by constructing an *iso-error map* of the field error distribution. This map can be reconstructed from the normalized error Fourier coefficients and phases.



The computations and contour map are programmed using MatLab.



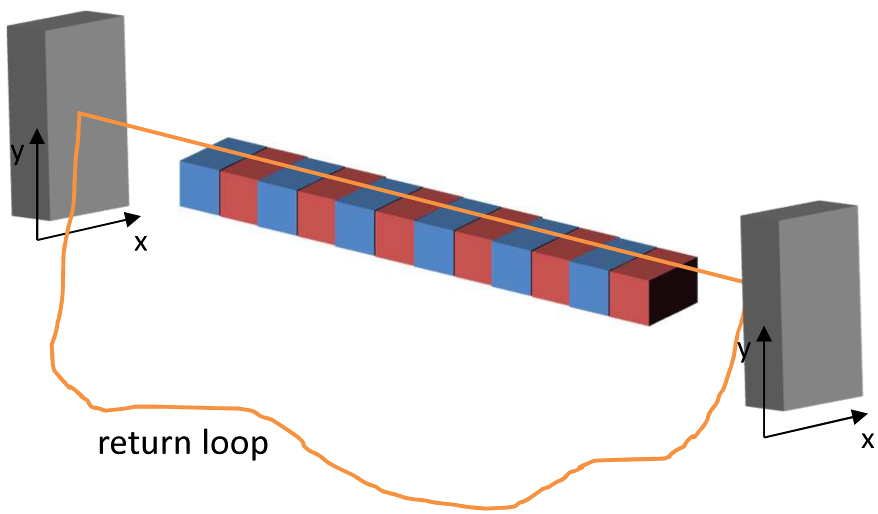
The iso-error plot is replotted for only the allowed multipoles (n=6, 10, 14 and 18) and the first three unallowed multipoles (n=3,4 and 5). It can be seen that it is virtually identical with the previous plot, indicating that the unallowed multipole errors > 6 are not important.



When the iso-error curve is replotted with the unallowed multipole errors reduced to zero and the allowed multipole phases adjusted to eliminate the skew terms, the $\Delta B/B < 10^{-4}$ region is dramatically increased. This illustrates the importance of the first three unallowed multipole errors which are primarily the result of magnet fabrication and assembly errors.

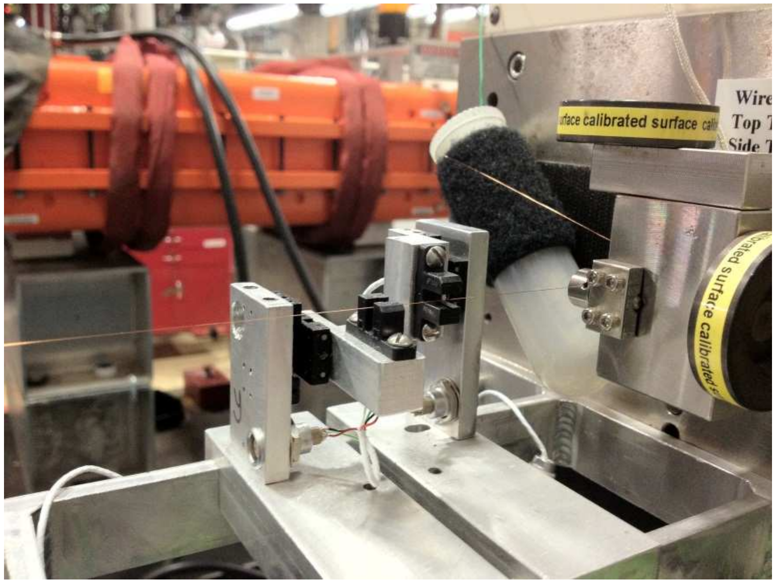
Stretched-wire

It is based on a Cu-Be or Ti-Al wire stretched between two synchronized XZ stages. The movable wire and a fix return part form a loop connected to a low noise voltmeter. This is moved at a constant speed (10 to 40 mm/s) along various paths built up from successive straight and circular segments while triggering the voltmeter at constant path length intervals. The circular motion can be used to measure harmonics.



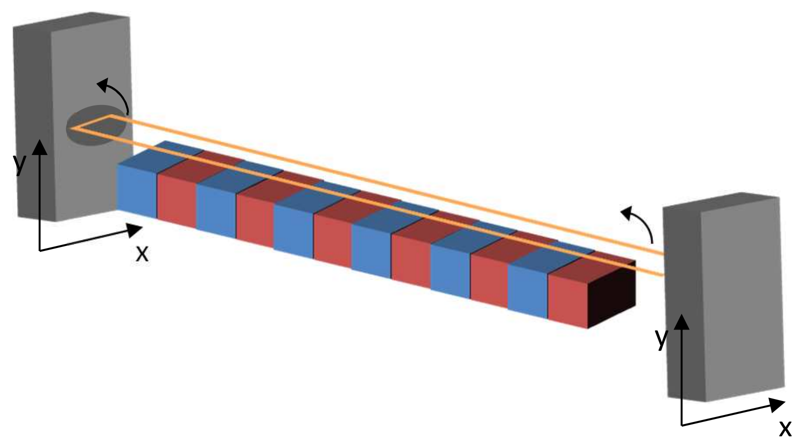
* ESRF webpage

Stretched-wire



Flipping coil

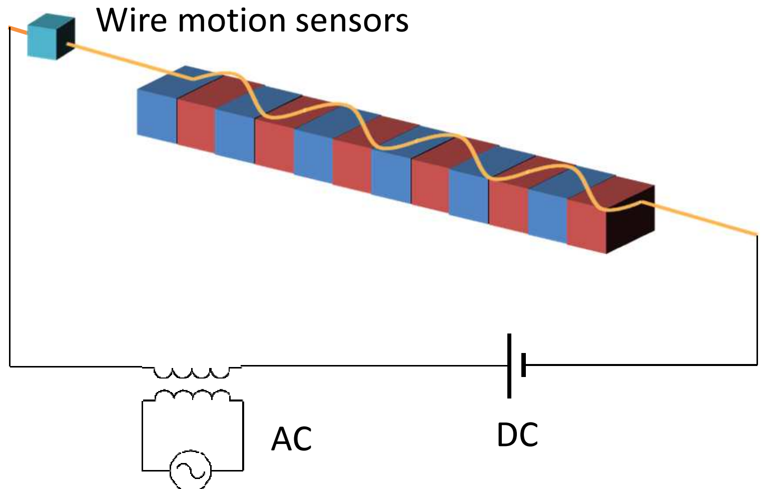
This system is composed of two X-Y-Z stages with an installed rotor to flip the long loop coil. One pair of the -axis is employed to stretch the coil and thus prevent coil sagging. The other two pairs of the and -axes are employed to position and move the coil on the transverse plane of the horizontal and vertical axes.



C. S. Hwang, “Integral Magnetic Field Measurement USing an Automatic Fast Long-Loop-Flip Coil System”

Vibrating wire

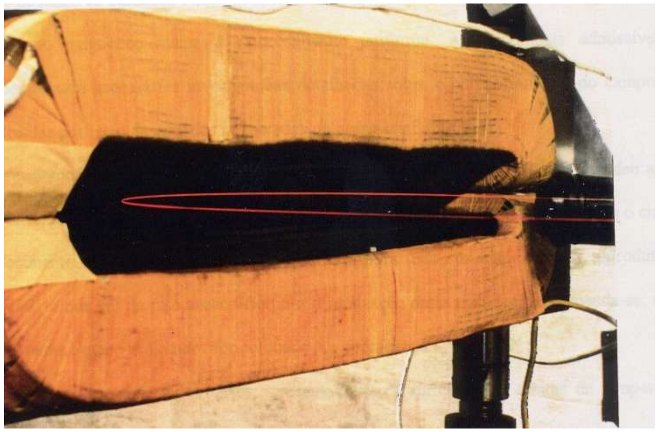
The vibrating wire technique uses a section of wire stretched through the testing region as a magnetic field probe. Lorentz forces between DC current flowing through the wire and the wiggler magnetic field cause the wire wiggling resembling the beam trajectory. Applying AC current with frequencies matching vibrating mode resonances and measuring amplitude and phase of the excited standing waves, one can obtain magnetic field characteristics along the wire, i.e., along the path reproducing the beam trajectory.

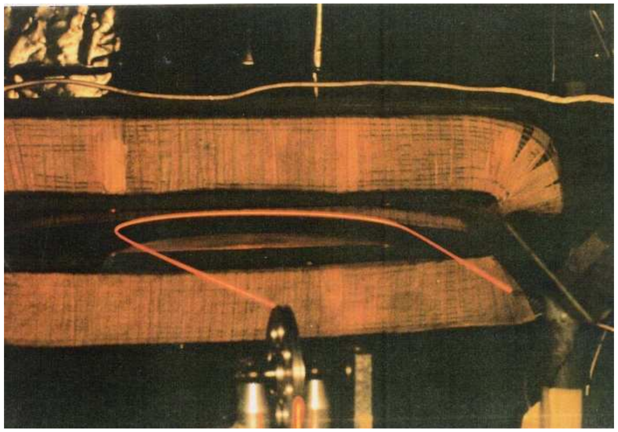


* A. Temnykh, "VIBRATING WIRE AND FLIPPING COIL MAGNETIC MEASUREMENT OF A CESR-C 7-POLE WIGGLER MAGNET"

Floating wire

Floating wire is a old technique that uses a incandescent **Ni** wire to visualize the path of a electron beam. The technique is based on the equivalence of the equations governing the position of equilibrium of a flexible wire carrying an electric current in a magnetic field, and the trajectory of a charged particle passing through the same magnetic field.





*

*Luciana Reyes Pires Kassab – PhD Thesis

U. Vogel , "Floating Wire Technique for Testing Magnetic Lenses"

Summary

- Magnetic measurements is a specialized area.
- Few (if any) of you will be involved intimately in this area and will need to understand the concepts in detail.
- However, the field is important since the quality of the magnets manufactured using the design principles covered in this course cannot be evaluated without a good magnetic measurements infrastructure.
- Few institutions maintains this infrastructure and often has to resurrect this capability whenever the needs arise.

Next…

- Simulations
- Magnet fabrication