RF Cavities and Linacs

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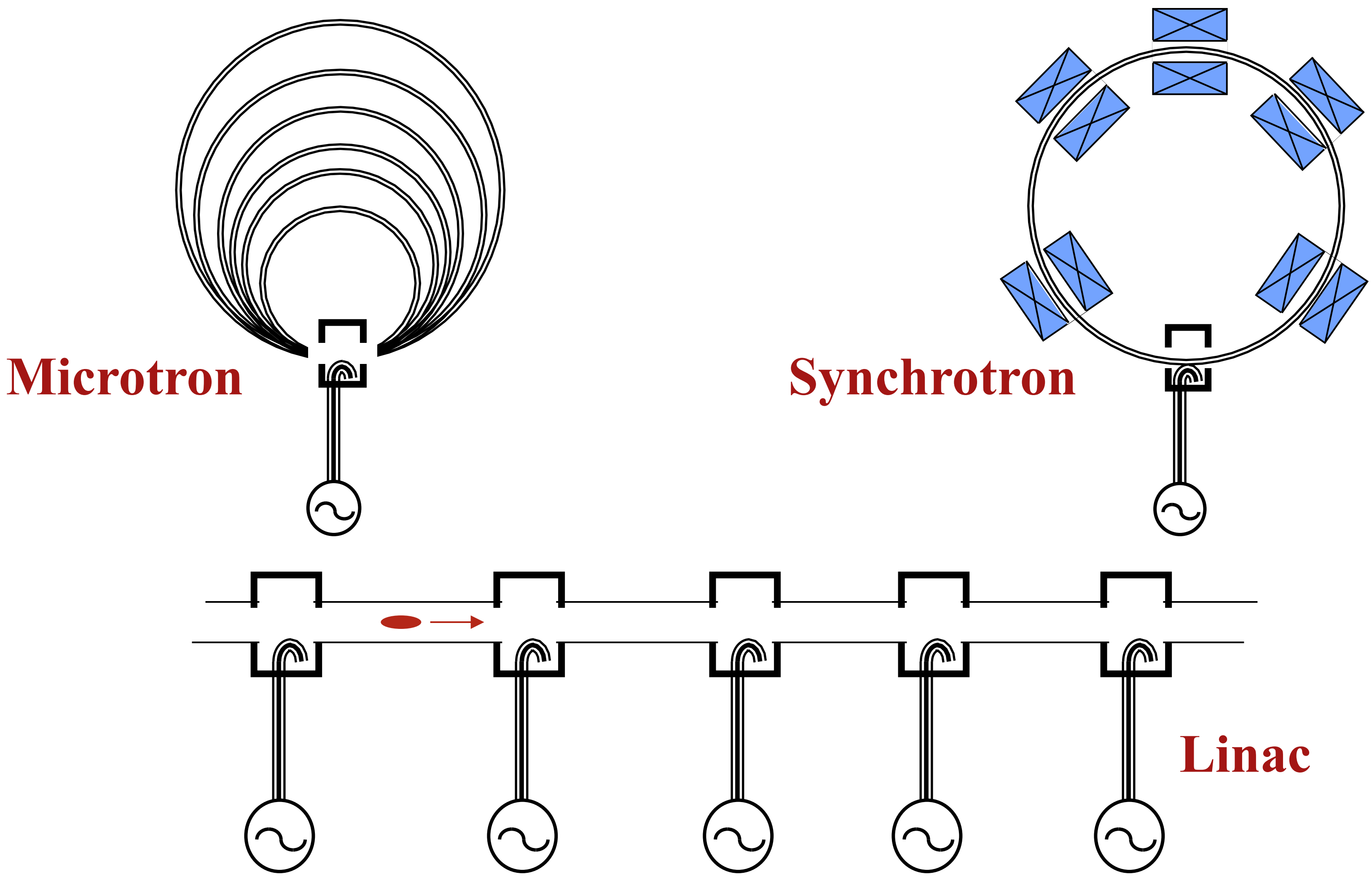
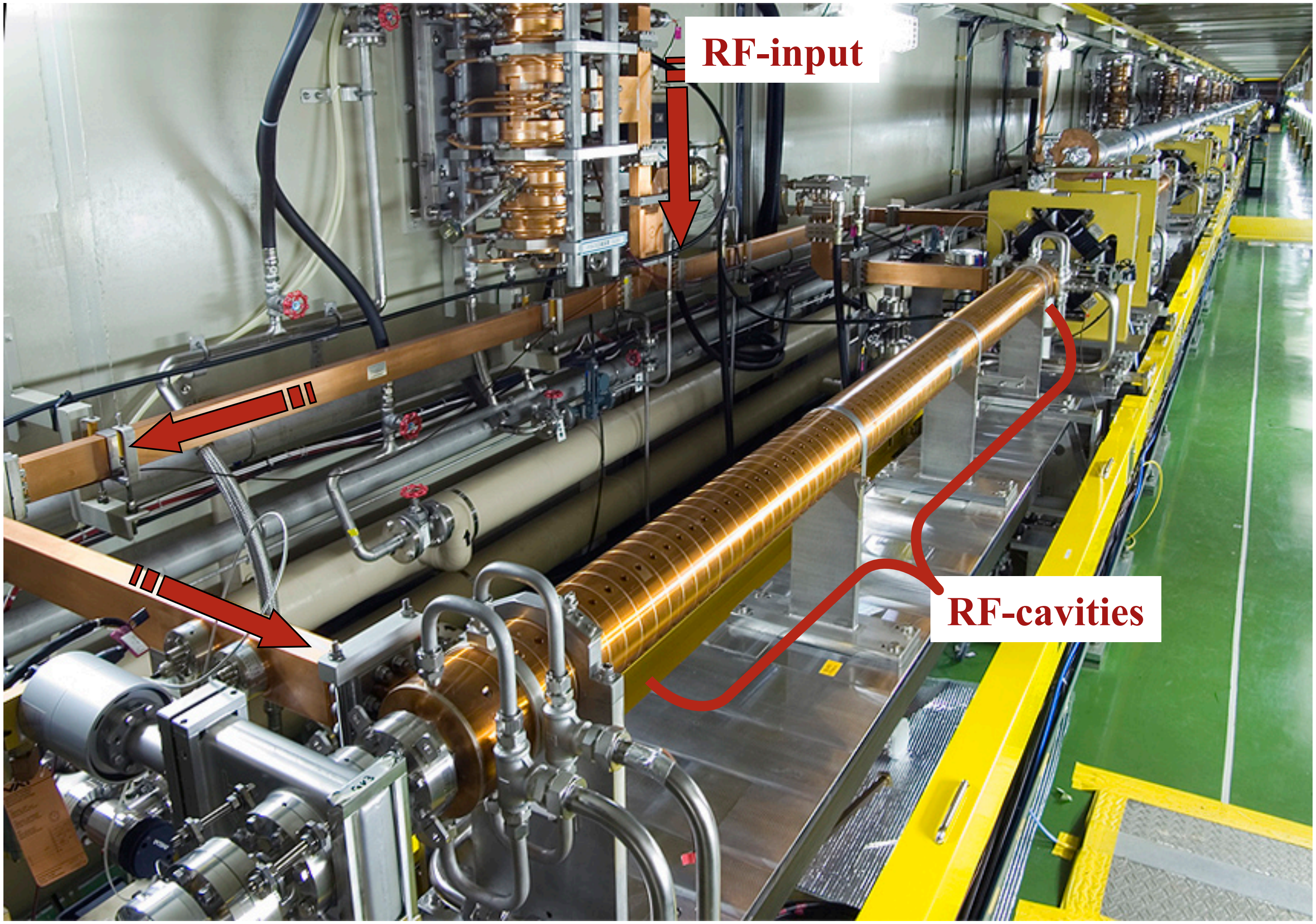
RF-cavities for acceleration

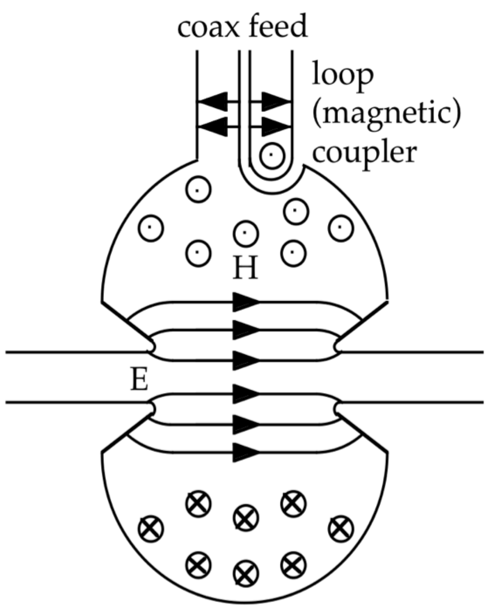


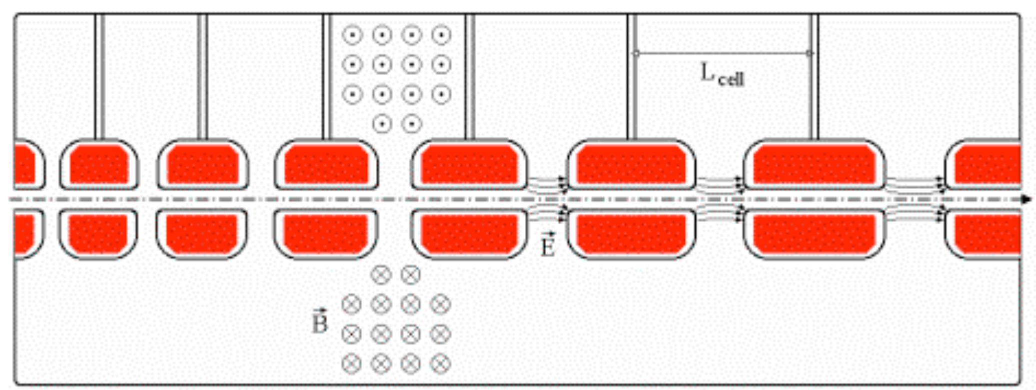
S-band (~3 GHz) RF linac



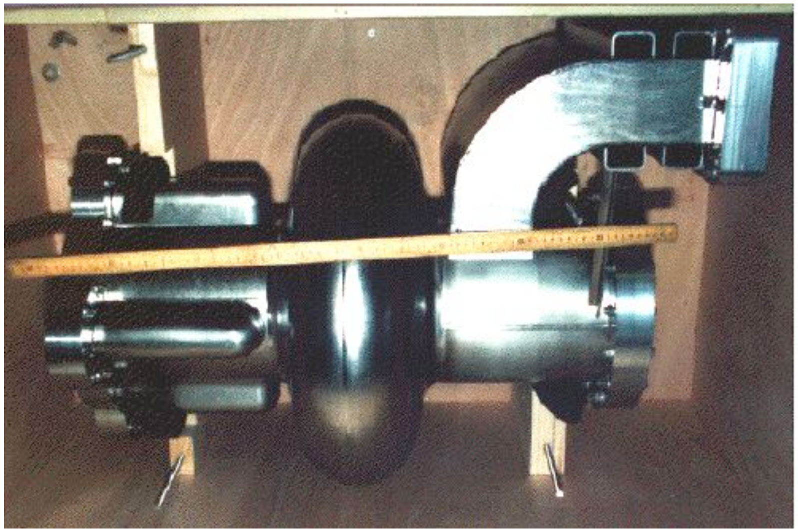
RF Cavities

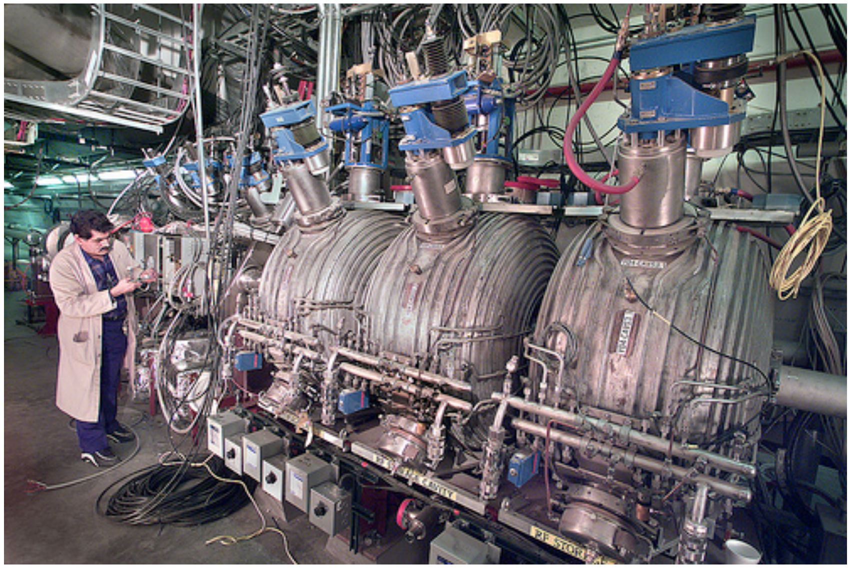
- Radiofrequency (RF) cavities are used to
  - Store energy for accelerating beam
  - Extract beam energy (for example as a beam pickup)
  - Modulate beam energy or position as a beam kicker

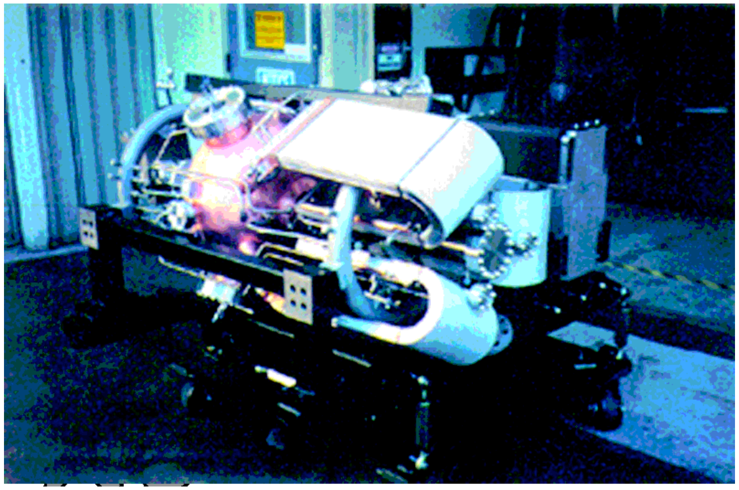




Example RF Cavities







Accelerating Cavity

Lorentz force $\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$

An accelerating cavity needs to provide an electric field E longitudinal with the velocity of the particle

Magnetic fields provide deflection but no acceleration

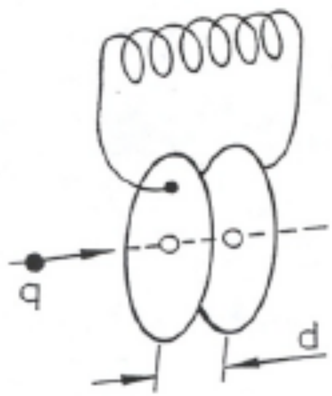
DC electric fields can provide energies of only a few MeV

Higher energies can be obtained only by transfer of energy from traveling waves →resonant circuits

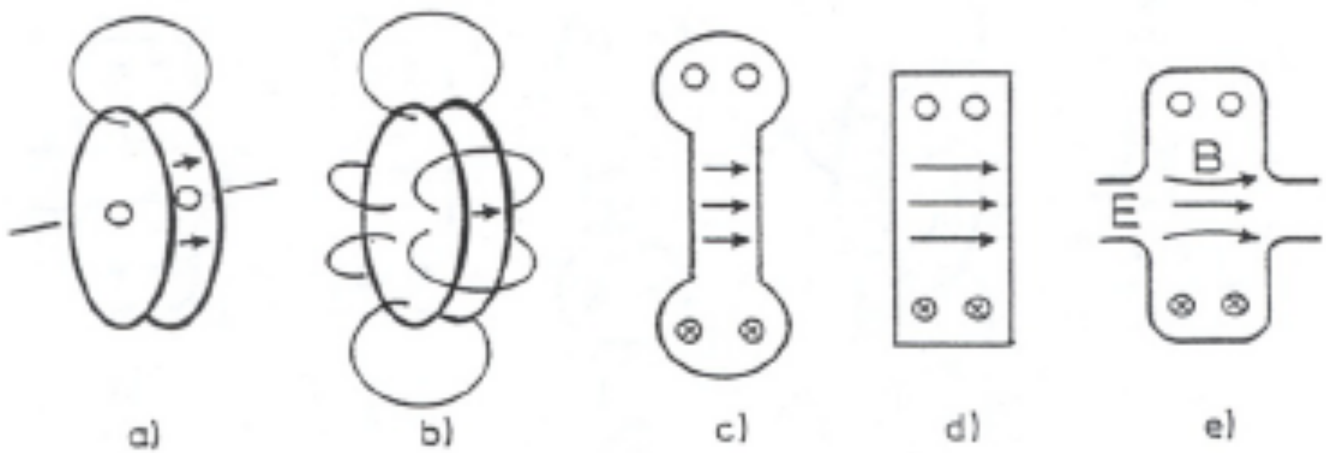
Transfer of energy from a wave to a particle is efficient only is both propagate at the same velocity

Energy Gain $$E = \frac{1}{L}\int E_z(z)\cos(\omega z / \beta c)dz$$

Parallel Resonant Circuit Model



- Imagine two capacitive plates with a parallel inductor.
- This creates a resonator with resonant frequency $\omega_o = \frac{1}{\sqrt{LC}}$
- If the inductor becomes many single loops of wire, this eventually becomes an accelerating cavity



Ohm's Law Generalized

- Basic approach is the Fourier analysis of a circuit
- Start with $\tilde{V} = Ve^{j(\omega t+\varphi)}$
- Instead of V = IR where the quantities are real we write

$$\tilde{V}(\omega) = \tilde{I}(\omega)\overleftrightarrow{Z}(\omega)$$

- We know this works for resistors.

V(t) = R I(t) ==> Z_R is real = R

- What about capacitors & inductors?

Impedance of Capacitors

- For a capacitor

$$I = C\left(\frac{dV}{dt}\right) \Rightarrow \tilde{I} = C\frac{d}{dt}Ve^{j(\omega t+\varphi)} = j\omega C\tilde{V}$$

- So our generalized Ohm's law is

$$\tilde{V} = \tilde{I}\overleftrightarrow{Z}_C$$

where

$$\tilde{Z}_C = \frac{1}{j\omega C}$$

Impedance of Inductors

❖ For a capacitor

$$V = L\left(\frac{dI}{dt}\right) \Rightarrow \tilde{V} = L\frac{d}{dt}Ie^{j(\omega t+\varphi)} = j\omega L\tilde{I}$$

❖ So our generalized Ohm's law is

$$\tilde{V} = \tilde{I}\overleftrightarrow{Z}_L$$

Where

$$\tilde{Z}_L = j\omega L$$

Combining impedances

❖ The algebraic form of Ohm's Law is preserved

==> impedances follow the same rules for combination in series and parallel as for resistors

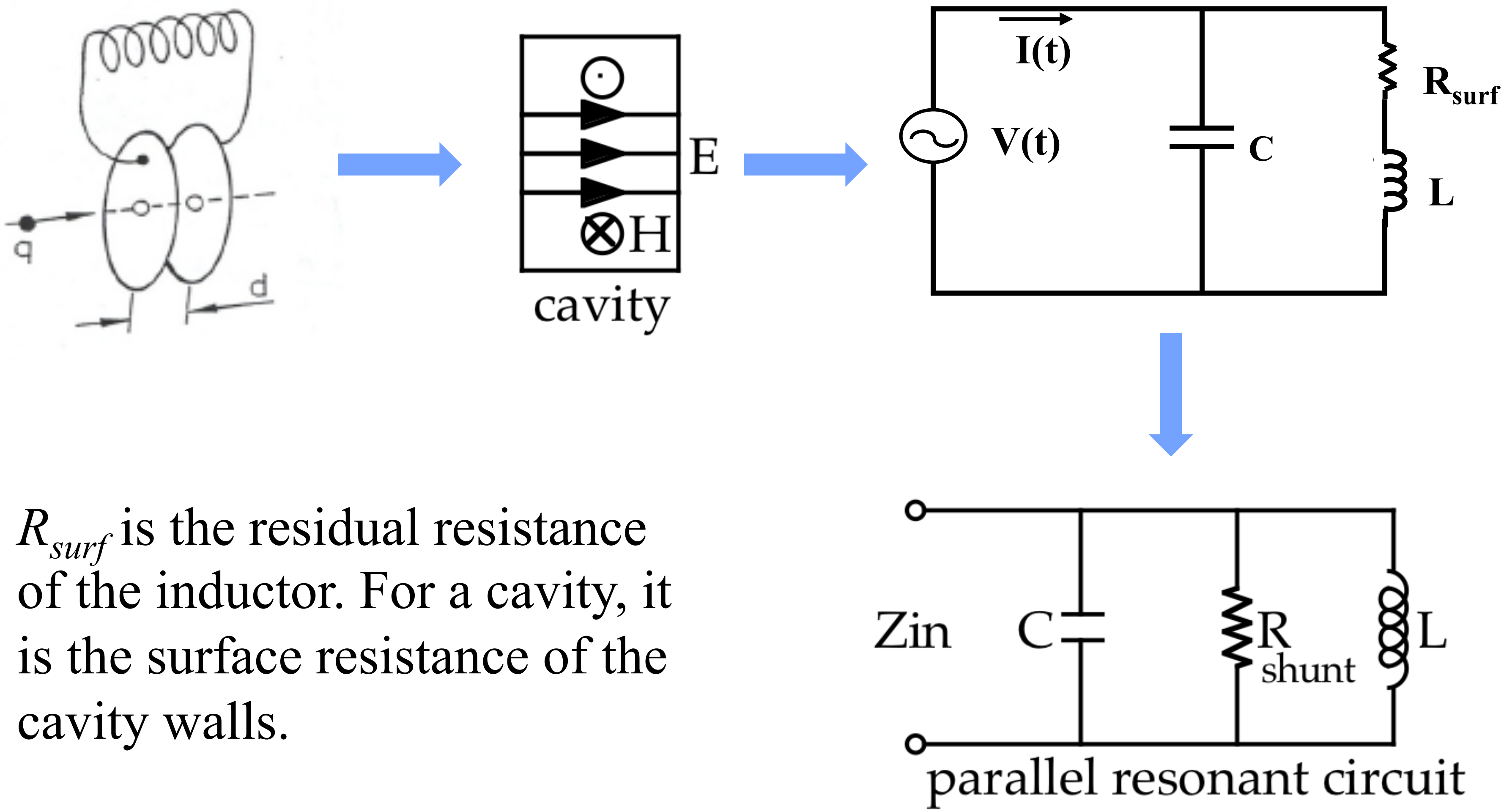
❖ For example

$$\tilde{Z}_s = \tilde{Z}_1 + \tilde{Z}_2$$

$$\tilde{Z}_p = \left[1/\tilde{Z}_1 + 1/\tilde{Z}_2\right]^{-1} = \frac{\tilde{Z}_1\tilde{Z}_2}{\tilde{Z}_1 + \tilde{Z}_2}$$

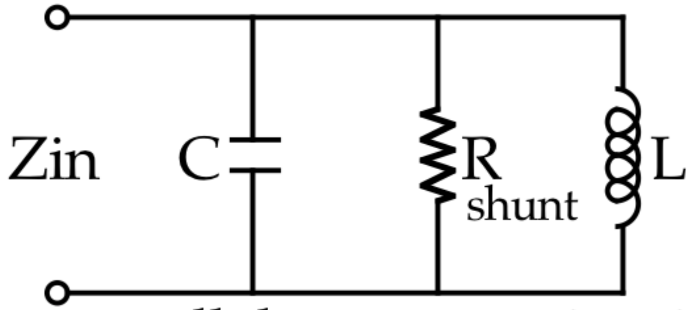
❖ We can now solve circuits using Kirkhoff's laws, *but in the frequency domain*

Circuit analog for a resonant cavity mode



R_{surf} is the residual resistance of the inductor. For a cavity, it is the surface resistance of the cavity walls.

Parallel Resonant Circuit Model



parallel resonant circuit

$$Z_{in} = \left(\frac{1}{R} + \frac{1}{j\omega L} + j\omega C\right)^{-1}$$

$$= \frac{R}{1 + jQ\left(\frac{\omega}{\omega_o} - \frac{\omega_o}{\omega}\right)}$$

$$\approx \frac{R}{1 + jQ2\left(\frac{\delta\omega}{\omega_o}\right)}$$

- Treat impedance of an isolated cavity mode as a parallel LRC circuit
- Voltage gained is given by

$$P_{loss} = \frac{1}{2}\frac{V^2}{R_s}$$

$$\frac{R}{Q} = \sqrt{\frac{L}{C}} = \frac{1}{\omega C} = \omega L$$

$$\omega_o = \frac{1}{\sqrt{LC}} \qquad Q = \frac{R}{\omega_o L} = \omega_o RC$$

Basic principles and concepts

- Superposition
- Energy conservation
- Orthogonality (of cavity modes)
- Causality

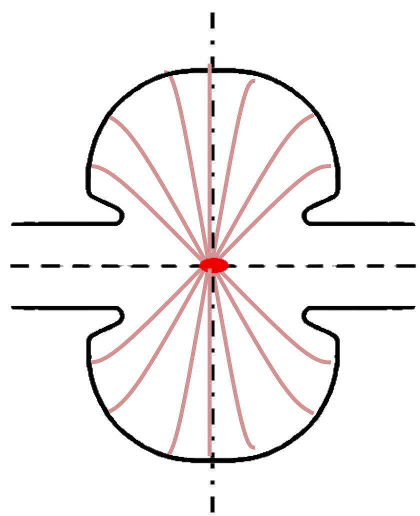
Basic principles: Reciprocity & superposition

❖ If you can kick the beam, the beam can kick you

==>

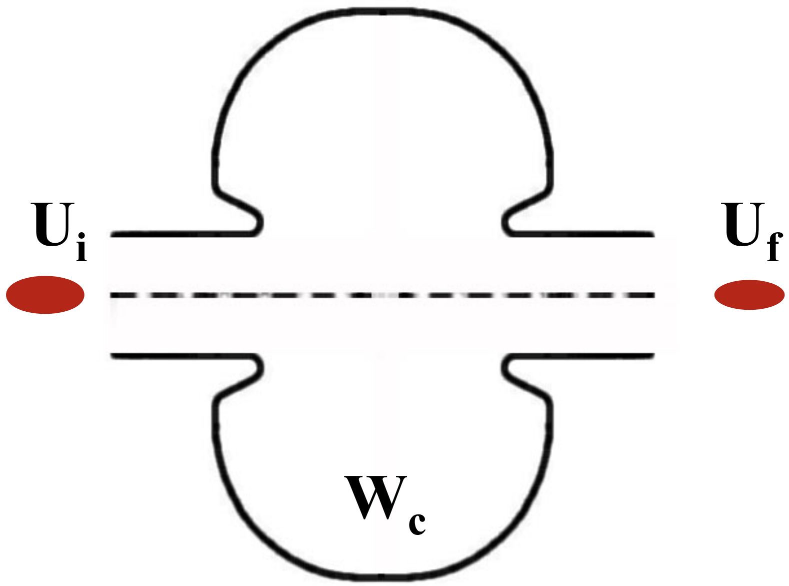
$$\text{Total cavity voltage} = V_{generator} + V_{beam\text{-}induced}$$

$$\text{Fields in cavity} = \mathbf{E}_{generator} + \mathbf{E}_{beam\text{-}induced}$$



Basic principles: Energy conservation

- Total energy in the particles and the cavity is conserved
  - Beam loading



$$\Delta W_c = U_i - U_f$$

Basics: Orthogonality of normal modes

- Each mode in the cavity can be treated independently in computing fields induced by a charge crossing the cavity.
- The total stored energy is equals the sum of the energies in the separate modes.
- The total field is the phasor sum of all the individual mode fields at any instant.

Basic principles: Causality

- There can be no disturbance ahead of a charge moving at the velocity of light.
- In a mode analysis of the growth of the beam-induced field, the field must vanish ahead of the moving charge for each mode.

Translate circuit model to a cavity model: Directly driven, re-entrant RF cavity

Outer region: Large, single turn Inductor

$$L = \frac{\mu_o \pi a^2}{2\pi(R+a)}$$

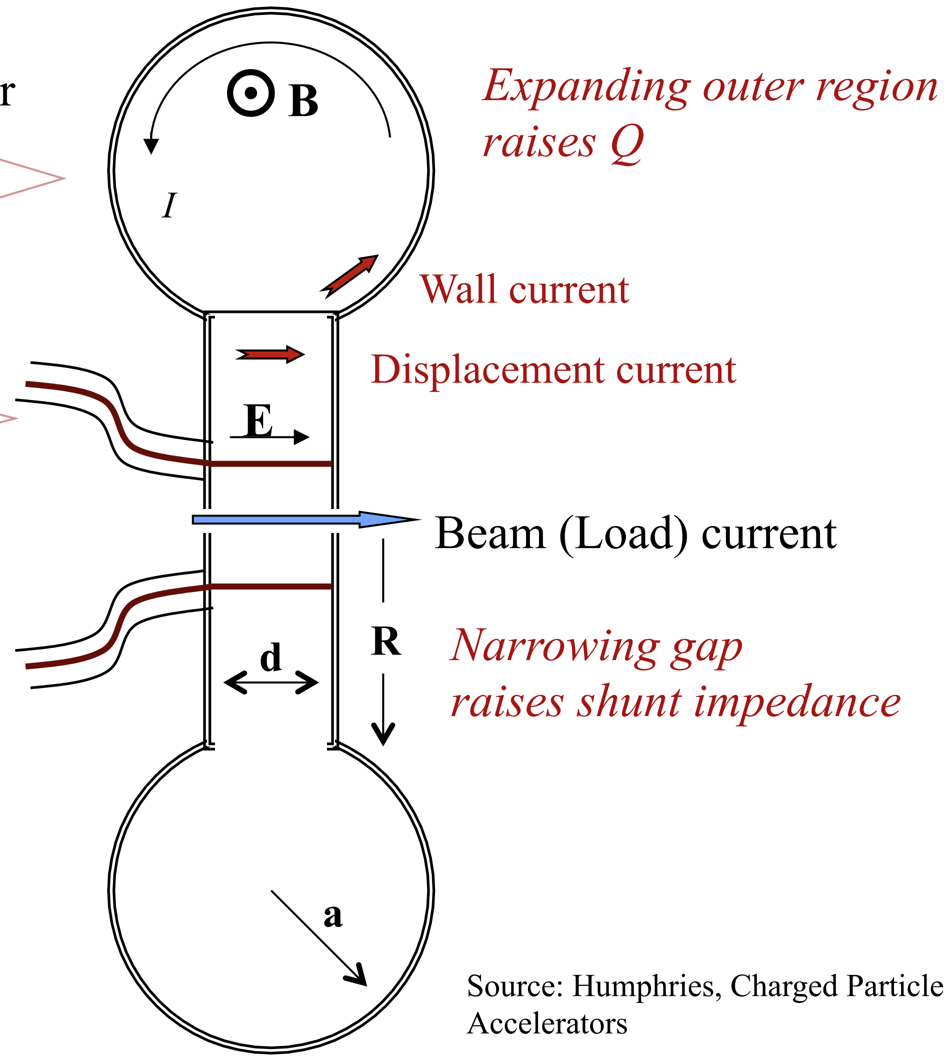
Central region: Large plate Capacitor

$$C = \varepsilon_o \frac{\pi R^2}{d}$$

$$\omega_o = 1 \Big/ \sqrt{LC} = c\left[\frac{2((R+a)d}{\pi R^2 a^2}\right]^{1/2}$$

Q – set by resistance in outer region

$$Q = \sqrt{L/C} \Big/ R$$



Source: Humphries, Charged Particle Accelerators

Quality Factor Q_0:

$$Q_0 \equiv \frac{\text{Energy stored in cavity}}{\text{Energy dissipated in cavity walls per radian}} = \frac{\omega_0 U}{P_{diss}}$$

$$= \omega_0 \tau_0 = \frac{\omega_0}{\Delta\omega_0}$$

$$Q_0 = \frac{\omega\mu_0}{R_s} \frac{\int_V dV \, |\mathbf{H}|^2}{\int_A da \, |\mathbf{H}_{||}|^2}$$

Lower surface resistance gives higher Q. For a given R/Q, this gives higher R. Lower external power is required for a given voltage V.

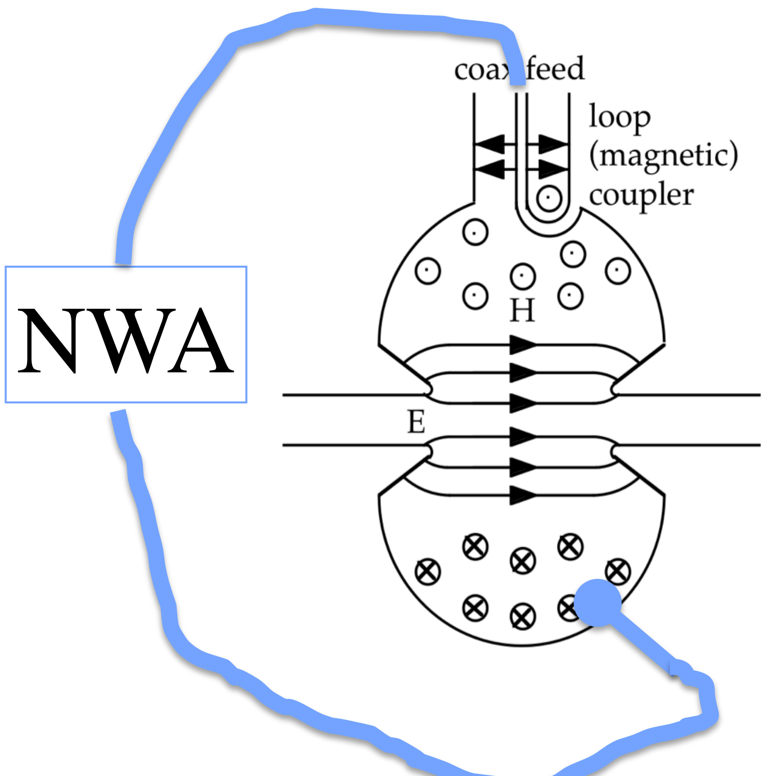
$$\mathscr{E} = \frac{\mu_o}{2} \int_v |H|^2 dv = \frac{1}{2} L \, I_o I_o^*$$

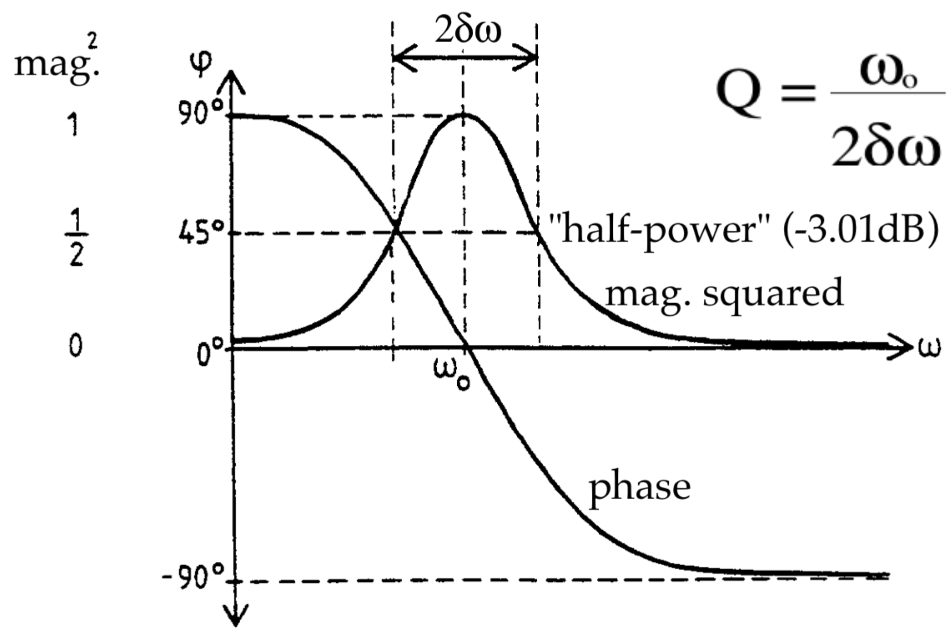
$$\langle \mathscr{P} \rangle = \frac{R_{surf}}{2} \int_s |H|^2 ds = \frac{1}{2} I_o I_o^* R_{surf}$$

$$R_{surf} = \frac{1}{Conductivity \circ Skin\ depth} \sim \omega^{1/2}$$

Transmission (S21) measurement

- Mode resonant frequency and Q can be measured in a transmission measurement.
- This is usually done with a network analyzer coupled to two cavity probes.

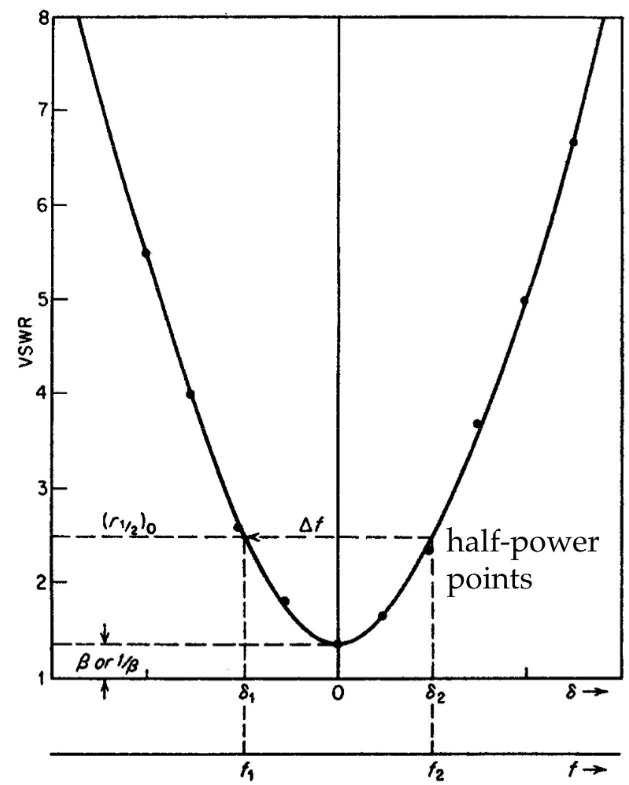




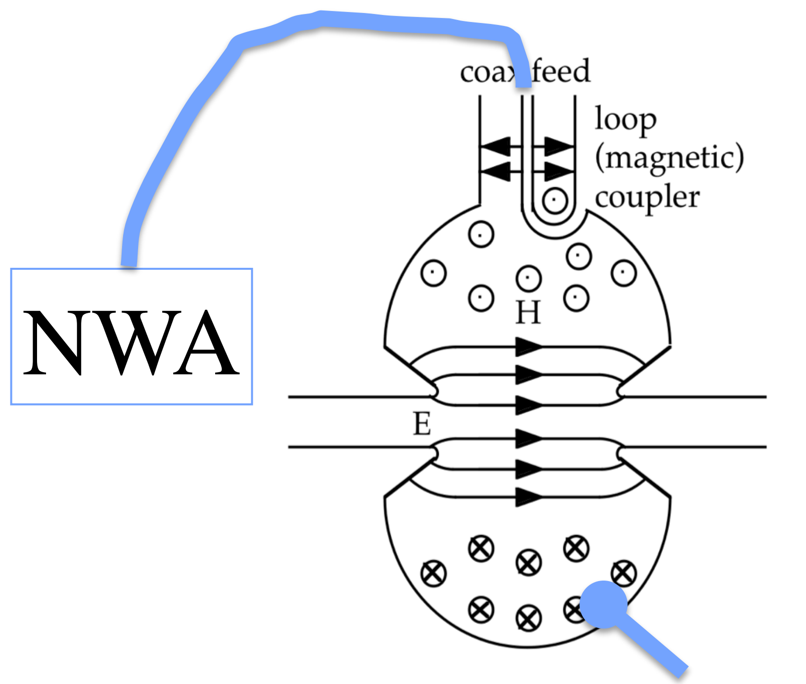
$$Q = \frac{\omega_o}{2\delta\omega}$$

Reflection (S11) near resonance

- The cavity frequency, Q, and coupling factor can be measured in reflection

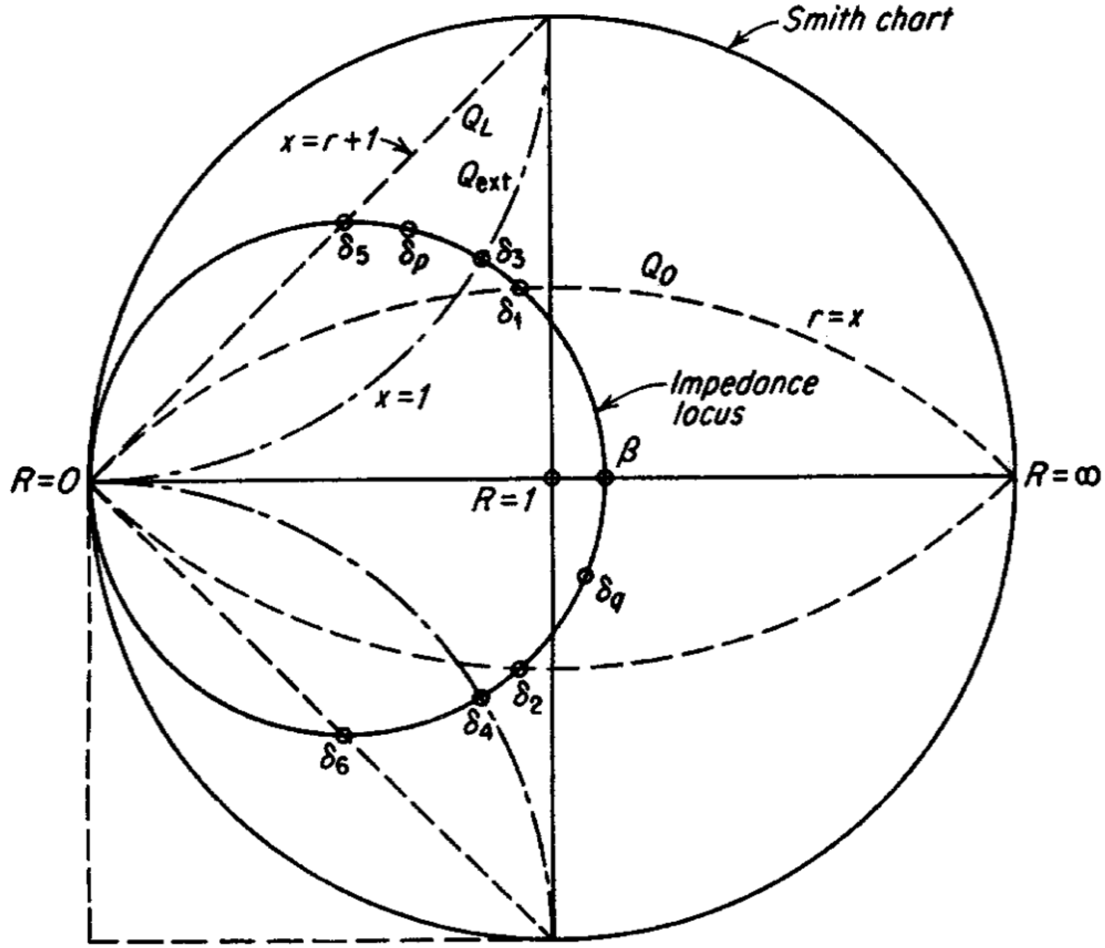


VSWR close to resonance



Smith Chart View

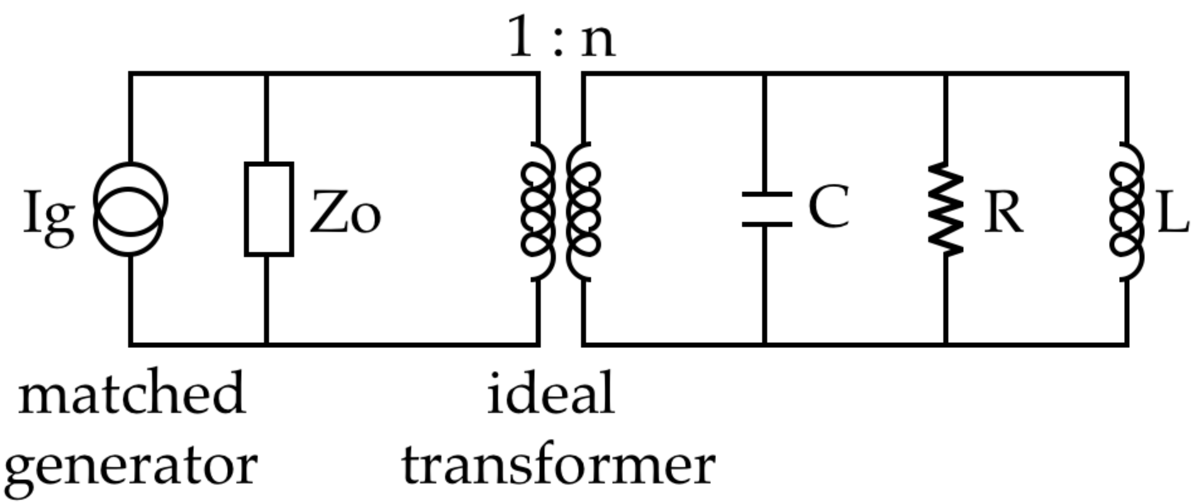
- The Smith Chart (Phillip Smith) is a polar display of the complex reflection coefficient i.e. phase and amplitude)
- A resonator appears as a circle in this format



Identification of the half-power points from the Smith chart.

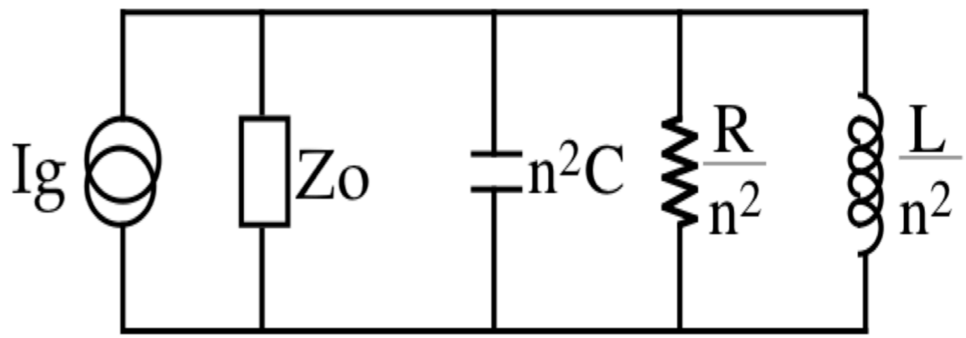
Cavity Coupling

- To accelerate beam, we have to power the cavity.
- Typical power sources include klystrons, tetrodes, etc.
- It is most efficient to match the load (cavity+beam) to the power source.
- Model the generator as a matched current source and coupling as a n turn transformer.

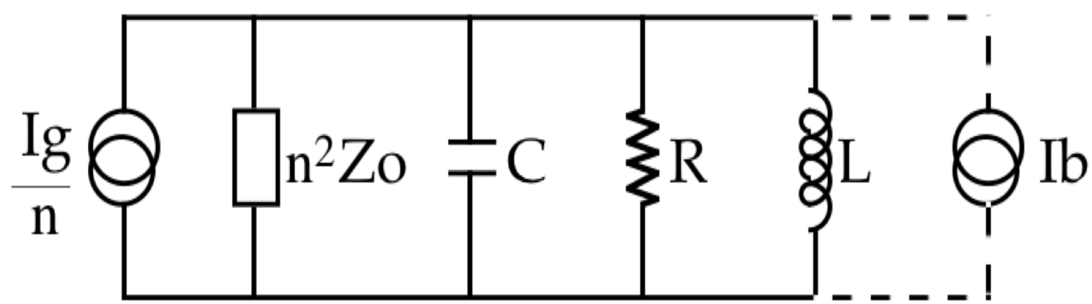


Coupling to external generator/load

- Transform the cavity impedance to the load



- Transform the generator current to the load



Cavity Coupling

- This model allows efficient power transfer to the cavity/ beam.
- Some definitions

$$\beta = \frac{\text{power loss in ext. cct}}{\text{power loss in cavity}} = \frac{Q_o}{Q_{ext}} = \frac{R}{n^2 Z_o}$$ Coupling beta

$$\frac{1}{Q_L} = \frac{1}{Q_o} + \frac{1}{Q_{ext}}$$ Loaded Q

$$Q_o = (1+\beta) Q_L$$ Loaded Q

Optimal coupling (no beam)

$$\beta = 1 \quad Q_L = \frac{Q_o}{2} \qquad n^2 = \frac{R}{Z_o}$$

Resonant Modes

Electromagnetic modes satisfy Maxwell equations

$$\left(\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\begin{Bmatrix}\vec{E}\\ \vec{H}\end{Bmatrix} = 0$$

With the boundary conditions (assuming the walls are made of a material of low surface resistance)

no tangential electric field $\vec{n}\times\vec{E} = 0$

no normal magnetic field $\vec{n}\bullet\vec{H} = 0$

Resonant Modes

Assume everything $\sim e^{-i\omega t}$

$$\left(\nabla^2 + \frac{\omega^2}{c^2}\right)\begin{Bmatrix}\vec{E} \\ \vec{H}\end{Bmatrix} = 0$$

For a given cavity geometry, Maxwell equations have an infinite number of solutions with a sinusoidal time dependence

For efficient acceleration, choose a cavity geometry and a mode where:

Electric field is along particle trajectory

Magnetic field is 0 along particle trajectory

Velocity of the electromagnetic field is matched to particle velocity

Example: PEP-II Cavity

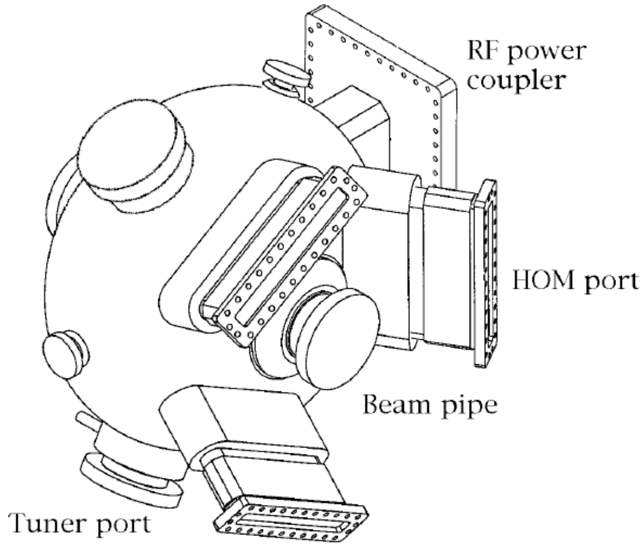
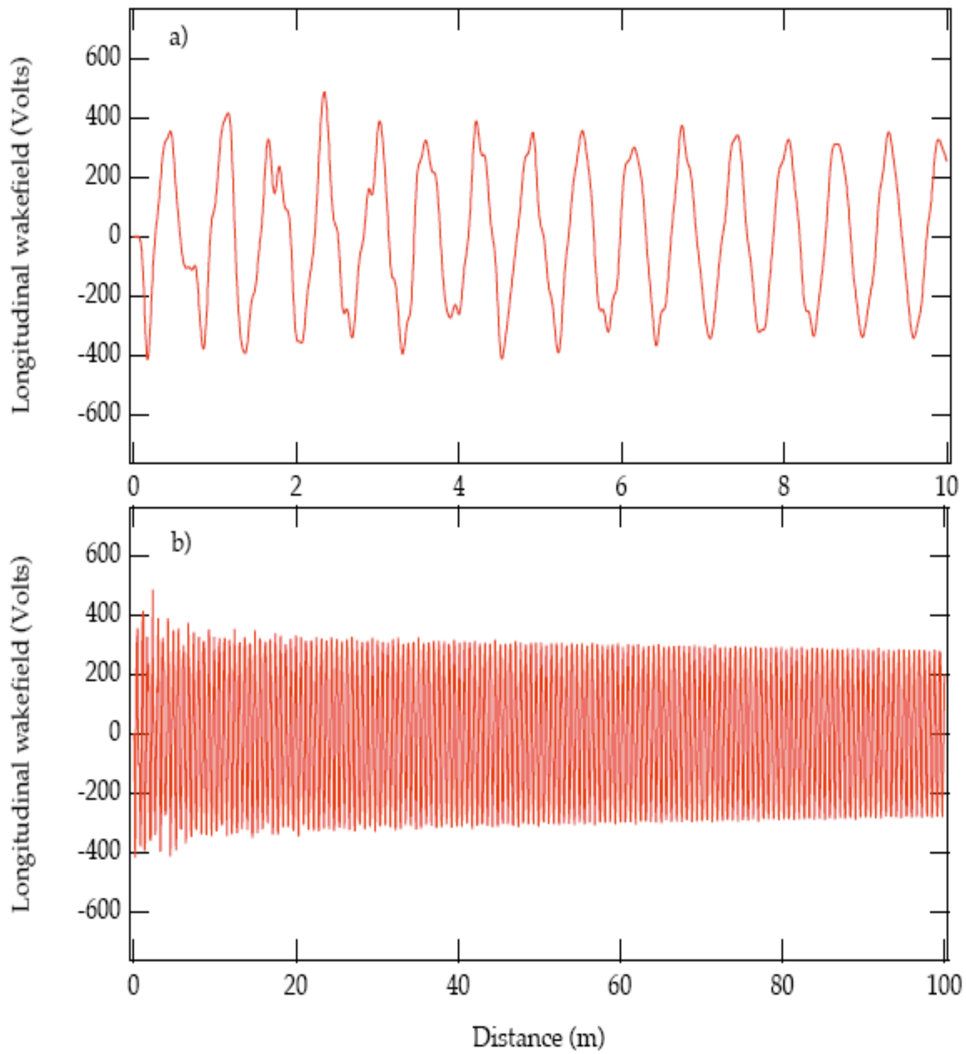


FIG. 1. CAD drawing of the PEP-II cavity. The waveguides are designed to couple to the cavity HOMs.

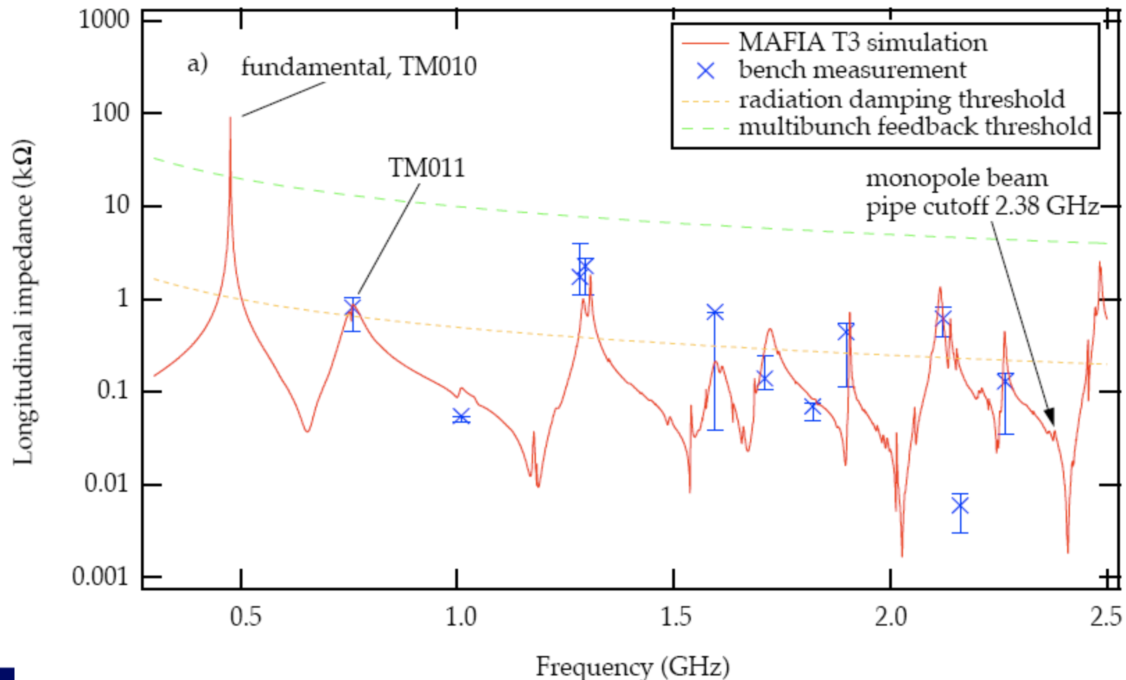
- Cavities have large number of resonant modes.
- Frequencies extend up to cutoff frequency of the beam pipe (TE or TM)
- Ringing is usually dominated by cavity fundamental mode.

Modeled ringing of the PEP-II cavity.

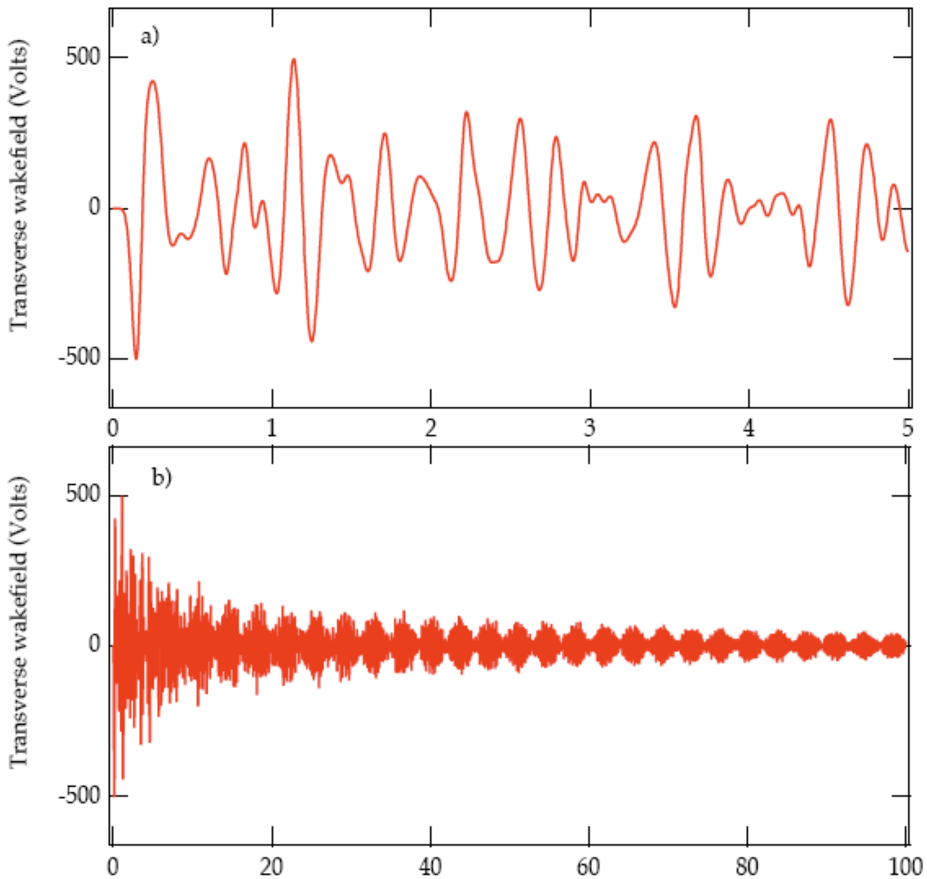


PEP-II Impedance

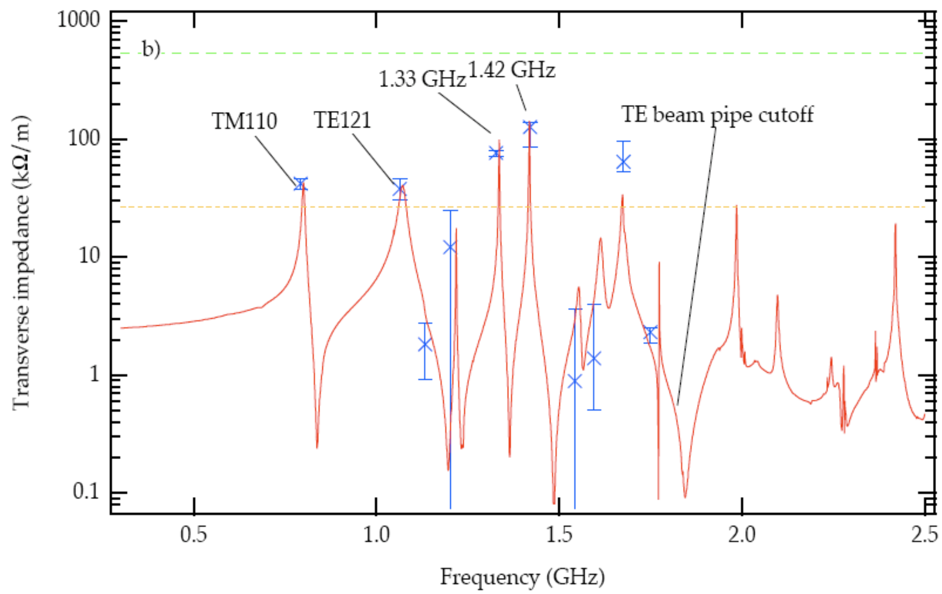
- The time domain wake in frequency domain is called the beam impedance
- Each mode can be defined by a frequency, Q, and shunt impedance
- Usually higher order modes are not wanted and some attempt is made to damp them.



PEP-II Transverse Wake

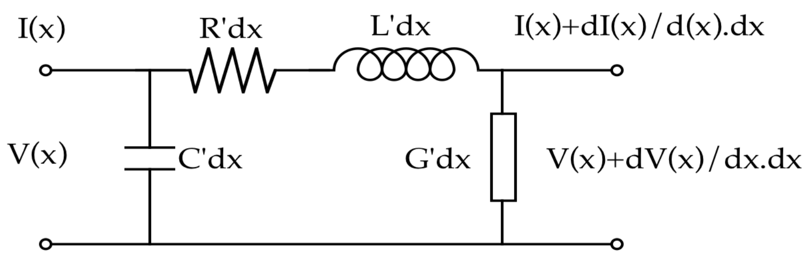


- A similar situation occurs for cavity modes which can give transverse beam deflections.

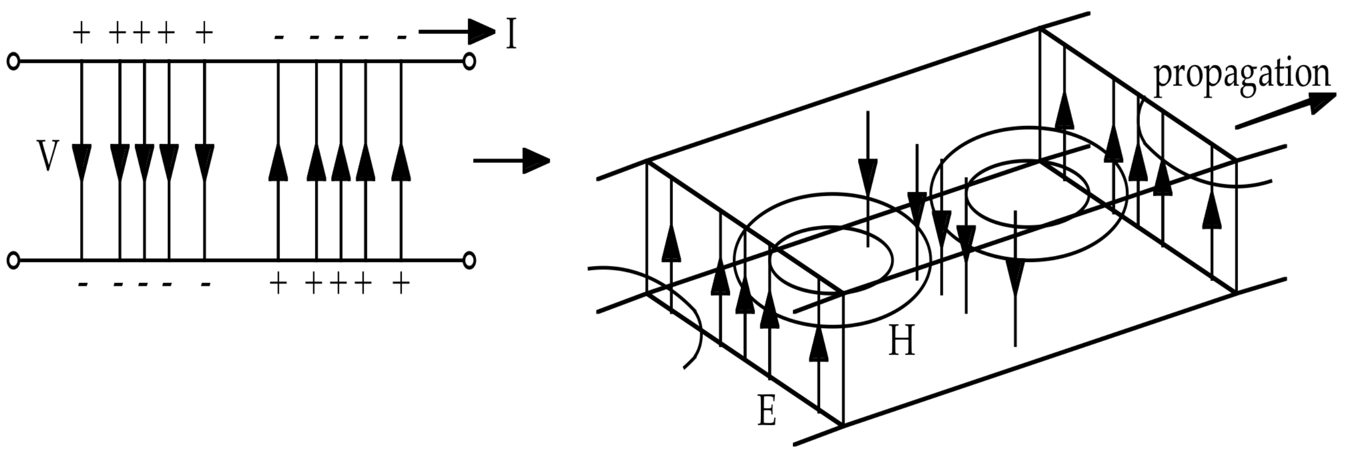


Cavity modes: transmission line analogy

❖ Consider the equivalent circuit for a transmission line

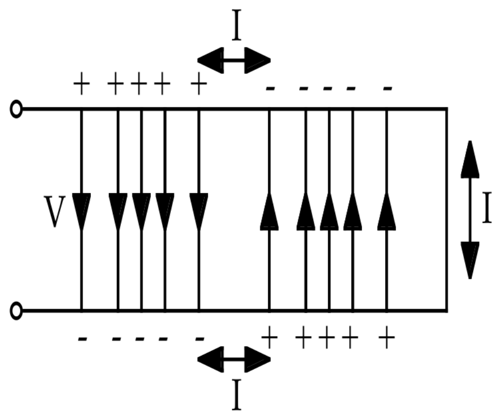


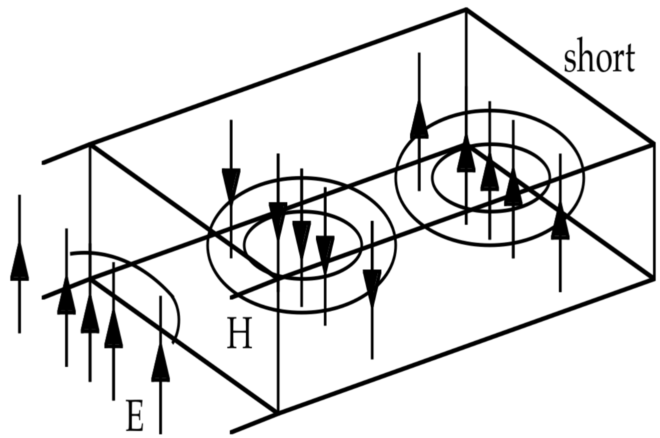
• This gives a field pattern



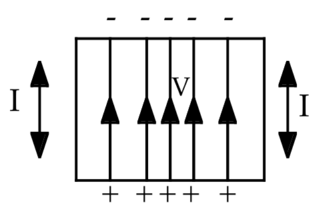
Cavity modes: transmission line analogy

❖ If we short one end of the line, forward and reflected waves give a standing wave

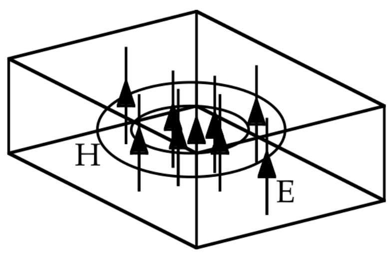




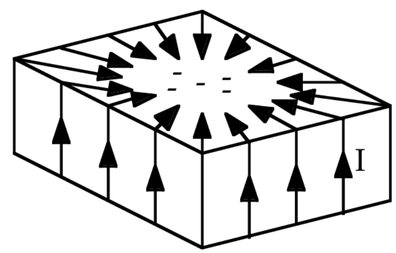
- If we short the other end of the line, we make a cavity



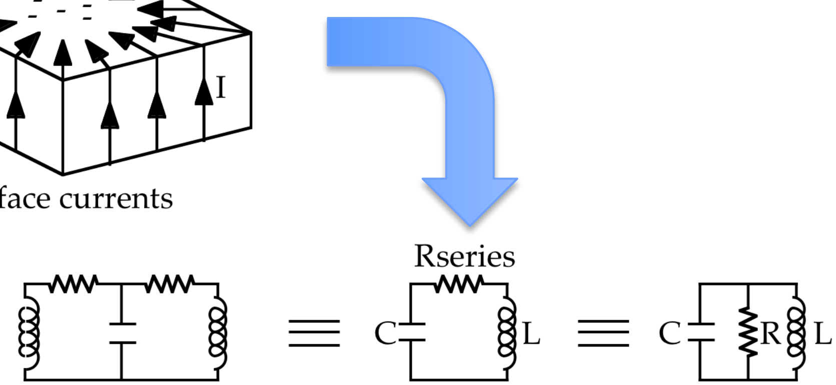
shorted line



standing wave



surface currents

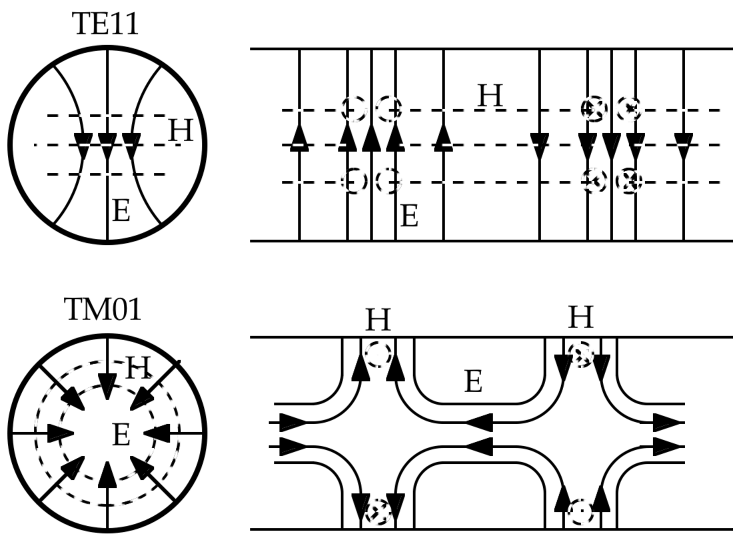


equivalent circuit

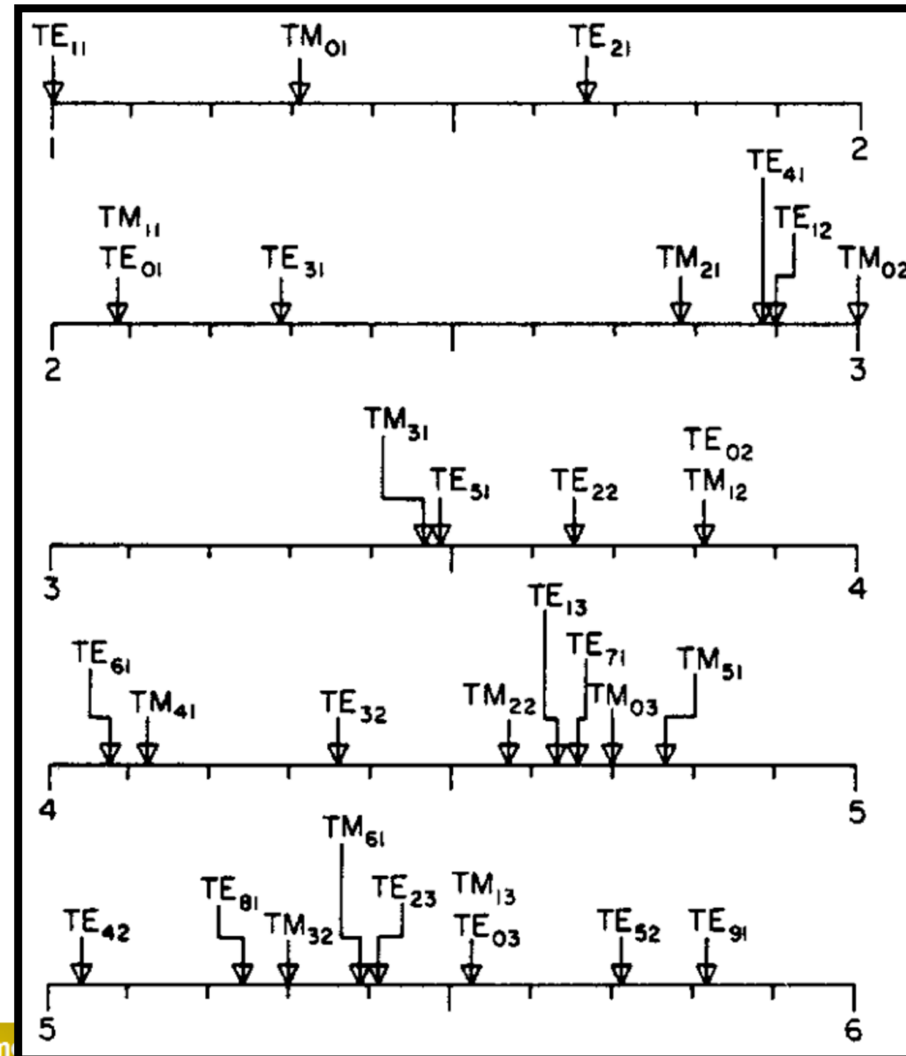
series circuit

parallel circuit

- Consider the two lowest modes of a cylindrical waveguide (TE=transverse electric field, TM=transverse magnetic field)



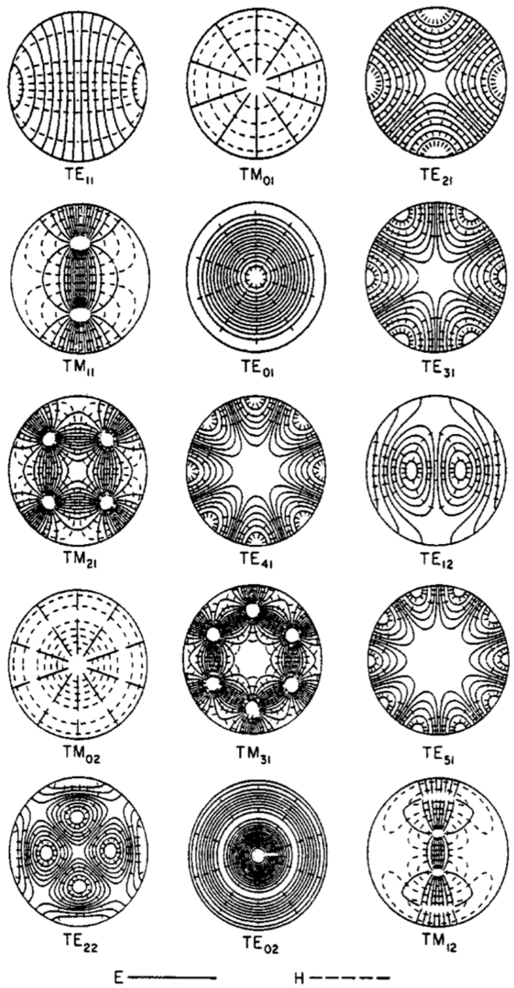
First two modes in circular waveguide

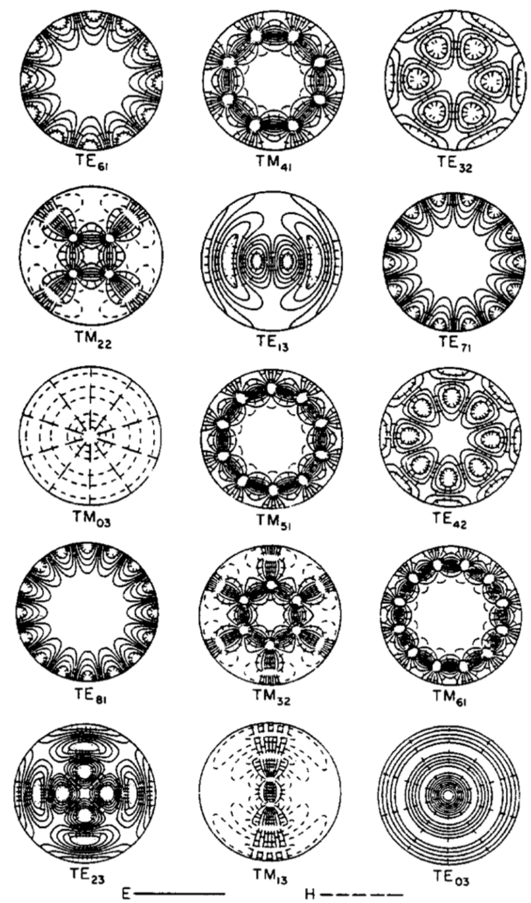


Cut-off frequencies for modes in circular waveguide, normalized to that of the lowest mode (TE11).

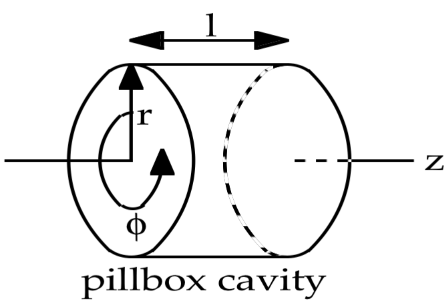
Pillbox Cavities

- Many different modes; only TM modes have field along beam direction and can interact with beam





- The transverse variations of the longitudinal field are solutions of Maxwell's equations within a circular boundary condition and are Bessel functions of the first kind.



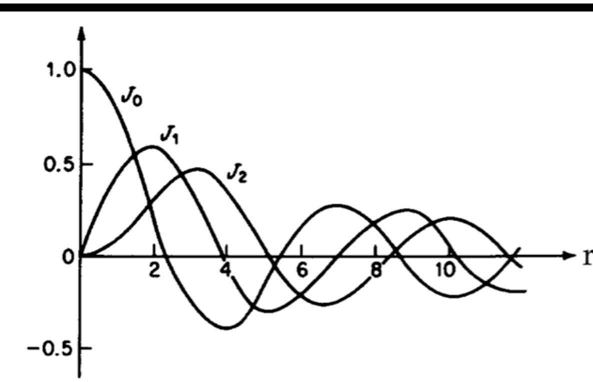
$$E_z(r, \phi) = E_o J_m(k_{mn} r) \cos m\phi$$

J_m are the first order Bessel functions
$k_{mn} = x_{mn}/r$ is the transverse wave number
x_{mn} are the roots of the Bessel functions J_m

- For TM_{mnz} modes the fields are:

$$E_z(r,z,t, \phi) = E_o J_m(\frac{x_{mn}}{a} r) e^{j\omega t} \cos(m\phi)\cos(k_z z)$$

$$H_\phi(r,z,t, \phi) = H_o J'_m(\frac{x_{mn}}{a} r) e^{j\omega t} \cos(m\phi)\cos(k_z z)$$



Low-order Bessel functions of the first kind

Monopole modes

- Modes which have no azimuthal variation are labelled “monopole” modes and TM modes of this type have longitudinal electric field on axis and thus can interact strongly with the beam.
- The radial distribution of E_z follows J_0, where the zeros satisfy the boundary condition that $E_z = 0$ at the conducting wall at radius a. Similarly H_f and E_r (if present) follow J'_0 and are zero in the center and have a finite value at the wall.

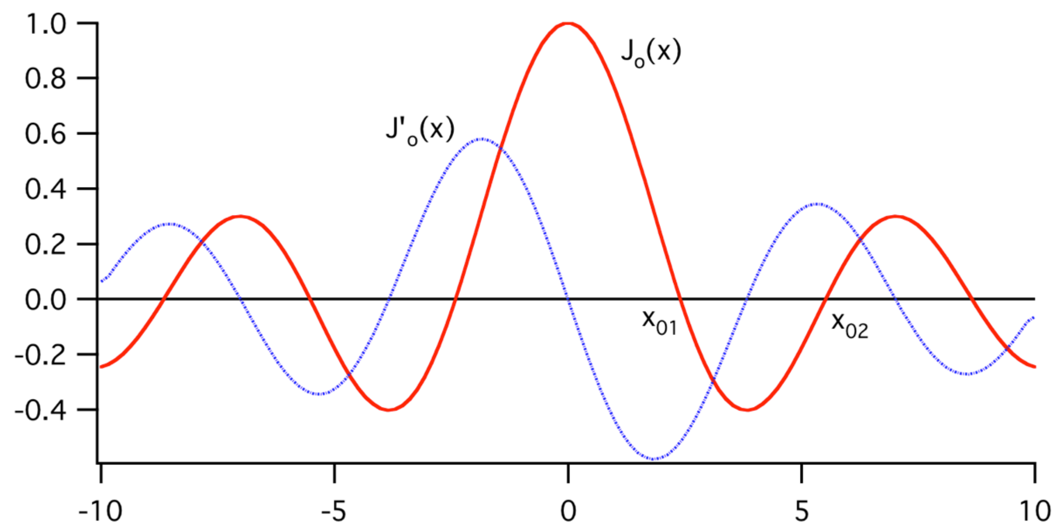
For TM_{oni} modes:

$$E_z = E_o J_o(k_{on} r)\cos(k_z z) \text{ where } k_{on} = x_{on}/a \text{ and } k_z = i\pi/\text{length } (i \geq 0)$$

$$H_\phi = H_{\phi o} J'_o(k_{on} r)\cos(k_z z) \qquad x_{o1} = 2.405$$

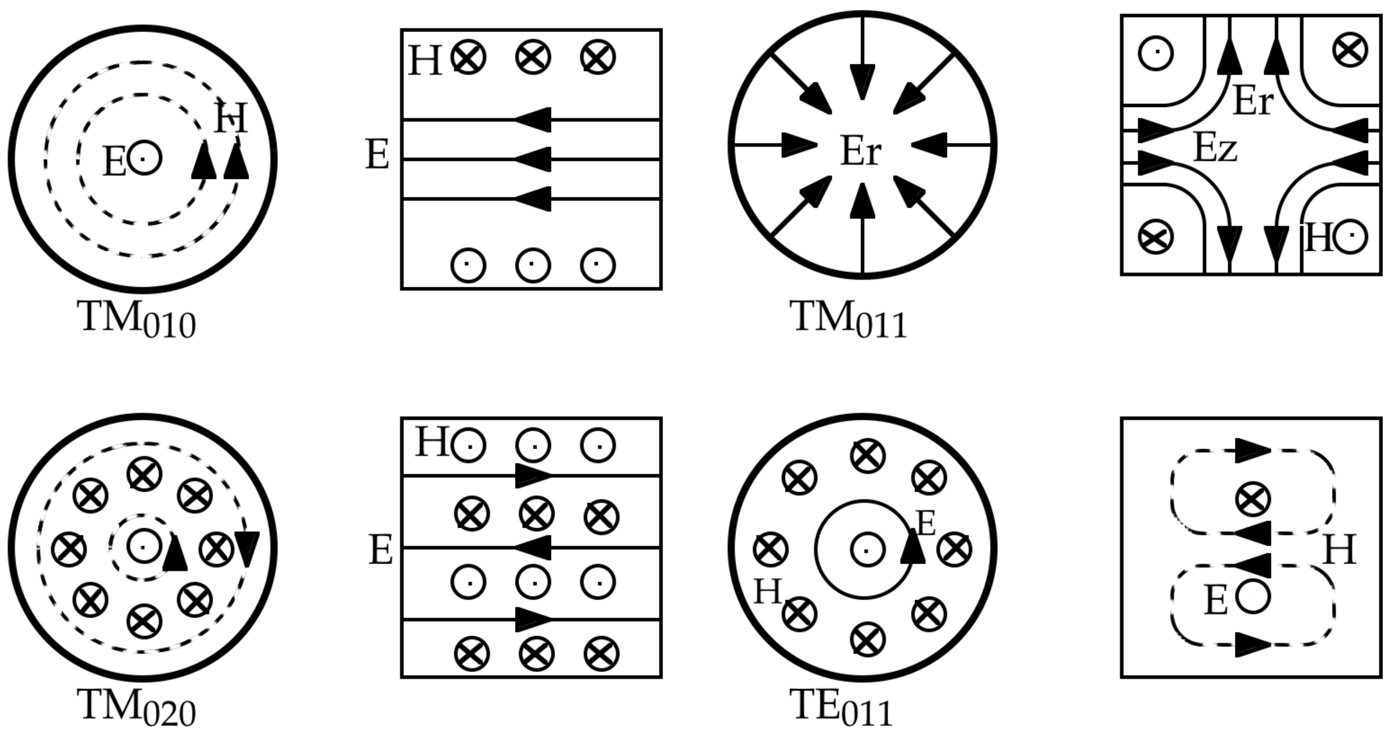
$$E_r = E_{ro} J'_o(k_{on} r)\sin(k_z z) \qquad x_{o2} = 5.520$$

$$x_{o3} = 8.654$$



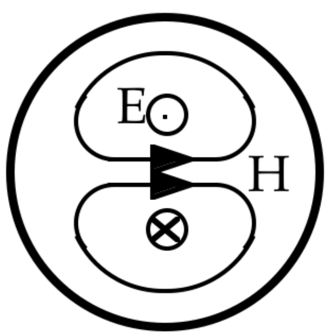
Monopole modes

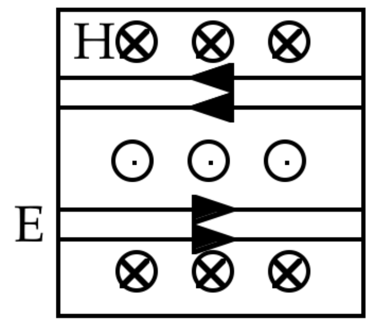
❖ A few example field patterns



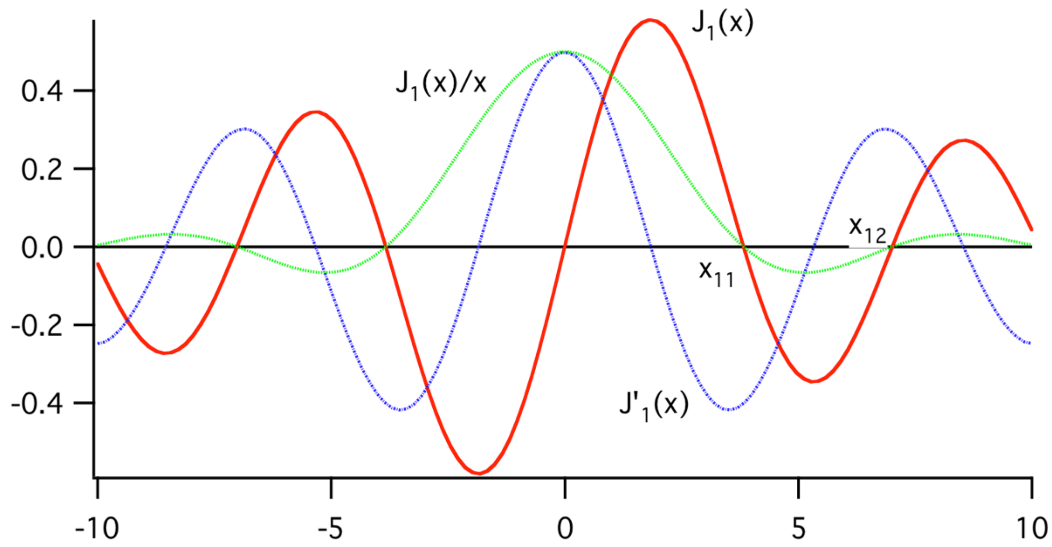
Dipole modes

- Characterized by one full period of variation around the azimuth (m=1)
- For TM modes this means there is no longitudinal field on axis and that the field strength grows linearly with radius close to the center, with opposite sign either side of the axis.
- This transverse gradient to the longitudinal field gives rise to a transverse voltage kick which is proportional to the beam current and the beam offset.
- Therefore, no transverse kick to the beam without a longitudinal field gradient (Panofsky-Wenzel Theorem)





- Field distributions for dipole modes



For TM_{1ni} modes:

$$E_z = E_o J_1(k_{1n} r)\cos(\phi)\cos(k_z z) \text{ where } k_{1n} = x_{1n}/a \text{ and } k_z = i\pi/\text{length } (i \geq 0)$$

$$H_\phi = H_{\phi o} J_1'(k_{1n} r)\cos(\phi)\cos(k_z z) \qquad x_{11} = 3.383171$$

$$|H_r| = H_{ro}\frac{J_1'}{r}(k_{1n} r)\sin(\phi)\cos(k_z z) \qquad x_{12} = 7.01559$$

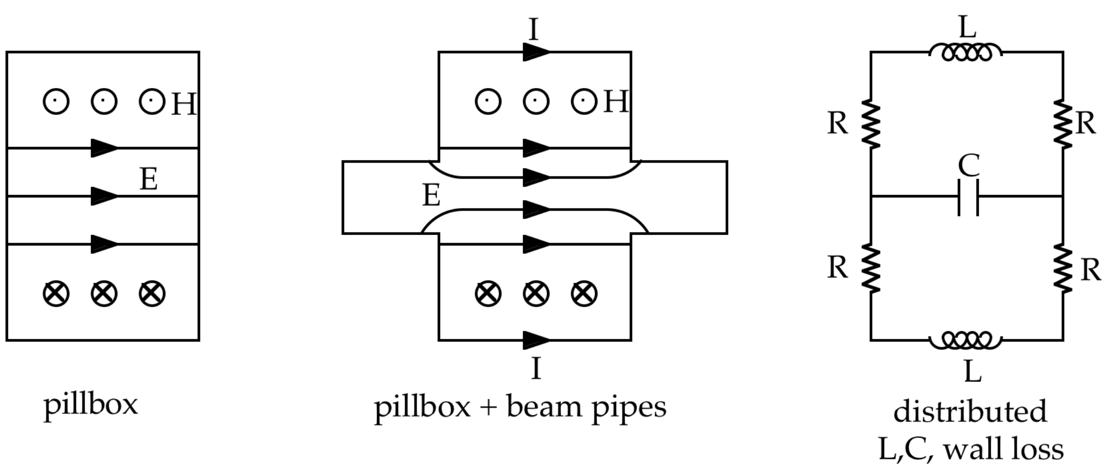
$$x_{13} = 10.17347$$

Higher-order modes (m>1):

- For modes with higher azimuthal order (m=2, quadrupole, m=3, sextupole etc.), the fields close to the beam axis become progressively weaker as the stored energy is concentrated towards the outer edge of the cavity.
- Modes with even m have no sign reversal across the axis and have a small transit time factor. Modes with odd m may couple weakly to transverse motion of the beam but are generally not problematic.

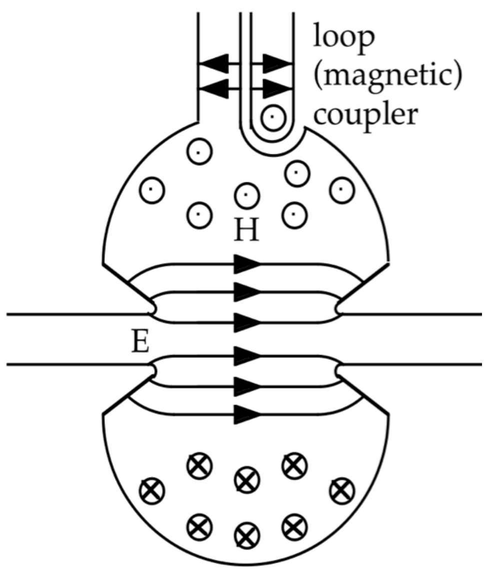
Real Cavities

- In practice the simple pillbox cavity shape can be improved upon to maximize the shunt impedance for acceleration.
- The shape must also be modified to allow passage of the beam and addition of one or more couplers



Normal Conducting Cavities

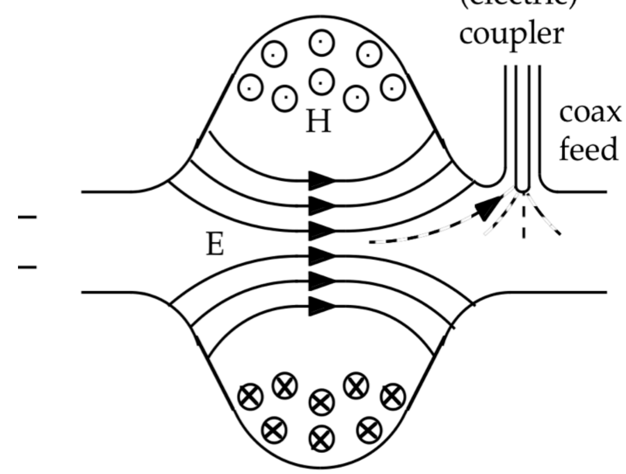
- Since R/Q=sqrt(L/C), maximize L and minimize C, while keeping the optimum interaction length (max. T), and maximum Q
- The "nose-cone" or re-entrant cavity increas the volume occupied by the magnetic field and the surface area carrying the current an decreasing the surface area in the capacitiv region (nose tips).
- Limiting factors to achievable gradient are wall-power dissipation and E-field strength a the nose-tips.



"nose-cone" copper cavity

Super Conducting Cavities

- Smooth shape is determined by the need to avoid field emission from the surface. R/Q is low but Q is very high
- Gradient limited by
  - Field emission from surface impurities
  - Ultimately limited by surface magnetic field



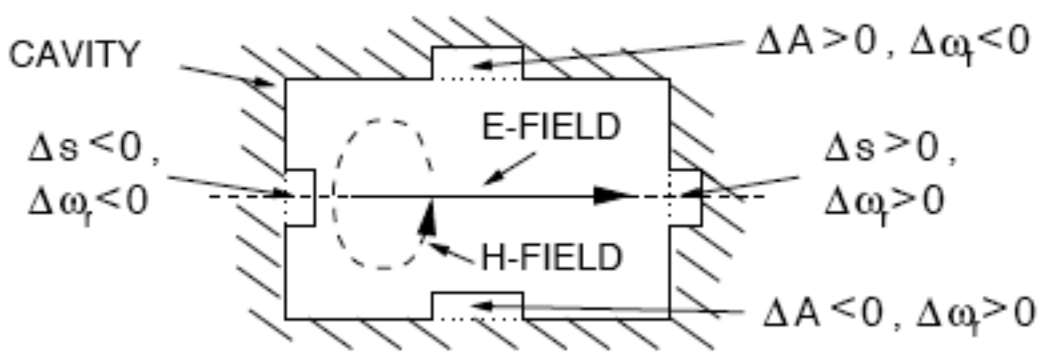
"bell-shaped" superconducting cavity

Cavity Tuning

- Mode resonant frequency can be adjusted by introducing perturbations
- Effect depends on whether perturbation affects electric or magnetic fields (change in capacitance or inductance).

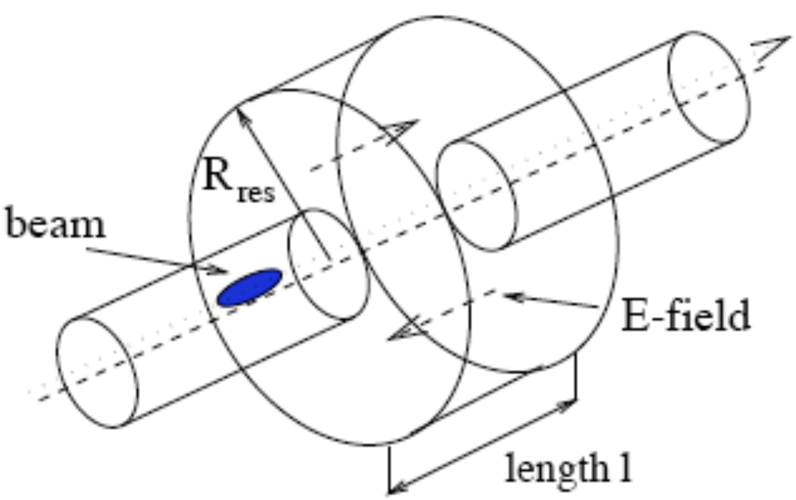
$$(L + \Delta L) = \frac{\mu H}{I}(A + \Delta A)\,. \qquad (C + \Delta C) = \frac{Q}{E\,(s + \Delta s)}.$$

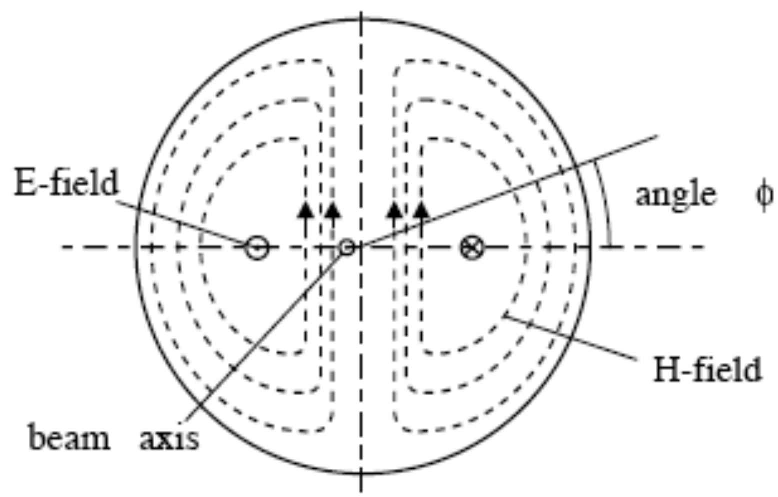
$$\omega_r + \Delta\omega_r = [(L + \Delta L)(C + \Delta C)]^{-\frac{1}{2}}$$



Cavity BPMs

- Use the linear dependence of excited field in the m=1 dipole mode as a measure of beam position.
- Full (x,y) measurement requires each mode polarization, usually achieved with two cavities.





Cavity BPM Sensitivity

❖ Dipole mode parameters (assume pillbox)

$$Q_{110} = \frac{1}{2\pi} \cdot \frac{a_{11}}{1 + (R_{\rm res} \cdot l^{-1})} \cdot \frac{\lambda_{110}}{\delta} \qquad \text{with} \qquad \delta = \sqrt{\frac{1}{\pi \cdot f_{110} \cdot \mu \cdot \kappa}}$$

$$\left(\frac{R}{Q}\right)_{110} = \frac{(V_{110}^{\max})^2}{2\omega_{110} \cdot W_{110}} = \frac{2 \cdot Z_0 \cdot l \cdot (J_1^{\max})^2 \cdot {T_{tr}}^2}{\pi \cdot R_{\rm res} \cdot J_0^2(a_{11}) \cdot a_{11}} \approx 130.73 \cdot \frac{l}{R_{\rm res}} \cdot {T_{tr}}^2$$

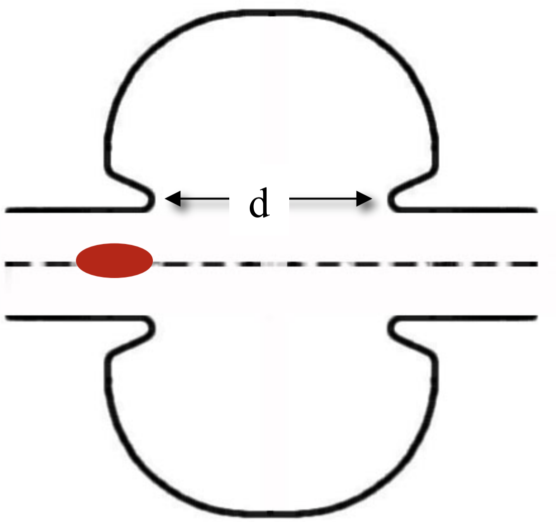
- Dipole mode sensitivity for offset dx

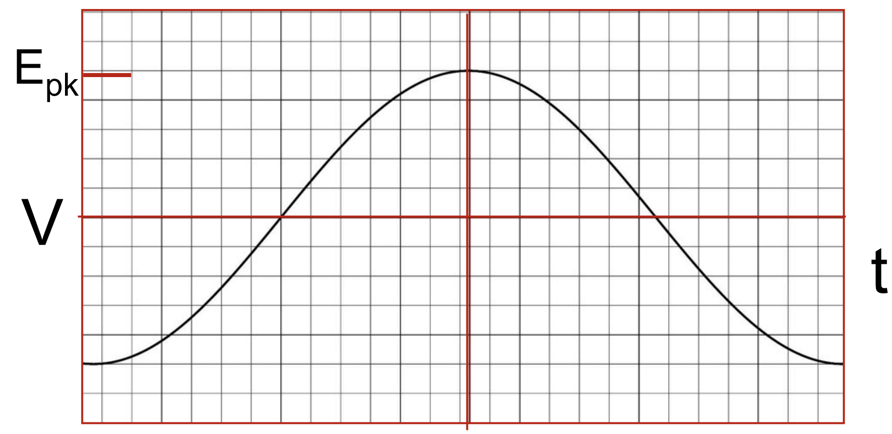
$$V_{110}^{in}(\delta x) = \left(\frac{R}{Q}\right)_{110} \cdot \omega \cdot q \cdot \left\langle \frac{a_{11} \cdot \delta x}{2 \cdot J_1^{max} \cdot R_{\rm res}} \right\rangle = \delta x \cdot q \cdot \frac{l \cdot {T_{tr}}^2}{{R_{\rm res}}^3} \cdot 0.2474 \left[\frac{\rm Vm}{\rm pC}\right]$$

Figure of Merit: Accelerating voltage

- The voltage varies during time that bunch takes to cross gap
  - reduction of the peak voltage by Γ (transt time factor)

$$\Gamma = \frac{\sin\left(\vartheta/2\right)}{\vartheta/2} \quad \text{where} \quad \vartheta = \omega d/\beta c$$





For maximum acceleration $T_{\mathrm{cav}} = \frac{d}{c} = \frac{T_{\mathrm{rf}}}{2}$ ==> $\Gamma = 2/\pi$

Computing shunt impedance

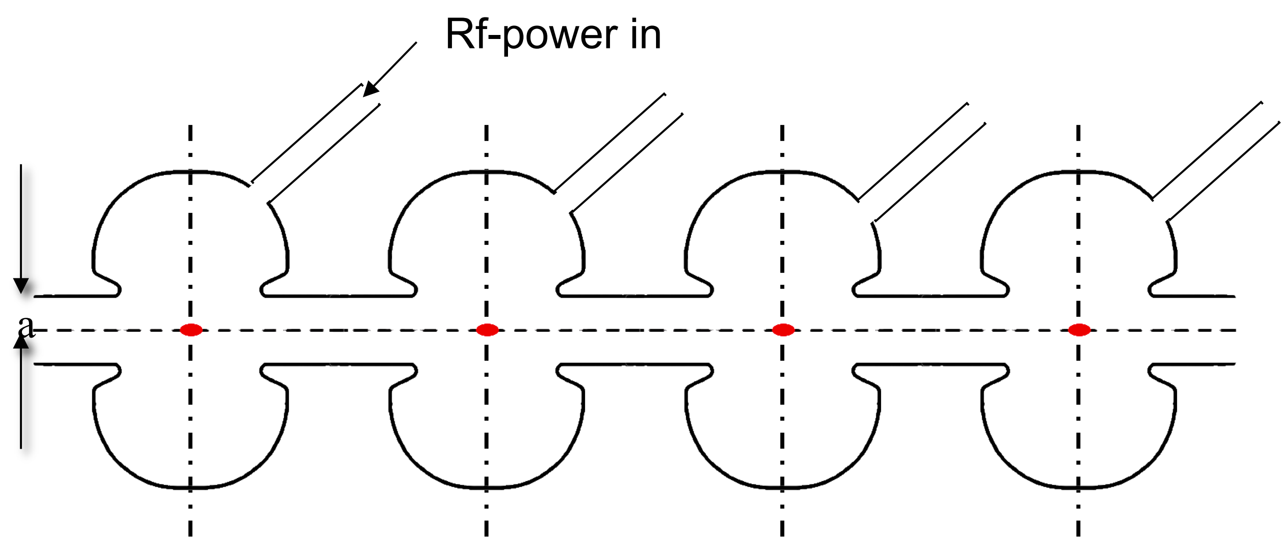
$$\mathscr{R}_{in} = \frac{V_o^2}{2\mathscr{P}}$$

$$\langle \mathscr{P} \rangle = \frac{R_{surf}}{2} \int_s |H|^2 ds$$

$$R_{surf} = \frac{\mu\omega}{2\sigma_{dc}} = \pi Z_o \frac{\delta_{skin}}{\lambda_{rf}} \quad \text{where } Z_o = \sqrt{\mu_o / \varepsilon_o} = 377\Omega$$

The on-axis field E and surface H are generally computed with a computer code such as SUPERFISH for a complicated cavity shape

Multicell cavities: towards the RF linac

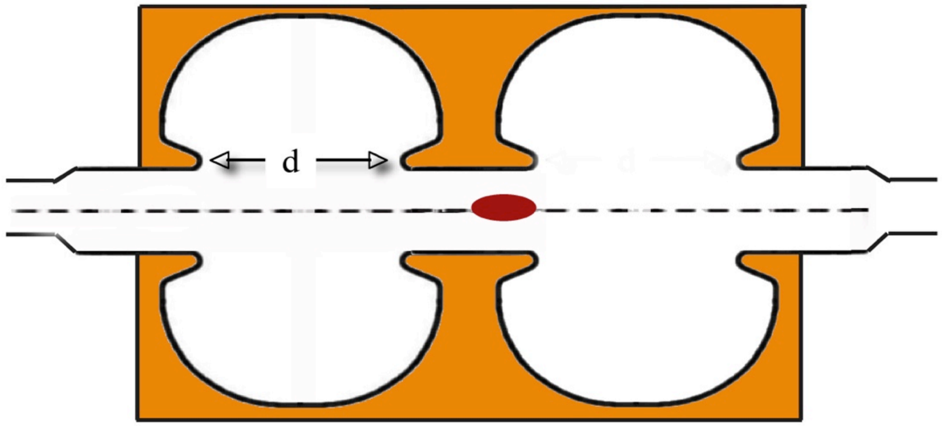


In warm linacs "nose cones" optimize the voltage per cell with respect to resistive dissipation

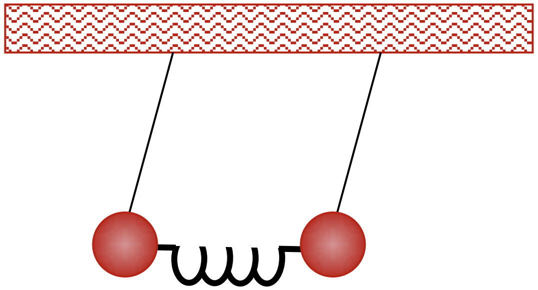
$$Q = \frac{\sqrt{L/C}}{\mathscr{R}_{surface}}$$

Usually cells are feed in groups not individually…. and

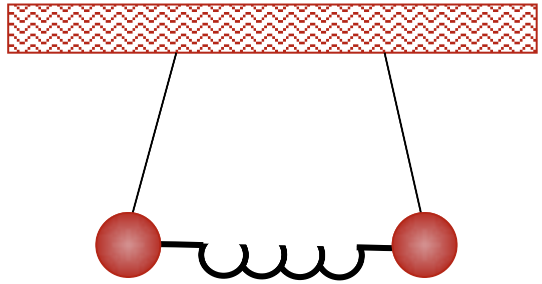
Linacs cells are linked to minimize cost



==> coupled oscillators ==>multiple modes

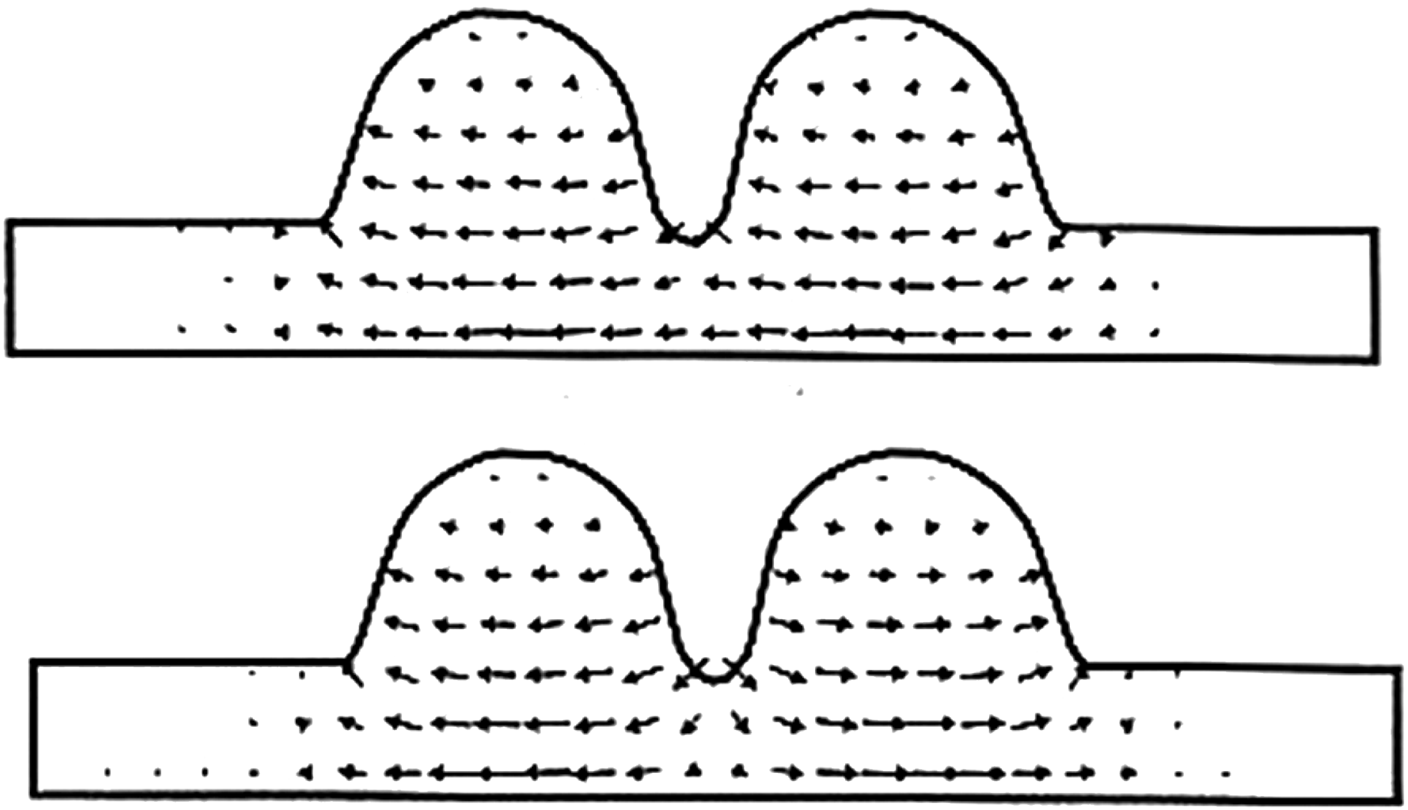


Zero mode

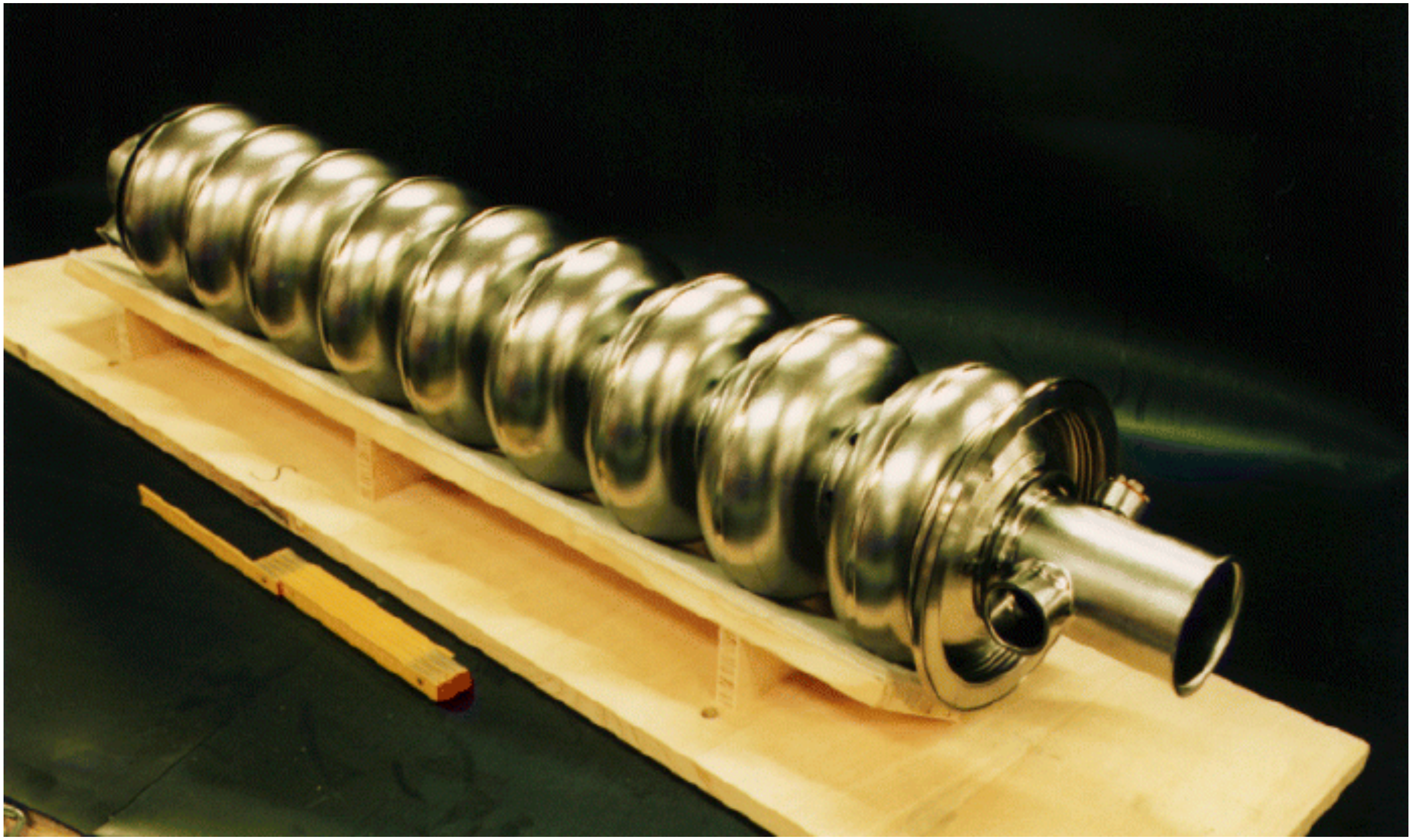


π mode

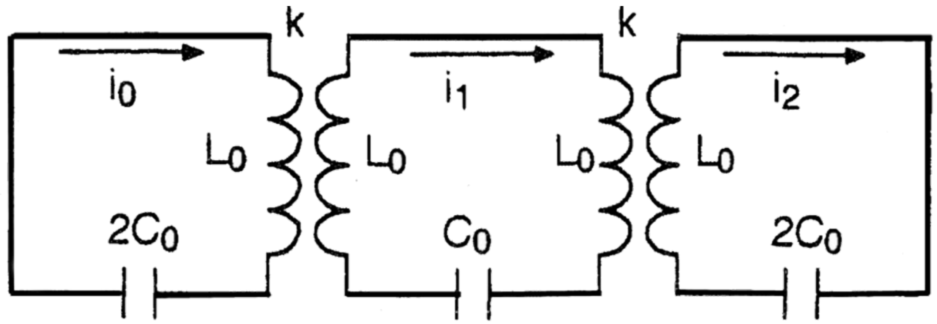
Modes of a two-cell cavity



9-cavity TESLA cell



Example of 3 coupled cavities



$$x_0\left(1-\frac{\omega_0^2}{\Omega^2}\right)+x_1 k=0 \qquad \text{oscillator } n=0$$

$$x_1\left(1-\frac{\omega_0^2}{\Omega^2}\right)+(x_0+x_2)\frac{k}{2}=0 \qquad \text{oscillator } n=1$$

$$x_2\left(1-\frac{\omega_0^2}{\Omega^2}\right)+x_1 k=0 \qquad \text{oscillator } n=2$$

$x_j = i_j\sqrt{2L_o}$ and $\Omega =$ normal mode frequency

Write the coupled circuit equations in matrix form

$$\mathbf{L}\mathbf{x}_q = \frac{1}{\Omega_q^2}\mathbf{x}_q \quad \text{where} \quad \mathbf{L} = \begin{pmatrix} 1/\omega_o^2 & k/\omega_o^2 & 0 \\ k/2\omega_o^2 & 1/\omega_o^2 & k/2\omega_o^2 \\ 0 & k/\omega_o^2 & 1/\omega_o^2 \end{pmatrix} \quad \text{and} \quad \mathbf{x_q} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$$

- Compute eigenvalues & eigenvectors to find the three normal modes

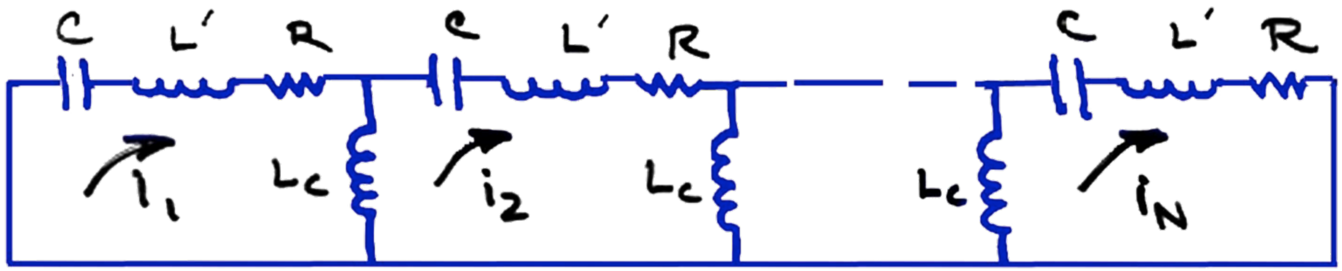
$$\text{Mode q} = 0: \text{ zero mode} \quad \Omega_0 = \frac{\omega_o}{\sqrt{1+k}} \quad \mathbf{x_0} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

$$\text{Mode q} = 1: \ \pi/2 \text{ mode} \quad \Omega_1 = \omega_o \quad \mathbf{x}_1 = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$$

$$\text{Mode q} = 2: \ \pi \text{ mode} \quad \Omega_2 = \frac{\omega_o}{\sqrt{1-k}} \quad \mathbf{x}_2 = \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$$

For a structure with N coupled cavities

- ==> Set of N coupled oscillators
  - N normal modes, N frequencies
- From the equivalent circuit with magnetic coupling

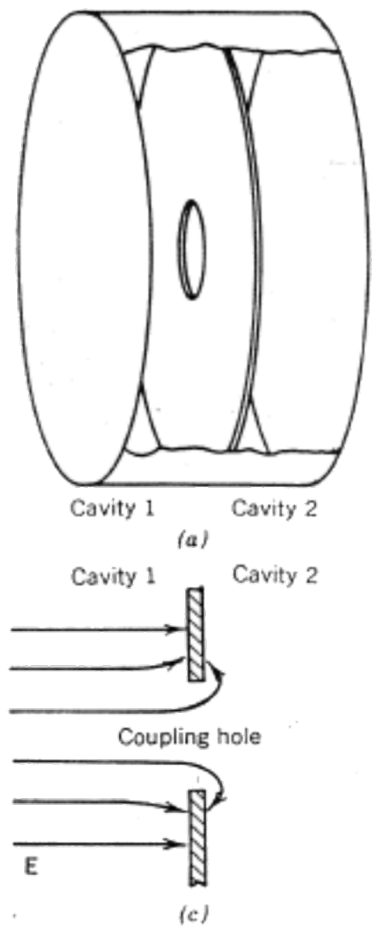


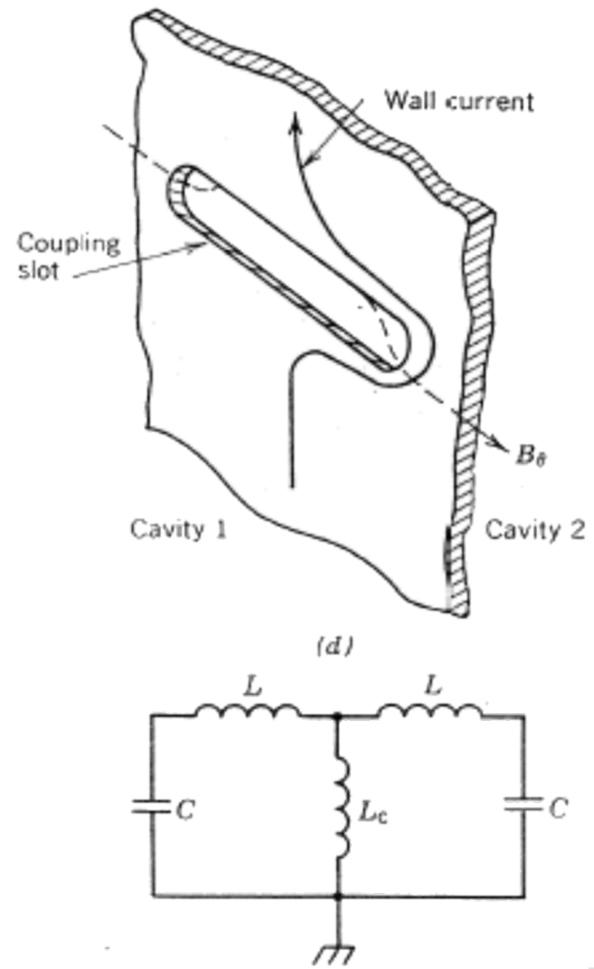
$$\omega_m = \frac{\omega_o}{\left(1 - B\cos\frac{m\pi}{N}\right)^{1/2}} \approx \omega_o\left(1 + B\cos\frac{m\pi}{N}\right)$$

where B= bandwidth (frequency difference between lowest & high frequency mode)

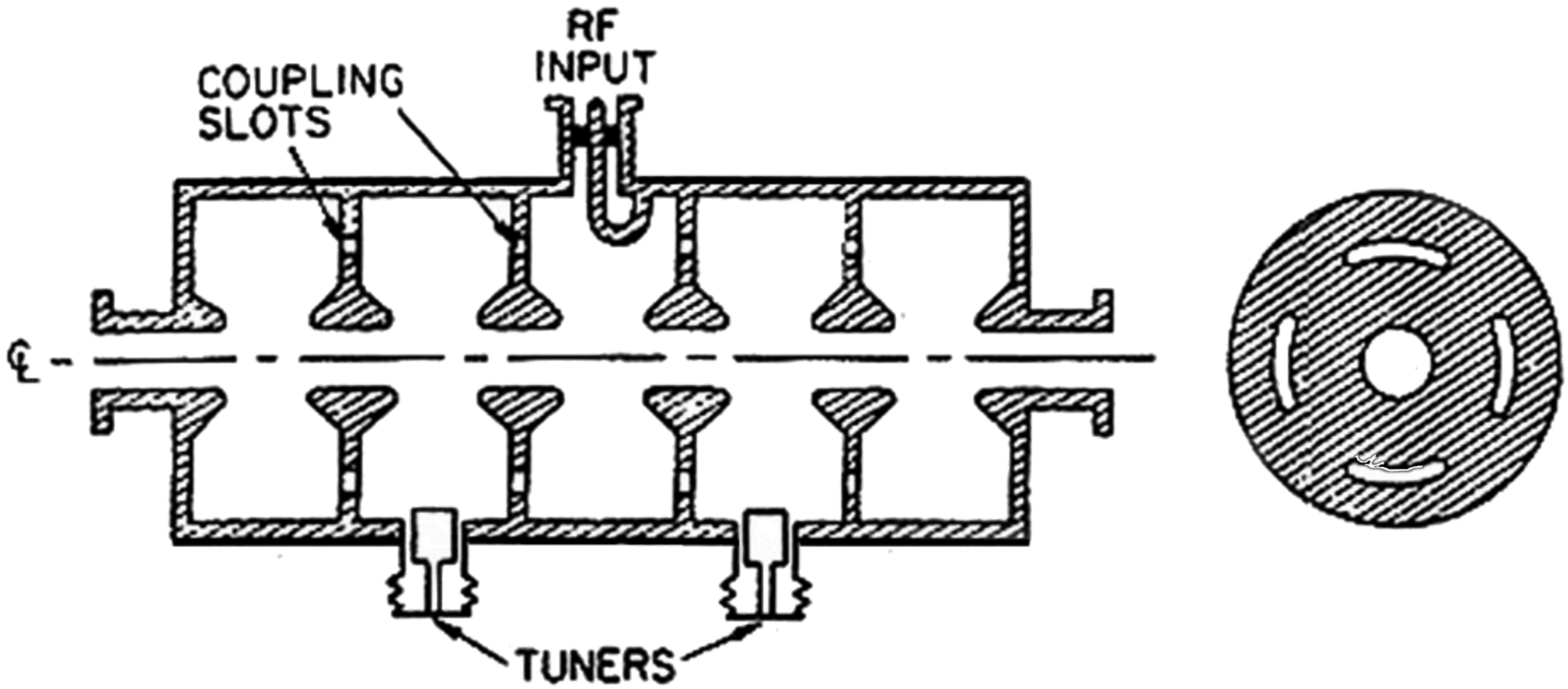
- Typically accelerators run in the π-mode

Magnetically coupled pillbox cavities





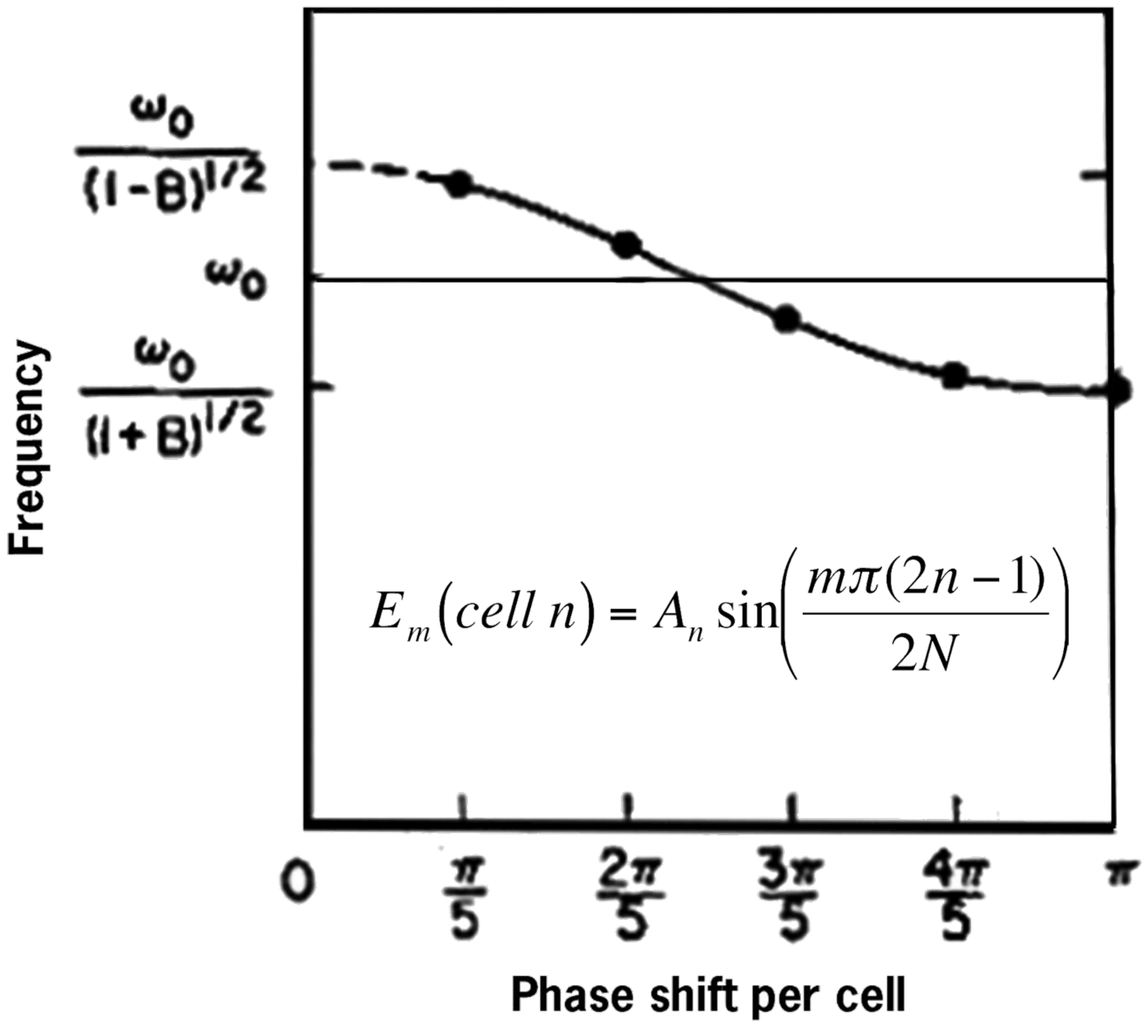
5-cell π-mode cell with magnetic coupling



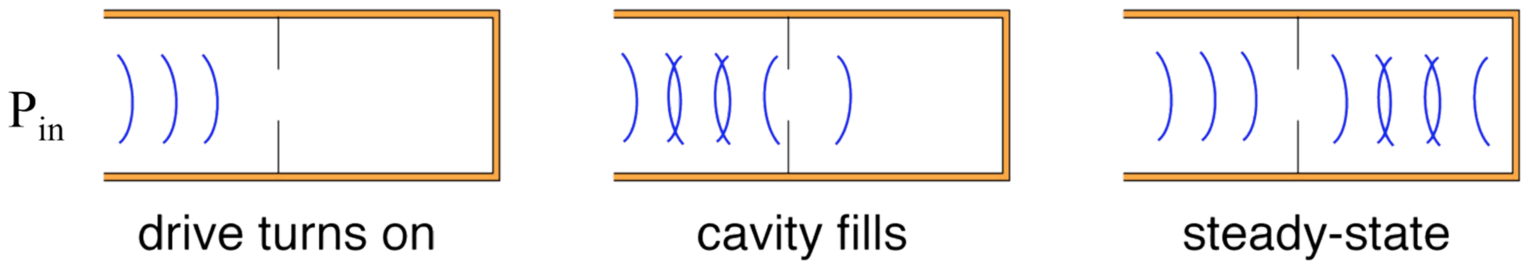
The tuners change the frequencies by perturbing wall currents ==> changes the inductance ==> changes the energy stored in the magnetic field

$$\frac{\Delta\omega_o}{\omega_o} = \frac{\Delta U}{U}$$

Dispersion diagram for 5-cell structure



Effects of wall-losses & external



- Define "wall quality factor", Q_w, & "external" quality factor, Q_e
- Power into the walls is $P_w = \omega U / Q_w$.
- If P_{in} is turned off, then the power flowing out $P_e = \omega U / Q_e$
- Net rate of energy loss $= \omega U / Q_w + \omega U / Q_e = \omega U / Q_{loaded}$

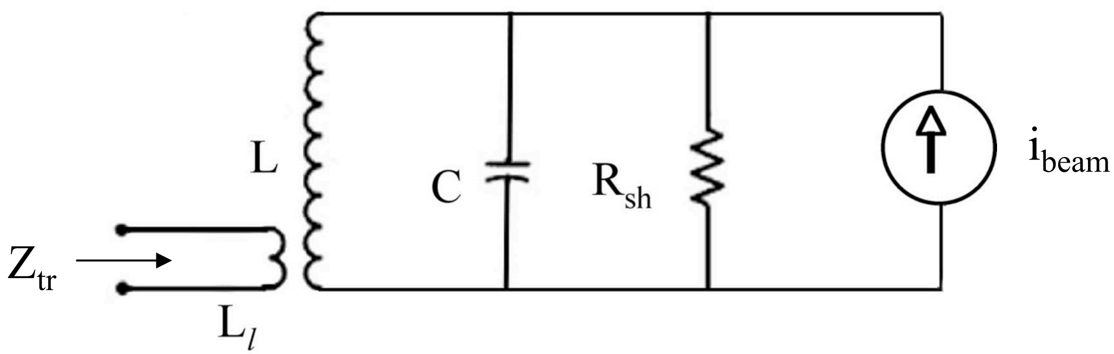
Fill time & coupling

- Loaded fill time

$$T_{fill} = 2Q_L/\omega$$

- Critically coupled cavity: $P_{in} = P_w \Longrightarrow 1/Q_e = 1/Q_w$

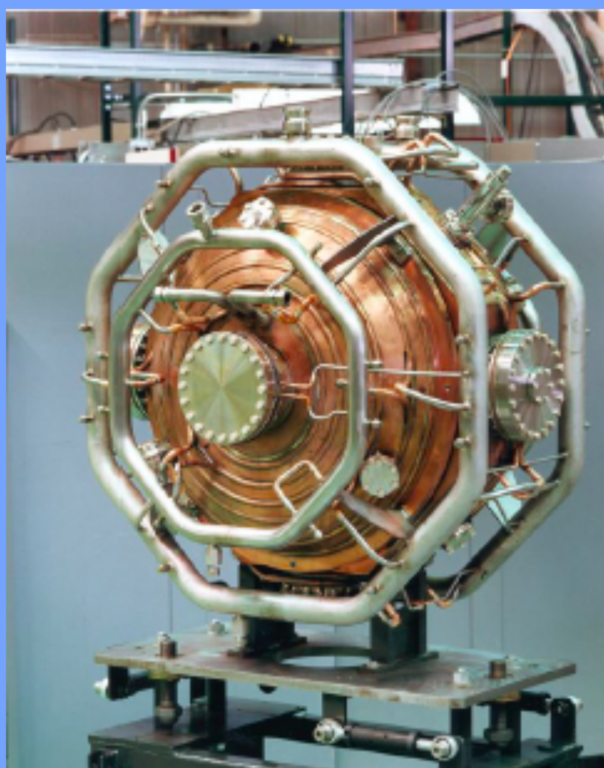
- In general, the coupling parameter $ß = Q_w / Q_e$



RF Cavity Options

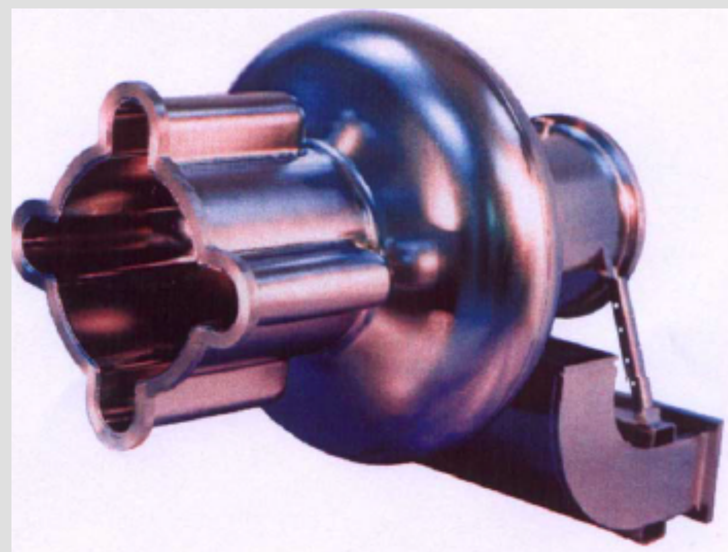
❖ Normal Conducting Cavity

- Power dissipated in cavity walls. Typical values of <100 kW.
- Water-cooled.
- Relatively compact.
- HOM-damped models available.



❖ Superconducting Cavity

- Low-loss, High Q
- Most RF power goes into beam
- Cryogenic system with several hundred watts at 2 K required.
- Cryomodule requires significant beamline space.
- HOM-damped models available.



Comparison of SC and NC RF

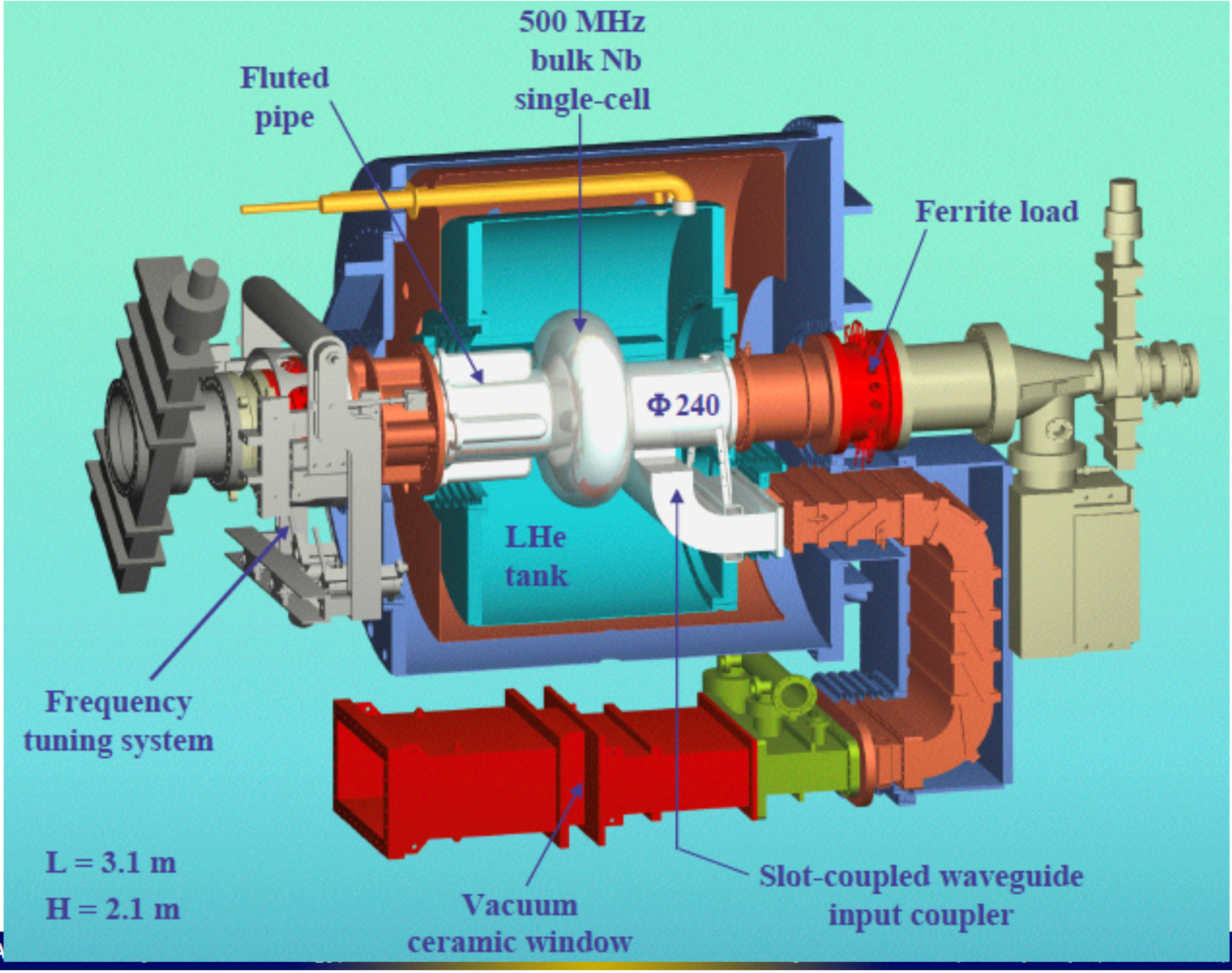
Superconducting RF

- High gradient ==> 1 GHz, meticulous care
- Mid-frequencies ==> Large stored energy
- Large stored energy ==> less beam loading and small ΔE/E
- Large Q ==> high RF power transfer to beam; high efficiency

Normal Conductivity RF

- High gradient ==> high frequency (5 - 17 GHz)
- High frequency ==> low stored energy
- Low stored energy ==> larger beam loading and ~10x larger ΔE/E
- Low Q ==> large power dissipation in RF structure walls: reduced efficiency

SRF Cryomodule



Passive SC Third Harmonic Cavity

- The harmonic cavity provides a reactive effect on the beam (does not power the beam.) Use the beam to excite fields in the cavity.
- This is called passive mode and is used in most light sources with HCs.
- SCRF is an excellent option since cavity impedance is very high (few cells) and little power is dissipated.

Figure 8: Cryomodule assembly

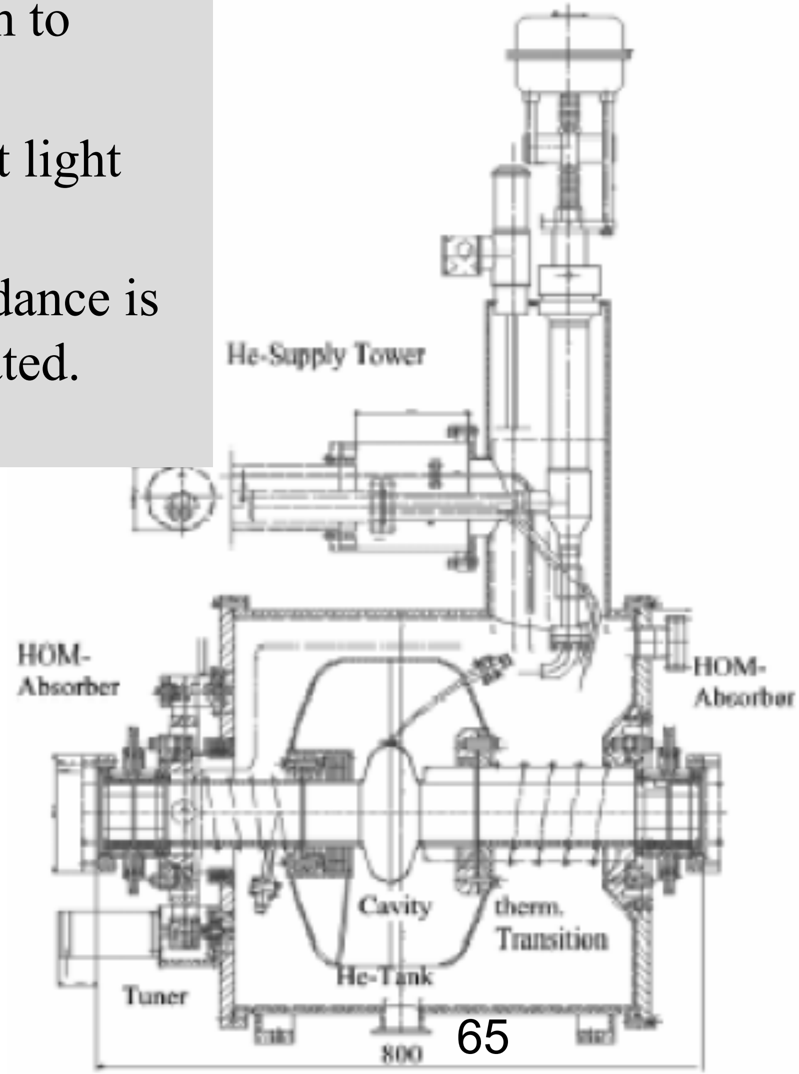


Figure 1: Layout of cryomodule

Figure of merit for accelerating cavity: Power to produce the accelerating field

Resistive input (shunt) impedance at ω_o relates power dissipated in walls to accelerating voltage

$$R_{in} = \frac{\langle V^2(t) \rangle}{\mathscr{P}} = \frac{V_o^2}{2\mathscr{P}} = Q\sqrt{L/C}$$

Linac literature more commonly defines "shunt impedance" without the "2"

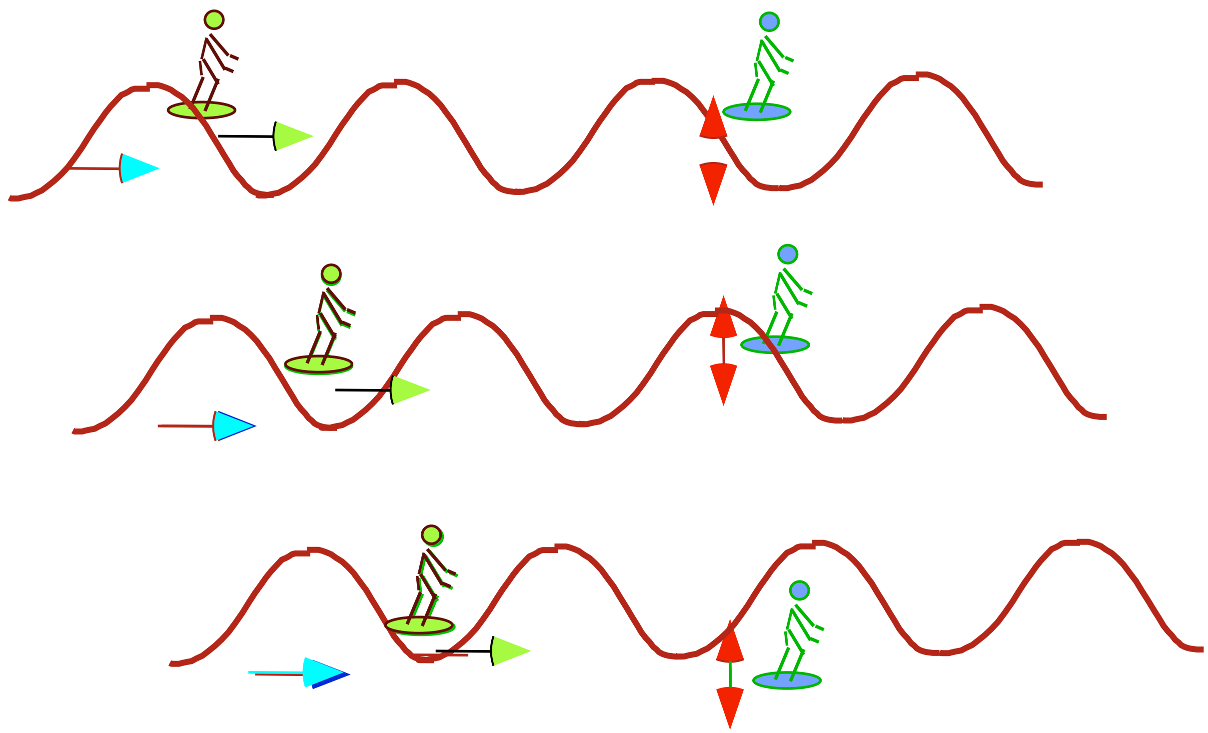
$$\mathscr{R}_{in} = \frac{V_o^2}{\mathscr{P}} \sim \frac{1}{R_{surf}}$$

For SC-rf $\mathscr{P}$ is reduced by orders of magnitude

BUT, it is deposited @ 2K

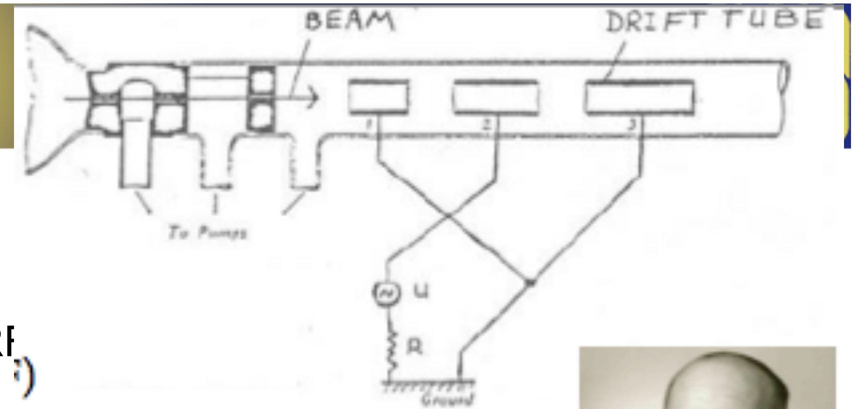
Traveling wave linacs

Surfing analogy of the traveling wave acceleration mechanism

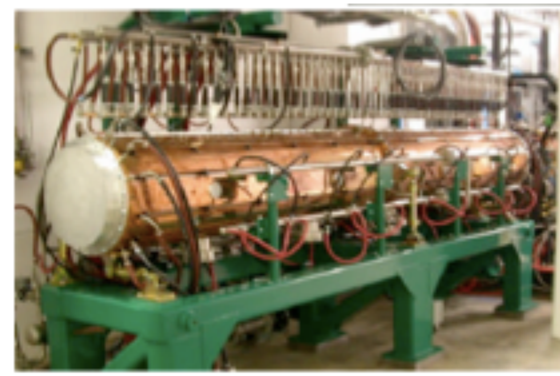


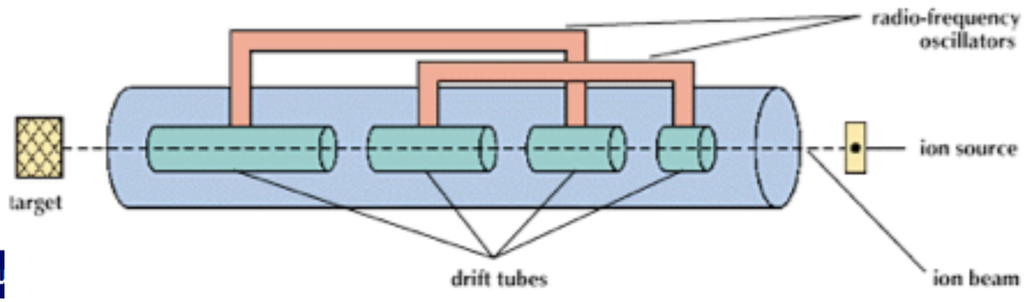
To "catch" the wave the surfer must be synchronous with the phase velocity of the wave

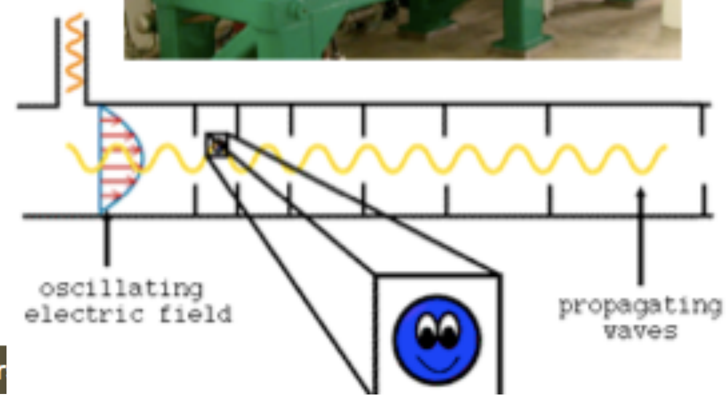
Acceleration in a linac



- Original idea by **Ising** (1924), first working linac by **Wideröe** (1928) and high energy (1.3 MeV) linac by **Sloan** and **Lawrence** (1931)
- Series of drift tubes alternately connected to high (RF) frequency voltage oscillator
- Particles get accelerated in gap, no effect inside tube (act like Faraday cage)
- Field reversed and then exit tube to be reaccelerated until they reach energy
- For constant RF frequency, drift tubes' length increases with velocity up to relativistic limit (electrons)
- Synchronization of particle and RF field assured by **phase focusing**
- **Beams** (1933) developed first cavity structure linac (waveguides). **Hansen** and **Varian** brothers (1937) developed first klystron (frequencies up to 10GHz).
- **Alvarez** (1946) developed first DTL resonant cavity



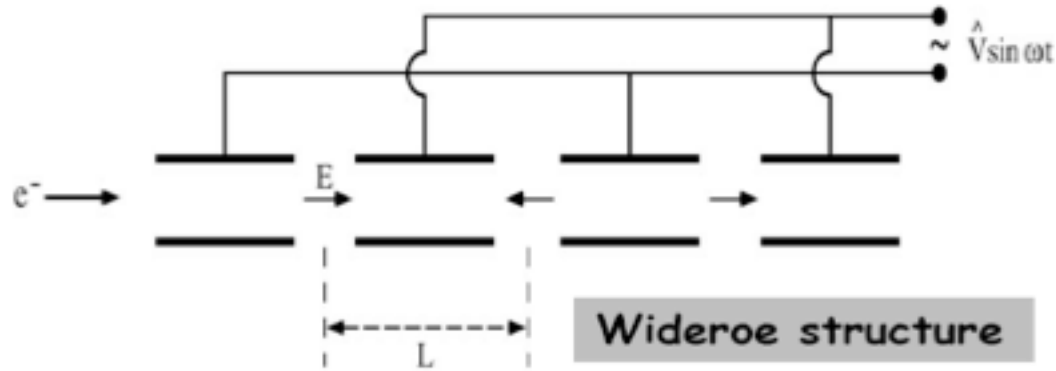


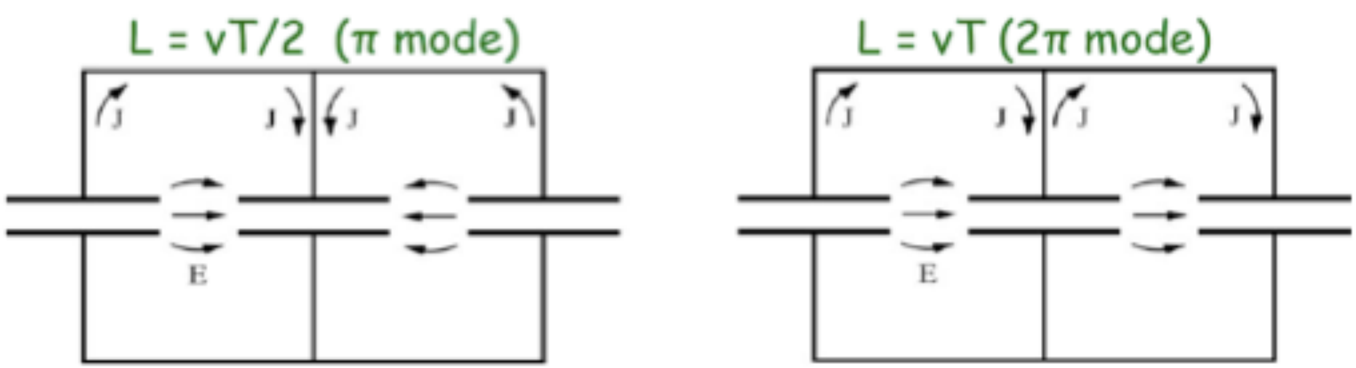


RF Synchronism gives acceleration

- The use of RF fields allows an arbitrary number of accelerating steps in gaps and electrodes fed by RF generator
- The electric field is not longer continuous but sinusoidal alternating half periods of acceleration and deceleration
- The synchronism condition for RF period T_{RF} and particle velocity *v*

$$L = vT_{RF}/2 = \beta c \frac{\pi}{\omega_{RF}} = \beta\lambda/2$$

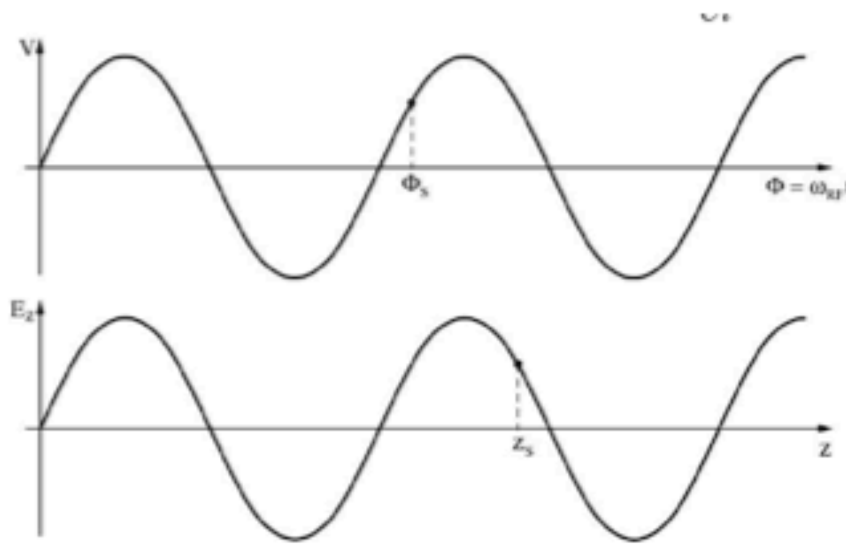




RF Focusing

In order to have stability, the time derivative of the Voltage and the spatial derivative of the electric field should satisfy

$$\frac{\partial V}{\partial t} > 0 \Rightarrow \frac{\partial E}{\partial z} < 0$$



In the absence of electric charge the divergence of the field is given by Maxwell's equations

$$\nabla\vec{E} = 0 \Rightarrow \frac{\partial E_x}{\partial x} + \frac{\partial E_z}{\partial z} = 0 \Rightarrow \frac{\partial E_x}{\partial x} > 0$$

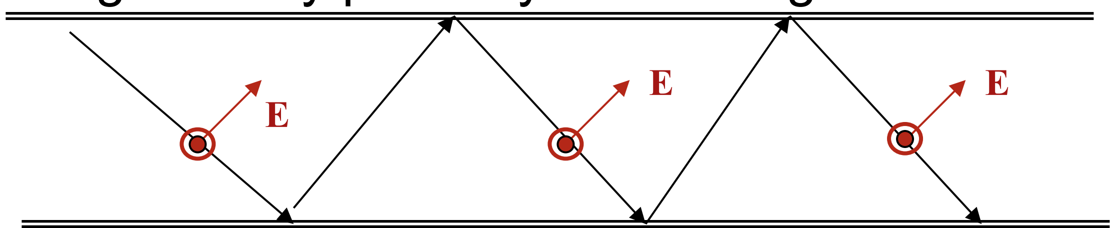
where x represents the generic transverse direction.

External focusing is required by using quadrupoles or solenoids

Longitudinal E-field needed
to accelerate particles in vacuum

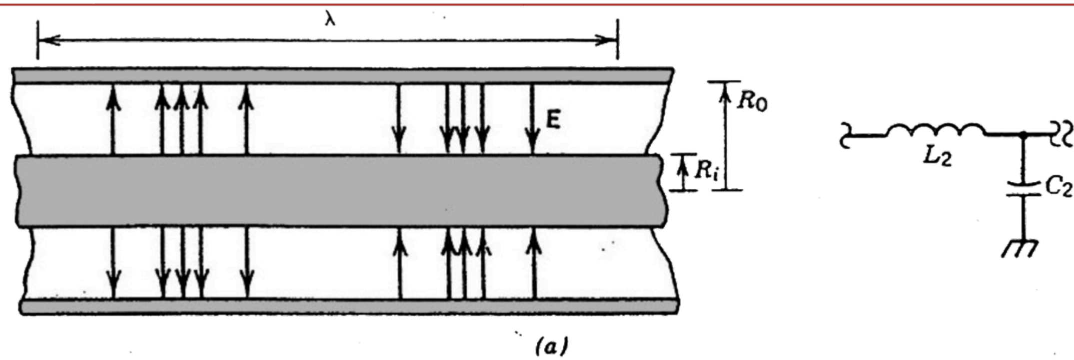
- Example: the standing wave structure in a pillbox cavity

- What about traveling waves?
  - Waves guided by perfectly conducting walls can have E_{long}

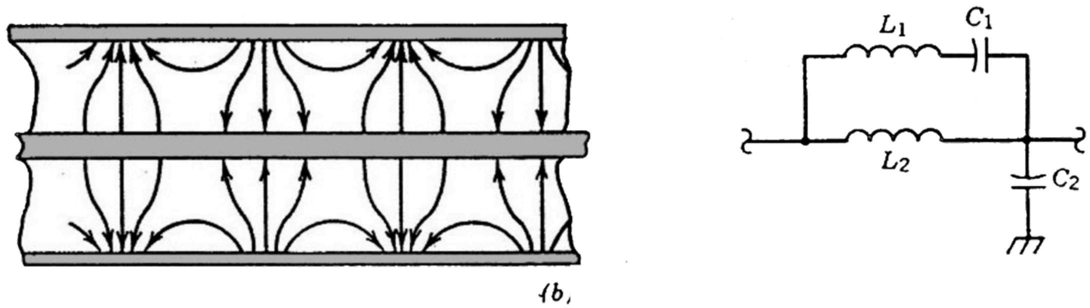


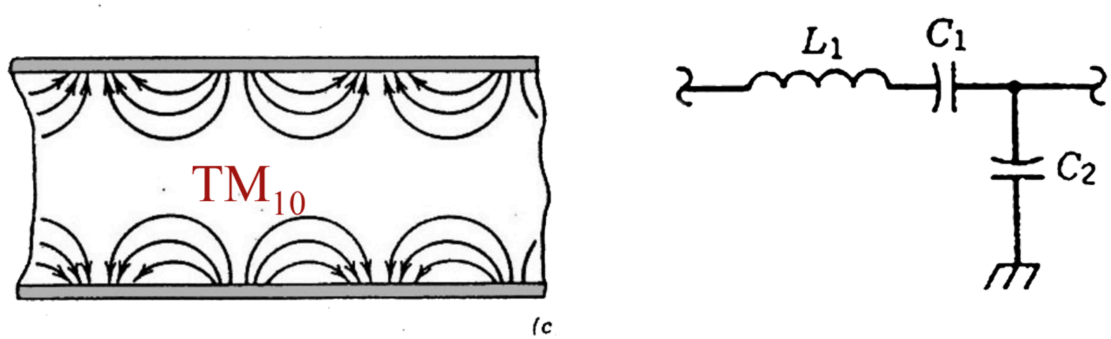
- But first, think back to phase stability
  - To get continual acceleration the wave & the particle must stay in phase
  - Therefore, we can accelerate a charge with a wave with a synchronous phase velocity, $v_{ph} \approx v_{particle} < c$

Propagating modes & equivalent circuits



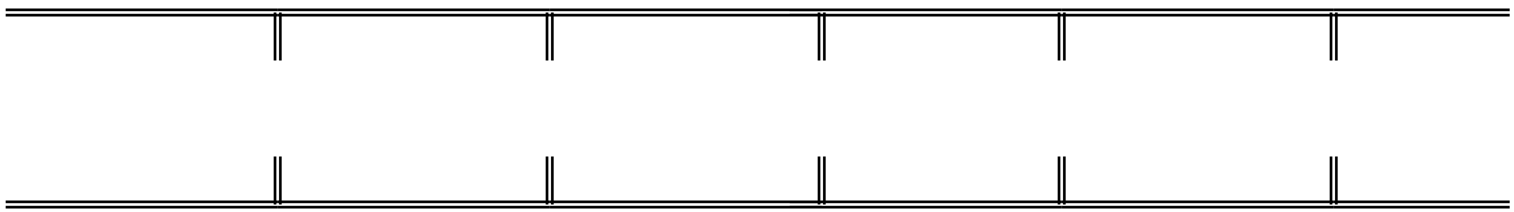
All frequencies can propagate





Propagation is cut-off at low frequencies

To slow the wave, add irises



In a transmission line the irises

a) Increase capacitance, C

b) Leave inductance ~ constant

c) ==> lower impedance, Z

d) ==> lower v_{ph}

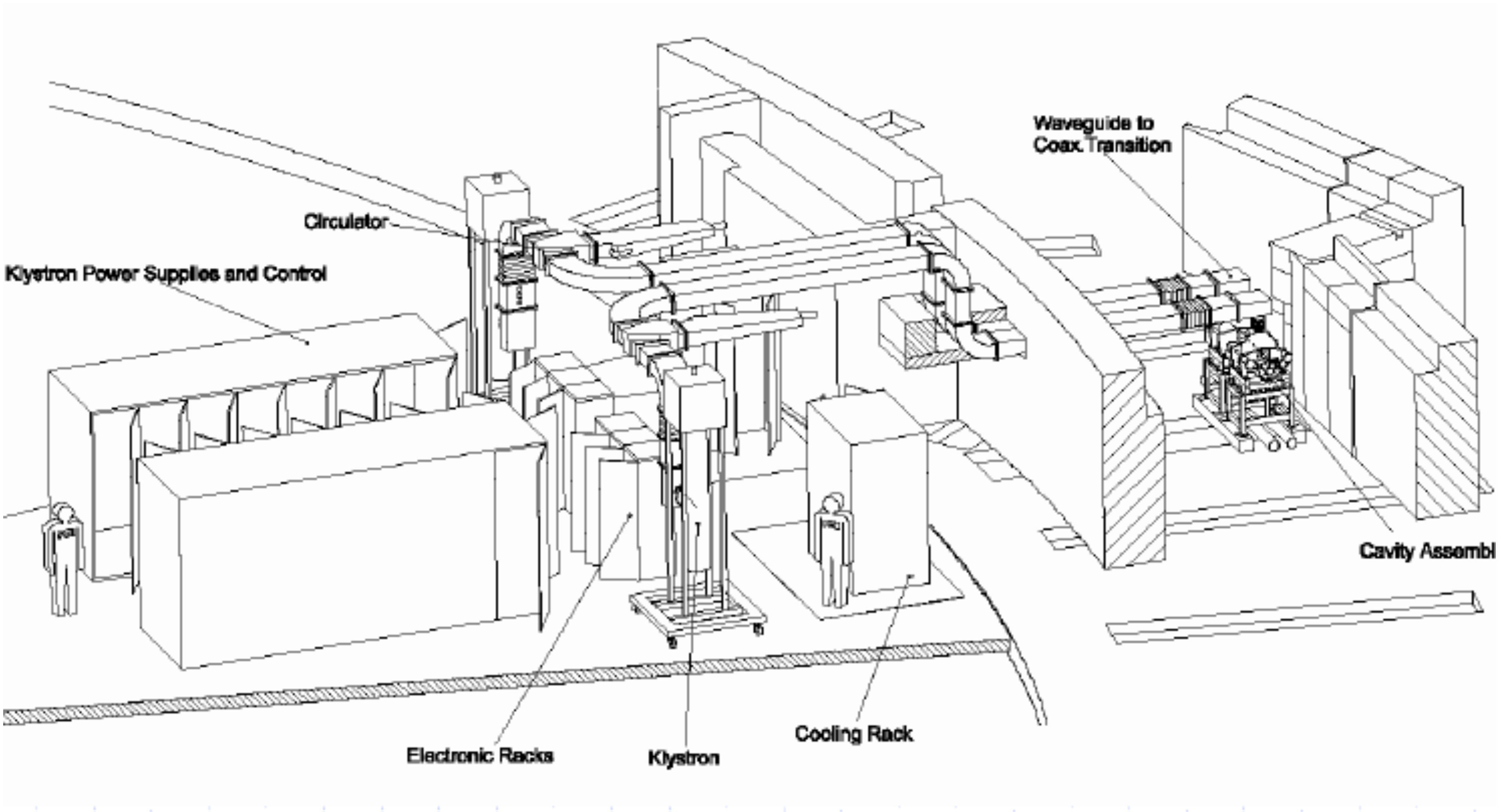
Similar for TM01 mode in the waveguide

$$\frac{\omega}{k} = \frac{1}{\sqrt{LC}}$$

$$Z = \frac{L}{C}$$

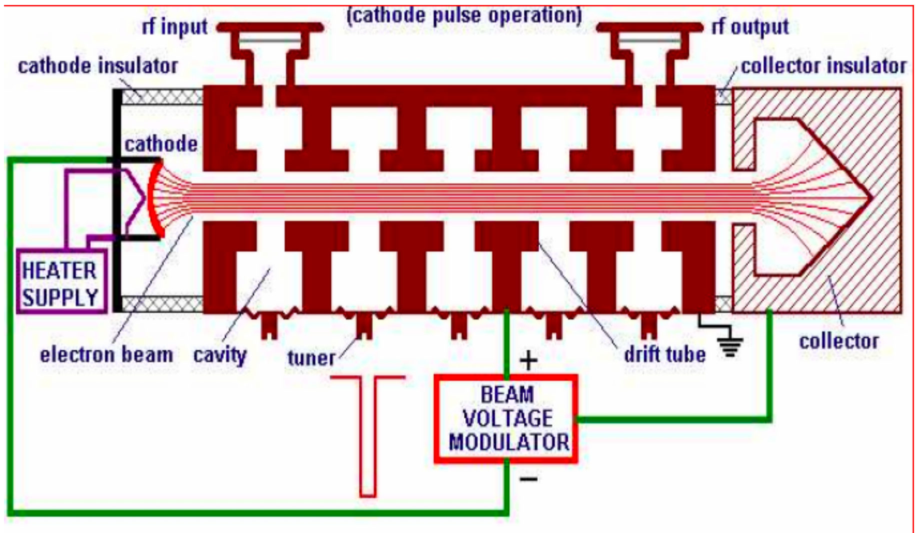
RF Sources and Systems

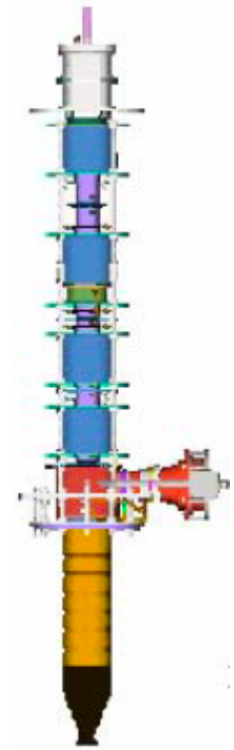
Example Storage Ring RF system



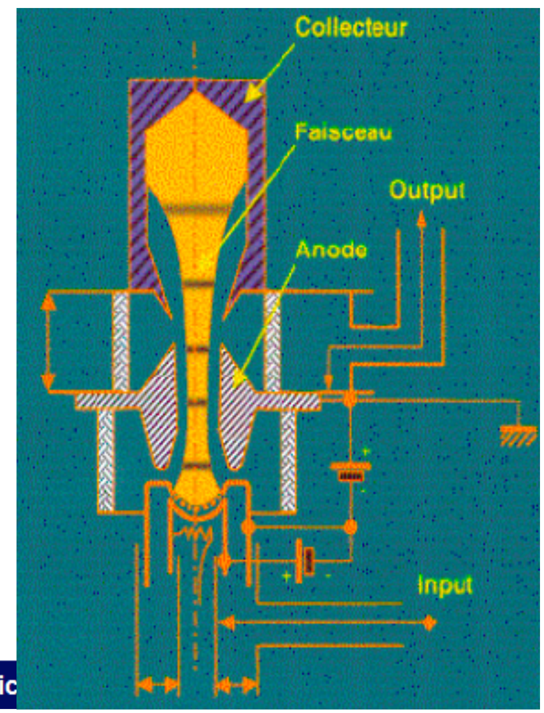
RF Power Sources: electron beam tubes

❖ Provide CW RF power of several hundred watts at a frequency of 350-700 MHz

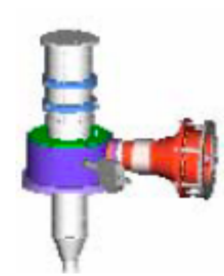




Klystron is the proven standard.



IOT (Inductive Output Tube) is slowly being adopted.



Soleil 190 kW Solid State System

- Combines 680 300 W solid state units 352 MHz.