Fundamentals of Accelerator Physics and Technology with Simulations and Measurements Lab

U.S. Particle Accelerator School

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Computer Session – Longitudinal Motion

The following sections describe three *Mathcad*[1] worksheets and some things to try with them. The worksheets demonstrate phase space motion in the longitudinal degree of freedom. A few hints on using *Mathcad* are found on the last page. Questions to be answered and turned in for grade are typeset in **Bold Type**.

1   Longitudinal Phase Space Motion

The difference equations for longitudinal motion of a charged particle in a linac or synchrotron can be written as

$$\Delta E_{n+1} = \Delta E_n + eV(sin\phi_n - sin\phi_s)$$

$$\phi_{n+1} = \phi_n + (2\pi h\eta/E)\Delta E_{n+1}$$

These equations and their derivation are explained in the *Introductory Notes*. The *Mathcad* worksheet named **RFPhaseSpaceMI.mcd** allows the user to enter various initial conditions (i.e. various ϕ_0, ΔE_0) of particles and "track" their motion for a specified number of iterations.

In order to do the calculations in *Mathcad*, the equations had to be written such that the indices of order *n* appear on the right-hand side, and indices of order *n+1* appear on the left. You should verify that the difference equations here are equivalent to the ones derived in class.

In the worksheet program, the user can vary the parameters of the basic equations of motion to produce phase space plots in ΔE_0 - ϕ coordinates. The default parameters are typical of *Fermilab Main Injector* operation at its injection energy. By inputting various initial conditions one can explore the phase space region under various parameter regimes. For example:

• The synchronous phase is initialized to 0. For this value locate the separatrix.

Q1) **What value (approximately) of ΔE will the Radiofrequency system support under these conditions?**

Q2) **What happens to the particles that find themselves outside of the separatrix – are they lost from the synchrotron?**

• Look at the motion of a particle that is *very near* the separatrix.

Q3) **How does the motion behave near the "unstable fixed points"?**

• Change the synchronous phase to 30˚ (what happens in a synchrotron when this phase change occurs?). Under these conditions…

Q4) **What value (approximately) of ΔE will the Radiofrequency system support now?**

Q5) **What happens to the particles that find themselves outside of the separatrix – are they lost from the synchrotron?**

Q6) **How does the motion behave near the "unstable fixed points"?**

Next, leaving the other parameters as they are, change the energy of the particle from 8 GeV to 150 GeV. Where is the stable phase space region located now? (note: the transition energy of the Main Injector is about 22; what happens at transition?).

Q7) **What maximum ΔE will the RF system support now?**

The user should feel free to play with the parameters of the program. Put in the parameters of you "favorite" accelerator and explore phase space. Be sure to play with changing the scales of the plot axes in order to see the results.

EXTRA

Since the difference equations are of a non-linear system, one can push the parameters until resonance islands, chaos, and other non-linear effects are found. Go back to the "stationary bucket" case ($\phi_s = 0$). Increase the voltage by a factor of 10 and look at the phase space; another factor of 10; another; and so on. Can you find other regions of stability in the phase space? Is there a value of the voltage for which the entire phase space becomes unstable?

2   Synchrotron Tune vs. Oscillation Amplitude

In the previous worksheet, it should have been noticed that the motion i.e. the frequency at which particles circulate in phase space, slows down when the particle is near the separatrix, especially near the "unstable fixed points". *Mathcad* has a built-in Fast Fourier Transform (FFT) function that allows one to analyze data for frequency content. The worksheet called **RFfftMI.mcd** performs this calculation.

This worksheet is similar to the previous one, but tracks a single particle. The motion of that particle is then analyzed using the FFT function, and the frequency (or, in this case, the synchrotron "tune") of the particle's motion is estimated by looking at what tune value corresponds to the peak of the FFT coefficients.

From class, the synchrotron tune for particles with small oscillation amplitude was given. This expression is reproduced in *Mathcad* so that the result of the FFT can be directly compared with the analytical result obtained by solving the equations of motion of a particle's longitudinal motion.

• Track a particle for 254 turns (note: an FFT calculation requires that one track for 2^n-2 turns, where n is an integer) which has initial conditions $\phi_0 = 0$ and $\Delta E_0 = 1$ MeV. These should be the default values when the worksheet is loaded.

Q8)   What synchrotron tune does the FFT of the tracking data show? How does this compare with the analytically predicted value?

• Try varying the number of turns tracked, such as 126, 510, 2046, etc.

Q9) **For what number of turns is the FFT result within 5% of the predicted result?**

• Next, tracking over 1022 turns, slowly raise the initial ΔE from 1 MeV (steps of 1-2 MeV are good) and keep track of the synchrotron tune. How does the tune behave?

Q10) **Make a plot of ν_s vs. ΔE_0 for ΔE from 1 MeV to 30 MeV. Note the value of ΔE at the separatrix.**

• Study the synchronous tune vs. amplitude for other synchronous phases, ϕ_s.

3   Injection to Acceleration

The third Mathcad worksheet demonstrates the effects of non-adiabatic variation of system parameters. This program, entitled ***Inj2AccMI.mcd***, tracks a single particle in longitudinal phase space, and varies the RF synchronous phase linearly from $\phi_s = 0$ (injection value) to a final value ϕ_{sf} (equal to 30° in the worksheet) used for acceleration.

The "injection porch" length – the time particles spend at the injection energy – is input with the parameter N_{porch}. The time it takes to reach the final acceleration rate is N_{parab}. The program plots ϕ_s versus turn number. The phase space plot begins by plotting only the first 900 turns (controlled by N_f).

Q11) **What is the synchrotron period in this example?**

• Vary N_f from 900 to the final value of 3400 in steps of 300. Note how the particle's phase space trajectory follows the synchronous phase as it moves from 0 to 30 degrees. Note also how the size of the particle's phase space ellipse remains essentially constant along the way. Thus, its "longitudinal emittance" is preserved in the process. The phase changes slowly enough that the motion is said to remain adiabatic.

• Next, change the "ramp parabola" i.e. decrease the number of turns it takes to make the RF synchronous phase reach 30°. Notice how the final particle emittance increases as the rate of phase change increases. The process becomes non-adiabatic and the emittance is not preserved. In general, if the motion is to be adiabatic, then the system parameters must change much more slowly than the period of the motion.

Q12) **What is the approximate fewest number of turns one can make the phase change in and still preserve the initial emittance?**

4 Some General *Mathcad* Hints

• To change a parameter, simply click on it until a cursor appears to its right, and then delete or alter it to how you want it. When you hit RETURN, Mathcad will update the entire worksheet.

• If you wish to update the worksheet manually, deselect Automatic Calculation in the MATH menu.

• If you wish to recalculate the worksheet without moving the cursor, just hit "Control-=".

• If you wish to move an equation, expression, or text area around, first click on the item. Put the cursor near the edge of the item's surrounding box; a hand should appear. Now you can drag the item to its new location.

• Mathcad has a Help feature which can be obtained by selecting HELP TOPICS under the "?" icon in the upper right hand corner of the screen. There is also a "?" icon in one of the tool palettes.

• An expression like "y:=x" is a definition of y in terms of x. An expression like "y=0.56" is the result of a calculation of y.

• To terminate a calculation, press "Control-." (Control-period) to abort the process. One can also hit the "Escape" key.