MATLAB Physics - II

- **MATLAB should now be ready for you to use.**
- **You should have the textbook(s), the scripts and the demos/templates**
- **Lecture/class notes – email list (sign up) will be used for communications**
- **Homework and “project”.**
  - **The goal is not just to execute existing scripts but to read and understand the script instructions.**
  - **To that end, practice and finish by writing a “project”.**

Course Topics

- **There are 3 main topics to be covered**
  - **First, MATLAB itself as a tool**
  - **Second; scattering of particles, stochastic processes and Monte Carlo methods**
  - **Third; interaction of particles and magnetic and electric fields. Radiation of particles, NR and SR. Beam elements – dipoles and quadrupoles. Simple beam design.**

Writing a script

- Homework assignment – due by last class
- Write a script about some topic you are working on or intersted in:
  - There are 200 examples available to you
  - Use the "template2" script if you wish to get started
- The project assignment attempts to make the course work less passive and more an active involvement in your interests.
- Use "help" and other tools like "demos". Send us e-mail if stumped. Note – data analysis is a good option (help dlmread, etc.). Look at fitting examples.

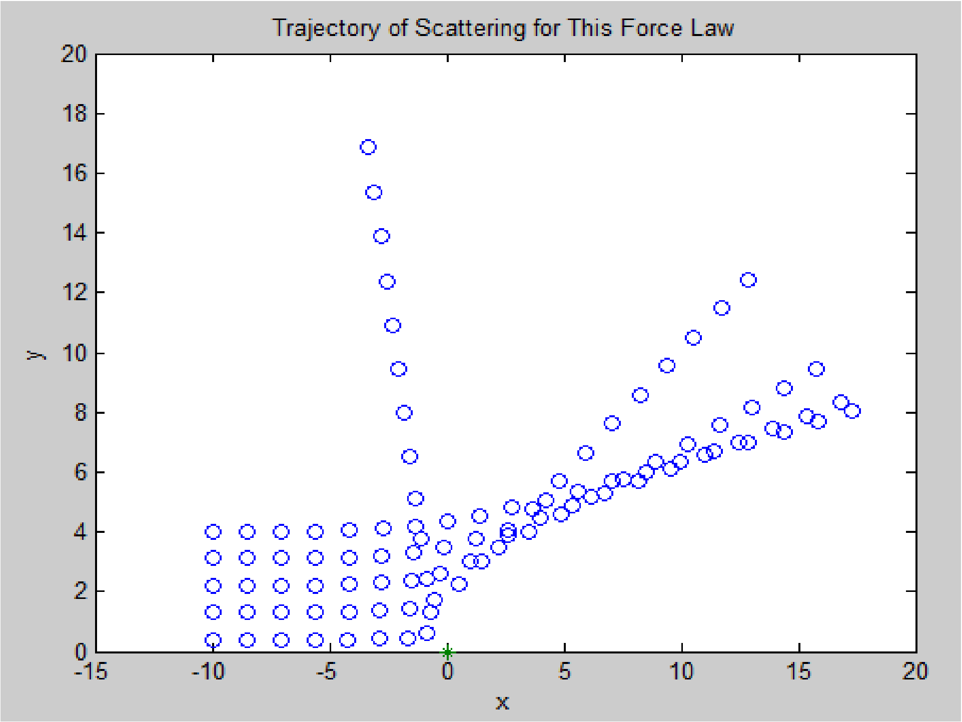
- **Numerical integration for ODE – impact parameter. First order RK solutions.**

The MATLAB tool, "ode45" is used which is a numerical solver for a set of ordinary differential equations. In this case, there are four unknowns, the x and y position and the x and y velocity, which are called y(i) in the script. The initial conditions are that x = -10 and y = b, the impact parameter with initial x velocity = v_0 = $\sqrt{2}$ and initial y velocity = 0. The mass and kinetic energy of the classical probe particle are taken to be equal to one. The equations as input to "ode45" are;

$$dx/dt = v_x,\ dy(2)/dt = y(1)$$
$$dy/dt = v_y,\ dy(4)/dt = y(3)$$
$$dv_x/dt = (x/r)(q/r^n) = dy(1)/dt$$
$$dv_y/dt = (y/r)(q/r^n) = dy(3)/dt \qquad (2.7)$$

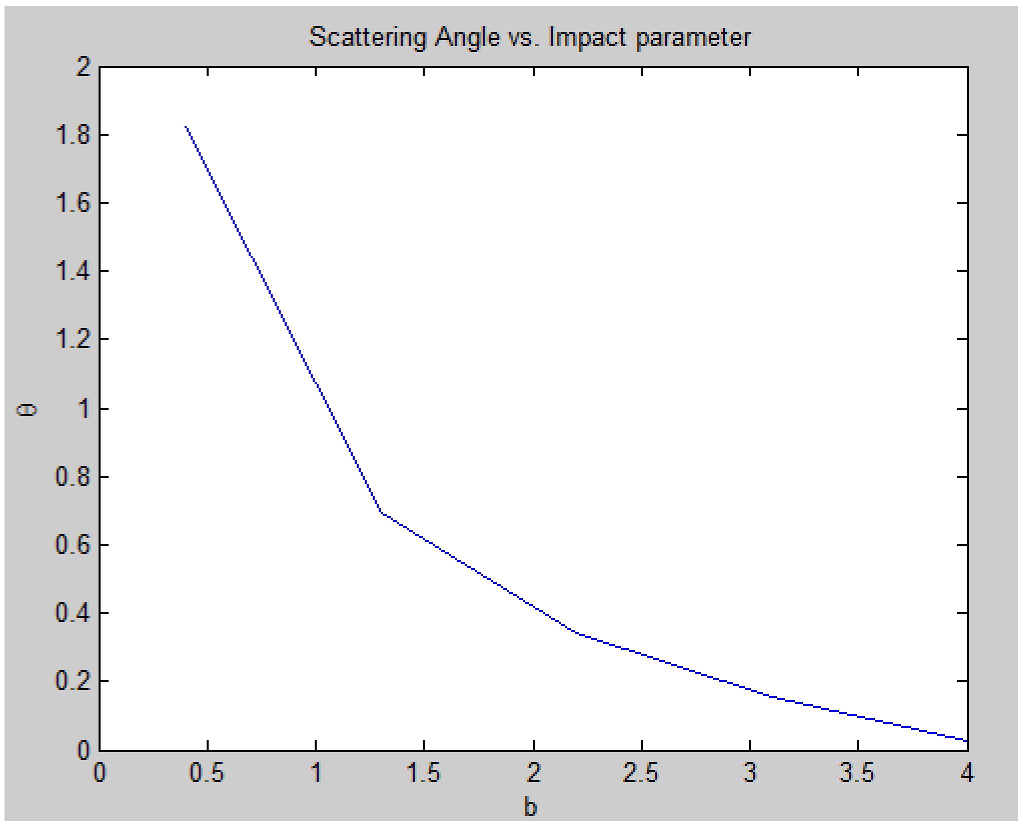
Impact parameter

- 1/r^2 – repulsive. Note asymptotic velocity in and out are =. Elastic scattering. No recoil. Movies!
- Run DG, “Scatt_Force_Law”



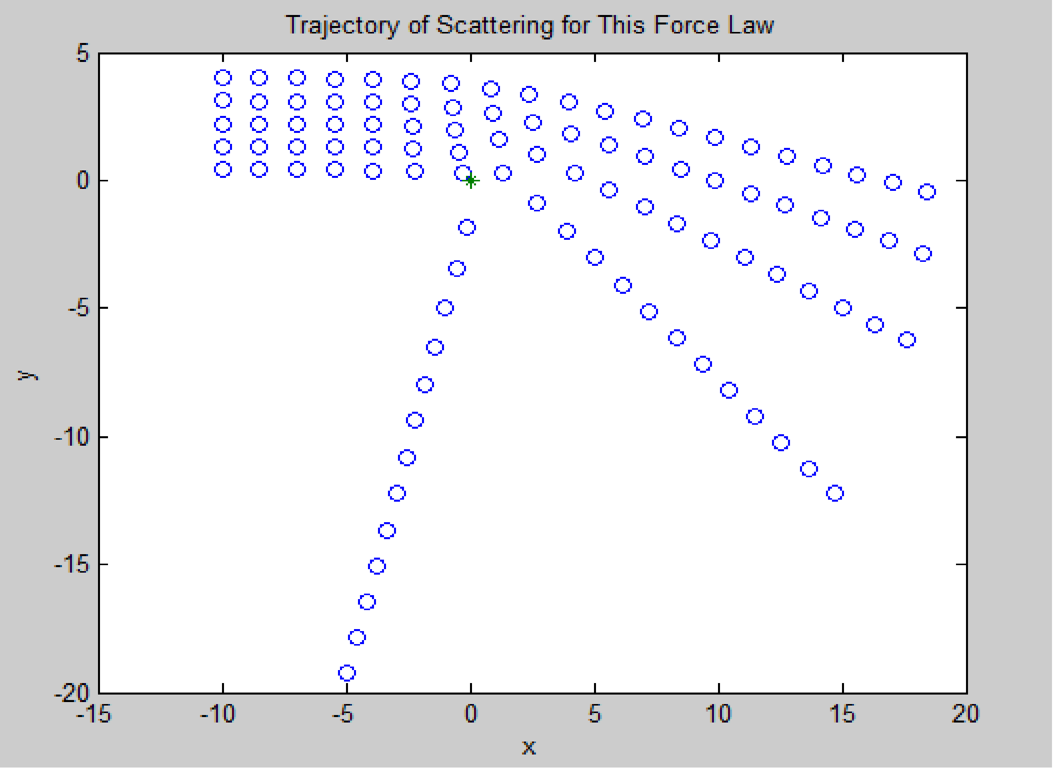
b and theta

- As b → theta ← since one gets futher from the force center



Attractive

- The relation in detail between b and theta depends on the strength and range of the force (compare 1/r to 1 /r^4). Scattering and force laws...



Rutherford Cross Section

- **Small angle Rutherford angular distribution is easily found.**

$$d\sigma = 2\pi\, b db = (2\pi\, b/\theta)(db/d\theta)\theta\, d\theta$$

$$d\sigma / d\Omega \sim (b/\theta)(db/d\theta)$$

$$F \sim 1/b^2$$

$$dP \sim F(b/v), \theta \sim dP/P \sim 1/b$$

$$d\sigma / d\Omega \sim 1/\theta^4$$

2 Body Collisions - Classical

- **Conservation of momentum and kinetic energy. Particle of mass m, energy T, strikes particle at rest (M).**

$$v_{in}^2 = v_2^2 M + v_1^2$$
$$\bar{v}_{in} = \bar{v}_1 + \bar{v}_2 \qquad (2.10)$$

These equations can be solved for the recoil velocity as a function of the angle of the recoiling target, ϕ .

$$v_2 = 2\cos\phi / (1 + M / m) \qquad (2.11)$$

Once the recoil angle is chosen, the recoil velocity is solved for, and then the scattered projectile velocity, v_1 follows from momentum conservation as does the scattering angle θ of the projectile.

$$v_1 = \sqrt{1 + v_2^2 - 2 v_2 \cos\phi}$$
$$v_1 \sin\theta = v_2 \sin\phi \qquad (2.12)$$

NR Scattering

- **Scattering off light and heavy targets. Run "cm_NR_scatt"**

The user can vary the masses and see how the velocity partition between recoil and projectile particles is altered when M is varied.

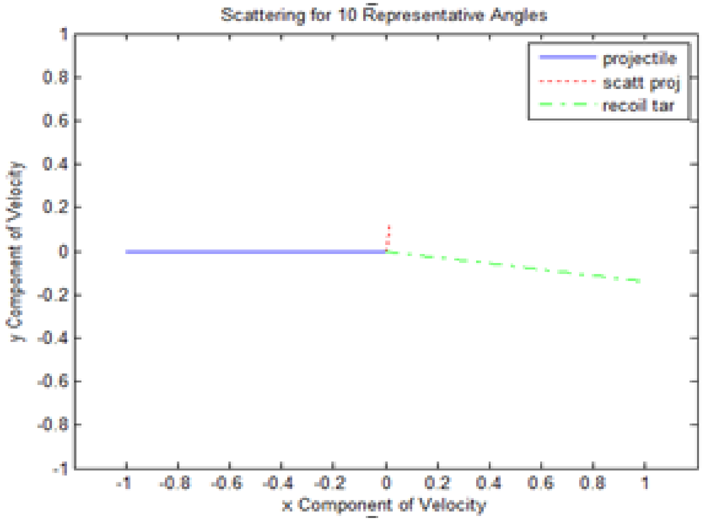


Figure 2.14: Scattering of a projectile and the recoil momentum and angle for the case of equal target and projectile mass. This is a snapshot of a movie covering several scattering angles.

Energy Transfer in a Collision

- **Energy transfer and target mass**

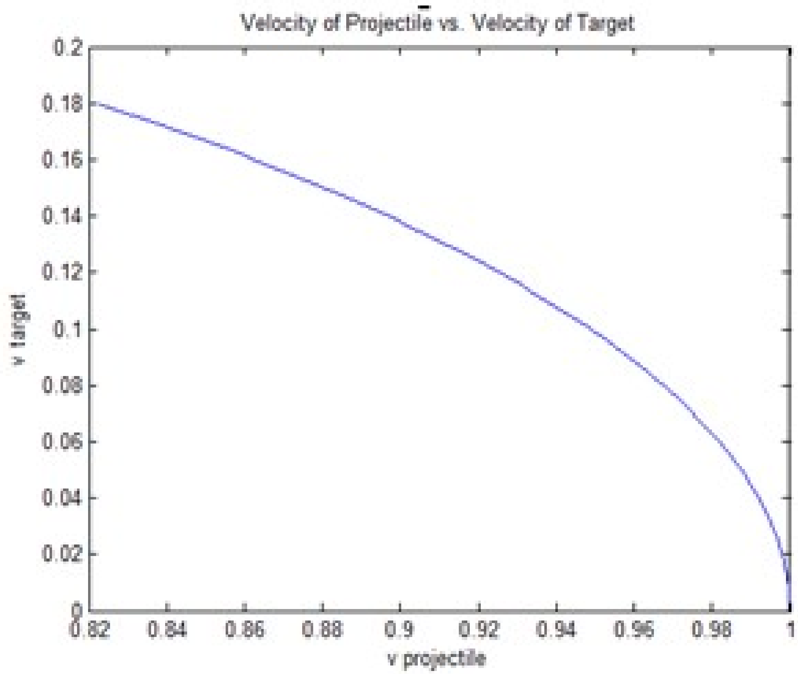
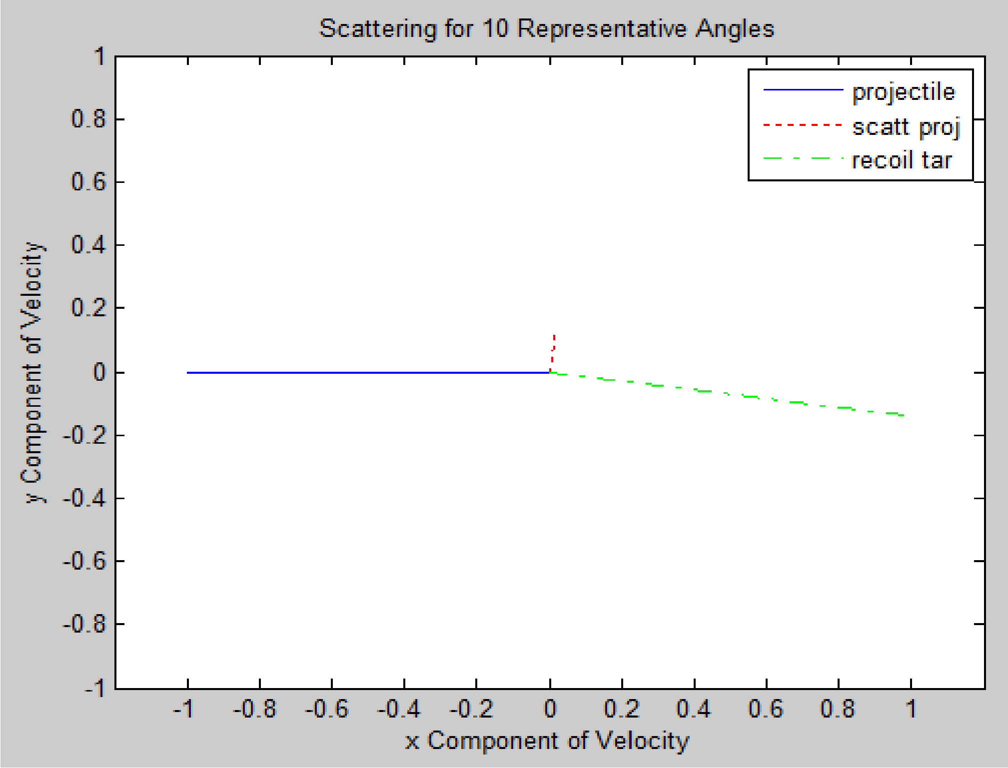


Figure 2.15: Target velocity as a function of scattered projectile velocity in the case of M = 10. There is a maximum velocity less than the projectile, which the target can attain that depends on M.

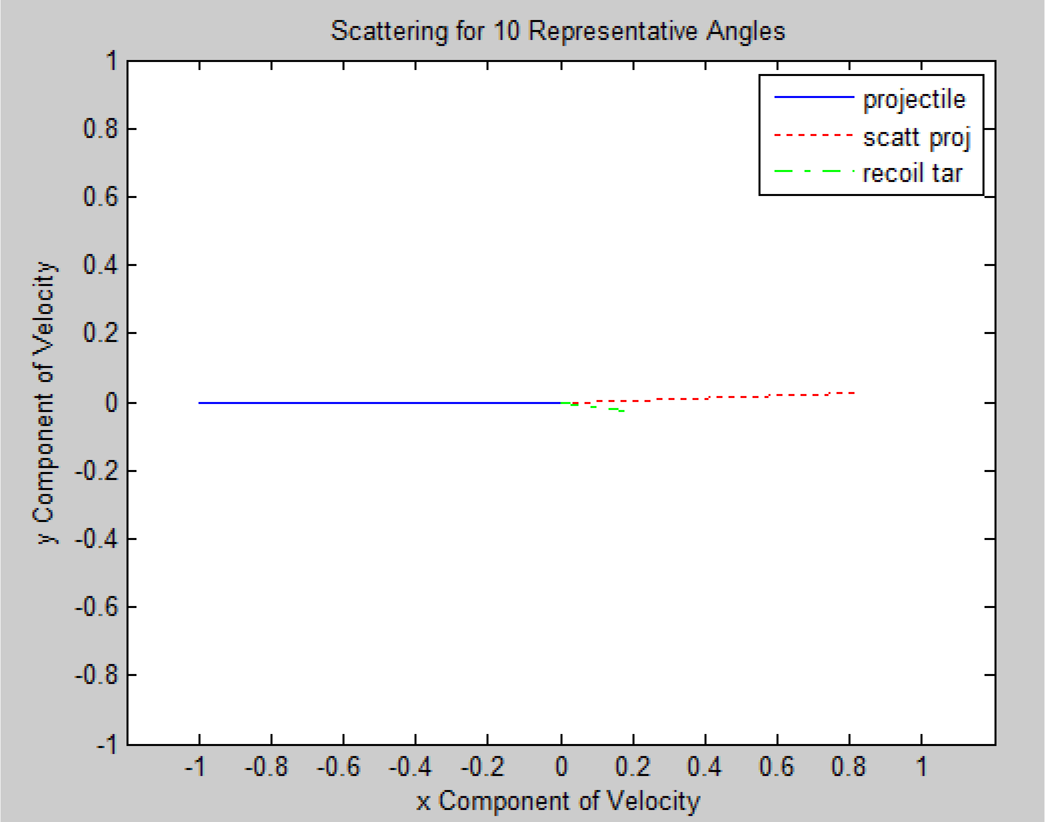
Varying M

- **Equal masses – 90 degree opening angle. All energy can be lost. Example - billiards**



Varying M

- **Billiards bumpers – vs smartcar/hummer collision – little energy lost**
- **Recall :**
  - **E loss,e**
  - **Scatter -Z**



SR – Scattering and Decay

- **In SR particle can decay, turning mass into energy**
- **In SR = mass particle do not have a 90 degree opening angle.**

There are two general cases. For the decay of a particle of mass M into two particles of mass m, the center of momentum, CM, quantities are simply:

$$M_{cm} = M, \beta_{cm} = P/\varepsilon, \gamma_{cm} = \varepsilon/M$$
$$\varepsilon^* = M/2 \tag{7.6}$$

The CM energy is simply the mass M particle, and the daughters of mass m share the CM energy equally. The mass M particle moves in the lab frame with momentum P.

In the case of elastic scattering, m + M -> m + M, the initial state has a target of mass M at rest and a projectile, mass m, moving with momentum P and energy ε. The CM in this case is:

$$M_{cm}^2 = m^2 + M^2 + 2M\varepsilon,$$
$$\beta_{cm} = P/(\varepsilon + M), \gamma_{cm} = (\varepsilon + M)/M_{cm}$$
$$P^* = PM/M_{cm} \tag{7.7}$$

The CM quantities for final state objects are indicated by a * superscript. In both cases, they are defined by a single "scattering/decay angle", θ^*. Making a Lorentz transformation back to the laboratory:

$$P_T = P^* \sin\theta^*$$
$$P_L = \gamma_{cm}(P^* \cos\theta^* + \beta_{cm}\varepsilon^*) \tag{7.8}$$

Equal Mass case

- Useful to “play” with SR scattering/decay. Run “sr_dec_scat”

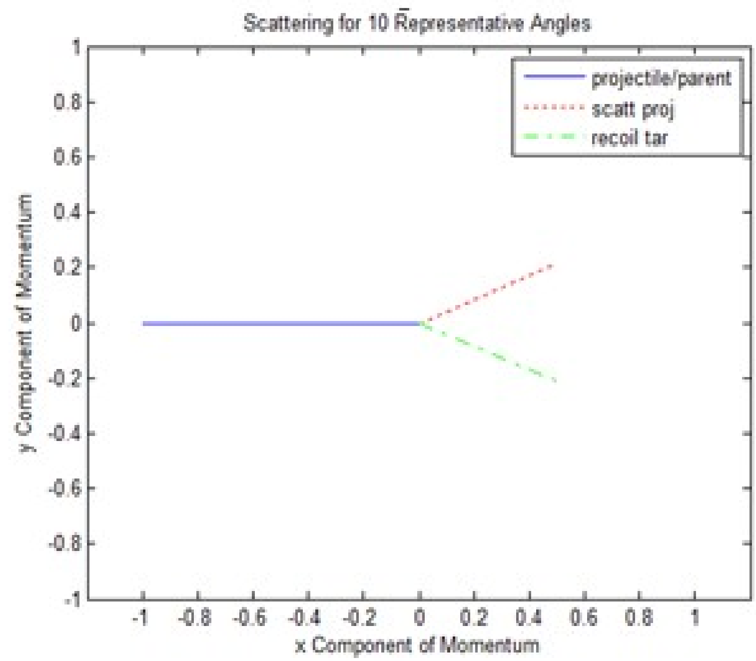


Figure 7.10: Scattering of equal mass particles. The angle between the scattered projectile and the recoiling target is less than ninety degrees for high momentum projectiles.

Field of a SR Charge

- **In electrostatics, E for a point charge is a sphere. In SR the field becomes a “pancake”. Run “E_SR”**

$$\vec{E} \sim [q(1-\beta^2)/(1-\beta^2\sin^2\theta)^{3/2}]\vec{r}/r^3$$

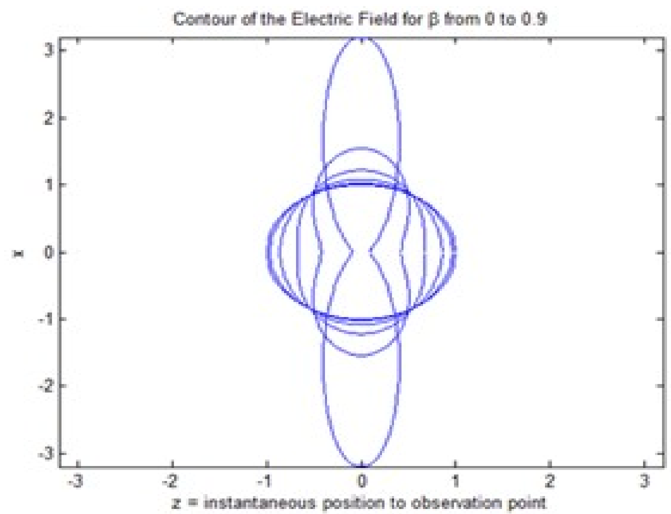


Figure 7.13: Contours of the electric field of a uniformly moving charge.

E Field and MIP

- **Energy transfer does not depend on particle energy. Hence dT/dx is a constant and a minimum at high energies. At low energies dT/dx ~ 1/v^2.**

$$\Delta t = b / v \rightarrow b / \gamma v$$

$$F = e^2 / b^2 \rightarrow \gamma e^2 / b^2$$

$$\Delta P = e^2 / bv \rightarrow e^2 / bc$$

$$\Delta T = \Delta P^2 / 2m = e^4 / 4Tb^2 \rightarrow e^4 / 2mcb^2$$

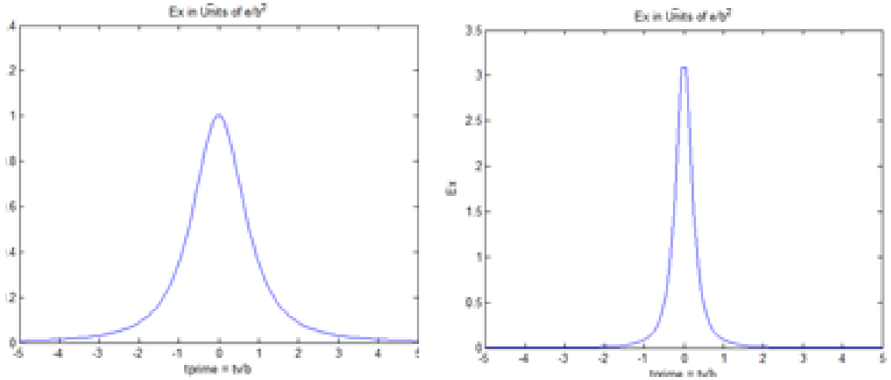


Figure 7.14 Transverse fields of a moving charge observed at the point x = d, y = 0 = z. At left the charge moves with $\beta = 0.1$, while at right it has velocity $\beta = 0.95$.

E of Fast Moving Charge

Run E_Move_Charge_SR" - movies

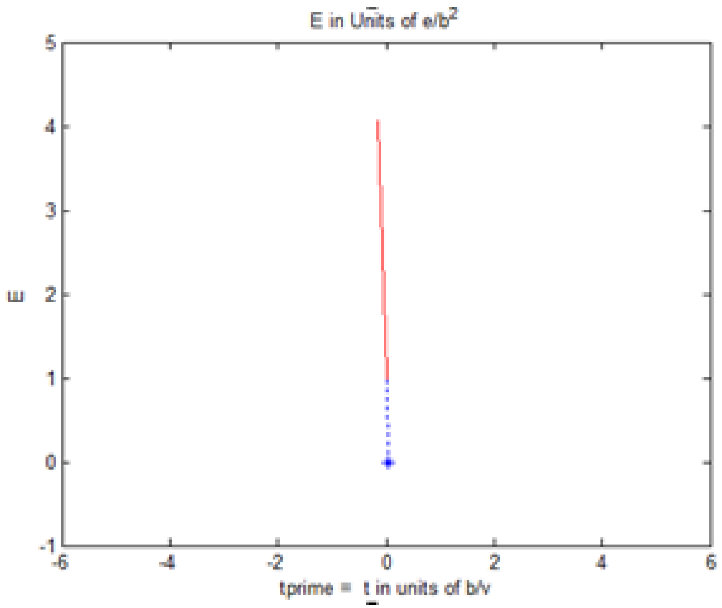


Figure 7.15: A frame of the movie for a charged particle with $\beta = 0.95$ when the charge is near the observation point. The blue line goes from the charge, blue *, to that point, while the red vector shows the size and direction of the electric field.

Energy Loss

- A charged particle also ionizes the electrons as it goes through matter (protons??)
- At relativistic velocities ~ all energy loss is about the same ~ 1.5 MeV/(gm/cm^2). - using density scale, dT/dρx. A "mip"
- At lower speeds the energy loss scales as $1/\beta^2$ – note p therapy for deep cancers....
- Track a beam of muons as it traverses material – check "stopping"

Range and Energy Loss

- **Loma Linda; dT/dx ~ 1/T for NR particles. Run "Range_Energy"**

$$T = \sqrt{T_o^2 - 2rz}$$

$$R = T_o^2 \,/\, 2r$$

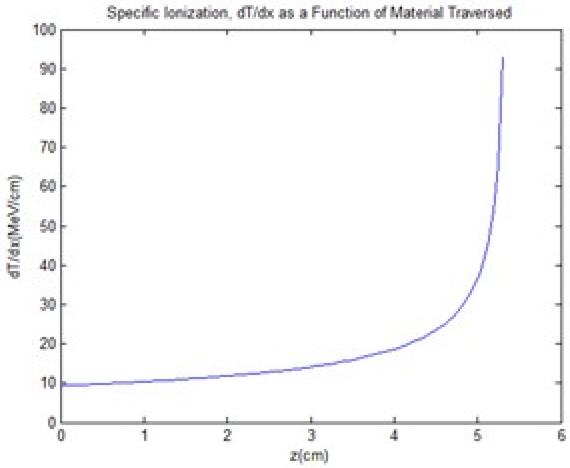
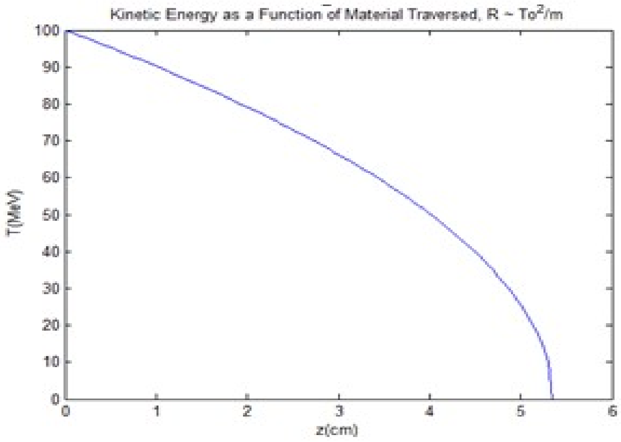


Figure 7.17: Energy deposited by a 100 MeV proton in water at a specific location as a function of the distance travelled.



igure 7.16: Kinetic energy of a 100 MeV proton in water as a function of the distance avelled.

Relativistic Radiation

- **For acceleration parallel or perpendicular to velocity, the radiation grows rapidly with energy and is thrown forward along the velocity. Run “Rel_Radiate”**

$$dp_{\perp} / d\Omega \sim \sin^2\theta / (1-\beta\cos\theta)^5$$

$$dp_{T} / d\Omega \sim 1/(1-\beta\cos\theta)^3[1-\sin^2\theta/\gamma^2(1-\beta\cos\theta)^2] \quad (7.12)$$

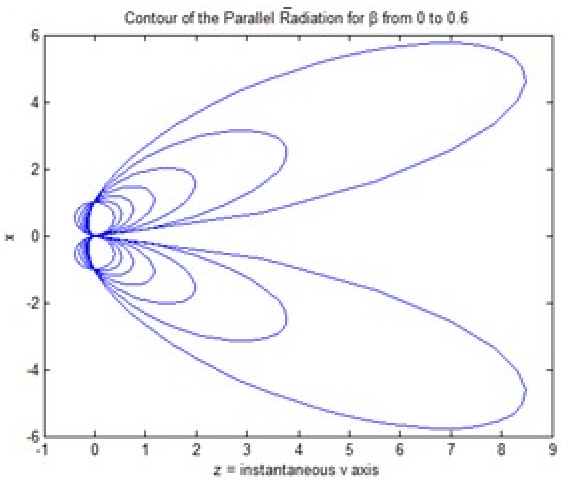


Figure 7.18: Parallel acceleration angular pattern as a function of the source velocity.

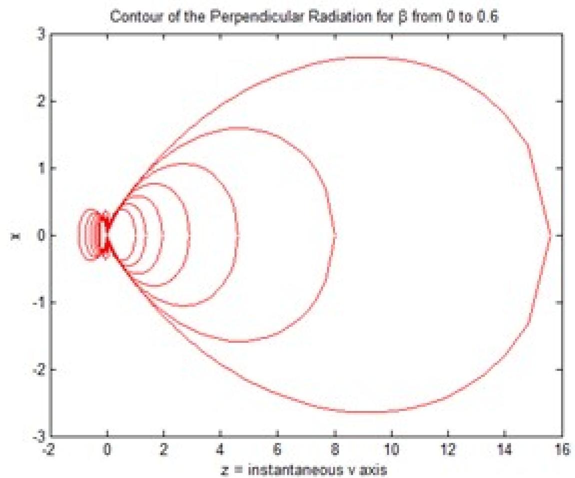


Figure 7.19: Perpendicular acceleration angular pattern as a function of the source velocity.

e and muons in material

- **Mip and shower – energy loss mechanisms. Run "HF_Movie_e_u"**
  - **For a relativistic particle (e) radiation dominates**
  - **For a NR particle, dT/dx dominates**

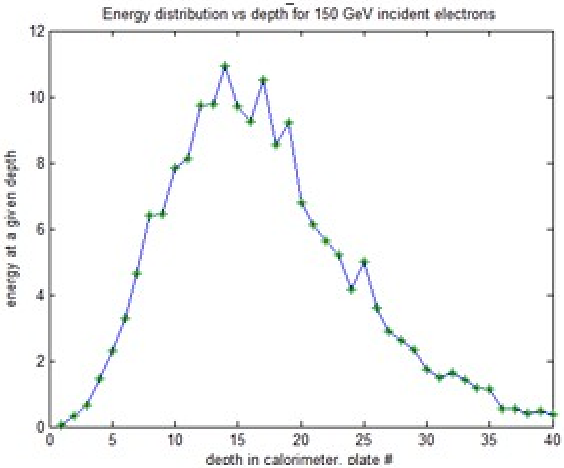


Figure 7.23: Energy deposit for a 156 GeV electron as a function of depth in a block of lead. There are 40 plates, 1/8" thick, for a total of 5 inches.

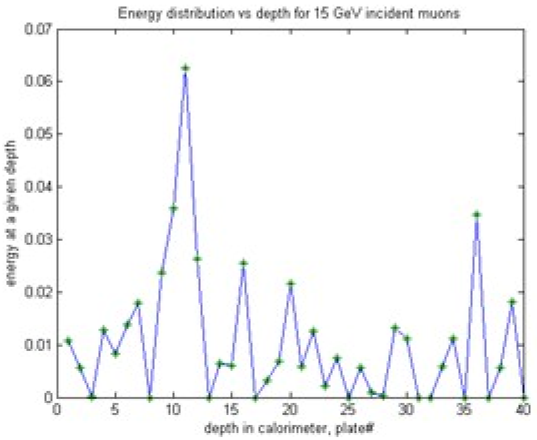


Figure 7.24: Energy deposit for a 15 GeV muon passing through a block of lead.

MC – Power Law

- MC models – choose system dynamics statistically and propagate through some physical apparatus. Run “MC Analytic”

One solvable problem is the power law distribution of a variable x, where α is the desired power. The range of the variables is x_{min} to x_{max} and r is a random variable giving uniformly distributed values from zero to one.

$$\int_{x_{min}}^{x} t^{\alpha} dt \Big/ \int_{x_{min}}^{x_{max}} t^{\alpha}\, dt = r \qquad 1.3$$

$$x = \left[x_{min}^{\alpha+1} + r\left(x_{max}^{\alpha+1} - x_{min}^{\alpha+1}\right)\right]^{1/(\alpha+1)}$$

Exponential and BW

- **Analytic solutions for exp, BW, power and Gauss**

$$\int_{x_{min}}^{x} e^{-t/\tau} dt \Big/ \int_{x_{min}}^{x_{max}} e^{-t/\tau} dt = r \qquad 1.4$$

$$x = -\tau \ln[e^{-x_{min}/\tau} + r(e^{-x_{max}/\tau} - e^{-x_{min}/\tau})]$$

$$\frac{\int_{x_{min}}^{x} \left[\frac{1}{(t-x_o)^2 + (\Gamma/2)^2}\right] dt}{\int_{x_{min}}^{x_{max}} \left[\frac{1}{(t-x_o)^2 + (\Gamma/2)^2}\right] dt} \qquad 1.5$$

$$\varphi_{min} = 2(x_{min} - x_o)/\Gamma$$

$$\varphi_{max} = 2(x_{max} - x_o)/\Gamma$$

$$x = x_o + \Gamma/2 \tan\left\{\tan^{-1}(\varphi_{min}) + r\left[\tan^{-1}(\varphi_{max}) - \tan^{-1}(\varphi_{min})\right]\right\}$$

MC - Gaussian

- **Gaussian requires 2 random number calls, returning 2 variable choices.**

A Gaussian distribution can be achieved by using two random numbers and the fact that the joint probability of two uncorrelated variables is the product of the probabilities. In Eq. 1.6 the standard deviation of the Gaussian is σ. The joint probability can have a "radius" chosen from an exponential and an azimuthal angle chosen randomly from zero to 2π. The means of the Gaussian can simply be added to the resulting x and y values.

$$
\begin{aligned}
& d\underline{P}(x)\, d\underline{P}(y) \\
& = e^{-x^2/2\sigma^2}\; e^{-y^2/2\sigma^2}\; dxdy \qquad\qquad 1.6 \\
& = e^{-r^2/2\sigma^2} r dr d\varphi \\
& = e^{-u/2\sigma^2} \frac{du\, d\varphi}{2},\; u = r^2
\end{aligned}
$$

Numerical Monte Carlo

- Look at “MC_Numeric” to see simple accept/reject method.
- Look at “Euler_Angles” to see the definition. A solid like a top requires 3 variables in general. Note the matrix multiplication and “plot3” use θ,ϕ
- Look at “Euler” to see how to go from a frame where the z axis is the direction of a particle momentum to a frame where the particle moves in the lab with spherical angles $,\theta\ \phi$
- Look at functions “Gauss” and “PowerLaw”

MC - Numeric

- **Most distributions must be chosen from using numerical methods. e.g. Compton scattering**

$$pick\ r_1 := x_{min} + r_1(x_{max} - x_{min})$$

$$pick\ r_2:\ if\ \ r_2 < \underline{P}(x)/\underline{P}(x)_{max}\ \ then\ \ accept\ x$$

$$if\ \ r_2 > \underline{P}(x)/\underline{P}(x)_{max}\ \ then\ reject\ x$$

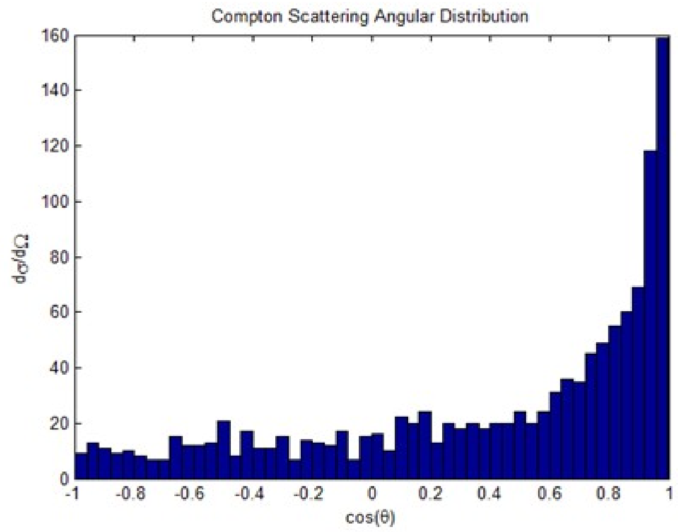


Figure 1.11: Angular distribution in Compton scattering for a specific, user defined, photon energy.

Random Walk

- **Stochastic processes and sqrt behavior. Multiple scattering is an example of a stochastic process. Run "rand_walk"**

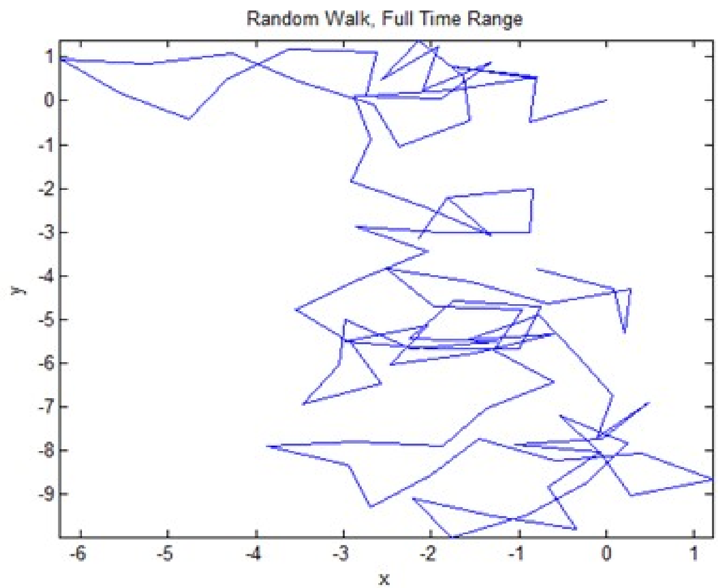


Figure 2.18: The path of a random walk in x and y with 100 steps of length one taken.

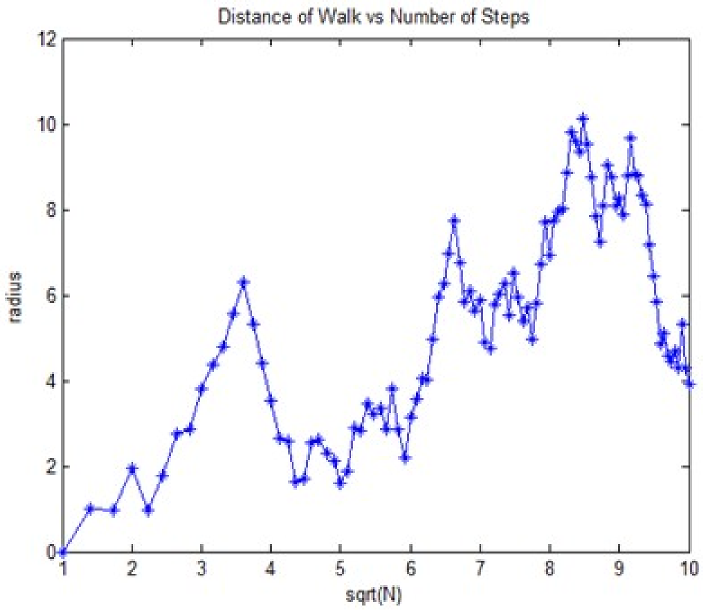


Figure 2.19: The radius from the starting point of a random walk with 100 steps of length one taken.

Multiple Scattering

- When a charged particle goes through material it scatters off the charge of the nuclei.
- Many small angle scatters lead to a resultant, stochastic scattering called multiple
- The characteristic length for the process is called the "radiation length", Xo. The Xo value depends strongly on the atomic number, Z, as expected.

Multiple Scattering - II

- Consider a particle with momentum components px, py, and pz and energy e
- The scattering angle for traversing a length L of material is distributed as a Gaussian with typical angle – no e loss assumed

$$\theta_{MS} = 21MeV\sqrt{L / X_o} / p\beta$$

$$e = \gamma m, p = \gamma\beta m$$

- Use Gauss, rand and Euler to pick the angle w.r.t. Particle direction and then to the lab.

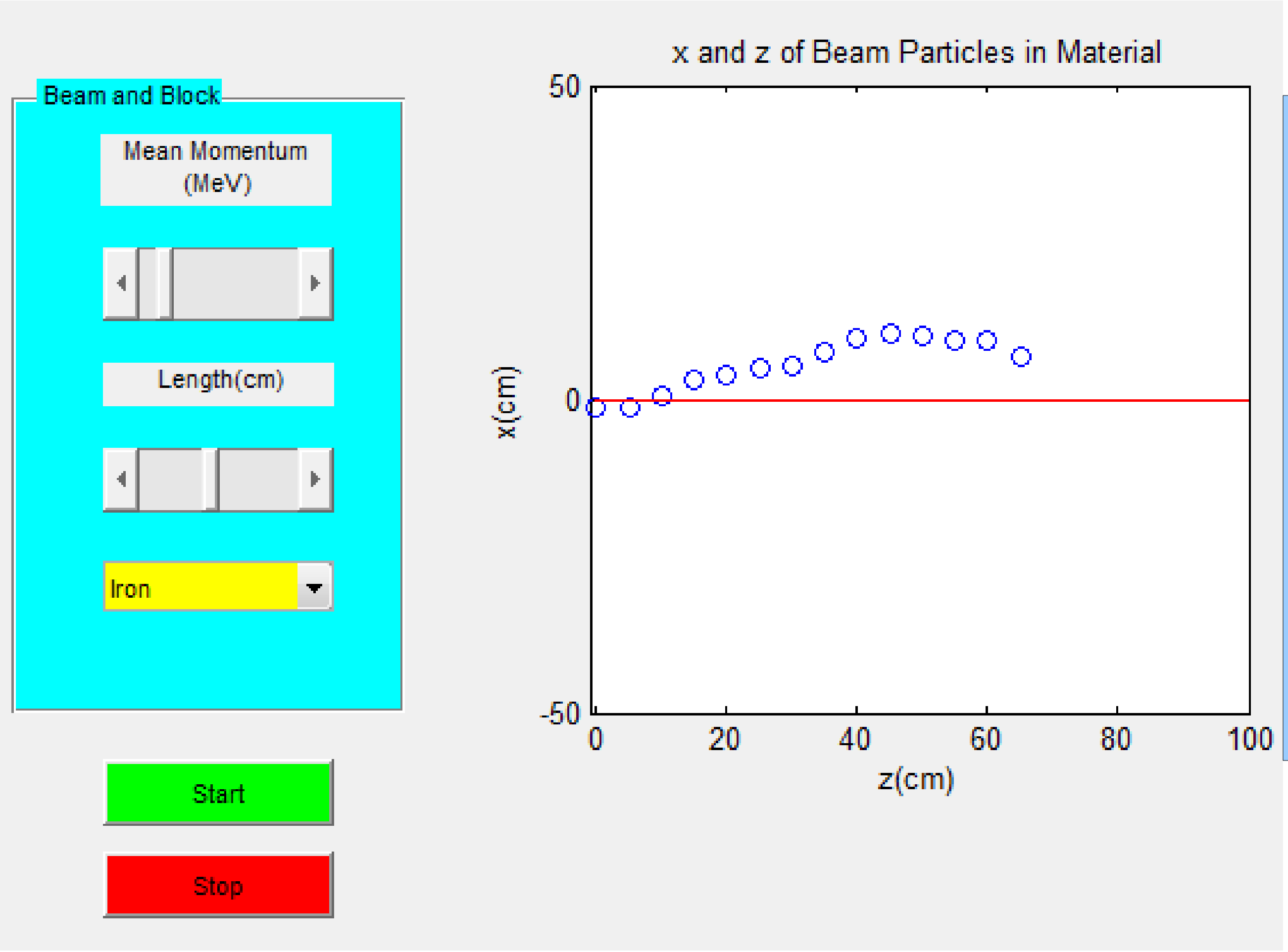
Muon Beam in Material

- **Transformations from lab frame to/from "CM" frame are needed → Euler angles. Look at "Mult_Scatt"**

The script uses the functions "Gauss", "PowerLaw", "Mult_Scatt" and "Euler". The first two functions were described above and provide Gaussian distributed or power law distributed variables. The function "Euler" transforms a momentum vector, p, from a frame with the z axis oriented along spherical angles ϑ and φ to the "laboratory" frame, p', where the z axis has those angles. A check for the special case where the momentum is along the z axis, $p_x = p_y = 0$, yields the momentum components in the primed system which are as expected, seen as the third column of the transformation matrix.

$$\begin{bmatrix} p_x' \\ p_y' \\ p_z' \end{bmatrix} = \begin{bmatrix} \cos\theta\cos\phi & -\sin\phi & \sin\theta\cos\phi \\ \cos\theta\sin\phi & \cos\phi & \sin\theta\sin\phi \\ -\sin\theta & 0 & \cos\theta \end{bmatrix} \begin{bmatrix} \begin{bmatrix} p_x \\ p_y \\ p_z \end{bmatrix} \end{bmatrix} \quad 1.8$$

GUI for Muon Beam



Sliders for Beam momentum And length of Material. Menu For the type Of material.

Particle may stop Before end of Block.

Multiple Scattering

- Plot of beam "spot" at start and end of a block of material. Command line plots. Run "Monte_Muon3"

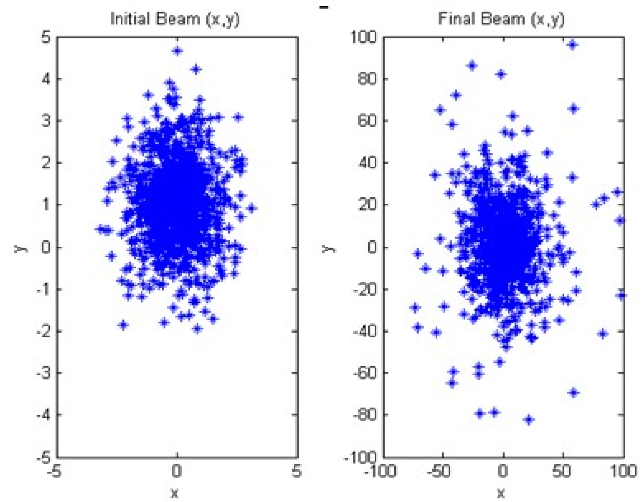


Figure 1.15: Distributions of muon positions in the beam at the start of the traversal of the iron, (left), and after complete traversal of the iron block (right). The scales are a factor of 20 different.