Longitudinal Beam Dynamics, and Real Accelerators with Errors

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Longitudinal Beam Dynamics

- Path length difference due to momentum deviation

$$\frac{\Delta C}{C} \approx \frac{\Delta T}{T} = (\alpha_c - \frac{1}{\gamma^2})\delta = \eta_c \delta$$

- Rate of change of the relative energy (u) over man revolutions (ignoring damping)

$$\frac{du}{dt} = f_0 \frac{qV_0}{E_s}(\sin(\phi+\phi_s) - \sin(\phi_s))$$

- Change of synchrotron phase

$$\frac{d\phi}{dt} = \omega_{RF} \frac{\eta_c}{\beta_s^2} u$$

- Equivalent Hamiltonian

$$H(u,\phi,t) = \frac{1}{2}\omega_{RF}\frac{\eta_c}{\beta_s^2}u^2 + \frac{qf_0V_0}{E_s}[\cos(\phi+\phi_s) + \phi\sin\phi_s - \cos\phi_s]$$

kinetic energy

potential energy

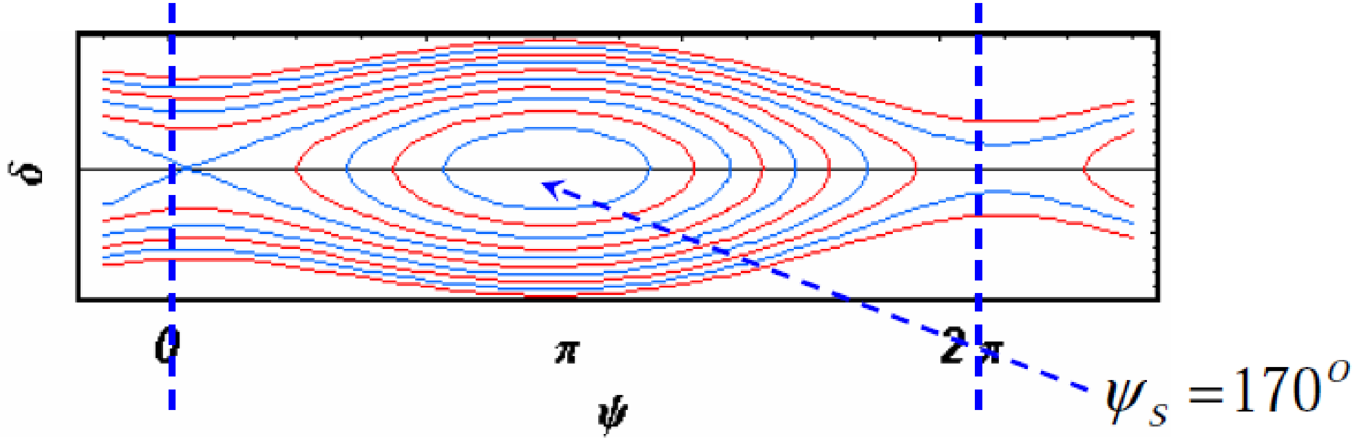


Figure by J. Wu at SLAC

Longitudinal Beam Dynamics

- **Energy acceptance**

$$u_{max}=\sqrt{\frac{2\beta_s^2}{\pi h \eta_c}\frac{qV_0}{E_s}\left[\left(\frac{\pi}{2}-\phi_s\right)\sin\phi_s-\cos(\phi_s)\right]}$$

- **Synchrotron oscillation frequency (small oscillation limit – a simple harmonic oscillator)**

$$f_s=f_0\sqrt{\frac{h\eta_c}{2\pi\beta_s^2}\frac{-qV_0\cos\phi_s}{E_s}} \qquad \nu_s=\sqrt{\frac{h\eta_c}{2\pi\beta_s^2}\frac{-qV_0\cos\phi_s}{E_s}}$$

An estimate: $\eta_c \sim 10^{-3}$ $h\sim 10^2$ $V_0 \sim 1\,MV$ $\phi_s \sim \pi$ $E_s \sim 1\,GeV$

$$\Rightarrow \quad \nu_s = f_s/f_0 : 10^{-3} \sim 10^{-2}$$

- **The longitudinal oscillation is slow:** $f_0 \sim MHz \Rightarrow f_s \sim kHz$

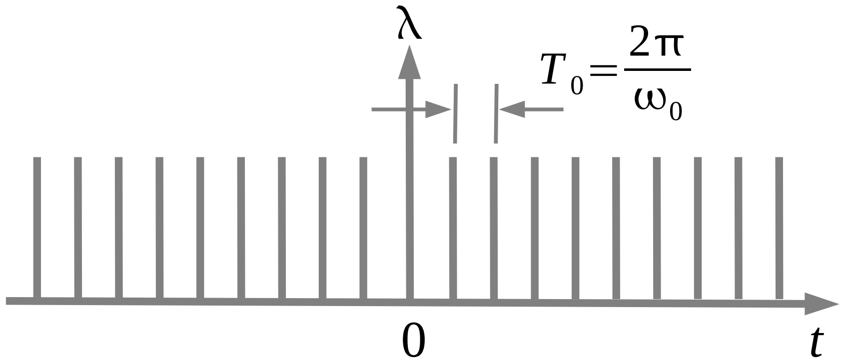
- **Synchrotron oscillations are typically seen as sidebands of revolution signals. Due to synchro-betatron coupling, synchrotron oscillations can also show up as sidebands around betatron oscillation peaks.**

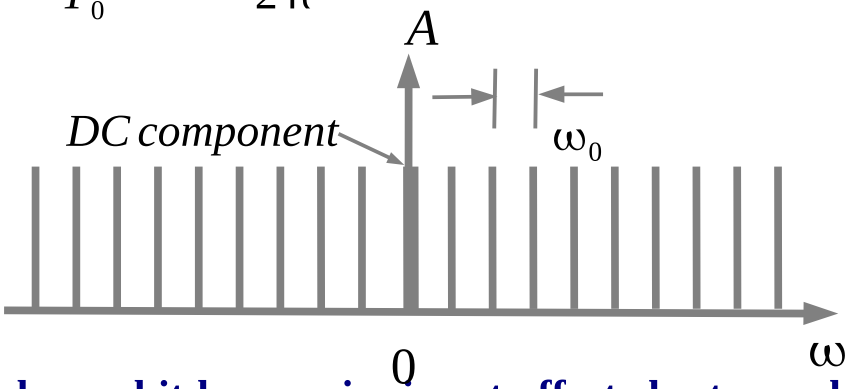
Beam Spectrum: Single Particle on Central Orbit

- For a charge circulating in a storage ring, the line charge density at a detector

$$\lambda = \frac{e}{\beta_0 c} \sum_{l=-\infty}^{\infty} \delta(t - l\,T_0) = \frac{e}{2\pi\,R_0} \sum_{n=-\infty}^{\infty} \cos(n\,\omega_0 t)$$

$$\beta_0 = \frac{v_0}{c},\quad T_0 = revolution\,time\,,\quad \omega = \frac{2\pi}{T_0},\quad R_0 = \frac{C}{2\pi}$$

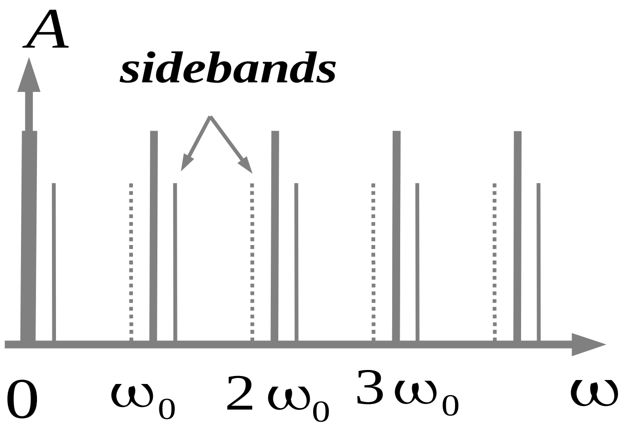




  - For a rigid bunch of beam, the spectrum at low orbit harmonics is not affected; at very high frequencies, the spectrum tapers off with a characteristic “cut-off” frequency at order of w_c = 1/s, where s is the bunch duration

- Circulating beam with betatron and/or synchrotron oscillations
  - Sideband frequencies:

$$\omega_n \;=\; (n \pm (\,fractional\,tune))\,\omega_0$$



- Finite line-width is observed for a multi-particle beam

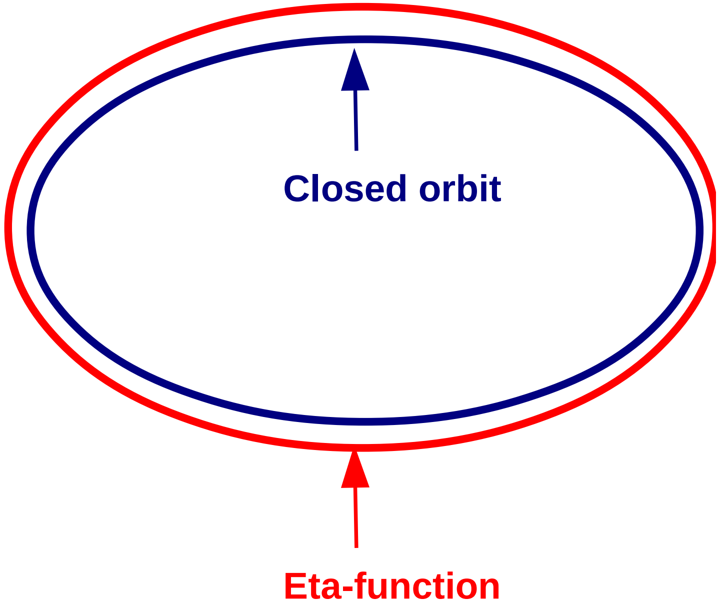
R. Littauer, “Beam Instrumentation”, AIP Conf. Porc. 105, 869 (1982).

Periodic Dispersion Functions: Eta-Functions

- **Consider the linear motion for off-momentum particle**

$$x'' + K_x(s)\,x = \frac{1}{\rho}(s)\,\delta$$

where $K_x(s+C) = K_x(s),\qquad \rho(s+C) = \rho(s)$



The general solution is

$$x(s) = x_\beta(s) \;+\; \eta(s)\,\delta$$

Betatron motion **Eta function**

Eta-functions can be considered as the closed orbit for the off-momentum particle

$$\eta(s+C) = \eta(s),\qquad \eta'(s+C) = \eta'(s)$$

Q: What is the impact of eta-function on the measured beam size using synchrotron radiation?

Periodic Dispersion Functions: Eta-Functions

Consider a one-turn matrix at the location s

$$M_{one-turn} = \begin{pmatrix} C_x & S_x & d_x \\ C'_x & S'_x & d'_x \\ 0 & 0 & 1 \end{pmatrix}$$

The eta functions at this location can be found by solving

$$\begin{pmatrix} \eta_x \\ \eta'_x \\ 1 \end{pmatrix} = \begin{pmatrix} C_x & S_x & d_x \\ C'_x & S'_x & d'_x \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \eta_x \\ \eta'_x \\ 1 \end{pmatrix}$$

Solution

$$\eta(s) = \frac{\sqrt{\beta(s)}}{2\sin\frac{\mu}{2}} \int_s^{s+C} \frac{\sqrt{\beta(\tau)}}{\rho(\tau)} \cos\left(\phi(\tau) - \phi(s) - \frac{\mu}{2}\right) d\tau$$

$$\eta'(s) = \frac{1}{2\sin\frac{\mu}{2}} \int_s^{s+C} \sqrt{\frac{\beta(\tau)}{\beta(s)}} \frac{1}{\rho(\tau)} \left(\sin\left(\phi(\tau) - \phi(s) - \frac{\mu}{2}\right) - \alpha(s)\cos\left(\phi(\tau) - \phi(s) - \frac{\mu}{2}\right)\right) d\tau$$

Real Accelerators with Errors

- **Modern accelerators are very complex systems with a very large number of components**

- **Physics models and numerical simulation codes for accelerators are effective and powerful in designing modern accelerators; however, ideal accelerators do not exist**

- **The performance of the accelerators will be influenced by imperfections of physics models, the errors in manufacturing, assembling, and installing accelerator components, and in stability of accelerator mechanical and electrical systems**

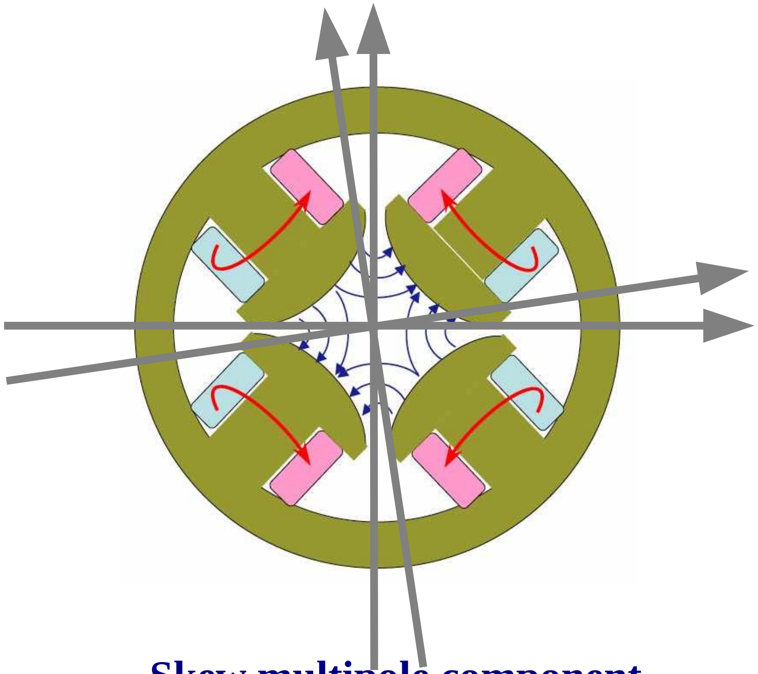
- **Various of errors can be detected and measured by diagnostic systems. Correction of these errors can lead to significant improvement in accelerator performance**

Power Supply Mispowering and Fluctuations

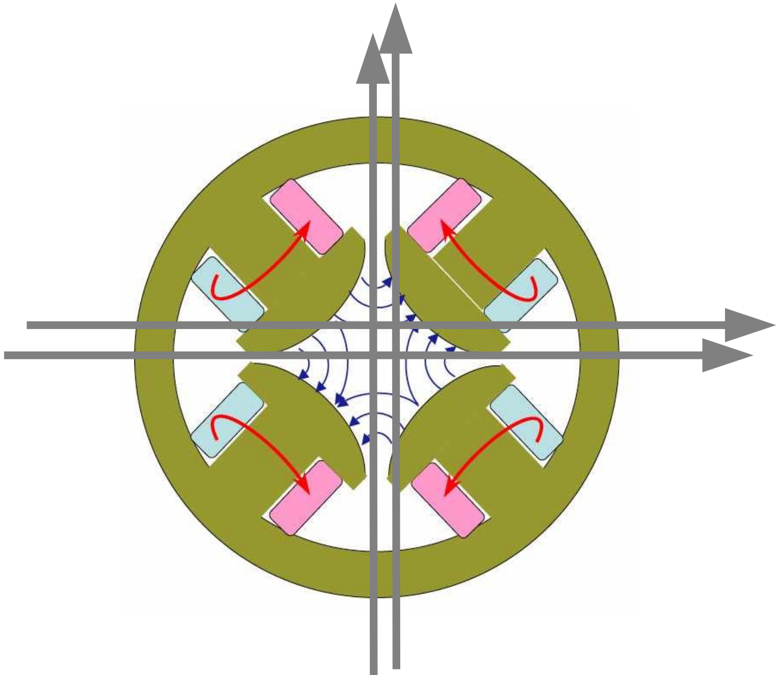
- Magnets are energized by power supplies. Mispowering and fluctuation of power supply current can degrade the performance of an accelerator
  - Mispowering and current fluctuation of dipole magnet power supplies can change the beam energy and closed orbit
  - Mispowering and current fluctuation of quadrupole magnet power supplies can change betatron tune and create beating of beta functions around the storage ring
  - Mis-powering effects are DC effects which can be measured and corrected directly
    - Tune-shifts and beta function beating can be determined using the LOCO method (Linear optics from closed orbits)
    - DC orbit distortion can be compensated using trim dipole correctors
  - Current fluctuation of power supplies is managed by requiring power supplies to have a very good stability
    - Typical power supply stability requirements: 10^{-3} for transport lines and 10^{-4} to 10^{-5} for storage ring main magnets
    - Eddy current effects in the vacuum chambers screen high frequency jitters (>~ 100 Hz)

Magnet Misalignment

- A normal multipole when misaligned with a small tilt angle along the longitudinal direction (z-direction), it generates a "skew" component of the same order with its amplitude proportional to the title angle
- A multipole magnet (2n-pole magnet) when misaligned with a small displacement in the x-y plane, it generates multipole-like field components which can be found in lower order multipoles



Skew multipole component



Multipole feeddown effect

pbpl.physics.ucla.edu/Research/Technologies/Magnets/Electromagnets/Quadrupoles/quad.jpg

Other Perturbations to Accelerator Operation

- **Large accelerators are sensitive to variations of earth magnetic field, motion of the moon, and traffic on neighboring roads and railways, etc.**

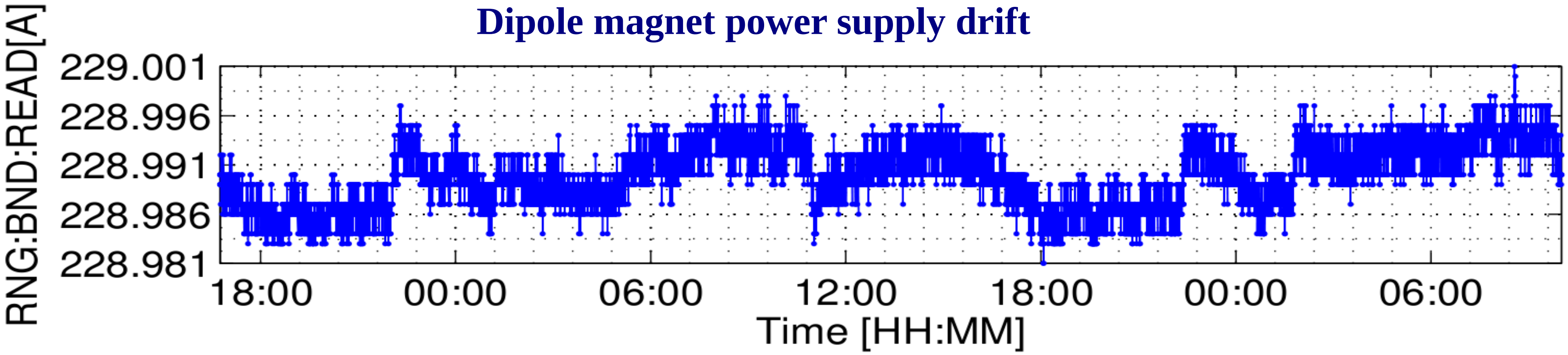
- **All accelerators are sensitive to changes of their environments**
  - **Stray electromagnetic fields from electric equipment**
  - **Fields generated by nearby accelerators**
  - **Temperature variations in cooling water and air**
  - **Voltage fluctuation of the AC line power**
  - **Ground motion and vibration**
  - **Seasonal changes in temperature and ground water level**

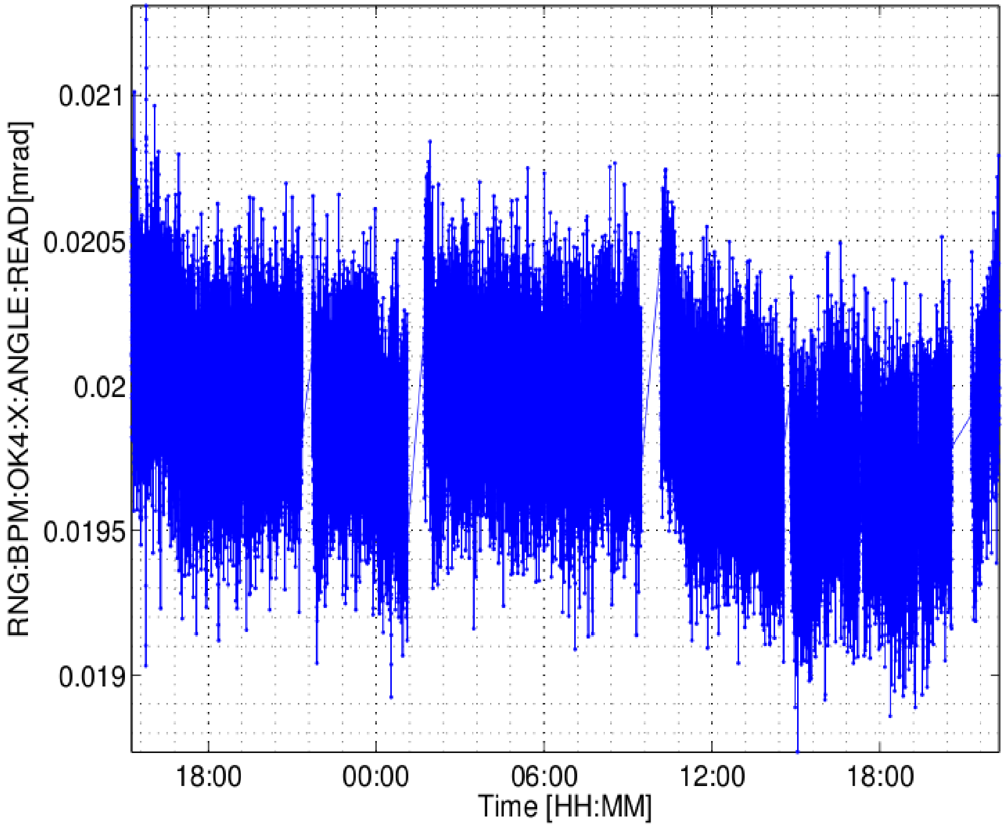
- **Performance of accelerators can also be influenced by the electromagnetic fields generated by insertion devices (wigglers/undulators) and particle detectors used in nuclear and high energy physics research**

- **In order to measure and compensate for undesirable effects in a real accelerator, beam diagnostic systems are deployed along the accelerators**

Duke Storage Ring: Power Supply Stability and Orbit Stability

Dipole magnet power supply drift





Beam angle drift at collision point at High Intensity Gamma-ray Source (HIGS)

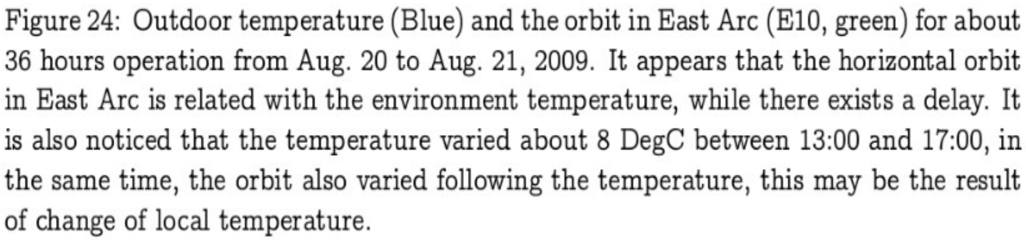
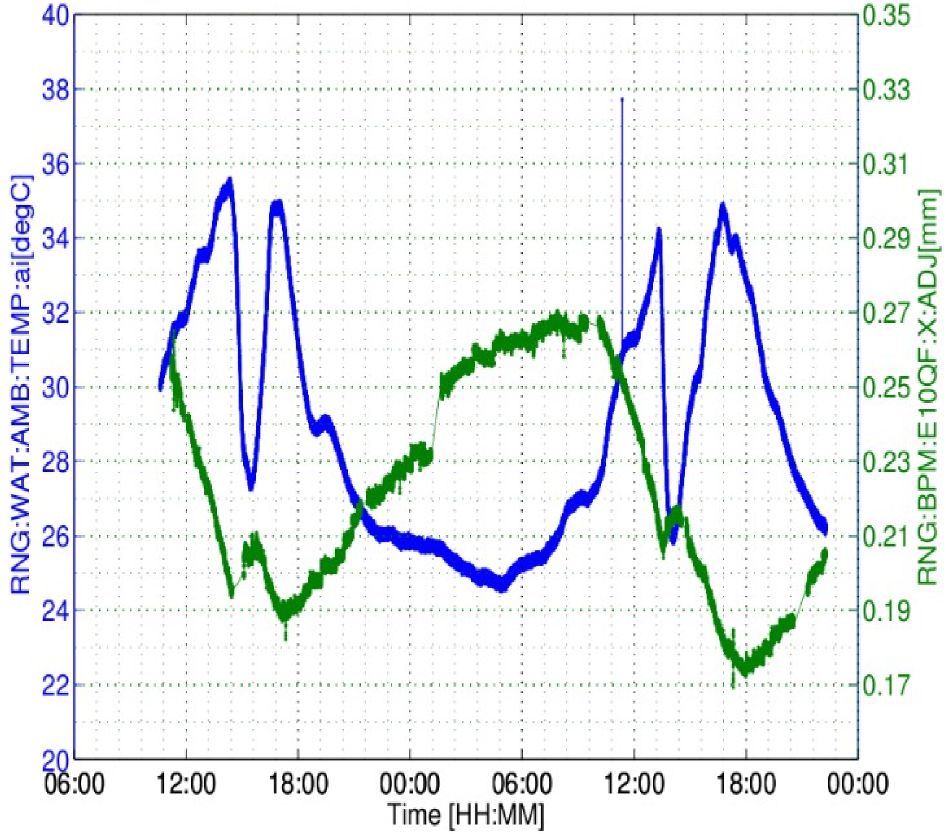


Figure 24: Outdoor temperature (Blue) and the orbit in East Arc (E10, green) for about 36 hours operation from Aug. 20 to Aug. 21, 2009. It appears that the horizontal orbit in East Arc is related with the environment temperature, while there exists a delay. It is also noticed that the temperature varied about 8 DegC between 13:00 and 17:00, in the same time, the orbit also varied following the temperature, this may be the result of change of local temperature.

Slide prepared by H. Hao

Duke Storage Ring: Orbit Stability

Beam Orbit vs. Temperature



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Beam Orbit vs. Booster Injection

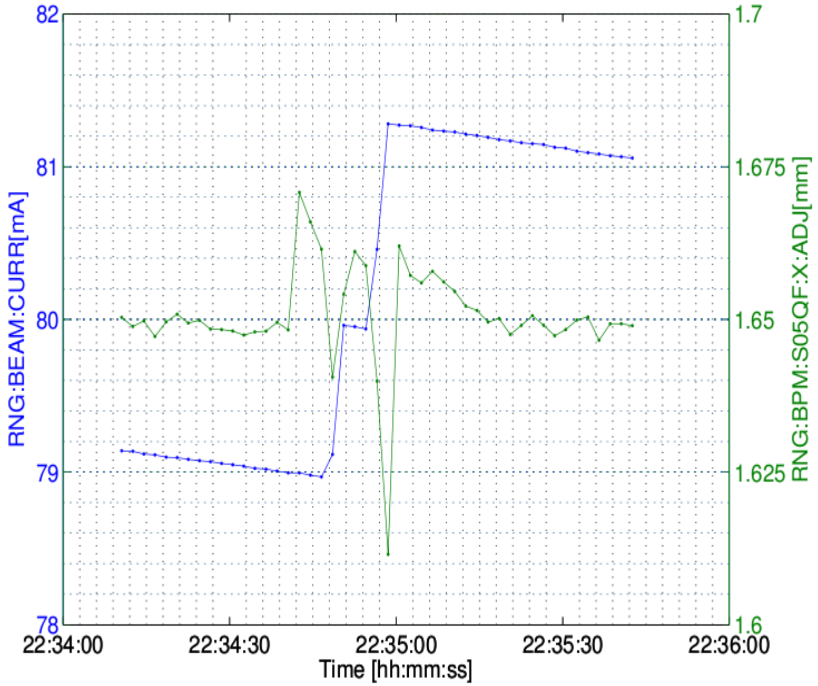


Figure 2: A typical injection process. The left axis is the beam current data, right axis is the storage ring orbit data, time interval between two adjacent points in the figure is 2 seconds. It appears that before the electron beam was injected to the storage ring, the beam orbit was perturbed by an external force which is probably the booster magnetic field for about 6 seconds. After the electron beam injection into the storage ring was stopped, the slow orbit feedback worked and corrected the orbit to desired location, this process toke about 15 to 20 seconds. In the analysis of beam orbit, the injection part is taken out because it does not represent the beam orbit condition of stable operation.

Slide prepared by H. Hao

Dipole Field Error

- A dipole field error, generated by a displacement of a magnet, mis-powering of an individual dipole magnet, or other means, will cause a distortion of the closed orbit

- Consider a dipole error at location s_i with a kick angle q_i, it produces a closed orbit distortion around the storage ring:

$$u(s)=\frac{\sqrt{\beta_u(s)}\sqrt{\beta_u(s_i)}}{2\sin(\pi\nu_u)}\cos\left(|\phi_u(s)-\phi_u(s_i)|-\frac{\mu_u}{2}\right)\theta_i, \qquad u=x,y$$

  - At integer resonance, $\nu_u=integer$ there is no closed orbit
  - Closed orbit distortion is large if the dipole field error occurs at a location with a large beta function
  - Closed orbit distortion at the same location of the single kick

$$u(s_i)=\frac{1}{2}\beta_u(s_i)\frac{\cos(\pi\nu_u)}{\sin(\pi\nu_u)}\theta_i$$

- Closed orbit distortion due to N kicks around the storage ring

$$u(s)=\frac{\sqrt{\beta_u(s)}}{2\sin(\pi\nu_u)}\sum_{i=1}^{N}\sqrt{\beta_u(s_i)}\cos\left(|\phi_u(s)-\phi_u(s_i)|-\frac{\mu_u}{2}\right)\theta_i, \qquad u=x,y$$

- A dipole filed error at a location of non-zero dispersion leads to a change of the circumference of the closed orbit

$$\Delta C=\eta_x(s_i)\theta_i$$

Orbit Correction

- **Beam position monitors (BPMs) are used to measure the orbit distortion**
- **A set of dipole correctors distributed around the storage ring are used to make the orbit correction**
- **Consider a orbit correction scenario with *n* BPMs and *m* orbit correctors; the orbit corrections observed by BPMs are**

$$u_{corr}(s_j)=\frac{\sqrt{\beta_u(s_j)}}{2\sin(\pi\nu_u)}\sum_{i=1}^{m}\sqrt{\beta_u(s_i)}\cos\left(|\phi_u(s_j)-\phi_u(s_i)|-\frac{\mu_u}{2}\right)\theta_i \;=\; \sum_{i=1}^{m}M_{ji}\theta_i\,,$$

$$i=1,2,\cdots,m,\quad j=1,2,\cdots,n,\quad s_i=BPM\ locations\,,\quad s_j=corrector\ locations$$

- **If the orbit distortion before correction measured at BPM locations are** $u_{uncorrected}(s_j)$
- **Orbit correction can be made if enough correctors are available**

$$u_{corr}(s_j)=\sum_{i=1}^{m}M_{ji}\theta_i=-u_{uncorrected}(s_j)\;\Longrightarrow\; M\,\vec{\theta}=-\vec{u}_{uncorrected}\;\Longrightarrow\;\vec{\theta}$$

- **In practice, we try to minimize the residual error after orbit correction while keeping corrector strengths reasonable**

$$\Delta u_{corr}^2\equiv\sum_{j=1}^{n}\left(\sum_{i=1}^{m}M_{ji}\theta_i+u_{uncorrected}(s_j)\right)^2$$

Quadrupole Error and Compensation

- Quadrupole errors (gradient errors) can be generated due to misalignment of higher-order multipoles or by mis-powering of the quadrupole magnets

- Quadrupole errors produce a betatron tune shift

$$\Delta\nu = \frac{1}{4\pi}\beta\,\Delta K_1 L_{quad}$$

$$\Delta K_1 = change\,of\,quadrupole\,strength, \quad L_{quad} = quadrupole\,length$$

- Change to the beta function due to the quad error

$$\frac{\Delta\beta(s)}{\beta(s)} = -\frac{\beta(s_i)}{2\sin(2\pi\nu)}\cos\left(2\left|\phi(s)-\phi(s_i)\right|-\mu\right)\Delta K_1 L_{quad}$$

$$s_i = location\,of\,quad\,error, \quad \beta,\phi = undisturbed\,values$$

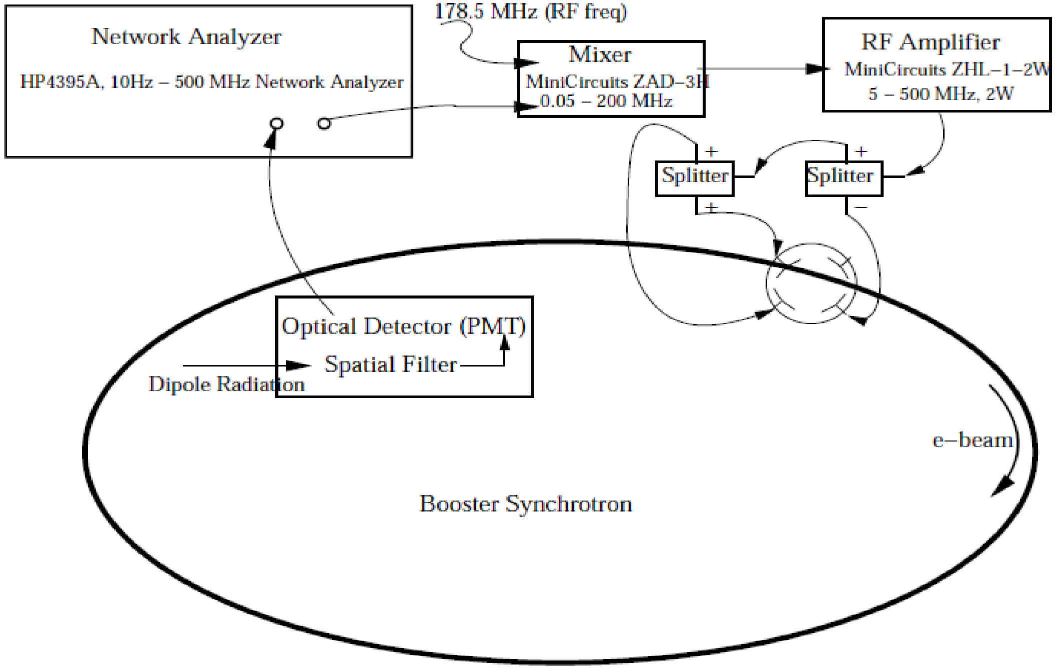
  - At integer and half-integer resonances, betatron motion is NOT stable due to quadrupole errors

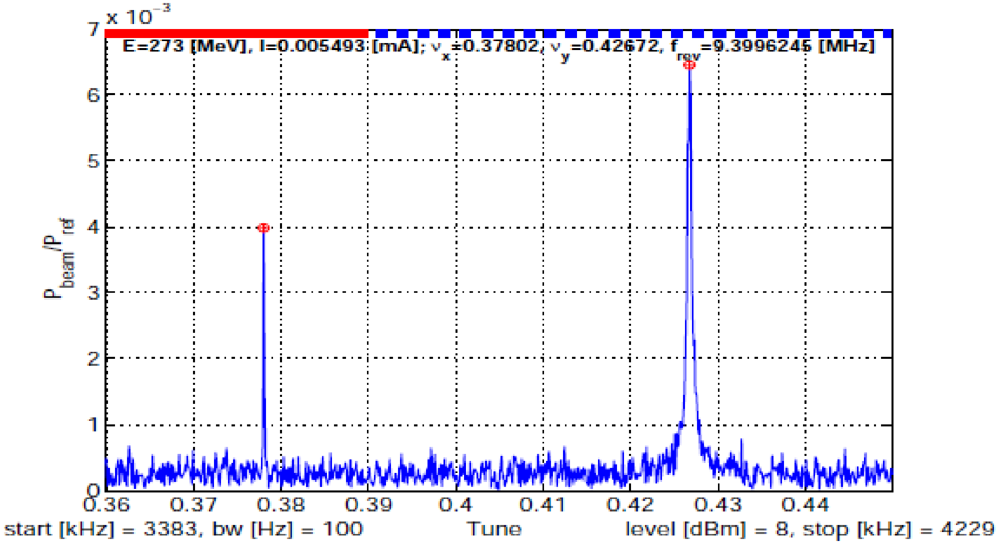
$$\nu = integer, half - integer$$

- Tilted/rotated normal quadrupoles produce skew quadrupole terms, which can cause horizontal and vertical motions to become coupled. Skew quadrupole correctors can be used to compensate/correct the coupling effect.

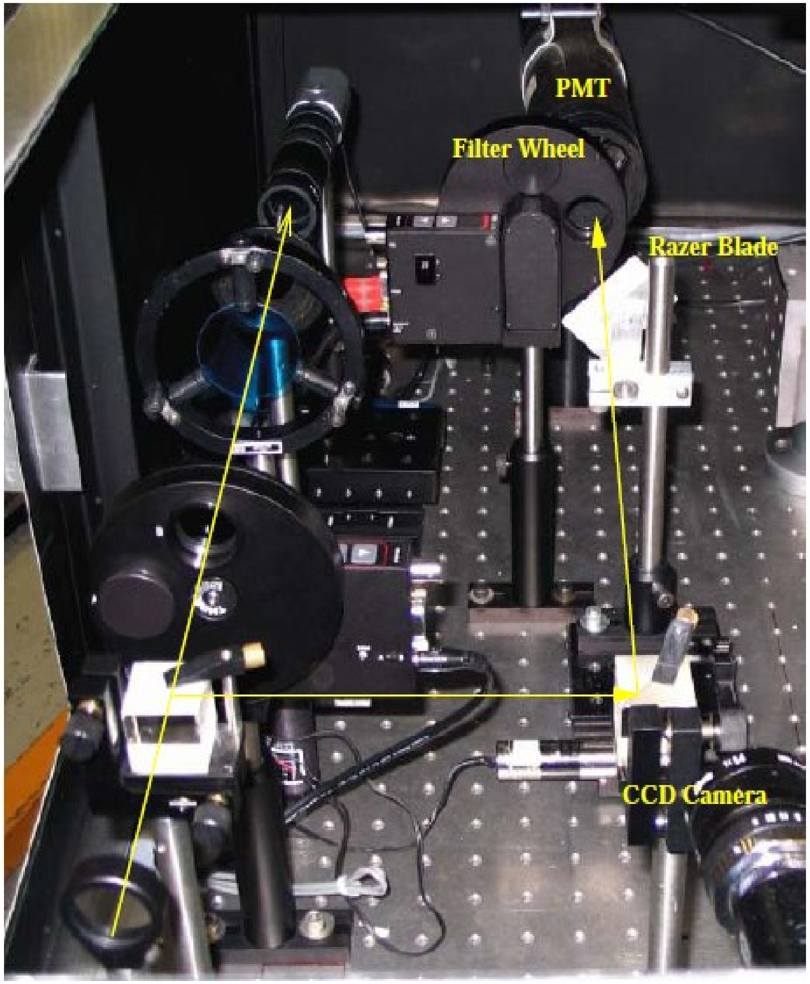
Tune Measurements (Betatron Tunes)

- **Betatron tunes can be measured using a network analyzer which drives/excites the betatron motion and detects the beam response at a pickup**





Tune Measurement System for Duke Booster Synchrotron

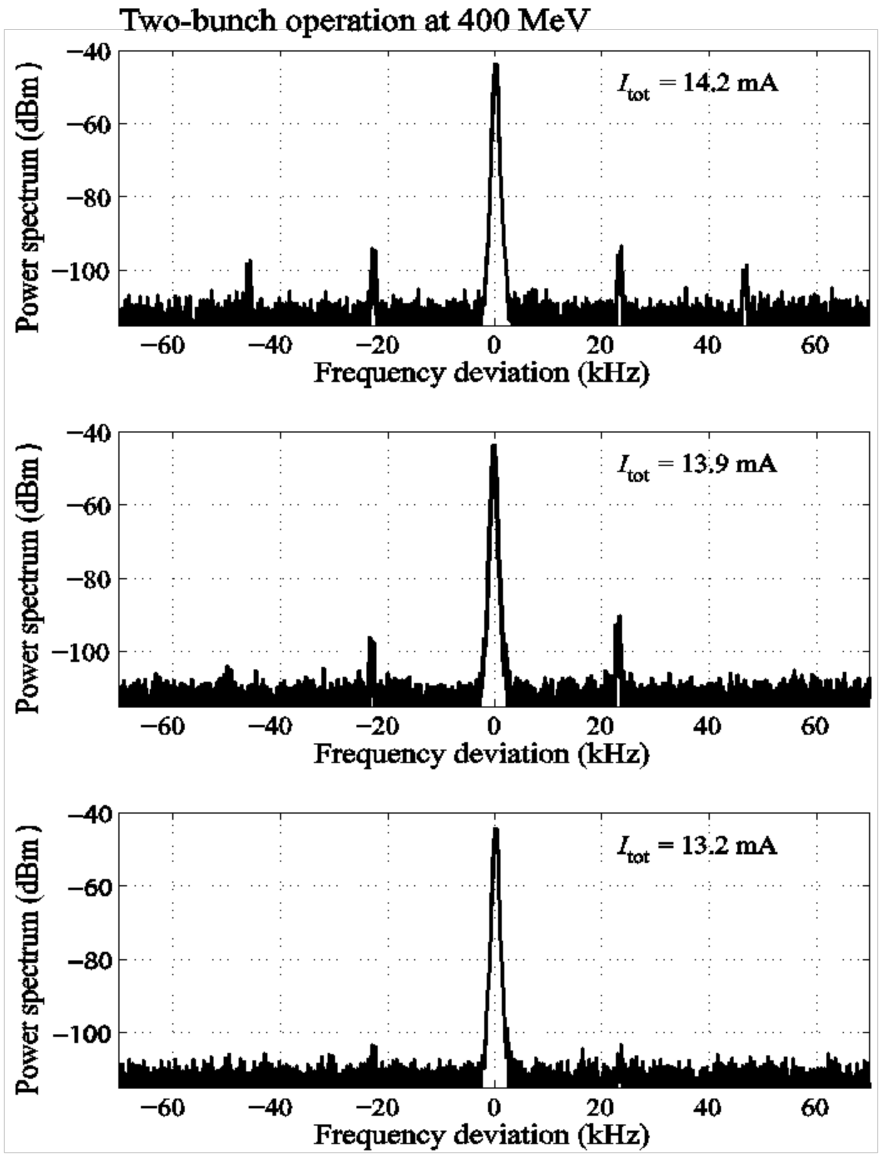


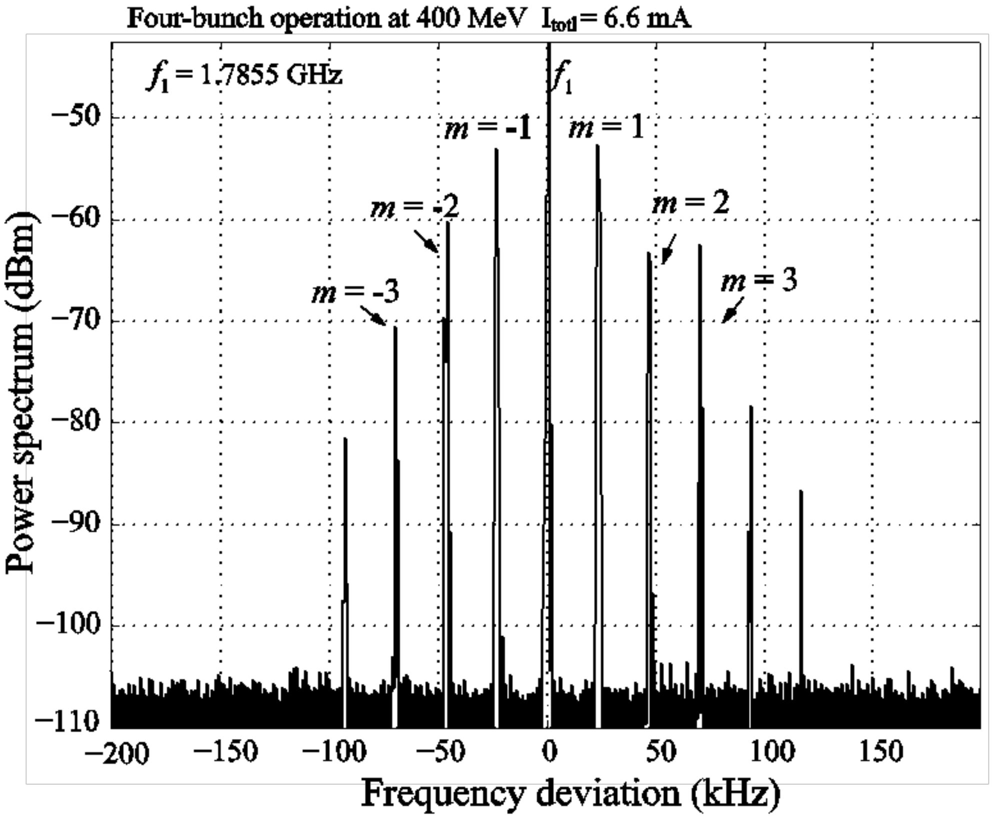
Y.K. Wu *et. al*, PAC07, p.4063 (2007).

Tune Measurements (Synchrotron Tunes)

- **Synchrotron tunes can be measured using a spectrum analyzer connected to a beam pickup device (e.g. the sum signal)**
- **For longitudinally unstable/semi-unstable beam excitation is usually unnecessary**

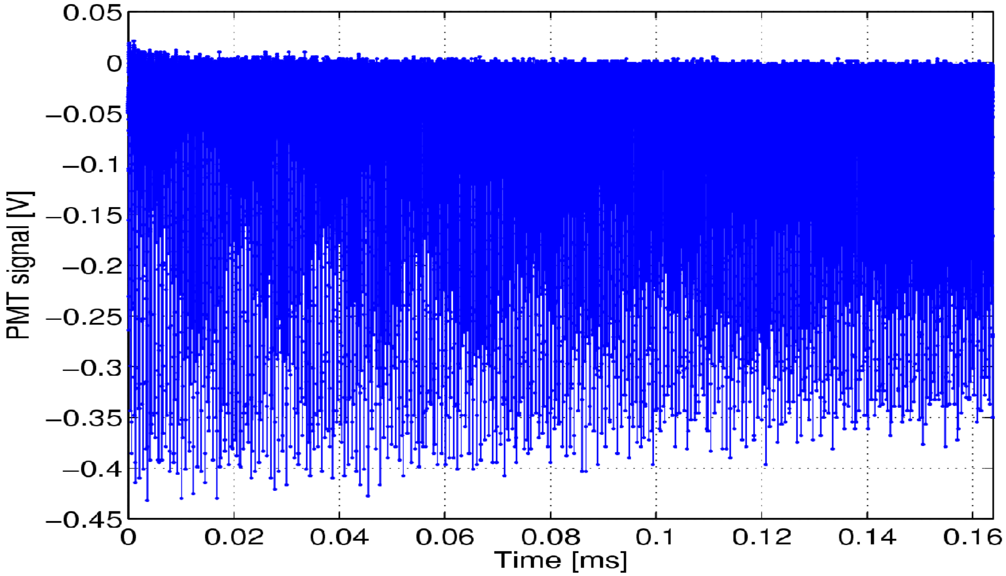
Duke Storage Ring: 2-bunch, 4-bunch Operation

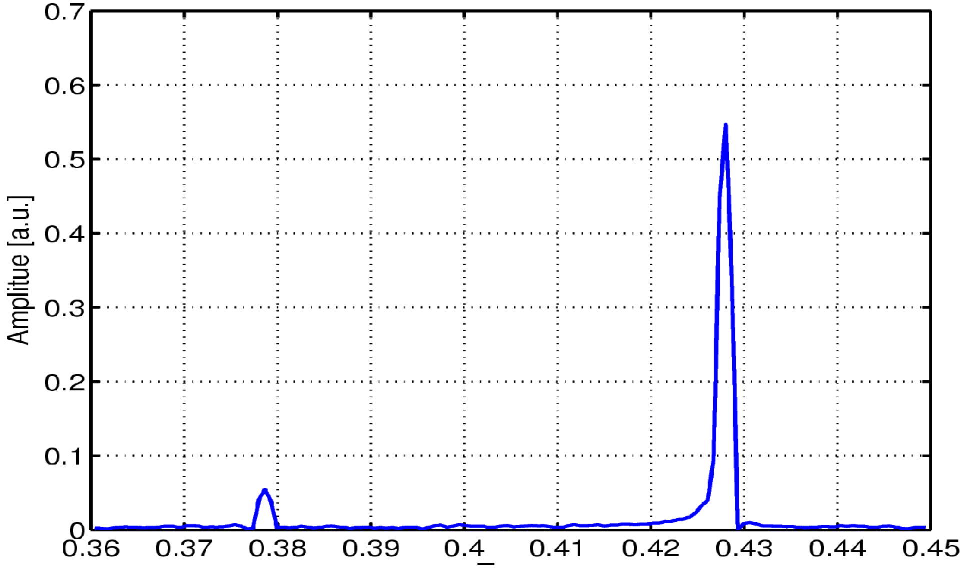


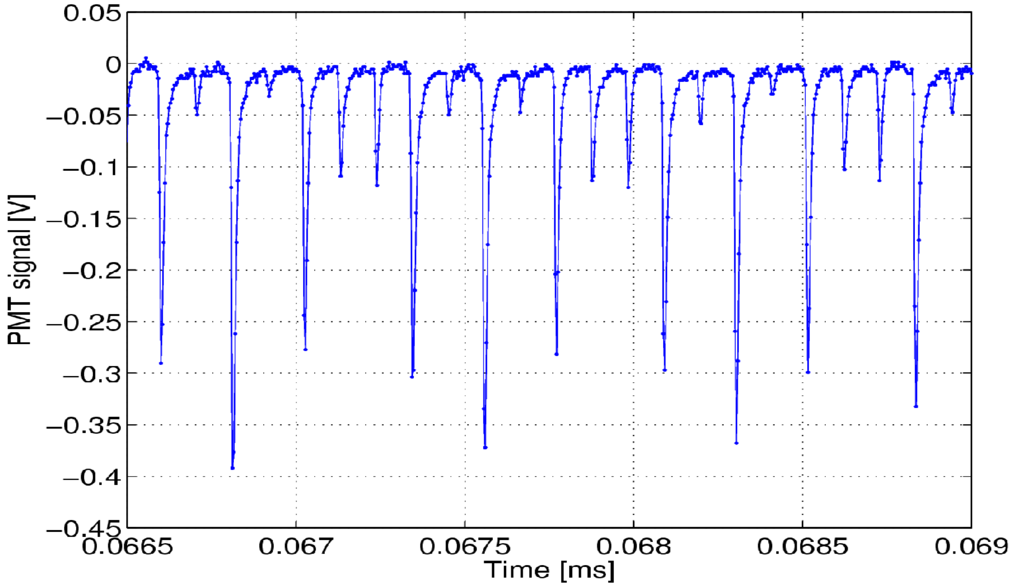


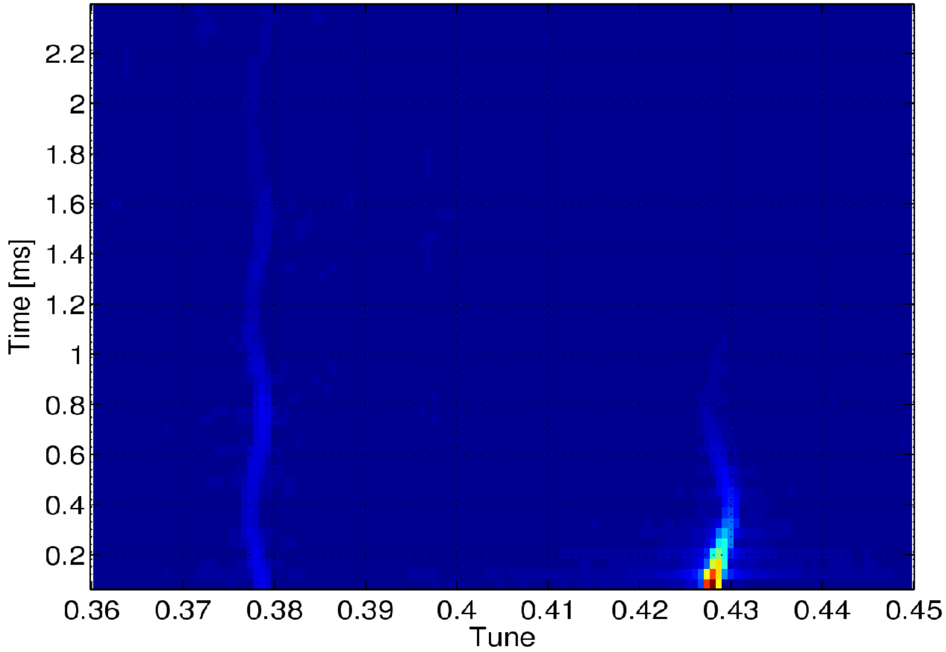
Tune Measurements

- **Betatron tunes can also be measured using turn-by-turn data**









Duke Booster Synchrotron: 273 MeV, Vertical Kicker 1 kV, 250 MHz Sampling, PMT signal

Chromaticity Measurements

- **Chromaticity is determined by measuring betatron tunes as a function beam energy**