U.S. Particle Accelerator School

Education in Beam Physics and Accelerator Technology

Elettra Sincrotrone Trieste

RF Technology

S. Di Mitri (90min.)

Review of Relativstic Formulas

$$\vec{\beta} = \frac{\vec{v}}{c}, \quad c = 2.998 \times 10^8 \, m/s$$

$$\gamma = \frac{1}{\sqrt{1 - \left|\vec{\beta}\right|^2}}$$

$$\left.\right\} \quad \left|\vec{\beta}\right| \approx 1 - \frac{1}{2\gamma^2} \quad \text{for } \gamma \gg 1$$

$$\vec{p} = \vec{\beta}\gamma m_e c$$

$$T = \frac{\left|\vec{p}\right|^2}{2m_e}$$

$$E = T + m_e c^2$$

$$\left.\right\} \quad \gamma = \frac{E}{m_e c^2} = 1 + \frac{T}{m_e c^2}$$

$$x' = \frac{p_x}{p_z} = \frac{\left|\vec{p}\right|}{p_z} \frac{x'}{\sqrt{x'^2 + y'^2 + 1}}$$

$$y' = \frac{p_y}{p_z} = \frac{\left|\vec{p}\right|}{p_z} \frac{y'}{\sqrt{x'^2 + y'^2 + 1}}$$

$$p_z = \frac{\left|\vec{p}\right|}{\sqrt{x'^2 + y'^2 + 1}}$$

Typically
$p_z \approx (10^3\text{-}10^5)\, p_x, p_y$

RF Acceleration

D. Nguyen, USPAS 2014

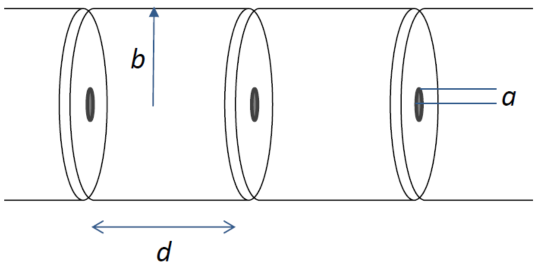
$$\vec{F} = \frac{d\vec{p}}{dt} = q\left(\vec{E} + v \times \vec{B}\right) \qquad \Delta T = \int \vec{F} d\vec{s} = -\int_0^L eE_z dz$$

To accelerate charged particles, the RF wave must have electric fields along the direction of propagation of the particle and the wave itself. However, EM waves in free space only have electric field that is transverse to direction of propagation.

To get non-zero acceleration of charged particles co-propagating with the electromagnetic wave, we have to do the following:

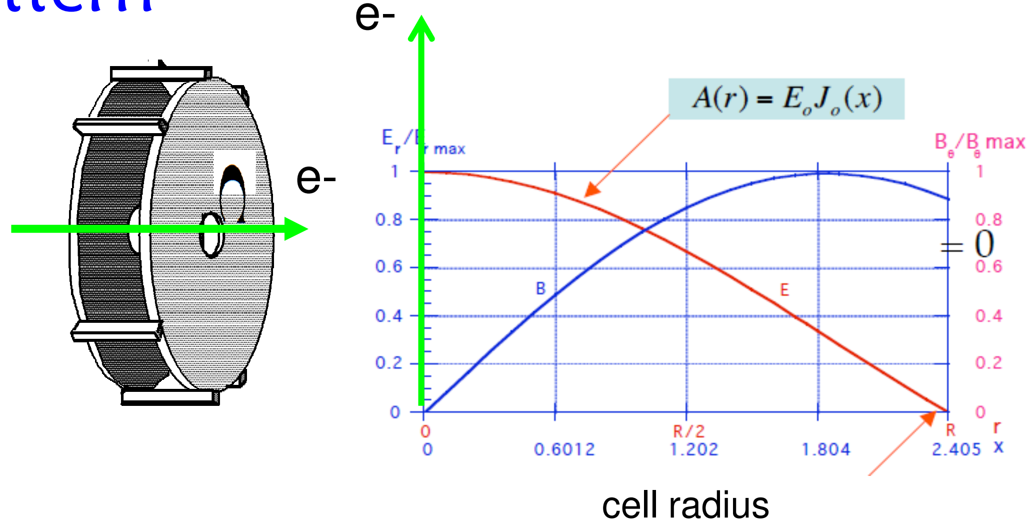
1. Use a resonant cavity that has transverse magnetic (TM) modes. The TM_{010} mode has axial electric field to accelerate particles along the axial direction.

2. Load the cavity with disk-and-washers to slow the phase velocity of the RF wave to the speed of light *c*, so that a charged particle traveling at speed slightly less than *c* will have non-zero acceleration accumulated over many RF cycles.



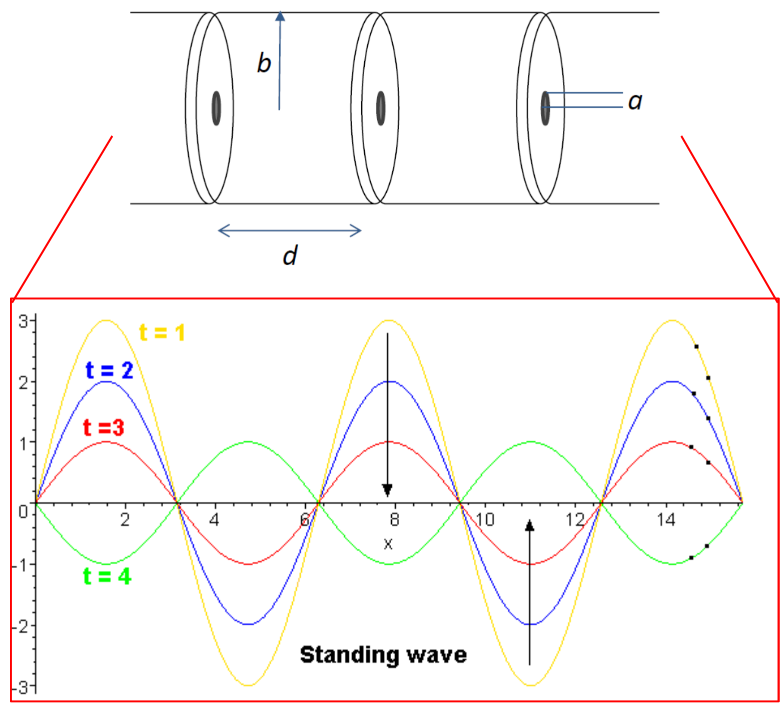
Standing Wave, Field Pattern

- Static picture of field pattern in a **cell**, as function of radius («pill-box» model):

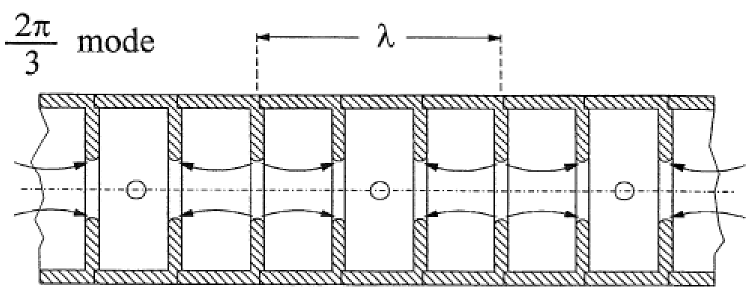


- Dynamical picture of the on-axis accelerating mode **E_z along the structure.** It is factorized in a temporal and spatial dependence. For the **reference particle at <z>=0:**

$$\boxed{E_z \underset{\substack{r=0\\n=0}}{\cong} \hat{E}_z \cos(\omega_{RF}t + \phi_0)\cos(k_0 z) = \\ = \hat{E}_z \cos(\omega_{RF}t + \phi_0) \equiv \hat{E}_z \cos(\phi_{RF})}$$



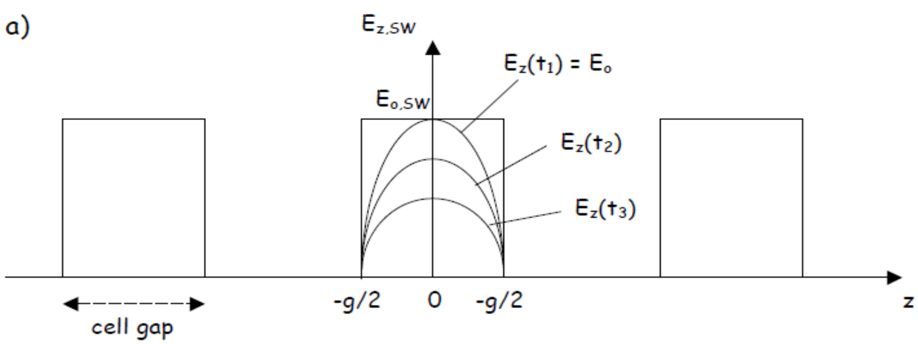
Standing Wave, Single Kick Model



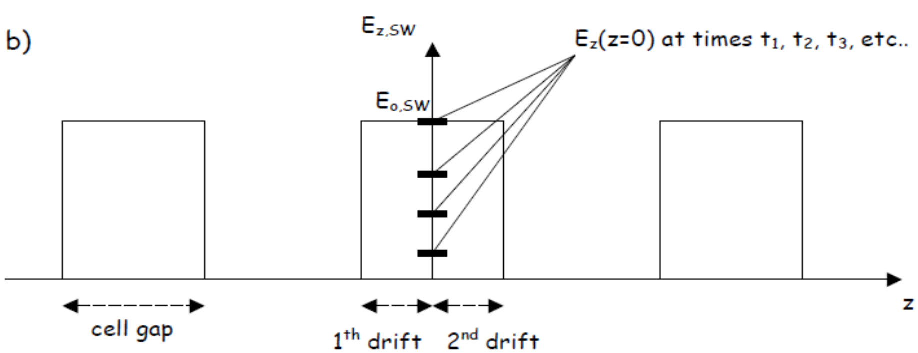
- ❑ E_z is "resonating" with multiple **reflections** back and forth. **Loss-free propagation** can only occur if the field wavelength is an integer multiple of the iris separation (constructive interference between reflections):

$$\boxed{\lambda_z = pd, \quad p = 1,2,3,...; \ k_z d = \frac{2\pi}{p}}$$

a)



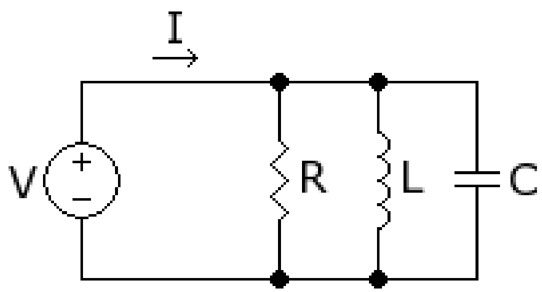
b)



- ❑ **Energy gain** in the approximation of **single kick**:

$$\Delta\gamma = -\int_0^{N_d} \frac{e}{m_e c^2} E_z dz = -\frac{e}{m_e c^2} \hat{E}_z N_d d \cos\phi_{RF} =$$
$$\equiv \frac{e}{m_e c^2} V_0 \cos(\phi_{RF})$$

Resonant Cavity, RLC Circuit Model



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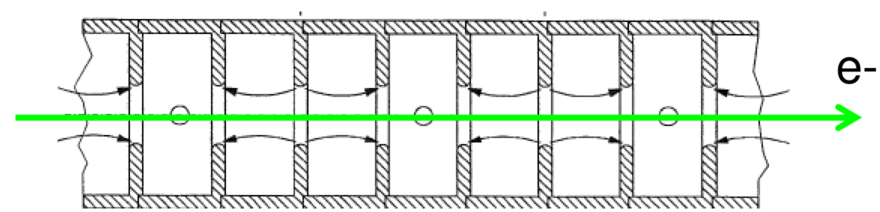


$$U = \frac{1}{2} C V^2 \qquad P_d = \frac{V^2}{R}$$

$$Q = \frac{\omega_{RF} U}{P_d} = \frac{\text{Energy stored in the cavity}}{\text{Energy dissipated in one RF period}}$$

$$V = \sqrt{P_d R_s} \equiv \sqrt{P_d r_s L}$$ **Accelerating voltage**

$$E_z^2 = \left(-\frac{dV}{dz}\right)^2 \cong \left(\frac{V}{L}\right)^2 = \frac{P_d r_s}{L} \cong r_s \frac{dP_d}{dz}$$ **Accelerating field**

$$\frac{r_s}{Q} = \frac{E_z^2}{\frac{dP_d}{dz}} \frac{P_d}{\omega_{RF} U} = \frac{E_z^2}{\frac{dP_d}{dz}} \frac{\frac{dP_d}{dz}}{\omega_{RF} \frac{dU}{dz}} = \frac{E_z^2}{\omega_{RF} u}$$

This ratio depends on the cavity ***geometry only***.

- **High Q, Quality Factor ~ high efficiency for energy storing**
- **High r_s, Shunt Impedance per unit length ~ high accelerating field vs.dissipated power**

Continuity Equation

$$\boxed{\frac{\partial u}{\partial t}+\frac{\partial P}{\partial z}+\frac{dP_d}{dz}+nevE_z=0}$$

Time-variation of the energy stored per unit length

Energy flux per unit length

Dissipated power per unit length

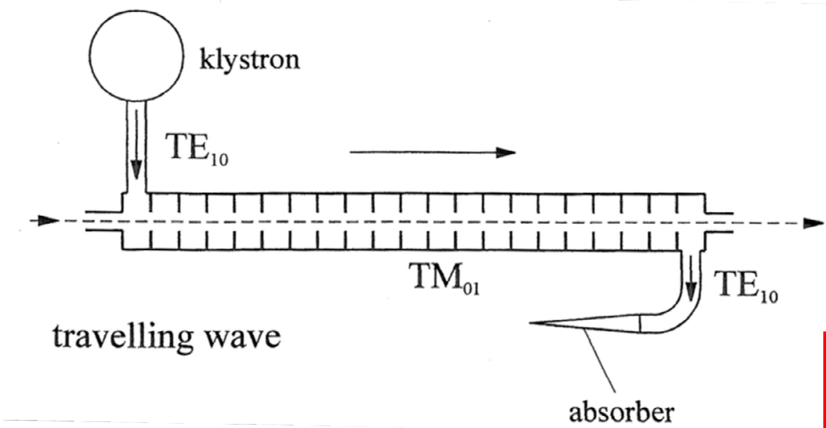
Energy absorbed by the beam, per unit length ("beam loading")

- **In a SW**, we assume dP/dz=0 and neglect nevE_z. Thus we find:

$$\frac{\partial u}{\partial t}=-\frac{dP_d}{dz}; \quad \Rightarrow \quad \frac{dU}{dt}=-P_d=-\frac{\omega_{RF}}{Q}U; \quad \Rightarrow \quad \boxed{U=U_0 e^{-\frac{\omega_{RF}}{Q}t}}$$

The energy stored in the structure decays exponentially with characteristic time Q/ω_{RF}.

Traveling-Wave Structure



In a traveling wave (TW) structure, the accelerating wave is **co-propagating with the particles**. Still parasitic power reflections have to avoided with "resonant" operating modes as in a SW.

$$\boxed{E_z \underset{\substack{r=0\\n=0}}{\cong} \hat{E}_z \cos(\omega_{RF} t - k_z z + \phi_0) \equiv \hat{E}_z(z)\cos(\phi_{RF})}$$

- To keep particles and accelerating wavefront synchronous (***phase-matched***) all along the structure, we must have $v_p \cong \beta c$. This is ensured by the irises that "slow down" v_p acting as a capacitive load.

- All previous formulas for SW are now valid with the prescription: **P_d (dissipated) → P (propagating)**.

- Since P is now propagting along the structure, the **filling time** & **group velocity** are defined:

$$\boxed{v_g := \frac{P}{dU/dz}} \qquad \boxed{t_f := \int_o^L \frac{dz}{v_g(z)}}$$

Constant Impedance (TW–CI)

$$\boxed{\frac{\partial u}{\partial t}+\frac{\partial P}{\partial z}+\frac{dP_d}{dz}+nevE_z=0}$$

- In a TW, we assume Pd=0 and nevE_z=0. Thus we find:

$$\frac{\partial u}{\partial t}=-\frac{dP}{dz}; \quad \Rightarrow \quad P=-uv_g=-\frac{Qv_g}{\omega_{RF}}\frac{dP}{dz}; \quad \Rightarrow \quad \frac{dP}{P}=-\frac{\omega_{RF}}{Qv_g}dz$$

$$P(z):=P_0e^{-2\tau\frac{z}{L}},$$

$$\tau:=\frac{\omega_{RF}L}{2Qv_g}$$

Attenuation factor

- From the r_s/Q ratio:

$$\frac{r_s}{Q}=\frac{E_z^2}{\omega_{RF}u} \quad \Rightarrow E_z^2=\frac{r_s}{Q}\omega_{RF}u=const.\times u(z)$$

- From the **accelerating field**:

$$E_z^2=-r_s\frac{dP}{dz} \quad \boxed{\Rightarrow E_z(z)=\sqrt{2\tau\frac{P_0r_s}{L}e^{-\tau\frac{z}{L}}}}$$

$$t_f:=\int_o^L\frac{dz}{v_g(z)}=\frac{L}{v_g}=2\tau\frac{Q}{\omega_{RF}}$$

N.B.: we are using the fact that v_g is ***constant*** all along the structure.

Constant Gradient (TW–CG)

$$\boxed{\frac{\partial u}{\partial t} + \frac{\partial P}{\partial z} + \frac{dP_d}{dz} + nevE_z = 0}$$

- In a TW, we assume Pd=0 and nevE_z=0. Thus we find:

$$\frac{\partial u}{\partial t} = -\frac{dP}{dz} = -\frac{E_z^2}{r_s} \equiv const.; \quad \Rightarrow \quad P(z) = P_0 - \frac{E_z^2}{r_s} z = P_0 + \frac{P(L) - P_0}{L} z;$$

- From the r_s/Q ratio:

$$\frac{r_s}{\pi Q} = \frac{E_z^2}{\omega_{RF} u} \quad \Rightarrow u = \frac{E_z^2 Q}{r_s \omega_{RF}} = const.$$

$$P(L) := P_0 e^{-2\tau} \qquad v_g = \frac{P}{u} \propto z$$

$$t_f := \int_o^L \frac{dz}{v_g(z)} = \ldots \cong 2\tau \frac{Q}{\omega_{RF}}$$

- From the accelerating field:

$$\boxed{E_z(z) = -r_s \frac{dP}{dz} = \sqrt{\left(1 - e^{-2\tau}\right) \frac{P_0 r_s}{L}}}$$

N.B.: τ is well-defined when $v_g(L)$ is known.

Summary

- **SWs** have practically a higher acceleration efficiency, but longer filling time time than **TWs**.

- **TW-CI** is simpler to build (all cells are identical). It usually provides a larger energy gain than a TW-CG, but with a higher peak field. This may induce discharges in the structure.

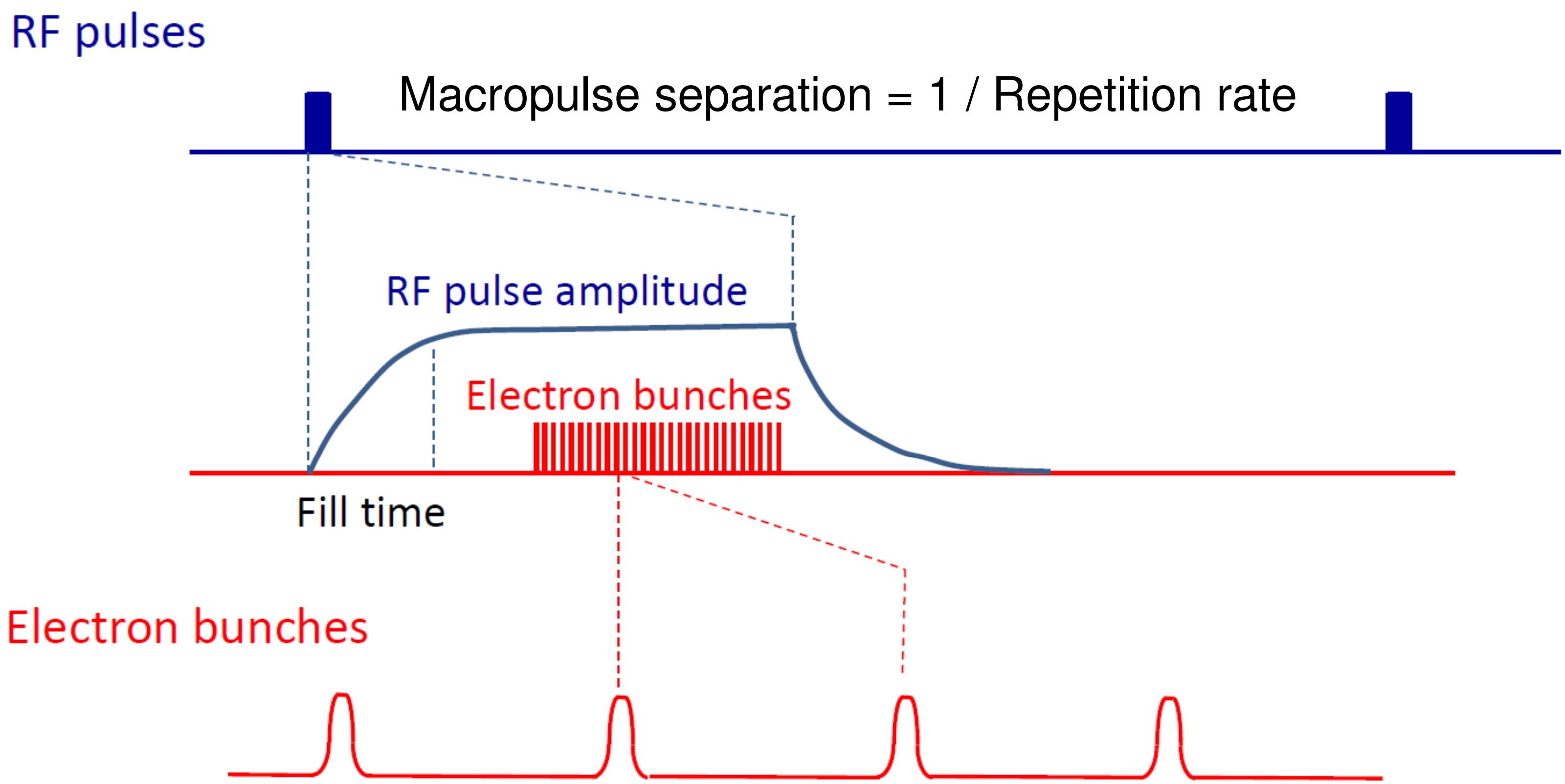
- **TW-CG** has cells with varying iris radius. It can provide the same energy gain than in a TW-CI but with a lower peak field. Commonly used in recent times.

- *N.B.:* the e.m. wave in a SW can be shown to be the superposition of 2 identical e.m. waves propagating thorugh the structure in opposite directions. This implies that, for the same structure's length L:

$$R_{s,SW} = \frac{E_0^2 L}{\frac{dP_d}{dz}} = \frac{E_0^2 L}{2\frac{dP}{dz}} = \frac{R_{s,TW}}{2}$$

Micro- and Macro-Pulses

D. Nguyen,
USPAS 2014

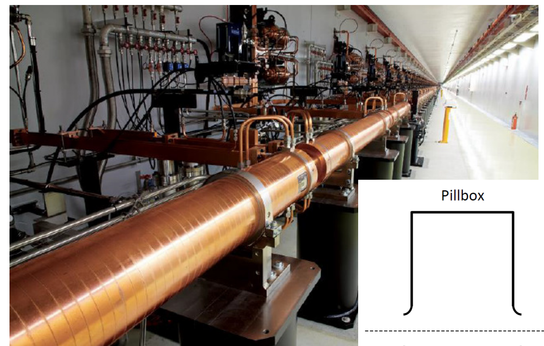


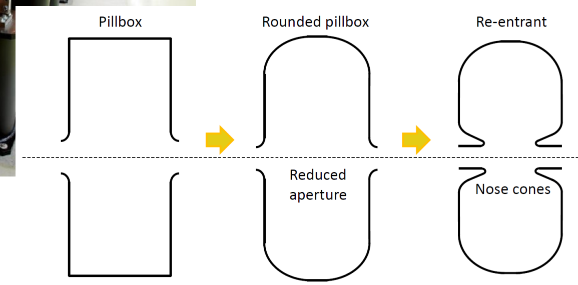
Bunch separation = n / f
n is the number of RF buckets between electron bunches

Normal- vs. Super-Conducting Tech

"**Normal-Conducting**" (NC): warm linacs, the thermal load threshold usually limits the maximum rep. rate.

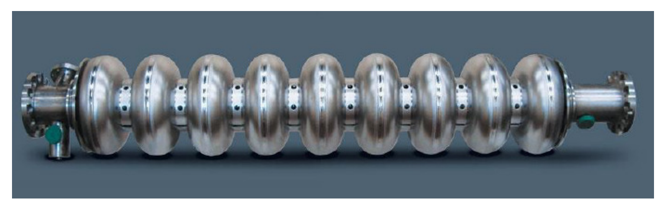
For any target E_z, the power consumption is minimized by a high r_s. This is maximized through inner cells' shaping. Example: r_s~mΩ, Q~10^4.





"**Super-Conducting**" (SC): He-liquid frozen structures that allow higher rep. rates than NC linacs.

The heat load is minimized by a large Q. This is made large with cell's surface treatment. Example: r_s~nΩ, Q~10^{10}.



RF Frequency

Ansatz: keep $L = V_0/E_z$ fixed, and scale all other dimensions as f^{-1}:

$$r_s = \frac{\hat{E}_z^2 L}{P_D} \propto \begin{cases} f^{1/2} & for\ NC \\ f^{-1} & for\ SC \end{cases}$$

$$Q = \frac{\omega_{RF} U}{\pi P_D} \propto \begin{cases} f^{-1/2} & for\ NC \\ f^{-2} & for\ SC \end{cases}$$

$$\boxed{\frac{r_s}{Q} \propto f}$$

- NC linacs maximize E_z through a high r_s, thus perform better at high f, but at the expense of small cells' iris.
- Single bunch interactions with walls (short-range wakefields) become strong. They impose attention to the beam dynamics.

- SC linacs minimize the heat load with a large Q, thus perform better at low f, but at the expense of higher order field modes.
- Inter-bunch interactions with walls (long-range wakefields) become strong. They can be suppressed with mode dampers.

Sort of available f (GHz): L-band (1.3), S-band (2.9), C-band (5.7) and X-band (11.4)

Exercise, NC TW

Parameter	S-band, NC	X-band, NC
Working frequency, f [GHz]	2.998	11.424
Type	TW, Constant impedance	TW, Constant gradient
Phase Advance per Cell	$2\pi/3$	$2\pi/3$
Length, L [m]	3	1
Average Iris Radius, *a* [mm]	10.0	3.5
Group Velocity, v_g [c]	0.01	0.01 (average)
Filling Time [μs]	?	?
Quality Factor, Q	12000	8600
Attenuation Factor, τ [neper]	?	?
Shunt Impedance, r_s [MΩ/m]	55	80
Input Peak Power, P_{RF} [MW]	45	80
RF Pulse Duration [μs]	2.0	1.5
Acc. Gradient, eV_0/L [MV/m]	?	?
Repetition Rate [Hz]	50	50

Design and parameters optimization really depends on what you take care more of….

Pulse Format (examples)

Normal-conducting RF Linac

Traveling-wave linac

Example: S-band SLAC, C-band SACLA

Water-cooled copper
Accelerating gradient ~ 30 MV/m
Maximum RF pulse ~ 3 μs
Fill time < 1 μs

Standing-wave linac

Example: L-band ISIR (Osaka)

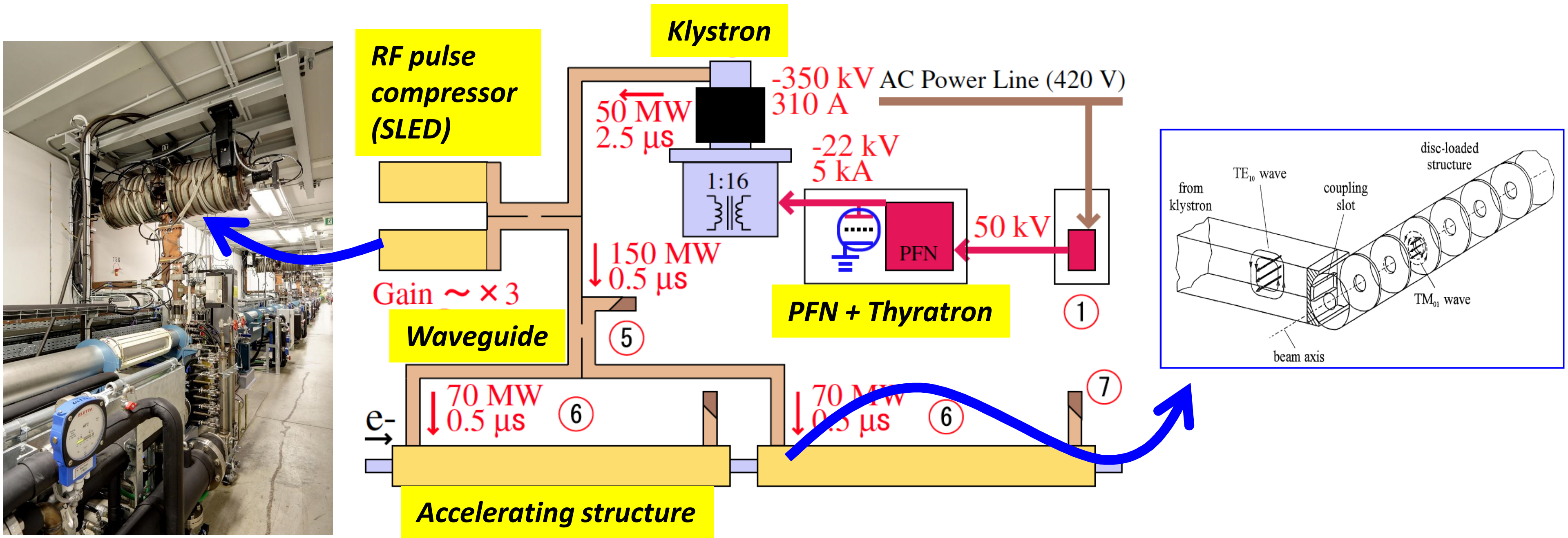
Water-cooled copper
Accelerating gradient ~ 20 MV/m
Maximum RF pulse ~ 30 μs
Fill time ~ 2 μs

Super-conducting RF Linac

Example: L-band TESLA (DESY)

Liquid He cooled niobium
Accelerating gradient ~ 15 MV/m
Long pulse to continuous-wave (CW)
Fill time ~ 500 μs

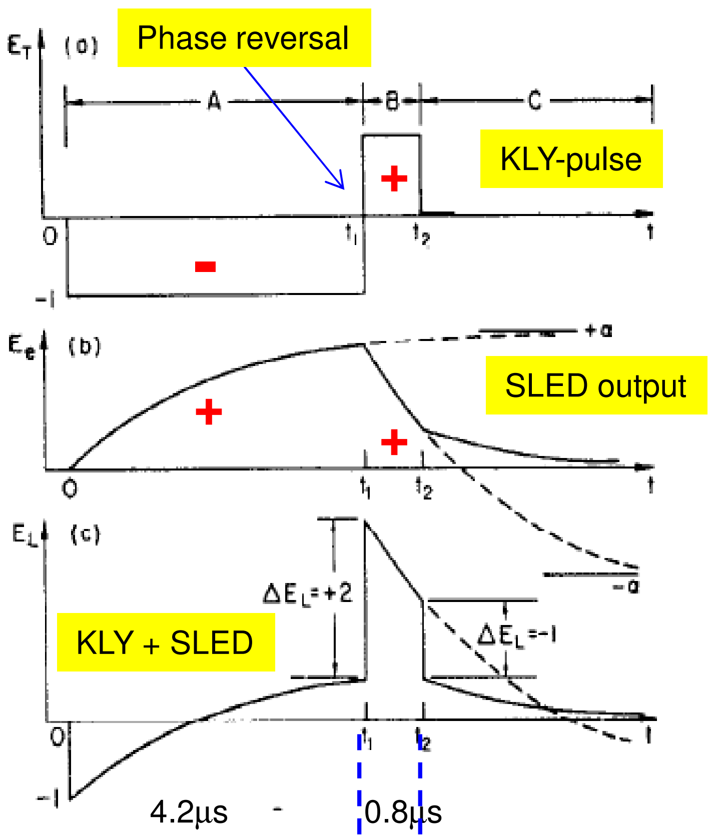
RF Generation

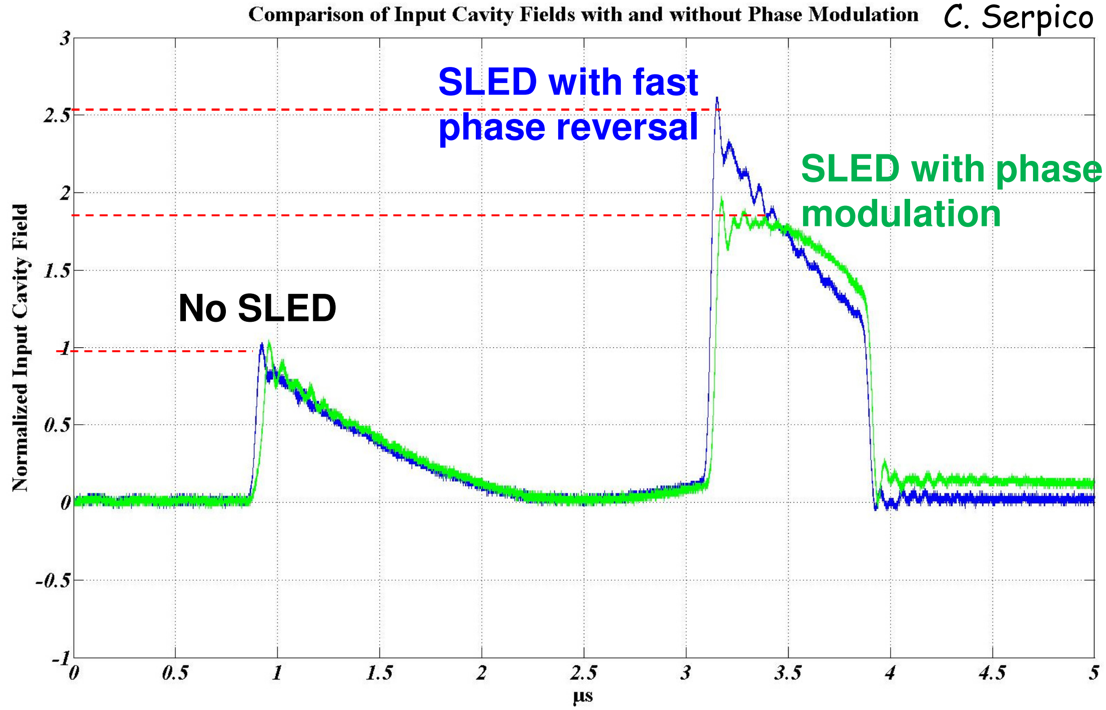


- Typical klysrtron power is **30 – 70 MW, over a few μs**.
- **SLED** cavities (see next slide) allow to gain a **factor of 3–4 in peak power**, reducing the RF pulse length to sub-μs duration.
- If the total power is enough to reach the maximum peak field tolerable by the cavity (before inducing discharges in the structure due to surface emission), the waveguide can be split into two lines to **supply multiple structures**.
- The final **RF pulse duration** has to be **longer** than the structure **filling time**.

SLED («Slac Energy Doubler via RF Pulse Compression»)

- The SLED is a high Q-cavity which stores klystron energy during a large fraction of each RF pulse, and then discharges it rapidly into the accelerator. So, the output RF pulse is shortened, the peak power and the accelerating voltage both increase.

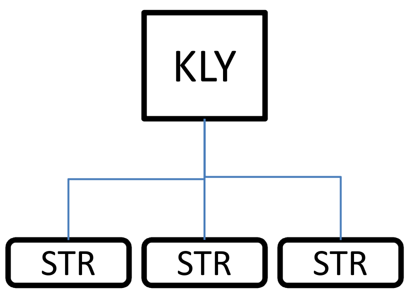




Pictures by C. Serpico

RF Distribution and Linac Layout

1. Target: final <E>
2. Goals: minimize L, minimize P_{RF}, operate at high rep.rate
3. Means: high E_z, high r_s, high Q.
4. Trade-off: RF power (cost of RF suppliers and electricity) vs. space (cost of accelerating structures and linac building).
5. One possible strategy:
   - fix the structure's geometry (f, L, r_s);
   - balance the P_{RF} per structure with the number of klystrons



acc. gradient

$$\boxed{E_f \cong NGL = eNV_0 = eN\sqrt{P_{RF} r_s L}}$$

\# of structures

structure's length

$M \equiv$ number of structures per klystron

Trends…

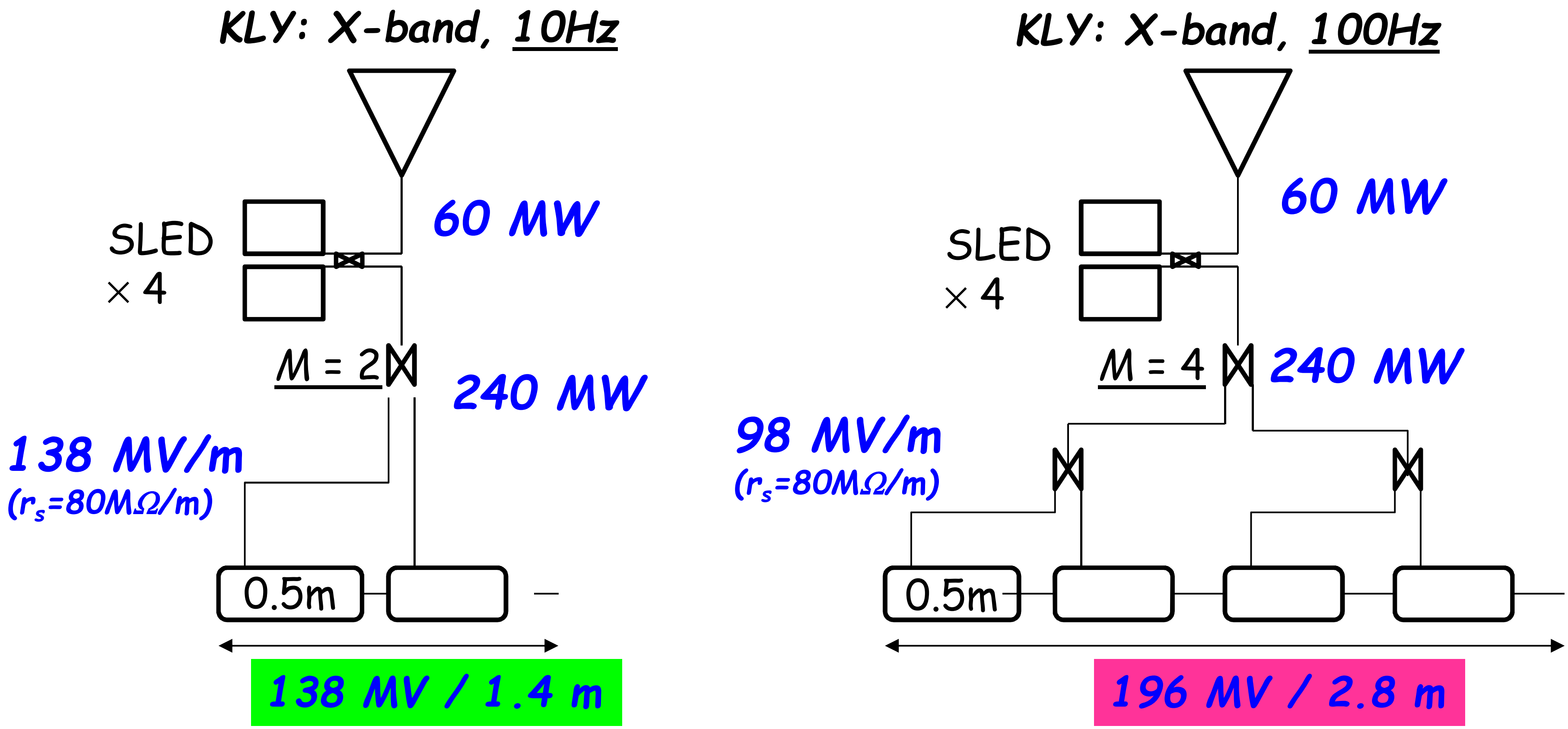
- **Small *M*** (<2:1) implies high P_{RF} that is:
  - high V_0, small N
  - a *compact* linac but *many klystrons*

- **Large *M*** (>4:1) implies low P_{RF} that is:
  - low V_0, large N
  - *few klystrons* but a *long linac*

- This choice is limited by the **structure breakdown limit** at high accelerating gradients.
- Many klystrons and modulators may imply a tighter packing of RF components, thus more space needed in the "klystron gallery".
- Well suited for **compact, NC** linacs at **low repetition rate**.

- This choice is limited by the **available space** for the linac.
- Many structures may imply a more complex cooling system.
- Well suited for **SC** linacs or **NC** linacs at **high repetition rate**.

Examples



Assume E_0 = 100MeV, E_f = 3GeV, and a periodic linac layout.

M=2: (3000–100)MeV/138MeV = *21 KLYs*, *21 girders* and a **29 m** long linac *@10Hz*

M=4: (3000–100)MeV/196MeV = *15 KLYs*, *30 girders and a* **42 m** long linac *@100Hz*

Estimate of Power Consumption

The *average* power sent to the structure is $\langle P_{RF,0} \rangle = P_{RF,0} \cdot \Delta t_{RF} \cdot rep.rate$

Case M = 2: $\langle P_{RF,0} \rangle$ = 120MW · 0.7μs · 10Hz = <u>0.84 kW / structure</u>
Case M = 4: $\langle P_{RF,0} \rangle$ = 60MW · 0.7μs · 100Hz = <u>4.20 kW / structure</u>
We do not expect strong cooling requirements.

For such a structure, the attenuation factor is $\tau \approx 1.5$ neper. Being TW-CG, we evaluate that ($1 - e^{-3}$) = 5% of input power goes to the load, while <u>95% of that is used for acceleration</u>. Thus, we have designed an efficient RF acceleration system.

i. Klystron power = 60MW · 1.4μs · 100Hz / 0.8 (efficiency) = 10.5 kW
ii. Modulator power = 10.5kW / 0.9 (efficiency) = 11.7 KW
iii. Using 1 klystron per 4 structures (M = 4), we require 11.7KW · 15 RF stations = <u>0.18 MW</u>
iv. We should add about 0.1 MW for the injector, 0.1 MW for the water cooling, 0.1 MW for the magnets and 0.1 MW for basic building power. In total we require <u>0.6 MW for a system this size</u>.

Modeling in Tracking Codes ($\beta \rightarrow 1$)

In many tracking codes for ultra-relativistic particles, the following approximations are routinely adopted both for SW and TW structures:

1) cylindrically symmetric, spatially periodic RF structure;
2) the accelerating field radial dependence is ignored (particles are assumed to be traveling close to the electric axis: $E_z(r)=E_z(0)$);
3) higher harmonics of E_z are neglected (negligible acceleration respect to the fundamental mode $n=0$);
4) the filling time is short enough respect to the RF pulse to ensure steady-state acceleration;
5) the transient time factor = 1;
6) the TW structure is a "constant-gradient" with attenuation factor = 0;
7) for any cell of finite length, acceleration is provided in the "drift-kick-drift" scheme (single energy kick in the cell's center):

In this simplified picture, the **total energy gain** per structure is just:

$$\boxed{\Delta\gamma(L) = \frac{|e|}{m_e c^2} V_0 \cos(\phi_{RF})}$$