



MATLAB Physics - I



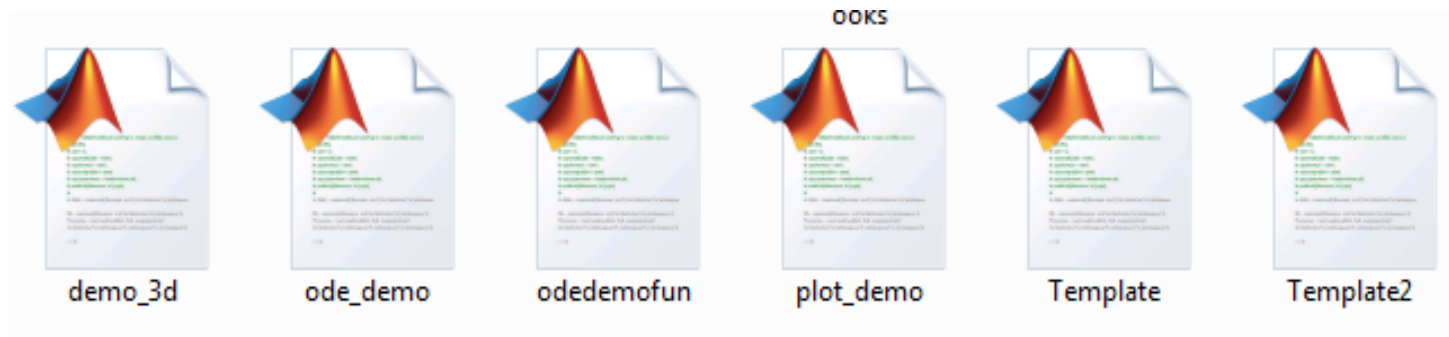
- **MATLAB and Symbolic Math should be installed on an accessible computer – you will execute the scripts.**
- **There is a textbook available**
 - **The book is supplied by UPAS**
 - **There is a CD with the .m file scripts for the demonstrations used in the course**
- **Lecture/class notes – email list for communication of the class**
- **Distribute all scripts by data stick – 200 examples**
- **Homework and “project”**



Programming in MATLAB



- **Templates and demos available - local**



| | | | | |
|--|----------|-------------------|---------------|------|
| | GUI_DEMO | 5/16/2014 4:13 PM | MATLAB Figure | 6 KB |
| | GUI_DEMO | 5/19/2014 9:41 AM | MATLAB Code | 6 KB |



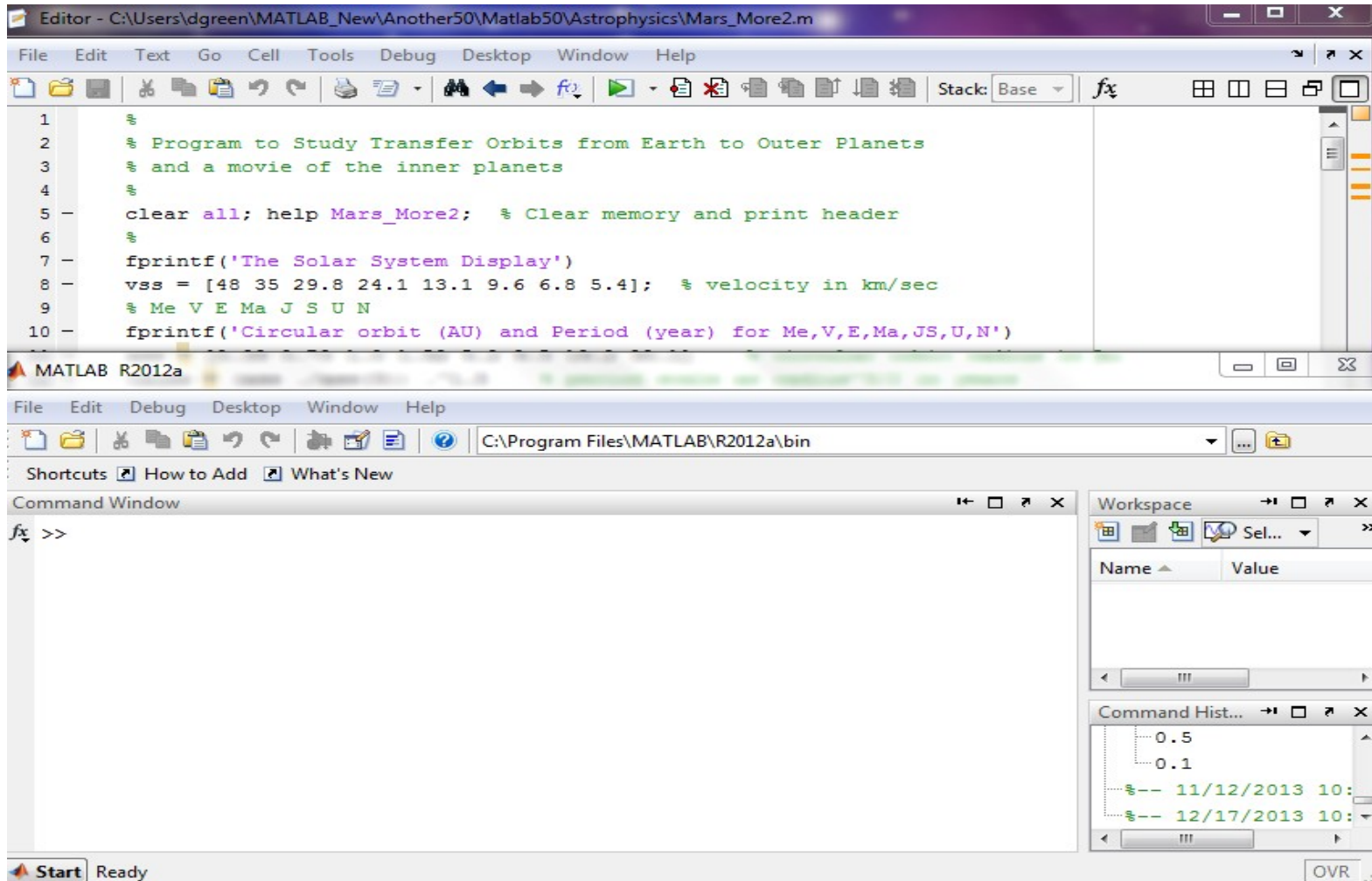
Matlab Windows



- **Workspace, command history, command window and editor – desktop layout**
 - **Open .m files, editor color coded**
 - **Debug options**
 - **In command window – set path to scripts (.m files)**
- **Setup toolbars, status busy/ready. Help**
- **Use product help and function browser**
- **Use demos – e.g. getting started, quick start**



Desktop Layout





- **Getting Started**
 - **Functions**
 - **Examples**
 - **Demos**
- **Quick start**
 - **Language**
 - **Math**
 - **Graphics**
 - **Programming**



Help Page – Top of Tree



The screenshot shows the MATLAB Help browser window. The left sidebar contains a tree view with the following items: Release Notes, Installation, **MATLAB** (selected), Neural Network Toolbox, and Symbolic Math Toolbox. The main content area displays the MATLAB logo and navigation options:

- Functions:**
 - [By Category](#)
 - [Alphabetical List](#)
- Handle Graphics:**
 - [Object Properties](#)

What's New

- [MATLAB Release Notes](#)
Summarizes new features, bug fixes, upgrade issues, etc.
- [General Release Notes for R2012a](#)
For all products, highlights new features, installation notes, bug fixes, and compatibility issues

Documentation Set

- ▶ [Getting Started](#)
- ▶ [User Guides](#)
- [Ways to Get Function Help](#)
Provides instructions for using help functions, the Help browser, and other resources
- [Examples in Documentation](#)
Lists major examples in the MATLAB documentation
- [Programming Tips](#)
Provides helpful techniques and shortcuts for programming in MATLAB

Product Demos

- [MATLAB Demos](#)



Demos and Getting Started



Help

File Edit View Go Favorites Desktop Window Help

Search

Contents Search Results

- Release Notes
- Installation
- MATLAB
 - Getting Started
 - User's Guide
 - Functions
 - Examples
 - Demos**
 - Release Notes
- Neural Network Toolbox
- Symbolic Math Toolbox

MATLAB DEMOS

MATLAB® is a high-level language and interactive environment that enables you to perform computationally intensive tasks faster than with traditional programming languages such as C, C++, and Fortran.

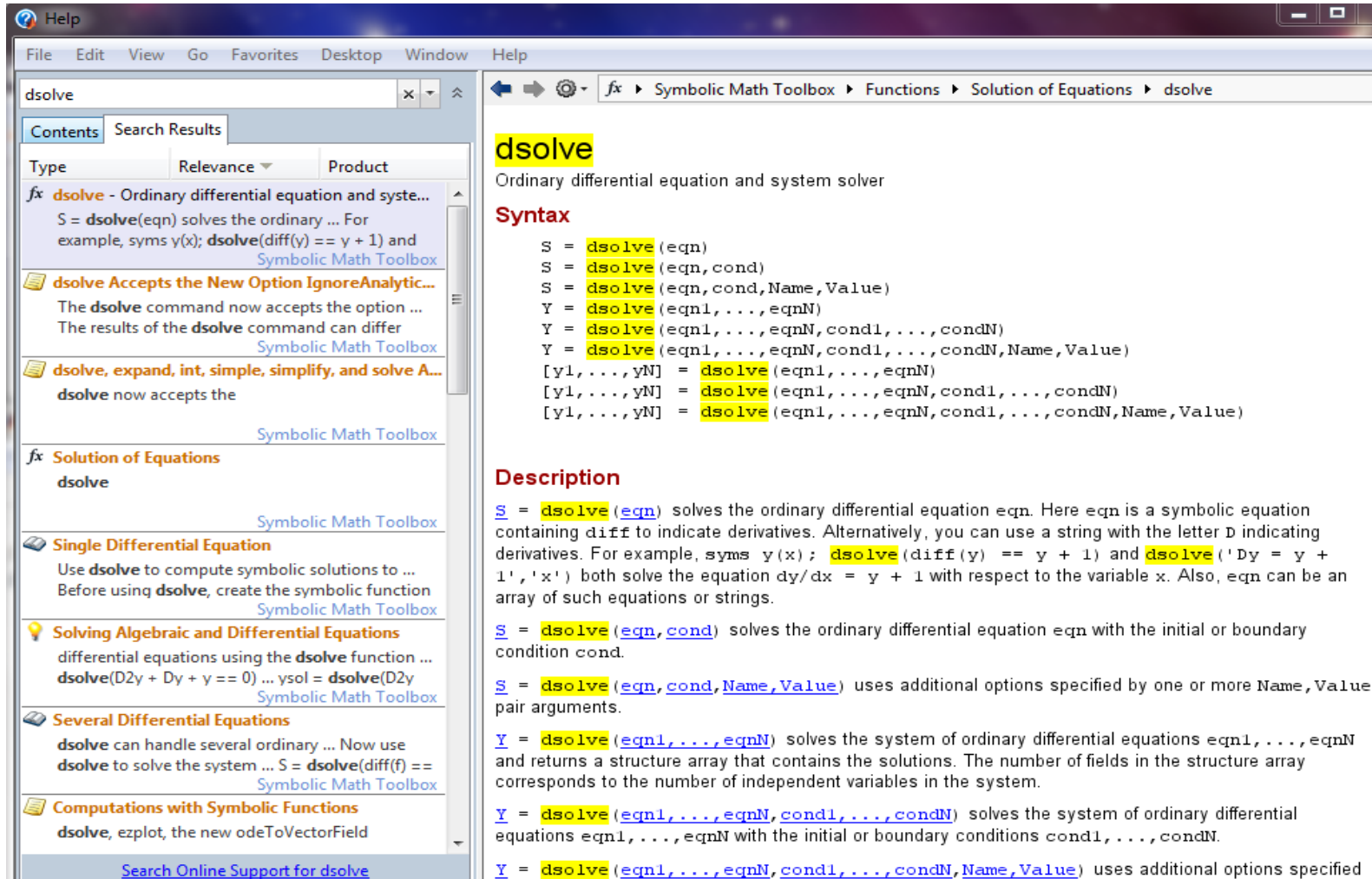
[Product page at mathworks.com](#)

Getting Started

- Getting Started with MATLAB (5 min, 18 sec)
- Working in The Development Environment (4 min, 7 sec)
- Writing a MATLAB Program (5 min, 45 sec)



Drill Down in HELP - Examples



The screenshot shows the MATLAB Help window for the `dsolve` function. The window title is "Help" and the breadcrumb path is "fx > Symbolic Math Toolbox > Functions > Solution of Equations > dsolve". The left pane shows a table of contents with entries like "dsolve - Ordinary differential equation and syste...", "dsolve Accepts the New Option IgnoreAnalytic...", "dsolve, expand, int, simple, simplify, and solve A...", "Solution of Equations", "Single Differential Equation", "Solving Algebraic and Differential Equations", "Several Differential Equations", and "Computations with Symbolic Functions". The right pane contains the following content:

dsolve

Ordinary differential equation and system solver

Syntax

```
S = dsolve (eqn)
S = dsolve (eqn, cond)
S = dsolve (eqn, cond, Name, Value)
Y = dsolve (eqn1, ..., eqnN)
Y = dsolve (eqn1, ..., eqnN, cond1, ..., condN)
Y = dsolve (eqn1, ..., eqnN, cond1, ..., condN, Name, Value)
[y1, ..., yN] = dsolve (eqn1, ..., eqnN)
[y1, ..., yN] = dsolve (eqn1, ..., eqnN, cond1, ..., condN)
[y1, ..., yN] = dsolve (eqn1, ..., eqnN, cond1, ..., condN, Name, Value)
```

Description

S = `dsolve`(`eqn`) solves the ordinary differential equation `eqn`. Here `eqn` is a symbolic equation containing `diff` to indicate derivatives. Alternatively, you can use a string with the letter `D` indicating derivatives. For example, `syms y(x); dsolve(diff(y) == y + 1)` and `dsolve('Dy = y + 1', 'x')` both solve the equation $dy/dx = y + 1$ with respect to the variable `x`. Also, `eqn` can be an array of such equations or strings.

S = `dsolve`(`eqn`, `cond`) solves the ordinary differential equation `eqn` with the initial or boundary condition `cond`.

S = `dsolve`(`eqn`, `cond`, `Name`, `Value`) uses additional options specified by one or more `Name`, `Value` pair arguments.

Y = `dsolve`(`eqn1`, ..., `eqnN`) solves the system of ordinary differential equations `eqn1`, ..., `eqnN` and returns a structure array that contains the solutions. The number of fields in the structure array corresponds to the number of independent variables in the system.

Y = `dsolve`(`eqn1`, ..., `eqnN`, `cond1`, ..., `condN`) solves the system of ordinary differential equations `eqn1`, ..., `eqnN` with the initial or boundary conditions `cond1`, ..., `condN`.

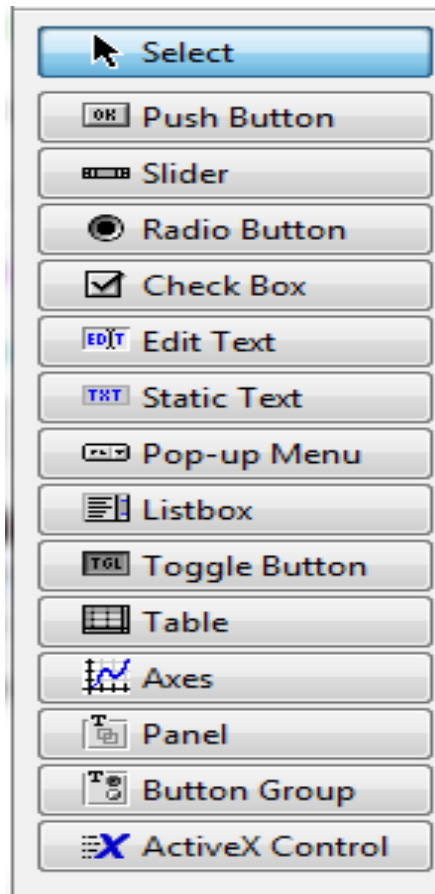
Y = `dsolve`(`eqn1`, ..., `eqnN`, `cond1`, ..., `condN`, `Name`, `Value`) uses additional options specified



Programming - GUI



- Demo exists – many tools supplied**



| | | |
|---------------------|--------------------------|--|
| HandleVisibility | on | |
| HitTest | on | |
| HorizontalAlignment | center | |
| Interruptible | on | |
| KeyPressFcn | | |
| ListboxTop | 1.0 | |
| Max | 1.0 | |
| Min | 0.0 | |
| Position | [9.8 18.308 20.2 2.3...] | |
| SelectionHighlight | on | |
| SliderStep | [0.01 0.1] | |
| String | Push Button | |
| Style | pushbutton | |
| Tag | pushbutton1 | |
| TooltipString | | |
| UIContextMenu | <None> | |
| Units | characters | |
| UserData | [0x0 double arr... | |
| Value | [0.0] | |
| Visible | on | |



MATLAB Displays



- **MATLAB has a suite of displays**
- **2d**
 - **Errorbar, plot, loglog, semilogx, semilogy, polar, bar, pie, hist, scatter**
 - **title, xlabel, ylabel, legend, grid, hold, axis**
- **3d**
 - **Contour, surface, mesh, quiver**



Help - Graphics



Help

File Edit View Go Favorites Desktop Window Help

Search

Contents Search Results

MATLAB

- Getting Started
- User's Guide
- Functions
 - Desktop Tools and Development Environment
 - Data Import and Export
 - Mathematics
 - Data Analysis
 - Programming and Data Types
 - Object-Oriented Programming
 - Graphics**
 - Basic Plots and Graphs
 - Plotting Tools
 - Annotating Plots
 - Specialized Plotting
 - Bit-Mapped Images
 - Printing
 - Handle Graphics
 - 3-D Visualization
 - Surface and Mesh Plots
 - View Control
 - Lighting
 - Transparency
 - Volume Visualization
 - GUI Development
 - External Interfaces
- Examples
- Demos
- Release Notes
- Neural Network Toolbox
- Symbolic Math Toolbox

Graphics

| | |
|--|--|
| Basic Plots and Graphs | Linear line plots, log and semilog plots |
| Plotting Tools | GUIs for interacting with plots |
| Annotating Plots | Functions for and properties of titles, axes labels, legends, mathematical symbols |
| Specialized Plotting | Bar graphs, histograms, pie charts, contour plots, function plotters |
| Bit-Mapped Images | Display image object, read and write graphics file, convert to movie frames |
| Printing | Printing and exporting figures to standard formats |
| Handle Graphics | Creating graphics objects, setting properties, finding handles |

[Back to Top of Section](#)

Basic Plots and Graphs

| | |
|---|--|
| box | Axes border |
| errorbar | Plot error bars along curve |
| hold | Retain current graph when adding new graphs |
| line | Create line object |
| LineStyle (LineStyle Specification) | Line specification string syntax |
| loglog | Log-log scale plot |
| plot | 2-D line plot |
| plot3 | 3-D line plot |
| plotyy | 2-D line plots with y-axes on both left and right side |
| polar | Polar coordinate plot |
| semilogx | Semilogarithmic plot |



Help – 3 d



The screenshot shows the MATLAB Help window with the following structure:

- File Edit View Go Favorites Desktop Window Help
- Search
- Contents Search Results
- Release Notes
- Installation
- MATLAB
 - Getting Started
 - User's Guide
 - fx Functions
 - Desktop Tools and Development Environmen
 - Data Import and Export
 - Mathematics
 - Data Analysis
 - Programming and Data Types
 - Object-Oriented Programming
 - Graphics
 - Basic Plots and Graphs
 - Plotting Tools
 - Annotating Plots
 - Specialized Plotting
 - Bit-Mapped Images
 - Printing
 - Handle Graphics
 - 3-D Visualization**
 - Surface and Mesh Plots
 - View Control
 - Lighting
 - Transparency
 - Volume Visualization
 - GUI Development
 - External Interfaces
 - Examples
 - Demos
 - Release Note

The main content area displays the following sections:

3-D Visualization

- [Surface and Mesh Plots](#) Plot matrices, visualize functions of two variables, specify colormap
- [View Control](#) Control the camera viewpoint, zooming, rotation, aspect ratio, set axis limits
- [Lighting](#) Add and control scene lighting
- [Transparency](#) Specify and control object transparency
- [Volume Visualization](#) Visualize gridded volume data

[Back to Top of Section](#)

Surface and Mesh Plots

- [Surface and Mesh Creation](#) Visualizing gridded and triangulated data as lines and surfaces
- [Domain Generation](#) Gridding data and creating arrays
- [Color Operations](#) Specifying, converting, and manipulating color spaces, colormaps, colorbars, and backgrounds

Surface and Mesh Creation

- [hidden](#) Remove hidden lines from mesh plot
- [mesh](#) Mesh plot
- [meshc](#) Plot a contour graph under mesh graph
- [meshz](#) Plot a curtain around mesh plot
- [peaks](#) Example function of two variables
- [surf](#) 3-D shaded surface plot
- [surface](#) Create surface object
- [surfz](#) Contour plot under a 3-D shaded surface plot



Plot details, Axis definition



Various line types, plot symbols and colors may be obtained with `plot(X,Y,S)` where `S` is a character string made from one element from any or all the following 3 columns:

| | | | | | |
|---|---------|---|------------------|--------|---------|
| b | blue | . | point | - | solid |
| g | green | o | circle | : | dotted |
| r | red | x | x-mark | -. | dashdot |
| c | cyan | + | plus | -- | dashed |
| m | magenta | * | star | (none) | no line |
| y | yellow | s | square | | |
| k | black | d | diamond | | |
| w | white | v | triangle (down) | | |
| | | ^ | triangle (up) | | |
| | | < | triangle (left) | | |
| | | > | triangle (right) | | |
| | | p | pentagram | | |
| | | h | hexagram | | |

For example, `plot(X,Y,'c+:')` plots a cyan dotted line with a plus at each data point; `plot(X,Y,'bd')` plots blue diamond at each data

```
>> help axis
```

```
axis Control axis scaling and appearance.
```

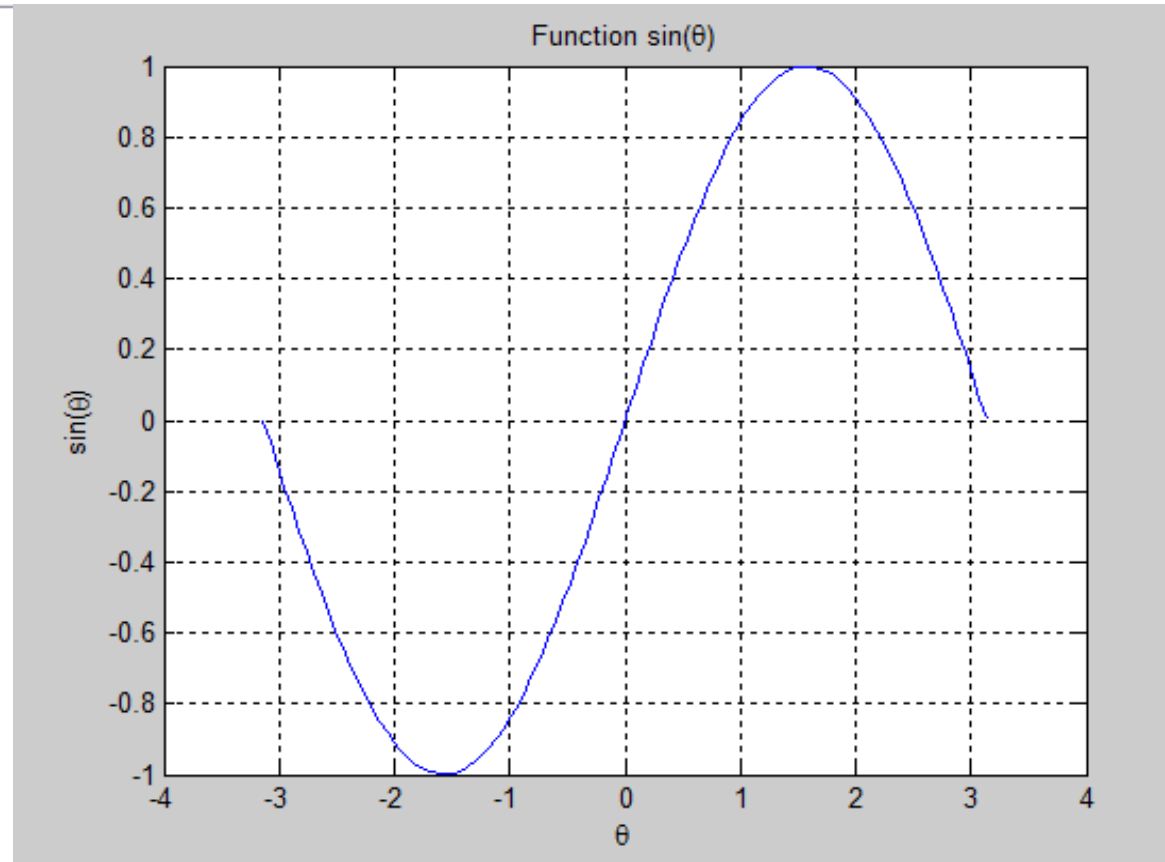
```
axis([XMIN XMAX YMIN YMAX]) sets scaling for the x- and y-axes  
on the current plot.
```



plot(x,y) –



```
%  
% plot demo  
%  
x = linspace(-pi,pi);  
y = sin(x);  
plot(x,y)  
title('Function sin(\theta)')  
xlabel('\theta')  
ylabel('sin(\theta)')  
grid
```





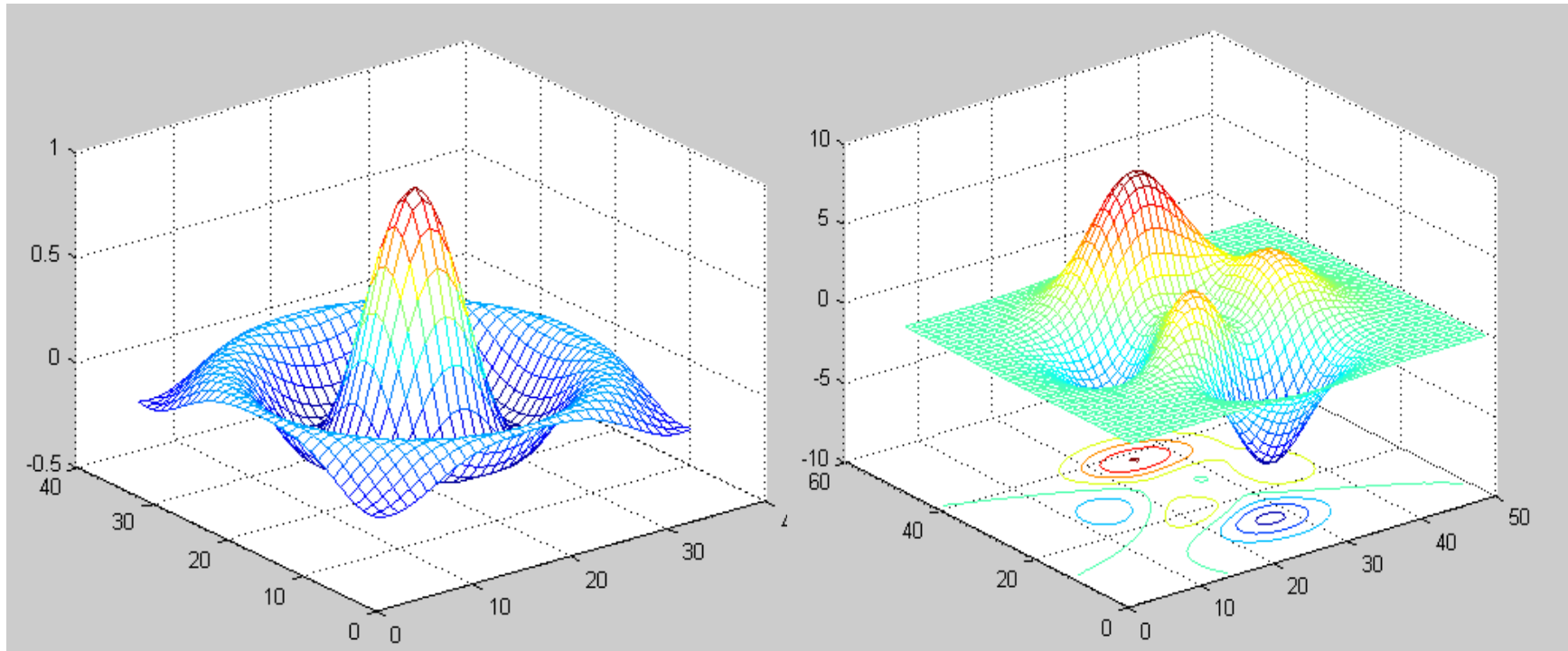
3-d Plots



```
%  
% demo for simple 3-d plots  
%  
figure(1)  
[X,Y] = meshgrid(-8:.5:8);  
R = sqrt(X.^2 + Y.^2) + eps;  
Z = sin(R)./R;  
mesh(Z);  
%  
figure(2)  
[X,Y] = meshgrid(-3:.125:3);  
Z = peaks(X,Y);  
meshc(Z);  
%  
figure(3)  
[X,Y,Z] = peaks(30);  
surf(X,Y,Z);  
axis([-3 3 -3 3 -10 5]);  
%  
figure(4)  
[X,Y] = meshgrid(-2:.2:2);  
Z = X.*exp(-X.^2 - Y.^2);  
[DX,DY] = gradient(Z,.2,.2);  
contour(X,Y,Z)  
hold on  
quiver(X,Y,DX,DY)  
hold off
```

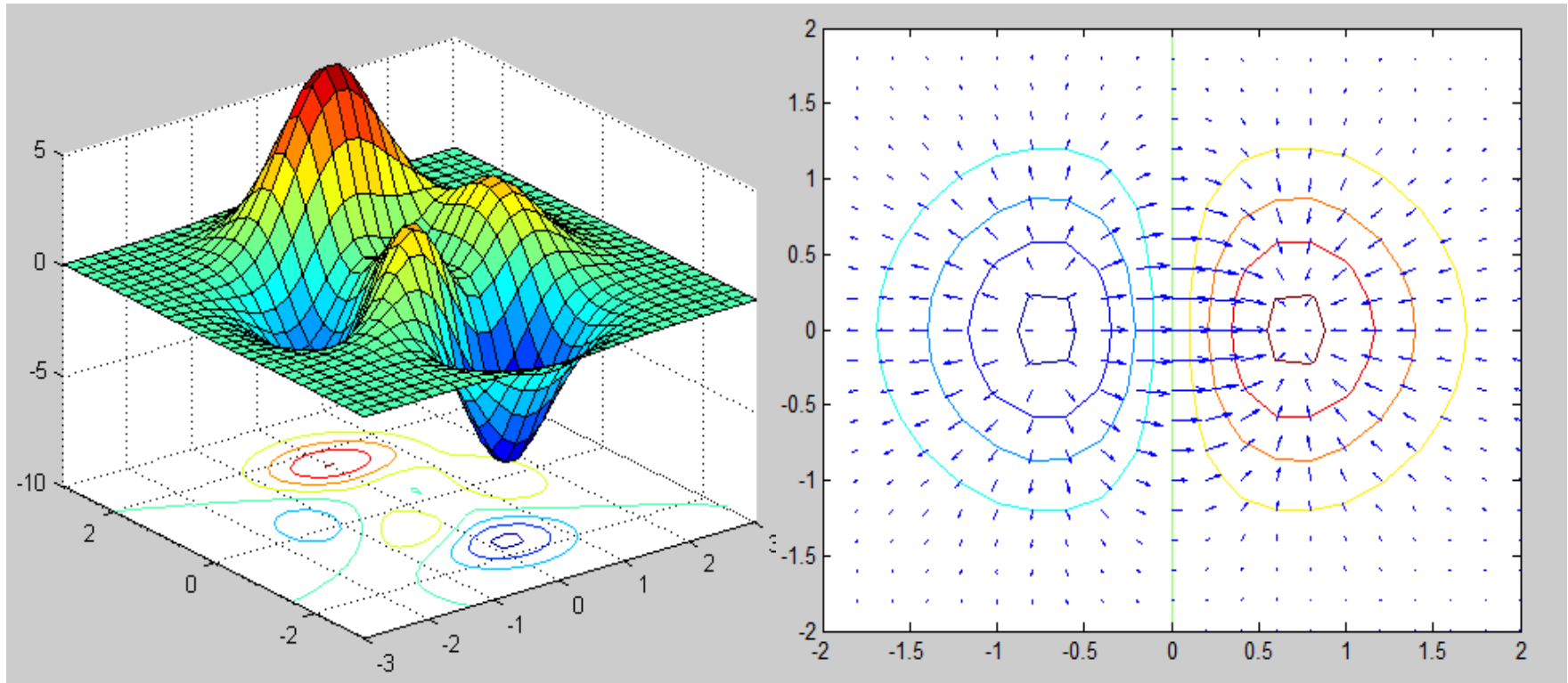


Mesh and Meshc





Surfc and quiver





MATLAB Symbolic Tools



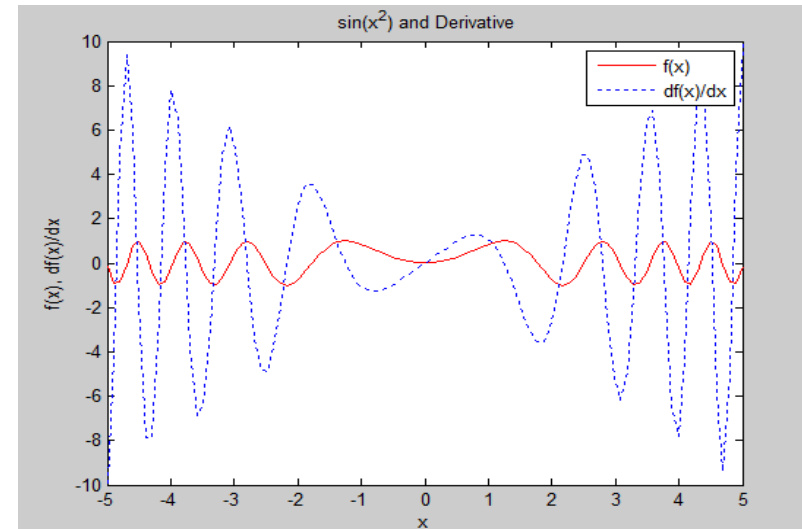
- **MATLAB has many, many tools. You will have to browse through the options.**
 - **Diff, int (calculus)**
 - **Matrix inversion**
 - **Eigenvalues**
 - **Taylor**
 - **Fourier – our scripts**
- **Other examples and script in the textbook**



SM_Diff and SM_Int



- Use MATLAB `diff`, `int` you need never do another integral



```
>> SM_Int
Symbolic Math - Integration

Symbolic Math Indefinite Integration: enter f(x), + - * / ^ cos sin tan sqrt,
An Example, Plot Function and Integral

fin =

tanh(x)

log(cosh(x))
```



SM_Diff Script “loop” -blue



- ~ standard “loop”
- Use “while”
- “break”
- “for”
- “if”
- “end”

```
while irun > 0
    krun = menu('Another Function?','Yes', 'No');
    if krun == 2
        irun = -1;
        break
    end
    %
    if krun == 1;
        clear( 'x', 'fin', 'fpr')
        syms x a b n fin fpr Y N yy
        iloop = iloop + 1;
        fin = input(' Enter f(x): ');
        %
        fpr = diff(sym(fin));
        yy = simple(fpr);
        pretty(yy)
        %
        xx = linspace(-5,5);
        for i = 1:100
            x = xx(i);
            fx(i) = eval(fin);
            dfx(i) = eval(fpr);
        end
        figure(iloop)
        plot(xx,fx,'r-',xx,dfx,':b')
        xlabel('x');
        ylabel('f(x), df(x)/dx')
        title(' Function and Derivative')
        legend('f(x)', 'df(x)/dx')
    %
end
end
```



Eigenvalues



```

>> SM_Eigen
Symbolic Math - Square Matrices: Determinants, Inverses Eigenvalues, Eigenve

Symbolic Matrices: Eigenvalues and Eigenvectors
An Example - Rotation by x
A =

[ cos(x), -sin(x) ]
[ sin(x),  cos(x) ]

Determinint of A
1
Inverse of A
+-      +-
|  cos(x), sin(x) |
|              |
| -sin(x), cos(x) |
+-      +-

Eigenvalues of A, Diagonals
+-      +-
| exp(-x i),  0 |
|              |
|  0,  exp(x i) |
+-      +-

Eigenvectors of A, Columns
+-      +-
| -i,  i |
|              |
|  1,  1 |
+-      +-

Enter A, [A11, A12; A21, A22]: [sin(x)^2, cos(x); -cos(x) , sin(x)^2]

```

You need never solve another eigenvalue problem



Eigen Script



- Used diff and int for the 2 previous examples
- Now use MATLAB matrix tools, det, inv and eig

```
fprintf(' An Example - Rotation by x')
A = [cos(x), -sin(x) ; sin(x), cos(x)]
fprintf('Determinint of A \n ')
Adet = det(A);
y = simple(Adet);
pretty(y)
%
% next the inverse;
%
fprintf('Inverse of A \n ')
Ainv = inv(A);
y = simple(Ainv);
pretty(y)
%
% then the eigenvalues and eigenvectors ;
%
fprintf('Eigenvalues of A, Diagonals \n ')
[Veig,Aeig] = eig(A);
y = simple(Aeig);
pretty(y)
fprintf('Eigenvectors of A, Columns \n ')
y = simple(Veig);
pretty(y)
-
```



Taylor Series

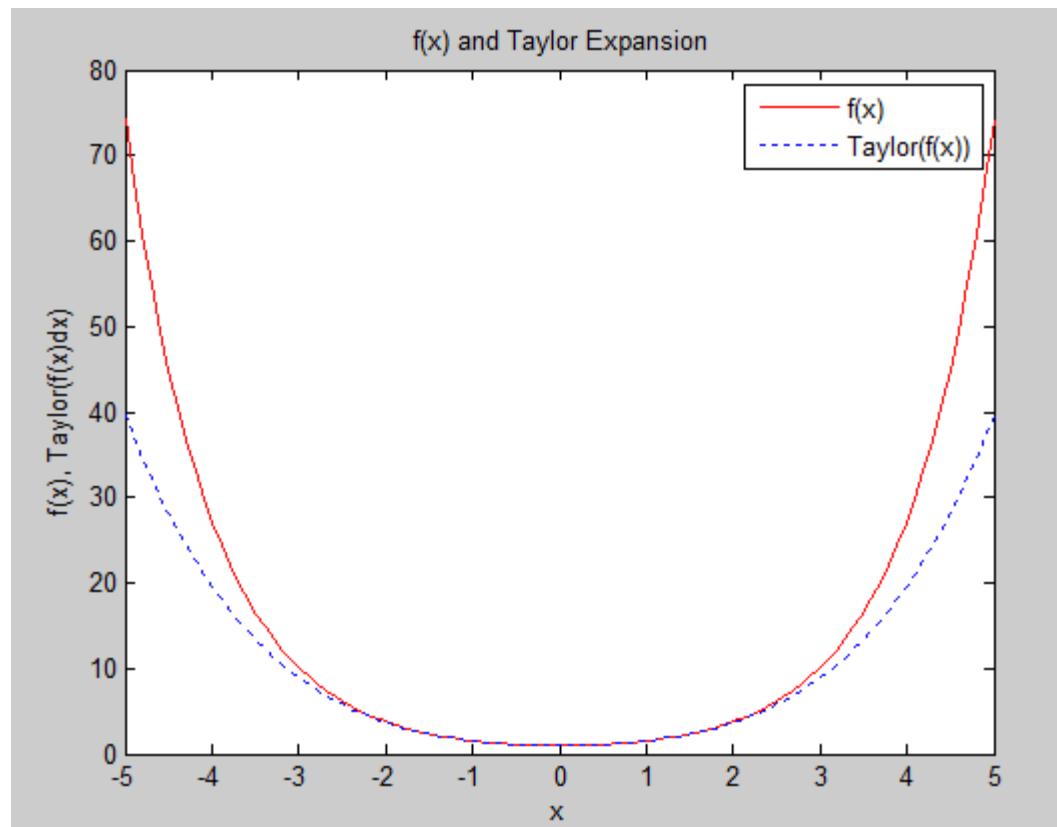


- See convergence of the series graphically – select the number of terms

```
ftay = taylor(sym(fin),x,aoff,'Order',nterm);  
yy = simple(ftay);  
pretty(yy)
```

```
Enter f(x): cosh(x)  
Enter Number of Terms: 5  
Enter a, Expansion About x = a: 0
```

```
  4    2  
  x    x  
-- + -- + 1 |  
24   2
```





Fourier - 1



- **Examples worked out already – square, triangle and sawtooth. See convergence with number of terms**

```
>> SM_fourier_ex
Fourier - fourier series for functions, period = [-1/2,1/2]

x = ao/2 + sumk( ak*cos(kwt) + bk*sin(kwt) )
ak = 2*int(x(u)*cos(2*pi*k*u)du, bk = 2*int(x(u)*sin(2*pi*k*u)du
u = t/T, [-1/2,1/2], w = 2*pi/T, w*t = 2*pi*u
z = pi*k, bk = 0, ak = 2*(1-cosz)/z^2
Cos Coefficients

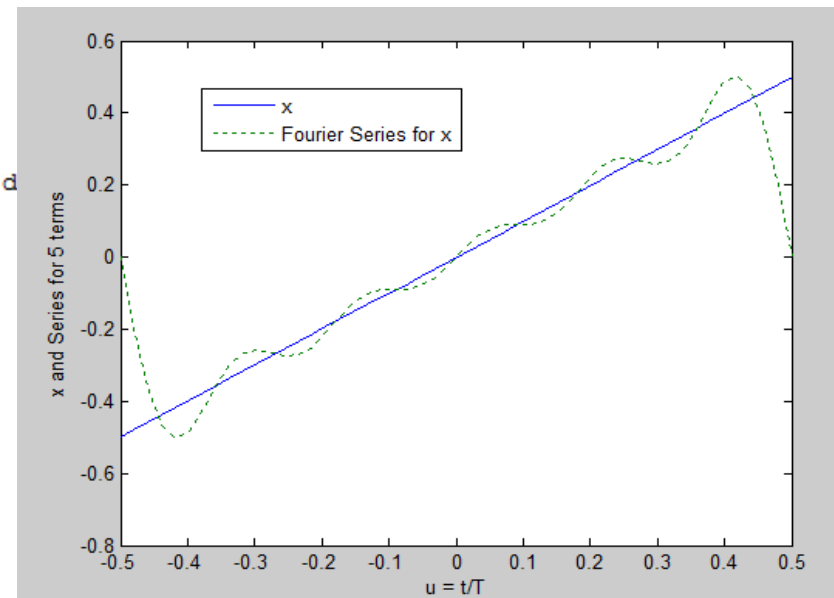
a =

    0    0    0    0    0

Sin Coefficients

b =

    0.3183   -0.1592    0.1061   -0.0796    0.0637
```





Fourier - 2



- Use `int`, `sym` and `eval` to do a general Fourier series. Any analytically solvable series!

```
>> SM_fourier_try
Fourier - fourier series for analytic function - input

Period is T = 1, Enter x(t) for First and Second 1/2 Period
Enter x(t) for t < 1/2 as an equation; e.g., t*cos(t)
: t*cos(t)
Enter x(t) for t > 1/2 as an equation; e.g., t*cos(t)
: t*cos(t)
Enter Number of Terms in the Series : 5
Even and Odd Fourier Coefficients

a =

[ 2*cos(1) + 2*sin(1) - 2, (2*(2*cos(1) + 2*cos(1/2) + 2*sin(1) + sin(1/2) + pi^2*(8*cos(1)

b =

[ 0, (2*(pi*(2*cos(1) + cos(1/2) - 4*sin(1) - 4*sin(1/2)) - pi^3*(8*cos(1) + 4*cos(1/2)))/

Full Fourier Series

xx = |

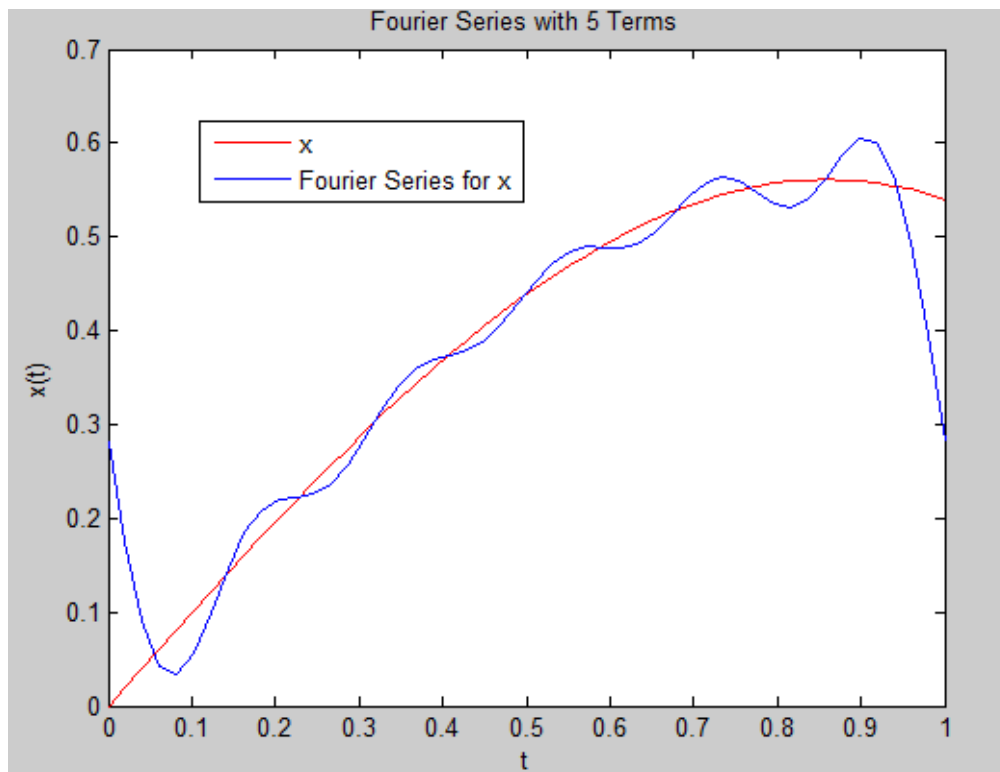
cos(1) + sin(1) - sin(2*pi*t)*((2*(pi*(cos(1/2) - 4*sin(1/2)) - 4*pi^3*cos(1/2)))/(4*pi^2 -
```



Fourier - 3



- Example for $f(t) = t \cdot \cos(t)$, 5 terms



$$x = a_0/2 + \sum_k [a_k \cos(k\omega t) + b_k \sin(k\omega t)]$$

$$a_k = 2 \int x(u) \cos(2\pi k u) du$$

$$b_k = 2 \int x(u) \sin(2\pi k u) du$$

$$u = t/T, [-1/2, 1/2]$$

$$\omega = 2\pi/T, \omega t = 2\pi u$$



Numeric Tools - I



- **Most problems must be solved numerically**
- **Numeric tools for a vector x**
 - **Length**
 - **Min**
 - **Max**
 - **Mean**
 - **Std**
 - **sort**



Numeric Tools- II



- **MATLAB utility “quad” does numeric integration. Try “int” first – default to “quad” if it fails.**

quad

Numerically evaluate integral, adaptive Simpson quadrature

Syntax

```
q = quad(fun,a,b)
q = quad(fun,a,b,tol)
q = quad(fun,a,b,tol,trace)
[q,fcnt] = quad(...)
```

Description

Quadrature is a numerical method used to find the area under the graph of a function, that is, to compute a definite integral.

$$q = \int_a^b f(x) dx$$

`q = quad(fun,a,b)` tries to approximate the integral of function `fun` from `a` to `b` to within an error of $1e-6$ using recursive adaptive Simpson quadrature. `fun` is a function handle. See [Function Handles](#) in the MATLAB Programming documentation for more information. Limits `a` and `b` must be finite. The function `y = fun(x)` should accept a vector argument `x` and return a vector result `y`, the integrand evaluated at each element of `x`.



MATLAB Equation Solvers



- **MATLAB has several ways to solve equations**
 - Symbolic solutions should be tried first
 - Define symbolic variables using “syms”
 - “solve” is for algebraic equations
 - “dsolve” is for ordinary differential equations
 - “ode45” is for numerical solutions to ODE
 - “pde” is for partial differential equations in 1 space (x) and 1 time (t) dimension



Help for solve – has examples



fx Symbolic Math Toolbox Functions Solution of Equations solve

solve

Equations and systems solver

Syntax

```

S = solve(eqn)
S = solve(eqn,var,Name,Value)
Y = solve(eqn1,...,eqnN)
Y = solve(eqn1,...,eqnN,var1,...,varN,Name,Value)
[y1,...,yN] = solve(eqn1,...,eqnN)
[y1,...,yN] = solve(eqn1,...,eqnN,var1,...,varN,Name,Value)

```

Description

`S = solve(eqn)` solves the equation `eqn` for the default variable determined by `symvar`. You can specify the independent variable. For example, `solve(x + 1 == 2, x)` solves the equation $x + 1 = 2$ with respect to the variable `x`.

`S = solve(eqn,var,Name,Value)` uses additional options specified by one or more `Name,Value` pair arguments. If you do not specify `var`, the solver uses the default variable determined by `symvar`.

`Y = solve(eqn1,...,eqnN)` solves the system of equations `eqn1,...,eqnN` for the variables determined by `symvar` and returns a structure array that contains the solutions. The number of fields in the structure array corresponds to the number of independent variables in a system.

`Y = solve(eqn1,...,eqnN,var1,...,varN,Name,Value)` uses additional options specified by one or more `Name,Value` pair arguments. If you do not specify `var1,...,varN`, the solver uses the default variables determined by `symvar`.

`[y1,...,yN] = solve(eqn1,...,eqnN)` solves the system of equations `eqn1,...,eqnN` for the variables determined by `symvar` and assigns the solutions to the variables `y1,...,yN`.

`[y1,...,yN] = solve(eqn1,...,eqnN,var1,...,varN,Name,Value)` uses additional options specified by one or more `Name,Value` pair arguments. If you do not specify `var1,...,varN`, the solver uses the default variables determined by `symvar`.

- Use help and search for solve
- Symbolic solution
- You need never solve another ODE



dsolve



• Solves ODE symbolically with IC

```
fx Symbolic Math Toolbox Functions Solution of Equations dsolve
```

dsolve
Ordinary differential equation and system solver

Syntax

```
S = dsolve(eqn)
S = dsolve(eqn,cond)
S = dsolve(eqn,cond,Name,Value)
Y = dsolve(eqn1,...,eqnN)
Y = dsolve(eqn1,...,eqnN,cond1,...,condN)
Y = dsolve(eqn1,...,eqnN,cond1,...,condN,Name,Value)
[y1,...,yN] = dsolve(eqn1,...,eqnN)
[y1,...,yN] = dsolve(eqn1,...,eqnN,cond1,...,condN)
[y1,...,yN] = dsolve(eqn1,...,eqnN,cond1,...,condN,Name,Value)
```

Description

S = `dsolve(eqn)` solves the ordinary differential equation `eqn`. Here `eqn` is a symbolic equation containing `diff` to indicate derivatives. Alternatively, you can use a string with the letter `D` indicating derivatives. For example, `syms y(x); dsolve(diff(y) == y + 1)` and `dsolve('Dy = y + 1','x')` both solve the equation $dy/dx = y + 1$ with respect to the variable `x`. Also, `eqn` can be an array of such equations or strings.

S = `dsolve(eqn,cond)` solves the ordinary differential equation `eqn` with the initial or boundary condition `cond`.

S = `dsolve(eqn,cond,Name,Value)` uses additional options specified by one or more `Name,Value` pair arguments.

Y = `dsolve(eqn1,...,eqnN)` solves the system of ordinary differential equations `eqn1,...,eqnN` and returns a structure array that contains the solutions. The number of fields in the structure array corresponds to the number of independent variables in the system.

Y = `dsolve(eqn1,...,eqnN,cond1,...,condN)` solves the system of ordinary differential equations `eqn1,...,eqnN` with the initial or boundary conditions `cond1,...,condN`.

Y = `dsolve(eqn1,...,eqnN,cond1,...,condN,Name,Value)` uses additional options specified



Dsolve - example



- Use symbolically for ODE. Initial conditions

```
>> syms a y(t)
>> Dy = diff(y);
>> D2y = diff(y, 2);
>> dsolve(D2y == -a*y)

ans =

C7*exp((-a)^(1/2)*t) + C8*exp(-(-a)^(1/2)*t)

>> dsolve(D2y == -a*y , Dy(0) == 0, y(0) ==1)

ans =

exp((-a)^(1/2)*t)/2 + exp(-(-a)^(1/2)*t)/2

>> simple(ans)

simplify:

cosh((-a)^(1/2)*t)
```



SM_ODE3



- **SM_ODE2 allows a choice of a few well known equations. SM_ODE3 is free form.**

```
>> SM_ODE3
  Program to symbolically solve ODE

Enter Single Differential Eq to Solve y(t); e.g., D2y+a*y=0
: Dy+a*y^3=0
Enter Condition on Function; e.g. y(a) = b,
: y(0)=b
Symbolic Solution, y(t) and v(t)

y =

(2^(1/2)*(1/(a*t + 1/(2*b^2))))^(1/2)/2
-(2^(1/2)*(1/(a*t + 1/(2*b^2))))^(1/2)/2

v =

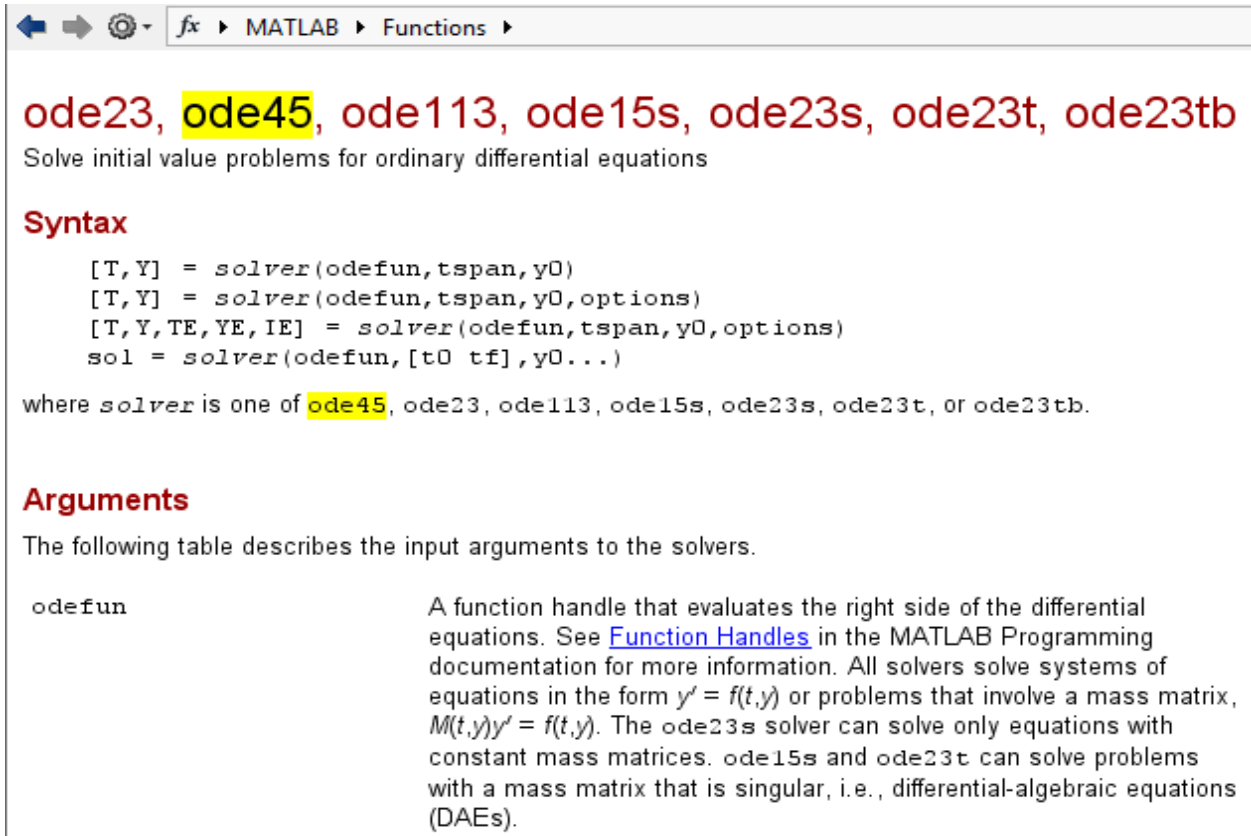
-(2^(1/2)*a)/(4*(1/(a*t + 1/(2*b^2))))^(1/2)*(a*t + 1/(2*b^2))^2
(2^(1/2)*a)/(4*(1/(a*t + 1/(2*b^2))))^(1/2)*(a*t + 1/(2*b^2))^2
```



ode45



• Numerical solution of ODE



ode23, **ode45**, ode113, ode15s, ode23s, ode23t, ode23tb

Solve initial value problems for ordinary differential equations

Syntax

```
[T, Y] = solver(odefun, tspan, y0)
[T, Y] = solver(odefun, tspan, y0, options)
[T, Y, TE, YE, IE] = solver(odefun, tspan, y0, options)
sol = solver(odefun, [t0 tf], y0...)
```

where *solver* is one of **ode45**, ode23, ode113, ode15s, ode23s, ode23t, or ode23tb.

Arguments

The following table describes the input arguments to the solvers.

| | |
|--------|---|
| odefun | A function handle that evaluates the right side of the differential equations. See Function Handles in the MATLAB Programming documentation for more information. All solvers solve systems of equations in the form $y' = f(t, y)$ or problems that involve a mass matrix, $M(t, y)y' = f(t, y)$. The <code>ode23s</code> solver can solve only equations with constant mass matrices. <code>ode15s</code> and <code>ode23t</code> can solve problems with a mass matrix that is singular, i.e., differential-algebraic equations (DAEs). |
|--------|---|

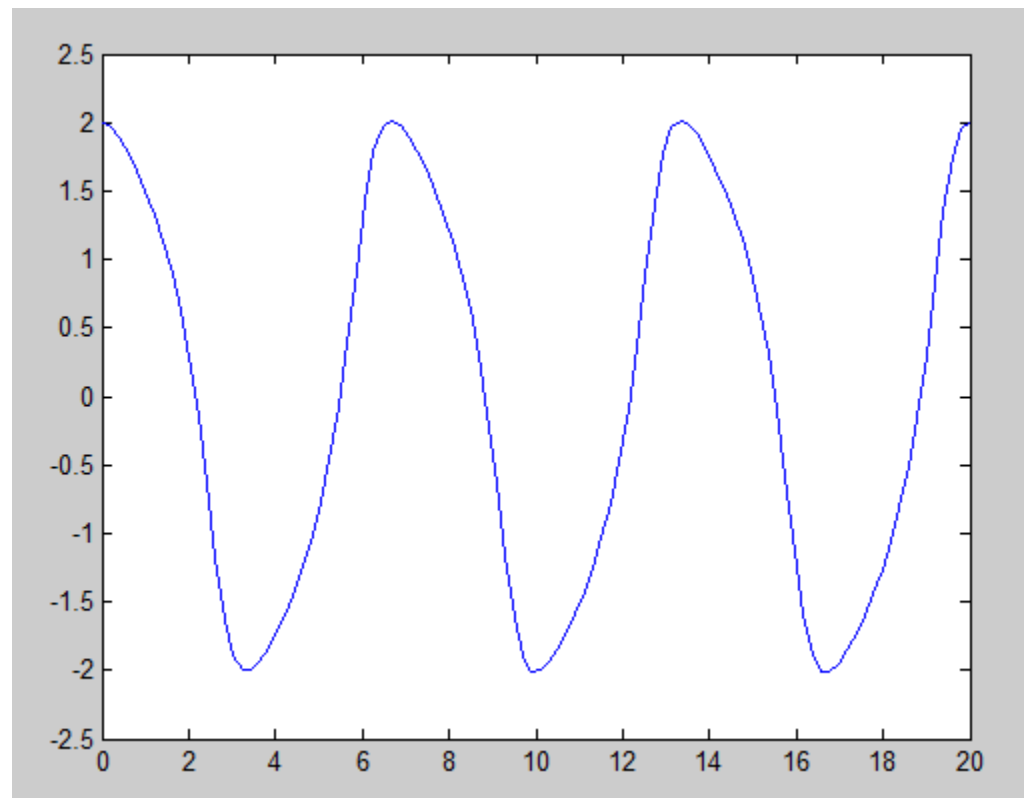


ode45 - 2



**Need to define
a function
which
specifies the
ODE**

```
>> sol = ode45(@vdp1,[0 20],[2 0]);  
>> x = linspace(0,20);  
>> y = deval(sol,x,1);  
>> plot(x,y);
```





ode45 - 3



- **The physics is all in the function specified. The boundary conditions are specified in the ode45 call as is the time evolution of the system. We do examples later.**

An example of a nonstiff system is the system of equations describing the motion of a rigid body without external forces.

$$\begin{aligned}y_1' &= y_2 y_3 & y_1(0) &= 0 \\y_2' &= -y_1 y_3 & y_2(0) &= 1 \\y_3' &= -0.51 y_1 y_2 & y_3(0) &= 1\end{aligned}$$

To simulate this system, create a function `rigid` containing the equations

```
function dy = rigid(t,y)
dy = zeros(3,1);    % a column vector
dy(1) = y(2) * y(3);
dy(2) = -y(1) * y(3);
dy(3) = -0.51 * y(1) * y(2);
```



● PDE solver in MATLAB

PDE Solver Syntax

The basic syntax of the solver is:

```
sol = pdepe(m,pdefun,icfun,bcfun,xmesh,tspan)
```

The input arguments are

m Specifies the symmetry of the problem. **m** can be 0 = slab, 1 = cylindrical, or 2 = spherical. It corresponds to *m* in [Equation 10-2](#).

pdefun Function that defines the components of the PDE. It computes the terms *c*, *f*, and *s* in [Equation 10-2](#), and has the form

```
[c,f,s] = pdefun(x,t,u,dudx)
```

where *x* and *t* are scalars, and *u* and *dudx* are vectors that approximate the solution *u* and its partial derivative with respect to *x*. *c*, *f*, and *s* are column vectors. *c* stores the diagonal elements of the matrix *c*.

icfun Function that evaluates the initial conditions. It has the form

```
u = icfun(x)
```

When called with an argument *x*, *icfun* evaluates and returns the initial values of the solution components at *x* in the column vector *u*.



• Example from 1-d Quantum Mechanics

```
sol = pdepe(m,@Sch_pde,@Sch_ic,@Sch_bc,x,t);  
%  
psixt = sol(:,:,1);  
%  
%-----  
function [c,f,s] = Sch_pde(x,t,u,DuDx)  
%  
global xm dxo k Vo a  
hbar = 0.666; % units eV,Å, 10^-15 sec  
mec2 = 511000.0; % eV - electron mass  
hbarc = 2000.0; % ev*Å  
c = j .*hbar;  
f = -(hbarc .*hbarc .*DuDx) ./ (2.0 .*mec2); % kinetic energy  
%s = 0.0; % free particle  
if x < 0 || x > a  
    s = 0;  
else  
    s = Vo .*u; % constant potential energy  
end  
%
```



Special Functions



- You need never again look up a special function! Search MATLAB help (mfun)
- Run script DG/SM ” Matlab_Functions”

| Function Name | Definition | mfun Name | Arguments |
|--|--|--|---|
| Bernoulli numbers and polynomials | Generating functions: $\frac{e^{xz}}{e^z - 1} = \sum_{n=0}^{\infty} B_n(x) \cdot \frac{z^{n-1}}{n!}$ | bernoulli (n) bernoulli (n, t) | $n \geq 0$ $0 < t < 2\pi$ |
| Bessel functions | BesselJ, BesselY—Bessel functions of the first kind. BesselK, BesselI—Bessel functions of the second kind. | BesselJ (v, x) BesselY (v, x) BesselI (v, x) BesselK (v, x) | v is real. |
| Beta function | $B(x, y) = \frac{\Gamma(x) \cdot \Gamma(y)}{\Gamma(x+y)}$ | Beta (x, y) | |
| Binomial coefficients | $\binom{m}{n} = \frac{m!}{n!(m-n)!}$ $= \frac{\Gamma(m+1)}{\Gamma(n+1)\Gamma(m-n+1)}$ | binomial (m, n) | |
| Complete elliptic integrals | Legendre's complete elliptic integrals of the first, second, and third kind. This definition uses modulus k. The numerical ellipke function and the MuPAD functions for computing elliptic integrals use the parameter $m = k^2 = \sin^2 \alpha$. | EllipticK (k) EllipticE (k) EllipticPi (a, k) | a is real, $-\infty < a < \infty$. k is real, $0 < k < 1$. |
| Complete elliptic integrals with complementary modulus | Associated complete elliptic integrals of the first, second, and third kind using complementary modulus. This definition uses modulus k. The numerical ellipke function and the MuPAD functions for computing elliptic integrals use the parameter $m = k^2 = \sin^2 \alpha$. | EllipticCK (k) EllipticCE (k) EllipticCPi (a, k) | a is real, $-\infty < a < \infty$. k is real, $0 < k < 1$. |

