Lecture 7

Transverse Beam Optics, Part III

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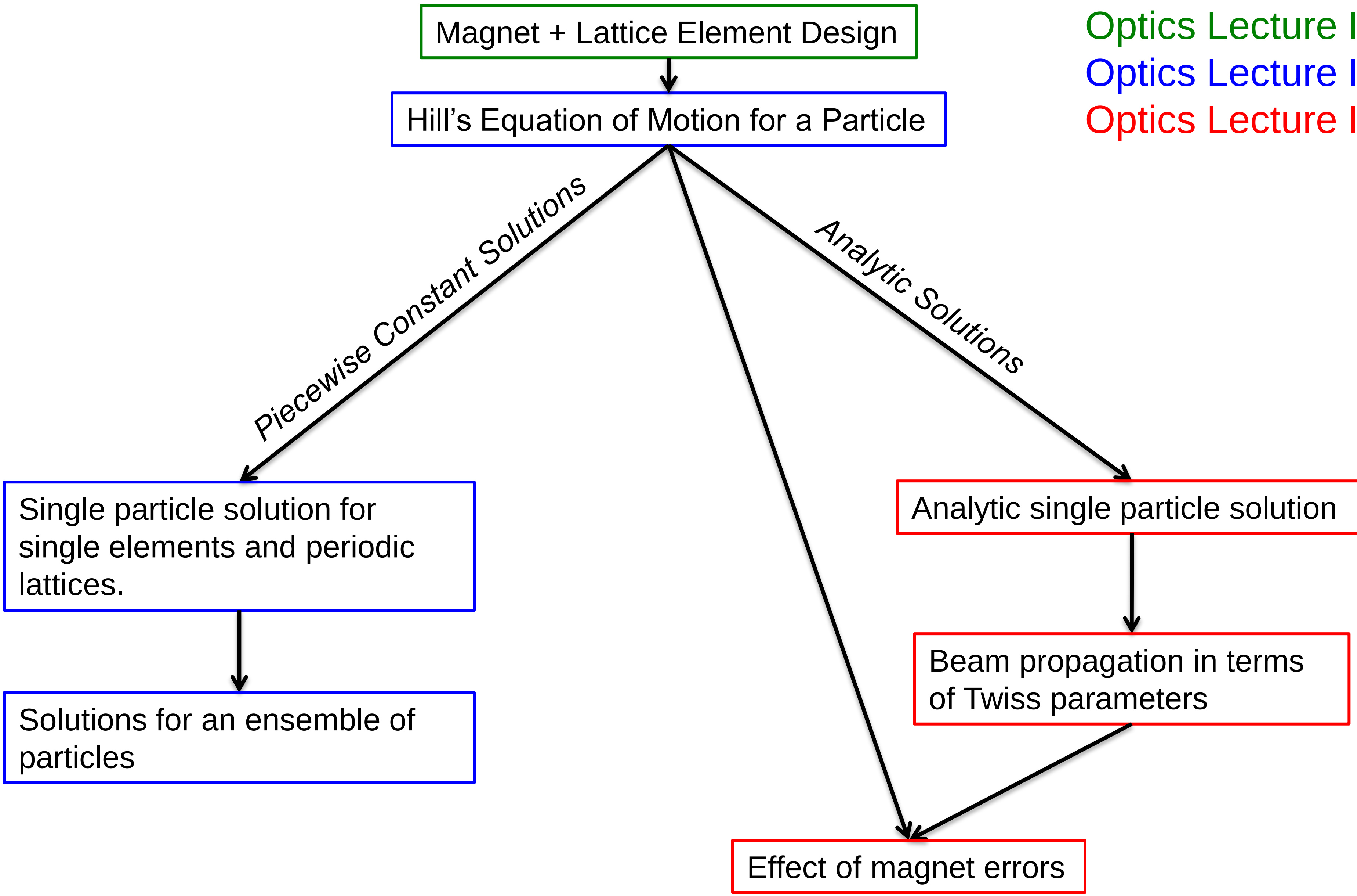
Map of Optics Lectures I - III



Optics Lecture I
Optics Lecture II
Optics Lecture III

Part 1: Fully Analytic Solution to Hill's EOM in terms of Twiss Parameters

Analytic Solution of Hill's Equation

So far we have solved the homogenous Hill's equation for piecewise constant magnetic strengths K. But it is useful to solve the full Hill's equation with the s-dependence:

$$u'' + K(s)u(s) = 0$$

The best thing is to make an intelligent guess:

$$u = \sqrt{\varepsilon\beta(s)}\cos(\varphi(s) - \phi)$$ (Wiedemann 5.55)

Position-dependent amplitude and phase (since k=k(s))

The Twiss Parameters and Particle Phase Advance

From the analytic solution, we find the identify the following important relations:

(Wiedemann 5.60 – 5.63)

(**Derivation**)

Phase advance is related to Beta function, β(s)

$$\varphi = \int_0^s \frac{1}{\beta(s)} ds$$

Beta function, β(s), is determined exclusively by magnetic lattice, K(s)

$$\frac{1}{2}\beta\beta'' - \frac{1}{4}\beta'^2 + \beta^2 K = 1$$

We can also define useful quantities related to the Beta function:

$$\alpha = -\frac{1}{2}\beta'$$

$$\gamma = \frac{1+\alpha^2}{\beta}$$

Units

- φ dimensionless {radians}
- ε length {meters $\times$ radians or millimeters $\times$ milliradians }
- β length {meters}
- α dimensionless {radians}
- γ length^{-1} {meters^{-1}}

Equation of an Ellipse

Now, taking our solution and its derivative, and eliminating the phase variable, we find the following important relation:

(**Derivation**)

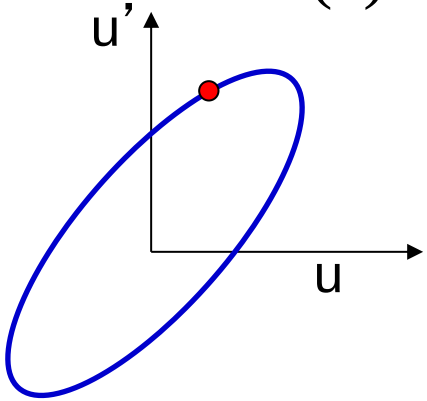
$$\boxed{\varepsilon = \gamma u^2 + 2\alpha u u' + \beta u'^2}$$

(Wiedemann 5.64)

This is the equation for an ellipse with area $\pi\varepsilon$! Thus our original choice of an ellipse to represent a beam in phase was not arbitrary.

$$u(s) = \sqrt{\varepsilon\beta(s)}\cos(\varphi(s) - \phi)$$

The solution to Hills equation represents a particle tracing out an ellipse in phase space.



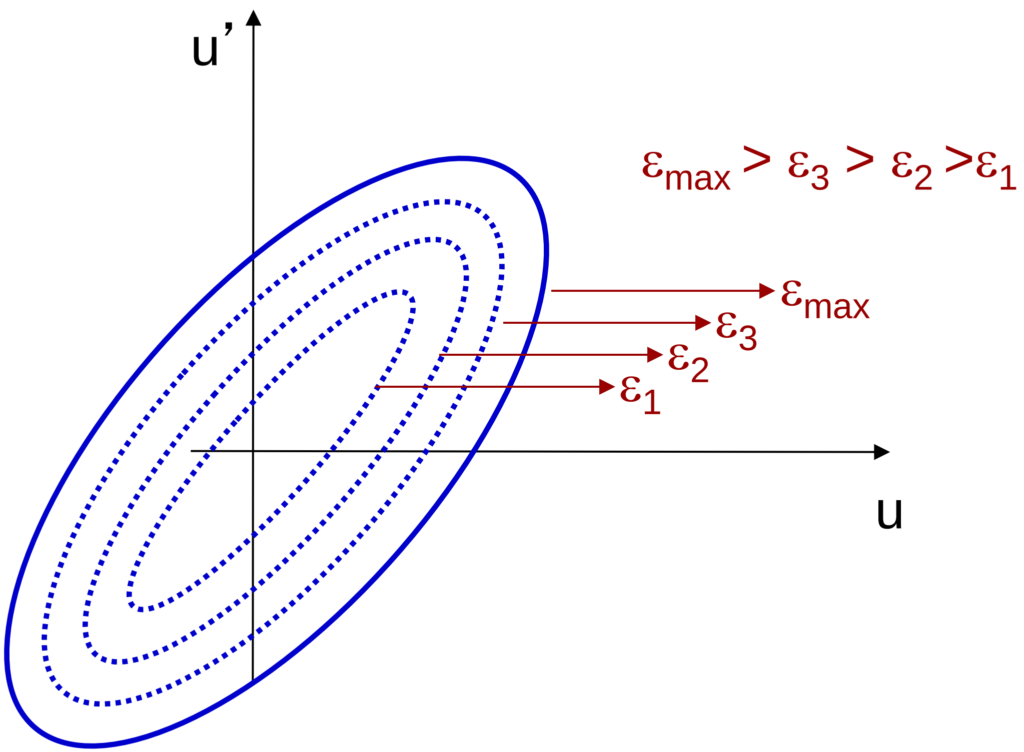
"Betatron Motion"

Representation of the Beam

Even though our solution to Hill's equation is for a single particle, if we take the outermost particle, i.e, highest emittance (ε_{max}) particle, then all other particles trace out smaller ellipses of the same shape inside the outermost ellipse. Therefore the outermost ellipse can be used to represent the whole beam:

The Twiss functions ($\beta(s)$, $\alpha(s)$, $\gamma(s)$), govern the shape of the ellipse at any location, s.

The emittance, ε, together with $\beta(s)$, determines the beam size.

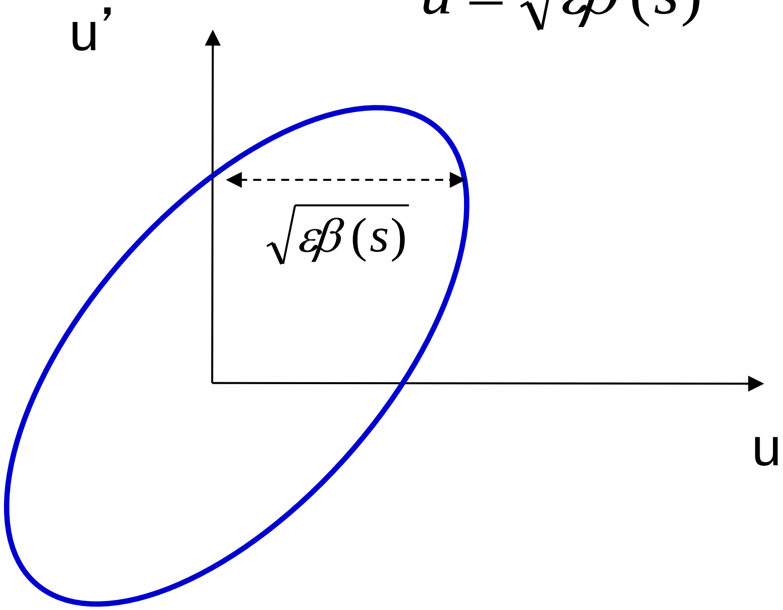


The Beam Envelope

In practice, we often care most about the outermost particle on the outermost ellipse – this one is the most likely to be lost.

In other words, we care about the "Beam Envelope", given by the amplitude term in our solution to Hill's equation:

$$u = \sqrt{\varepsilon\beta(s)}$$



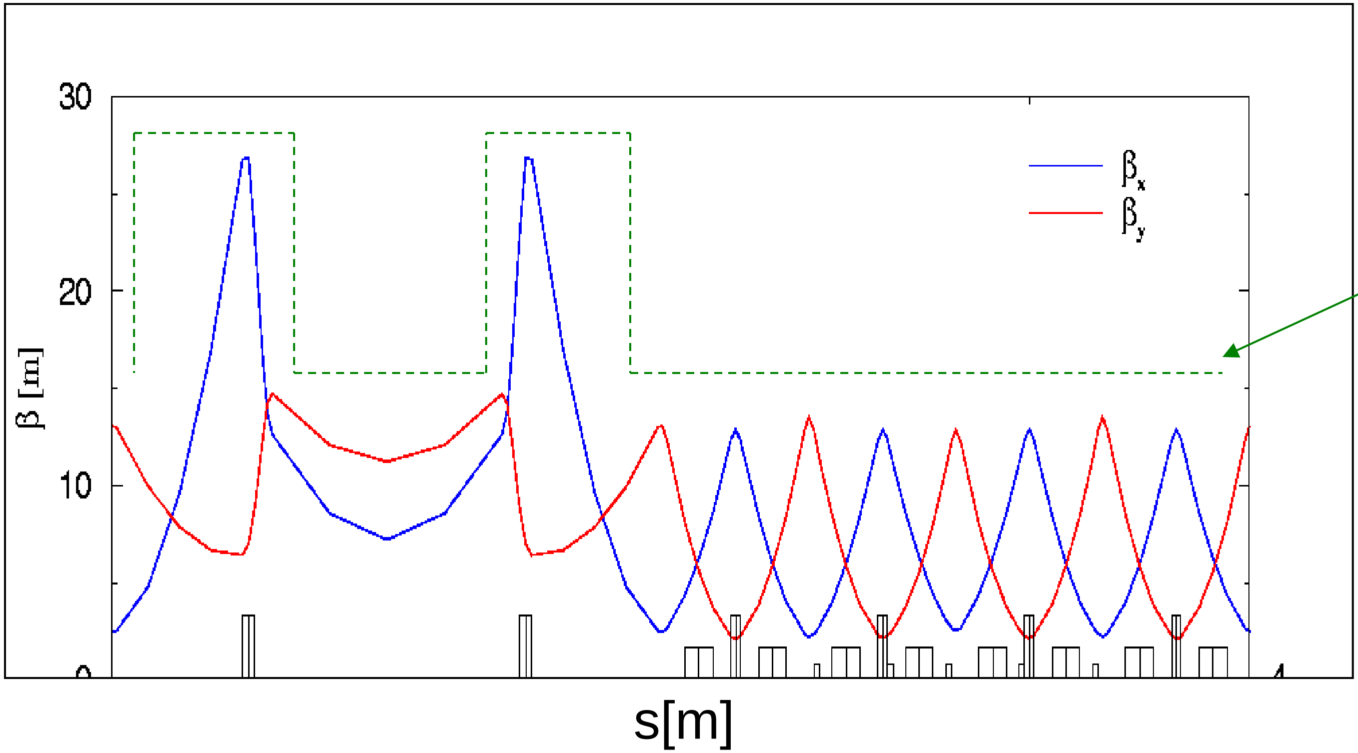
- ✓ ε is a conserved in linear transport systems.
- ✓ $\beta(s)$ is the "beta function", and is also often called the "envelope function"
- ✓ Together, ε and $\beta(s)$ determine the beam size.

$$\frac{1}{2}\beta\beta'' - \frac{1}{4}\beta'^2 + \beta^2 K = 1$$

Example Betatron Function

Once we know ε and $\beta(s)$, we can determine the beam size and requisite aperture of the machine.

Horizontal and Vertical Beta Functions for the SNS Ring

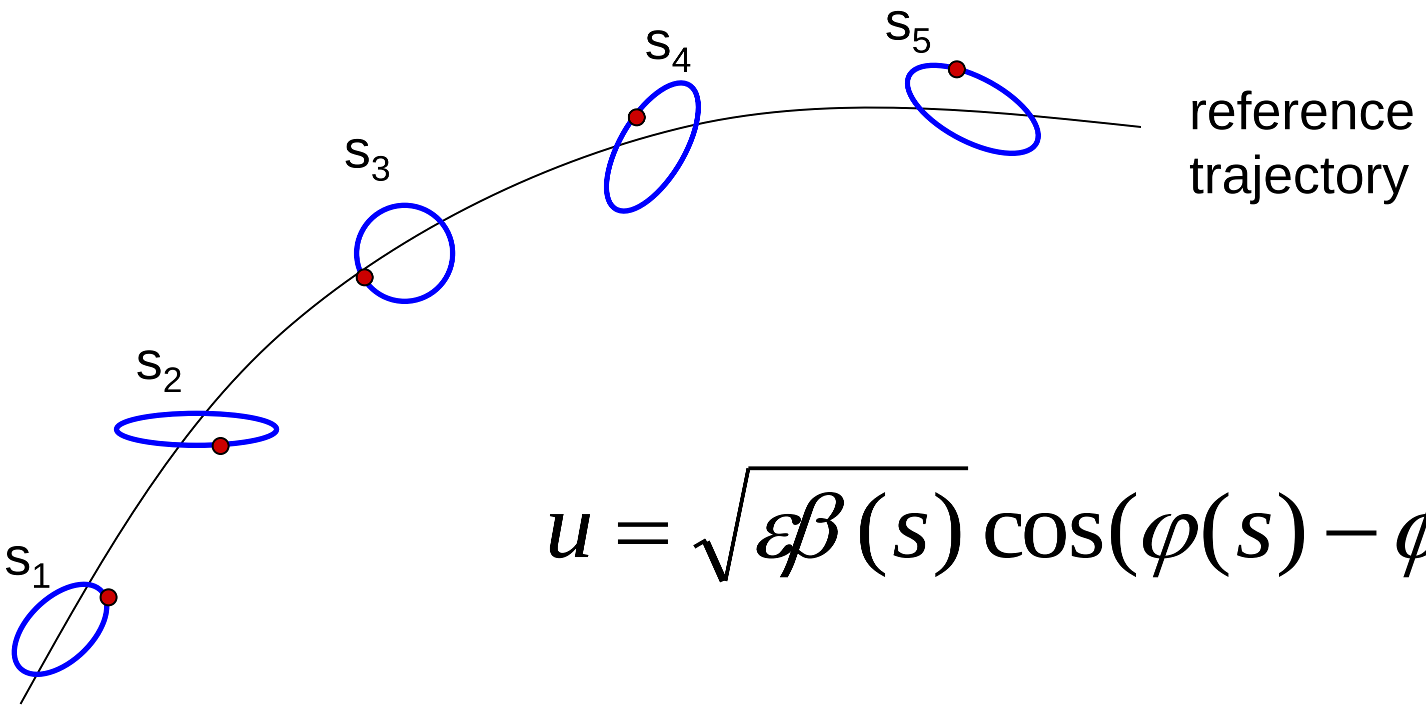


Beam pipe aperture scales with beta function.

$$\boxed{u = \sqrt{\varepsilon\beta(s)}}$$

The Whole Picture

The whole story is more complicated. As the ellipse transforms along the beam line, the particle executes oscillation on the ellipse.



$$u = \sqrt{\varepsilon\beta(s)}\cos(\varphi(s) - \phi)$$

The motion of the particle in real space, u, is “quasi-harmonic”.

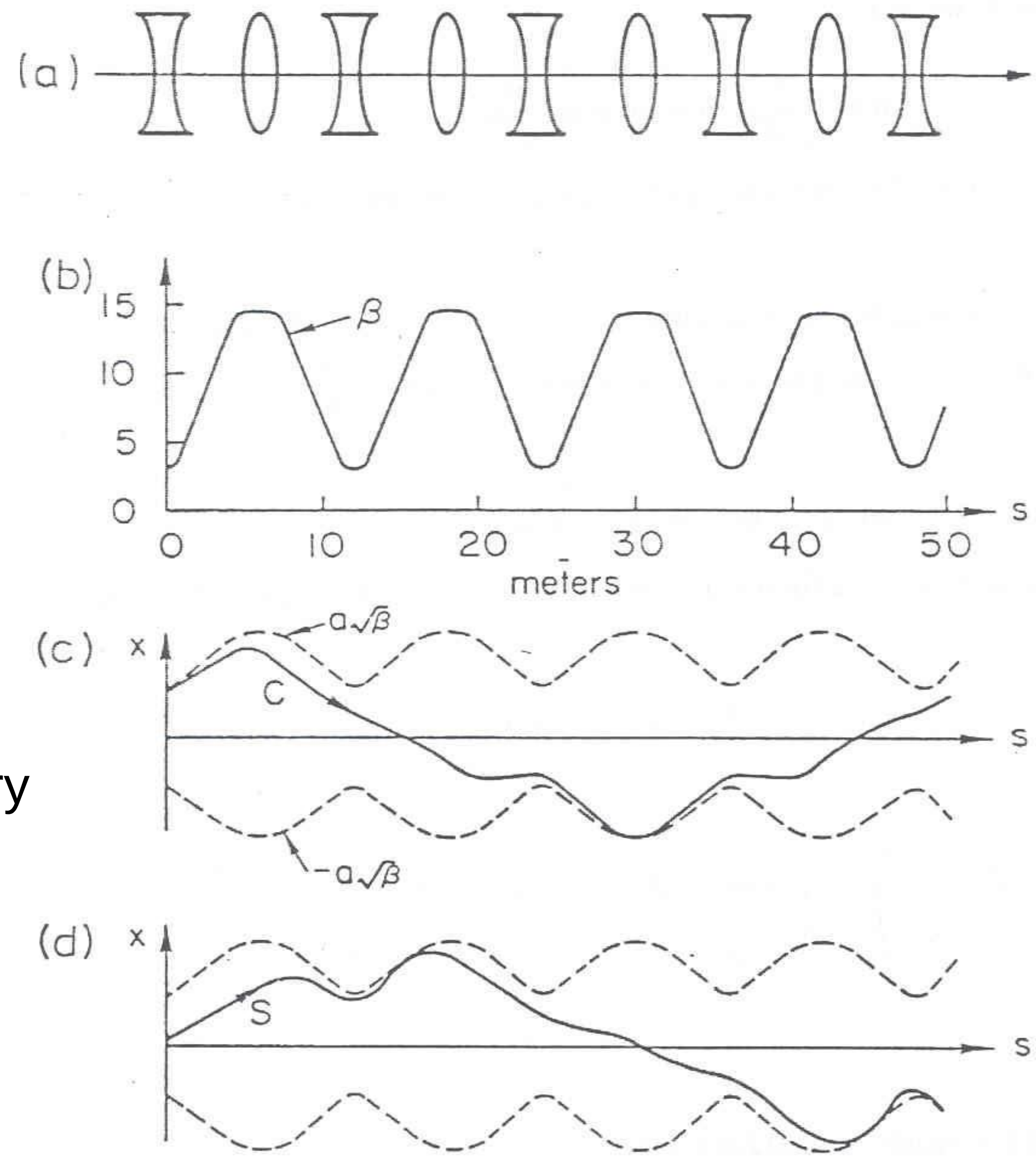
The Whole Picture (Continued)

FODO Lattice arrangement

Beta function

Individual particle cosine-like trajectory

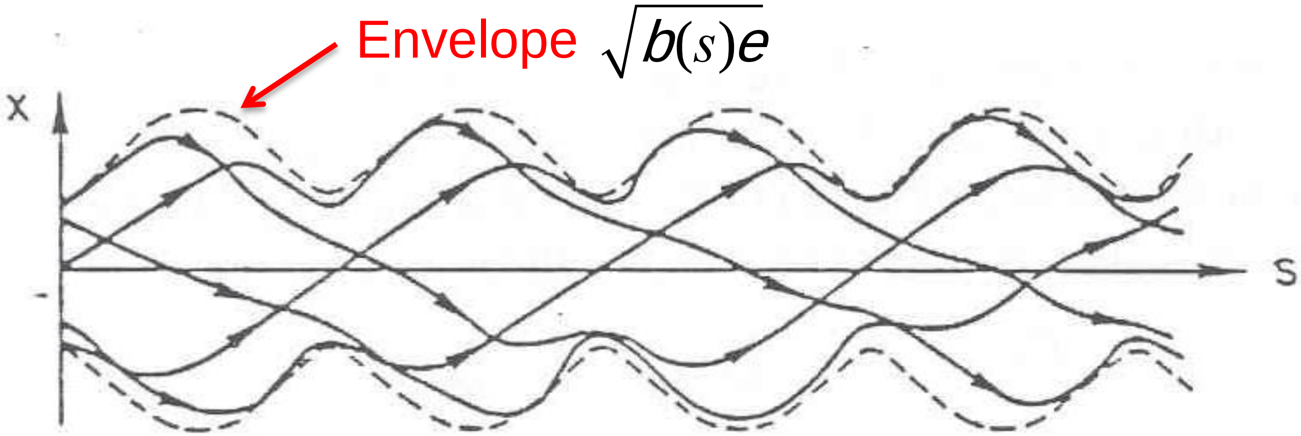
Individual particle sine-like trajectory



The Beam Envelope

In reality we do not know the location of individual particles in the lattice. But we can measure the beam envelope, which is composed of different particles at any moment.

From Hill's equation, it is the amplitude term: $\sqrt{b(s)e}$



Transfer Matrix in Terms of Twiss Parameters

It's often the case that we have knowledge of $\beta(s)$ instead of K(s). In this case, it is possible for us to write a general transfer matrix for (u, u') in terms of the Twiss parameters, and their initial conditions.

Initial condition (s=0): $\alpha(0)=\alpha_o;\ \beta(0)=\beta_o;\ \varphi(0)=0$

Then the transfer matrix from 0 to location s is:

(**Derivation**) Note: φ is the phase advance from 0 to s.

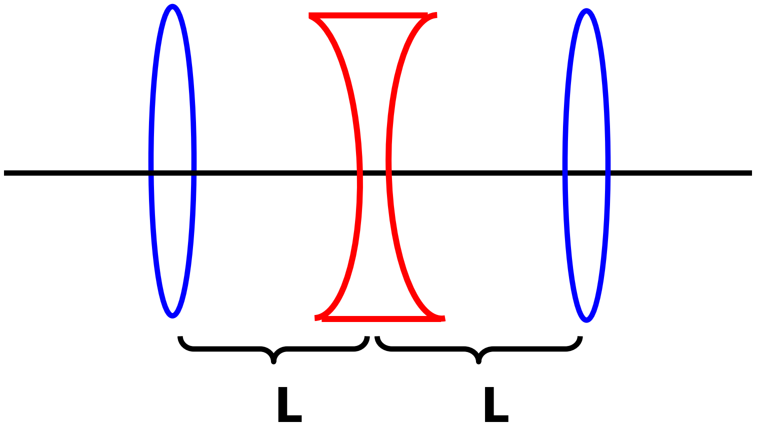
$$\boxed{M=\begin{pmatrix} \sqrt{\frac{\beta}{\beta_o}}(\cos\varphi+\alpha_o\sin\varphi) & \sqrt{\beta\beta_o}\sin\varphi \\ \frac{\alpha_o-\alpha}{\sqrt{\beta\beta_o}}\cos\varphi-\frac{1+\alpha_o\alpha}{\sqrt{\beta\beta_o}}\sin\varphi & \sqrt{\frac{\beta_o}{\beta}}(\cos\varphi-\alpha\sin\varphi) \end{pmatrix}}$$

(Wiedemann 5.76)

This matrix gives the same result for (u,u') as the general M matrix in terms of K(s), found earlier. We choose which one to use based on whether we know K(s), or β(s).

Symmetric Periodic FODO Lattice

The symmetric and periodic FODO lattice is a case where we can analytically determine the beta functions. We enforce periodicity by requiring that the input Twiss parameters be equal to the output Twiss parameters.



$$M_{\text{FODO}} = M_{\text{Half QF}} M_{\text{Drift}} M_{\text{QD}} M_{\text{Drift}} M_{\text{Half QF}}$$

<u>Starting with the assumption of periodicity we can derive:</u>

- The shape of the beta function.
- The value of the beta function through the FODO cell.
- The optimum phase advance of the FODO cell in order to minimize the peak beta function.

Symmetric, Periodic FODO Cell

- Consider the FODO thin lens transfer matrix we found in the peicewise constant solutions. It should give the same solution for (u, u') as the Twiss matrix.

Thin lens FODO:

$$M_{\mathrm{FODO}} = \begin{pmatrix} 1-\frac{L^2}{2f^2} & 2L(1+\frac{L}{2f}) \\ \frac{-L}{2f^2}(1-\frac{L}{2f}) & 1-\frac{L^2}{2f^2} \end{pmatrix}$$

Requirements:

1. α at beginning and end of FODO must the same and equal to zero (symmetric diagonal terms), e.g., $\alpha=\alpha_0=0$
2. Since $\alpha=\alpha_0=0$, and $\alpha=-\beta'/2$, then $\beta=\beta_0$
3. Since α is proportional to the derivative of β, then β must be an extremum at ends $\beta=\beta_0=\beta_{MAX}$
4. There must be another extremum in β at the defocusing quad.
5. In between the quads, β evolves according to the drift equation.

Twiss Transformation Matrix for FODO cell

With these facts, our transformation matrix reduces to:

$$
M=\begin{pmatrix} \cancelto{1}{\sqrt{\frac{\beta}{\beta_o}}}(\cos\varphi+\cancelto{0}{\alpha_o}\sin\varphi) & \sqrt{\beta\beta_o}\,\sin\varphi \\ \cancelto{0}{\frac{\alpha_o-\alpha}{\sqrt{\beta\beta_o}}}\cos\varphi-\frac{1+\cancelto{0}{\alpha_o}\alpha}{\sqrt{\beta\beta_o}}\sin\varphi & \cancelto{1}{\sqrt{\frac{\beta_o}{\beta}}}(\cos\varphi-\cancelto{0}{\alpha}\sin\varphi) \end{pmatrix}
$$

FODO

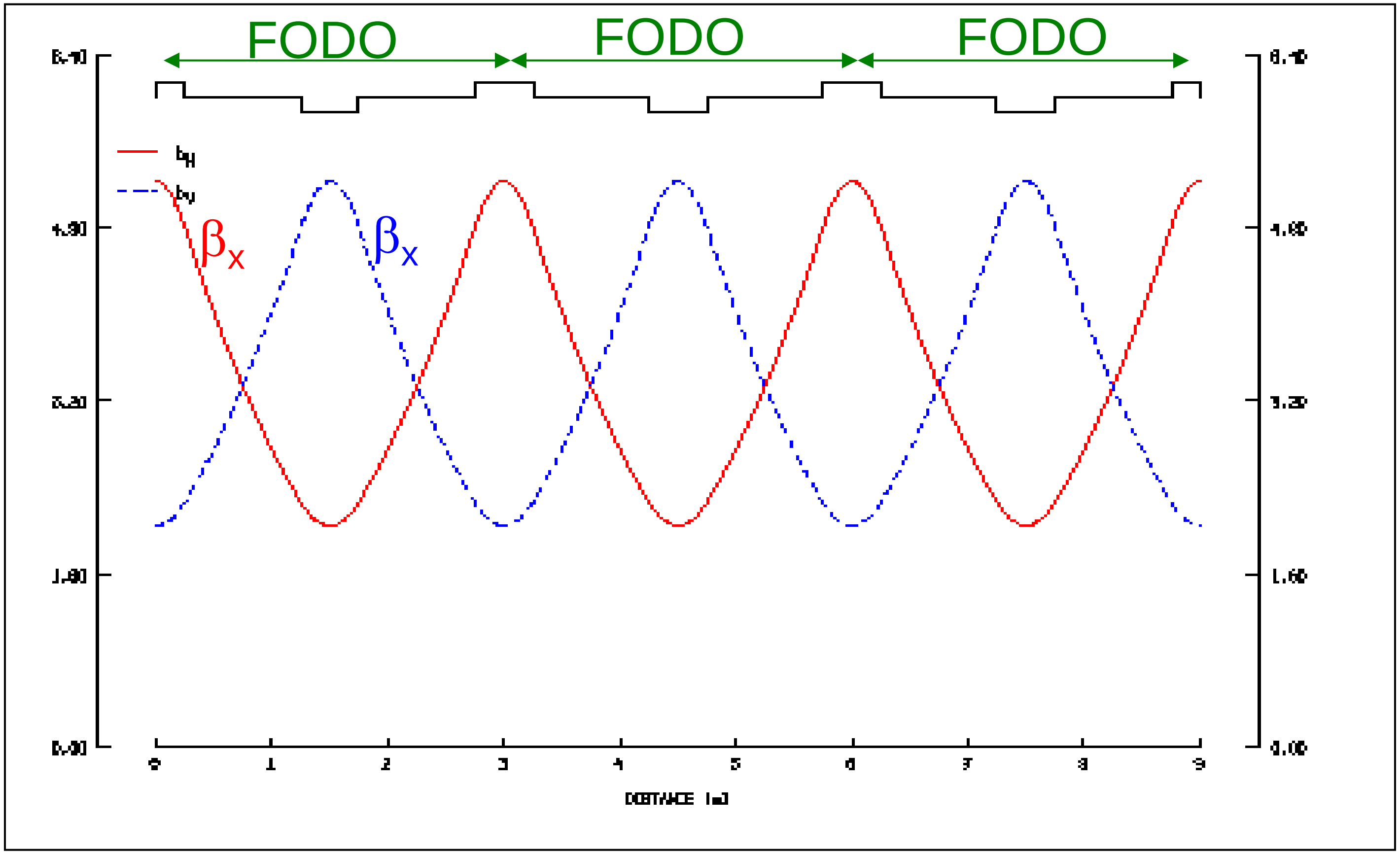
$$
M_{FODO}=\begin{pmatrix} \cos\varphi & \beta\sin\varphi \\ -\frac{1}{\beta}\sin\varphi & \cos\varphi \end{pmatrix}
$$

This is the complete transformation matrix through a periodic FODO cell!

(Wiedemann 7.6)

Beta Function in a FODO Cell

The Betatron function in a FODO cell is symmetric in the two planes, and reaches the maximum and minimum values in the center of the quads:



What does the beam envelope in the (x,y) plane look like?

Periodic Betatron Function in a FODO Cell

- To use our periodic, symmetric FODO, we need to calculate the initial β.
- This is one of the few analytically solvable cases for β.
- Assume that the thin lens approximation is valid, and that we know quad strength K and length L.
- Start with:

$$\beta = \beta_0 C^2 - 2SC\alpha_0 + S^2\gamma_0$$

(Wiedemann 5.22, Lecture 6)

Result: (**Derivation**)

$$\beta^{+} = \frac{L\kappa(\kappa+1)}{\sqrt{\kappa^2-1}}$$

$$\beta^{-} = \frac{L\kappa(\kappa-1)}{\sqrt{\kappa^2-1}}$$

where $\kappa = \frac{2f}{L}$

In center of QF, we have: $\beta_x = \beta^+ \text{ and } \beta_y = \beta^-$

In center of QD, we have: $\beta_x = \beta^- \text{ and } \beta_y = \beta^+$

Finding the Minimum Peak Beta Function

Relating diagonal elements in the Twiss and thin lens matrix, we have:

$$\cos\varphi = 1 - 2\frac{L^2}{f^2} = \frac{\kappa^2 - 2}{\kappa^2}$$

Or, using half-angle formula:

$$\sin\frac{\varphi}{2} = \frac{1}{\kappa}$$

So real solutions only exist when $\sin\frac{\varphi}{2} \leq 1$

$$\Rightarrow \frac{1}{\kappa} = \frac{L}{f} \leq 1 \quad \Rightarrow \quad L \leq f$$

Finding the Minimum Peak Beta Function

In order to keep our beam pipe aperture as small as possible, we would like the maximum beta function (β^+) to be a small as possible. So we want to vary κ to find the "minima of the maxima":

Find roots for:

$$\frac{d\beta^+}{d\kappa} = \frac{d}{d\kappa}\left(\frac{L\kappa(\kappa+1)}{\sqrt{\kappa^2-1}}\right)$$

Result is:

$$\kappa^2 - \kappa - 1 = 0, \quad \kappa_{\min} = \frac{1+\sqrt{5}}{2} = 1.6180$$

$$\frac{\beta_{\max}}{2L} = 1.665$$

$$\frac{\beta_{\min}}{2L} = 0.393$$

Relating diagonal elements in the Twiss and thin lens matrix, we have:

$$\cos\varphi = 1 - 2\frac{L^2}{f^2} = \frac{\kappa^2 - 2}{\kappa^2}$$

$$\Rightarrow \varphi = 76.3°$$

Periodic Focusing Channels

In an accelerator, we often deal with periodic, closed, lattices. These include rings and synchrotrons.

$$u''+K(s)u(s)=0$$

$$K(s)=K(s+L_p),\ \ \text{where } \mathrm{L_p} \text{ is the length of the period}$$

For a periodic, closed, system, we must have periodic solutions:

$$\beta(s)=\beta(s+L_p)$$

$$\alpha(s)=\alpha(s+L_p)$$

Does the particle return to the same phase position on every turn, i.e., u(s)=u(s+Lp)?

General Periodic Transformation Matrix

For a general periodic lattice, we only require $\alpha=\alpha_0$, and $\beta=\beta_0$.

$$M = \begin{pmatrix} \sqrt{\frac{\beta}{\beta_o}}(\cos\varphi + \alpha_o \sin\varphi) & \sqrt{\beta\beta_o}\sin\varphi \\ \frac{\alpha_o - \alpha}{\sqrt{\beta\beta_o}}\cos\varphi - \frac{1+\alpha_o\alpha}{\sqrt{\beta\beta_o}}\sin\varphi & \sqrt{\frac{\beta_o}{\beta}}(\cos\varphi - \alpha\sin\varphi) \end{pmatrix}$$

$$\Big\downarrow \; \alpha=\alpha_0,\ \beta=\beta_0$$

$$M_{\text{Periodic}} = \begin{pmatrix} \cos\varphi + \alpha\sin\varphi & \beta\sin\varphi \\ -\gamma\sin\varphi & \cos\varphi - \alpha\sin\varphi \end{pmatrix}$$ (Wiedemann 7.25)

The FODO matrix was a special case of this where the diagonal elements had to be equal such that $\alpha=\alpha_0=0$.

Review of Transport Matrices

In the last two lectures, we have come up with a number of transport, matrices, M. Let's review:

Matrix tracking (u,u') through a piecewise constant lattice (K is known):

$$M_n = \begin{pmatrix} \cos(\sqrt{K}l) & \frac{1}{\sqrt{K}}\sin(\sqrt{K}l) \\ -\sqrt{K}\sin(\sqrt{K}l) & \cos(\sqrt{K}l) \end{pmatrix} = \begin{pmatrix} C(s) & S(s) \\ C'(s) & S'(s) \end{pmatrix}$$

$$M_{total} = M_n M_{n-1} \ldots M_1$$

Specific application to a thin lens FODO lattice, with focal lengths f1=-f2=f.

$$M_{\text{FODO}} = \begin{pmatrix} 1 - \frac{L^2}{2f^2} & 2L(1 + \frac{L}{2f}) \\ \frac{-L}{2f^2}(1 - \frac{L}{2f}) & 1 - \frac{L^2}{2f^2} \end{pmatrix}$$

Review of Transport Matrices

For piecewise constant lattices, we can also transform the Twiss parameters themselves through the lattice, assuming K(s) is known:

$$\begin{pmatrix} \beta \\ \alpha \\ \gamma \end{pmatrix} = \begin{pmatrix} C^2 & -2SC & S^2 \\ -CC' & (S'C+SC') & -SS' \\ C'^2 & -2S'C' & S'^2 \end{pmatrix} \begin{pmatrix} \beta_o \\ \alpha_o \\ \gamma_o \end{pmatrix}$$

where the C and S refer to the cosine-like and sine-like solutions found for the piece-wise constant lattice.

Review of Transport Matrices

Matrix for tracking (u,u') through a lattice when the lattice Twiss are already known:

$$M=\begin{pmatrix}\sqrt{\frac{\beta}{\beta_o}}(\cos\varphi+\alpha_o\sin\varphi) & \sqrt{\beta\beta_o}\sin\varphi \\ \frac{\alpha-\alpha_o}{\sqrt{\beta\beta_o}}\cos\varphi-\frac{1+\alpha_o\alpha}{\sqrt{\beta\beta_o}}\sin\varphi & \sqrt{\frac{\beta_o}{\beta}}(\cos\varphi-\alpha\sin\varphi)\end{pmatrix}$$

Specific application to a periodic lattice:

$$M_{\mathrm{Periodic}}=\begin{pmatrix}\cos\varphi+\alpha\sin\varphi & \beta\sin\varphi \\ -\gamma\sin\varphi & \cos\varphi-\alpha\sin\varphi\end{pmatrix}$$

And for a FODO lattice with symmetry points at the end-points:

$$M_{FODO}=\begin{pmatrix}\cos\varphi & \beta\sin\varphi \\ -\frac{1}{\beta}\sin\varphi & \cos\varphi\end{pmatrix}$$

Part 2: Lattice Tune, Beam Mismatch, Lattice Errors and Introduction to Resonances

Particle "Tune" in a Ring

Though the Twiss parameters return to the same values on every turn, **a particle does not return to the same coordinates (u, u')**. It "phase advances" every turn, meaning it shows up on a different part of the same ellipse. The **Twiss ellipse at a location s is the same on every turn**.

The phase advance in one period is related to the particle "Tune", ν.

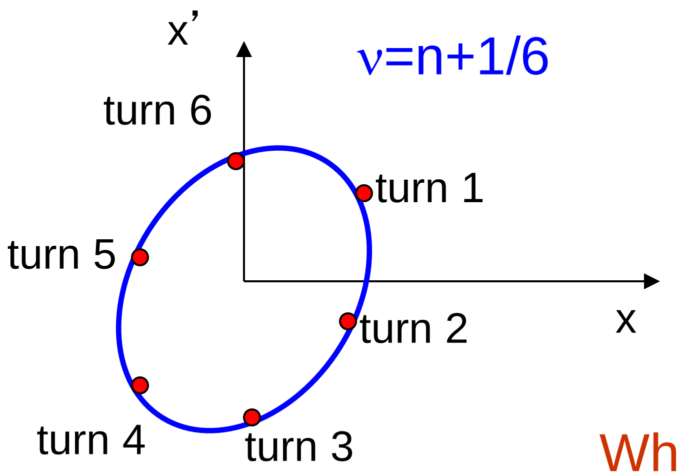
$$\boxed{\nu = \frac{1}{2\pi}\varphi_{total} = \frac{1}{2\pi}\oint \frac{ds}{\beta(s)}}$$

The tune is the total phase advance a particle undergoes during one turn around the ring, normalized to 360 degrees.

Equivalently, the tune is the number of "phase space oscillations" a particle undergoes in one turn.

More on the Tune…

For a fixed location in the ring, a particle will appear at different places on the ellipse on consecutive turns, according to the tune:



Tune = integer piece + fractional piece

From this picture we can not tell the integer piece of the tune. But it looks like the fractional piece is around 1/6 .

What does the turn-by-turn motion look like in real space?

Example: ν=6.33   It means that in one turn around the ring, a particle executes 6 and one-third oscillations around the ellipse. The fractional piece is the most important!

Some fractional tunes are allowed (stable), and some are not. More on this later…

The Smooth Approximation

For rings and synchrotrons, a useful approximation for finding the average beta function from the tune, or conversely the tune from the beta function, is given by:

For a ring of radius R, the approximate tune is:

$$\nu = \frac{1}{2\pi}\oint\frac{ds}{\beta(s)} \approx \frac{2\pi R}{2\pi}\frac{1}{\beta_{\text{ave}}} = \frac{R}{\beta_{\text{ave}}}$$

$$\therefore \nu \approx \frac{R}{\beta_{\text{ave}}}$$

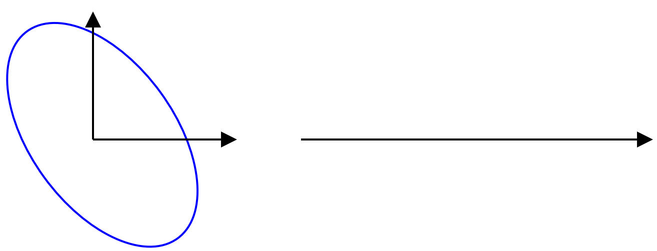
This is also called a “uniform focusing” approximation. It is useful for quick calculations and theoretical analysis.

Beta Function in a (Nonperiodic) Transport Line

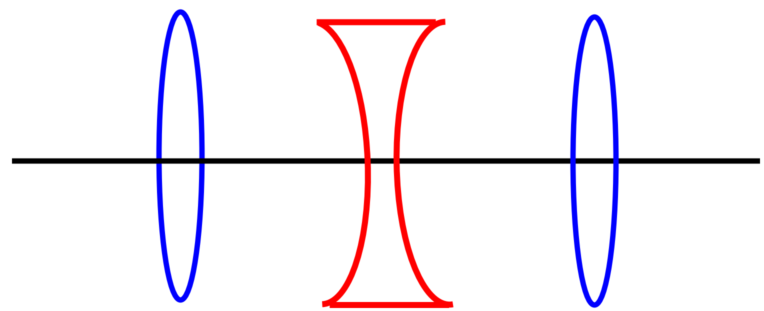
In a transport line, the starting Twiss parameters (beam ellipse) determine the initial conditions from which to calculate the subsequent values. We can not find $\beta(s)$ for the lattice without the initial conditions: $(\beta_o, \alpha_o, \gamma_o)$

$$\frac{1}{2}\beta\beta'' - \frac{1}{4}\beta'^2 + \beta^2 K = 1$$

$(\beta_o, \alpha_o, \gamma_o)$



Incoming beam



Subsequent transport

The $\beta(s)$ in the lattice is a function of both K(s) (magnetic lattice) and the initial Twiss parameters. Some initial conditions will produce better solutions than others (smaller beam envelopes).

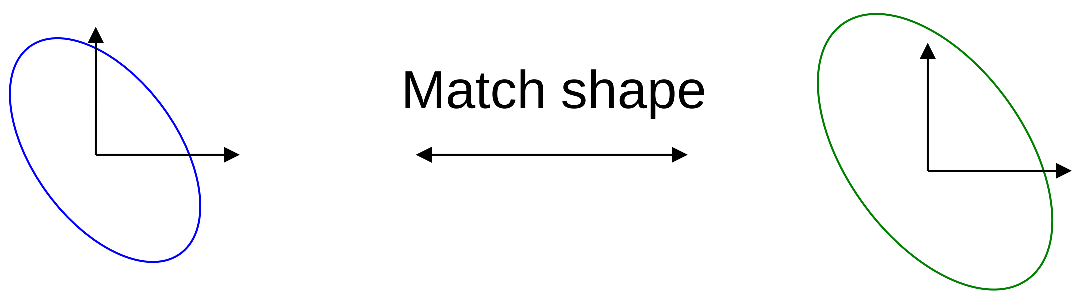
Beam Mismatch

A closed, periodic system is a special case where we do not require an initial condition to calculate the beta function, $\beta(s)$. In this case, $\beta(s)$ is uniquely determined by the fact that we must require the initial and end conditions to match $\beta(s) = \beta(s+L_p)$. In the jargon of differential equations, transport lines are initial value problems, while rings have periodic boundary conditions.

For a periodic lattice ($K(s)=K(s+L_p)$), $\beta(s)$ is uniquely determined.

In any lattice, the phase space ellipses determined by the beam distribution should match those determined by the lattice Twiss parameters at the injection point:

$(\beta_o, \alpha_o, \gamma_o)$ $(\beta_{lattice}, \alpha_{lattice}, \gamma_{lattice})$

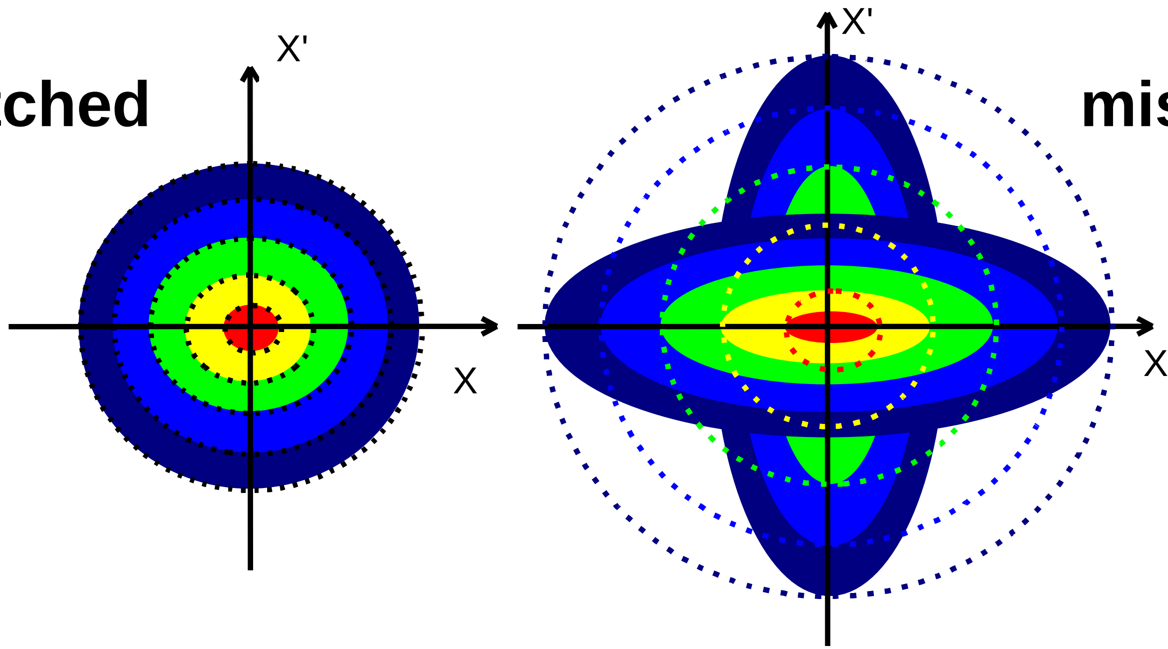


Incoming Distribution Beam Ellipse Lattice Ellipse

Beam Mismatch in a Periodic Lattice

If our initial beam does not match the Twiss parameters imposed by the lattice, we say that the beam is mismatched.

matched



mismatched

Though the emittance is conserved, the beam width and momentum spread are larger than for the matched case.

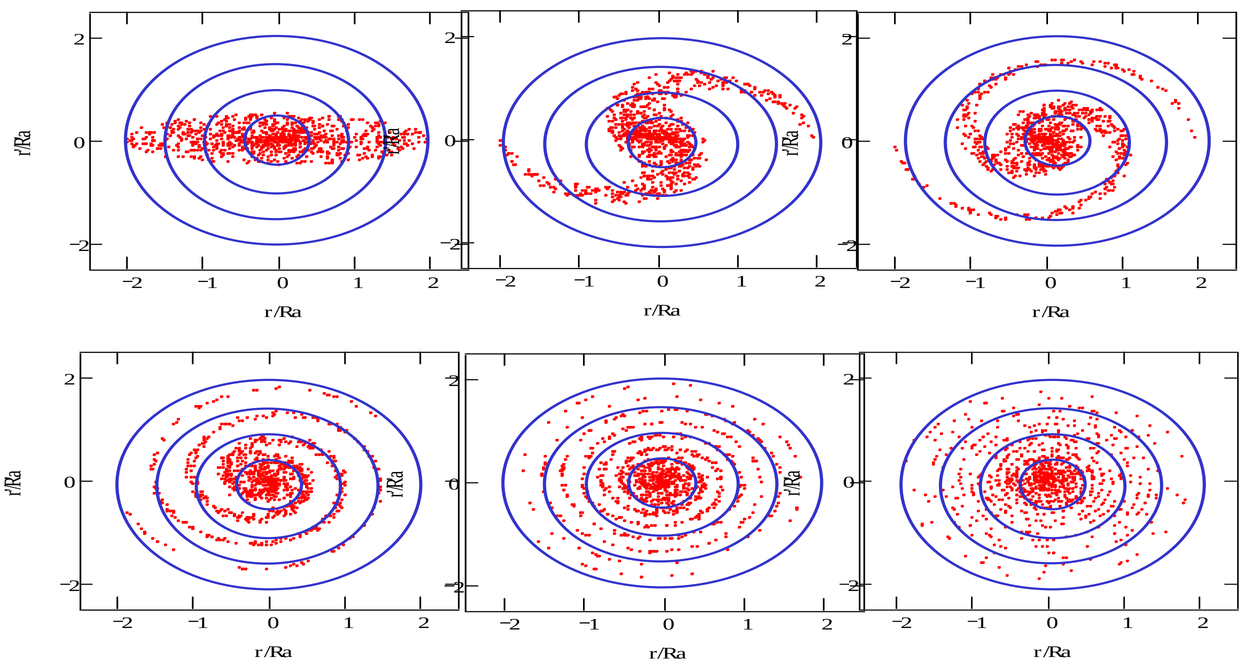
Mismatched beams do not return to the same Twiss ellipse turn-by-turn. Thus they are more likely to intercept the aperture and cause loss.

Effect of Mismatch in a Nonlinear Lattice

So far we have considered only the linear, homogeneous Hill's equation (we set all higher order terms to zero). In reality, some lattices have significant higher order terms.

$$u'' + K(s)u(s) = O[u^2] + ...$$

When nonlinear terms are included, a mismatched beam will dilute in phase space, and emittance will grow..

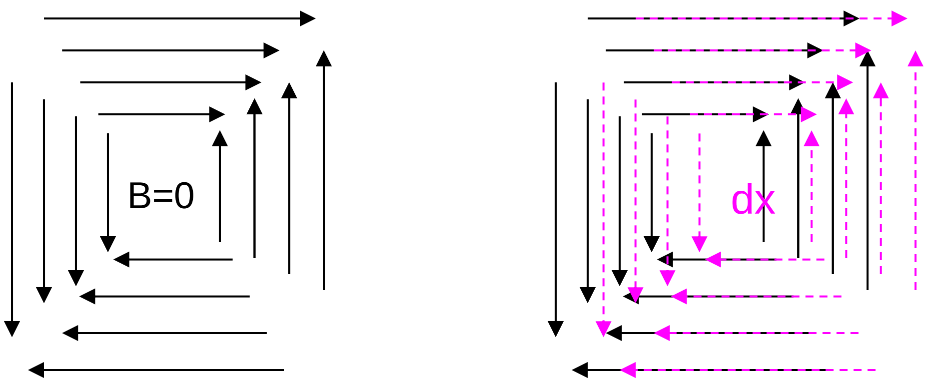


Magnet Errors

So far we have considered only perfect magnetic lattices. In reality there is no such thing. A machine has many sources of lattice "errors":

- Magnet misalignments – offset and roll
- Magnet strength errors
- Magnet field imperfections
- etc, etc…

For instance, a quad displaced by dX gives rise to a dipole field error:



There is a dBy field difference at each location x, and By$\neq$ 0 @ x=0

In general, a magnet of order N can give rise to magnetic error terms of order N or less when displaced or rotated.

Dipole Errors

A dipole error is represented by a kick to a beam at a certain location. Dipole errors change the closed orbit of the beam, i.e., the reference trajectory.

To find the new closed orbit, we must solve Hill's equation again, this time with a particular solution of a dipole error at a location s_o.

$$u'' + K(s)u(s) = \delta(s - s_o)\theta_{error}$$

And solutions will be of the form:

$$u(s) = \underbrace{C(s)u_o + S(s)u_o'}_{\text{Normal betatron oscillations}} + P(s)$$

New perturbed closed orbit

Closed Orbit Distortion due to Dipole Error

The new equilibrium trajectory (closed orbit) of a particle in the presence of a dipole error θ at location s_o is:

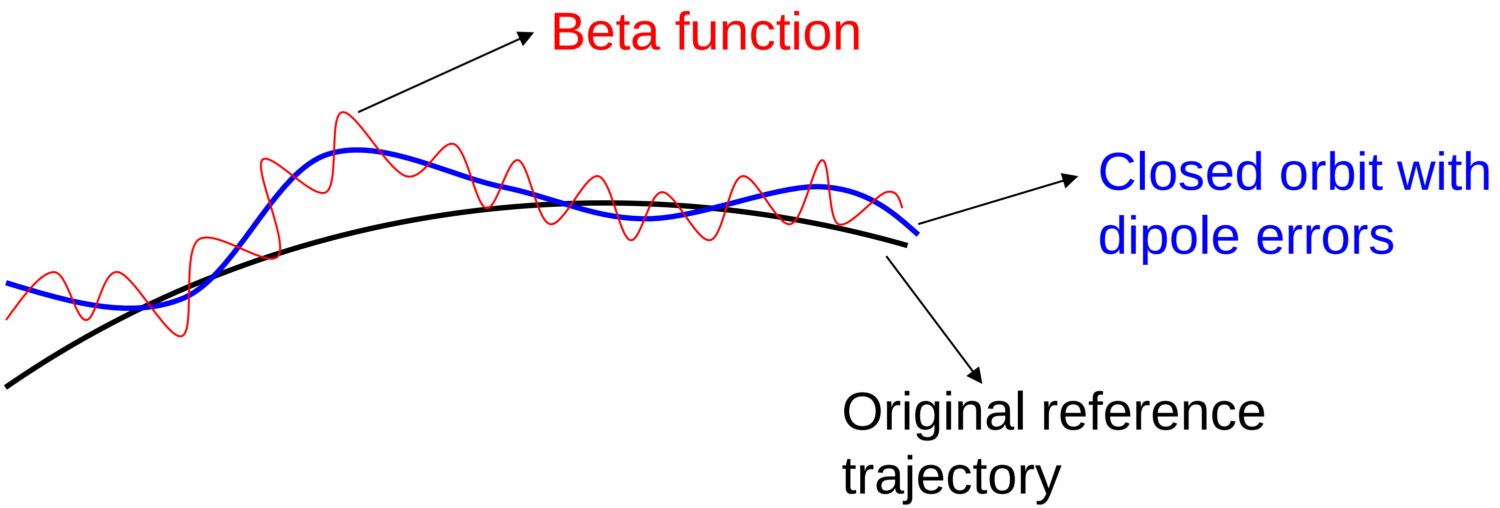
$$u_{co}(s) = \frac{\sqrt{\beta(s_o)\beta(s)}}{2\sin(\pi\nu)}\theta\cos(\pi\nu - \varphi(s) + \varphi(s_o))$$

And in the presence of many such dipole errors we have:

$$u_{co}(s) = \frac{\sqrt{\beta(s)}}{2\sin(\pi\nu)}\sum_i \sqrt{\beta_i}\theta_i\cos(\pi\nu - \varphi(s) + \varphi_i)$$

Closed Orbit Distortion

Recall that the betatron oscillations occur about the reference trajectory, or in the case of a ring, the closed orbit. Therefore, the new closed orbit caused by the dipole error becomes the origin of the betatron motion:



Closed Orbit Instability

Look closely again at the expression for the closed orbit in the presence of a dipole error:

$$u_{co}(s) = \frac{\sqrt{\beta(s)}}{2\sin(\pi\nu)} \sum_i \sqrt{\beta_i}\,\theta_i \cos(\pi\nu - \phi(s) + \phi_i)$$

When sin($\pi\nu$) = 0, the orbit blows up!

This is called the "Closed Orbit Instability", and occurs whenever the "Integer Resonance" Condition is met:

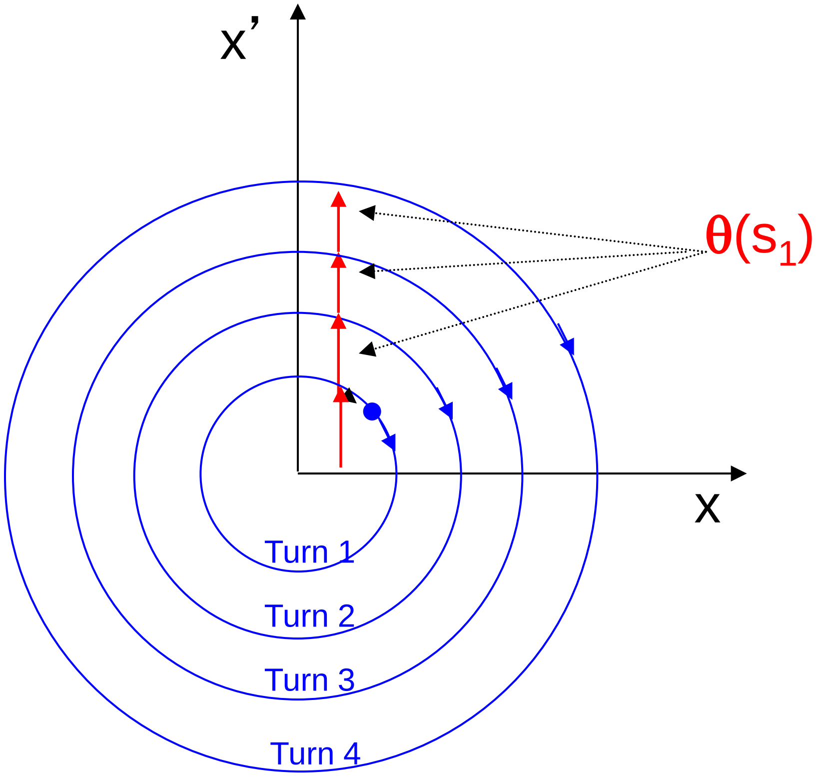
$$\sin(\pi\nu) = 0$$

$$\Rightarrow \pi\nu = n\pi \quad \text{or} \quad \nu = \mathrm{n}$$

Dipole errors will always be present in any accelerator. Therefore, we should never set up a lattice with an integer tune value, as this will cause the beam trajectory to be unstable.

Phase Space for Dipole Error

Consider what is going on in the phase space. The dipole error causes a change in the u' variable, at the location s. If the beam has integer tune, the kicks add up coherently:



What would this look like if $\nu \neq n$?

What is the effect of $\theta(s_1)$ if $\nu = 0.5$?

(**Homework**)

Quadrupole Errors and Tune Shift

As dipole errors affect the reference trajectory (closed orbit), quadrupole errors affects the net focusing of the beam and thus the the tune and beta function.

A quadrupole error can be represented as a matrix:

$$M_{\text{error}} = \begin{pmatrix} 1 & 0 \\ -K(s_1)ds_1 & 1 \end{pmatrix}$$

(**Derivation**)

Including this matrix in the one turn map and enforcing periodicity gives the quad tune shift due to the error:

$$\Delta\nu = \frac{1}{4\pi}\beta(s_1)K(s_1)ds_1$$

This gives us an easy way to measure the β at a quadrupole:
Vary the quadrupole to produce a small deviation k(s), then measure the tune change, and calculate β at the quad.

Beta Function Response to Quad Error

The Beta function in the ring is also altered by a quadrupole error. The expression for the deviation of the beta function from nominal:

$$\boxed{\frac{\Delta\beta(s)}{\beta(s)} = -\frac{1}{2\sin(2\pi\nu)}\int \beta(s_1)K(s_1)\cos(2\pi\nu_o - 2\varphi(s) + 2\varphi(s_1))ds_1}$$

And this time we have $\beta\rightarrow\infty$ when $\sin(2\pi\nu)=0$, i.e., whenever the tune is a half-integer value:

$$\sin(2\pi\nu) = 0$$

$$\Rightarrow 2\pi\nu = n\pi \quad \text{or} \quad \nu = \frac{n}{2}$$

What does the phase space of a particle with half-integer tune look like in the presence of a quad error at a fixed location s_1?

(**Example**)

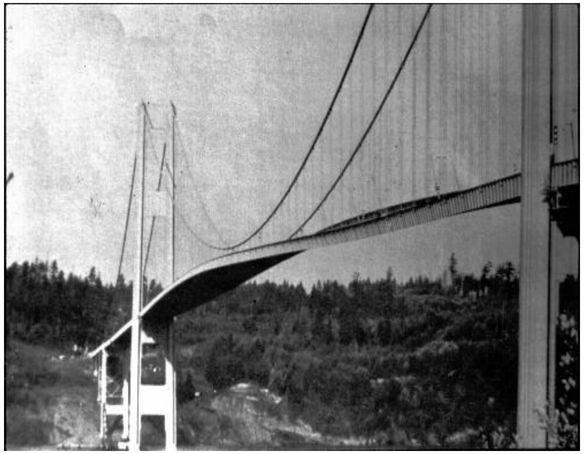
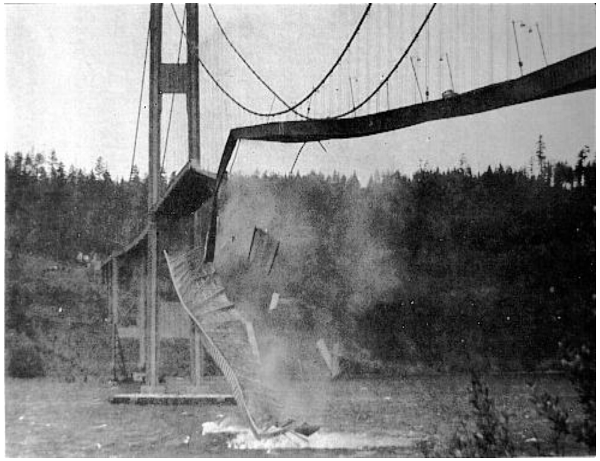
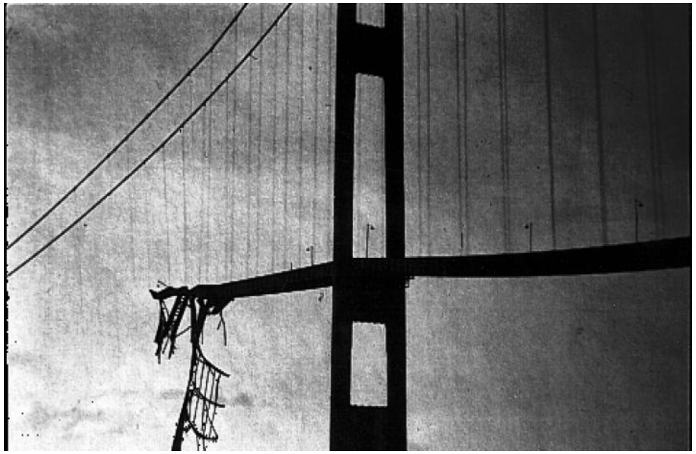
General Resonance Condition

Hill's equation is quasiharmonic, and whenever we have a harmonic system, the danger of exciting a resonance exists. Multiple sources of resonant "driving terms" exist in accelerators:

- **Linear magnet imperfections**
- **Time varying fields**
- **Non-linear magnets**
- Collective Effects
- etc, etc..

A resonance occurs when the frequency of the external force approaches the natural frequency of the system.

Tacoma Narrow bridge 1940

Resonance excitation between wind gusts and natural frequency of the bridge.

General Resonance Condition

So far we have seen resonances for dipole and quadrupole driving terms. We can write a general resonance condition for the tunes in both planes:

$$\boxed{kn_x + ln_y = m}$$

Where (k, l, m) are integers, and |k|+|l| is the "order of the resonance". For multiple superperiods, m→Nm, where N= # superperiods.

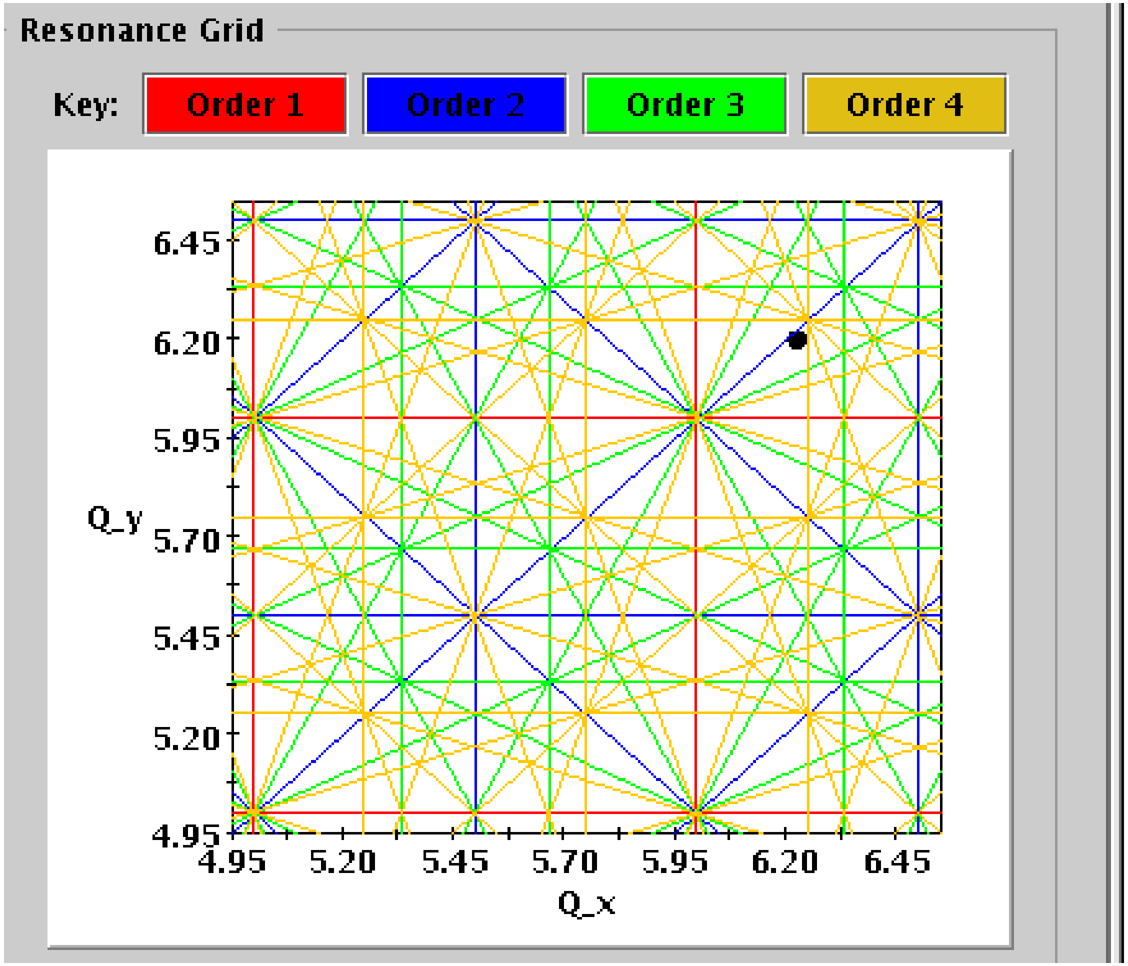
k	l	Driving Field	Resonance Name
1	0	dipole	"Integer resonance"
0	1	dipole	"Integer resonance"
1	1	skew quad	"Half-integer"
2	0	upright quad	"Half-integer"
0	2	upright quad	"Half-ingeger"
3	0	sextupole	"Third order resonance"
...		…	…

In general, the strength of the resonance decreases as the resonance order increases.

The Tune Resonance Diagram

A tune diagram is a convenient way to map out the unstable tune areas. We draw a line for every important resonance:

This is a tune diagram up to 4th order for the SNS ring. The main tune "working point" is (6.23, 6.20).



We could populate the diagram endlessly, but since resonances become weaker with increasing order gets, it's not necessary.