Homework 7 Solution for #4

Exercise 4.

a) The radius of the RING

$$\rho = \frac{C}{2\pi} = \frac{40 \times 10^6\, m}{2\pi} \simeq ~~3.662\times10~~ 6.366\times10^6\, m$$

With "Beam rigidity" relation

$$B\rho = \frac{10}{2.998} \beta p(GeV) \Rightarrow B\rho = 3.336 \beta p(GeV/c)$$

$$\Rightarrow p = \frac{B\rho}{3.336\beta} = \frac{8.7\,T \times 6.366\times10^6 m}{3.336\times1}$$

$$\simeq 16.6\times10^6\, GeV/c$$

$$\Rightarrow \boxed{E = 1.66\times10^7\, GeV = 1.67\times10^{16}\, eV}$$

b) For Lep. the bending radius 3026 m

Length of LEP 26.7 km

$3026 \times 2\pi \simeq 19\, km < 26.7\, km$.

Straight section. $26.7\,km - 19\,km \simeq 7.7\,km$.

SR power in bending dipole

$$P = \frac{P_{ave} \times 26.7\,km}{19\,km} = \frac{20\,MW \times 26.7\,km}{19\,km} \simeq 28.1\,MW$$

$\because$ SR P: $P(kW) = 88.46 \frac{E^4(GeV)\, I(A)}{\rho(m)}$

$$\Rightarrow I(A) = \frac{P(kW) \times \rho(m)}{88.46 \times E^4(GeV)} = \frac{28.1\times10^3 \times 3026}{88.46 \times (100)^4}$$

$$= 95.8\times10^{-5}\,A \simeq 9.6\times10^{-3}\,A$$

For Proton

$$P_p = 6.03 \, \frac{E^4(TeV)\, I(A)}{\rho(m)}$$

$$= \frac{6.03 \times (1.66 \times 10^4)^4 \times 9.6 \times 10^{-3}}{6.366 \times 10^6}$$

$$= \frac{6.03 \times 1.66^4 \times 9.6}{6.366} \times 10^{16-3-6}$$

$$= 69 \times 10^7 = 6.9 \times 10^8 \ KW$$

c)

$$\frac{P_{LEP}}{L_{LEP}} = \frac{P_{RING}}{L_{RING}}$$

$$P_{RING} = \frac{P_{LEP}}{L_{LEP}} \cdot L_{RING}$$

$$= \frac{20\,MW}{26.7\,km} \times 40\,Mm \simeq 3 \times 10^4 \,MW$$

For $P(kW) = 88.46 \, \frac{E^4(GeV)\, I(A)}{\rho(m)}$

$$\Rightarrow E = \left[\frac{P(kW) \cdot \rho(m)}{88.46 \times I(A)} \right]^{\frac{1}{4}}$$

$$= \left[\frac{3 \times 10^7 \times 6.366 \times 10^6}{88.46 \times 9.6 \times 10^{-3}} \right]^{\frac{1}{4}}$$

$$= \left[\frac{3 \times 6.366}{88.46 \times 9.6} \right]^{\frac{1}{4}} \times 10^4 \simeq 3.87 \times 10^3 \,GeV$$

$$= 3.87 \,TeV$$

d) $N = \frac{0.1nC}{1.6\times10^{-19}C} = 6.25\times10^{8}$

$I = \frac{Q}{T} = \frac{0.1nC \times 2\times10^{6}}{~~6.366~~ / 40\times10^{6}/3\times10^{8}} \simeq 1.5\times10^{-3}A$

Normalized emittance $\varepsilon_0 = \beta\gamma\varepsilon$

$\Rightarrow \varepsilon = \frac{\varepsilon_0}{\beta\gamma} = \frac{0.4\times10^{-6}}{(1.66\times10^{7}/0.938)} \simeq 2.26\times10^{-14}\ \pi\cdot m\cdot rad$

$f_{coll} = N_{bunch}\cdot f_0$

$= 2\times10^{6}\times\frac{3\times10^{8}}{40\times10^{6}} = 1.5\times10^{7}Hz$

$L = \frac{f_{coll}\cdot N\cdot N}{4\varepsilon\beta^*} = \frac{1.5\times10^{7}\times(6.25\times10^{8})^2}{4\times2.26\times10^{-14}\times12.8}$

$\simeq 0.51\times10^{37}m^{-2}s^{-1}$

$= 5.1\times10^{34}cm^{-2}s^{-1}$

$\Delta Q_y = \frac{r_p}{2}\frac{\beta_y^* N}{\gamma A_{int}}$ $\quad A_{int} = \varepsilon_x\beta_y^*$

$\Rightarrow \Delta Q_y = \frac{r_p N}{2\varepsilon_0}$ $\quad r_p = \frac{e^2}{4\pi\varepsilon_0 m_p c^2} \simeq 1.54\times10^{-18}m$

(classic radius of proton)

$\Rightarrow \Delta Q_y = \frac{1.54\times10^{-18}\times6.25\times10^{8}}{2\times0.4\times10^{-6}} \simeq 1.2\times10^{-3}$