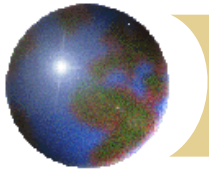


## Fundamentals of Detector Physics and Measurements Lab - III

Carl Bromberg  
Michigan State University  
Dan Green  
Fermilab

June 18-22, 2012



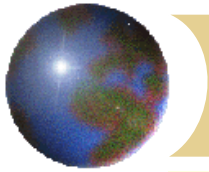
# Outline

## ✚ Lecture I

- ▣ Constants, atoms, cross sections
- ▣ Photoelectric, TOF
- ▣ PMT, SiPM Scint, Cerenkov

## ✚ Lecture II

- ▣ Collisions, cross sections
- ▣ Multiple scattering, radiation length
- ▣  $dE/dx$ , MIP, Range
- ▣ Critical Energy



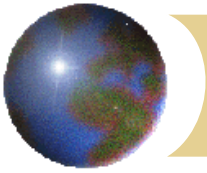
# Outline II

## ✚ Lecture III

- ✚ B fields, trajectories
- ✚ Quadrupoles, focal length
- ✚ Drift and Diffusion
- ✚ Pulse formation in unity gain and gas gain

## ✚ Lecture IV

- ✚ Radiation NR, Thompson, Compton
- ✚ Relativistic radiation
- ✚ Bremm, Pair Production



# LHC Accelerator - Dipoles



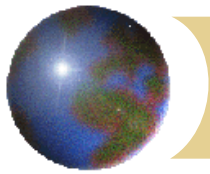
wrt Tevatron (at design)

Energy  $\times 7$

luminosity  $\times 20$

Large increase  $\rightarrow$  discovery

- The LHC at 1.9 K is colder than the CMB at 2.3 K.
- The LHC is the highest energy collider in the world
- The LHC has the worlds largest cryogenic plant.
- The LHC is designed to have the highest reaction rate of any collider, design value is 1 GHz.
- The associated experiments are the largest and most complex scientific instruments ever built - a 20 year effort.



# LHC by the Numbers - Beams

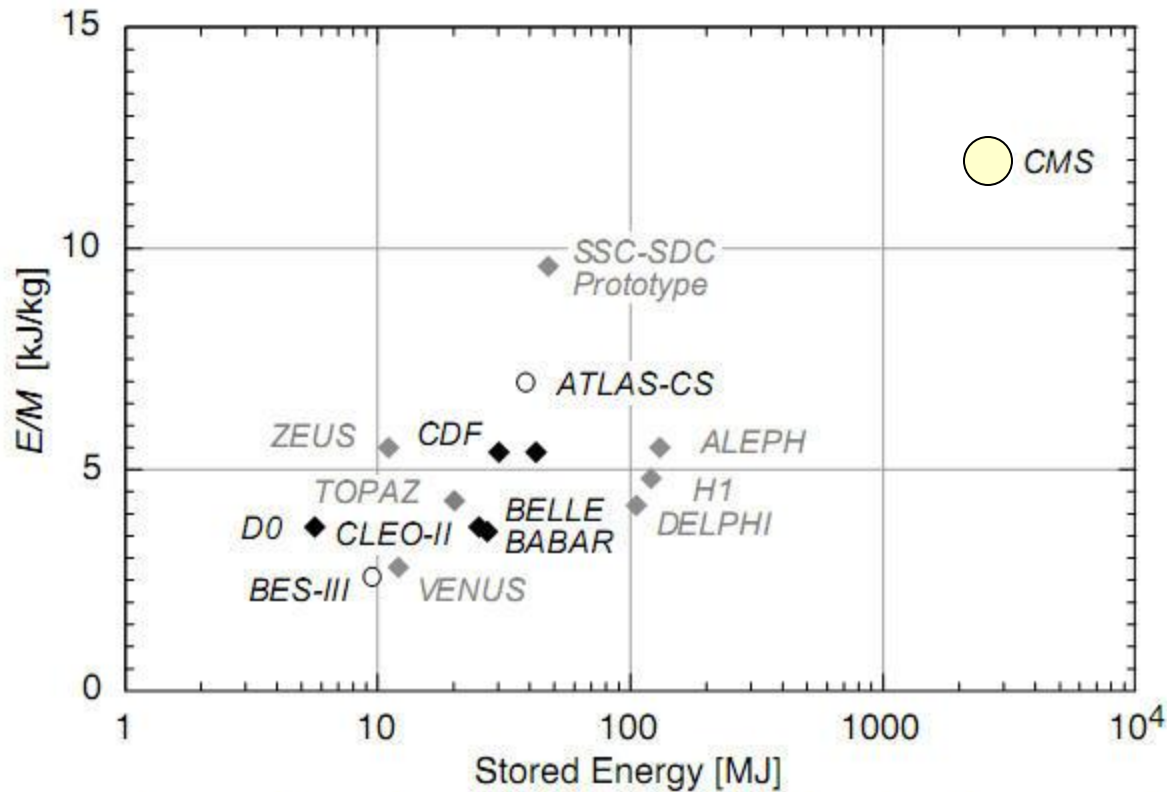
- 2808 r.f. bunches per beam, spaced by 25 nsec = 7.5 m
- Bunch intensity  $\sim 1.2 \times 10^{11}$  p per bunch
- Each bunch  $\sim 5$  cm long,  $\sim 16$  microns wide
- Each beam carries 362 MJ – the 2 beams have sufficient energy to melt  $\sim 1$  ton of cryogenic (2 K) copper.
- Quench detection, abort and collimation. Beam dump systems are critical for safe operation.
- “Store” beams for  $\sim 10$  hours. Protons travel  $\sim 10$  billion km around the LHC ring  $\sim 65$  AU  $\sim$  round trip to Pluto -> need a good vacuum, better than the vacuum found on the Moon –  $10^{-10}$  Torr  $\sim 3$  million molecules/cm<sup>3</sup>. Compare to No.

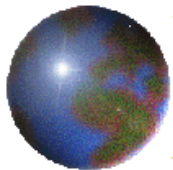


# Solenoid Magnet

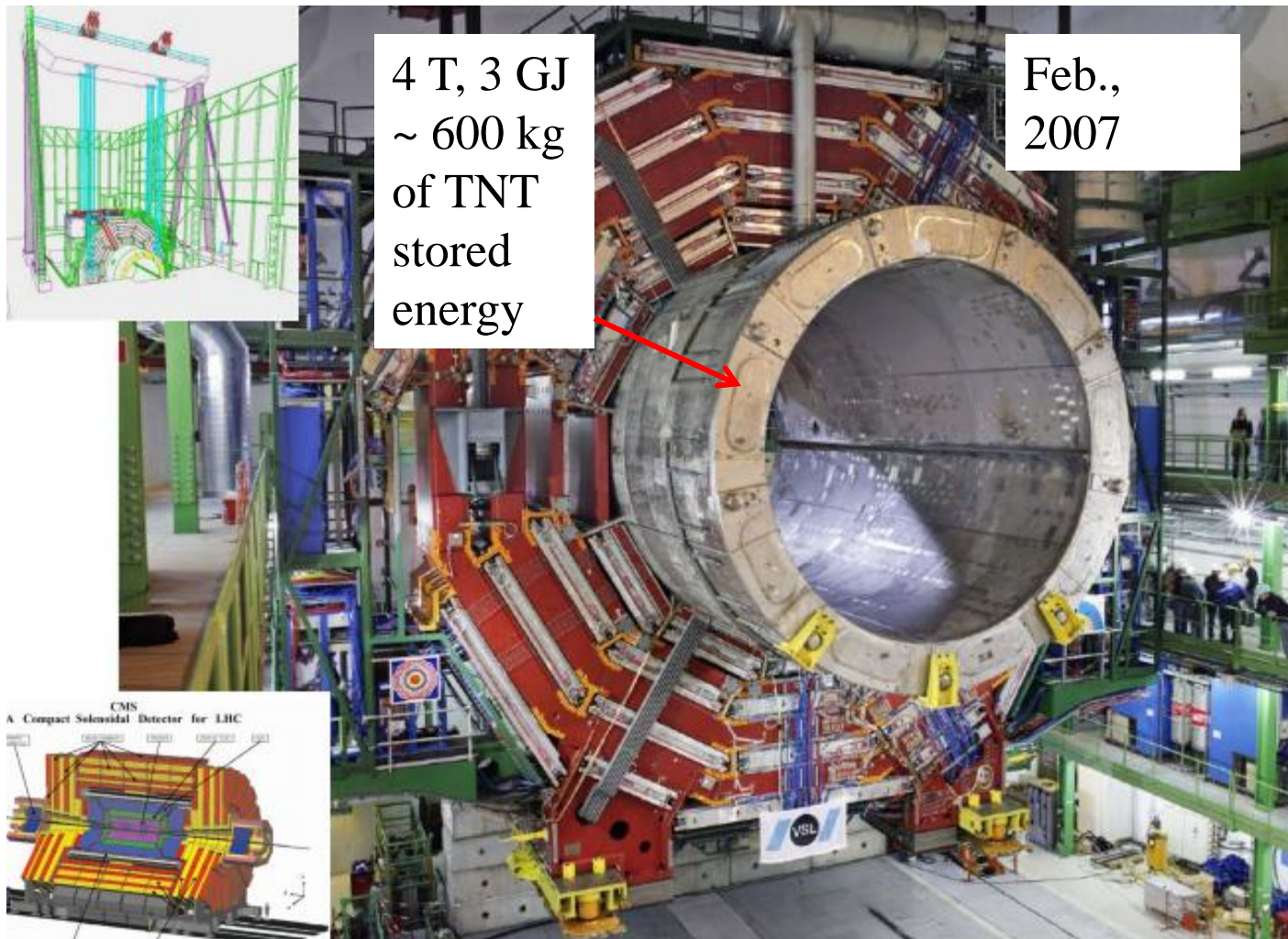
$$B = 4\pi nI / c$$

Conductors of size 2 cm in z all stacked by 4 in r for a total of n= 200 turns/m. Then to achieve a field of 5 T, 20.8 kA of current is required – 3 GJ @ 4T (CMS)





# World's Largest Solenoid





# Dipole Fields

$$B(\text{centerline}) = \frac{4\pi nI}{c} \left[ \frac{(L/2)}{\sqrt{(L/2)^2 + a^2}} \right] \equiv B_o$$

$$B_o \rightarrow 4\pi nI / casL \rightarrow \infty, \text{CGS}$$

$$B \rightarrow \mu_o nI, \text{MKS}$$

$$\vec{\nabla} \times \vec{B} = 0$$

$$\partial_y B_z = \partial_z B_y \sim \pm B_o / \ell$$

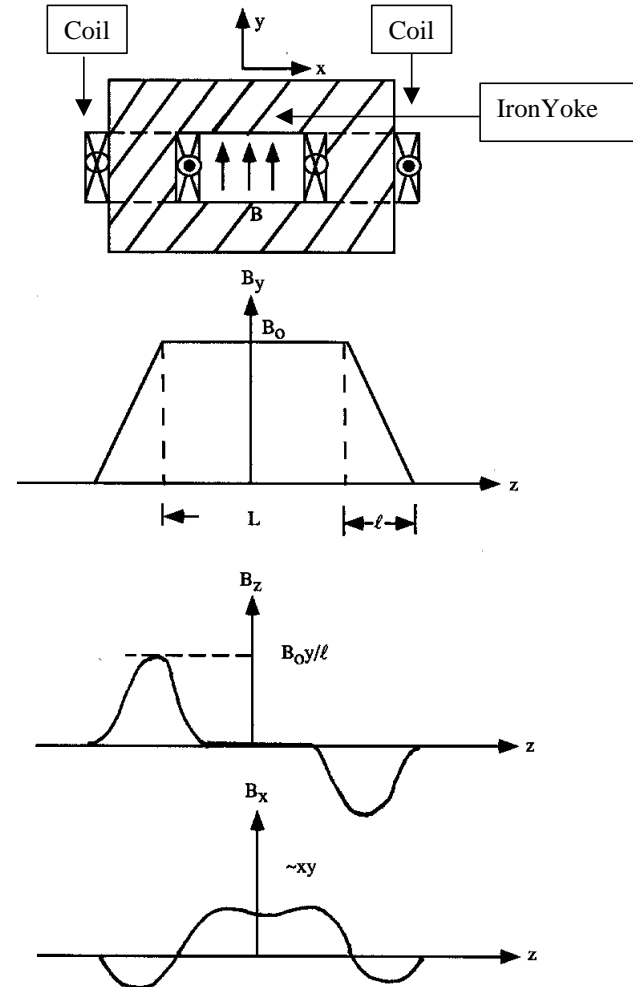
$$B_z \sim (\partial_z B_y) y \sim \pm B_o y / \ell$$

$$\vec{\nabla} \cdot \vec{B} = 0$$

$$\partial_x B_x \sim -\partial_z B_z \sim -y(\partial_z^2 B_y)$$

$$B_x \cong -xy(\partial_z^2 B_y)$$

The fields cannot abruptly go to zero. Any real dipole has “fringe fields” at their boundaries (lenses)







# Forces and Trajectories

$$\vec{F} = m\vec{a} = q(\vec{v} \times \vec{B}) = d\vec{p} / dt$$

$$\vec{F} \cdot \vec{v} = 0 \Rightarrow |\vec{p}| = \text{const}$$

$$F_{\text{cent}} = mv^2 / r = qvB$$

$$p = mv, r = a$$

$$a = p / eB$$

$$\omega_c = eB / m$$

$$d\vec{p} / dt = \frac{q}{m} (\vec{p} \times \vec{B})$$

NR force law

Energy does not change in  
a B field

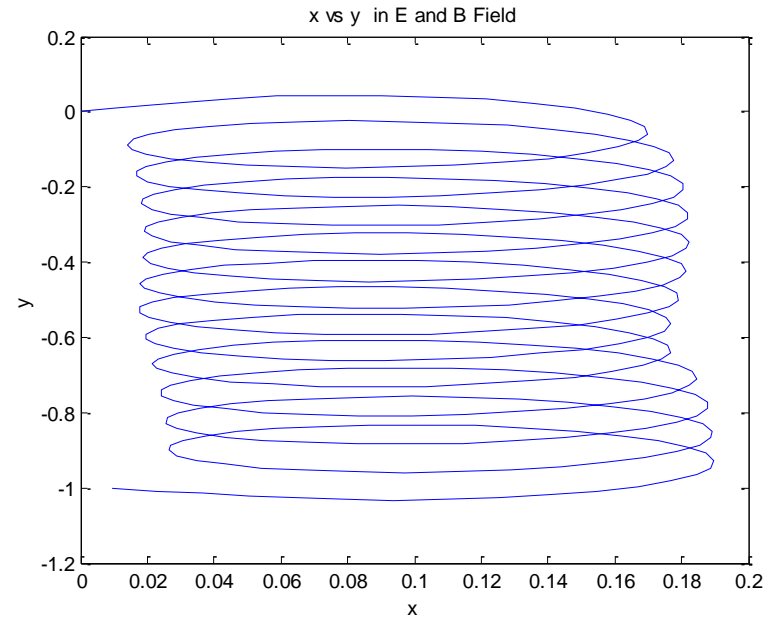
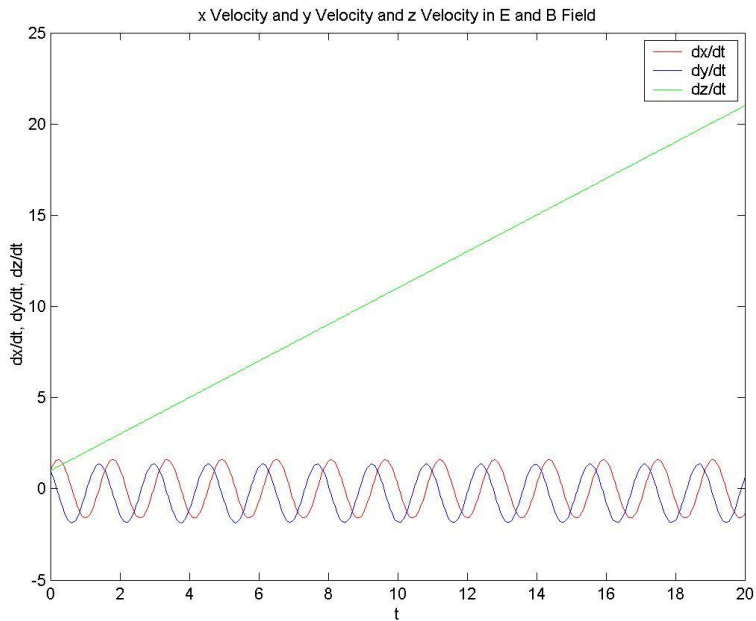
Circular orbit, radius  $\sim p$   
and  $1/B$ .

Cyclotron frequency –  
does not depend on  
particle energy

SR force law – important  
Kinematic quantity is  
momentum  $\vec{p} = \gamma\vec{\beta}m$



# Demo – E and B (NR)



Electric field accelerates a charged particle.  
Magnetic field bends it into a circular path -  
constant E and B fields in the demo.



# Momentum Resolution

$$\phi_B = (\Delta p_T)_B / p$$

$$(\Delta p_T)_B = eLB = 0.03 BL \left( \frac{\text{GeV}}{\text{kGm}} \right)$$

$$\phi_B \sim L / a = eLB / p$$

$$1/p = \phi_B / (\Delta p_T)_B$$

$$d(1/p) = \frac{d\phi_B}{(\Delta p_T)_B} \sim \frac{(dx_T / L)}{(\Delta p_T)_B}$$
$$= dp / p^2$$

$$(\Delta p_T)_{MS} = \frac{E_S}{\sqrt{2}} \sqrt{L / X_o}$$

$$dp / p \sim (\Delta p_T)_{MS} / (\Delta p_T)_B$$

Suppose tracking detectors measure the helical trajectory with accuracy  $dx$  over a region of field  $B$  of length  $L$ . Over that path the  $B$  field imparts a transverse momentum impulse, causing the momentum transverse to the field to rotate by a “bend angle”.

The measured angular error leads to a fractional momentum error which goes as  $\sim p$ .

At low momentum, since no detector is massless, multiple scattering limits the measurement error of the bend angle, leading to a  $\sim$  constant fractional momentum error.



# Sagitta and Track Momentum

The sagitta  $s$  is:

$$s = a(1 - \cos(\phi / 2)) \sim a\phi^2 / 8$$

$$\phi = L / a$$

$$s \sim L^2 / 8a = 0.3B(T)L^2(m) / 8P_T$$

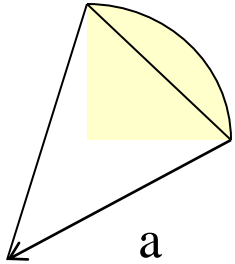
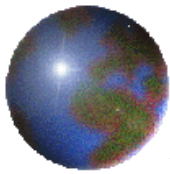


Figure of merit is  $BL^2$ . A large B field is desired but a large field volume is most important.

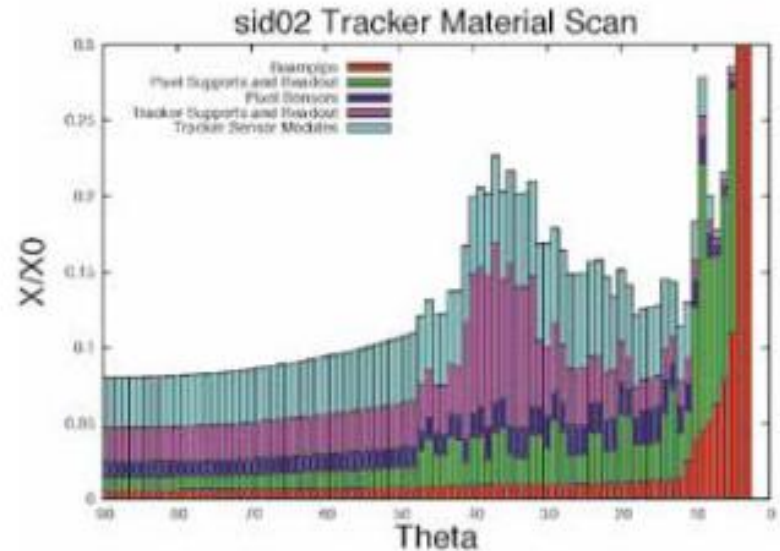
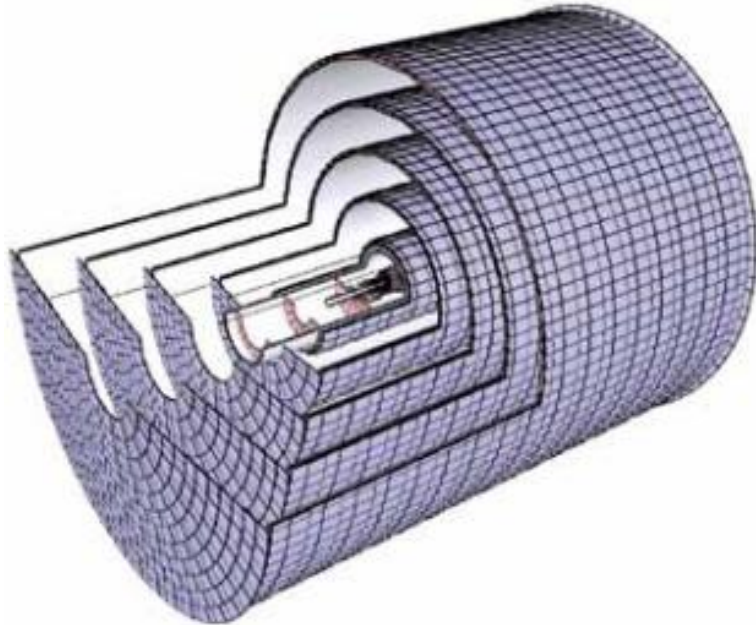
e.g.  $L = 1\text{m}$ ,  $B = 5\text{T}$ , Impulse =  $1.5\text{ GeV}$ ,  $P_T = 1000\text{ GeV} \rightarrow s = 0.3\text{ mm} = 300\text{ um}$ . Si strip width  $d \sim 400\text{ um}$ ,  $dx \sim 115\text{ um}$ .  $dx/L \sim 0.000115$ , so  $dp/p \sim 7.7\%$  for  $p = 1\text{ TeV}$ .

Note that for a uniform distribution, binary readout:

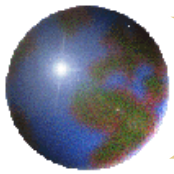
$$\sigma^2 = \int_{-d/2}^{d/2} x^2 dx / d = d^2 / 12, \text{ with charge sharing across strips one can do better}$$



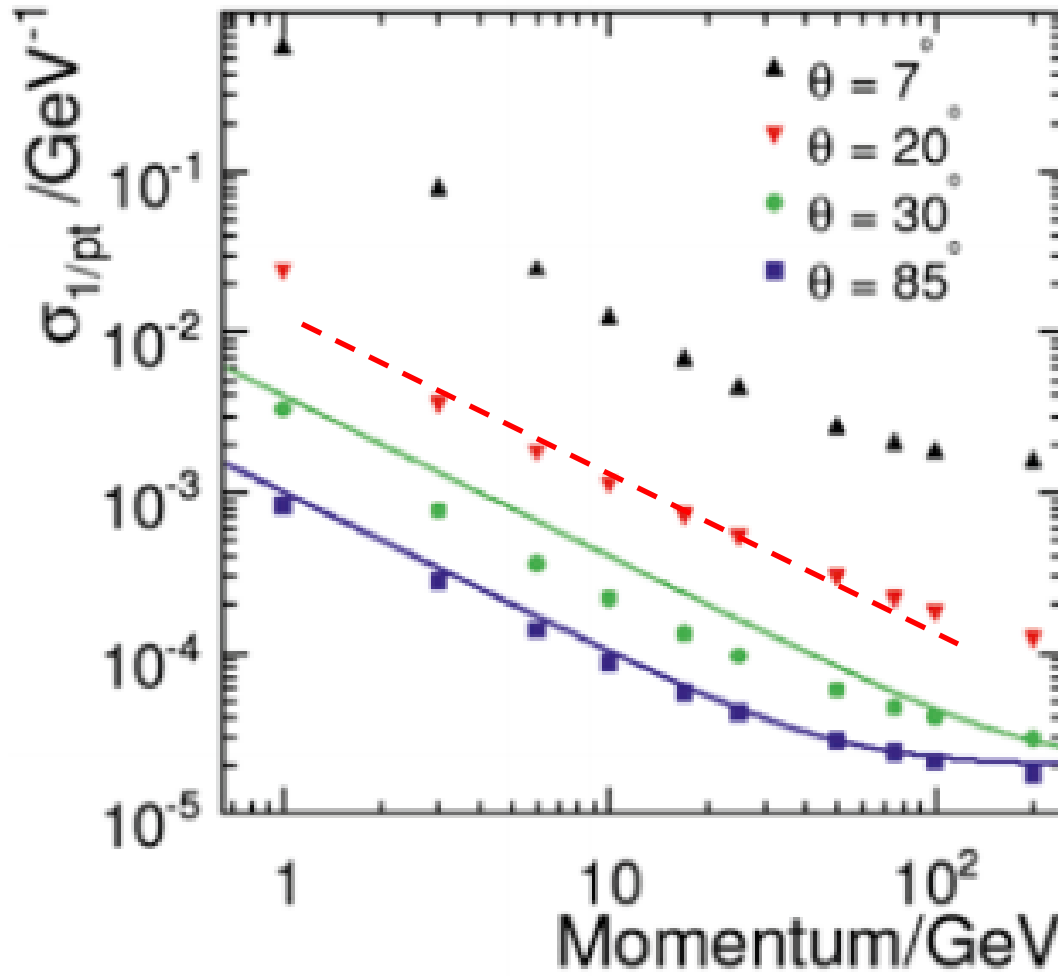
# ILC Si Tracking



Designs for ILC tracking detectors are all Si (typically) with pixels for vertex detection (discussed later) and strips for tracking in the B field. A major effort is made to keep the  $X_0$  in the designs to  $\sim 0.1$  for accurate measurement of the momentum of “soft” particles of low momentum.



# ILD – Tracking Resolution



The bending power is  $v \times B$  – weaker in the forward direction

The resolution due to both measurement error (high momentum) and multiple scattering (low momentum) is seen, constant and  $1/p$  for  $d(1/p)$



# P Resolution vs. P - CMS

$$(\Delta P_T)_B = erB = 0.3rB \quad T, \text{ m units}$$

$$d(1/P_T) = dP_T / P_T^2 = d(\Delta\phi_B) / (\Delta P_T)_B \sim ds / er^2 B \quad \text{FOM} \sim r^2 B$$

1.5 mrad at 1 TeV, 7.7 %

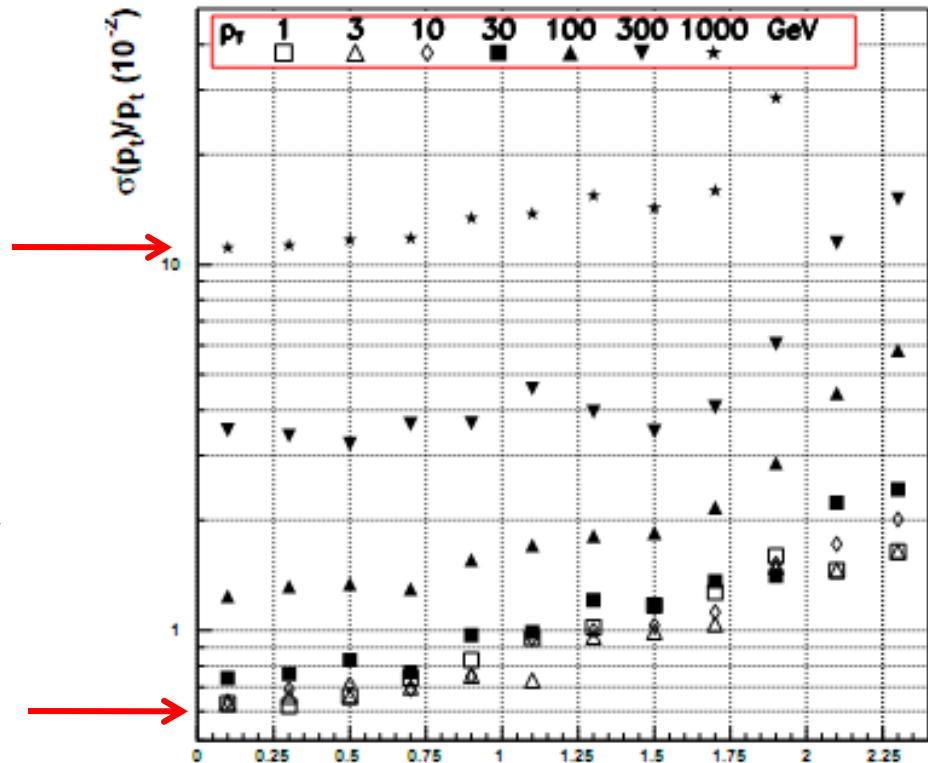
$$dP_T / P_T \sim (\Delta P_T)_{MS} / (\Delta P_T)_B$$

$$(\Delta P_T)_{MS} = E_s \sqrt{\sum L_i / 2X_o}$$

0.1 Xo, 6.4 MeV - MS, res = 0.0042 MS. Crossover at 54 GeV, 0.6 % total

$$(\Delta P_T)_B = 2(P_T)_{loop} = erB / 2$$

“loopers” < 0.75 GeV





# Exact Solutions

$$ds=vd t$$

$$p d\vec{p} / ds=q(\vec{p} \times \vec{B})$$

$$z=z_o + \alpha_z s, s=0 \text{ at } z=z_o, \alpha_z = \text{constant}$$

$$\begin{pmatrix} \alpha_x \\ \alpha_y \end{pmatrix} = \begin{pmatrix} \cos \phi_B & \sin \phi_B \\ -\sin \phi_B & \cos \phi_B \end{pmatrix} \begin{pmatrix} \alpha_{x_o} \\ \alpha_{y_o} \end{pmatrix}$$

$$(x-x_o) = -(a/p)(p_y - p_{y_o})$$

$$(y-y_o) = (a/p)(p_x - p_{x_o})$$

$$(x-x_o - a\alpha_{y_o})^2 + (y-y_o + a\alpha_{x_o})^2 =$$

$$a^2(\alpha_x^2 + \alpha_y^2) \equiv a_T^2$$

Can use for numerical, stepping, results for any B field.

The exact path is helical.  
Circular in a plane  
perpendicular to the B field  
direction and a straight line  
in direction of the B field  
(constant direction cosign).

s = path length

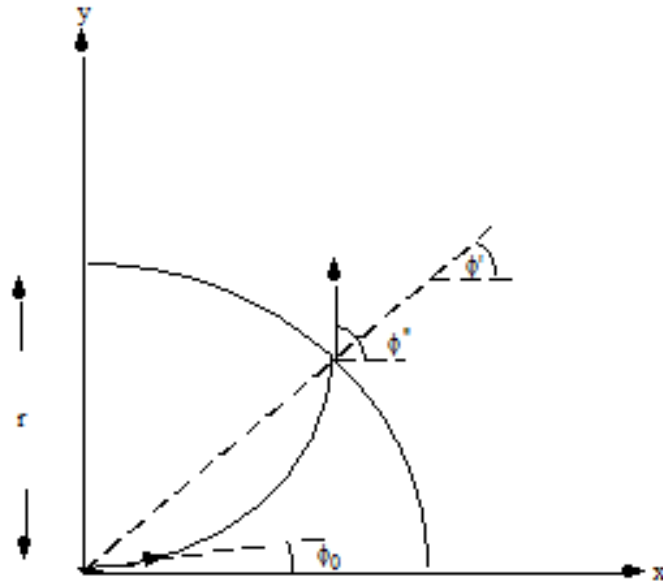
The (x,y) plane has a  
rotation matrix for the 2  
direction cosigns with angle  
= the bend angle.

The radius of curvature  
refers to the momentum  
transverse to the B direction.





# Approximate Solutions



$$p_T = p \sin \theta$$

$$a_T = p_T / qB_o$$

$$\tan \phi_o = p_{y_o} / p_{x_o}$$

$$\sin(\phi' - \phi_o) = -(r / 2a_T)$$

$$\tan \phi'' = \left[ \frac{\sin \phi_o - (r / a_T) \cos \phi'}{\cos \phi_o + (r / a_T) \sin \phi'} \right], \phi'' - \phi_o = \phi_B$$

Fig.7.4: Definitions used in constructing the path of a particle in a purely axial solenoidal field emitted from the origin at angle  $\phi_o$ . The particle at radius  $r$  is located at angle  $\phi'$  and its momentum vector at that point has an azimuthal angle  $\phi''$ .

At high momentum the bend angles are small and simple approximations are useful

$$\phi_o = 0$$

$$\phi' \sim r / 2a_T$$

$$\phi'' \sim -r / a_T \sim \phi_B$$



# Quadrupoles

$$\vec{\nabla} \times \vec{B} = 0 \Rightarrow \frac{\partial B_x}{\partial y} = \frac{\partial B_y}{\partial x} = B'$$

$$B_y = B'x$$

$$B_x = B'y$$

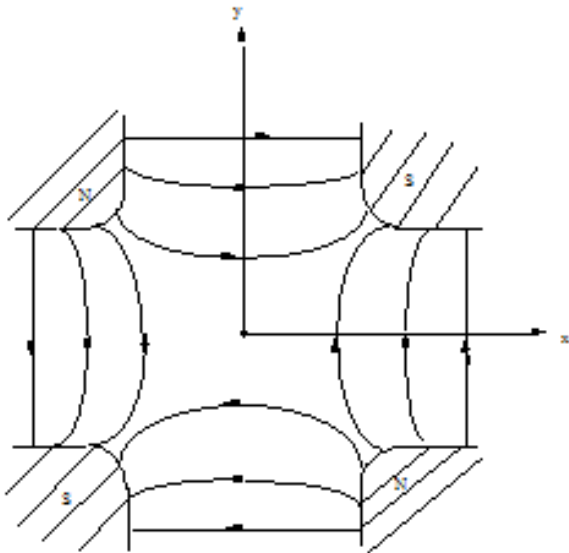


Fig. 7.8: Field orientation for a quadrupole.

$$\vec{F} = d\vec{p} / dt = \gamma m v d\vec{v} / ds = q(\vec{v} \times \vec{B})$$

$$= \gamma m v^2 d^2 \vec{x} / ds^2 = p v d^2 \vec{x} / ds^2$$

$ds = v dt$ ,  $v$  is constant

$$\frac{d^2 x}{ds^2} = + kx$$

$$\frac{d^2 y}{ds^2} = - ky$$

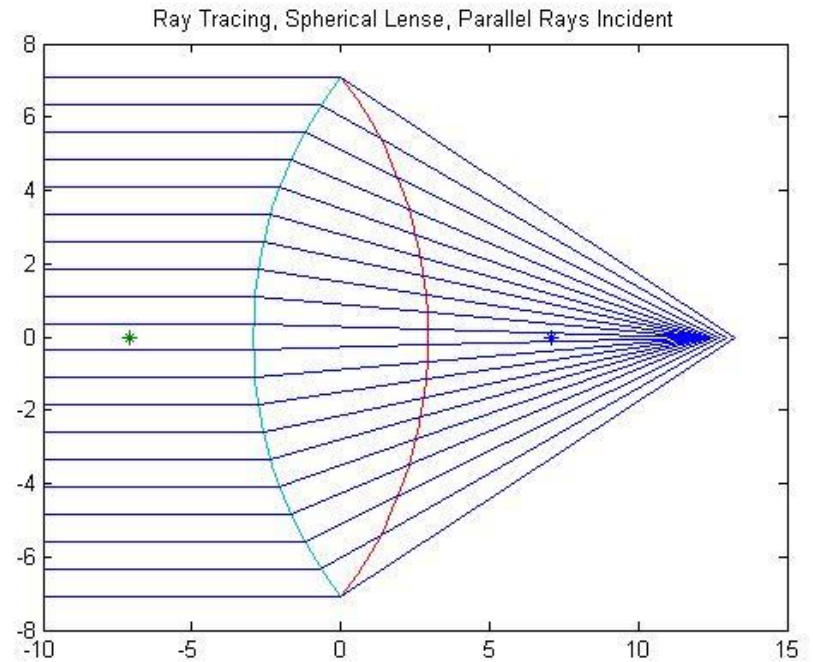
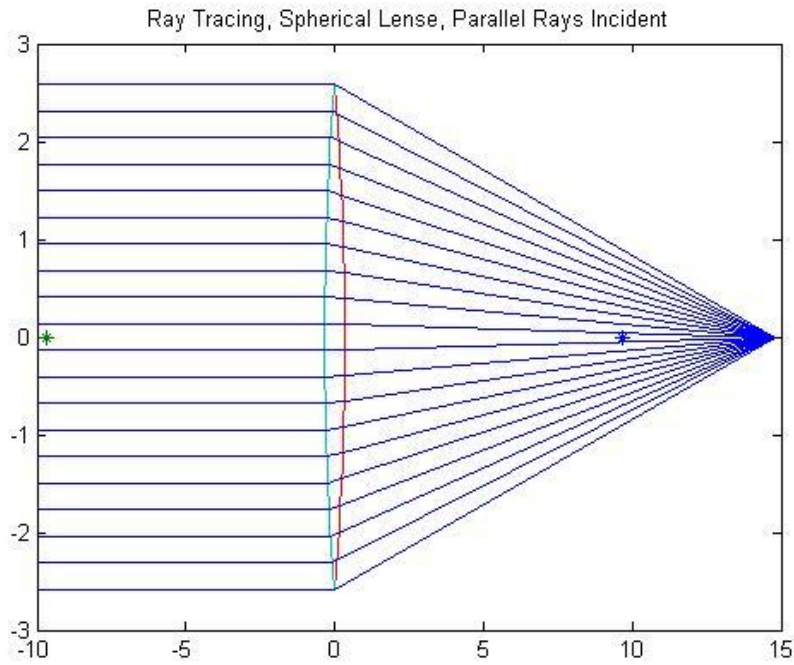
$$k = qB' / p$$

Simple harmonic motion. Spring constant goes as  $\sim B$  gradient and  $\sim 1/p$ . Dimension of  $[k] = 1/L^2$

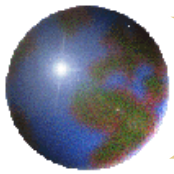
For accelerators and beamlines besides bending and steering the particles they need to be focused in order to transport them a long distance



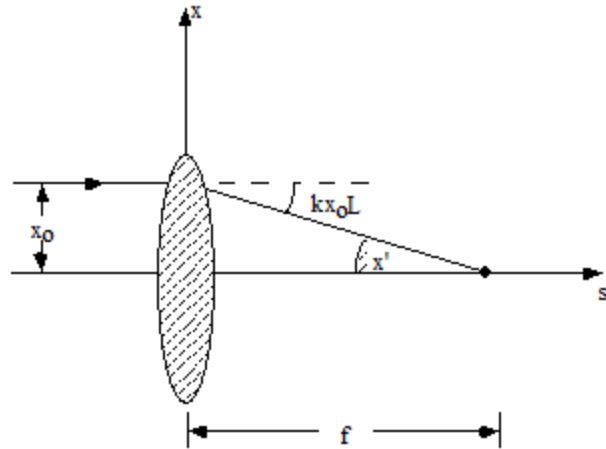
# Demo – Thin/Thick Lens



In optics there are changes in focal length as the lens becomes thicker. There is a similar behavior in a magnetic lens. Obviously there is also a “chromatic aberration” because the lens strength  $\sim 1/p$ .



# Quad Doublet



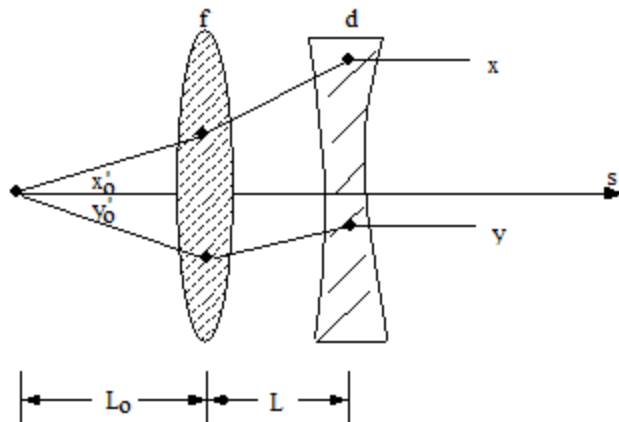
$$1/f = kL$$

Quad focal length

$$\begin{pmatrix} x \\ x' \end{pmatrix} = \begin{pmatrix} \cos \varphi_Q & \sin \varphi_Q / \sqrt{k} \\ -\sqrt{k} \sin \varphi_Q & \cos \varphi_Q \end{pmatrix} \begin{pmatrix} x_o \\ x'_o \end{pmatrix},$$

$$= M_Q \begin{pmatrix} x_o \\ x'_o \end{pmatrix}$$

$$\varphi_Q = \sqrt{k} L$$



$$M_Q \rightarrow \begin{pmatrix} 1 & L \\ -kL & 1 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 0 \\ \pm 1/f & 1 \end{pmatrix}$$

Thin lens approx

$$M_{doublet} = M_Q(F) M_D(L) M_Q(D) M_D(L_o).$$

$$f = L/2 + \sqrt{(L/2)^2 + L_o L}$$

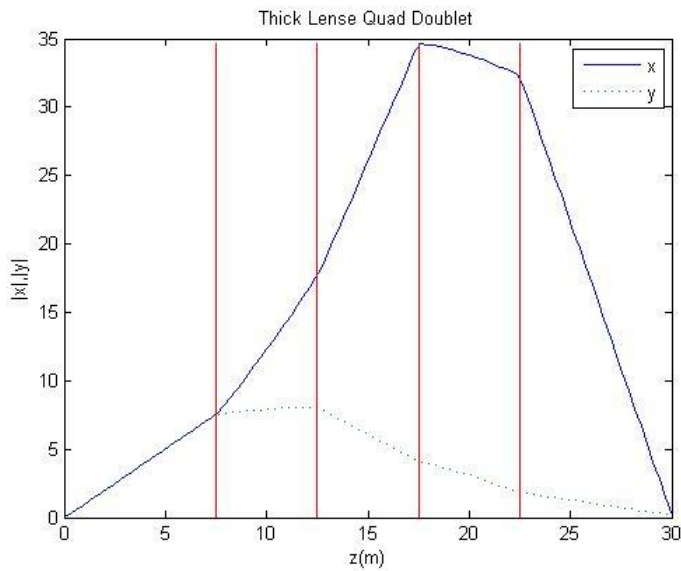
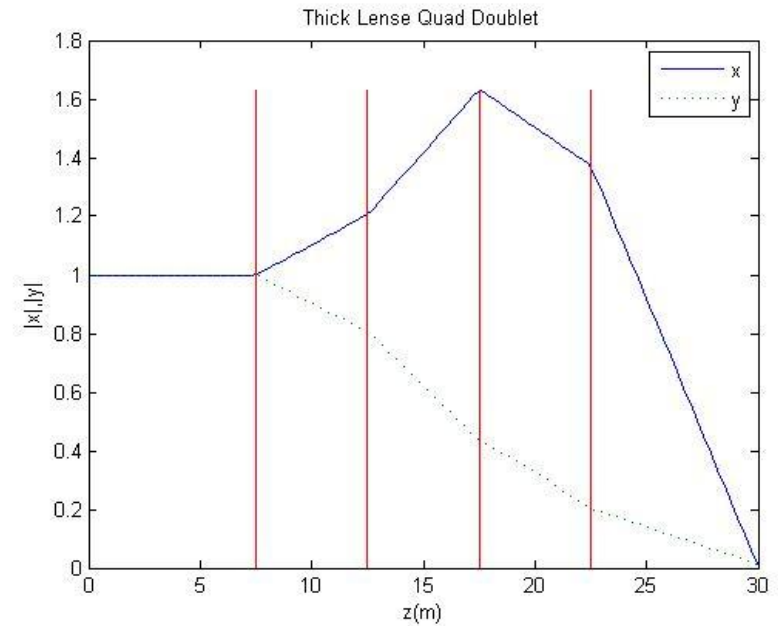
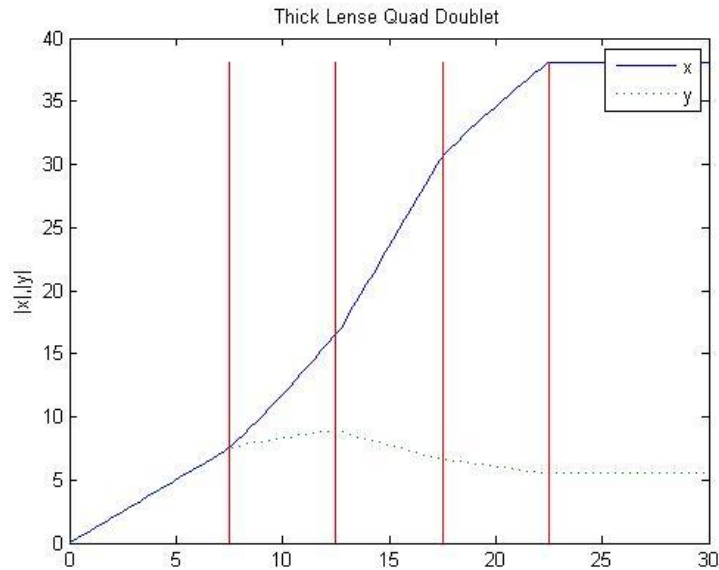
“point to parallel”

$$x = x'_o (L_o + L + L L_o / f)$$

$$y = y'_o (L_o + L - L L_o / f)$$



# Demo – Quad Doublet



Full thick lens  
quadrupole matrices.  
There are 3 options;  
point to parallel,  
parallel to point and  
point to point.



# Mobility in Gases

$$\langle L \rangle^{-1} \cong N_o \rho \sigma / A$$

$$\sigma \sim \pi a_o^2$$

$$\langle v_d \rangle \sim \left( \frac{eE}{m_e} \right) \left( \frac{A / N_o \rho \sigma}{v_T} \right) = \frac{eEA}{N_o \rho \sigma \sqrt{3kT m_e}}$$

$$\mu \equiv \frac{\langle v_d \rangle}{E / \rho} \sim \text{const}, \langle v_d \rangle \equiv \mu E (P_o / P)$$

$$P_o = 1 \text{ATM} = 760 \text{Torr (STP)}$$

$$\mu \sim (e / M) \left[ \frac{A}{N_o \sigma v_T} \right],$$

$$\mu_{Ar} = 1.7 (\text{cm}^2 / \text{V} \cdot \text{sec})$$

$$\mu_{CO_2} = 1.1 (\text{cm}^2 / \text{V} \cdot \text{sec})$$

e.g.  $E=1 \text{ kV/cm}$ ,  $\text{CO}_2$ ,  $v_d=11 \text{ m/sec}$

Consider the motion of ionization e in collision with molecules in a gas. They have a thermal velocity which is random and are accelerated between collisions by an electric field.

Drift velocity is the acceleration times the mean time between collisions.  $\sim L/vt$ .

Mobility is the drift velocity per E field per STP density.



# Drift and Mean Free Path

Thermal energy  
and velocity

$$T_T = Mv_T^2 / 2 \sim \frac{3}{2} kT$$

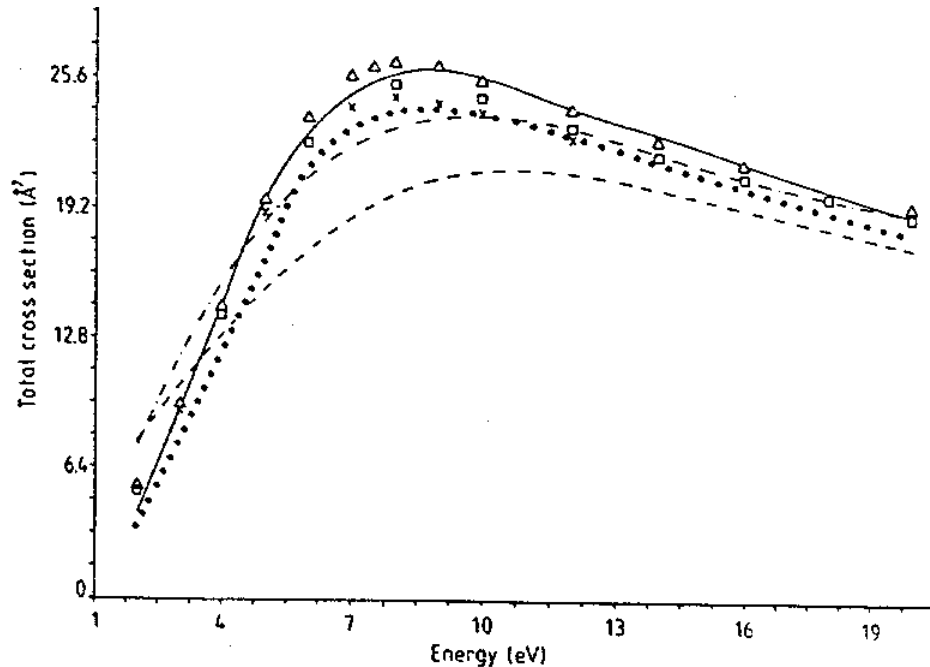
$$v_T \sim \sqrt{\frac{3kT}{M}}$$

Drift velocity

$$\vec{a} = e\vec{E} / M$$

$$\langle v_d \rangle \sim a\tau$$

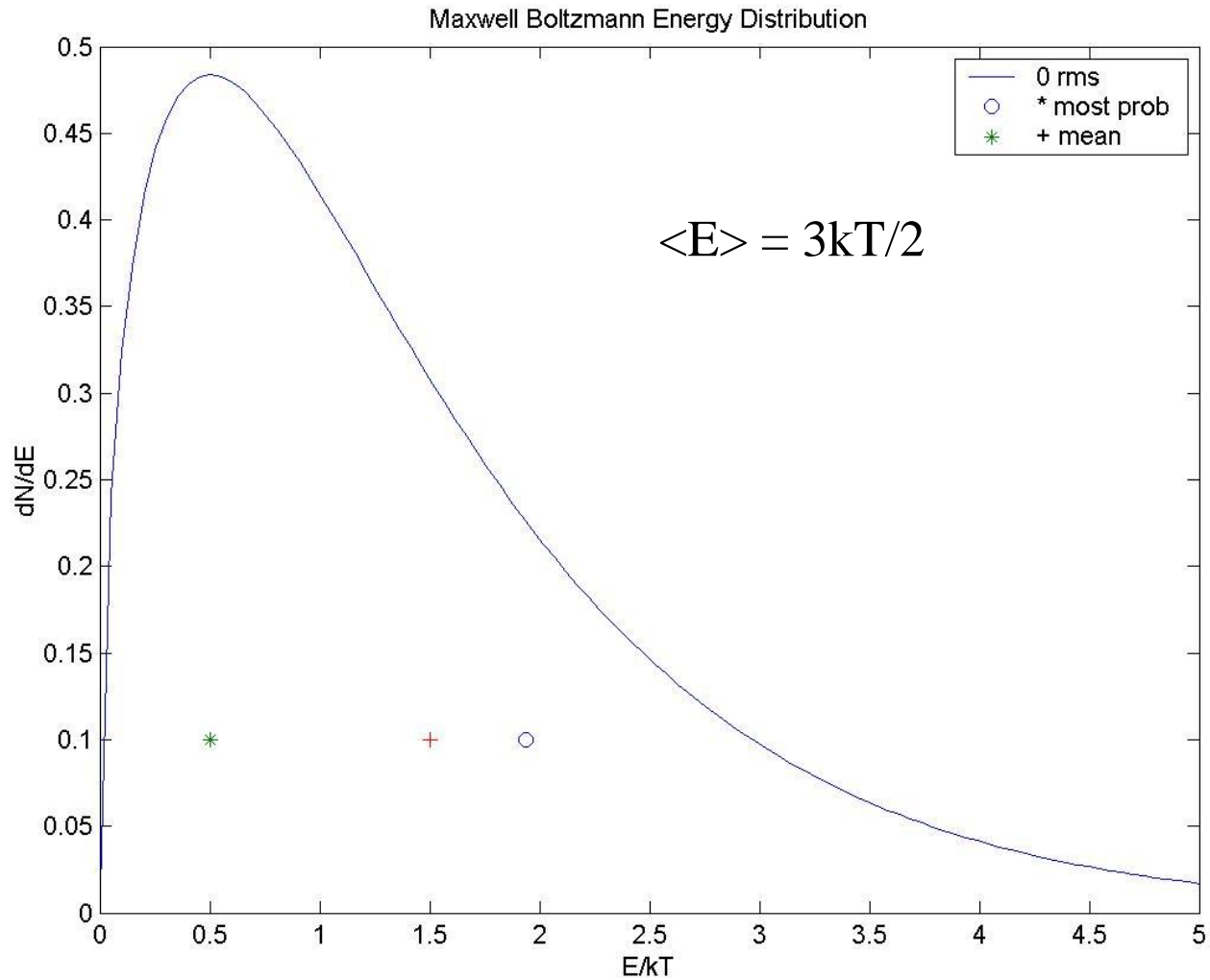
$$\sim \left( \frac{eE}{M} \right) \left( \frac{\langle L \rangle}{v_T} \right)$$



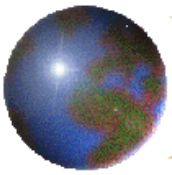
Example of collision cross  
section of e on a gas – in  
units of Å<sup>2</sup>.



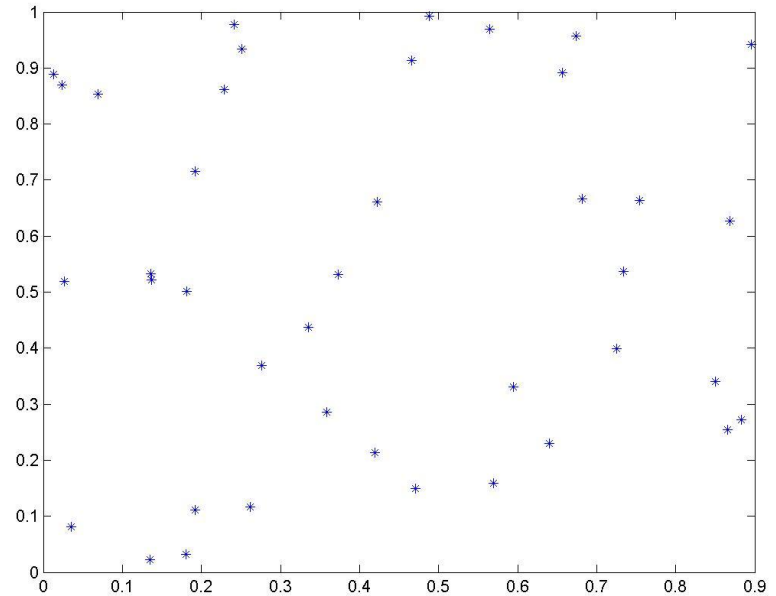
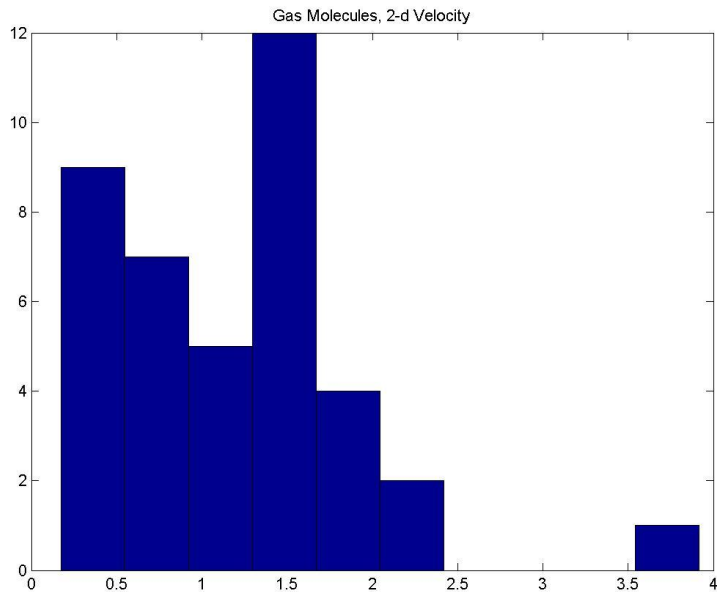
# Demo – Maxwell - Boltzmann



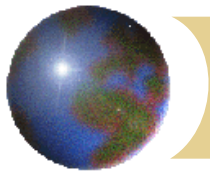




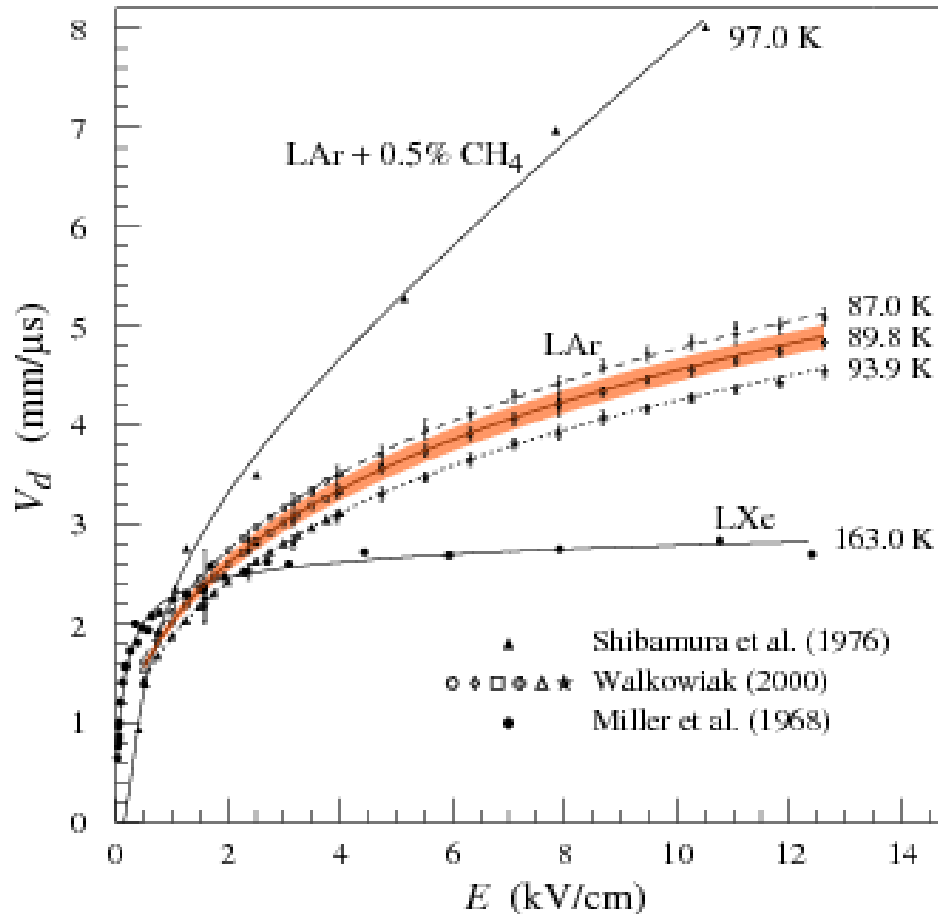
# Demo – Maxwell – Boltzmann II



40 particles in a box – vs  $10^{23}$   
One can vary the volume of the box  
and track the number of wall  
collisions and the momentum  
transfer ~ pressure – ideal gas laws ?



# LAr Drift Velocity



The same analysis applies to liquids. Expect that drift velocity goes as  $\sim E$  and  $1/\sqrt{T}$ . However, cross section also depends on the collision energy.

Scale of velocity is  $\sim$  mm/usec



# Diffusion Eq.

$$\frac{\partial \rho}{\partial t} = -D \partial^2 \rho / \partial x^2$$

$$\rho(x, t) \sim \frac{1}{\sqrt{Dt}} e^{-x^2/4Dt}$$

$$\sigma_{x_T} \sim \sqrt{2Dt} \sim \sqrt{2v_T \langle L \rangle (x / \langle v_d \rangle)}$$

$$\sigma_{x_T} \sim \sqrt{\frac{v_T^2 x}{a}}, \langle v_d \rangle \sim a \langle L \rangle / v_T$$

$$\sigma_{x_T} \cong \sqrt{\left( \frac{2kT}{eE} \right) x}$$

$$\sigma_{x_T} \cong \left[ \sqrt{\frac{2kT}{eV_0}} \right] x$$

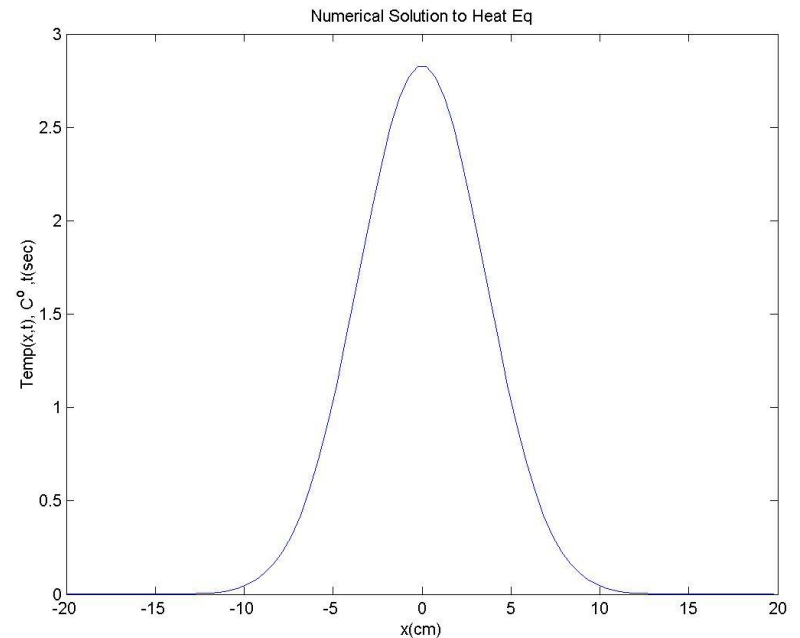
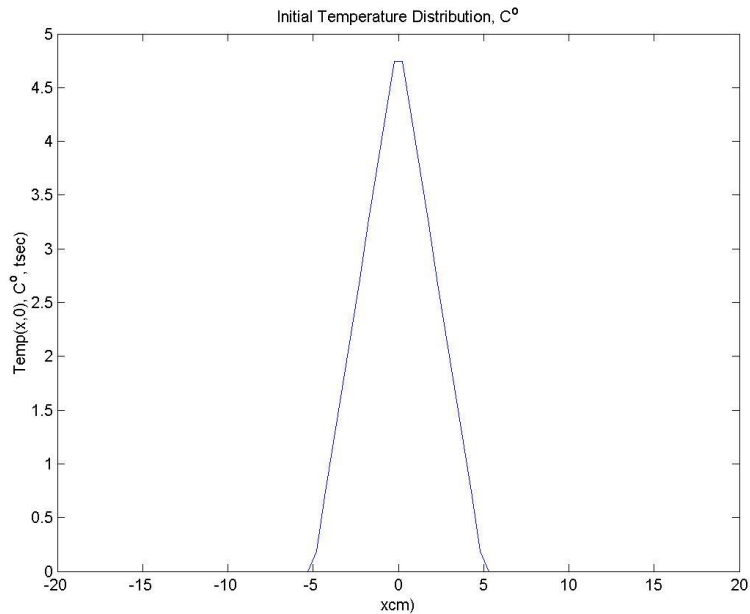
A pointlike charge distribution will spread out as it drifts due to the stochastic nature of the collisions.

The 1-D diffusion Eq. defines a diffusion coefficient D. The longitudinal spread of the charge goes as  $\sim \sqrt{t}$  which is characteristic of random processes.

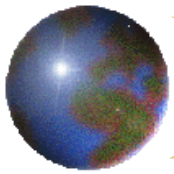
Competition between thermal energy  $\sim kT$  and electric acceleration  $\sim V$



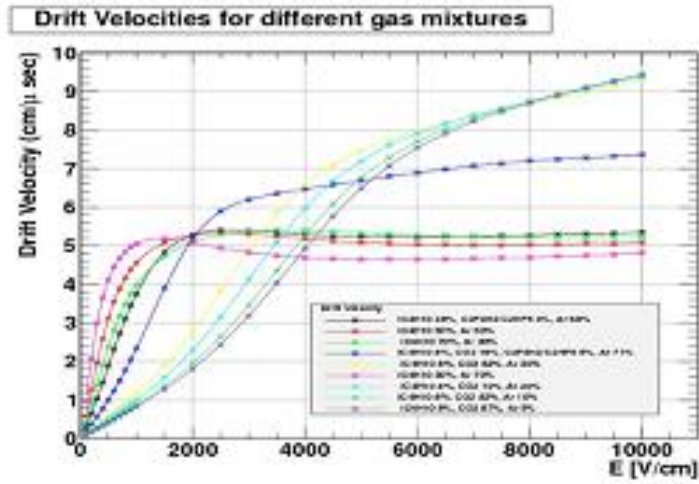
# Demo – Heat Diffusion



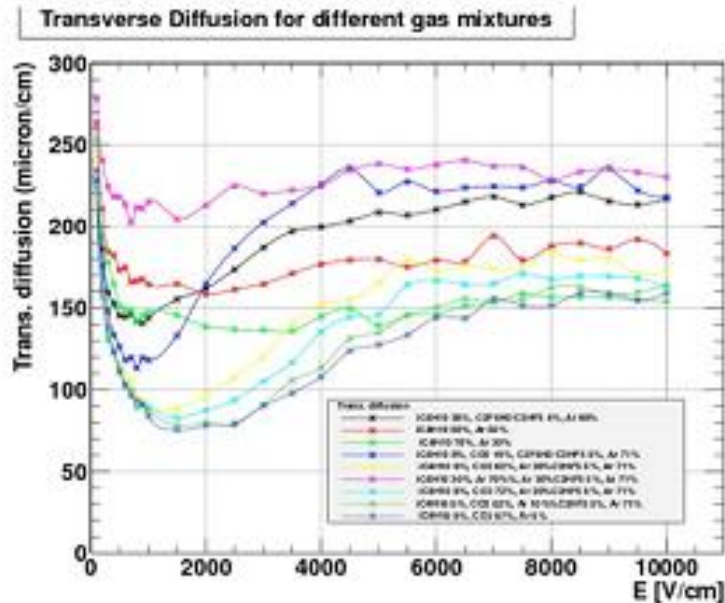
The heat equation has diffusion built in.  
An initial state of temperature spreads out  
with time. (n.b. wave packets in quantum  
mechanics).



# Drift/Diffusion in Gases



For gases with an external field of  $\sim 5$  kV/cm a  $v_d$  of  $\sim 5$  cm/usec is typical.



The transverse size for a 1 cm drift is typically  $\sim 0.2$  mm. This spread will limit the ultimate spatial resolution



# Pulse Formation - I

$$\begin{aligned}dU &= Q_o dQ(t) / C = Fdx \\ &= [q(t)E][\langle v_d \rangle dt]\end{aligned}$$

$$dQ = \frac{q(t)E}{V_o} (\langle v_d \rangle dt)$$

$$q(t) = q_s \text{ for } t < \tau'_d, = 0, t > \tau'_d, \tau'_d \equiv x_o / \langle v_d \rangle$$

$$I(t) = \frac{q_s}{\tau'_d}, t < \tau'_d = 0, t > \tau'_d$$

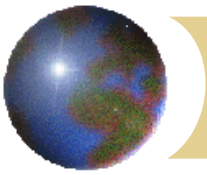
$$\begin{aligned}Q(t) &= q_s t / \tau'_d \\ &\leq q_s (x_o / d)\end{aligned}$$

Current is constant.  $q_s$  is ionization charge,  $Q(t)$  is  $\sim t$  and is the electrode charge

Look at ionization in “unity gain” devices.

Devices have energy  $U$  stored in an applied  $E$  field. Ionization charge causes a change in energy,  $dU$  as it moves in  $E$  to the collecting electrode.

Source charge  $q_s$  is produced at a point and drifts to electrodes a distance  $x_o$  in time  $\tau'_d$ .



# Pulse Formation - II

$$dQ(t) = q(t) \langle v_d \rangle dt [E / V_o] = [q(t) \mu E^2 / V_o] dt$$

$$\frac{dQ(t)}{dt} \equiv I(t) = q(t) \mu E^2 / V_o$$

$$q(t) = q_s \left( 1 - \frac{t}{\tau_d} \right), \quad t < \tau_d$$
$$= 0, \quad t > \tau_d$$

$$\tau_d = d / \langle v_d \rangle$$

$$I(t) = (q_s / \tau_d) (1 - t / \tau_d)$$

$$\int I(t) dt \equiv Q(t)$$

$$= q_s [y - y^2 / 2], \quad y = t / \tau_d$$

Line ionization. In this case the ionization is a line from electrode to electrode – a “gap” of length  $d$ .

$q(t)$  is charge remaining in the “gap” – decreases linearly as charge is swept up by the field.  $I(t)$  is electrode current, decreasing linearly.



# Lorentz Angle

$$\vec{F}_F = \vec{p} / \tau, \quad \vec{F} = q(\vec{E} + \vec{\beta} \times \vec{B}) + mc \vec{\beta} / \tau = m\vec{a}$$

$$\vec{a} = 0 \text{ when } \vec{v} = -\frac{q\tau}{m} (\vec{E} + \vec{\beta} \times \vec{B})$$

$$\langle \vec{v}'_d \rangle = \frac{-e\tau / m}{[1 + (\omega_c \tau)^2]} [\vec{E} + (\omega_c \tau) \hat{B} \times \vec{E} + (\omega_c \tau)^2 (\hat{B} \cdot \vec{E}) \hat{B}]$$

$$\omega_c = eB / m$$

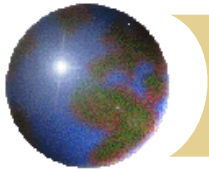
$$\vec{E} \perp \vec{B}$$

$$\tan \varphi_L = \omega_c \tau$$

Angle in E,B  
plane between  
E and  $v_d$ ,  
caused by B

For ionization drifting in combined E and B fields the path is more complicated. Take tau as the mean time between collisions where the ionization responds to E and B and use that to represent the force for diffusion. The new variable is the cyclotron frequency which describes the NR circular motion in the B field. Note that in strong B fields the diffusion of the ionization can be greatly reduced – TPC.



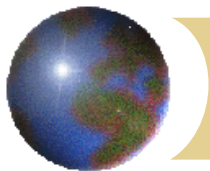


# B and Reduced Diffusion

$$\langle \vec{v}'_d \rangle = \frac{[\vec{v}_d + \omega_c \tau (\hat{B} \times \vec{v}_d)]}{[1 + (\omega_c \tau)^2]}$$

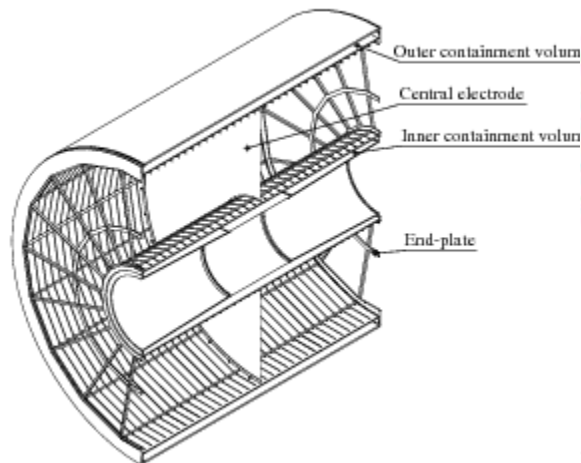
$$\langle v'_d \rangle = \langle v_d \rangle / \sqrt{1 + (\omega_c \tau)^2}$$

TPC – drift long distances with much reduced diffusion of the charge. The drift time is longer, but the diffusion of charge is reduced to that the position resolution of the ionization is improved.



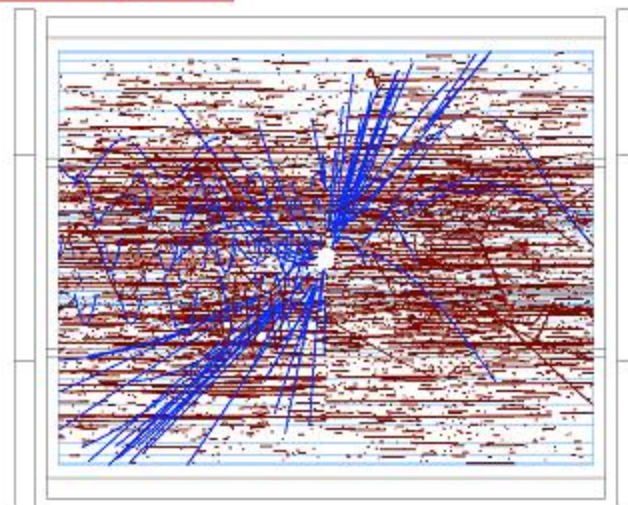
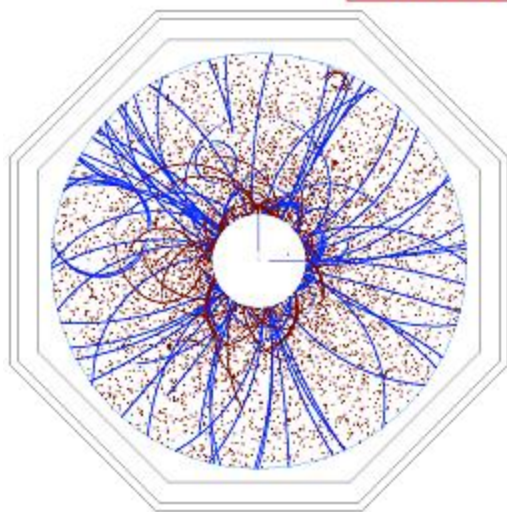
# TPC Construction

## Background: TPC

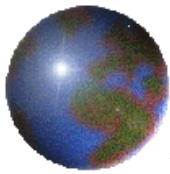


- ★ Simulated 2000 bunch crossings (BXs) of beam background
- ★ For TPC, conservatively take drift velocity to be  $4 \text{ cm } \mu\text{s}^{-1}$
- ★ Therefore fill TPC with 150 BXs of background shifted in  $z$
- ★ First order attempt to merge unresolvable hits
- ★ Superimpose on fully-hadronic top-pair events at 500 GeV

150 BXs of pair background

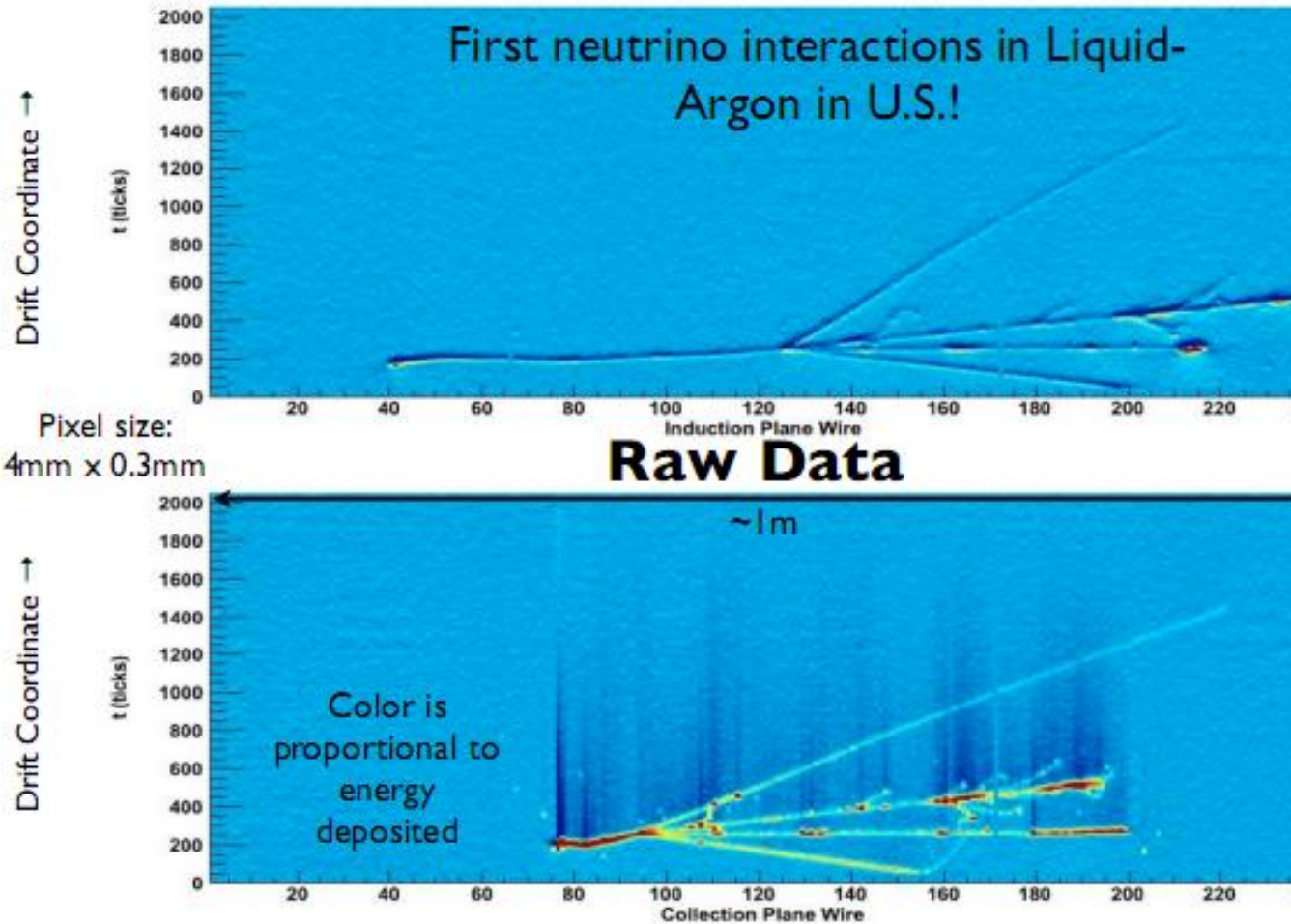


ILD proposal for ILC detector. The time to collect the charge is not a severe constraint at the ILC



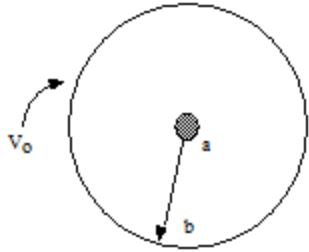
# ArgoNeut

## ArgoNeuT Neutrino Event





# Pulse Formation in PWC



$$E = 2\lambda / r$$

$$V_o = 2\lambda \ln(b/a), \quad V(r) = V_o \ln(r/a) / \ln(b/a)$$

$$E(r) = \frac{(V_o / r)}{[\ln(b/a)]}, \quad v_d = dr / dt = \mu(2\lambda / r)$$

E near sense wire

$$dN(r) = N(r) \alpha dr$$

$$N(r) = N_o e^{\alpha r}$$

$$\langle I \rangle_A \sim 26 \text{ eV}$$

E multiply by impact in high E near the wire  
 ~ point ionization. N(r) is the “gas gain” –  
 typical ionization potential in the gas.

$$dV = \frac{q_s}{CV_o} E(r) dr$$

$$V^- = \int_a^{Na} dV,$$

$$V^+ = \int_{Na}^b (-dV)$$

$$V^- = \frac{q_s}{C} \frac{\ln(N)}{\ln(b/a)}, \quad V^+ = \frac{q_s}{C} \frac{\ln(b/Na)}{\ln(b/a)} \gg V^-$$

$$\int_a^r r dr = \mu[2\lambda] \int_0^t dt$$

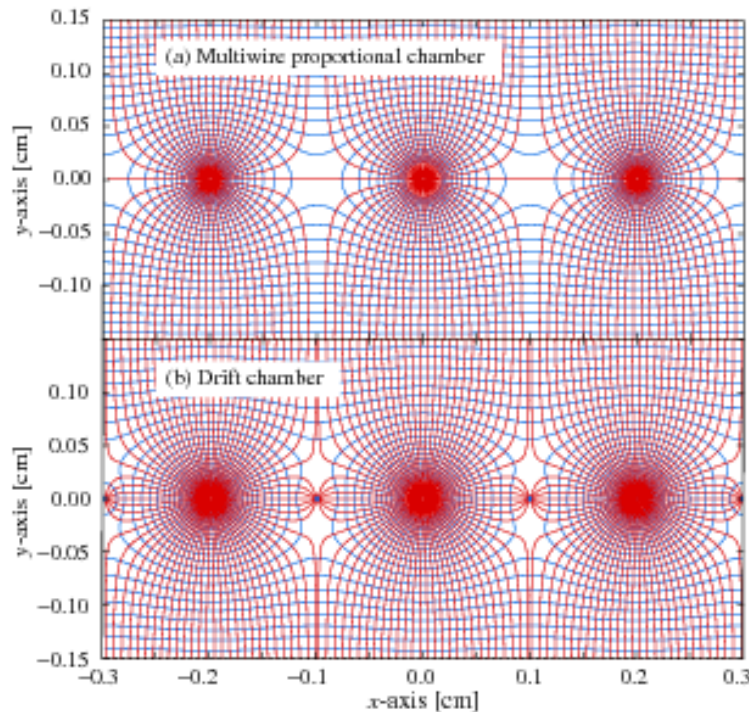
$$r = a\sqrt{1 + 4\mu\lambda t / a^2}$$

$$\equiv a\sqrt{1 + t / \tau_o}, \quad \tau_o = a^2 / 4\mu\lambda$$

Motion of e to the wire and positive ions to the wall. Ion motion makes for the detected signal. Ions with mobility  $\mu$  move in E field defined by  $\lambda$ .



# PWC/Drift Electrostatics

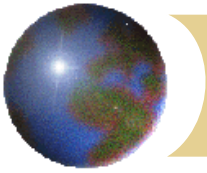


In a drift chamber there are both sense wires and field shaping wires to provide a  $\sim$  uniform E field  $\Rightarrow$  a constant drift velocity so that a time of arrival of the ionization  $\Rightarrow$  the distance from the wire.

Not a unity gain device like LAr. Gas multiplication near the wire in the high field region of the PWC. The pulse has a short rise time and a long tail. The characteristic time is set by drifting a distance  $\sim a$  in a field  $E(a)$  ;

$$\mu = \langle v_d \rangle / E \sim a / (\tau_o E(a))$$

$$\tau_o \sim a / \mu E(a) = a^2 / 2\mu\lambda$$



# Pulse Formation – II

$$I(t) = q_s \mu E^2 / V_o$$
$$= \frac{q_s \mu (2\lambda / r^2)}{\ln(b/a)}$$

$$I(t) = \frac{q_s (2\lambda\mu / a^2)}{\ln(b/a)(1+t/\tau_o)}$$

$$I(t) = \left[ \frac{q_s / 2\tau_o}{\ln(b/a)} \right] / (1+t/\tau_o) = I(0) / (1+t/\tau_o)$$

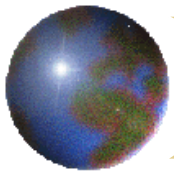
$$E(a) = 160 \text{ kV} / \text{cm}$$

$$\mu_A \sim 1.5 \text{ cm}^2 / (\text{V} \cdot \text{sec})$$

$$\tau_o = 8.3 \text{ nsec}$$

A typical time constant value

Time structure of the PWC pulse. There is a rapid rise followed by an inverse t falloff with a characteristic time constant.



# Induced Charges - Pads

$$E_{11} = 0$$

$$E_T(r) \sim (q_s / r^2)(2a / r) \sim \sigma(r)$$

$$q_p = \int_{-d/2}^{d/2} \int_{-\infty}^{\infty} (2q_s a / r^3) dy dz$$

$$= q_s (\theta_1 - \theta_2) / \pi, \quad \pm d / 2a = \tan(\theta_{1,2})$$

$$q_p / q_s = \frac{\tan^{-1}(d / 2a)}{\pi} \rightarrow \frac{1}{2}$$

The motion of the ions capacitively induces a charge on the other electrode. Used to get an additional “pad” signal orthogonal to the wire => “3d readout” – e.g. TPC

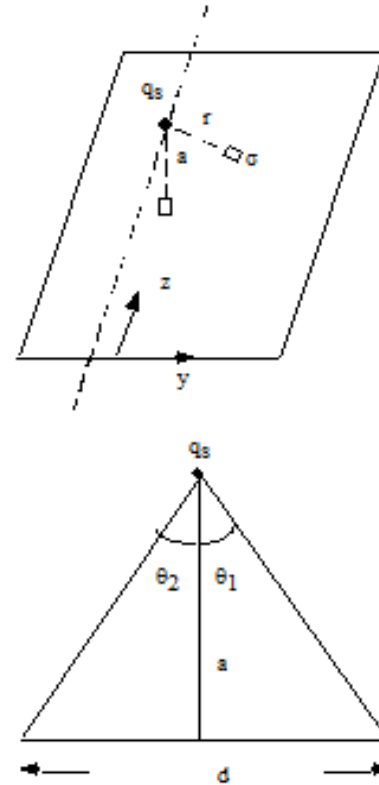
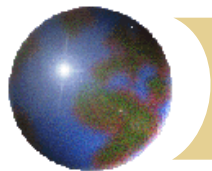


Fig. 8.15: a) Geometry for deriving the induced surface charge on a pad,  $\sigma$ , for source charge  $q_s$  at height =  $a$  above the pad. b) Geometry for infinitely long strip electrode of width  $d$ .



# Pad Charges – 3d

wire

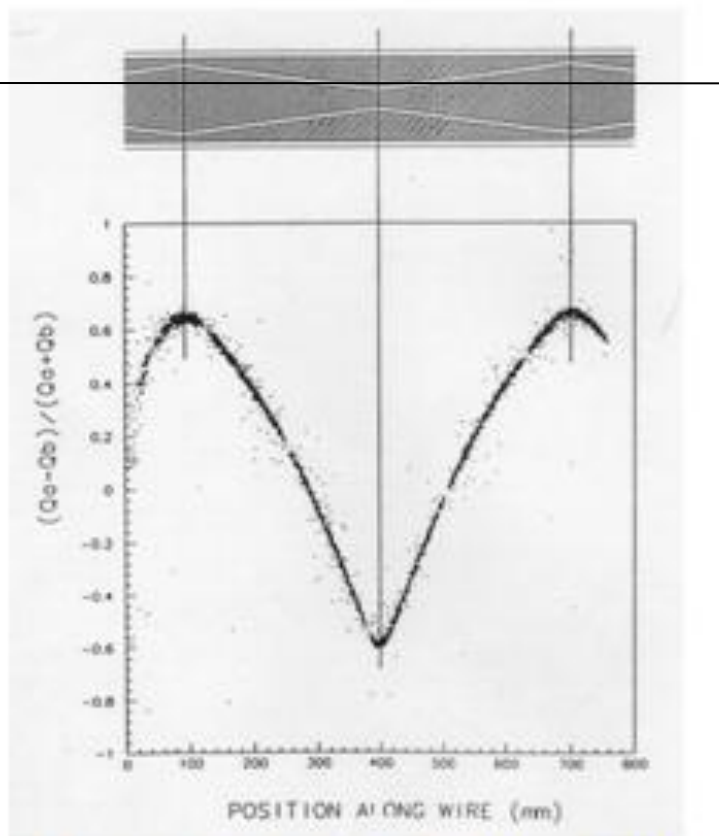


Fig. 8.16: Relationship of the pad charge ratio to the position along the wire for the structure shown in Fig. 8.6. (From Ref. 11, with permission.)

Use the electrodes needed for field shaping in a drift chamber to provide another independent coordinate. In this specific case, the distance along the wire.