Basics on FEL physics; undulators; high-level machine-design parameters

MV

last revised 21-June-2015

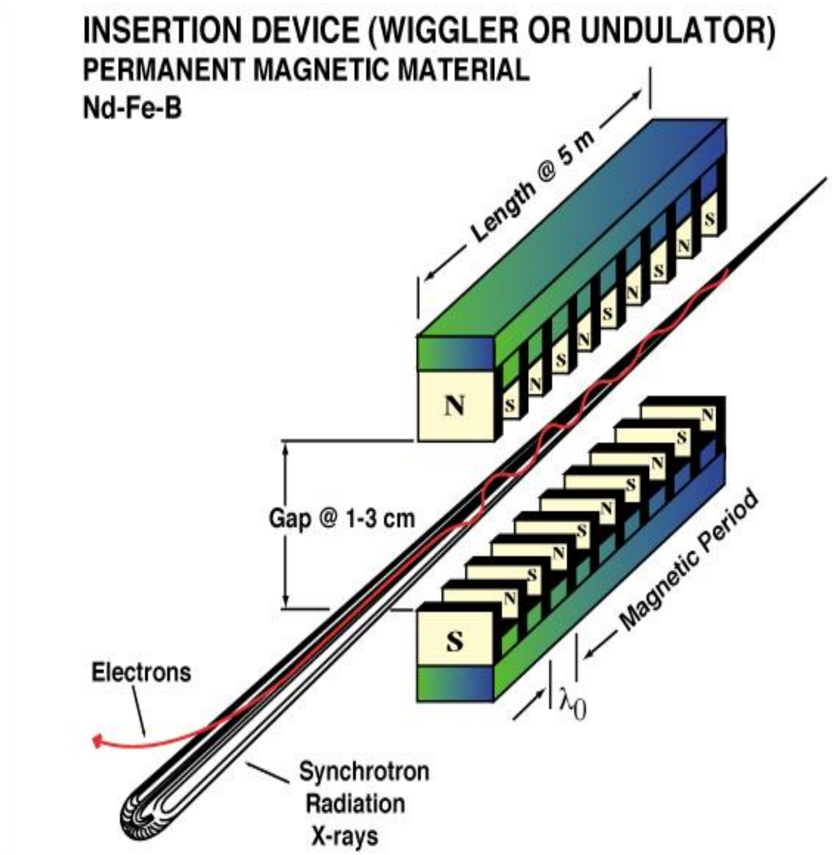
Outline

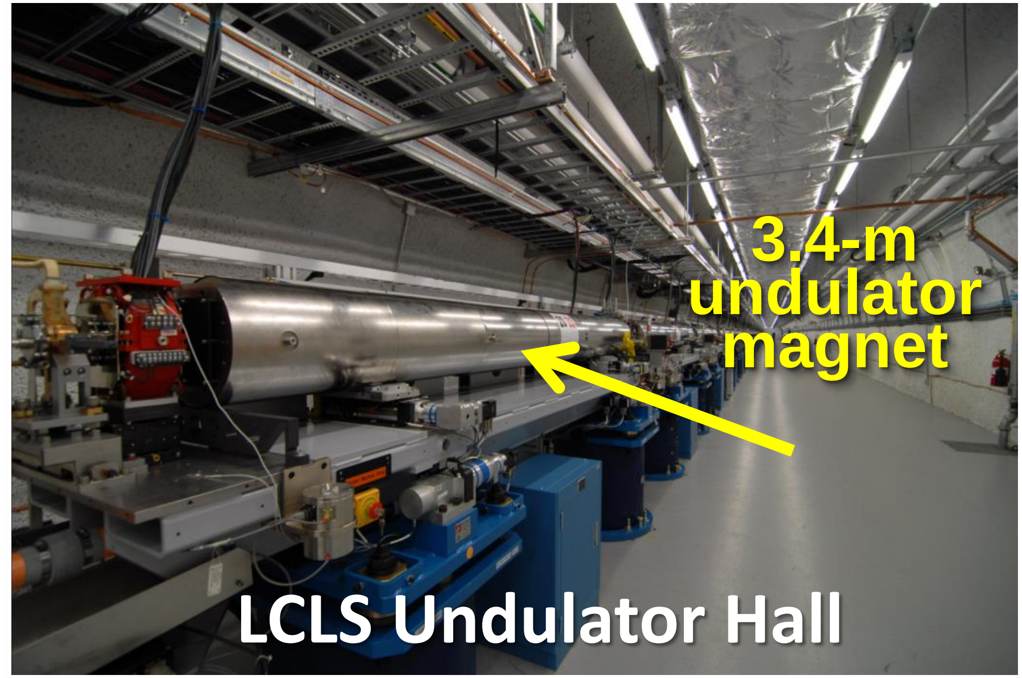
Main goal here is to make a case for the need of high-brightness beams

- **FEL physics: the basics**
  - Spontaneous undulator radiation; Incoherent vs. coherent radiation
  - Physical picture of FEL process (SASE)
  - Essentials of 1D FEL theory
  - Notion of e-beam brightness
  - Refresher on concept of emittance
  - Seeding & beam quality
- **Undulator technology**
- **Beam and high-level Linac design parameters**

FEL basics

The undulator as the center-piece of an FEL





Spectral bandwidth:

$$\frac{\Delta\lambda}{\lambda} \simeq \frac{1}{N_u}$$

Undulator radiation formula

(same as FEL resonance condition)

Undulator period

Radiation wavelength *(observed on-axis)*

$$\lambda = \frac{\lambda_u}{2\gamma^2}\left(1 + \frac{K^2}{2}\right)$$

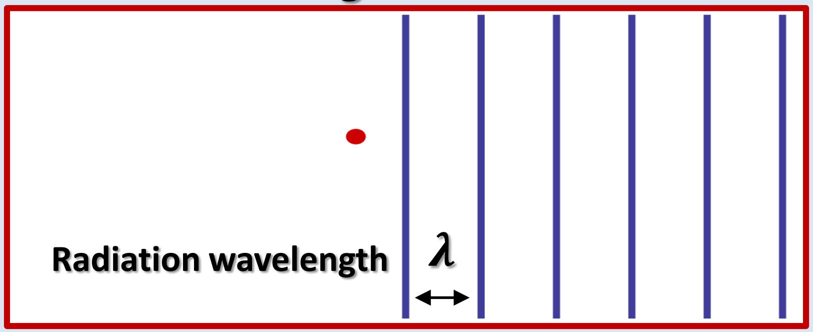
Relativistic factor, electron energy (E/mc^2)

Peak B-field

Undulator parameter: $K = \frac{eB_0\lambda_u}{2\pi mc} \simeq 0.934\lambda_u[cm]B[T]$ *(Planar undulator)*

Incoherent vs. Coherent Radiation Power

Single electron

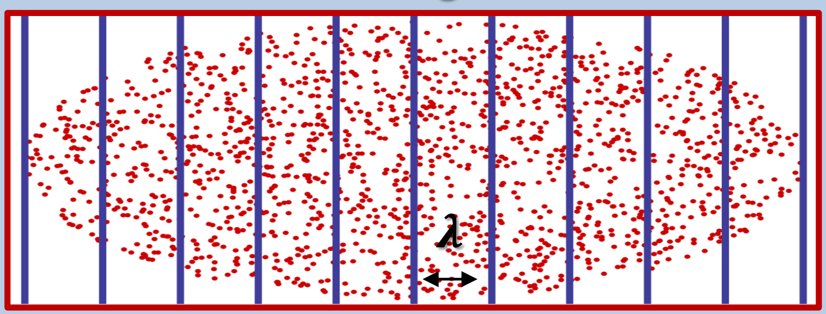


no. photons emitted by 1 electron through N_u undulator periods in $\sim 1/N_u$ bandwidth and $\sim 1/(\gamma^{*2} N_u)$ solid angle

$$N_{ph} \sim \pi\alpha \frac{K^2}{1 + K^2/2}$$

$\alpha = e^2/\hbar c$ (cgs)

Bunch length $\gg \lambda$

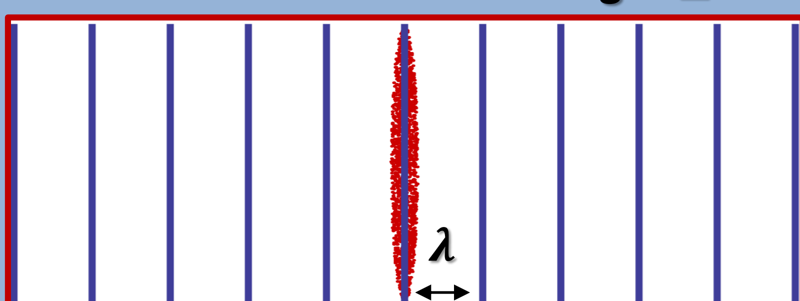


Linear *in no. of electrons/bunch*

$$N_{ph} \sim N_e \pi\alpha \frac{K^2}{1 + K^2/2}$$

Incoherent emission

"Nanobunch" beam length $\leq \lambda$



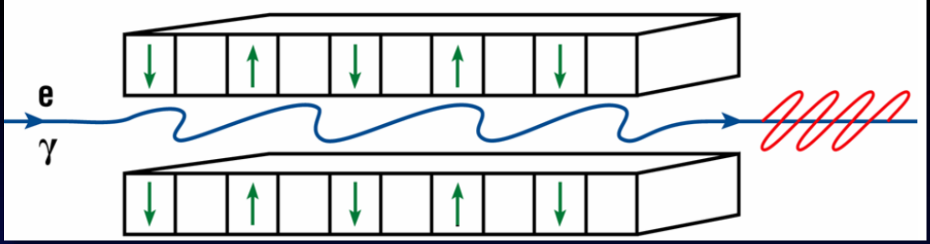
Quadratic *in no. of electrons/bunch*

$$N_{ph} \sim N_e^2 \pi\alpha \frac{K^2}{1 + K^2/2}$$

Fully coherent emission

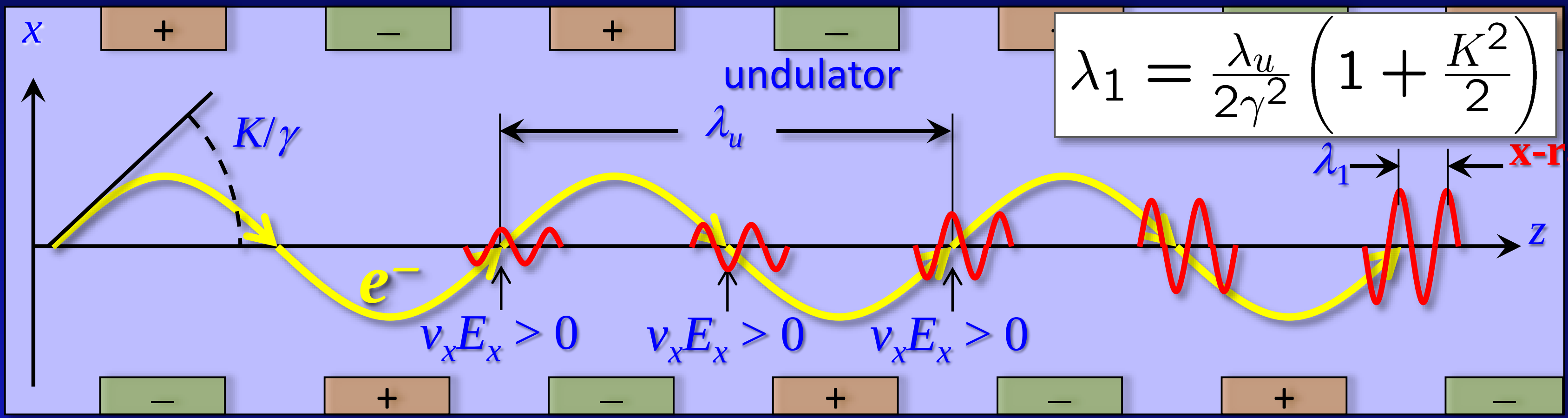
Can we generate nano-bunches???

A pictorial view of the FEL gain process

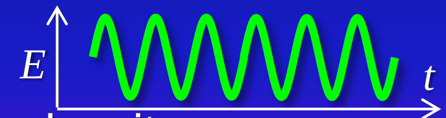


~10s-0.1 nm

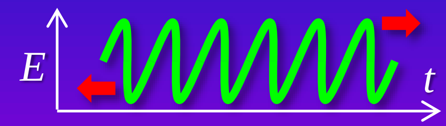
- Electrons **slip** behind EM wave by λ_1 per undulator period (λ_u)



- Due to sustained interaction, some electrons gain energy, while others lose ➔ energy modulation at λ_1 $\Rightarrow$



- e^- losing energy slow down, and e^- gaining energy catch up ➔ density modulation at λ_1 (microbunching) $\Rightarrow$



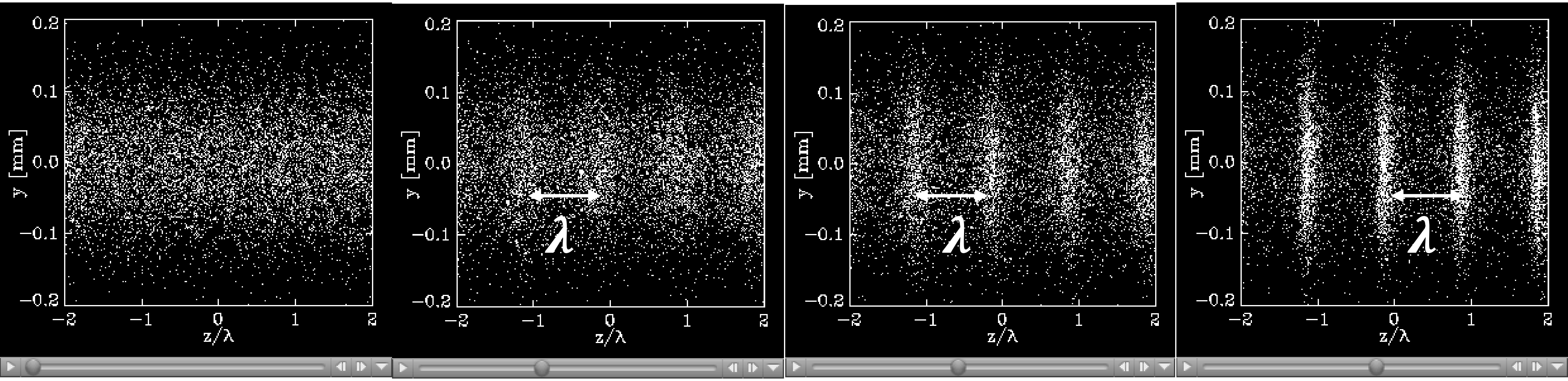
- Micro-bunched beam radiates coherently at λ_1, enhancing the process ➔ **exponential growth of rad. power** ($P_r \approx \rho P_e$)

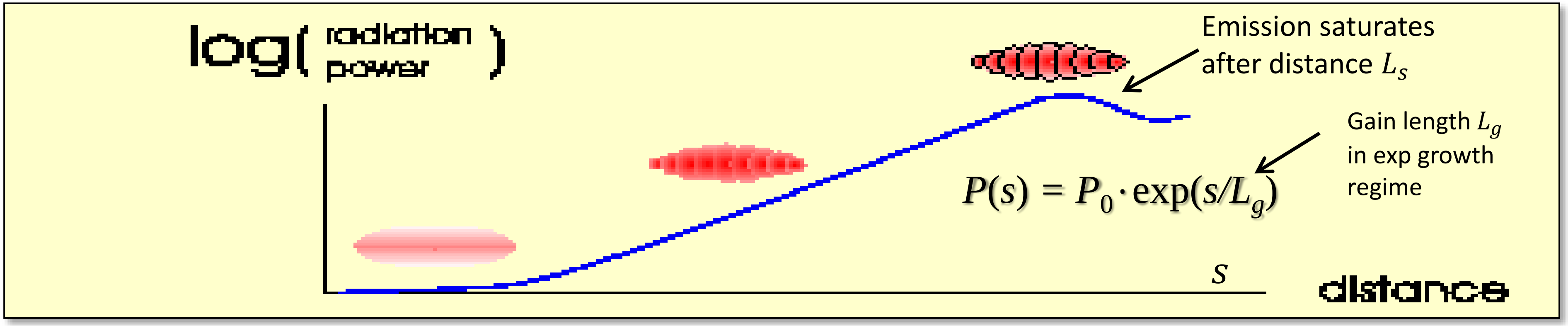
Courtesy of P. Emma

SASE FEL

- Self-Amplified Spontaneous Emission FEL
  - Fundamental mode of operation of x-FELs
  - Emission is jump-started by spontaneous u-radiation emission
  - bunching develops and grows exponentially to saturation as a result of the FEL process

Snap shots of small portion of e-beam developing bunching along undulator line*





Genesis simulations

Important facts/quantities that we need to keep in mind about FEL processes

$$P = P_0 e^{s/L_g}$$

- **Radiation power** grows exponentially along the undulator (typical behavior for instability-driven processes) until saturation

$$L_g$$

- **The FEL gain length**. The smaller the more efficient the FEL process, the sooner we achieve saturation, the shortest the undulator, the less $$$ we need to spend

$$L_s \sim 20 L_g$$

- **Saturation length**. The exponential growth regime of emitted power saturates after about 15-20 Lg (this sets the scale for the undulator length)

$$\ell_c \sim \lambda \frac{L_g}{\lambda_u}$$

- **Cooperation length**. The length over which electrons within the bunch can "communicate" with each other (i.e. how far ahead the radiation emitted by an electron goes by the time it travels through L_g)

- Lot of physics goes into determining L_g. For accurate determination do detailed numerical simulations (e.g GENESIS code)
- Simplified analytical models and formulas give useful insight.

1D Model for FELs

- Basic assumption: 'Cold beam': zero-emittance, zero-energy spread. Infinitely wide beam with uniform transverse density (no radiation diffraction effects)

e-beam peak current

Undulator parameter

$[JJ] = J_0(\xi) - J_1(\xi)$ ***with*** $\xi = \frac{K^2}{2(2+K^2)}$, J_0, J_1 ***=Bessel Functions***

$$\boxed{\boldsymbol{\rho = \left[\frac{I}{\gamma^3 I_A} \frac{\lambda_u^2}{2\pi\sigma_x\sigma_y} \frac{(K \times [JJ]\,)^2}{32\pi} \right]^{1/3}}}$$

Alfven current
$\boldsymbol{I_A \simeq 17kA}$

*Transverse rms beam size (gauss distribution) **

Notice
1/3 exponent

- **Pierce parameter ρ.** The jack of all trades of 1D FEL theory. Typical values $\rho \lesssim 10^{-3}$

$$\boxed{L_g \sim \frac{1}{4\pi\sqrt{3}} \frac{\lambda_u}{\rho}}$$

- **The FEL gain length** is inversely proportional to ρ
=> We want large ρ!

$$\boxed{L_s \sim \frac{\lambda_u}{\rho}}$$

- **Length to saturation**. About $\sim 4\pi\sqrt{3}L_g \sim 20\, L_g$ gain length. Large ρ is good.

$$\boxed{P_r \sim 1.6 \times \rho\, P_b}$$

- **Radiation peak power** at saturation is proportional to ρ and beam power: $P_b = E_b I/e$. Large ρ is good.

Alternate expression in terms of emitted radiation energy per pulse E_r and bunch energy E_b

$$\boxed{E_r \sim 1.6 \times \rho\, E_b}$$

Note: ρ is expressed in terms of a 'equivalent gaussian' beam, i.e. a beam with transverse peak density equal to that of the nominal uniform beam

FEL performance benefits from beam high current, small transverse emittance, small energy spread

- 1D FEL model -> for high-performance we need large ρ -> large beam density ($\frac{I}{\sigma_x \sigma_y} \propto \frac{Q}{\sigma_z \sigma_x \, \sigma_y}$)

- Two important beam requirements
  - *beyond cold-beam 1D theory*

$$\boxed{\sigma_\delta < \rho}$$

- E-beam relative energy spread σ_δ should be smaller (in fact a bit smaller) than ρ, say < 0.5ρ . Electrons with energy too different from nominal slip off the FEL resonance and do not contribute to lasing

$$\boxed{\varepsilon_\perp \lesssim \frac{\lambda}{4\pi}}$$

- E-beam transverse geometric rms emittance $\varepsilon_\perp$ should be on the order of, or smaller than, the radiation emittance $\varepsilon_r = \frac{\lambda}{4\pi}$.

Quick aside: Quantitative description of 3D effects by Ming Xie's FEL model

- Analytical treatment of a fairly complete 3D theory of FEL gain complicated but feasible

- Min Xie (mid ~90s) gave a simple parametrization of gain length based on numerical solutions of 3D theory
  - Very handy
  - Used extensively for FEL design optimization

$$\boldsymbol{L_g = L_{g0}(1 + \Lambda)}$$

3D-theory gain length (generally longer than L_{g0})

1D-theory gain length

$$\Lambda = \Lambda(X_\delta,\ X_d,\ X_\varepsilon)$$

M-X found polynomial approx. to this function

$$X_\delta = \frac{4\pi \boldsymbol{\sigma_\delta}}{\lambda_u} L_{g0}$$ *Scaled energy spread*

$$X_d = \frac{\lambda}{4\pi \boldsymbol{\sigma_r}^2} L_{g0}$$ *Scaled transverse size*

$$X_\varepsilon = \frac{4\pi \boldsymbol{\varepsilon_\perp}}{\beta_{twiss}\lambda} L_{g0}$$ *Scaled emittance*

1D limit recovered when $X_\delta, X_d, X_\varepsilon \to 0$ while keeping the beam charge density constant

Beam brightness as a measure of beam quality and FEL performance

- Recap: to get good performance we want a beam with
  - ***high current*** *(large no. of particles; short bunches)*
  - ***small energy spread***
  - ***small transverse emittance*** *(small divergence; large transverse particle density)*

- Can we come up with a figure of merit that captures all the desirable beam properties at once?

- Particle density in phase space?
  - -> 6D beam brightness

No. particles/bunch

$$B_6 = \frac{N}{\varepsilon_{nx}\varepsilon_{ny}\varepsilon_{nz}}$$

Normalized rms emittances in x,y, and z

Refresher on emittance

- The word "emittance" is sometimes a source of confusion because it can mean two different (although very much related) quantities

For simplicity here we assume no acceleration

- "**single-particle emittance ϵ_x**", a property of the orbit of a *single particle* in linear approximation (parametrization of orbit in terms of Twiss functions $\beta_x, \alpha_x = -\beta_x'(s)/2$)

$$\boldsymbol{x(s) = \sqrt{\epsilon_x \beta_x(s)}\, cos(\psi(s) + \psi_0)}$$

  - A better notation is $\boldsymbol{J_x = \epsilon_x/2}$, ($\epsilon_x$ has the meaning of an action, as in action-angles variables, and measures the amplitude of the betatron oscillations)
  - D*ynamical* invariant of linear motion,
  - i.e. $\boldsymbol{2J_x = \gamma_x(s)x^2 + 2\alpha_x(s)x\,x' + \beta_x(s)x'^2}$ is constant when for x and x' we substitute the particle orbit $x(s)$ and $x'(s)$ in *linear* approximation (Courant-Snyder invariant).

- "**rms emittance ε_x**", a statistical property of the whole beam (dependent on the 2nd moments of the beam distribution), the determinant of the *covariance* or "**σ"-matrix:**

$$\boldsymbol{\varepsilon_x^2} = \mathrm{Det}\,\boldsymbol{\sigma} = \mathrm{Det}\begin{bmatrix} \langle x^2\rangle & \langle xx'\rangle \\ \langle xx'\rangle & \langle x'^2\rangle \end{bmatrix} = \boldsymbol{\langle x^2\rangle\langle x'^2\rangle - \langle xx'\rangle^2}$$

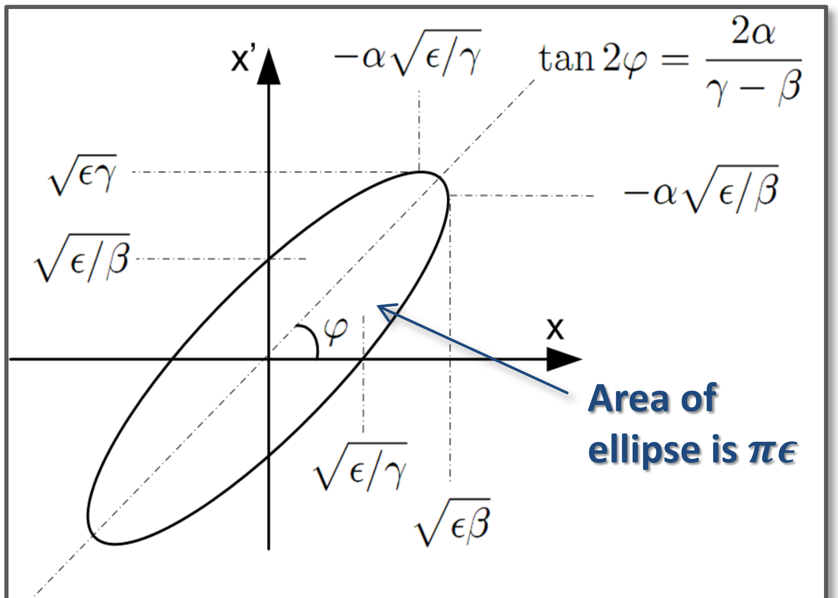
$$\boldsymbol{\langle x^2\rangle} \equiv \sum_{i=1}^{N} \frac{x_i^2}{N} = \int x^2 f(x,x')dxdx'$$

Note: this notation assumes that the centroids vanish $\langle x\rangle = \langle x'\rangle = 0$. More in general, one should write $\langle (x-\langle x\rangle)^2\rangle$ instead of $\langle x^2\rangle$, etc. to obtain the commonly accepted definition of rms emittance

Courant-Snyder ellipse and concept of 'matched beam'

The equation for the Courant-Snyder invariant defines an ellipse in the x/x' plane

$$2J_x = \epsilon_x = \gamma_x(s)x^2 + 2\alpha_x(s)x\,x' + \beta_x(s)x'^2$$

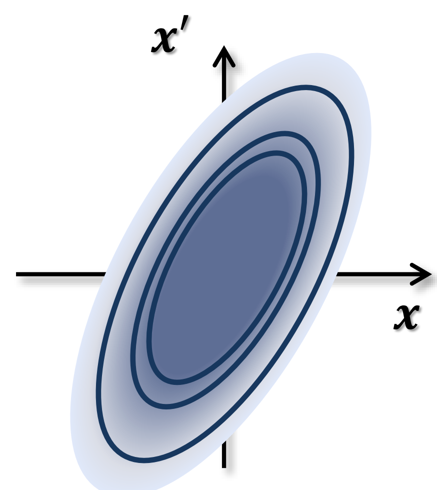


All particles on the border of the ellipse have the same action.

If the motion is linear along the linac, the same particles will still be sitting on the border of an ellipse (with different shape but same area $\pi\epsilon_x$)

How to build a matched beam

(i.e. matched to the design lattice functions)



Formal definition: for a matched beam the phase-space particle distribution $f(x, x')$ is a 1D function of $J_x = J_x(x, x')$:

$$f(x, x') = f(\,J_x(x, x')\,)$$

A tale of two ellipses: the beam 'rms ellipse'

In reality, actual beams are never exactly matched to the design Twiss lattice functions. The beam 'rms ellipse' is a useful concept to describe them.

Two examples of beam distributions

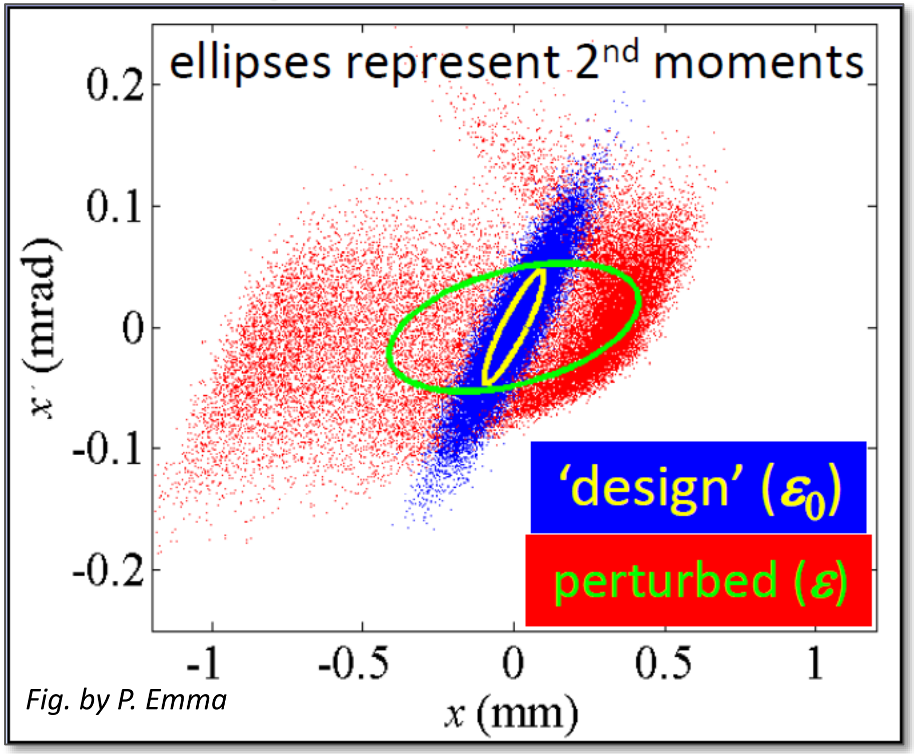


Fig. by P. Emma

- Introduce the beam rms parameters β_x^*, γ_x^*, α_x^* to represent the 2nd moments of the beam distribution

$$\beta_x^* \equiv \frac{\langle x^2 \rangle}{\varepsilon_x} \qquad \alpha_x^* \equiv -\frac{\langle x x' \rangle}{\varepsilon_x} \qquad \gamma_x^* \equiv \frac{\langle x'^2 \rangle}{\varepsilon_x}$$

Note 1 : ε_x is the rms emittance a defined earlier

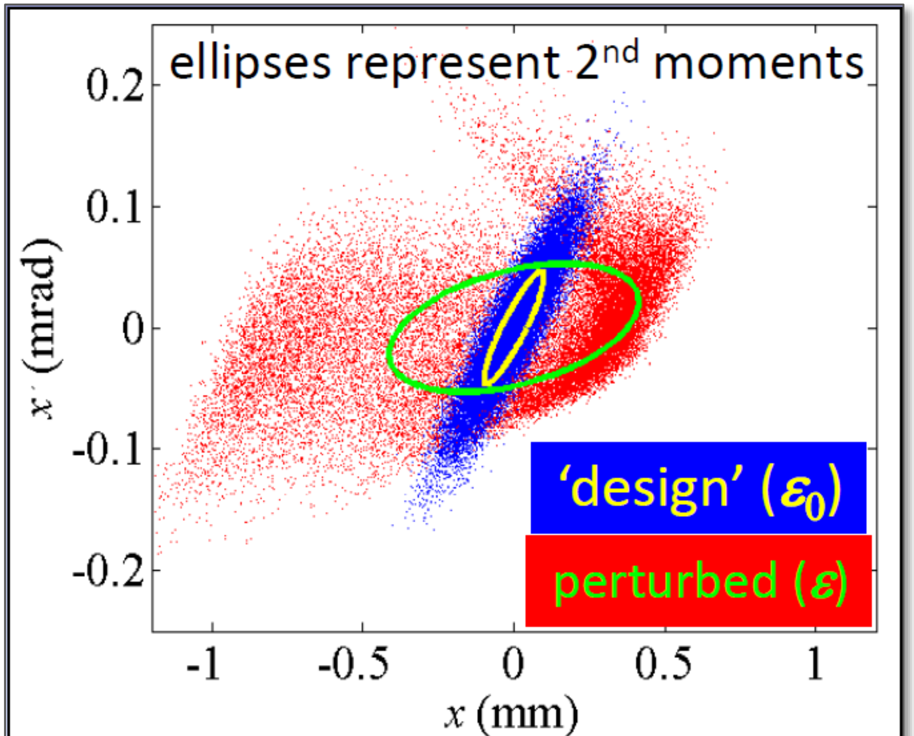
Note 2 : The rms beam parameters are not independent

$$\gamma_x^* \beta_x^* - \alpha_x^{*2} = 1$$

Definition of beam rms ellipse $\varepsilon_x = \gamma_x^* x^2 + 2\alpha_x^* x\, x' + \beta_x^* x'^2$

*For a matched beam we have $\alpha_x^* = \alpha_x$, $\beta_x^* = \beta_x$, $\gamma_x^* = \gamma_x$, i.e. the **Courant-Snyder ellipse is** concentric with the **beam rms ellipse***

Design *vs.* actual (perturbed) beam



- **Several things can go 'wrong' causing emittance growth and/or mismatch**
  - the beam is not injected right; errors in various linac components (magnets, rf structures); misalignments; collective effects, radiation effects, etc

- **The rms ellipses are a way to characterize how far from the design beam the actual beam has gone**

- **Geometrically, the rms ellipse of the actual beam can differ from that of the design beam because**
  - The area may have increased -> <u>rms emittance growth</u>
  - There may be a deformation and/or a tilt -> <u>mismatch</u>
  - Both of the above

Note: *mostly, we care about emittance growth; but a mismatch is not good either, partly because a beam with large mismatch is more susceptible to emittance growth down the line*

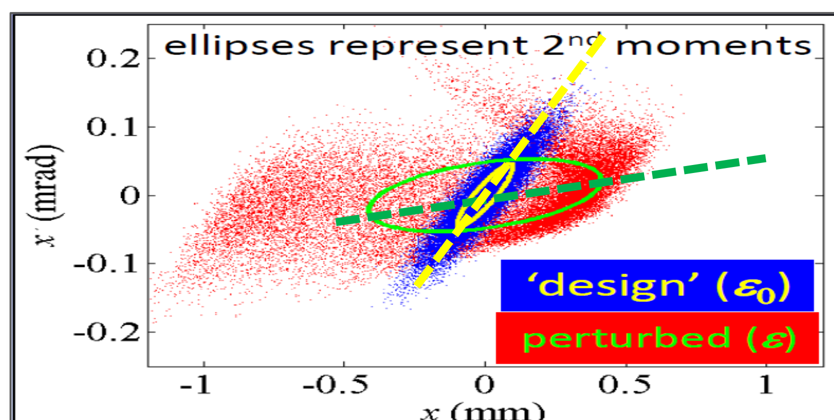
More on beam mismatch: how to quantify the degree of mismatch with just one number

Design-lattice Twiss functions at s

$$B_{\text{mag}}(s) \equiv \frac{1}{2}\left(\beta_x^* \gamma_x - 2\alpha_x^* \alpha_x + \gamma_x^* \beta_x\right)$$

Beam Twiss rms parameters at s

- **It is always $B_{\text{mag}} \geq 1$;**
  - $B_{\text{mag}} = 1 \rightarrow$ beam is matched (i.e. $\alpha_x^* = \alpha_x$, $\beta_x^* = \beta_x$, $\gamma_x^* = \gamma_x$).
- **B_{mag} is an invariant of linear motion**
  - the Twiss functions $(\alpha_x, \beta_x, \gamma_x)$ and rms parameters $(\alpha_x^*, \beta_x^*, \gamma_x^*)$ vary along the lattice from point to point so it's not trivial that B_{mag} should be invariant.
- B_{mag} is a measure of how far the design and unperturbed beam rms ellipses are from being concentric with one another



Formalism to describe emittance growth

- A perturbation to a particle orbit results into space and/or angular offsets Δx and $\Delta x'$ at some observation point downstream of the perturbation.

- **Q**: How do we calculate the rms emittance for the perturbed beam?
- **A**: Take determinant of covariance matrix:

$$\varepsilon_x^2 = Det\begin{bmatrix} \langle (x+\Delta x)^2\rangle & \langle (x+\Delta x)(x'+\Delta x)\rangle \\ \langle (x+\Delta x)(x'+\Delta x)\rangle & \langle (x'+\Delta x')^2\rangle \end{bmatrix}$$

- Consider the special (but relevant) case of vanishing correlations $\langle x\Delta x\rangle = \langle x\Delta x'\rangle = \langle x'\Delta x\rangle = \langle x'\Delta x'\rangle = 0$

$$\varepsilon_x^2 = \mathrm{Det}\begin{bmatrix} \langle x^2\rangle + \langle (\Delta x)^2\rangle & \langle xx'\rangle + \langle \Delta x\Delta x'\rangle \\ \langle xx'\rangle + \langle \Delta x\Delta x'\rangle & \langle x'^2\rangle + \langle (\Delta x')^2\rangle \end{bmatrix} =$$

$$= \underbrace{[\langle x^2\rangle\langle x'^2\rangle - \langle xx'\rangle^2]}_{\varepsilon_{x0}^2} + \underbrace{\left[\ \langle \boldsymbol{x^2}\rangle\langle \boldsymbol{\Delta x'^2}\rangle + \langle \boldsymbol{x'^2}\rangle\langle \boldsymbol{\Delta x^2}\rangle - \boldsymbol{2}\langle \boldsymbol{xx'}\rangle\langle \boldsymbol{\Delta x\Delta x'}\rangle\right]}_{\boldsymbol{\varepsilon_{x0}}(\ \boldsymbol{\beta_x}\langle \boldsymbol{\Delta x'^2}\rangle + \boldsymbol{\gamma_x}\langle \boldsymbol{\Delta x^2}\rangle + \boldsymbol{2\alpha_x}\langle \boldsymbol{\Delta x\Delta x'}\rangle\ )} + \underbrace{\langle \Delta x^2\rangle\langle \Delta x'^2\rangle - \langle \Delta x\Delta x'\rangle^2}_{{\varepsilon_\Delta}^2}$$

Emittance of unpertubed beam

Recall $\langle \boldsymbol{x^2}\rangle = \boldsymbol{\varepsilon_{x0}\ \beta_x}$, $\langle \boldsymbol{xx'}\rangle = -\boldsymbol{\varepsilon_{x0}\ a_x}$

This would be the emittance if ε_{x0} were negligible

Last word on formalism for describing emittance growth (for today …)

Useful to study e.g. CSR effects

$$\boxed{\varepsilon_x^2 = \varepsilon_{x0}^2 + \varepsilon_{x0}\left(\beta_x\langle\Delta x'^2\rangle + 2\alpha_x\langle\Delta x\Delta x'\rangle + \gamma_x\langle\Delta x^2\rangle\right) + \varepsilon_\Delta^2}$$

- Consider class of *small* perturbations involving only angular kicks, i.e. $\langle\Delta x^2\rangle = 0$ and $\langle\Delta x\Delta x'\rangle = 0$ (and therefore also, $\varepsilon_\Delta^2 = 0$)

Small perturbation approx.

$$\varepsilon_x^2 = \varepsilon_{x0}^2 + \varepsilon_{x0}\,\beta_x\langle\Delta x'^2\rangle \quad\Longrightarrow\quad \underbrace{(\varepsilon_x - \varepsilon_{x0})}_{\Delta\varepsilon_x}2\varepsilon_{x0} \simeq \varepsilon_{x0}\,\beta_x\langle\Delta x'^2\rangle$$

$$\boxed{\frac{\Delta\varepsilon_x}{\varepsilon_{x0}} \simeq \frac{\beta_x}{2\varepsilon_{x0}}\langle\Delta x'^2\rangle}$$

Valid for $\Delta\varepsilon_x \ll \varepsilon_{x0}$

Note: It turns out that this formula (derived in the special case of vanishing correlations, previous slide) has more general validity and can be applied *e.g.* to describe chromatic effects in quads; multipole errors in magnets, etc.

Note that in the general case when $\langle\Delta x'\rangle \neq 0$ one should use $\langle(\Delta x' - \langle\Delta x'\rangle)^2\rangle$ in place of $\langle\Delta x'^2\rangle$ to obtain the growth for the *central* rms emittance

Back to Brightness

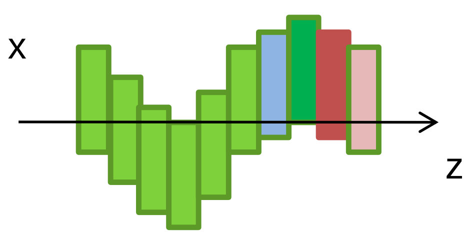
$$B_6 = \frac{N}{\varepsilon_{nx}\varepsilon_{ny}\varepsilon_{nz}}$$

- **Brightness is best expressed in terms of "normalized" emittances (linearly invariant in the presence of acceleration):** i.e. $\varepsilon_{nx} = \gamma\beta\varepsilon_x \simeq \gamma\varepsilon_x$
  - → Brightness is a linear invariant though a transport/accelerator line
- **Invariant longitudinal rms emittance** (no correlations) $\varepsilon_{nz} = \sigma_z \frac{\sigma_E}{mc^2}$
- **To function effectively, FELs need beams that meet minimum brightness requirements**
  - Below that minimum the performance of the FEL may not be affected (e.g. if $\frac{\sigma_E}{E} \ll \rho$ already, further reduction of σ_E won't improve FEL much, although the beam has larger B_6 brightness)
  - A concept of 4D (or 5D) brightness can then be more useful:

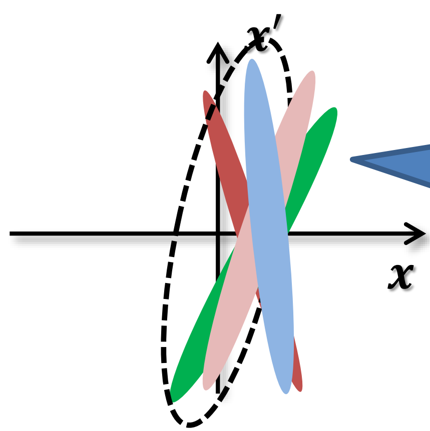
$$B_4 = \frac{Q}{\varepsilon_{nx}\varepsilon_{ny}} \quad \text{or} \quad B_5 = \frac{I}{\varepsilon_{nx}\varepsilon_{ny}}$$

← ***Beam peak current***

- **Slice vs. projected transverse emittance:**
  - For lasing what counts primarily is the rms emittance of the particles within a longitudinal beam slice on the order of the **cooperation length** (where the electrons 'talk' to each other)
  - However we should not let the projected emittance grow too much or else individual slices will not be all properly matched to the intended e-beam optics in the undulator.



Snapshot of beam in x/z plane
(various slices highlighted)



Individual slices may have the same (slice) emittance but if the slice rms ellipses are not concentric the emittance of the whole beam is larger (projected emittance)

Last word on Brightness: "What the injector giveth the Linac shall not take away"

- FELs targeting X-rays were made possible only by the invention of high-brightness sources (e.g. photo-guns).

- Example: LCLS photo—cathode injector*
  - $\boldsymbol{Q = 1nC}$, $\boldsymbol{\varepsilon_{nx} = 1\mu m}$, $\boldsymbol{B_4 = \frac{Q}{\varepsilon_{nx}\varepsilon_{ny}} = 1nC\ \mu m^{-2}}$
  - Longitudinal emittance is small but not very well known (bordering measurement resolution) ; if $\sigma_E \sim 2keV$, then $\boldsymbol{\varepsilon_{nz} \sim 3.5\mu m}$ (for $\sigma_t = 2.8ps$)

- Ideally a Linac would accelerate and transport beams into FELs with minimal degradation of beam brightness, while performing the needed beam manipulations (i.e. compression).

  - Keeping transverse emittance growth under control is an important task of Linac design/operation [offensive effects are Coherent and Incoherent Synchrotron radiation (CSR, ISR), chromaticities, magnet errors, misalignments, wakefields, etc.)]

  - The longitudinal phase space of the beam injected into the Linac tends to be 'colder' (i.e. small energy spread) than needed for lasing; we can afford wasting some longitudinal brightness and be OK. In fact, because of instabilities, we are better off giving up some brightness by actively heating the beam longitudinally (Laser Heater).

  - More desirable: increase the longitudinal emittance while decreasing the transverse emittance so that the 6D brightness stays the same (concept of emittance exchange; not yet implemented in user facilities)

*Y. Ding et al. PRL 102 254801 (2009)

Beyond SASE: more on beam quality requirements

- **To overcome the limitations in the degree of longitudinal coherence from SASE FEL and other undesirable SASE features (e.g. pulse-to-pulse fluctuations in pulse energy and spectrum) do "external" seeding**
  - i.e. use a fully (longitudinally) coherent radiation pulse to initiate the FEL process
- **Because suitable coherent conventional sources do not exist in the x-ray spectrum (yet) ingenious schemes have been proposed that use**
  - conventional laser radiation pulses (say $\lambda \sim 200nm$)
  - various beam manipulations (among these, High Gain Harmonic Generation has been demonstrated, down to few nm. It is the mode of operation in FERMI@Elettra)
- **Self-seeding**
  - **demonstrated for both hard and soft x-rays; still inherits some of SASE defects e.g. pulse energy fluctuations.**

- **Seeding poses additional demands on beam quality:**
  - Longitudinal coherence of radiation is spoiled by non-uniformity of e-beam energy profile (e.g. a quadratic energy chirp).
  - Seeding also favors a beam that has a long core with uniform profile (i.e. we want "flat-flat" beams);
  - Uncorrelated energy spread should be low to minimize laser power needed for external seeding, maximize max. order of harmonic

Undulators

How do we get short λ?
How do we get tuning range?

$$\lambda = \frac{\lambda_u}{2\gamma^2}\left(1 + \frac{K^2}{2}\right)$$

- **Generally the challenge is to reach short radiation wavelength λ.**
- **Two ways to generate short λ**
  - Shorter undulator period λ_u (min. value set by available technology and FEL performance)
  - Larger e-beam energy γ (max. value set by $$$)

- **Tuning range** (i.e. range of radiation spectrum that can be generated once the undulator is installed and λ_u is fixed)
  - **Vary beam energy** (can pose operational nuisances; not practical if same linac feeds multiple FELs, operating at the same time and targeting different radiation wavelengths)
  - **Vary undulator parameter K** by changing B_0 in undulator. Tuning range depends on undulator technology and requirements on the minimun undulator aperture (or 'gap')

Desired undulator features

- **Short period**
  - achieve short wavelength radiation with lower e-beam energy)
- **High B field**
  - FEL efficiency depends on $\mathrm{K} \propto B_0$
  - wide range of B-field allows for correspondingly wide tunability range)
- **Sufficiently wide gap**
  - to accommodate the e-beam and keep losses to a minimum
- **Variable configurations for polarization control**
- **Low cost**

unfortunately these tend to fight each other

Undulator technology options

- **Electromagnetic undulators/wigglers**
  - Not good for x-rays radiation, period is too long
- **Permanent magnet undulators (pure or hybdrid)**
  - Technology of choice for existing x-ray FELs
- **Superconducting undulators**
  - So far used only in storage-ring light sources. Emerging technology for FELs. Capable of shorter period, larger fields
- **RF undulators** (use interaction of e-beam with RF fields instead of static B-field to wiggle the electron trajectory)
  - At early R&D stage
- **Conventional-laser undulator** (e-beam interacting with laser pulse)
  - Period could be very small (~1um), meaning required e-energy would be very low (10s MeV) and very attractive; but this doesn't work as an FEL unless e-beam is of exceedingly high quality

Permanent Magnets (PM) Undulators:
simple law for peak B-field

- Useful equation for FEL design
- Equation valid for pure PM (PPM) undulators:

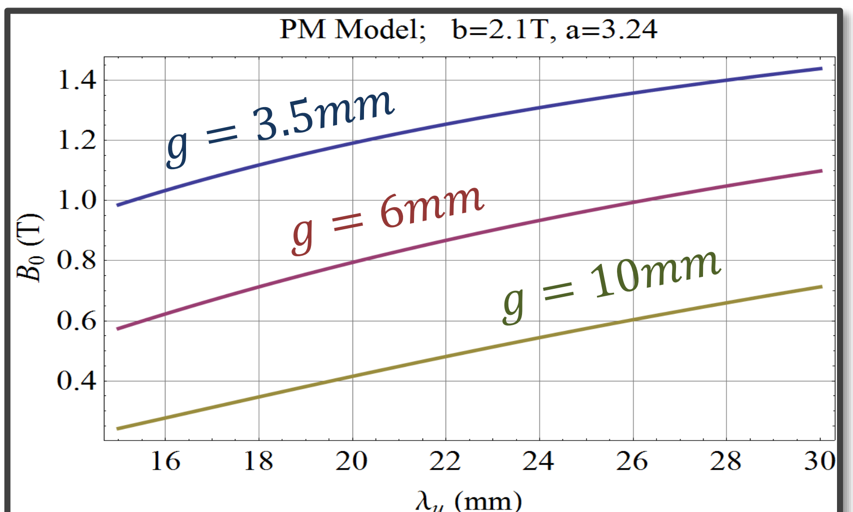
$$\boxed{B_0 = b e^{-a\left(\frac{g}{\lambda_u}\right)}}$$

u-gap

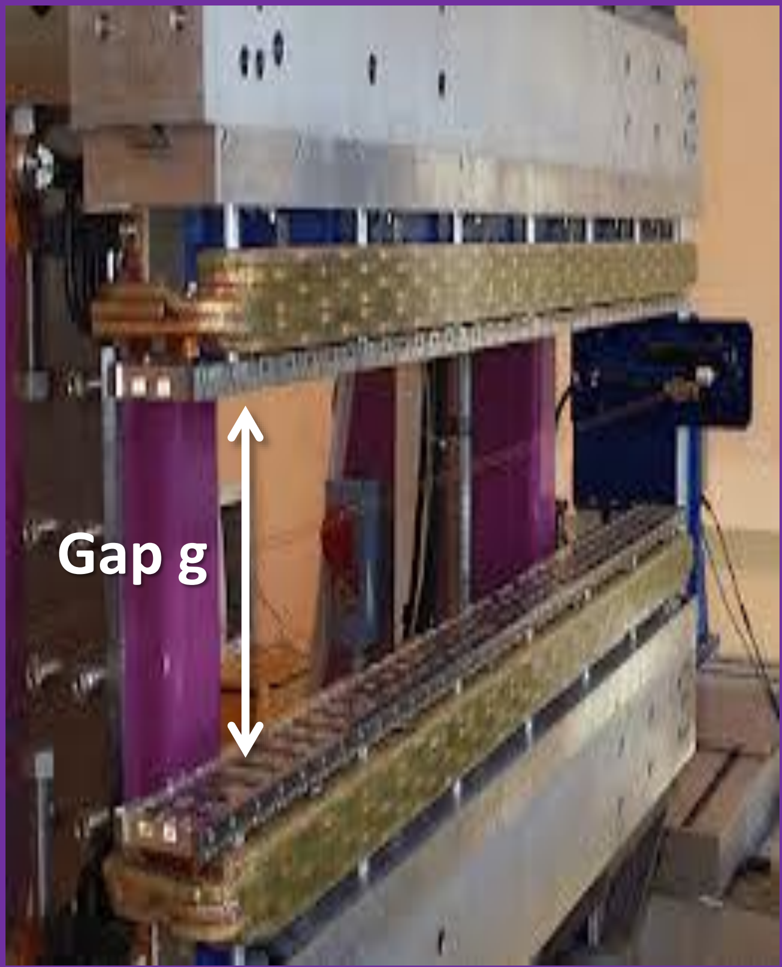
u-period

Typical values for coefficients*

- $b \sim 2.1T$
- $a \sim \pi$, why? – see Homework Exercise



A wide open undulator



*NdFeB allow, P. Elleaume et al.

Putting the FEL formulas to work

A first rough stab at scoping out choice of parameters for the beam/linac design;
Emphasis on radiation wavelength tuning range

How do we choose beam/machine parameters to achieve the desired FEL performance?

- A representative of the x-rays radiation user community (with $$ in his pockets) comes to you and says:
  - *"I want a SASE FEL generating up to 1.2keV photon energy (1nm) and tunable so that I can go as low as 250eV to reach the C K-edge. Also, I'd like multiple beamlines that can be operated independently and simultaneously. And by the way, 10^{12} photons/pulse @ 1nm would be good".*
- What kind of machine/beam would do the job?

- **Step 1**: Rough assessment of e-beam energy requirement.
  - Shortest radiation wavelength target: $\lambda = 1\text{nm}$.
  - Take $\lambda_u = 2cm$ (a guess for starters)
  - Undulator parameter on the order of unity: $K \sim 1$.
  - Use u-radiation resonance condition $\lambda = \frac{\lambda_u}{2\gamma^2}\left(1 + \frac{K^2}{2}\right)$
  - find $mc^2\gamma = mc^2\sqrt{\frac{\lambda_u}{2\lambda}(1 + \frac{K^2}{2})} = 0.51\sqrt{\frac{0.02}{2\times 10^{-9}}\frac{3}{2}} \sim 2\ GeV$
  - Independent beamline tunability -> we cannot vary e-energy to vary λ. We have to adjust the u-gap

Step 2: Choose undulator technology and min. gap

- **Select technology**
  - E.g. choose well tested Hybrid PM undulators
  - Ask the magnet designer for magnetic field model

$$B_0[g,\lambda_u] = 4.22[T]\exp\left(-5.08\times\left(\frac{g}{\lambda_u}\right)+1.54\times\left(\frac{g}{\lambda_u}\right)^2\right)$$

- **Choose min. undulator gap (corresponds to max B-field):** *e.g.* $g_{min} = 6\,mm$
  - Somewhat aggressive (*e.g.* LCLS $g_{min} = 7.5\,mm$; X-FEL has 10mm)
  - Determined by fear of beam losses. To be revisited depending on choice of rep. rate; study of beam losses, dark currents, collimation, etc.

- **For a given u-period (still to be determined) g_{min} gap corresponds to the maximum $K = K_{max}$**

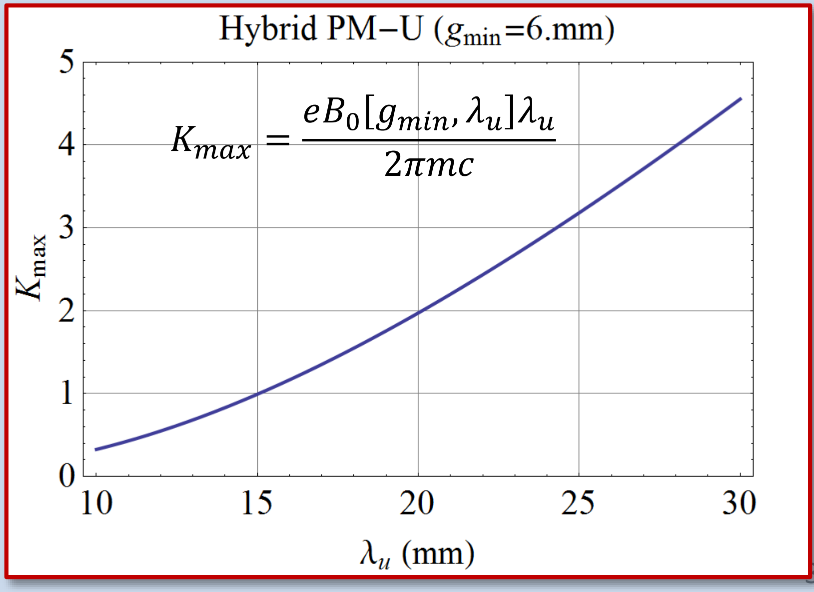
$$K = \frac{eB_0\lambda_u}{2\pi mc}$$

- **K_{max} corresponds to the longest radiation wavelength**

$$\lambda_{max} = \frac{\lambda_u}{2\gamma^2}\left(1+\frac{K_{max}^2(\lambda_u)}{2}\right)$$

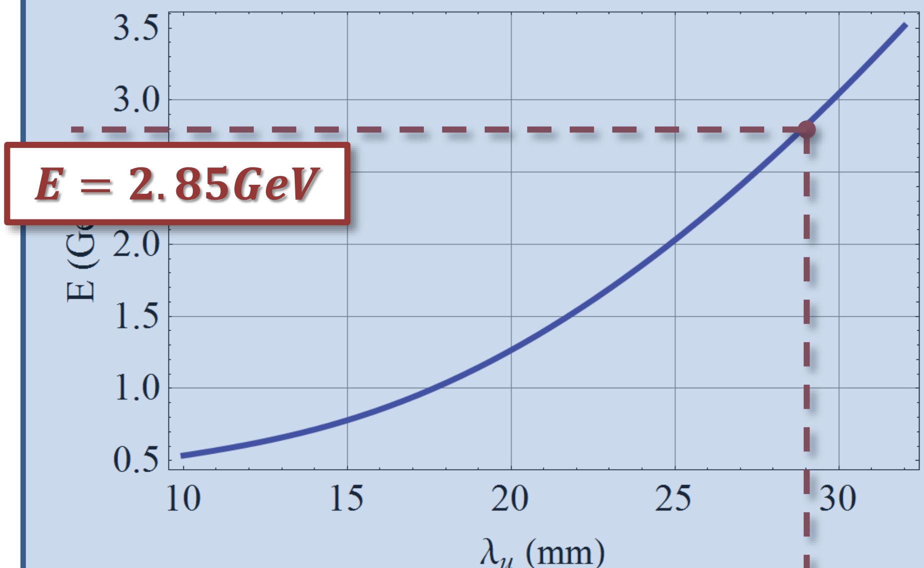
Use this equation to find γ as a function of λ_u, having set $\lambda_{max} = 4.8\,nm$

K_{max} as a function of u-period for a set minimum gap



Step 3: Determine undulator period, e-energy

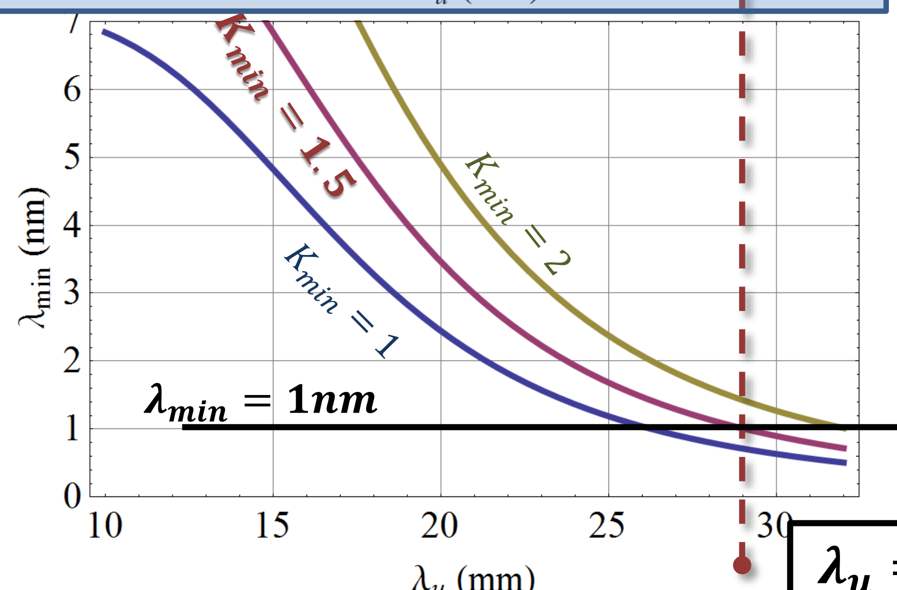
Electron energy as a function of λ_u for set choice of g_{min} and λ_{max}



$$\lambda_{max} = \frac{\lambda_u}{2\gamma^2}\left(1 + \frac{K_{max}^2(\lambda_u)}{2}\right)$$

- Set desired $K = K_{min} \simeq 1.5$ corresponding to $\lambda = \lambda_{min}$

$$\lambda_{min} = \frac{\lambda_u}{2\gamma(\lambda_u)^2}\left(1 + \frac{K_{min}^2}{2}\right)$$



$\lambda_u = 29\ mm$

- Plot λ_{min} as a function of λ_u

- Setting $\lambda_{min} = 1\ nm$ identifies the the u-period λ_u and e-energy

Step 4: Now we have the e-energy requirement: What about beam emittance, charge?

- Match e-beam geometric rms emittance to radiation emittance at 1nm (most demanding wavelength)

Normalized rms emittance

$$\boldsymbol{\varepsilon_{nx} = \gamma \frac{\lambda}{4\pi} = 5560 \times \frac{10^{-9}m}{4\pi} = 0.45\mu m}$$

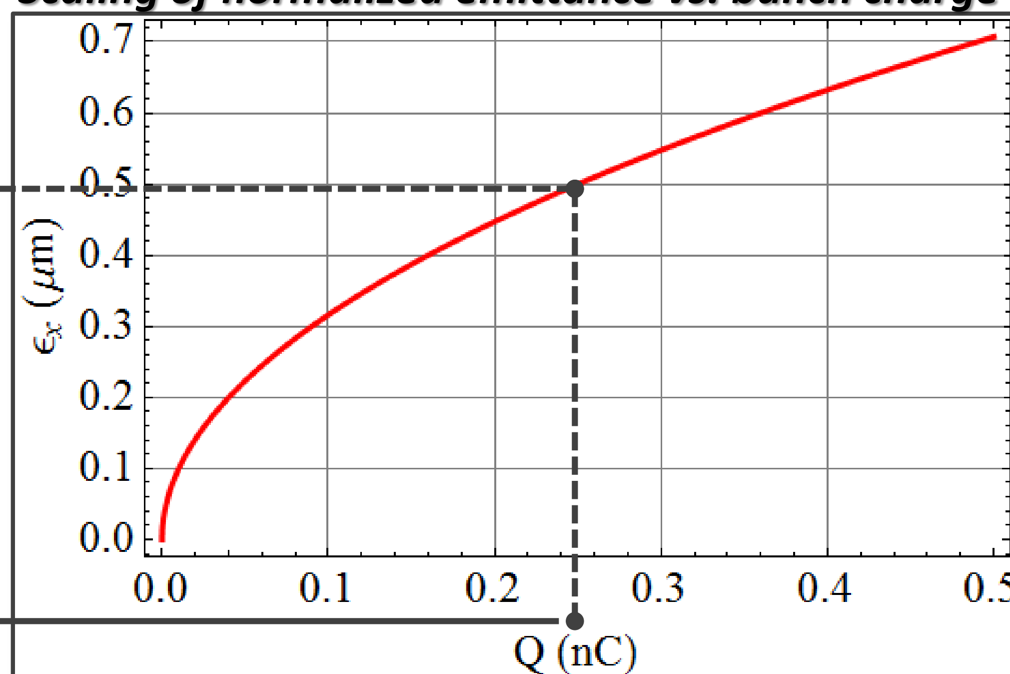
$$\boxed{\boldsymbol{\varepsilon_{\perp} \le \frac{\lambda}{4\pi}}}$$

- Call it $\varepsilon_{nx} = 0.5\mu m$ (slightly larger emittance is OK).

- The minimum emittance is set by gun. Use $\sim \sqrt{Q}$ scaling law for emittance, (roughly fitting measurements of SLAC gun):

Scaling of normalized emittance vs. bunch charge

This is the emittance we want $\boldsymbol{\varepsilon_{nx} = 0.5\mu m}$



This is the bunch charge we need

$\boldsymbol{Q \sim 250pC}$

Step 5: Estimate FEL performance using 1D and 3D (Ming-Xie's formulas) theory

Beam/Machine Parameters

(shortest wavelength)

λ (nm)	1.
I (A)	1000
γ	5577.3
E (MeV)	2850.
λ_u	0.029
Twiss β (m)	12
ϵ_{nx} (μm)	0.5
σ_δ (10^{-3})	0.0714286
σ_E (keV)	203.571
K	1.51345
Q (nC)	0.25

Not optimized

Somewhat arbitrary

- **Assume peak current I~1kA**
  - Somewhat arbitrary but not unreasonable
  - Refinement of this choice is part of Linac design optimization
  - If peak current at exit of injector I~40A (SLAC gun) the compression factor in Linac will be C~1000/40=25

20% degradation in gain length because of 3D effects

1D Gain Length

No. photons per pulse at 1nm

Can tolerate up to $\Delta E \sim 0.5 \times \rho E \sim 1 MeV$ energy spread

ρ (10^{-3})	L_{g0} (m)	3D L_g (m)	$(L_{g0}/L_g)^2$	E_{ph}/pulse (μJ)	N_{ph}/pulse
0.885542	1.50459	1.78756	0.841701	715.203	3.60485×10^{12}

- This only a first (rough) pass.
- Complete optimization should include K_{min}, betatron function, and exploration of various trade-offs (e.g. tuning range vs. no. photons per pulse) and will require many iterations.
- Selected working points then need to be checked against detailed numerical simulations
- Ultimately cost and other (e.g. rep rate) considerations should also be factored in (undulators, RF structure technology, RF power, etc).

Summary & highlights

- Undulator radiation /FEL resonance equation; undulator parameter"

$$\lambda = \frac{\lambda_u}{2\gamma^2}\left(1 + \frac{K^2}{2}\right) \qquad K = \frac{eB_0\lambda_u}{2\pi mc} \simeq 0.934\lambda_u[cm]B[T]$$

- FEL $\boldsymbol{\rho}$ (Pierce) parameter, 1D Theory FEL gainlength

$$\boxed{\rho = \left[\frac{I}{\gamma^3 I_A} \frac{\lambda_u^2}{2\pi\sigma_x\sigma_y} \frac{(K \times [JJ]\,)^2}{32\pi} \right]^{1/3}} \qquad \boxed{L_g \sim \frac{1}{4\pi\sqrt{3}} \frac{\lambda_u}{\rho}}$$

- Requirements for beam relative-energy spread and transverse rms emittance

$$\boxed{\sigma_\delta < \rho} \qquad \boxed{\varepsilon_\perp \lesssim \frac{\lambda}{4\pi}}$$

- E-beam brightness

$$B_6 = \frac{N}{\varepsilon_{nx}\varepsilon_{ny}\varepsilon_{nz}} \qquad B_5 = \frac{I}{\varepsilon_{nx}\varepsilon_{ny}} \qquad B_4 = \frac{Q}{\varepsilon_{nx}\varepsilon_{ny}}$$

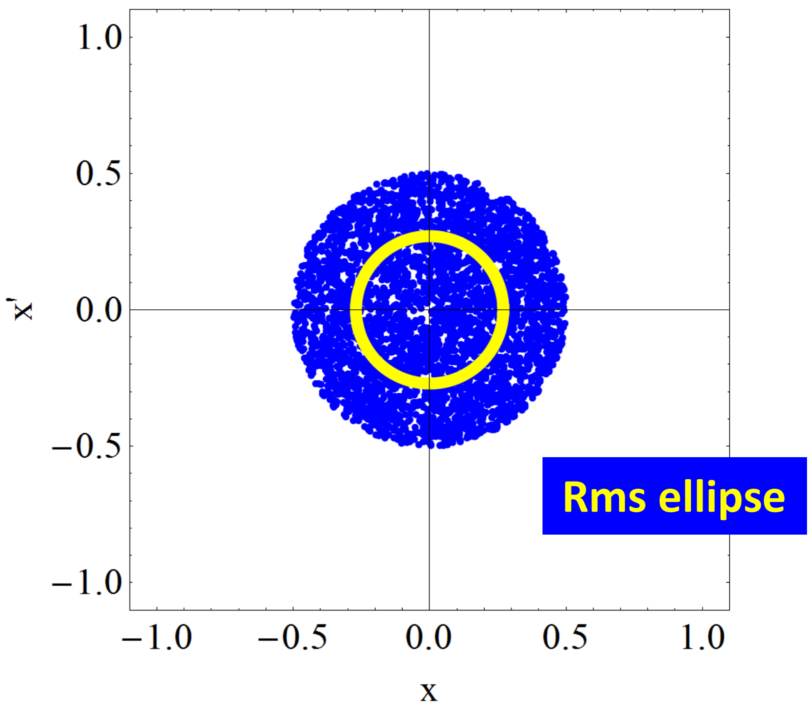
- Emittance growth due to angular kick perturbation

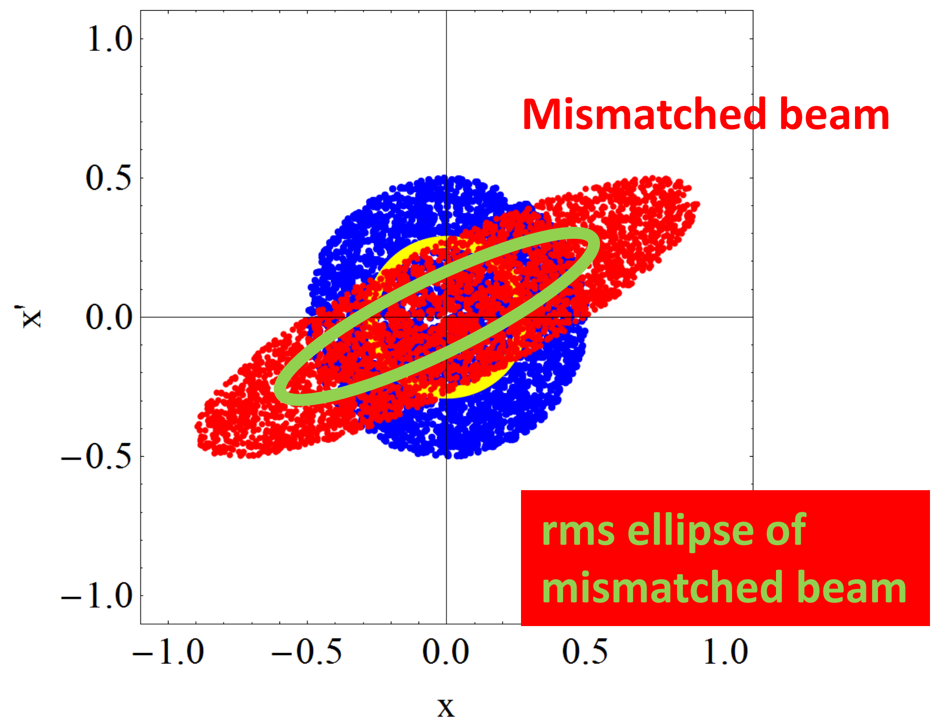
$$\boxed{\frac{\Delta\varepsilon_x}{\varepsilon_{x0}} \simeq \frac{\beta_x}{2\varepsilon_{x0}}\langle \Delta x'^2 \rangle}$$

Supplemental material

How mismatch can lead to emittance growth by filamentation (1)

Matched beam

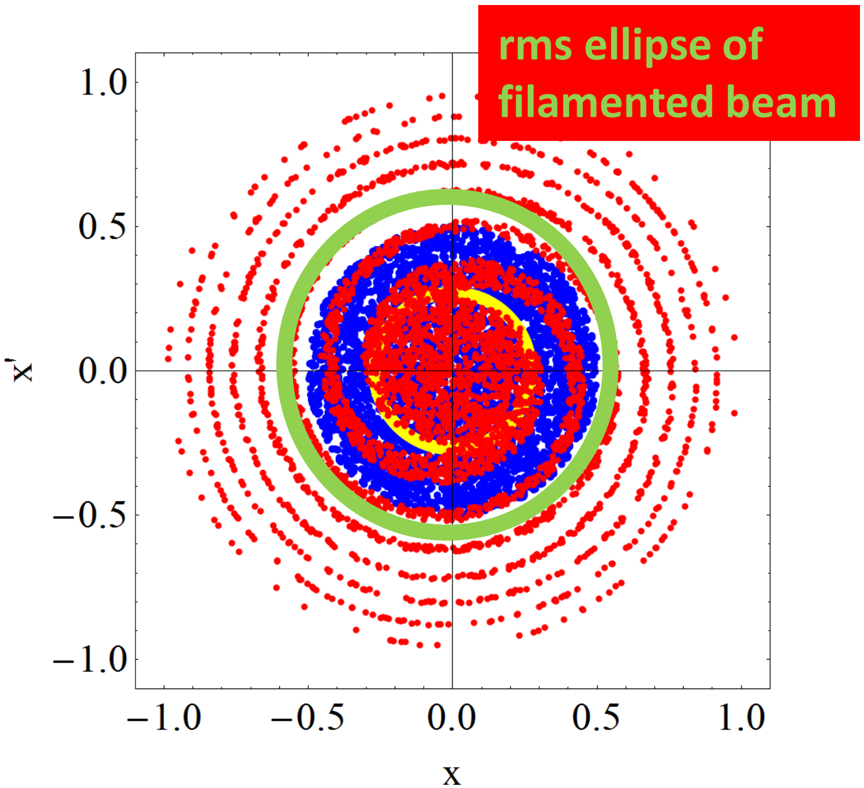




We use normalized coordinates
so that the ellipse is, in fact, a circle

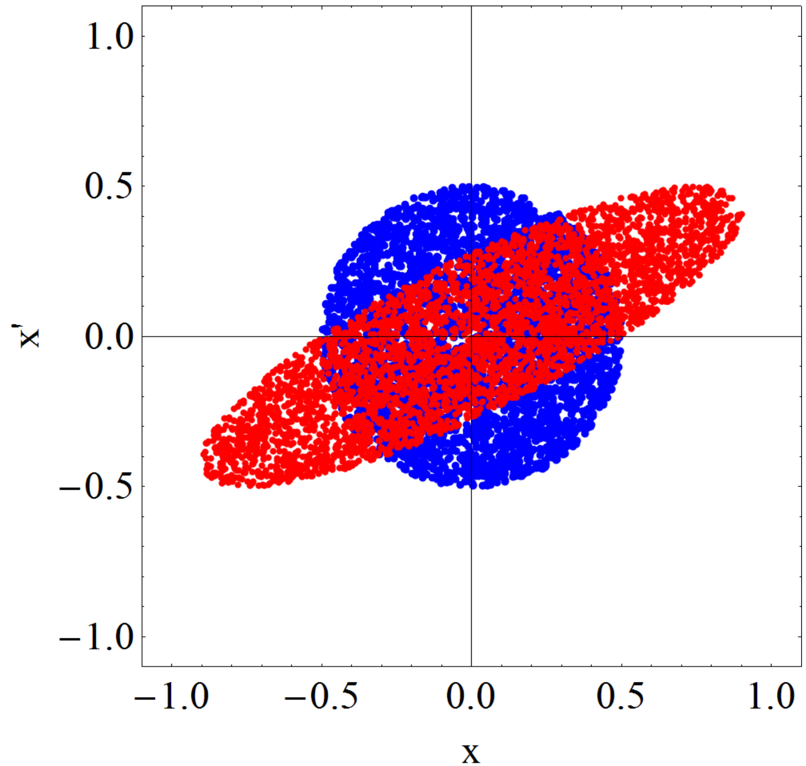
In this example, mismatched and matched seams have the same initial rms emittance ε_x

How mismatch can lead to emittance growth by filamentation (2)



The rms emittance of the filamented beam has become larger than that of the matched beam (interestingly, the fully filamented beam is ~matched: the green and yellow ellipses become concentric)

Because on anharmonicities the mismatched beam will fill a larger region of phase space as it is transported along the accelerator



animation

Interesting fact: the rms emittance of the filamented beam ε_{xf} is

$$\varepsilon_{xf} = B_{mag}\varepsilon_x$$

(mismatch before filamentation)

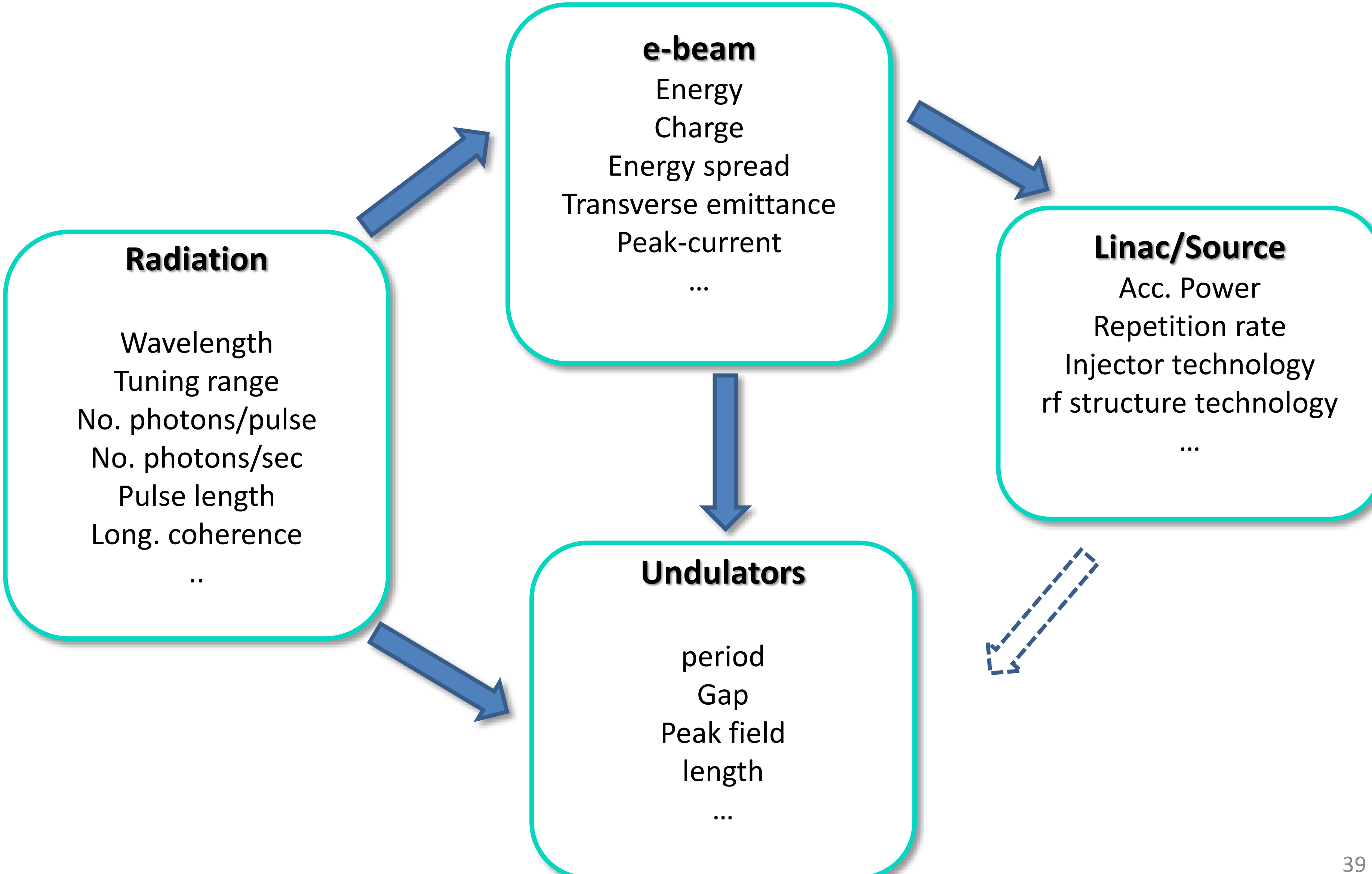
One last concept: the 'equivalent' emittance

- Because we care about how much the emittance has grown to $\varepsilon_x(s) > \varepsilon_{x0}$ as well as the mismatch $B_{mag}(s)$ accrued by the beam up to a certain point s in the lattice, a sensible measure of '***how bad***' a beam is the product of the two:

$$B_{mag}(s) \times \varepsilon_x\,(s) \equiv \varepsilon_{eq,x}(s)$$

- We call this product the 'equivalent' emittance ε_{eq}.
  - *Physical interpretation: this is the rms emittance that the beam eventually will exhibit at the end of a very long transport line as a result of anharmonicities downstream of* s *(see previous slide)*
  - *In a Linac, complete filamentation may never happen (too few betatron oscillations) but* ε_{eq} *remains a useful figure of merit to gauge beam quality.*

Desired radiation properties drive e-beam requirements and technology/design choices (e-source/Linac/undulators)



In actual design process the arrows will turn around: available technology sets limits to achievable radiation

e-beam

Energy
charge
Energy spread
Transverse emittance
Peak-current
...

Radiation

Wavelength
Tuning range
No. photons/pulse
No. photons/sec
Pulse length
Long. coherence
..

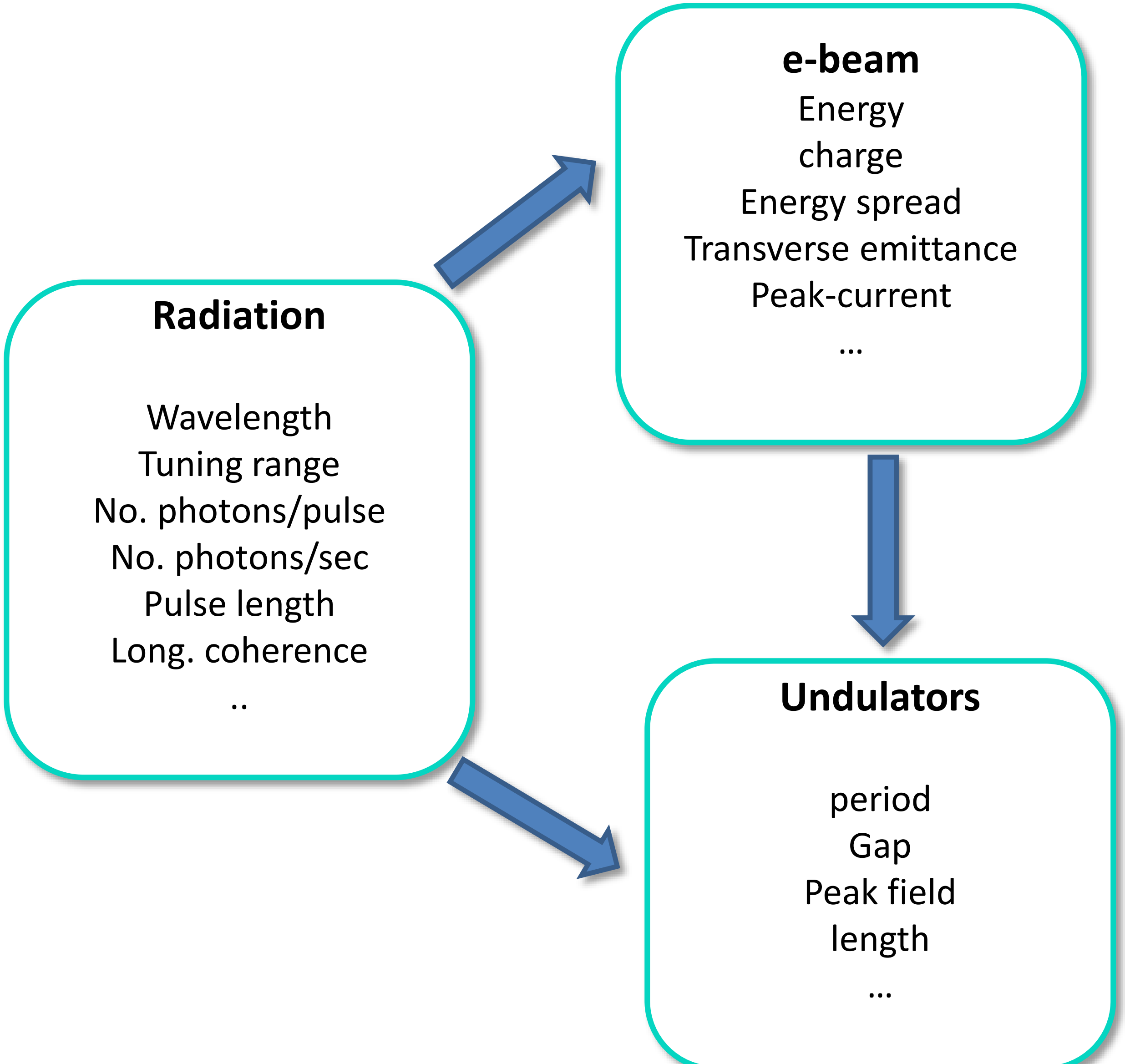
Linac/Source

Acc. Power
Repetition rate
Injector technology
rf structure technology
...

Undulators

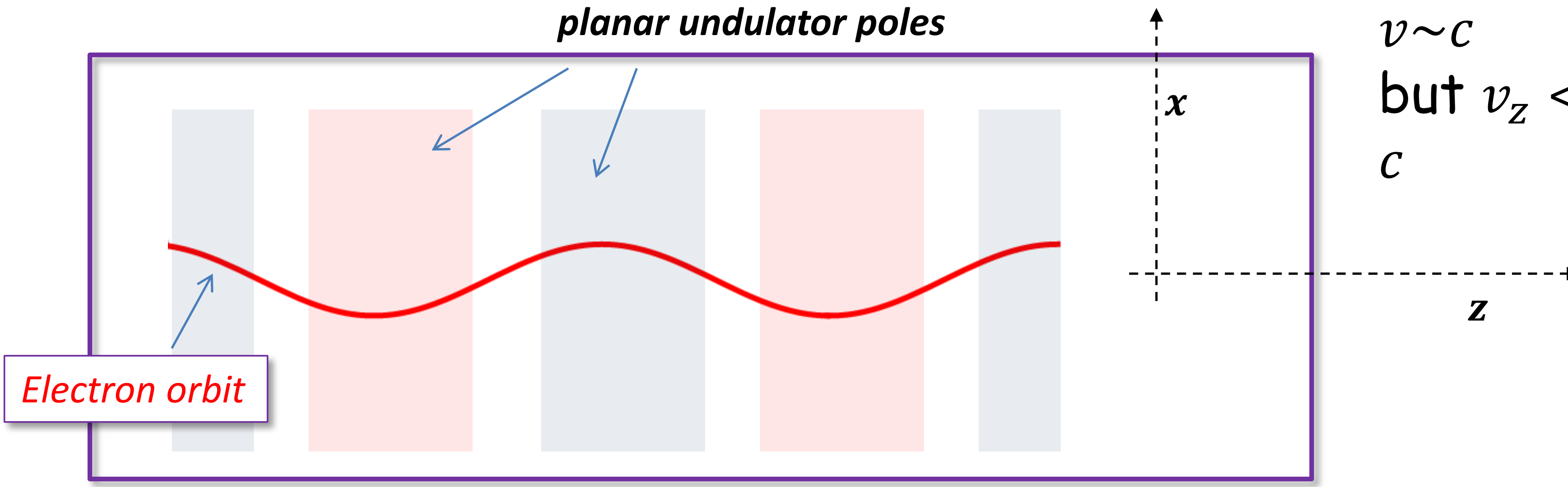
period
Gap
Peak field
length
...

In this Lecture we have shown a stripped-down (but not unrealistic) example of how radiation requirements may drive choice of e-beam, undulator parameters



- How we generate and deliver the e-beam to the FEL is the topic for the rest of this Course

The undulator radiation formula derived (kind of)



$v \sim c$

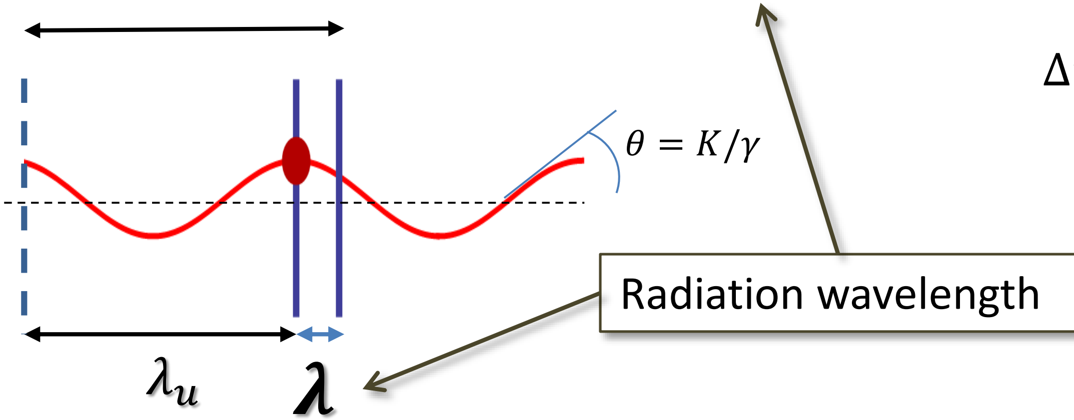
but $v_z < c$

$\boldsymbol{\Delta z}$ Distance travelled by light by the time electron goes through 1 period

$$\lambda = \boldsymbol{\Delta z} - \lambda_u = c\Delta t - \lambda_u = \frac{c\lambda_u}{\bar{v}_z} - \lambda_u \simeq \boldsymbol{\frac{\lambda_u}{2\gamma^2}\left(1 + \frac{K^2}{2}\right)}$$

$$\Delta t = \lambda_u / \bar{v}_z$$

$$\bar{v}_z = c\left[1 - \frac{1}{2\gamma^2}\left(1 + \frac{K^2}{2}\right)\right] < c$$

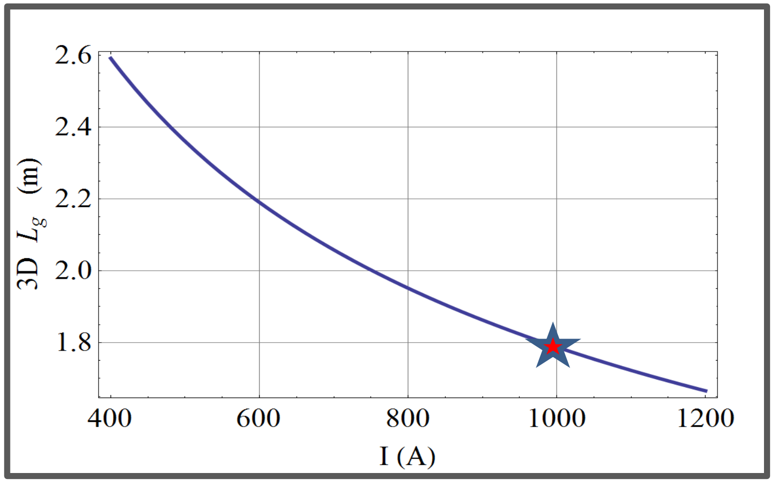


e-orbit in undulator: $x = \frac{K\lambda_u}{2\pi\gamma} \sin\left(\frac{2\pi}{\lambda_u} z\right)$

Sensitivity of 3D gain-length to main beam parameters (Ming-Xie's model)

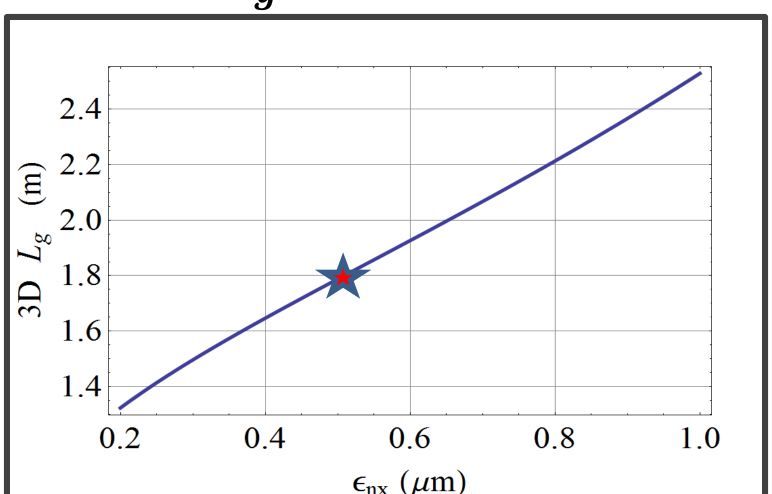
Shorter L_g is better

$3D\ L_g$ vs. current



- Vary current, emittance, energy spread (independently) around working point defined in previus slides

$3D\ L_g$ vs. rms emittance



$3D\ L_g$ vs. relative rms energy spread