USPAS June 2013

Design of Electron Storage and Damping Rings

Homework Problems 3

Lecture 5 Problems

1. Show that if the coupling strength κ in a storage ring is large compared to the difference $\Delta\nu$ between the fractional parts of the betatron tunes, then the emittance ratio in the storage ring is given approximately by:

$$\frac{\varepsilon_y}{\varepsilon_x} \approx 1 - \frac{4\pi\Delta\nu}{\kappa}.$$

2. Show that if the coupling strength κ in a storage ring is small compared to the difference $\Delta\nu$ between the fractional parts of the betatron tunes, then the emittance ratio in the storage ring is given approximately by:

$$\frac{\varepsilon_y}{\varepsilon_x} \approx \frac{\kappa^2}{\kappa^2 + 16\pi^2\Delta\nu^2}.$$

3. A quadrupole of focal length 0.5 m is located in a storage ring at a point where the horizontal and vertical beta functions are 20 m and 8 m, respectively, and the dispersion is zero. An alignment error leads to this quadrupole having a tilt of 10 mrad around the beam axis. The fractional parts of the horizontal and vertical tunes are 0.20 and 0.25, respectively. Neglecting other sources of coupling than the tilted quadrupole, estimate the emittance ratio in the ring.

Lecture 6 Problem

4. An electron storage ring contains a single bunch of N electrons. Over one turn, this bunch sees its own transverse wake field from just the previous turn.

   a. Show that the growth rate of the amplitude of coherent betatron oscillations of the bunch is given by:

$$\frac{1}{\tau} \approx \frac{cNr_e}{4\pi\gamma\nu} W_\perp \sin(2\pi\nu),$$

   where r_e is the classical radius of the electron, γ is the relativistic factor, ν is the betatron tune, and $W_\perp$ is the transverse wake function at a distance equal to the circumference of the ring.

   b. What is the coherent tune shift arising from the wake field?

   c. Comment on the dependence of the growth rate and the tune shift on the betatron tune.