Fundamentals of Accelerator Physics and Technology with Simulations and Measurements Lab

U.S. Particle Accelerator School
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Computer Session – Transverse Dynamics

This session, dealing with transverse dynamics, uses three *Mathcad*[1] worksheets. The first demonstrates the development of a beam envelope by tracking a particle repeatedly through a FODO structure. The second demonstrates the development of ellipses in phase space for the same FODO structure. The third demonstrates a local orbit distortion in a FODO lattice and how to correct it. A few hints on using *Mathcad* are found on the last page. Questions to be answered and turned in for grade are typeset in **Bold Type**.

1   Betatron Oscillation Envelope

For transverse motion of a particle (beam) to be stable, the trace of the 2X2 transport matrix for each degree of freedom (horizontal and vertical) must be less than or equal to 2 in absolute value: |TrM| ≤ 2. For a simple FODO structure made up of focusing elements (of focal lengths ±F separated by a distance L) the condition is that $F \geq L/2$.

In the *Mathcad* worksheet **Envelope.mcd** the FODO cell parameters are initially F = 20 and L = 25 meters. The worksheet shows eight half-cells, or four full FODO cells, and a particle with initial conditions $x_{0,0}$ and $x'_{0,0}$ is tracked through the system. The indices on the position $x_{i,j}$ are for the j-th passage through the i-th element of the structure. (to see the actual tracking algorithm you must scroll to the right in the Mathcad window.) The number of turns begins with N_{turn} = 1 and with $x_{0,0}$ = 5. Gradually increase N_{turn} to 2, 5, 10, 100 turns. While the particle starts out with a displacement of 5 mm, it ultimately reaches larger ones.

Q1)   What is the largest displacement the particle attains in the structure?

Q2)   Where in the FODO structure i.e. at the F, D, or O (drift) does this maximum displacement occur?

Return N_{turn} to 1. Adjust the focal length, F, until the parameter μ is 90°. Change the number of half-cells to N_{cell} = 20. Look at the plot of the trajectory.

Q3)   **How many full cells (50 meter units) does it take for the pattern to repeat itself?**

Q4)   **For what value of F does μ = 60°?**

Q5)   **How many full cells (50 meter units) does it take for the pattern to repeat itself? How would you interpret the parameter μ ?**

The FODO system will be unstable when |TrM| > 2 or when F < L/2. Leaving the other parameters alone, set F = 12.499. Note that the parameter μ becomes imaginary! Look at the particle's trajectory.

Q6)   **If the vacuum chamber is 7 <u>centimeters</u> away from the particle's ideal orbit, how far will the particle travel before it reaches the chamber wall?**

2   Betatron Phase Space

Open the next Mathcad worksheet, ***Csinvariant.mcd***. At first glance the screen will look very similar to the previous one. Scrolling down, you will see two plots, however, - a trajectory plot plus a phase space plot. As the particle traverses the FODO structure, its position and angle are kept track of and plotted in the phase space plot. For N_{turn} = 1, follow and understand how the lines in the two plots are related to one another. A change in slope on the phase space plot corresponds to a "kink" seen in the trajectory plot, etc. Change N_{turn} to 2, 3, 4, 5, 10, and 100 watching the two plots each time. Set N_{turn} back to 1 for a moment. Double click on the phase space plot – a formatting window will appear. Select the tab marked "Traces." For trace 1, change the "Type" column from "draw" to "points". Hit "OK" to return to the main window. Now, the phase space plot will only have points drawn on it. Again, change N_{turn} to 2, 3, 4, 5, 10, and 100. The phase space plot now has only the END POINTS of the lines which were previously drawn, and these points all lie on ellipses! The way the program is set up, the end points of all the lines are at the entrance and exit of each quadrupole magnet.

Q7)   Print out or make a sketch of this phase space plot and identify with each ellipse its corresponding quadrupole (F or D) and whether it is at the entrance or exit of the magnet.

3   Closed Orbit Error & Correction

Today's final exercise will be to look at how we can correct for a local orbit distortion. Open ***ClosedOrbit.mcd***. This file uses our "standard" F = 20 meters, L = 25 meters FODO lattice, but this time, at specific locations in the structure (initially position '10'), there is an additional element – a steering magnet – which gives the particle an angular deflection of amount θ_i each time the particle passes by. By looking at the plot of particle displacement, we see that when the particle starts with x = 0 and x' = 0, it begins a betatron oscillation when it passes by the steering magnet (see the SOLID trace in the plot).

   However, there is a particular orbit which, if the particle trajectory starts out just right, the orbit will be deflected by the steering magnet but when it returns to the beginning of the accelerator will end up with the same position and slope it started out with. Thus, this particular particle will follow the same path over and over again.

Change N_{turn} from 1 to 2, 5, 10, 100. Notice how the particle appears to oscillate about the new closed orbit generated by the steering magnet. Even though the magnet only steers the particle at one location in the accelerator, it can affect the displacement everywhere.

If F and L are in meters, x in millimeters, and x' in mrad, then θ_i is in mrad (0.05 initially).

Q8) **What steering error, θ_1, would generate a 3mm ($\pm$ 0.01) maximum displacement of the closed orbit in the beam pipe?**

Scroll further down. In addition to a steering "error" being defined at position '10' two steering correctors are defined at locations '14' and '16' These two are initially set to zero, and the trajectory is plotted below these definitions. Carefully adjust the strengths of these two correctors so that a particle whose trajectory starts with x = 0, x' = 0 before θ_1 ends up with x = 0 and x' = 0 after θ_3. Check the "closure" of your "orbit bump" by changing N_{turn} to some large value and seeing that the orbit indeed repeats itself.

Q9) **For θ_1 = 0.05, what values of θ_2 and θ_3 are required to bring the trajectory back to x = 0 and x' = 0?**

The use of three steering correctors to make local displacements of the beam trajectory is sometimes called a "three-bump." With a distribution of correctors around an accelerator, three-bump combinations of correctors can be used to "smooth" the accelerator orbit in the presence of magnet misalignments and field errors.

4   Some General *Mathcad* Hints

• To change a parameter, simply click on it until a cursor appears to its right, and then delete or alter it to how you want it. When you hit RETURN, Mathcad will update the entire worksheet.

• If you wish to update the worksheet manually, deselect Automatic Calculation in the MATH menu.

• If you wish to recalculate the worksheet without moving the cursor, just hit "Control-=".

• If you wish to move an equation, expression, or text area around, first click on the item. Put the cursor near the edge of the item's surrounding box; a hand should appear. Now you can drag the item to its new location.

 • Mathcad has a Help feature which can be obtained by selecting HELP TOPICS under the "?" icon in the upper right hand corner of the screen. There is also a "?" icon in one of the tool palettes.

• An expression like "y:=x" is a definition of y in terms of x. An expression like "y=0.56" is the result of a calculation of y.

• To terminate a calculation, press "Control-." (Control-period) to abort the process. One can also hit the "Escape" key.