Lecture 9a

Synchrotron Radiation

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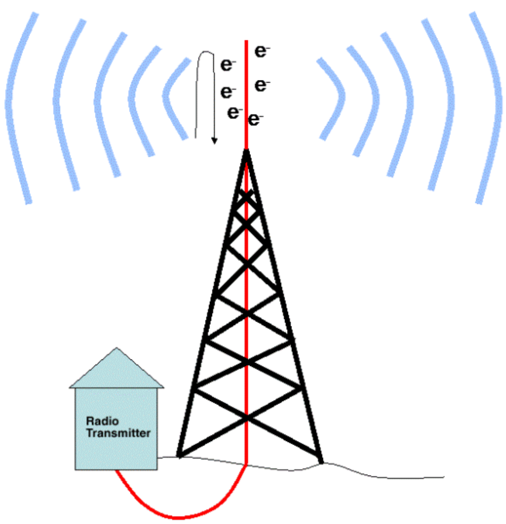
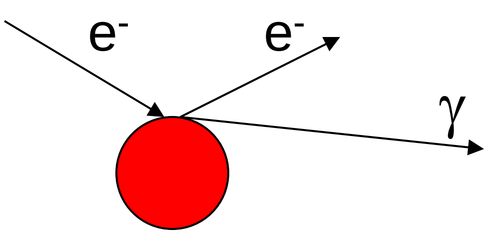
Radiation from an Accelerated Charge

- A charge that is accelerated emits electro-magnetic radiation
- Examples you may be familiar with:

  - EM radiated from an antenna: time-varying current runs up and down the antenna, and in the process emits radio waves



  - Bremsstrahlung: (braking radiation). An electron is accelerated when it collides with an atomic nucles, emitting a photon

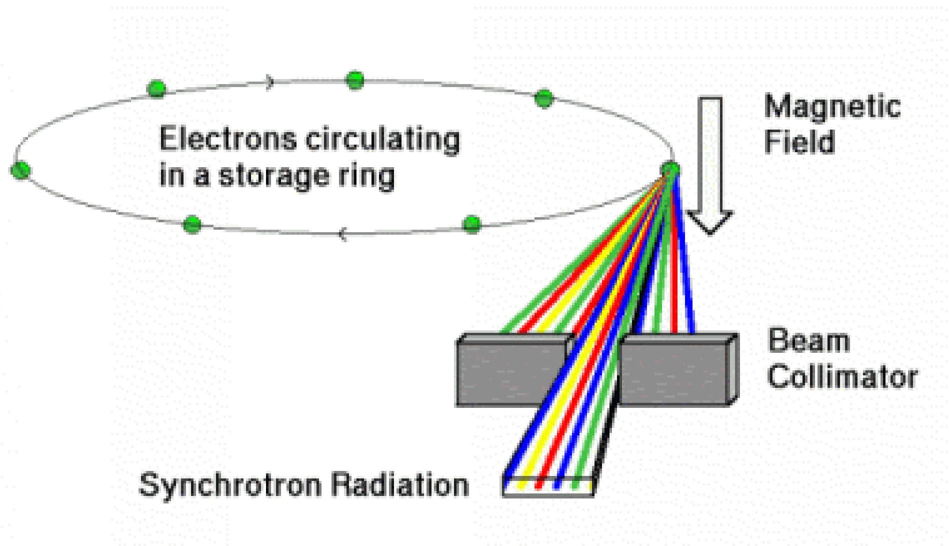


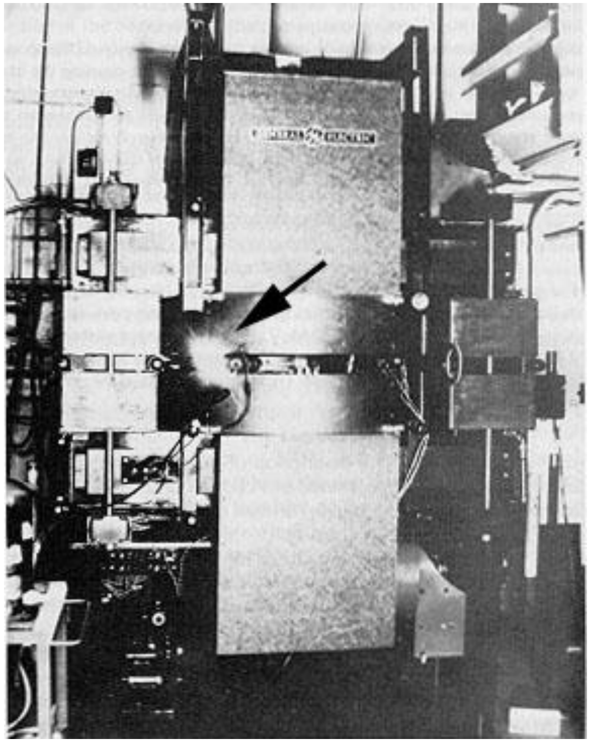
Atomic nucleus

Synchrotron Radiation

- Synchrotron radiation is electromagnetic radiation emitted when charged particles are radially accelerated (moved on a circular path).

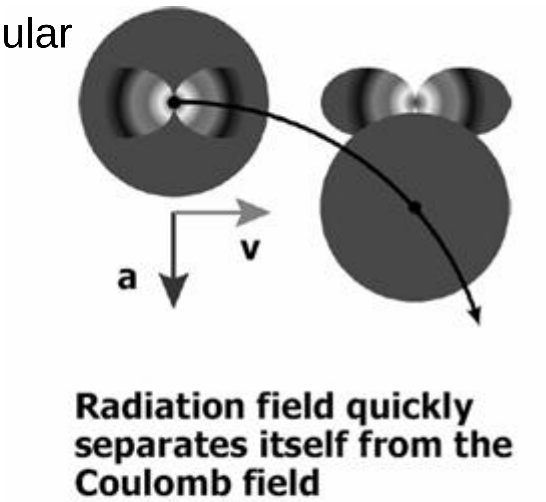
•Synchrotron radiation was first observed in an electron synchrotron in 1947: the 70 MeV synchrotron at General Electric Synchrotron in Schenectady, New York





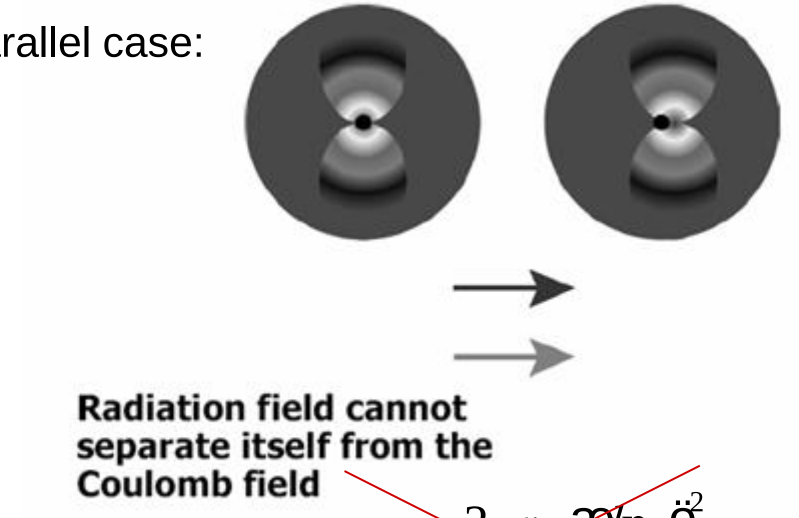
Longitudinal vs. Transverse Acceleration

Perpendicular case:



$$P_\perp = \frac{2}{3}\frac{r_c}{mc}\gamma^2\left(\frac{d\mathbf{p}_\perp}{dt}\right)^2$$

Parallel case:



$$\cancel{P_\parallel = \frac{2}{3}\frac{r_c}{mc}\left(\frac{d\mathbf{p}_\parallel}{dt}\right)^2}$$

negligible!

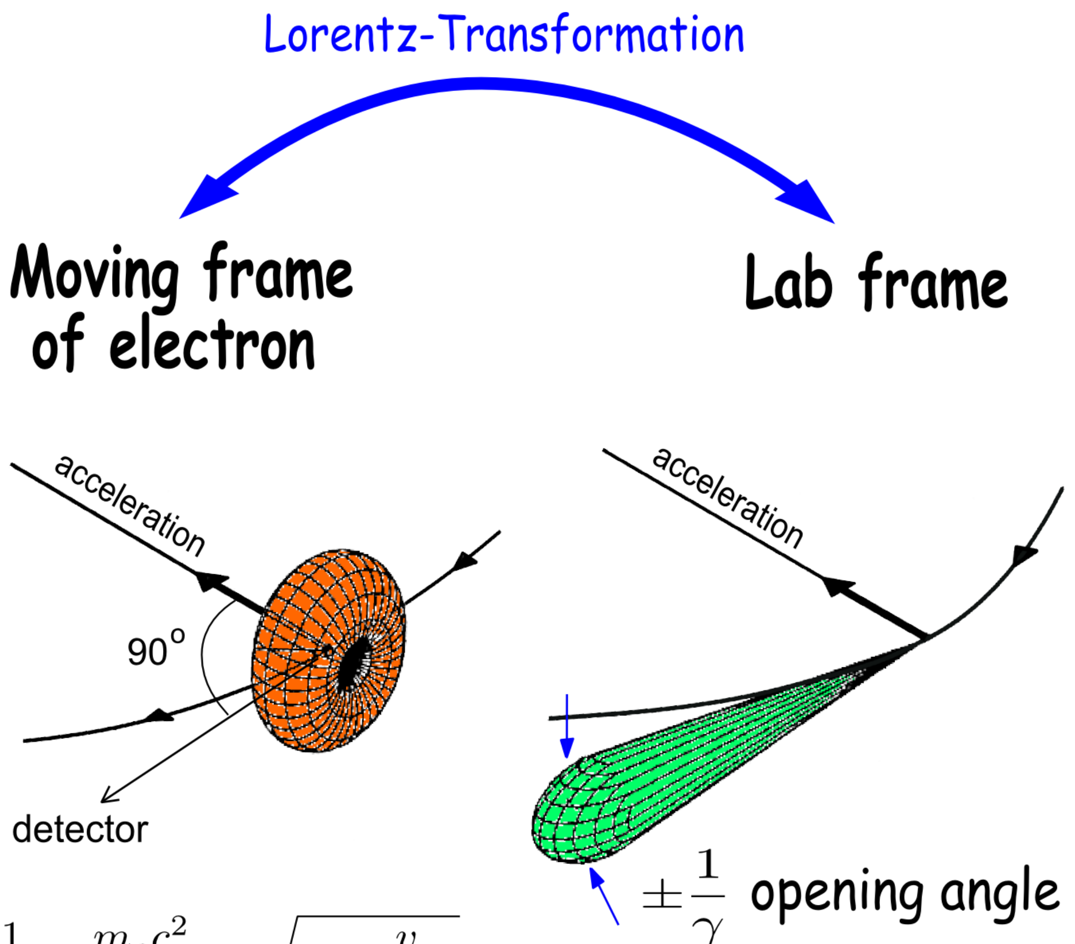
$$\boxed{P_\perp = \frac{2}{3} r_c mc^3 \frac{(\beta\gamma)^4}{\rho^2} \qquad \rho = curvature\ \ radius}$$

(Weidemann 21.34)

- **Radiated power for transverse acceleration increases dramatically with energy. This sets a practical limit for the maximum energy obtainable with a storage ring, but makes the construction of synchrotron light sources extremely appealing!**

Properties of Synchrotron Radiation: Angular Distribution

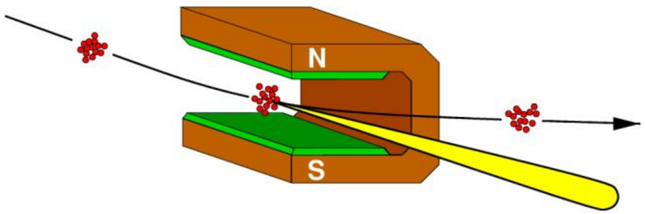
- **Radiation becomes more focused at higher energies.**



$$\frac{1}{\gamma} = \frac{m_o c^2}{E} = \sqrt{1 - (\frac{v}{c})^2}$$

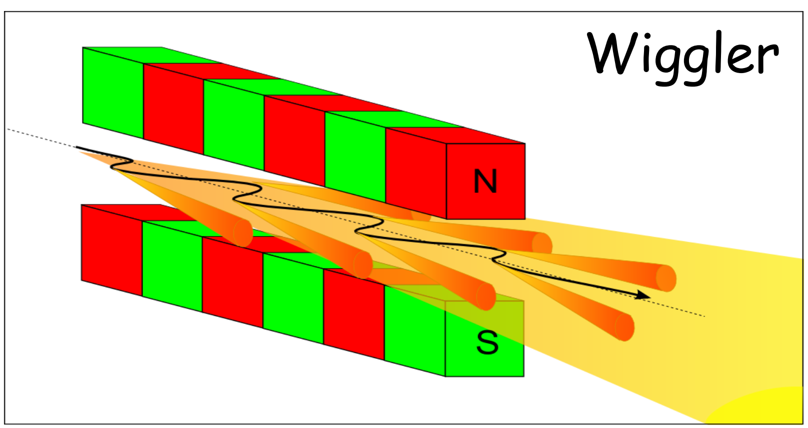
Accelerator Synchrotron Sources

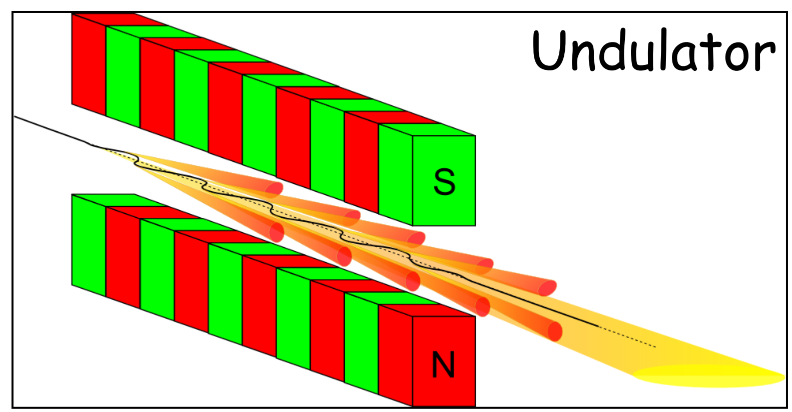
- Synchrotron radiation is generated in normal accelerator bending magnets



$$q = \pm \frac{1}{g}$$

- There are also special magnets called wigglers and undulators which are designed for this purpose.

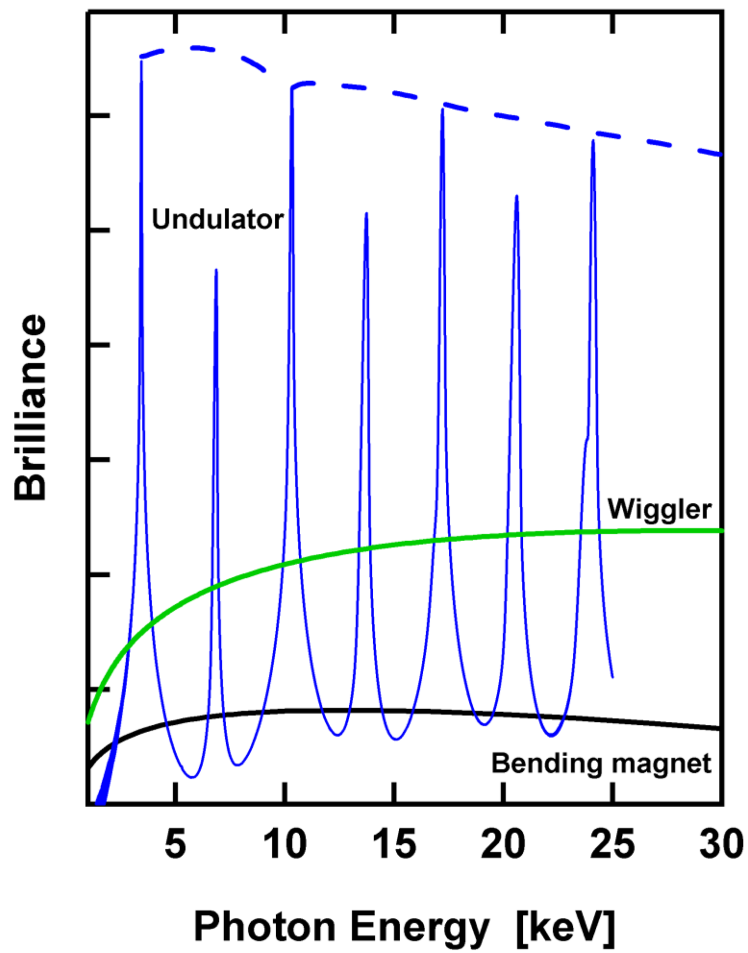




Insertion Devices

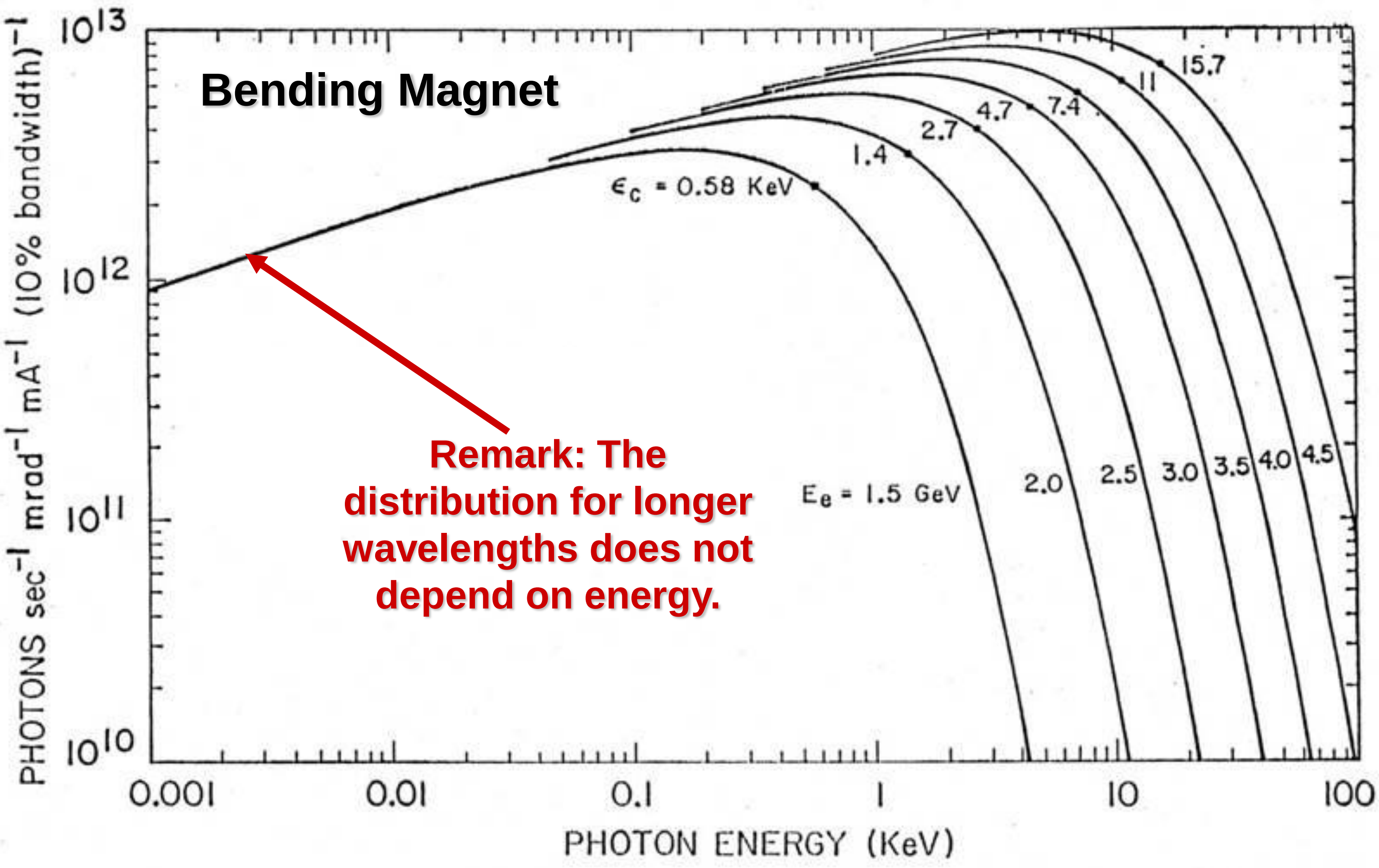
Undulator: Electron beam is periodically deflected by weak magnetic fields. Particle emits radiation at wavelength of the periodic motion, divided by γ^2. So period of cm for magnets results in radiation in VUV to X-ray regime.

Wiggler: Electron beam is periodically deflected by strong bending magnets. Motion is no longer pure sinusoid and radiation spectrum is continuous up to a critical cut off photon energy ($\varepsilon_{crit} \sim B\gamma^2$). Spectrum is infrared to hard X-rays.

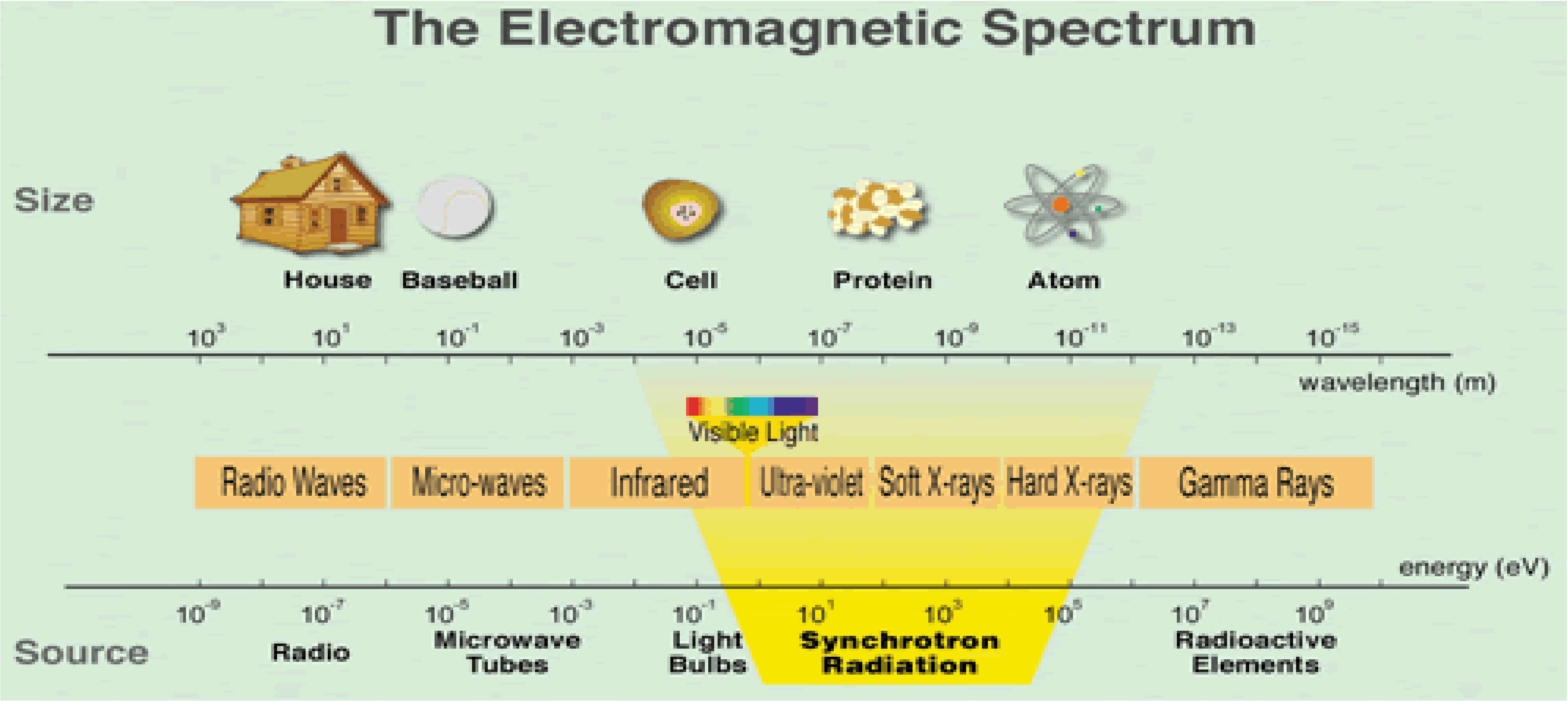


Synchrotron Radiation Spectrum

- For a wiggler or single bend magnet, the radiation spectrum depends on a single parameter, *the critical energy*



Properties of Synchrotron Radiation: Radiation Spectrum



SR Power and Energy Loss for Electrons

- Instantaneous Synchrotron Radiation Power for a single ***electron***

$$P_g[\mathrm{GeV/s}] = \frac{cC_g}{2p}\frac{E^4[\mathrm{GeV}^4]}{r^2[\mathrm{m}^2]}$$ (Weidemann 21.35)

$$C_\gamma = 8.8575\times 10^{-5}\,\frac{\mathrm{m}}{\mathrm{GeV}^3}$$

- Energy loss per turn for a single particle in an isomagnetic lattice with bending radius ρ is given by integrating P_γ over the lattice,

$$\Delta E[\mathrm{GeV}] = C_\gamma\,\frac{E^4[\mathrm{GeV}^4]}{\rho[\mathrm{m}]}$$ (Weidemann 21.41)

- The average Radiated Power for an entire beam is,

$$P_\gamma[\mathrm{MW}] = 8.8575\times 10^{-2}\,\frac{E^4[\mathrm{GeV}^4]}{\rho[\mathrm{m}]}\,I[\mathrm{A}]$$ (Weidemann 21.43)

- Radiated Power varies as the *inverse fourth power of particle mass.* Comparing radiated power from a proton vs. an electron, we have:

$$\frac{P_e}{P_p} = \left(\frac{m_p}{m_e}\right)^4 = 1836^4 = 1.1367\times 10^{13}$$ (Weidemann 21.38)

Examples

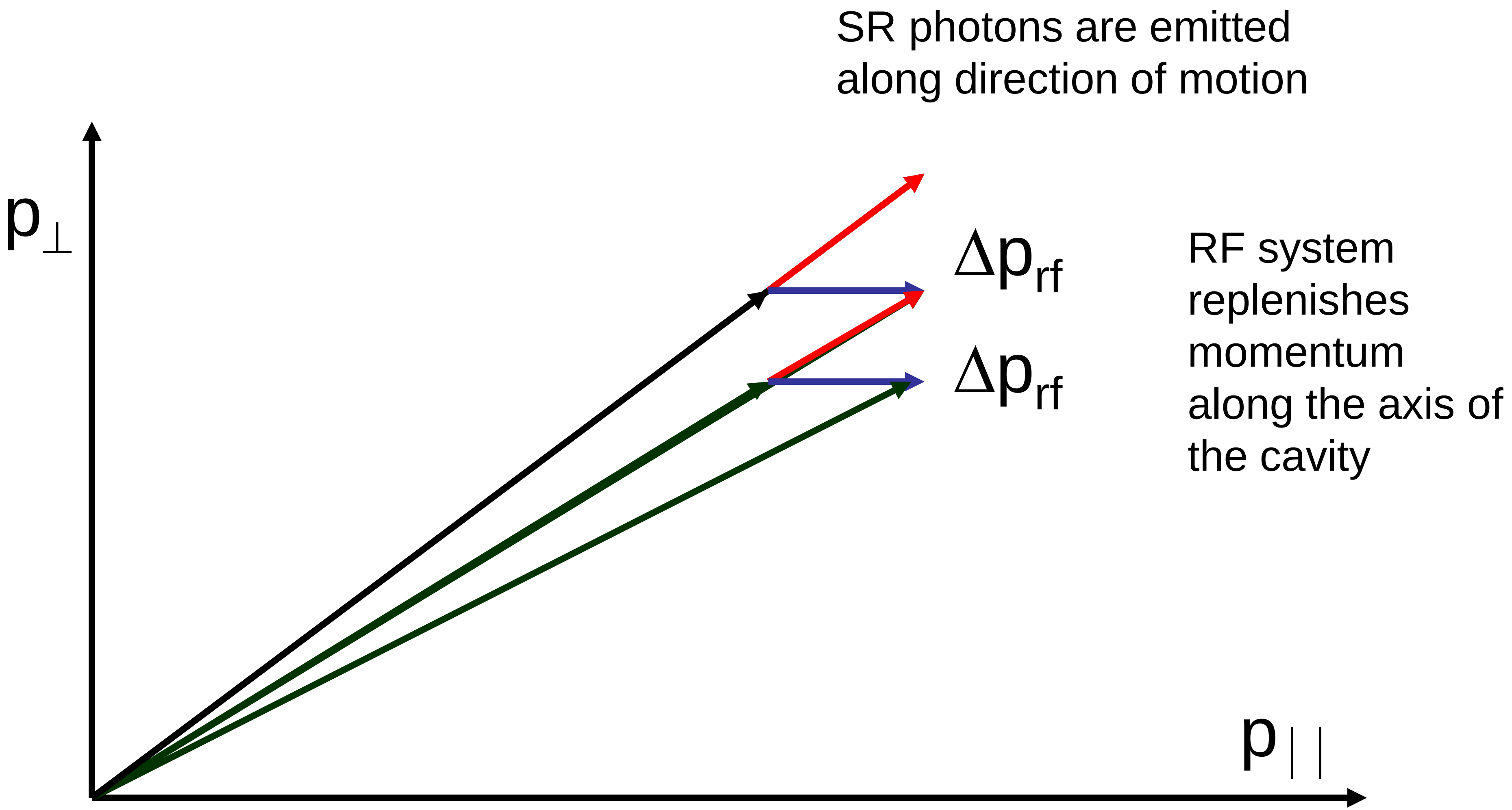
- Calculate SR radiated power for a 100 mA electron beam of 3 GeV in a storage ring with circumference 1 km (typical light source)
- Calculate SR radiated power for a 1 mA electron beam of 100 GeV in a storage ring with circumference 27 km (LEP storage ring)

Circular vs. Linear Electron Accelerators

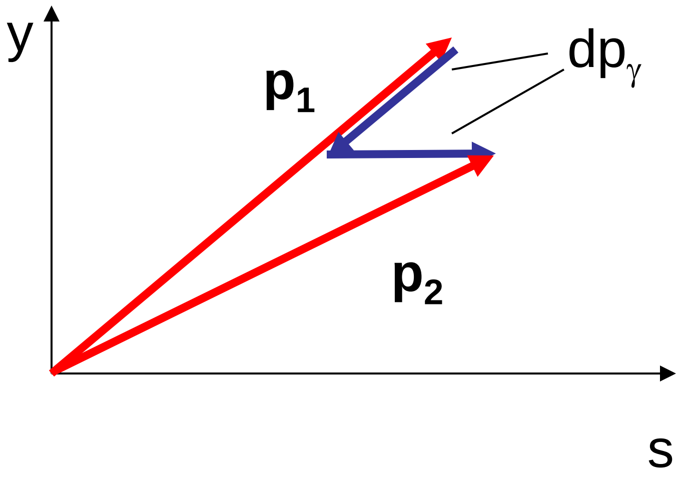
- At high enough electron energy, the radiated synchrotron power becomes impractical.
- Say you want to build the International Linear Collider as a circular collider, using the LEP tunnel
  - E=500 GeV, I=10 mA
- Gives P=13 GW!! This is ten times the power capacity of a commercial nuclear power plant
- Using two linacs avoids the necessity of bending these high energy beams, so synchrotron radiation is nearly eliminated

Consequences of Sychrotron Radiation: Radiation Damping

- Consider betatron motion in the vertical plane



Radiation Damping



$$\vec{p}_2 = \vec{p}_1 - d\vec{p}_\gamma + \left|dp_\gamma\right|\hat{s}$$

$$p_{2,\perp} = p_{1,\perp} - \left|dp_\gamma\right|\frac{p_{1,\perp}}{\left|\vec{p}_1\right|} = p_{1,\perp}\left(1 - \left|dp_\gamma\right| / \left|\vec{p}_1\right|\right)$$

$$p_{2,\parallel} = p_{1,\parallel} - \left|dp_\gamma\right|\frac{p_{1,\parallel}}{\left|\vec{p}_1\right|} + \left|dp_\gamma\right| = p_{1,\parallel}\left(1 - \left|dp_\gamma\right| / \left|\vec{p}_1\right| + \left|dp_\gamma\right| / p_{1,\parallel}\right)$$

$$y_2' = \frac{p_{2,\perp}}{p_{2,\parallel}} = \frac{p_{1,\perp}}{p_{1,\parallel}}\frac{\left(1 - dE_\gamma / E\right)}{\left(1 - dE_\gamma / E + dE_\gamma / cp_s\right)}$$

$$y_2' \approx y_1'\left(1 - dE_\gamma / E\right)$$ (Weidemann 8.13)

- The rate of change of slope with s is

$$y'' = \frac{dy'}{ds} = \frac{y_2' - y_1'}{ds} = \frac{y_1'\left(1 - dE_\gamma / E\right) - y_1'}{ds}$$

$$y'' = -y'\frac{1}{E}\frac{dE_\gamma}{ds}$$

Radiation Damping

- We see now another new term in the equation of motion, one proportional to the instantaneous slope of the trajectory y′:

$$y'' + y' \frac{1}{E} \frac{dE_\gamma}{ds} + ky = 0$$

- This looks like the damped harmonic oscillator equation from classical mechanics:

$$m\ddot{x} + b\dot{x} + kx = 0$$

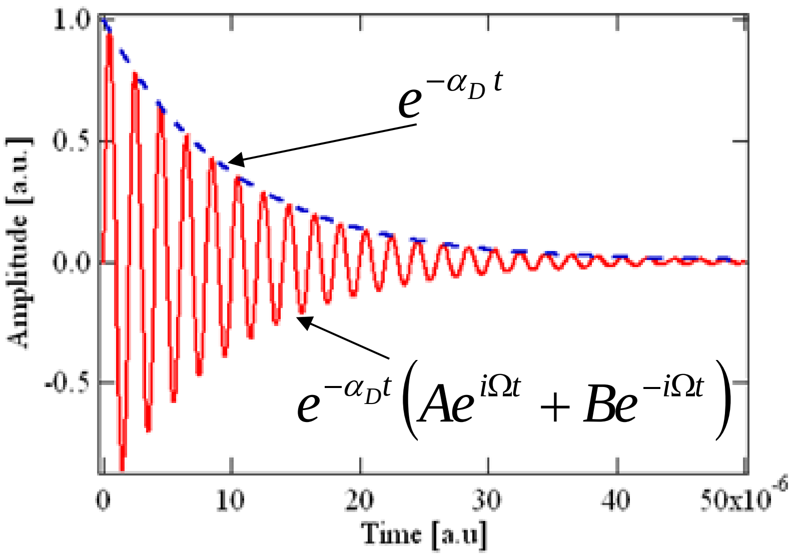
- Which is often written like this

$$\ddot{x} + 2\alpha\dot{x} + \omega_0^2 x = 0$$

- With

$$\alpha = \frac{b}{2m}$$

- The solution is a damped harmonic oscillator



$$x = Ae^{-\alpha t}\cos(\omega_1 t + \phi_0) \qquad \omega_1 = \sqrt{\omega_0^2 - \alpha^2}$$

Radiation Damping

- The resulting vertical betatron motion is *damped* in time.
- The damping term we derived is in units of m^{-1}. We need the damping rate in sec^{-1}. They are related by velocity: $\alpha[sec^{-1}] = c\beta\, \alpha[m^{-1}]$

$$a = \frac{cb}{2E}\frac{dE_g}{ds} = \frac{cb}{2E}\frac{dE_g}{cbdt} = \frac{1}{2E}\langle P_g \rangle, \text{ where } \langle P_g \rangle = \frac{dE}{dt}$$

$$a = \frac{1}{t_y} = \frac{1}{2t_0}$$

- Where we have defined $t_0 = \dfrac{E}{\langle P_g \rangle}$

- Motion in the horizontal and longitudinal planes are damped also, but their derivation is more complex.
- The damping rates are:

$$\boxed{\begin{aligned} \alpha_y &= \frac{1}{2\tau_0} = \frac{1}{2\tau_0} J_y \\ \alpha_x &= \frac{1}{2\tau_0}(1-\vartheta) = \frac{1}{2\tau_0} J_x \\ \alpha_z &= \frac{1}{2\tau_0}(2+\vartheta) = \frac{1}{2\tau_0} J_z \end{aligned}}$$

(Weidemann 8.27)

- And they are related by *Robinson's damping criterion* $\sum_i J_i = 4$

Radiation Damping

- The damping partition numbers depend on the lattice properties according to

$$\vartheta = \frac{\oint \frac{\eta}{\rho^3}\left(1+2\rho^2 k\right)ds}{\oint \frac{ds}{\rho^2}}$$

(Weidemann 8.25)

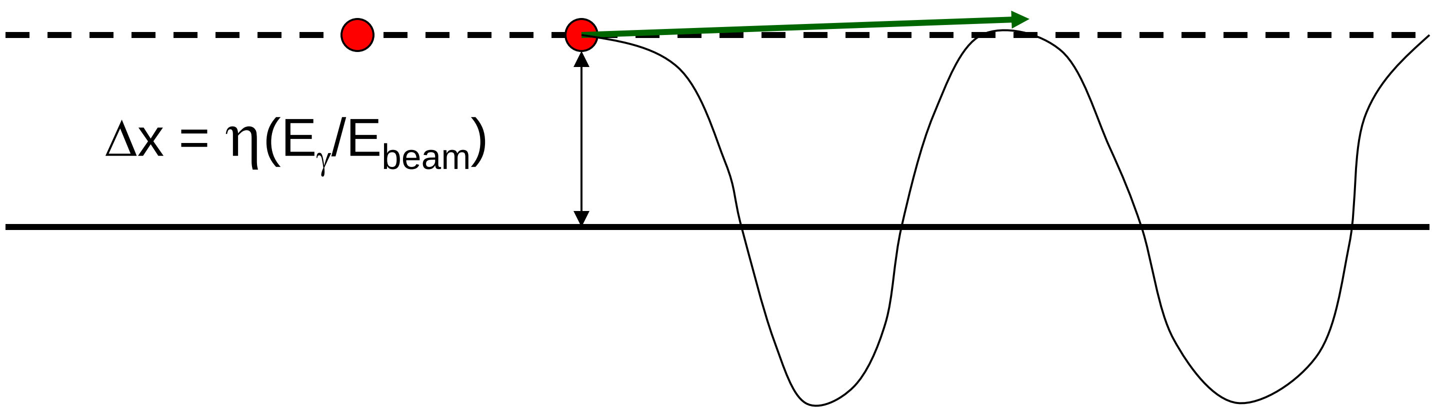
- You might imagine that oscillations in the beam would eventually be damped to zero, collapsing the beam to a single point in phase space. Is this possible?

Consequences of Synchrotron Radiation: Quantum Excitation

- Eventually, the individual beam particles become excited by the emission of synchrotron radiation, a process known as quantum excitation
- After emission of a SR photon, there is a change in reference path corresponding to the new particle energy.
- The particles position and angle in real-space do not change, but it acquires a betatron amplitude about the new reference orbit given by:

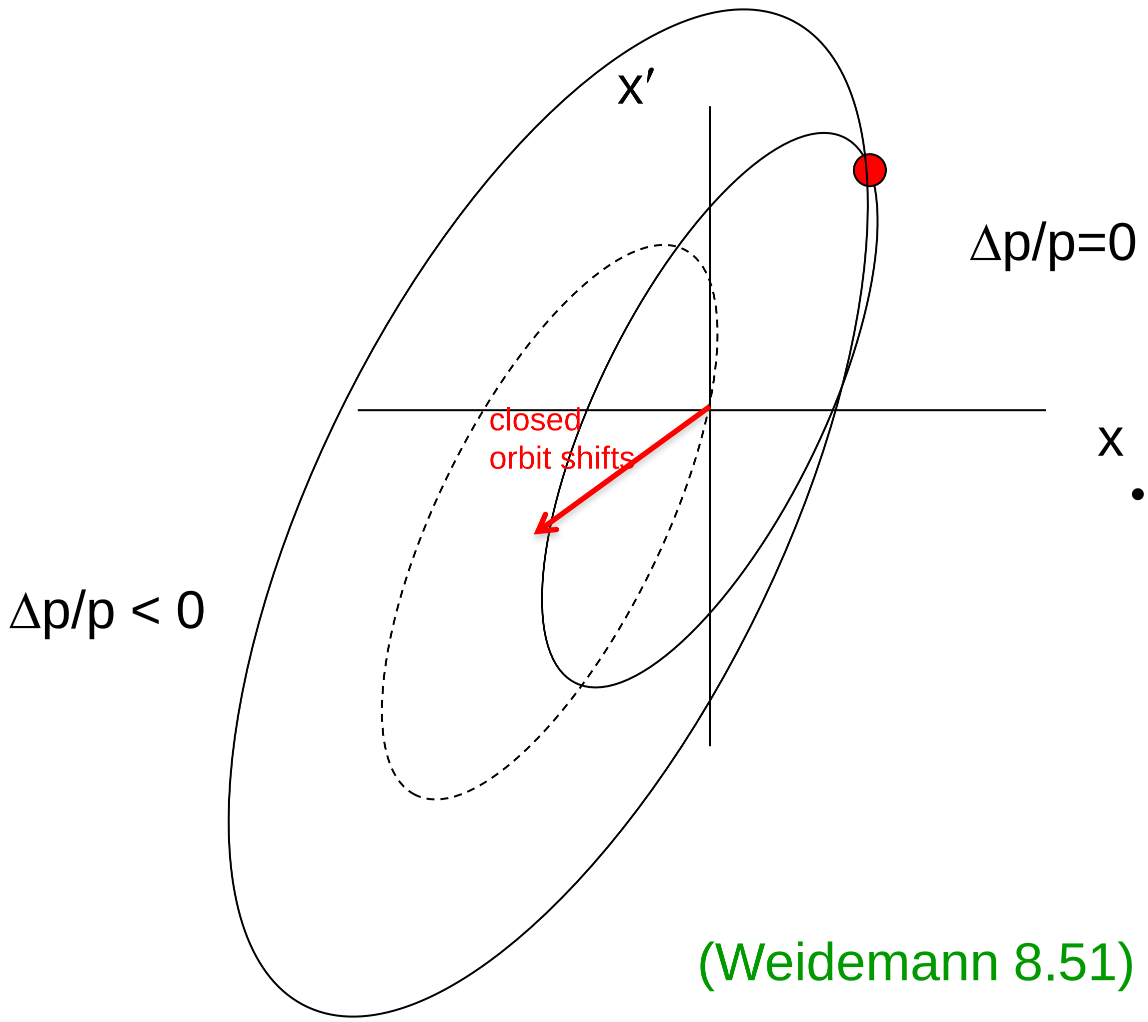
$$u_\beta = u_{\beta 0} + \eta \frac{E_\gamma}{E_0} \qquad u'_\beta = u'_{\beta 0} + \eta' \frac{E_\gamma}{E_0}$$

(Weidemann 8.50)



Quantum Excitation

- The particle oscillates at a larger betatron amplitude after emission of a SR photon



- The particles new amplitude and the average variation of phase space area is given by:

$$u = u_\beta + \eta \frac{\Delta E}{E}$$

$$a^2 = \gamma u^2 + 2\alpha u u' + \beta u'^2$$

(Weidemann 8.51) $$\langle \delta a^2 \rangle = \left(\frac{E_\gamma}{E} \right)^2 \left(\gamma \eta^2 + 2\alpha \eta \eta' + \beta \eta'^2 \right)$$

Equilibrium Beam Parameters

- The beamsize in an accelerator where synchrotron radiation is important eventually reaches emittance values in all three planes that are *an equilibrium between radiation damping and quantum excitation*
- The equilibrium beam energy spread in an electron storage ring depends only on the beam energy and bending radius

$$\frac{\sigma_\varepsilon^2}{E^2} = C_q \frac{\gamma^2}{J_z} \frac{\left\langle 1/\rho^3 \right\rangle}{\left\langle 1/\rho^2 \right\rangle}$$

$C_q = 3.84 \times 10^{-13}$ m

- The transverse beamsizes are given by

$$\varepsilon_u = \frac{\sigma_u^2}{\beta_u} = C_q \frac{\gamma^2}{J_u} \frac{\left\langle \mathcal{H}/\rho^3 \right\rangle}{\left\langle 1/\rho^2 \right\rangle}$$

(Weidemann 8.52, 8.58)

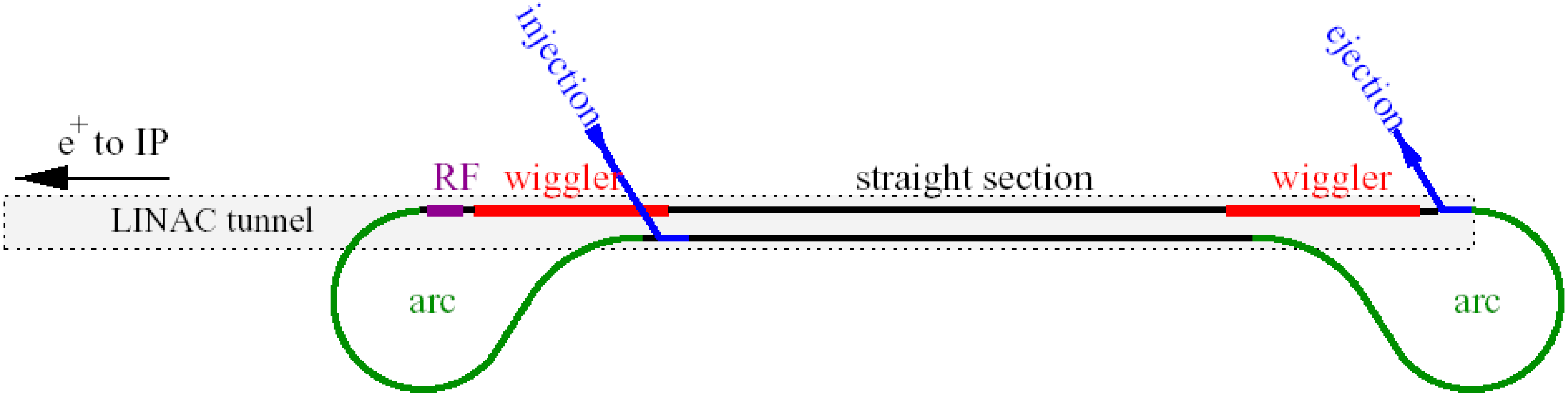
$$\mathcal{H}(s) = \beta\eta'^2 + 2\alpha\eta\eta' + \gamma\eta^2$$

- For the vertical plane, dispersion and therefore H are zero. Does the vertical emittance shrink to zero?
- No: the vertical beamsize is theoretically limited by $1/\gamma$ angular emission of synchrotron radiation. In practice it is limited by more mundane issues like orbit errors

Auxillary Slides

Damping Ring

- A Damping Ring has parameters tuned to minimize quantum excitation while providing damping, so that the equilibrium emittance can be reduced.
- This can be accomplished by producing more synchrotron radiation with strong bending fields (wiggler magnets) placed in dispersion-free straight sections



International Linear Collider Damping Ring

Colliders and Luminosity

- Two beams of opposite charge counter-rotating in a storage ring follow the same trajectories and have the same focusing
- The beams collide and produce particle reactions with a rate given by

$$R = \sigma_{physics} \mathcal{L}$$

- where

$$\mathcal{L} = f_{rev} \frac{N_1 N_2}{Area} = f_{rev} \frac{N_1 N_2}{4\pi \sigma_x \sigma_y}$$

- Beamsizes are reduced by special quadrupole configurations “low-beta” to reduce the beamsizes at the collision points