6. Lattice design/matching

Previous chapter

- We have developed transfer matrix for SCL elements
    - Drift, Quadrupole, solenoid, Rf gap
- We have introduced beam matrix and how to transport it in the SC linac elements (spreadsheet and trace3D)

This chapter

- We will introduce formalism for periodic transport
- We will show possible design layout for SC linac using
    - Quadrupole doublets or solenoid
- We will apply it and design accelerating period for SNS-like and FRIB like cases

Trace 3-D twiss conversion

- Trace 3D Transfer matrices are in (z,Δp/p) coordinates
- Trace 3D input file needs twiss parameters in (ϕ,ΔKE) Conversion between the two sets of coordinates are as follow

$$z = -\frac{\beta\lambda}{360}\Delta\phi \qquad \frac{\Delta p}{p} = \frac{\gamma}{\gamma+1}\frac{\Delta KE}{KE}$$

- Conversion factors A and B

$$A[\deg/mm] = \frac{0.36}{\beta\lambda}$$

$$B[keV/mrad] = \frac{\gamma+1}{\gamma}KE$$

Plug in the formulae:
βλ in meters and KE in MeV

- From (z,Δp/p) twiss to (ϕ,ΔKE) twiss for Trace3D

$$\begin{pmatrix}\alpha\\ \beta\\ \varepsilon\end{pmatrix} \rightarrow \begin{pmatrix}-\alpha\\ \frac{A}{B}\beta\\ AB\varepsilon\end{pmatrix}$$

Homework 6-1

- Consider a $^{4}He^{1+}$ at 100 MeV/u with initial twiss parameters of α=2.0 β=4.0m and ε=1 π.mm.mrad in a 500 MHz SC linac
  - Use Trace 3D to compute the final twiss parameters after a 3 meter long drift in (deg-keV) units
  - Give these twiss parameters in (mm-mrad) units
  - Use excel spreadsheet to compute final twiss parameters directly in (mm-mrad)

6-D beam matrix transport in periodic system (1)

- A stable periodic system has a transfer matrix of the form

$$M_{period} = \begin{bmatrix} \cos\mu + \tilde{\alpha}\sin\mu & \tilde{\beta}\sin\mu \\ -\tilde{\gamma}\sin\mu & \cos\mu - \tilde{\alpha}\sin\mu \end{bmatrix}$$

- Where μ is the called the phase advance and α, β, and γ tildes are the twiss parameters of the period. μ interval is]0;π[
- Writing a general transfer matrix as

$$M_{period} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

- And identifying the two matrices leads to

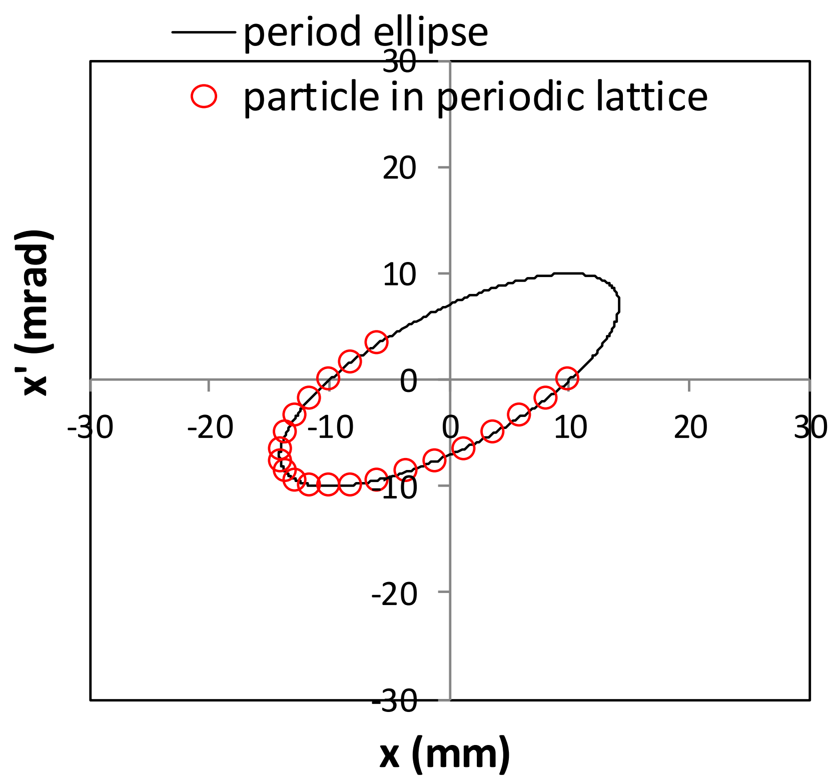
$$\cos\mu = \frac{a+d}{2} \qquad \tilde{\alpha} = \frac{a-d}{2\sin\mu} \qquad \tilde{\beta} = \frac{b}{\sin\mu} \qquad \tilde{\gamma} = \frac{1+\tilde{\alpha}^2}{\tilde{\beta}}$$

6-D beam matrix transport in periodic system (2) - spreadsheet

- One particle is tracked through a periodic system
- The particle follows the period ellipse

PERIOD		
α	-1	
β	2	m
μ	10	degrees
γ	1	rad/m

M	
0.81	0.35
-0.17	1.16



period #	x	x'
0	10.00	0.00
1	8.11	-1.74
2	5.98	-3.42
3	3.66	-5.00
4	1.23	-6.43
5	-1.23	-7.66
6	-3.66	-8.66
7	-5.98	-9.40
8	-8.11	-9.85
9	-10.00	-10.00
10	-11.58	-9.85
11	-12.82	-9.40
12	-13.66	-8.66
13	-14.09	-7.66
14	-14.09	-6.43
15	-13.66	-5.00
16	-12.82	-3.42
17	-11.58	-1.74
18	-10.00	0.00
19	-8.11	1.74
20	-5.98	3.42

6-D beam matrix transport in periodic system (2) – properties of period matrix

- Period matrix can conveniently be rewritten

$$M = I\cos\mu + J\sin\mu$$

- With

$$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \qquad J = \begin{bmatrix} \tilde{\alpha} & \tilde{\beta} \\ -\tilde{\gamma} & -\tilde{\alpha} \end{bmatrix}$$

- Useful relation

$$J^{-1} = -J \qquad J^2 = -I$$

- Matrix after k-periods Multiple periods

$$M^k = I\cos k\mu + J\sin k\mu \qquad M_{k-periods} = \begin{bmatrix} \cos k\mu + \tilde{\alpha}\sin k\mu & \tilde{\beta}\sin k\mu \\ -\tilde{\gamma}\sin k\mu & \cos k\mu - \tilde{\alpha}\sin k\mu \end{bmatrix}$$

- Inverse

$$M^{-1} = I\cos\mu - J\sin\mu$$

Homework 6-2

- Show that

$$M^2 = I\cos 2\mu + J\sin 2\mu$$

6-D beam matrix transport in periodic system (3) – beam transport & matching

- Beam matrix rotate along the period ellipse, by an angle equal to the phase advance of the period
- The smallest beam size in a periodic channel occurs for a "matched" beam (e.g. a beam with twiss parameters equal to the one of the period)
- Any beam with twiss parameters different than the ones of the period is called "mismatched"

6-D beam matrix transport in periodic system (4) – spreadsheet

PERIOD	xx'	yy'	zz'	
α	0	0	0	
β	0.5	1	1	m
μ	90	45	45	degrees

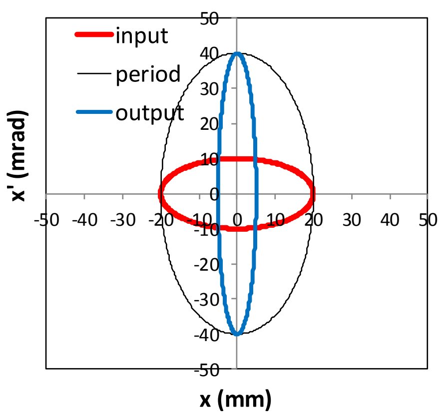
	S0		M		S1		
Sxx	400.0	0.0	6.13E-17	0.50	25.0	0.0	Sxx
	0.0	100.0	-2	0.00	0.0	1600.0	
Syy	564.0	0.0	0.71	0.71	317.5	-246.5	Syy
	0.0	70.9	-0.71	0.71	-246.5	317.5	
Szz	800.0	400.0	0.71	0.71	925.0	-275.0	Szz
	400.0	250.0	-0.71	0.71	-275.0	125.0	

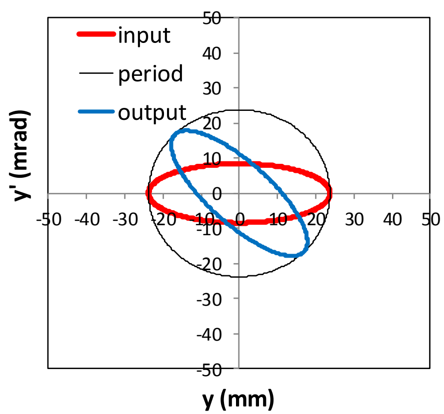
$$M_{period} = \begin{bmatrix} \cos\mu + \tilde{\alpha}\sin\mu & \tilde{\beta}\sin\mu \\ -\tilde{\gamma}\sin\mu & \cos\mu - \tilde{\alpha}\sin\mu \end{bmatrix}$$

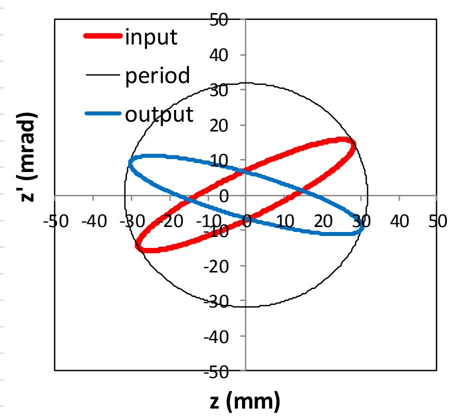
INPUT	xx'	yy'	zz'	
α	0.00	0.00	-2.00	
β	2.00	2.82	4.00	m
ε	200.00	200.00	200.00	pi-mm-mrad
γ	0.50	0.35	1.25	rad/m
half-size	20.00	23.75	28.28	mm
half-divergence	10.00	8.42	15.81	mrad
full-size	40.00	47.50	56.57	mm
full-divergence	20.00	16.84	31.62	mrad
θ	0.00	0.00	0.48	rad
θ	0.00	0.00	27.75	deg
upright ellipse beta	2.00	2.82	5.05	m
upright ellipse gamma	0.50	0.35	0.20	rad/m
major axis	20.00	23.75	31.79	mm
minor axis	10.00	8.42	6.29	mrad

Period ell	xx'	yy'	zz'	
α	0.00	0.00	0.00	
β	0.50	1.00	1.00	m
ε	800.00	564.00	1010.41	pi-mm-mrad
γ	2.00	1.00	1.00	rad/m
half-size	20.00	23.75	31.79	mm
half-divergence	40.00	23.75	31.79	mrad
full-size	40.00	47.50	63.57	mm
full-divergence	80.00	47.50	63.57	mrad
θ	-1.57	0.00	0.00	rad
θ	-90.00	0.00	0.00	deg
upright ellipse beta	2.00	1.00	1.00	m
upright ellipse gamma	0.50	1.00	1.00	rad/m
major axis	40.00	23.75	31.79	mm
minor axis	20.00	23.75	31.79	mrad
R	4.25	3.17	5.25	
Area fac	4.00	2.82	5.05	
Mismatch	1.00	0.68	1.25	

OUTPUT	xx'	yy'	zz'	
α	0.00	1.23	1.38	
β	0.13	1.59	4.63	m
ε	200.00	200.00	200.00	pi-mm-mrad
γ	8.00	1.59	0.63	rad/m
half-size	5.00	17.82	30.41	mm
half-divergence	40.00	17.82	11.18	mrad
full-size	10.00	35.63	60.83	mm
full-divergence	80.00	35.63	22.36	mrad
θ	-1.57	0.00	-0.30	rad
θ	-90.00	0.00	-17.25	deg
upright ellipse beta	8.00	2.82	5.05	m
upright ellipse gamma	0.13	0.35	0.20	rad/m
major axis	40.00	23.75	31.79	mm
minor axis	5.00	8.42	6.29	mrad







beam mismatch factors

- Quantifies the difference between two ellipses
- Consider two centered beam ellipses

ell1 $\gamma x^2 + 2\alpha x x' + \beta x'^2 = \varepsilon$

ell2 $G x^2 + 2A x x' + B x'^2 = \varepsilon$

- Area factor (AF) is how much the second ellipse must be enlarged so it covers the other ellipse
- Assuming the first ellipse is matched into a transport channel, the Mismatch factor (M) is how much larger the beam envelope for the second ellipse will be in the transport channel (i.e. M=0.1 means beam size will be 10% larger in the channel)

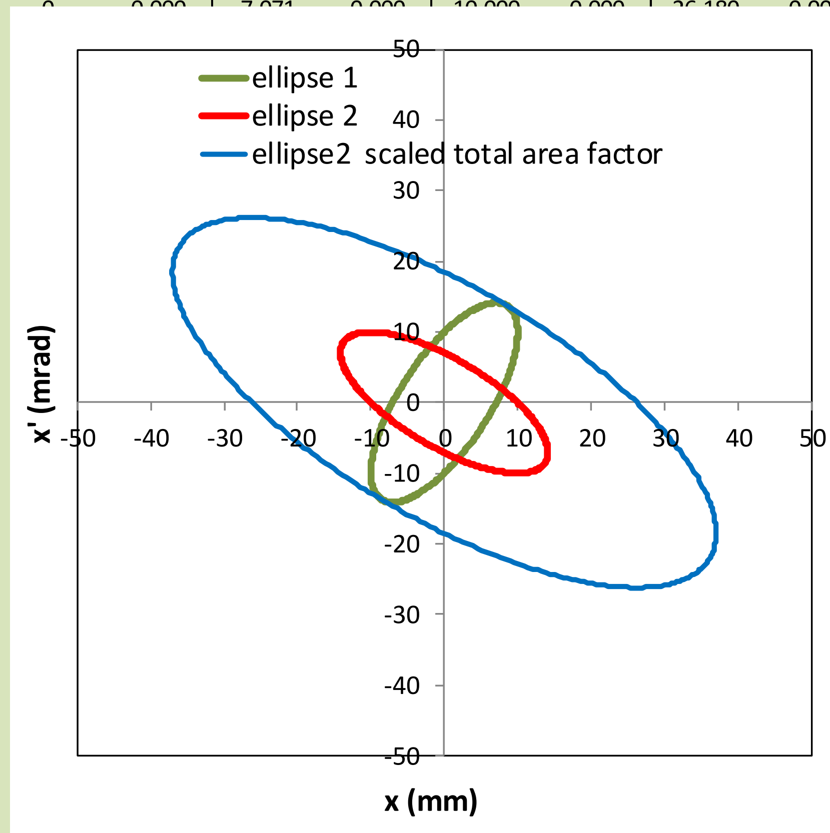
- Area factor: $$AF = \left[\frac{1}{2}\left(R + \sqrt{R^2 - 4}\right)\right]$$
- Mismatch factor: $$M = \sqrt{AF} - 1$$
- where R is given by: $$R = \beta G + \gamma B - 2\alpha A$$

beam mismatch factor (2) - spreadsheet

	Ellipse1	Ellipse2	Scaled Ell	
α	-1.00	1.00	1.00	
β	1.00	2.00	2.00	m
ε	100.00	100.00	685.41	pi-mm-mrad
γ	2.00	1.00	1.00	rad/m
half-size	10.00	14.14	37.02	mm
half-divergence	14.14	10.00	26.18	mrad
full-size	20.00	28.28	74.05	mm
full-divergence	28.28	20.00	52.36	mrad
θ	1.02	-0.55	-0.55	rad
θ	58.28	-31.72	-31.72	deg
upright ellipse beta	2.62	2.62	2.62	m
upright ellipse gamma	0.38	0.38	0.38	rad/m
major axis	16.18	16.18	42.36	mm
minor axis	6.18	6.18	16.18	mrad

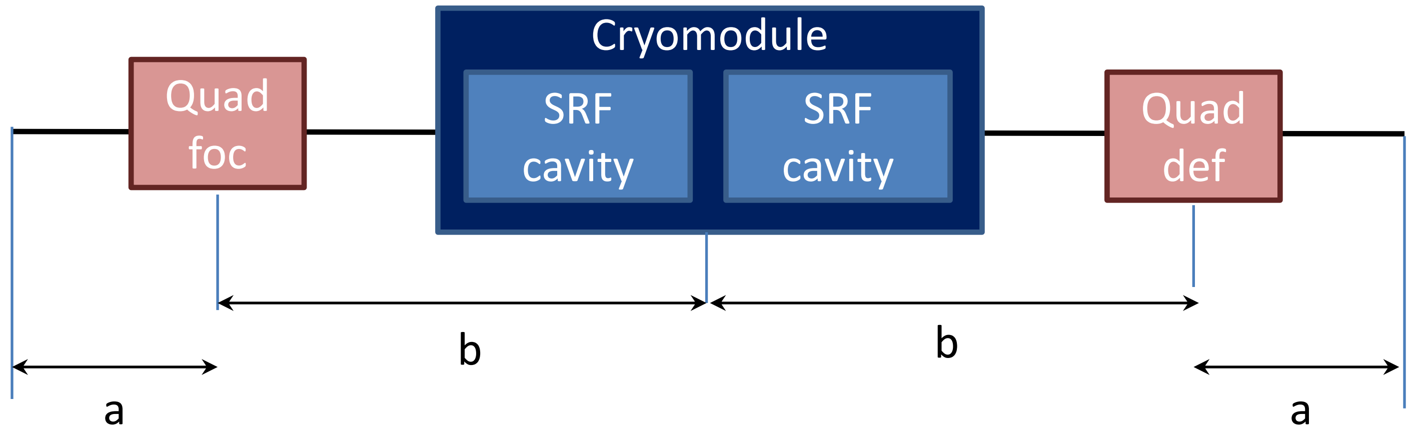
R	7.0	see Trace3D manual
Area factor	6.9	assumes emit ell1 and ell2 are the same
Mismatch factor	1.6	see Trace3D manual
total area factor	6.9	Enlarge area by that factor

(deg)	(rad)	x(mm)	x'(mrad)	x(mm)	x'(mrad)	x(mm)	x'(mrad)
0	0.000	7.071	0.000	10.000	0.000	26.180	0.000
24	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
25	0.436	9.397	5.977	4.837	4.226	12.663	11.064

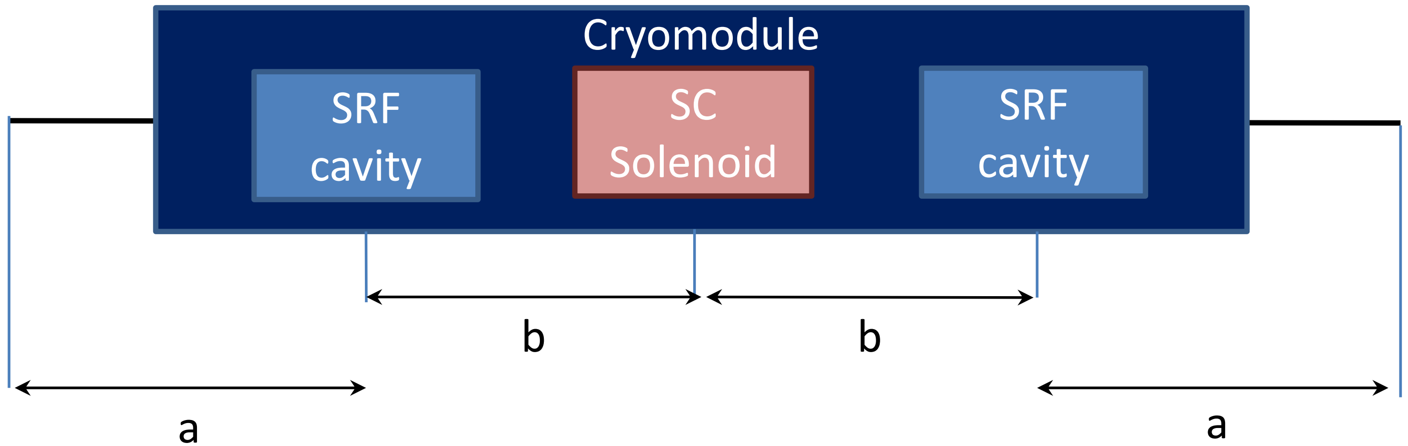


Superconducting Linac accelerating lattice (period)

- We consider two SRF linac lattice design
  - Cryomodule and room-temperature quadrupole doublet



  - Cryomodule and superconducting solenoid



SCL lattice design

- For the design, it is simpler to use thin lens approximation for all elements
- The kicks are carried in the middle of the elements

- Drift

$$M_{xx,yy} = \begin{pmatrix} 1 & L \\ 0 & 1 \end{pmatrix} \quad M_{zz} = \begin{pmatrix} 1 & L/\gamma^2 \\ 0 & 1 \end{pmatrix}$$

- Quadrupole

$$M_{xx,yy} = \begin{pmatrix} 1 & 0 \\ \pm k_q & 1 \end{pmatrix} \quad M_{zz} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$k_q = \sqrt{K}\sin\left(\sqrt{K}L\right)$$

$$K = \frac{G}{B\rho}$$

- Solenoid

$$M_{xx,yy} = \begin{pmatrix} 1 & 0 \\ -k_s & 1 \end{pmatrix} \quad M_{zz} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$k_s = K\sin(KL)$$

$$K = \frac{B}{2B\rho}$$

- RF cavity

$$M_{xx,yy} = \begin{pmatrix} 1 & 0 \\ k_{rfx} & 1 \end{pmatrix} \quad M_{zz} = \begin{pmatrix} 1 & 0 \\ -k_{rfz} & 1 \end{pmatrix}$$

$$k_{rfx} = K/(\beta\gamma)_f$$

$$k_{rfz} = 2\gamma^2 k_{rfx}$$

$$K = -\frac{\pi}{\beta^2\gamma^2\lambda}\frac{Q}{A}\frac{E_0 TL}{m_u c^2}\sin\phi_s$$

SCL lattice – cryomodule + quadrupole doublet (1)

- The transfer matrix for the lattice period is given by

$$M_{xx} = M_{drift}(a).M_{quad}(k_q).M_{drift}(b).M_{rf}.M_{drift}(b).M_{quad}(-k_q).M_{drift}(a)$$

$$M_{xx} = \begin{pmatrix} 1 & a \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ k_q & 1 \end{pmatrix}\begin{pmatrix} 1 & b \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ k_{rfx} & 1 \end{pmatrix}\begin{pmatrix} 1 & b \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ -k_q & 1 \end{pmatrix}\begin{pmatrix} 1 & a \\ 0 & 1 \end{pmatrix}$$

$$M_{xx} = \begin{pmatrix} 1-2bk_q(1+ak_q)(1+bk_{rfx}/2)+k_{rfx}(a+b) & 2\left(a+b-a^2bk_q{}^2\right)+k_{rfx}\left((a+b)^2-(abk_q)^2\right) \\ -2bk_q{}^2+k_{rfx}\left(1-b^2k_q{}^2\right) & 1+2bk_q(1-ak_q)(1+bk_{rfx}/2)+k_{rfx}(a+b) \end{pmatrix}$$

$$\mu_x = \text{acos}\left(1-2abk_q{}^2(1+bk_{rfx}/2)+k_{rfx}(a+b)\right)$$

- Myy is obtained by replacing kq→-kq
- When rf cavity is off (krfx=0)

$$M_{xx} = \begin{pmatrix} 1-2bk_q(1+ak_q) & 2\left(a+b-a^2bk_q{}^2\right) \\ -2bk_q{}^2 & 1+2bk_q(1-ak_q) \end{pmatrix} \qquad \mu_x = \text{acos}\left(1-2abk_q{}^2\right)$$

SCL lattice – cryomodule + quadrupole doublet (2)

- In the longitudinal direction

$$M_{zz} = M_{drift}(a+b).M_{rf}.M_{drift}(a+b)$$

$$M_{zz} = \begin{pmatrix} 1 & (a+b)/\gamma^2 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -k_{rfz} & 1 \end{pmatrix} \begin{pmatrix} 1 & (a+b)/\gamma^2 \\ 0 & 1 \end{pmatrix}$$

$$M_{zz} = \begin{pmatrix} 1-k_{rfz}(a+b)/\gamma^2 & (a+b)/\gamma^2\left(2-k_{rfz}(a+b)/\gamma^2\right) \\ -k_{rfz} & 1-k_{rfz}(a+b)/\gamma^2 \end{pmatrix}$$

$$\mu_z = \mathrm{acos}\left(1-k_{rfz}(a+b)/\gamma^2\right)$$

SCL lattice – cryomodule + solenoid (1)

- The transfer matrix for the lattice period is given by

$$M_{xx} = M_{drift}(a).M_{rf}.M_{drift}(b).M_{sol}(-k_s)M_{drift}(b).M_{rf}.M_{drift}(a)$$

$$M_{xx} = \begin{pmatrix} 1 & a \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ k_{rfx} & 1 \end{pmatrix}\begin{pmatrix} 1 & b \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ -k_s & 1 \end{pmatrix}\begin{pmatrix} 1 & b \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ k_{rfx} & 1 \end{pmatrix}\begin{pmatrix} 1 & a \\ 0 & 1 \end{pmatrix}$$

$$M_{xx} = \begin{pmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{pmatrix}$$

$$M_{11} = 1 + 2bk_{rfx} - bk_s - b^2 k_{rfx}k_s - a(1 + bk_{rfx})(k_s + k_{rfx}(-2 + bk_s))$$

$$M_{12} = -(a + b + abk_{rfx})(-2 + bk_s + a(k_s + k_{rfx}(-2 + bk_s)))$$

$$M_{21} = -(1 + bk_{rfx})(k_s + k_{rfx}(-2 + bk_s))$$

$$M_{22} = 1 + bk_{rfx} + b(k_{rfx} - (1 + bk_{rfx})k_s) - a(1 + bk_{rfx})(k_s + k_{rfx}(-2 + bk_s))$$

- Myy is identical to Mxx

$$\mu_x = \operatorname{acos}\left(\frac{M_{11} + M_{22}}{2}\right)$$

SCL lattice – cryomodule + solenoid (2)

- When rf cavity is off (krfx=0)

$$M_{xx} = \begin{pmatrix} 1-k_s(a+b) & (a+b)(2-k_s(a+b)) \\ -k_s & 1-k_s(a+b) \end{pmatrix} \qquad \mu_x = \mathrm{acos}(1-(a+b)k_s)$$

- In the longitudinal direction

$$M_{zz} = \begin{pmatrix} M_{55} & M_{56} \\ M_{65} & M_{66} \end{pmatrix}$$

$$M_{55} = 1-2bk_{rfz}/\gamma^2 - 2ak_{rfz}/\gamma^2(1-bk_{rfz}/\gamma^2)$$

$$M_{56} = -(a+b-abk_{rfz}/\gamma^2)(-2+2ak_{rfz}/\gamma^2)/\gamma^2$$

$$M_{65} = -2k_{rfz}(1-bk_{rfz}/\gamma^2)$$

$$M_{66} = 1-2bk_{rfz}/\gamma^2 - 2ak_{rfz}/\gamma^2(1-bk_{rfz}/\gamma^2)$$

$$\mu_z = \mathrm{acos}\left(\frac{M_{55}+M_{66}}{2}\right)$$

SCL design – basic procedure

1) Define cavity types (frequency, geometric beta, number of cells
2) Design cavities
3) Define number of cavities for each cavity type
4) Define accelerating lattice
   - Number of cavities per cryomodule
   - Type of transverse focusing
5) Tune first cell for each type
6) Assemble overall linac

   Scale transverse focusing to keep focusing strength similar from one period to the next (effect of change in magnetic rigidity as beam is accelerated and to the change of rf defocusing from one period to the next).

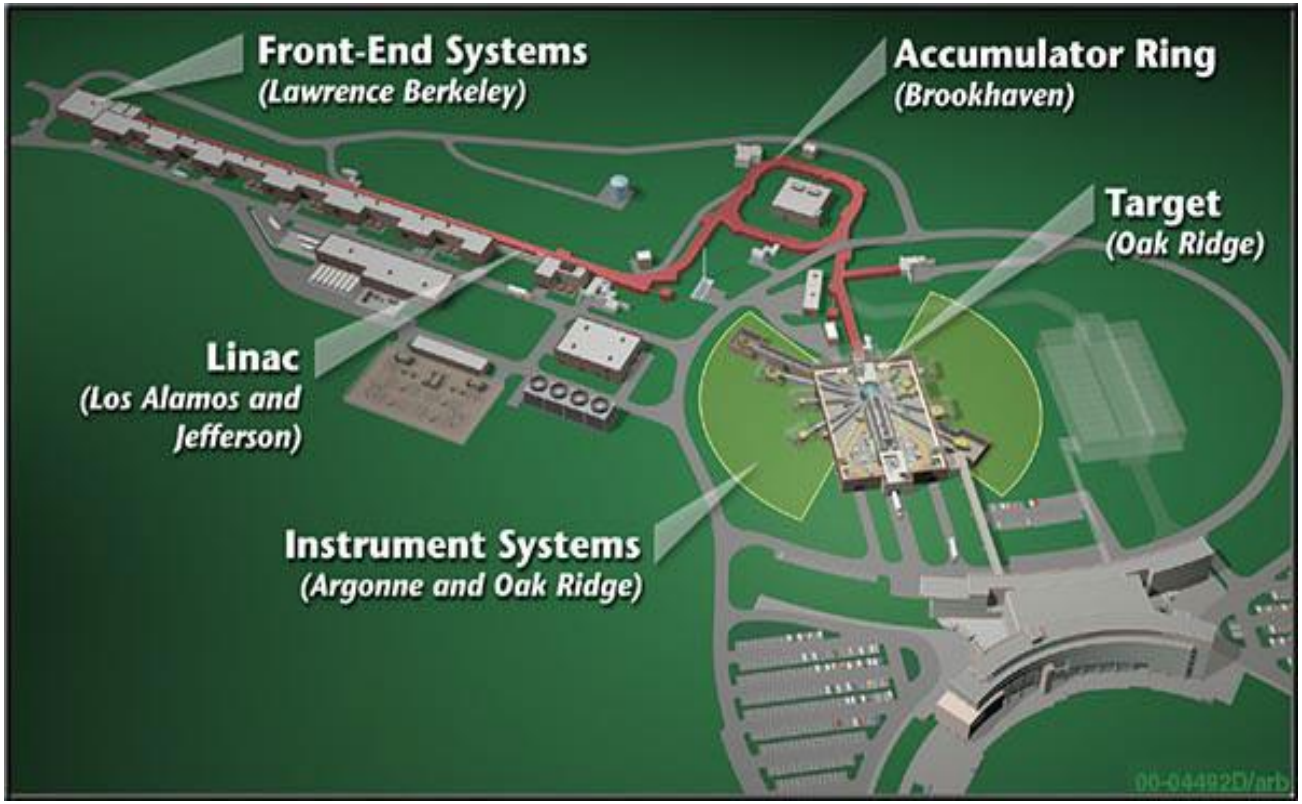
7) Match beam at the change of period if necessary

   change magnet strength and rf phases of neighboring elements to adapt the beam to the new lattice period (Trace3D has tools for this)

SCL lattice – example (0)

- Let's design an "SNS-like" linac

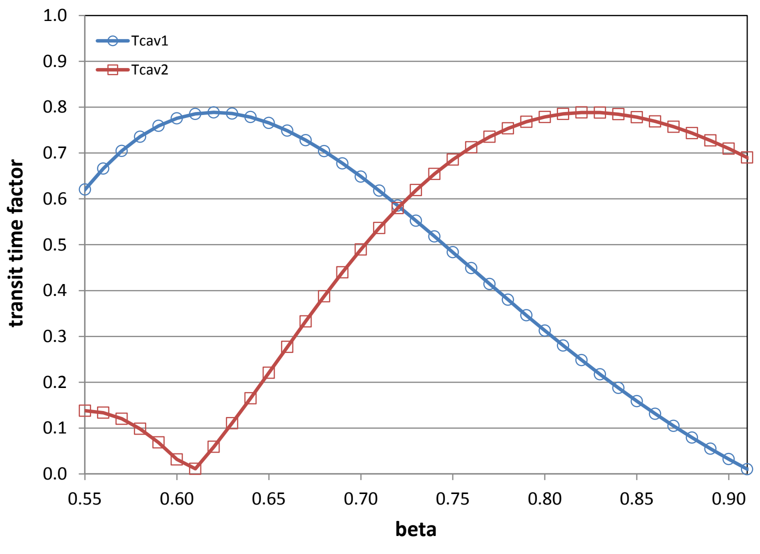
- **1.4 MW average proton power on a Hg target**
- **1-GeV energy**
- **1.4-mA avg current**
- **60-Hz repetition rate**
- **1-ms pulse (6% duty)**
- **170-m-circumference accumulator ring with 1060 injected turns**
- **1.5E14 protons on target in 695 ns**



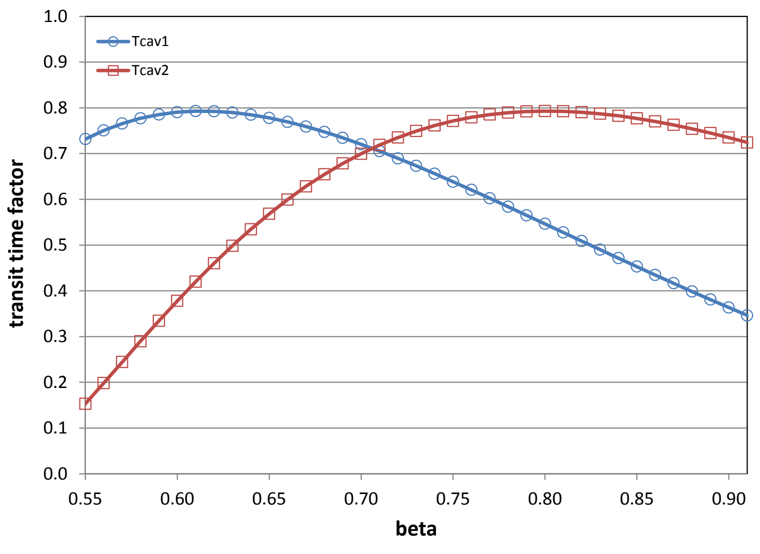
SCL lattice – example (1)

- Let's design an "SNS-like" linac with the requirements
  - Proton, 185MeV to 1.3 GeV, 805 MHz using two cavity types

- 1) Define cavity types
  - Start by finding beta range (kinematics spreadsheet)
    - KE=185 MeV gives beta=0.55
    - KE=1.0 GeV gives beta=0.908
  - Investigate suitable types of cavities over that range of betas (design betas spreadsheet) using f=805 MHz



6-cell β=0.61 and 6-cell β=0.81
Transition around b=0.72 = 414 MeV



4-cell β=0.59 and 4-cell β=0.77
Transition around b=0.7 = 375 MeV

SCL lattice – example (2)

- Carrying forward with the 6-cell option, β=0.61 and β=0.81

- 2) Design cavities
  - Design and optimize cavity geometries using superfish
    - We assume that one can get
      - E0=15 MV/m for β=0.61 cavity
      - E0=17.5 MV/m for β =0.81 cavity
    - Export transit time factor table T(β) from superfish
    - Polynomial fit of T(β) for convenience

- 3) Define number of cavities for each cavity type
  - Start at 185 MeV, finish at 1.3 GeV
  - Transition should be around 414 MeV (see previous slide)
  - Use “linac” spreadsheet to optimize number of cavities
  - Possible design
    - 33 β0.61 cavities at ϕ=-20 deg
    - 80 β0.81cavities at ϕ=-18 deg

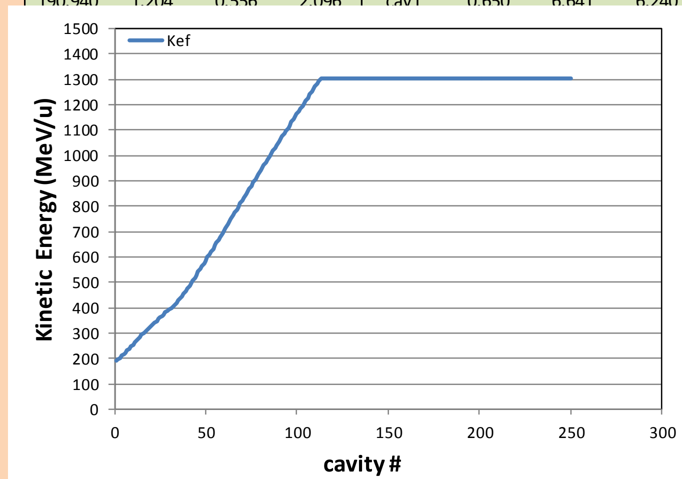
SCL lattice – example (3) – “linac” spreadsheet

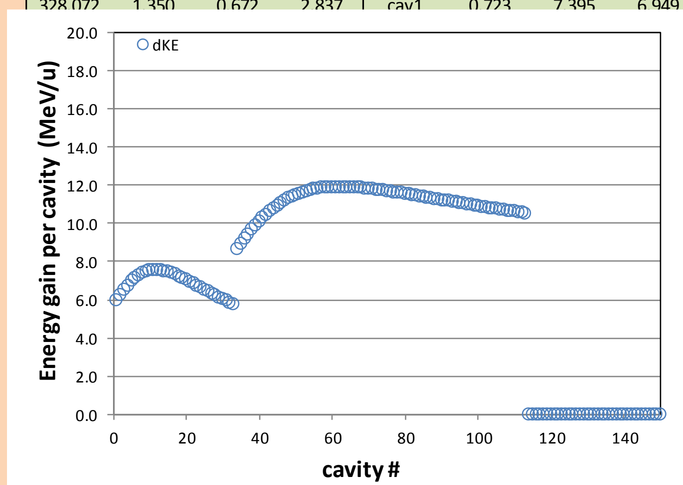
Beam	initial	
Mu	938.2723	MeV/c2
A	1	
Q	1	
beta	0.550	
gamma	1.197	
pc	617.564	MeV/u
E	1123.272	MeV/u
KE	185.000	MeV/u
pc tot	617.6	MeV
Etot	1123.3	MeV
KE tot	185.0	MeV
Brho	2.060	Tm

Cavities	cav1	cav2	
Ncav	33	80	
f	805.000	805.000	MHz
bcell	0.610	0.810	
Ncell	6	6	
E0 cav	15	17.5	MV/m
T0	3.6744E+02	3.7150E+02	
T1	-3.4052E+03	-2.5944E+03	
T2	1.2877E+04	7.3951E+03	
T3	-2.5535E+04	-1.1056E+04	
T4	2.8146E+04	9.1909E+03	
T5	-1.6420E+04	-4.0443E+03	
T6	3.9722E+03	7.3815E+02	
lambda	0.372	0.372	m
L cell	0.114	0.151	m
Lcav	0.6815	0.9050	m
Ep axis	23.6	27.5	MV/m

Linac		
N cav1	33	
Ncav2	80	
N total	113	
KEf cav1	410.5	MeV/u
Final KE	1302.8	MeV/u

Cav #	Amp	phi(deg)	Kei	gi	bi	Brhoi	cav type	TTF cav	E0TL cav	dKE	Kef	gf	bf	Brhof
1	1	-20	185.000	1.197	0.550	2.060	cav1	0.618	6.321	5.940	190.940	1.204	0.556	2.096
2	1	-20	190.940	1.204	0.556	2.096	cav1	0.650	6.641	6.240	197.180	1.210	0.563	2.133
3	1	-20									203.695	1.217	0.570	2.171
4	1	-20									210.456	1.224	0.577	2.211
5	1	-20									217.429	1.232	0.584	2.251
6	1	-20									224.582	1.239	0.591	2.291
7	1	-20									231.879	1.247	0.598	2.332
8	1	-20									239.289	1.255	0.604	2.373
9	1	-20									246.778	1.263	0.611	2.415
10	1	-20									254.318	1.271	0.617	2.456
11	1	-20									261.882	1.279	0.624	2.496
12	1	-20									269.446	1.287	0.630	2.536
13	1	-20									276.991	1.295	0.636	2.576
14	1	-20									284.498	1.303	0.641	2.615
15	1	-20									291.952	1.311	0.647	2.654
16	1	-20									299.341	1.319	0.652	2.692
17	1	-20									306.654	1.327	0.657	2.729
18	1	-20									313.885	1.335	0.662	2.766
19	1	-20									321.026	1.342	0.667	2.802
20	1	-20									328.072	1.350	0.672	2.837
21	1	-20	328.072	1.350	0.672	2.837	cav1	0.723	7.395	6.949	335.021	1.357	0.676	2.871
22	1	-20									341.869	1.364	0.680	2.905
23	1	-20									348.616	1.372	0.684	2.938
24	1	-20									355.259	1.379	0.688	2.970
25	1	-20									361.800	1.386	0.692	3.002
26	1	-20									368.238	1.392	0.696	3.033
27	1	-20									374.574	1.399	0.699	3.063
28	1	-20									380.810	1.406	0.703	3.093
29	1	-20									386.946	1.412	0.706	3.122
30	1	-20									392.984	1.419	0.709	3.150
31	1	-20									398.926	1.425	0.713	3.178
32	1	-20									404.774	1.431	0.715	3.205
33	1	-20									410.529	1.438	0.718	3.232
34	1	-18									419.184	1.447	0.723	3.272
35	1	-18									428.103	1.456	0.727	3.313
36	1	-18									437.277	1.466	0.731	3.355
37	1	-18									446.697	1.476	0.736	3.398
38	1	-18									456.350	1.486	0.740	3.442
39	1	-18									466.225	1.497	0.744	3.486
40	1	-18	466.225	1.497	0.744	3.486	cav2	0.670	10.604	10.085	476.310	1.508	0.748	3.531
41	1	-18	476.310	1.508	0.748	3.531	cav2	0.683	10.810	10.281	486.591	1.519	0.753	3.577





SCL lattice – example (4)

- 4) Define accelerating lattice
  - Number of cavities per cryomodule
  - Type of transverse focusing

  - Use “lattice” spreadsheet to investigate possible designs
  - We try to pack
    - 3-cavities per cryomodule for β0.61 cavities
    - 4 cavities per cryomodule for β0.81 cavities
  - We choose room-temperature doublet for focusing

  - Find focusing parameters for rf focusing and defocusing (“foc strength” spreadsheet)
    - For easiness, concentrate all the rf kick in one equivalent cavity
    - (multiply E0TL of a single cavity by number of cavities in cryomodule)
  - Adjust lengths, transverse focusing, and rf phase to define an acceptable lattice
    - Phase advances around 75 degrees are a good starting point
    - Avoid parametric resonance by maintaining transverse phase advance above half longitudinal phase advance

$$\mu_{x,y} > \frac{\mu_z}{2}$$

SCL lattice – example (5)

4) And 5)

- Start working on lattice for the first type of cavity β0.61
- Get E0TL for single cavity from "linac" spreadsheet
- Multiply by 3 to get E0TL for 3 rf cavities in a cryomodule (we're trying to pack 3 β0.61 cavities per cryomodule and we'll use this E0TL in "lattice" spreadsheet to check if the design is acceptable)
- Use this in "foc strength" spreadsheet to get krf x and krfz focusing values in "lattice" spreadsheet

Beam		in	out	
	Mu	938.2723	938.2723	MeV/c2
	A	1	1	
	Q	1	1	
	beta	0.550	0.569	
	gamma	1.197	1.216	
	pc	617.564	649.412	MeV/u
	E	1123.272	1141.092	MeV/u
	KE	185.000	202.820	MeV/u
	pc tot	617.6	649.4	MeV
	Etot	1123.3	1141.1	MeV
	KE tot	185.0	202.8	MeV
	Brho	2.060	2.166	Tm
rf cavity	f	805		MHz
	E0TL	18.963		MV
	Phi	-20		deg
	lambda	0.372		
	K	0.135		m-1
	krf x	0.194		m-1
	krf z	0.575		m-1
	f rf x	5.14		m
	f rf z	1.74		m

SCL lattice – example (6)

- 4) And 5) continued
- Use the krfx and krf z values in "lattice" spreadsheet and try to design a period layout so that the dimensions are reasonable (e.g. space between cavities, end of cryomodule, size of magnets etc..) and twiss parameters for the period are satisfactory

lattice	drift 1	0.4	0.279	m
Quad	quad	0	m	
doublet	drift 2	2.2	1.535	m
	rf cav	0	m-1	
	drift2	2.2	m	
	quad	0	m	
	drift 1	0.4	m	
	k quad	0.750	m-1	
	k rf x	0.194	m-1	
	k rf z	0.575	m-1	
Mxx	-3.702	6.034	**detx**	
	-2.810	4.310	1	
Myy	4.310	6.034	**dety**	
	-2.810	-3.702	1	
Mzz	-0.044	1.735	**detz**	
	-0.575	-0.044	1	
TWISS	**xx**	**yy**	**zz**	
alpha	-4.20	4.20	0.00	
beta	6.33	6.33	1.74	**m**
mu	72.31	72.31	92.50	**deg**
			zz	
alpha			0.00	
beta			0.0090	**deg/keV**
mu			92.50	**deg**

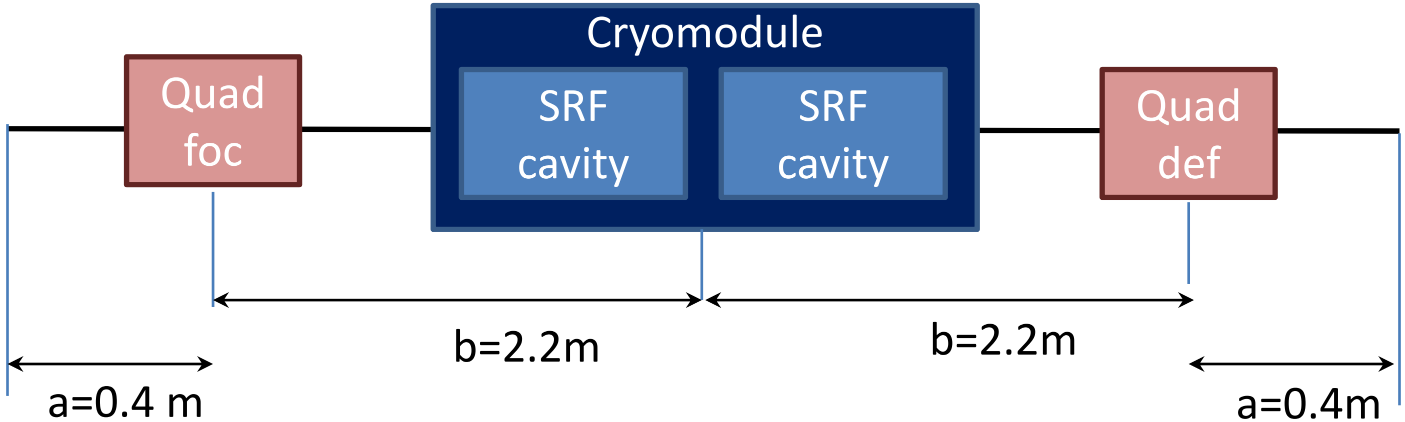
SCL lattice – example (7)

- 4) And 5) continued
- Verify that the focusing strength for the chosen magnets are reasonable

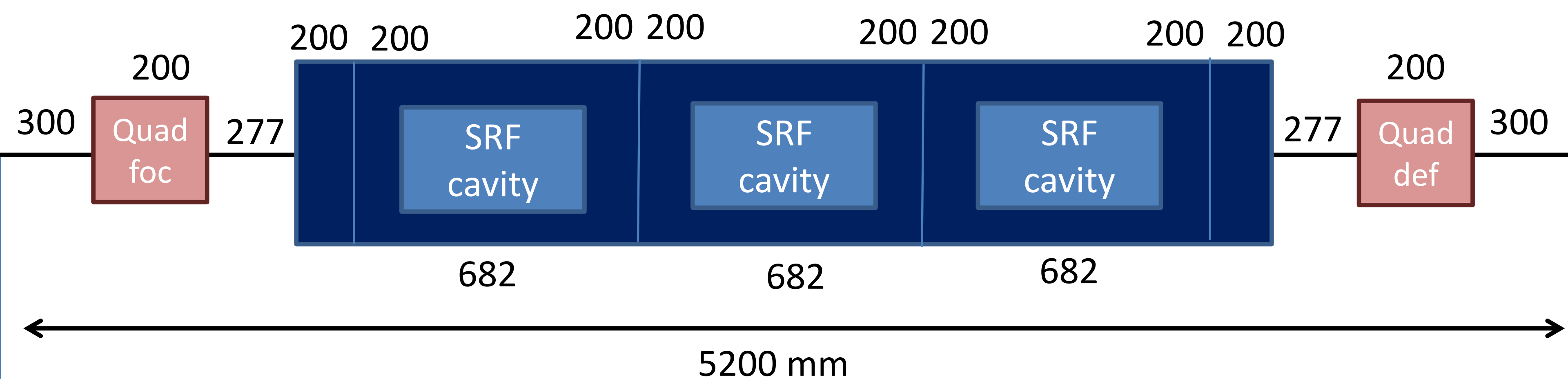
Beam			Tm
	Mu	938.2723	MeV/c2
	A	1	
	Q	1	
	beta	0.550	
	gamma	1.197	
	pc	617.564	MeV/u
	E	1123.272	MeV/u
	KE	185.000	MeV/u
	pc tot	617.6	MeV
	Etot	1123.3	MeV
	KE tot	185.0	MeV
	Brho	2.060	Tm
Quad	Length	0.2	m
	Gradient	7.9	T/m
	rad apert	5	cm
	B pole tip	0.395	T
	K	3.84	m-1
	k quad	0.75	m-1
	f quad	1.338	m

SCL lattice – example (8)

- The period for the low beta cavity type is 5.2 m long and looks like



- Dimensions for each of the elements needs to be sorted out and building a Trace-3D model. For example, one can assume that all SRF cavities needs to have 200mm of distance on either of its side. And that the end of cryomodules to transition from cold to room temperature requires another 200mm.



SCL lattice – example (9)

- Trace 3D input

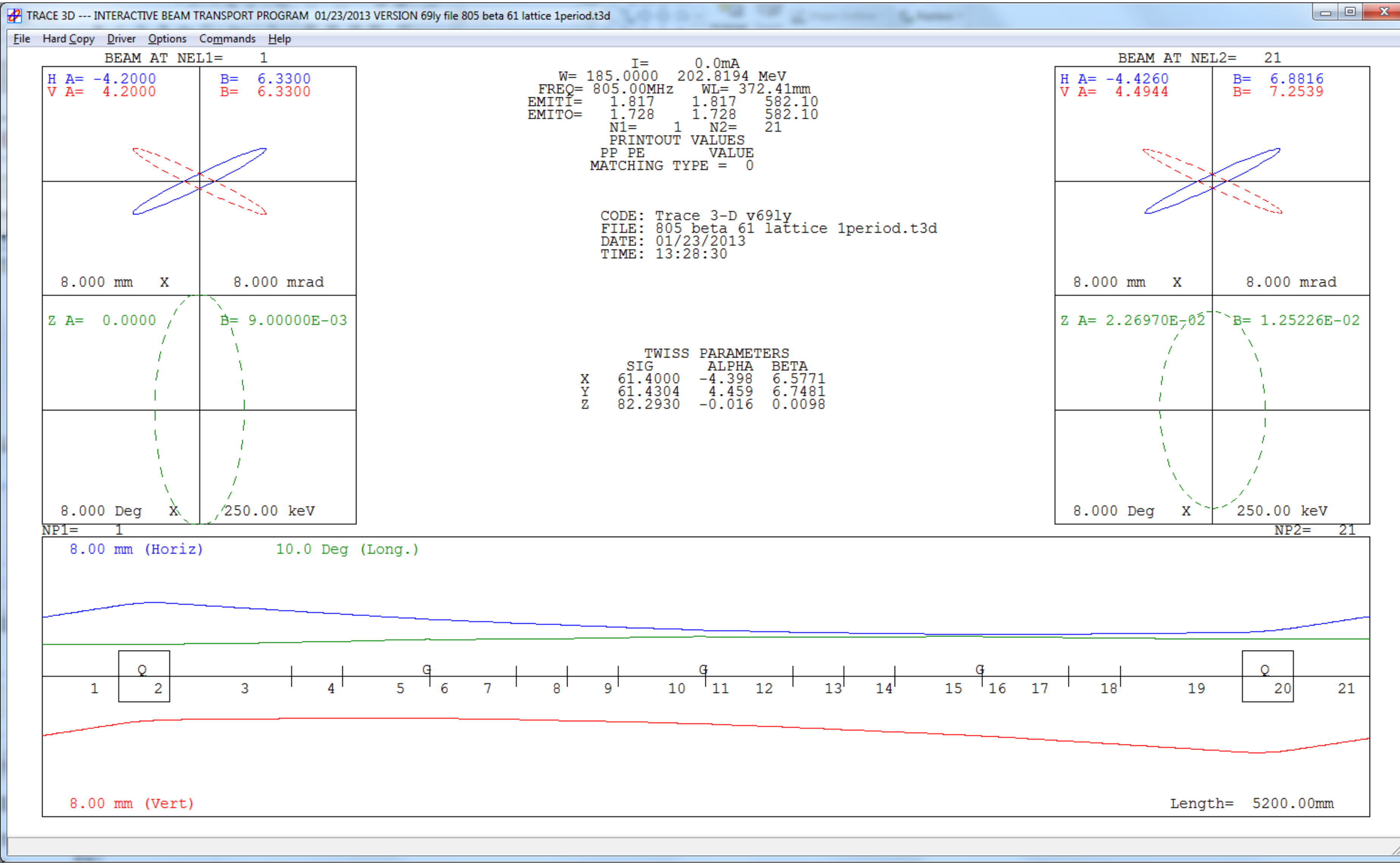
805 beta 61 lattice 1period.t3d - Notepad

File Edit Format View Help

```
&data
er= 938.2723 q= 1.,

freq=   805, pqext=  2.50, ichrom= 0,
xm= 8.00 xpm= 8.00, dpm=  8.0, dwm=  250.0, ym=  8.0 dpp=  10.0,
smax=   2.0, pqsmax=   2.0,

emiti=1.8175 1.8175 582.10000
beami= -4.2 6.33 4.2 6.33 0.0 0.009

w=185.00 xi=0, nel1=1, nel2=21, np1=1, np2=21
n1=1, n2=21
 nt( 1)= 1, a(1, 1)=  300
 nt( 2)= 3, a(1, 2)=  7.9,200
 nt( 3)= 1, a(1, 3)=  477
 nt( 4)= 1, a(1, 4)=  200
 nt( 5)= 1, a(1, 5)=  341.0
 nt( 6)= 10, a(1, 6)=  6.321,-20,0,1
 nt( 7)= 1, a(1, 7)=  341.0
 nt( 8)= 1, a(1, 8)=  200
 nt( 9)= 1, a(1, 9)=  200
 nt(10)= 1, a(1, 10)=  341.0
 nt(11)= 10, a(1, 11)=  6.321,-20,0,1
 nt(12)= 1, a(1, 12)=  341.0
 nt(13)= 1, a(1, 13)=  200
 nt(14)= 1, a(1, 14)=  200
 nt(15)= 1, a(1, 15)=  341.0
 nt(16)= 10, a(1, 16)=  6.321,-20,0,1
 nt(17)= 1, a(1, 17)=  341.0
 nt(18)= 1, a(1, 18)=  200
 nt(19)= 1, a(1, 19)=  477
 nt(20)= 3, a(1, 20)=  -7.9,200
 nt(21)= 1, a(1, 21)=  300
&end
```

SCL lattice – example (10)

- Trace 3D output

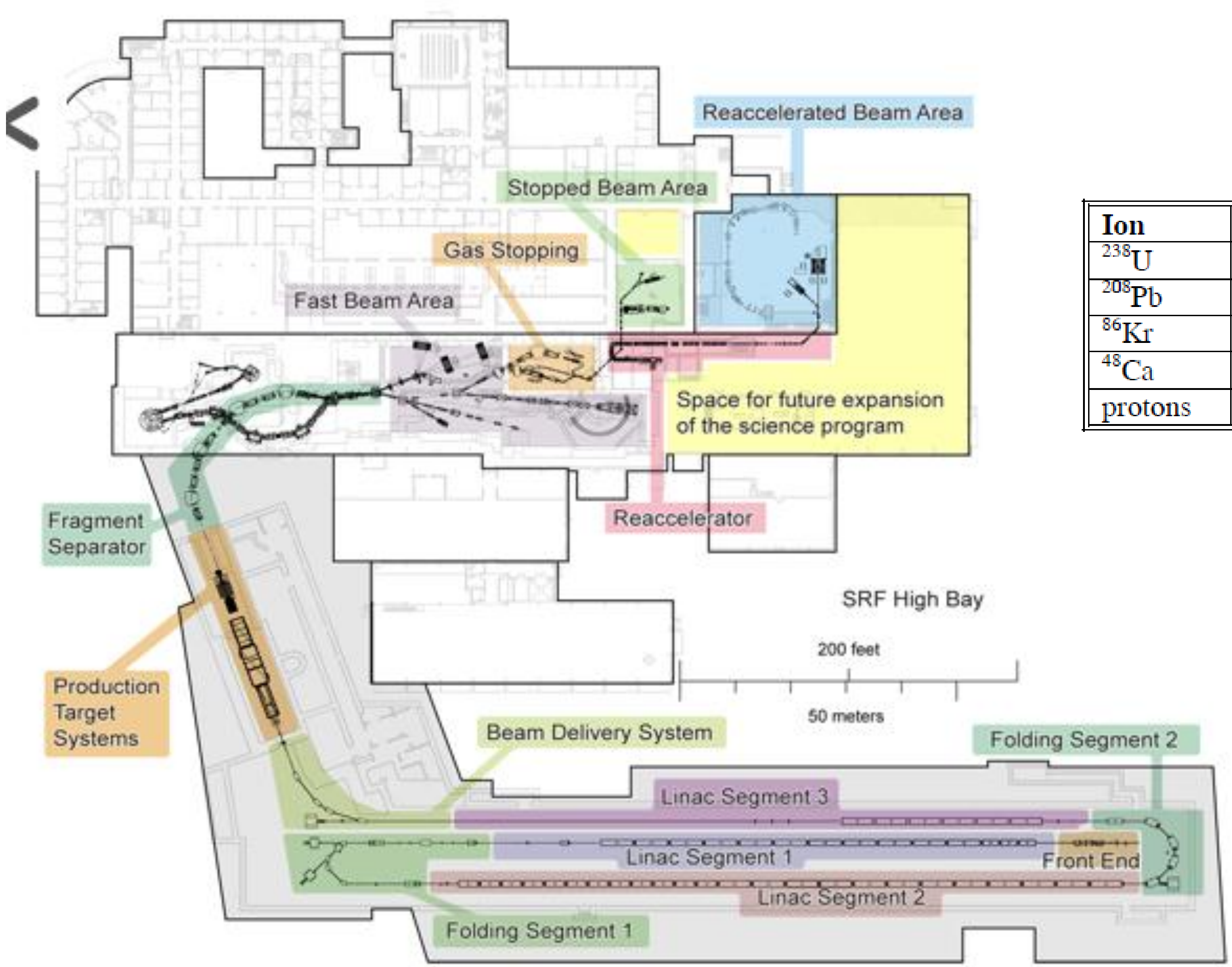


Homework 6-3

- Design a period for the second type of cavity β0.81 for energy of 410 MeV/u

SCL lattice – example2 (0)

- Let's design an "FRIB-like" linac segment



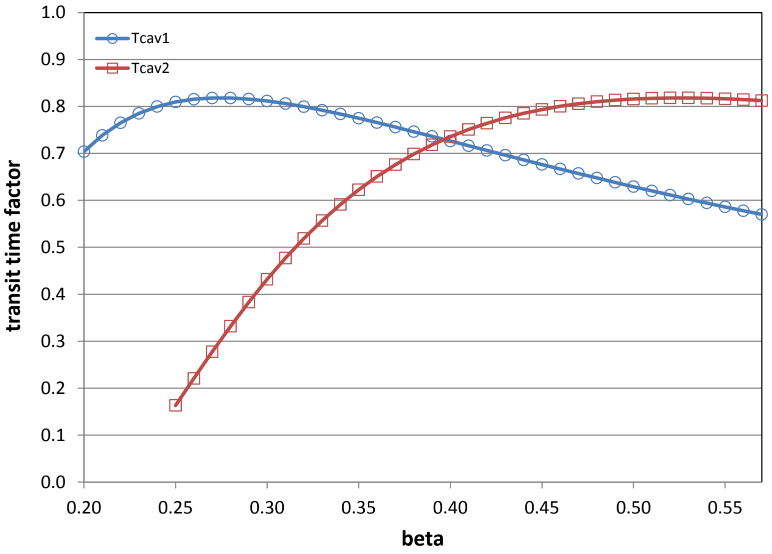
Ion	Energy (MeV/u)
^{238}U	210
^{208}Pb	210
^{86}Kr	265
^{48}Ca	270
protons	610

NSCL FRIB
U.S. Department of Energy Office of Science
National Science Foundation
Michigan State University

SCL lattice – example2 (1)

- Let's design an "FRIB-like" linac segment with the requirements
  - A/Q=3, 20MeV/u to 200 MeV/u, 322 MHz using two cavity types

- 1) Define cavity types
  - Start by finding beta range (kinematics spreadsheet)
    - KE=20 MeV/u gives beta=0.203
    - KE=200 MeV/u gives beta=0.566
  - Investigate suitable types of cavities over that range of betas (design betas spreadsheet) using f=322 MHz.



2-cell β=0.23 and 2-cell β=0.44
Transition around b=0.37 = 72 MeV/u

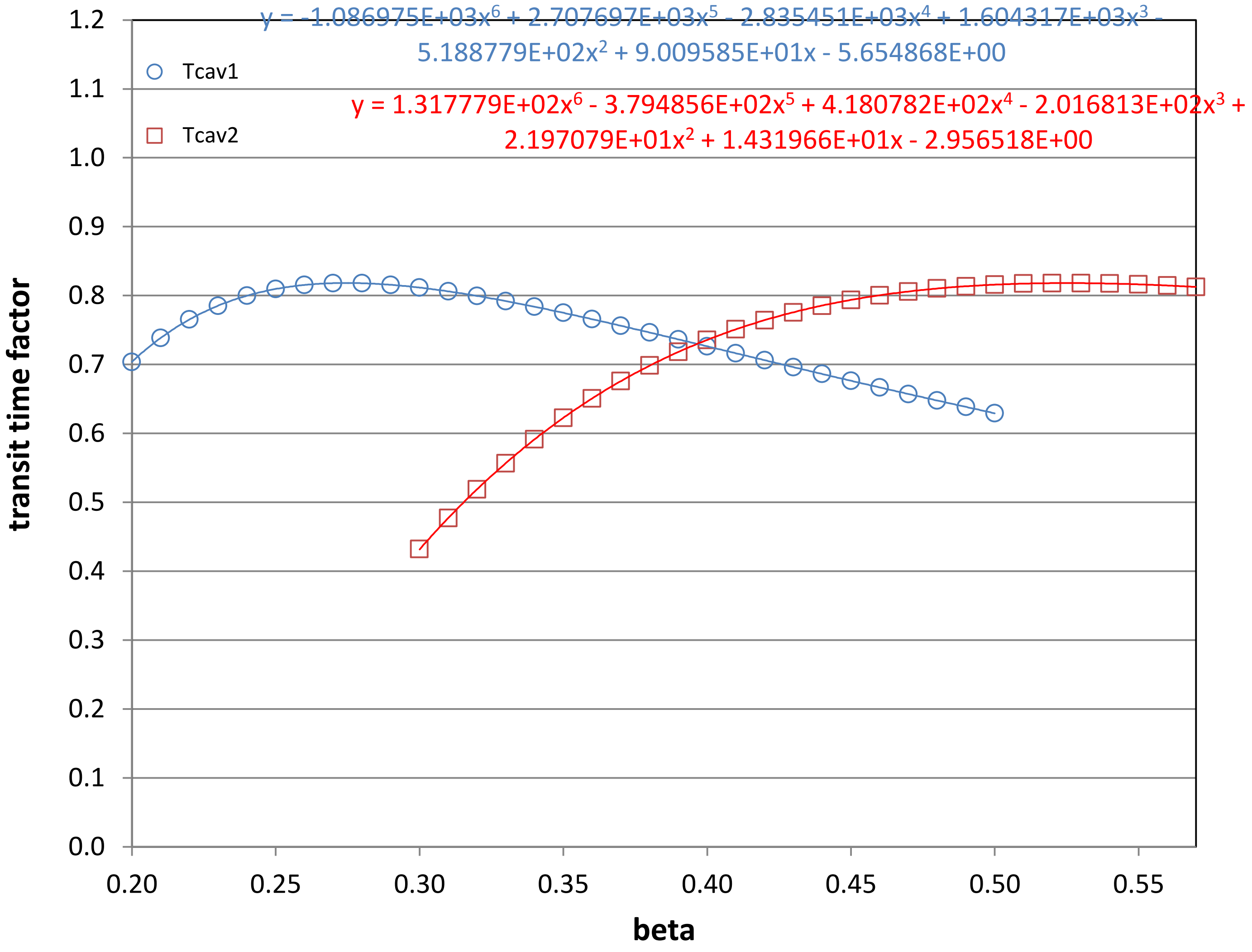
SCL lattice – example2 (2)

- Carrying forward with the 2-cell option, β=0.23 and β=0.44

- 2) Design cavities
  - Design and optimize cavity geometries. In this regime, the cavity type (half-wave, spoke) is not cylindrically symmetric.
    - We assume that one has
      - E0=10 MV/m for β=0.23 cavity
      - E0=10 MV/m for β =0.44 cavity
    - Use transit time factor table T(β) and do polynomial fit for convenience

- 3) Define number of cavities for each cavity type
  - Start at 20 MeV/u, finish at 200 MeV/u
  - Transition should be around 72 MeV (see previous slide)
  - Use linac spreadsheet to optimize number of cavities
  - Possible choice
    - 96 β0.23 cavities at ϕ=-20 deg
    - 128 β0.44cavities at ϕ=-20 deg

SCL lattice – example2 (3)



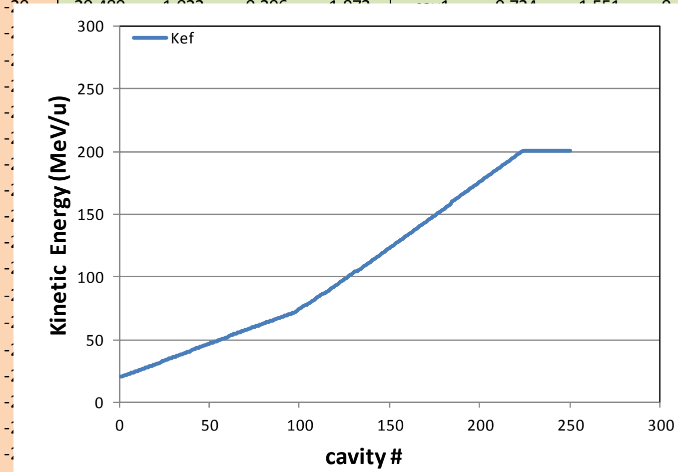
SCL lattice – example2 (4)

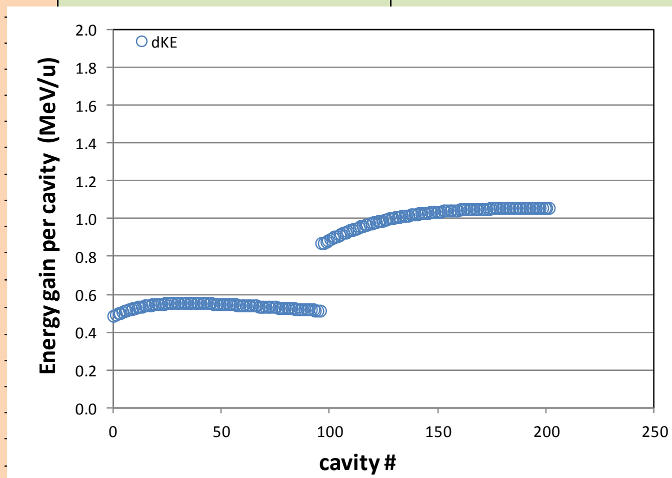
Beam	initial	
Mu	938.2723	MeV/c2
A	3	
Q	1	
beta	0.203	
gamma	1.021	
pc	194.759	MeV/u
E	958.272	MeV/u
KE	20.000	MeV/u
pc tot	584.3	MeV
Etot	2874.8	MeV
KE tot	60.0	MeV
Brho	1.949	Tm

Cavities	cav1	cav2	
Ncav	96	128	
f	322.000	322.000	MHz
bcell	0.230	0.440	
Ncell	2	2	
E0 cav	10	10	MV/m
T0	-5.6549E+00	-2.9565E+00	
T1	9.0096E+01	1.4320E+01	
T2	-5.1888E+02	2.1971E+01	
T3	1.6043E+03	-2.0168E+02	
T4	-2.8355E+03	4.1808E+02	
T5	2.7077E+03	-3.7949E+02	
T6	-1.0870E+03	1.3178E+02	
lambda	0.931	0.931	m
L cell	0.107	0.205	m
Lcav	0.2141	0.4097	m
Ep axis	15.7	15.7	MV/m
	0.1071		

Linac		
N cav1	96	
Ncav2	128	
N total	224	
KEf cav1	71.0	MeV/u
Final KE	200.7	MeV/u

Cav #	Amp	phi(deg)	Kei	gi	bi	Brhoi	cav type	TTF cav	E0TL cav	dKE	Kef	gf	bf	Brhof
1	1	-20	20.000	1.021	0.203	1.949	cav1	0.716	1.533	0.480	20.480	1.022	0.206	1.972
2	1										20.966	1.022	0.208	1.996
3	1										21.457	1.023	0.210	2.019
4	1										21.953	1.023	0.213	2.043
5	1										22.454	1.024	0.215	2.066
6	1										22.958	1.024	0.217	2.090
7	1										23.467	1.025	0.220	2.113
8	1										23.980	1.026	0.222	2.136
9	1										24.496	1.026	0.224	2.159
10	1										25.015	1.027	0.226	2.183
11	1										25.538	1.027	0.229	2.206
12	1										26.063	1.028	0.231	2.228
13	1										26.591	1.028	0.233	2.251
14	1										27.121	1.029	0.235	2.274
15	1										27.654	1.029	0.238	2.296
16	1										28.188	1.030	0.240	2.319
17	1										28.724	1.031	0.242	2.341
18	1										29.262	1.031	0.244	2.363
19	1										29.802	1.032	0.246	2.385
20	1	-20	29.802	1.032	0.246	2.385	cav1	0.806	1.727	0.541	30.343	1.032	0.248	2.407
21	1										30.885	1.033	0.250	2.429
22	1										31.428	1.033	0.253	2.450
23	1										31.972	1.034	0.255	2.472
24	1										32.517	1.035	0.257	2.493
25	1										33.063	1.035	0.259	2.514
26	1										33.610	1.036	0.261	2.536
27	1										34.157	1.036	0.263	2.556
28	1										34.704	1.037	0.265	2.577
29	1										35.252	1.038	0.267	2.598
30	1										35.801	1.038	0.269	2.618
31	1										36.349	1.039	0.271	2.639
32	1										36.898	1.039	0.272	2.659
33	1										37.447	1.040	0.274	2.679
34	1										37.995	1.040	0.276	2.699
35	1										38.544	1.041	0.278	2.719
36	1										39.093	1.042	0.280	2.738
37	1										39.641	1.042	0.282	2.758
38	1										40.190	1.043	0.284	2.777
39	1	-20	40.190	1.043	0.284	2.777	cav1	0.817	1.750	0.548	40.738	1.043	0.285	2.797





SCL lattice – example2 (5)

- 4) Define accelerating lattice
  - Number of cavities per cryomodule
  - Type of transverse focusing

  - Use “lattice” spreadsheet to investigate possible designs
  - We try to pack
    - 6-cavities per cryomodule for β0.23 cavities
    - 8 cavities per cryomodule for β0.44 cavities
  - We choose superconducting for focusing

  - Find focusing parameters for rf focusing and defocusing (“foc strength”)
    - For easiness, concentrate all the rf kick on each side of the solenoid in one equivalent cavity
    - (multiply E0TL of a single cavity by half the number of cavities in the cryomodule)
  - Adjust lengths, transverse focusing, and rf phase to define an acceptable lattice
    - Phase advances around 75 degrees are a good starting point
    - Avoid parametric resonance by maintaining transverse phase advance above half longitudinal phase advance

SCL lattice – example2 (6)

- The chosen dimensions need to be realistic (e.g. space between cavities, end of cryomodule, size of magnets etc..)

Beam		in	out	
	Mu	938.2723	938.2723	MeV/c2
	A	3	3	
	Q	1	1	
	beta	0.203	0.210	
	gamma	1.021	1.023	
	pc	194.759	201.812	MeV/u
	E	958.272	959.731	MeV/u
	KE	20.000	21.458	MeV/u
	pc tot	584.3	605.4	MeV
	Etot	2874.8	2879.2	MeV
	KE tot	60.0	64.4	MeV
	Brho	1.949	2.020	Tm
rf cavity	f	322		MHz
	E0TL	4.6		MV
	Phi	-18		deg
	lambda	0.931		
	K	0.040		m-1
	krf x	0.184		m-1
	krf z	0.385		m-1
	f rf x	5.44		m
	f rf z	2.60		m

lattice	drift 1	1.221	1.171	m
Solenoid	rf cav	0	m	
	drift 2	1.021	0.979	m
	sol	0	m-1	
	drift2	1.021	m	
	rf cav	0	m	
	drift 1	1.221	m	
	k sol	0.550	m-1	
	k rf x	0.184	m-1	
	k rf z	0.385	m-1	
Mxx	0.294	2.693	detx	
	-0.339	0.294	1	
Myy	0.294	2.693	dety	
	-0.339	0.294	1	
Mzz	-0.315	1.878	detz	
	-0.480	-0.315	1	
TWISS	xx	yy	zz	
alpha	0.00	0.00	0.00	
beta	2.82	2.82	1.98	m
mu	72.88	72.88	108.35	deg
			zz	
alpha			0.00	
beta			0.0951	deg/keV
mu			108.35	deg

SCL lattice – example2 (7)

- Verify that the focusing strength for the chosen magnets are reasonable

Beam			Tm
	Mu	938.2723	MeV/c2
	A	3	
	Q	1	
	beta	0.210	
	gamma	1.023	
	pc	202.010	MeV/u
	E	959.772	MeV/u
	KE	21.500	MeV/u
	pc tot	606.0	MeV
	Etot	2879.3	MeV
	KE tot	64.5	MeV
	Brho	2.021	Tm
Solenoid	Length	0.4	m
	B	4.85	T
	K	1.20	m-1
	k sol	0.55	m-1
	f quad	1.81	m

- Build Trace 3-D model to check overall design

SCL lattice – example2 (7)

- Trace 3D input

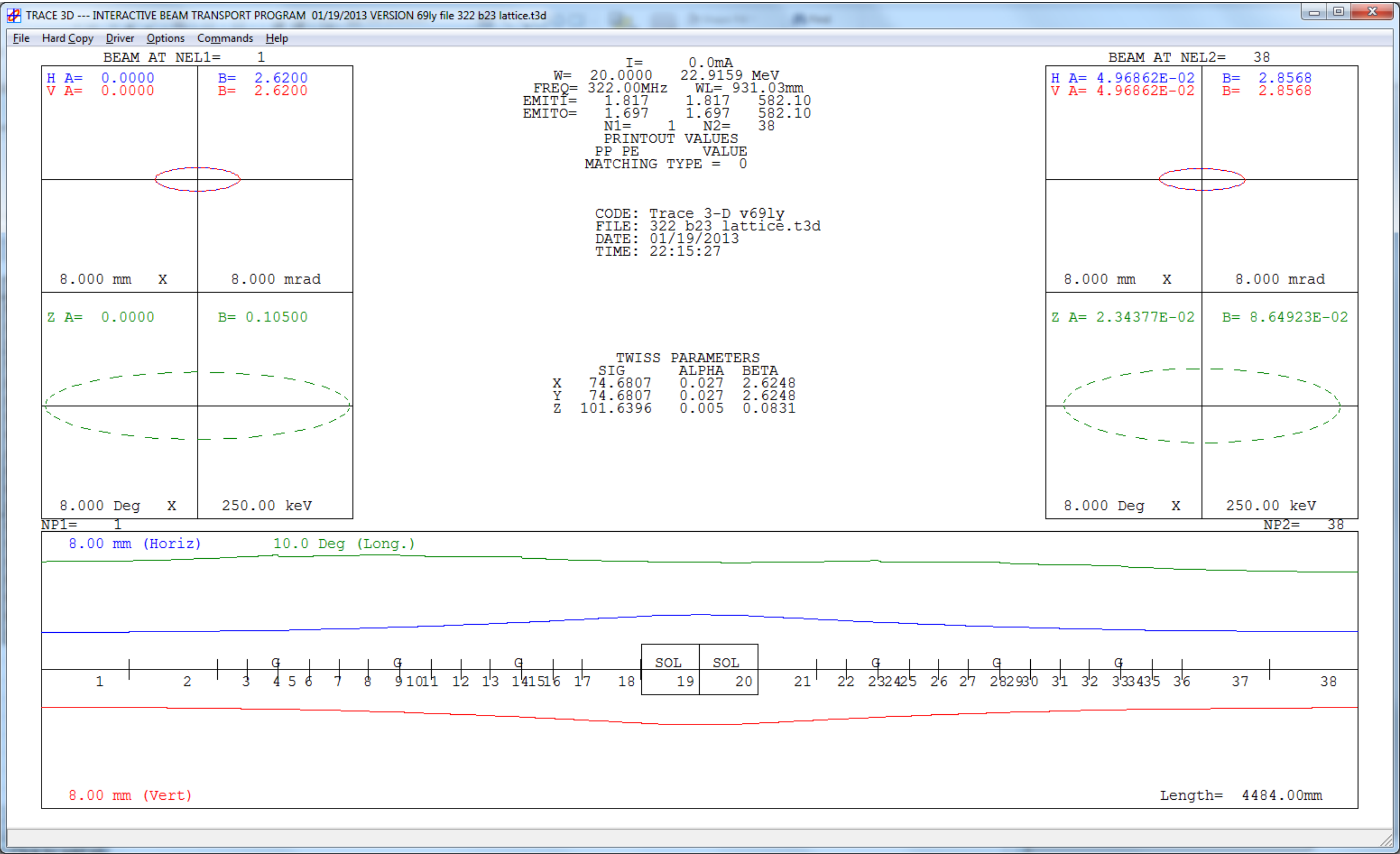
322 b23 lattice.t3d - Notepad

File Edit Format View Help

```
&data
er= 938.2723 q= -0.33333,

freq=   322, pqext=  2.50, ichrom= 0,
xm= 8.00 xpm= 8.00, dpm=  8.0, dwm=  250.0, ym=  8.0 dpp=  10.0,
smax=   2.0, pqsmax=   2.0,

emiti=1.8175 1.8175 582.10000
beami= 0.0 2.62 0.0 2.62 0.0 0.105
beamo= 1.047373 1.088715 1.438201 0.500933 -1.544348 1.141240

w=20.00 xi=0, nel1=1, nel2=38, np1=1, np2=38
n1=1, n2=38
 nt( 1)= 1, a(1, 1)=  300
 nt( 2)= 1, a(1, 2)=  300
 nt( 3)= 1, a(1, 3)=  100
 nt( 4)= 1, a(1, 4)=  107.0
 nt( 5)= 10, a(1,5)=  1.533,-18,0,1
 nt( 6)= 1, a(1, 6)=  107.0
 nt( 7)= 1, a(1, 7)=  100
 nt( 8)= 1, a(1, 8)=  100
 nt( 9)= 1, a(1, 9)=  107.0
 nt(10)= 10, a(1,10)=  1.533,-18,0,1
 nt(11)= 1, a(1,11)=  107.0
 nt(12)= 1, a(1,12)=  100
 nt(13)= 1, a(1,13)=  100
 nt(14)= 1, a(1,14)=  107.0
 nt(15)= 10, a(1,15)=  1.533,-18,0,1
 nt(16)= 1, a(1,16)=  107.0
 nt(17)= 1, a(1,17)=  100
 nt(18)= 1, a(1,18)=  200.0
 nt(19)= 5, a(1,19)=  48500,200.0
 nt(20)= 5, a(1,20)=  -48500,200.0
 nt(21)= 1, a(1,21)=  200.0
 nt(22)= 1, a(1,22)=  100
 nt(23)= 1, a(1,23)=  107.0
 nt(24)= 10, a(1,24)=  1.533,-18,0,1
 nt(25)= 1, a(1,25)=  107.0
 nt(26)= 1, a(1,26)=  100
 nt(27)= 1, a(1,27)=  100
 nt(28)= 1, a(1,28)=  107.0
 nt(29)= 10, a(1,29)=  1.533,-18,0,1
 nt(30)= 1, a(1,30)=  107.0
 nt(31)= 1, a(1,31)=  100
 nt(32)= 1, a(1,32)=  100
 nt(33)= 1, a(1,33)=  107.0
 nt(34)= 10, a(1,34)=  1.533,-18,0,1
 nt(35)= 1, a(1,35)=  107.0
 nt(36)= 1, a(1,36)=  100
 nt(37)= 1, a(1,37)=  300
 nt(38)= 1, a(1,38)=  300
&end
```

SCL lattice – example2 (10)

- Trace 3D output



SCL lattice – non linear effects in longitudinal phase space

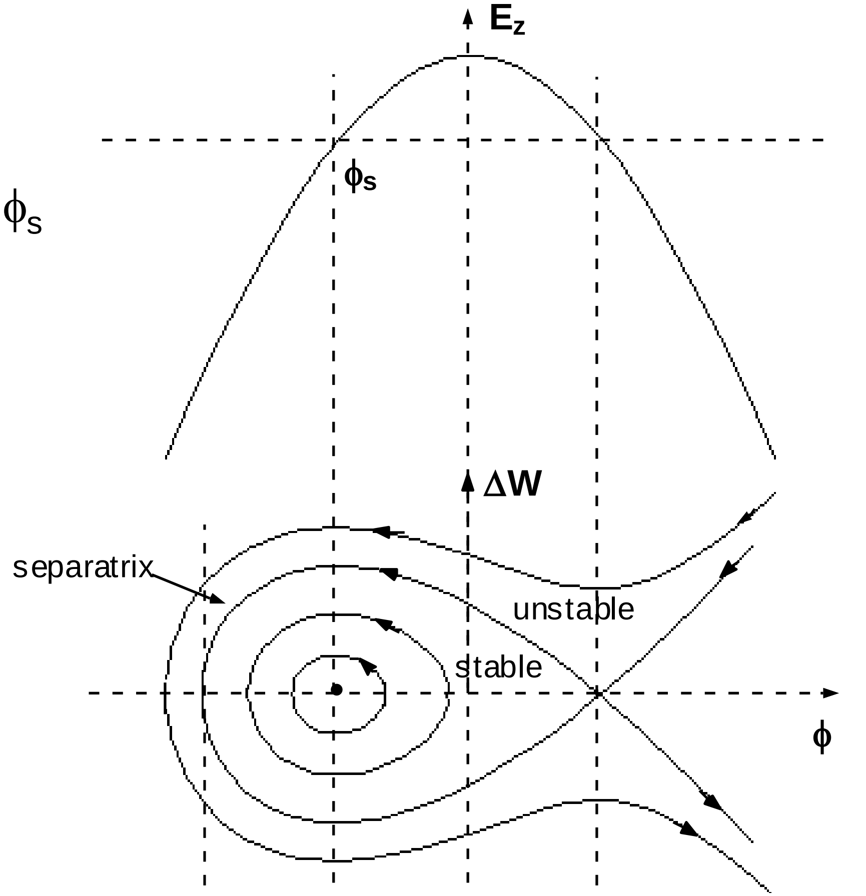
- We assumed that the longitudinal restoring force was linear with respect to the phase, which is a good approximation if the beam has a small spread in phases
- Taking into account the non-linearities from the energy gain cosine curve, the motion in phase space can be separated in stable and unstable regions

- As a rule of thumb, the linear approximation is justified if the phase extent of the beam is much smaller than ϕ_s along the linac

$$\frac{d(KE-KE_s)}{ds} = qE_0T(\cos\phi - \cos\phi_s)$$

$$\frac{d\phi}{ds} = -\frac{2\pi(KE-KE_s)}{mc^2\gamma_s^3\beta_s^3\lambda}$$

$$\frac{d^2\phi}{ds^2} + \frac{2\pi qE_0T}{mc^2\gamma_s^3\beta_s^3\lambda}(\cos\phi - \cos\phi_s) = 0$$



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