Fundamentals of Detector Physics and Measurements Lab - II

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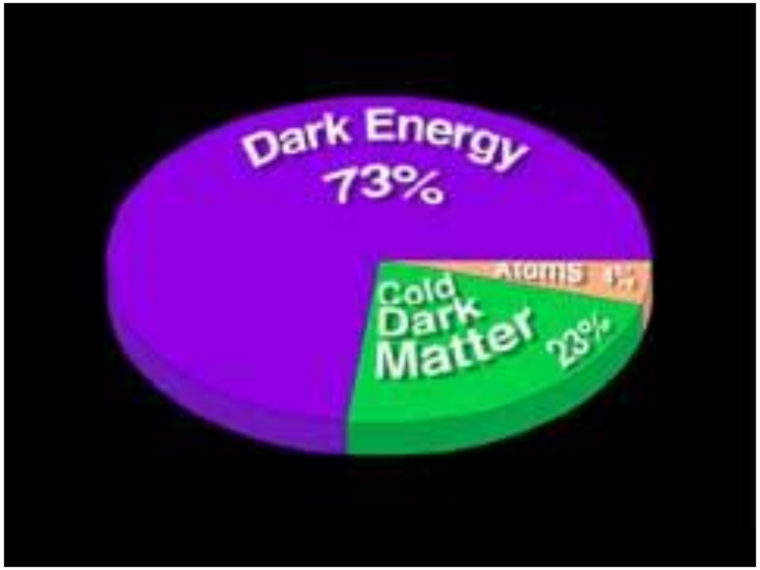
Outline

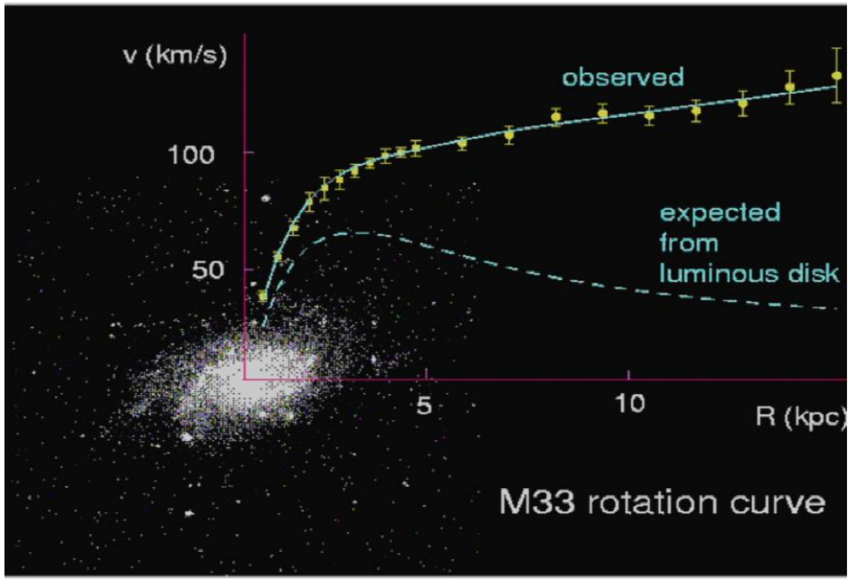
- Lecture I
  - Constants, atoms, cross sections
  - Photoelectric, TOF
  - PMT, SiPM Scint, Cerenkov
- Lecture II
  - Collisions, cross sections
  - Multiple scattering, radiation length
  - dE/dx, MIP, Range
  - Critical Energy

Outline II

- Lecture III
  - B fields, trajectories
  - Quadrupoles, focal length
  - Drift and Diffusion
  - Pulse formation in unity gain and gas gain
- Lecture IV
  - Radiation NR, Thompson, Compton
  - Relativistic radiation
  - Bremm, Pair Production

Dark Matter ?

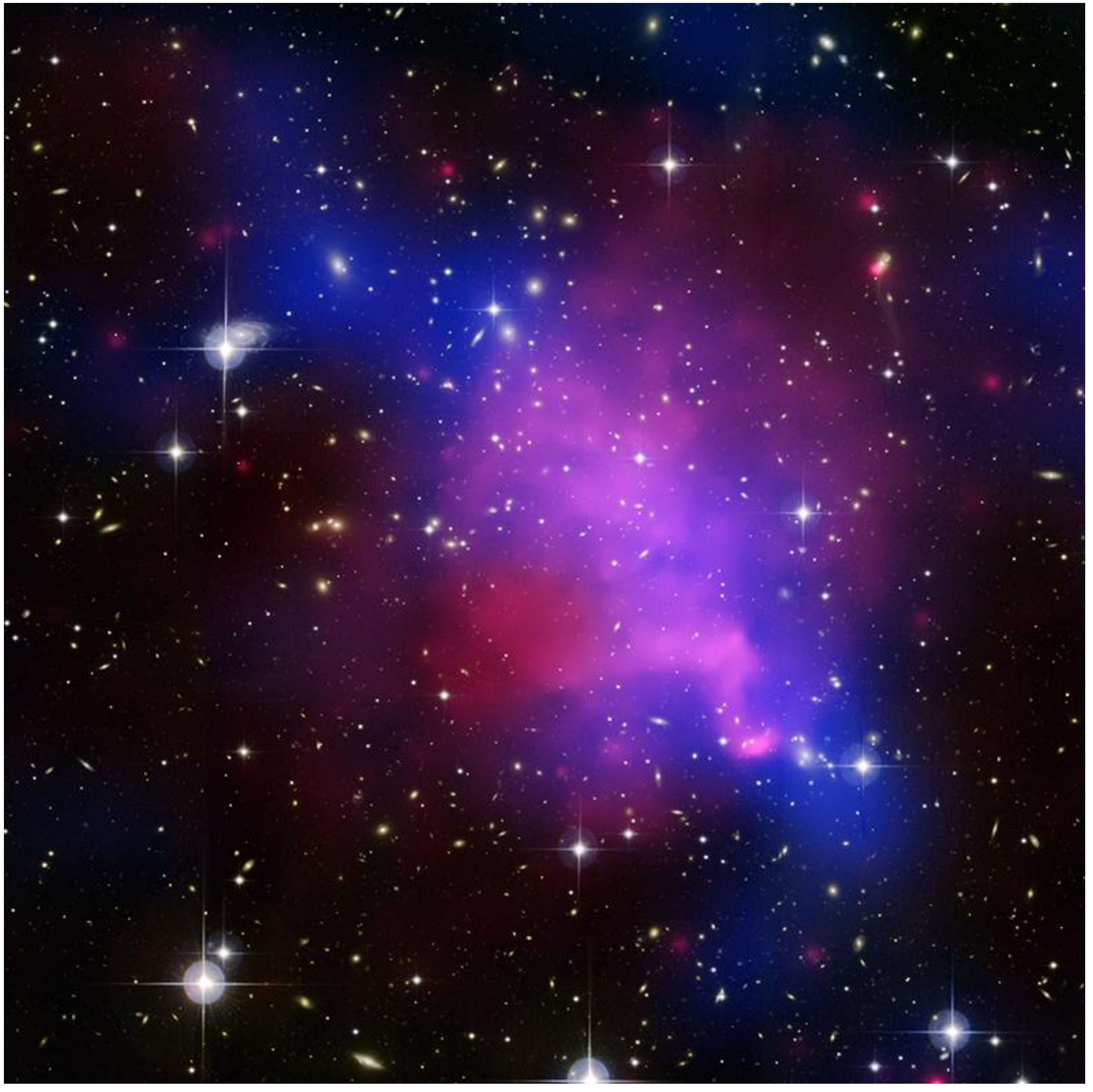




$$v \sim r, v \sim 1/\sqrt{r}$$

There is no SM candidate for DM. Can we produce DM in the LHC? Is it the SUSY LSP? Searches at LHC, in direct (recoil) measures – e.g. CDMS, in annihilations - e.g. FERMI, PAMELA , may find new aspects of DM. The LHC is actively searching for SUSY particles.

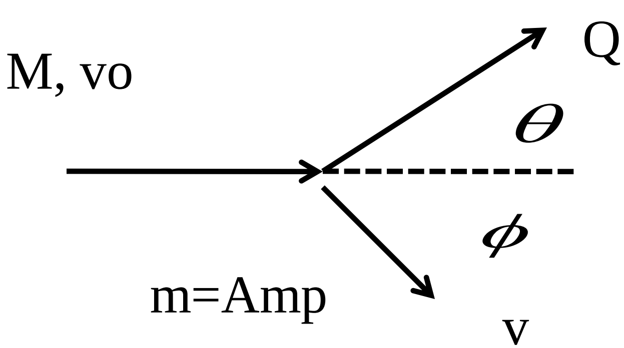
DM and Colliding Galaxies



Galaxy has a visible component and a dark component which interacts only gravitationally (see by lensing). Colliding galaxies show different interactions of the neutral DM and the visible matter, which has EM interactions.

Wimp – Elastic Recoils

- Cross sections ~ 10^-44 cm^2. M = wimp mass ~ 100 GeV.
- Measure the rate of recoils and the Q dependence.
- Vary the target mass m.
- Do not know the density of Wimps or the velocity distribution vo



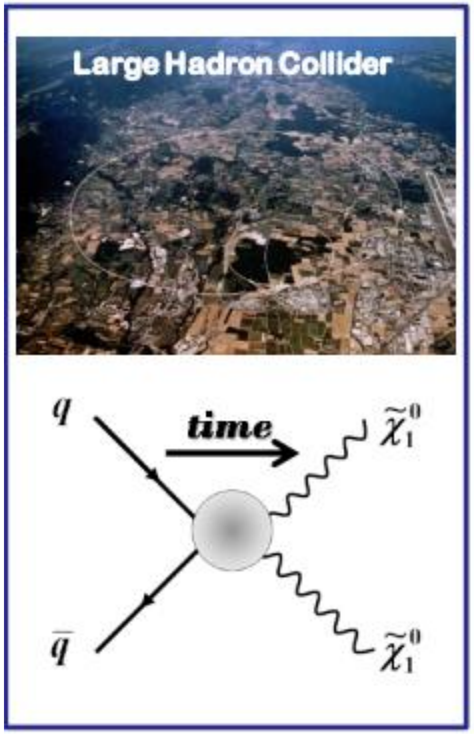
$$\mu = Mm/(M+m)$$

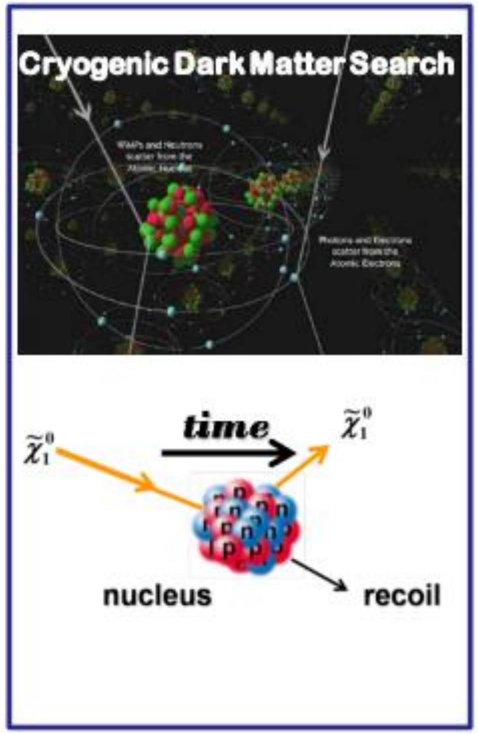
reduced mass

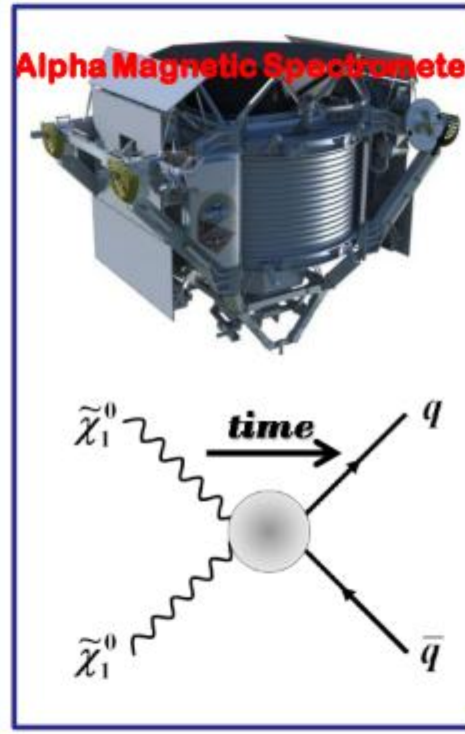
Wimp Recoils - II

- T + p conservation
- $v = (2\mu / m) v_0 \cos\theta$ Do not know vo or angle
- Spin independent cross section, Q = recoil K.E. Nuclear FF is ~1, coherent over nucleus.
- Use different nuclei to break “degeneracy”.
- Integrate over vo from min value to Inf. – vo ~ 200 km/sec => Q ~ 10 keV.

Dark Matter Searches -“Full Court Press”







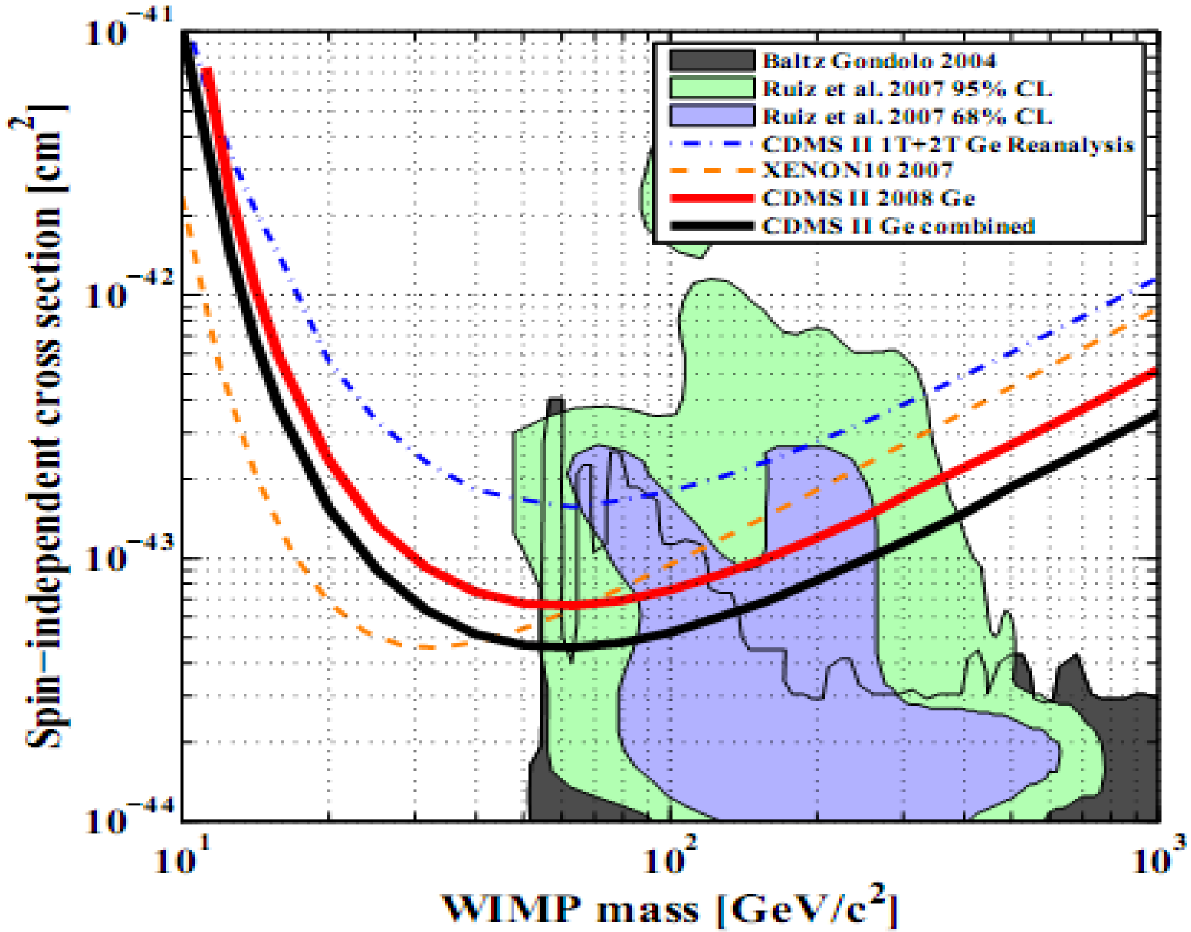
Produce it — **LHC**

Scatter it — **FNAL**

Observe it — **annihilate**

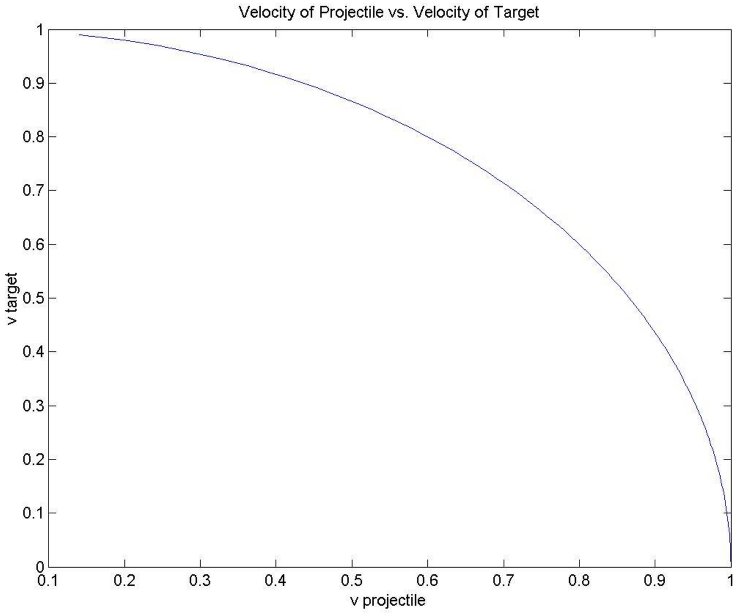
These 3 could converge, making a tremendous scientific breakthrough

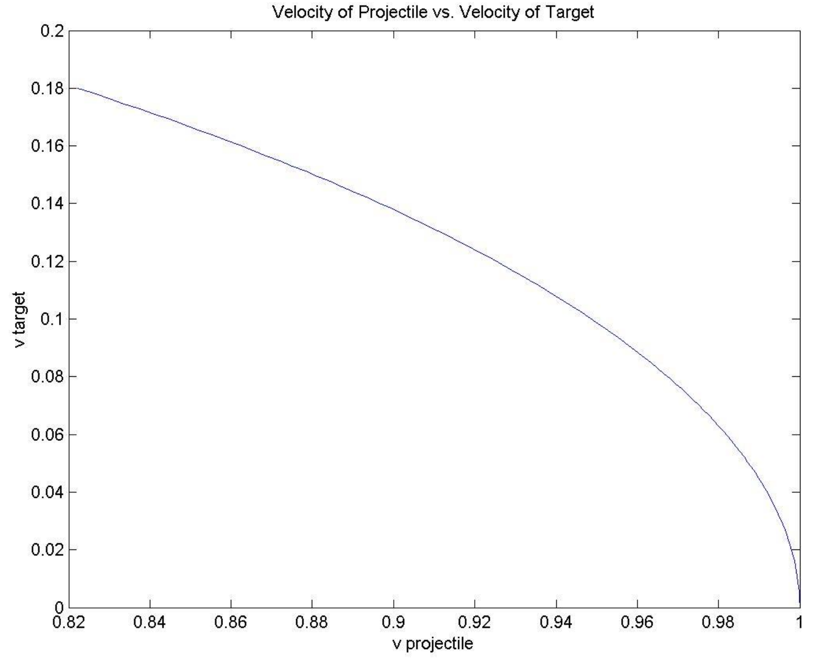
DM - CDMS



Solid State or Nobel Liquids – scaleable to the needed tonnage? Expect decisive results soon.

Demo -Scattering – Target Mass



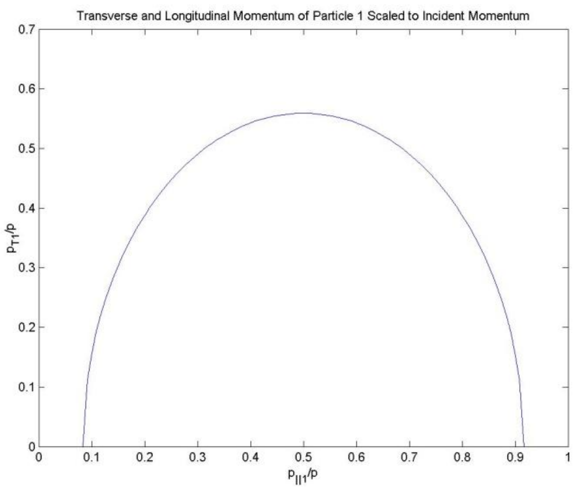


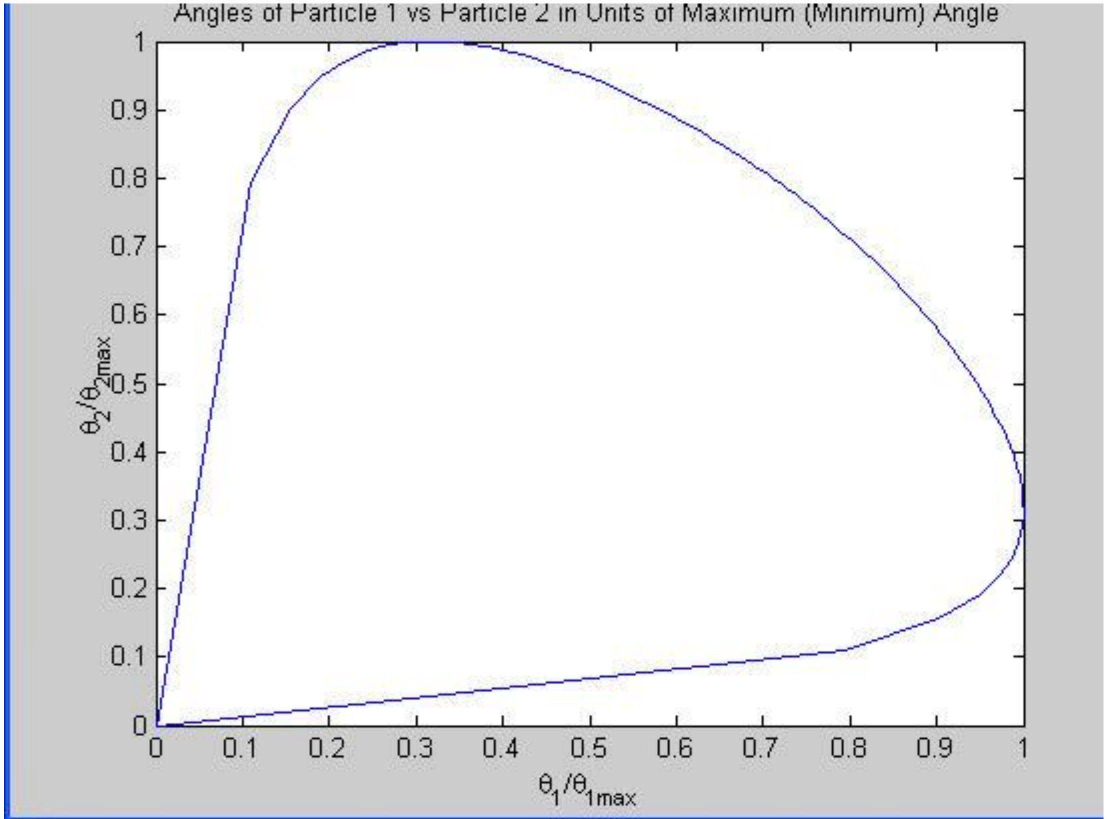
M = 1

M = 10

What are the differences in scattering as the mass ratio of projectile to target varies – think pool sharks…

Demo – SR and Decays



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Enter Mass of Target (Projectile m =1) or Decay Particle (Daughter m = 1) 3
Enter Momentum of Incoming Particle in m=1 Units 2

qq =

     2

CM Energy = 3, beta CM = 0.894427, gamma CM = 0.745356, P in CM = 1.11803
Maximum(Minimum) Angles for Particle 1 = 1.11157 (rad) and 2 = -1.11157
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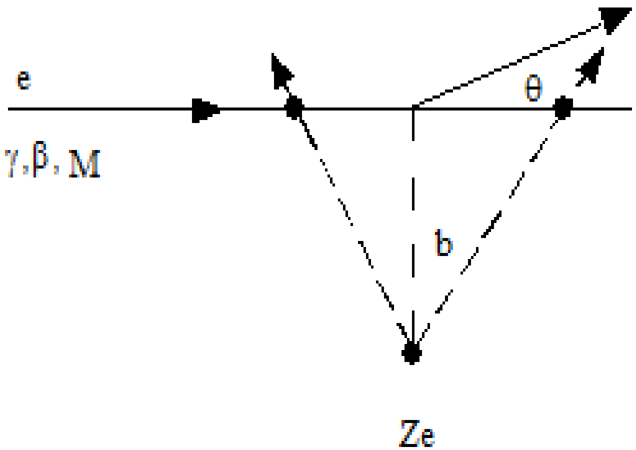
In SR you can convert mass to energy => unstable particles can decay into sets of lighter particles.

Coulomb Collision



$$F(b) = Ze^2 / b^2$$

$$\Delta t = 2b / v$$

$$\Delta p_T \sim F(b)\Delta t, \;\; \vec{F} \equiv d\vec{p} / dt$$

$$\theta \quad \sim \Delta p_T / p, \;\; \Delta p_T = 2Z\alpha / bv$$

$$\theta_R \quad \sim 2Z\alpha / pvb$$

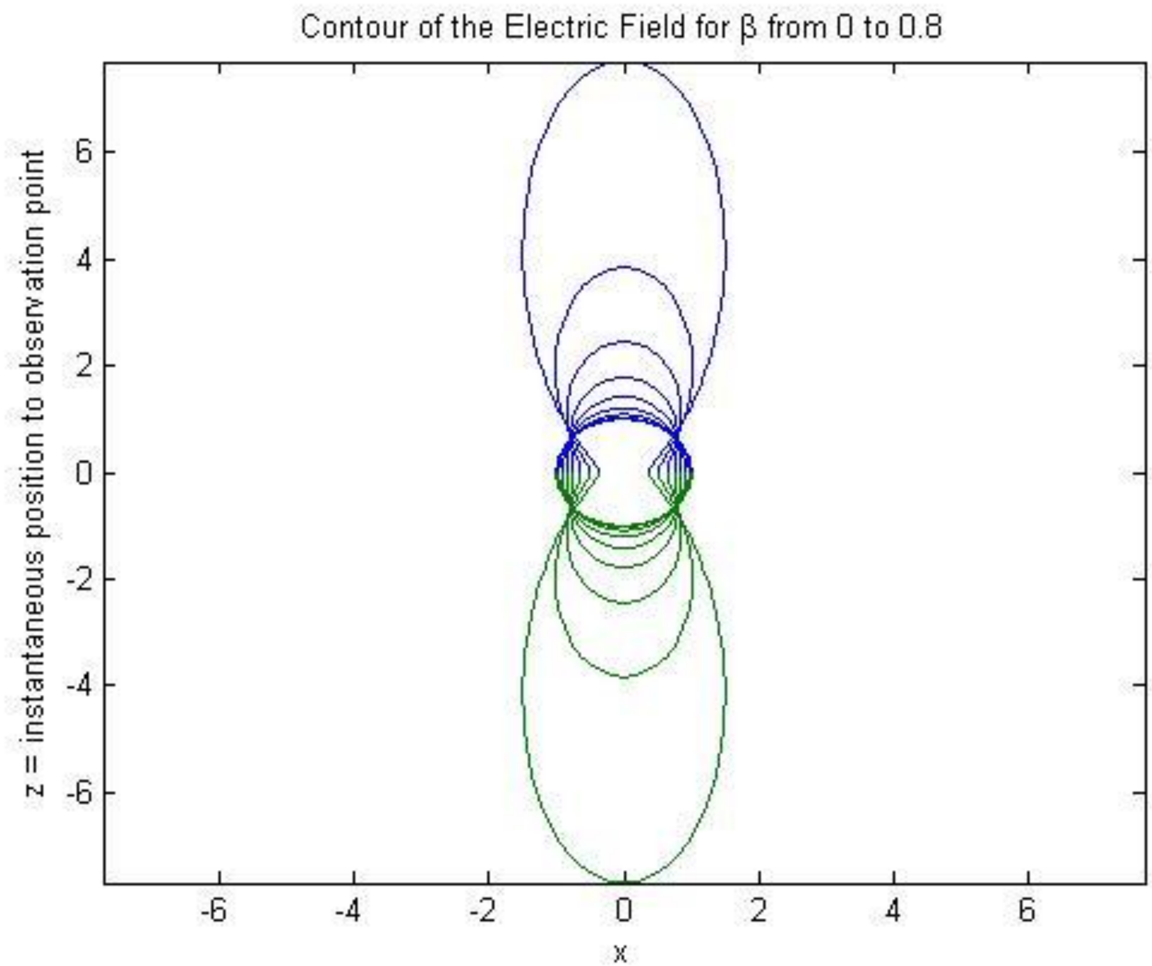
The force is active only for r ~ b which occurs for a time ~ 2b/v. The momentum transfer goes as 1/v due to scattering off the Coulomb field of the nucleus.

$$E_T(b) \sim \gamma$$

$$\Delta t \quad \sim 2b / \beta\gamma c$$

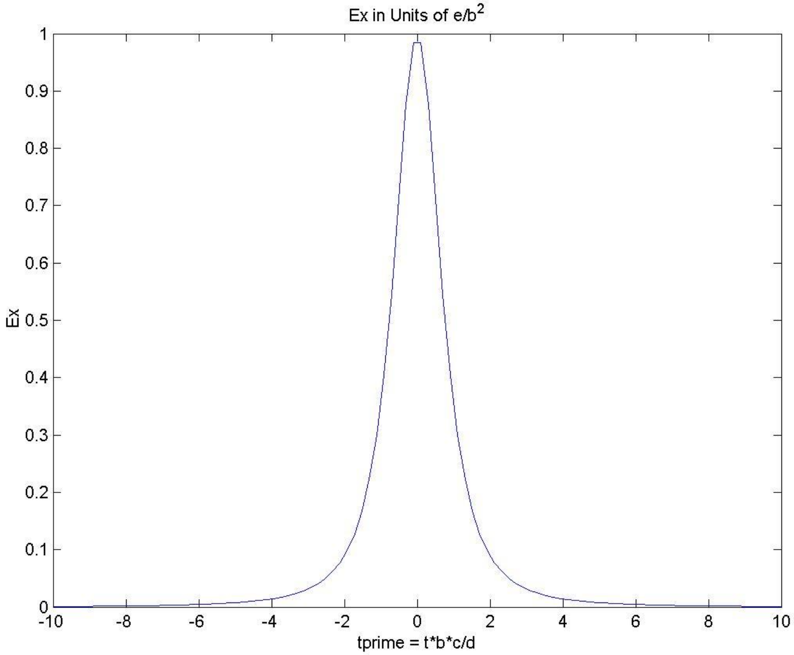
$$\Delta p_T \sim \; e\,E_T(b)\,\Delta t \sim cons\tan t$$

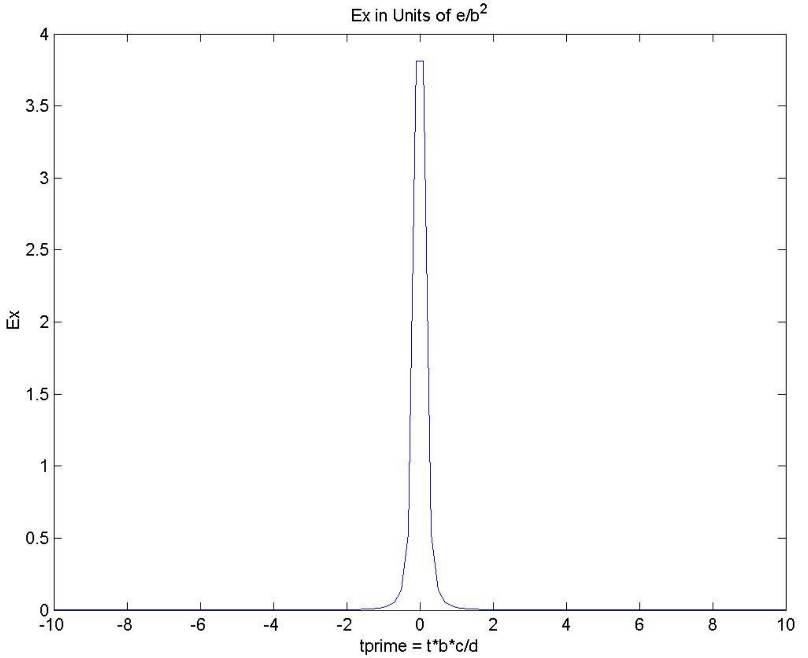
Demo – Electric Field in SR



As the velocity increases the transverse E field increases in peak value. However the time over which the field is active decreases by the same factor => MIP

Demo – Energy Transfer



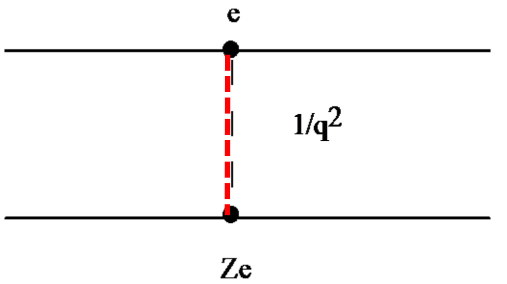


v/c = 0.01

v/c = 0.99

Note the growth of the vertical E field scale and the shrinkage of the time scale. Note time scale is in b/v units expected in NR mechanics.

Impact -> Cross section



$$dP \sim d\vec{b} = d\sigma$$

$$d\sigma \sim bdbd\varphi = \frac{d\sigma}{d\Omega}d\Omega, \quad d\Omega = \sin\theta\, d\theta d\varphi$$

$$\frac{d\sigma}{d\Omega} = \frac{b}{\sin\theta}\left(\frac{db}{d\theta}\right)$$

$$q \sim \Delta p_T \sim p\theta$$

$$A(q) \sim (Ze)e/q^2$$

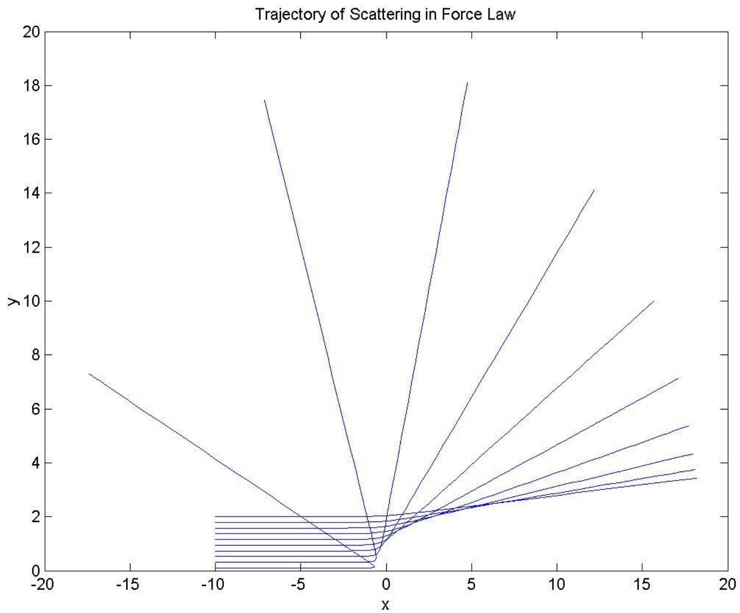
$$\left|A(q)\right|^2 \sim (Z\alpha)^2/\theta^4$$

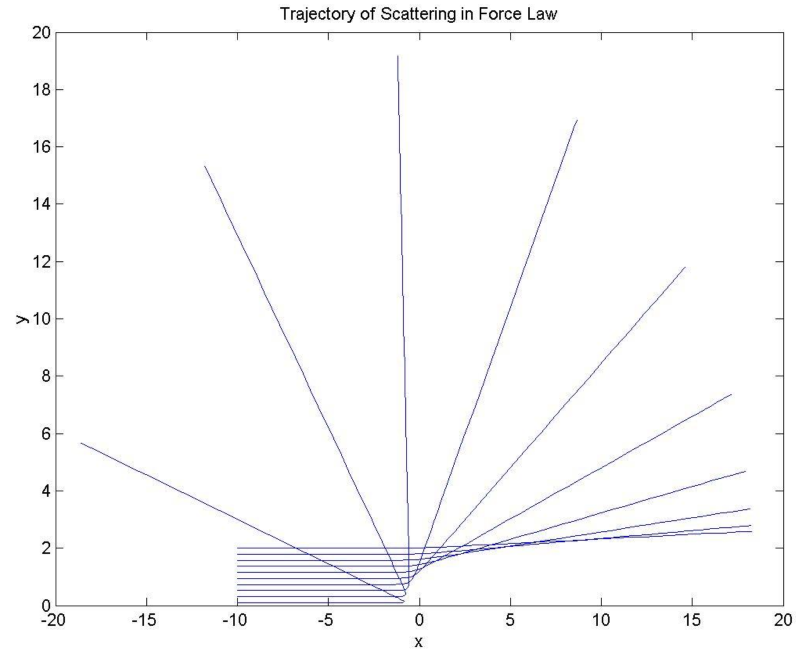
$$\frac{d\sigma_R}{d\Omega} \sim \left(\frac{Z\alpha}{Mv^2}\right)^2/\theta^4$$

Coulomb collision

Basic idea is that you cannot ‘aim” so that all area elements w.r.t. the target are equally probable. Then change variables from b (unknown) to angle (measured)

Demo - Rutherford



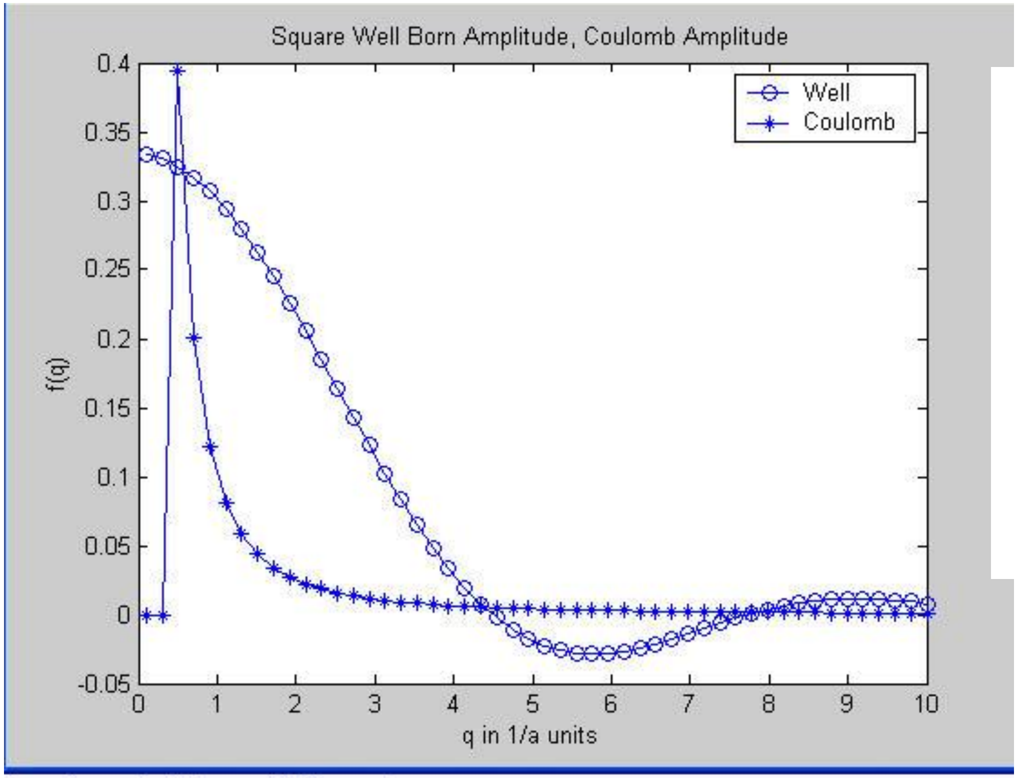


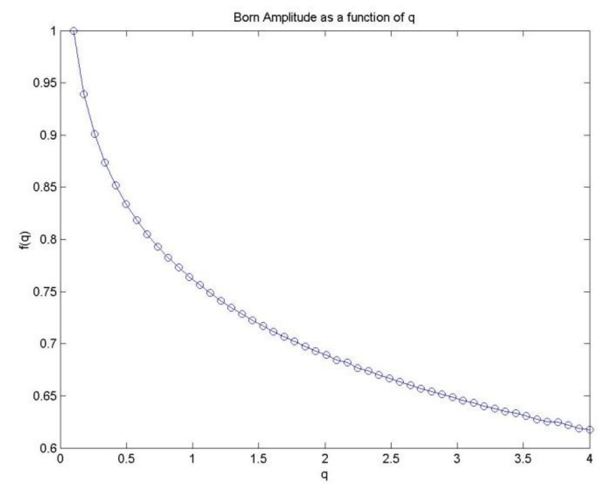
F~1/r^2, Attractive

F~ 1/r^4, repulsive

Different force laws give different relationships between b and scattering angle. Use scattering to infer the force law – particle physics.

Demo – Born Approx





V=1/r^3

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Example - 3-d Square Well, r < a

                         -sin(q a) + q a cos(q a)
                       - ------------------------
                                     3
                                    q
Example - Coulomb, No e Screening Screening

                                 cos(q r)
                               - --------
                                     2
                                    q
```

The Born approximation assumes plane waves for incoming and outgoing, scattered, waves. Again, different angular distributions for different targets.

Multiple Scattering

$$\theta_{\min} \sim (2Z\alpha)/pv\,a_o$$

$$\sigma \sim \int_{\theta_{\min}}^{\infty} \left(\frac{d\sigma}{d\Omega}\right) 2\pi\,\theta d\theta = \int_{o}^{a_o} 2\pi b db =$$

$$\sim \pi a_o^2 \sim 1/\theta^2{}_{\min} \sim \int_{\theta_{\min}}^{\infty} d\theta/\theta^3$$

$$\left\langle \theta^2 \right\rangle \equiv \frac{\int \theta^2 \frac{d\sigma}{d\Omega} d\Omega}{\int \frac{d\sigma}{d\Omega} d\Omega} \sim \frac{\int (d\theta/\theta)}{\int (d\theta/\theta^3)}$$

$$\sim 2\,\theta_{\min}^2 \left[\ln(\theta_{\max}/\theta_{\min})\right]$$

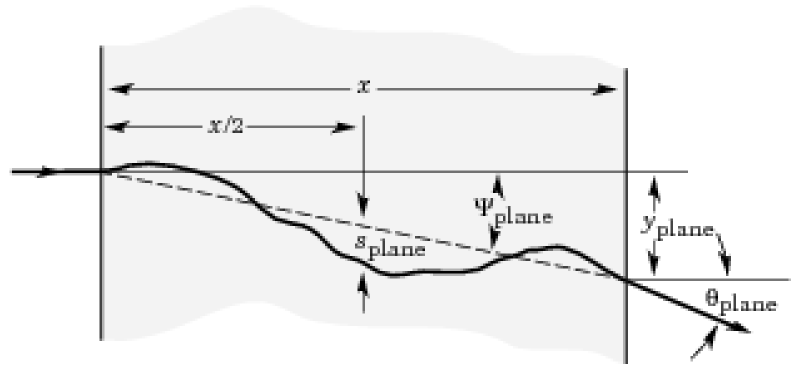
Formally Rutherford scattering cross section diverges. However, it is cut off when the e shield the nuclear charge – at impact parameter ~ the Bohr radius. There are many small angle scatters at large impact parameters. The mean scattering angle is therefore close in value to the minimum angle.

Multiple Scattering - II

$$< \theta^2_{MS} > = N < \theta^2 >$$

$$N = \left(N_o \rho \sigma / A \right) dx, \; (Lecture I)$$

$$= dx / < L >$$



$$< \theta^2_{MS} > \; = \left[\frac{N_o \rho dx}{A} \right] 2\pi \left[\frac{2Z\alpha}{p\beta c} \right]^2 \left[\ln (\;) \right]$$

$$\left\langle \theta^2_{MS} \right\rangle \; = \frac{dx}{X_o} m^2 / \left(\alpha \beta^2 p^2 \right)$$

$$X_o^{-1} = \frac{16}{3} \left(\frac{N_o \rho}{A} \right) \left(Z^2 \alpha \right) \left(\alpha^2 / m^2 \right) \left[\ln () \right]$$

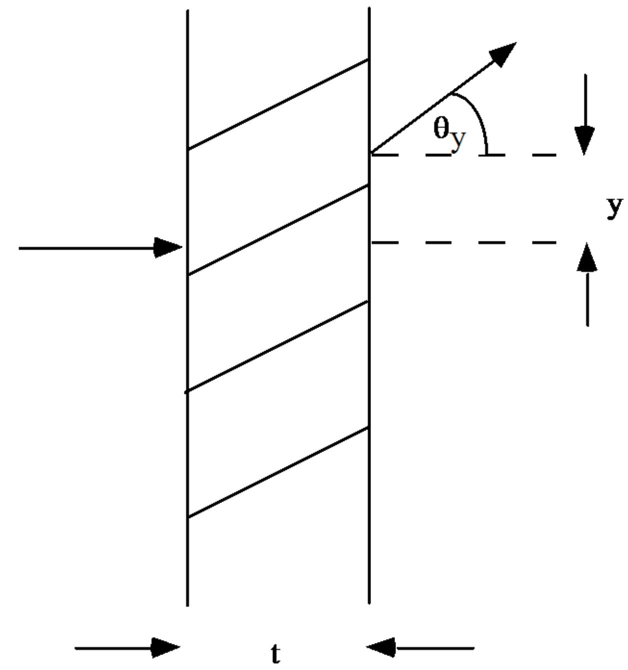
Consider N scatters at small angle. Use Coulomb cross section. Define a length = the radiation length (arbitrary for now).

Radiation Length

$$E_s \equiv \sqrt{\frac{4\pi}{\alpha}}\,(mc^2) = 21\ MeV$$

$$\sqrt{\langle \theta_{MS}^2 \rangle} = \frac{E_s}{p\beta}\sqrt{\frac{dx}{X_o}}$$

It is conventional to define the scattering energy



$$\theta_{MS} = (\Delta p_T)_{MS}/p, (\Delta p_T)_{MS} = \frac{E_s}{\beta}\sqrt{\frac{dx}{X_o}} = \frac{E_s}{\beta}\sqrt{t}$$

$$M_{MS} = \begin{bmatrix} \langle \theta_y{}^2 \rangle & \langle \theta_y y \rangle \\ \langle \theta_y y \rangle & \langle y^2 \rangle \end{bmatrix}, \begin{bmatrix} \theta_y \\ y \end{bmatrix}$$

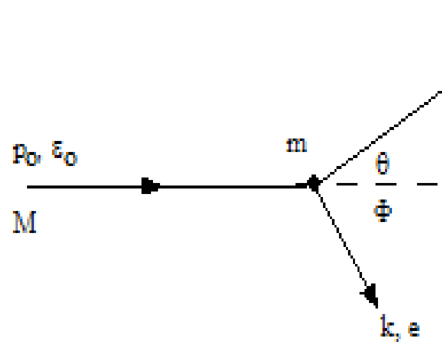
$$= \begin{bmatrix} 1 & t/2 \\ t/2 & t^2/3 \end{bmatrix} \langle \theta_y{}^2 \rangle$$

$$M'_{MS} = \begin{bmatrix} 1 & 0 \\ 0 & t^2/12 \end{bmatrix} \langle \theta_y{}^2 \rangle \begin{bmatrix} \theta_y \\ y_s \end{bmatrix}$$

Define an overall transverse momentum impulse given by the N scatters in the material.

Larger scatters lead to larger displacements – correlations.

NR/SR Collisions



90 degrees

Fig. 5.5: Kinematic definitions for scattering of a particle of mass M off a target of mass m which recoils with angle Φ with respect to the projectile direction.

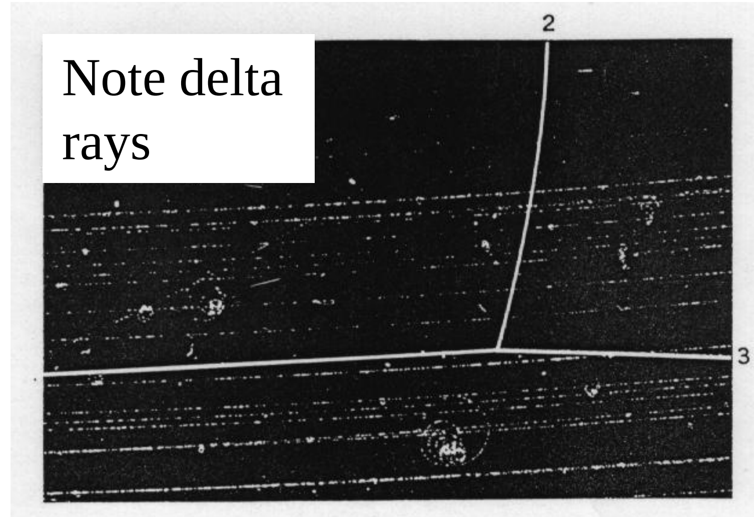
$$\vec{p}_o = \vec{p} + \vec{k}$$

$$\varepsilon_o + m = \varepsilon + e$$

$$T \equiv e - m$$

$$Q \equiv T / m$$

In NR collisions T and vector velocity are conserved. In SR energy and momentum are conserved.



$$Q = 2\left\{\frac{(\beta\gamma M\cos\Phi)^2}{(m+\gamma M)^2 - (\beta\gamma M\cos\Phi)^2}\right\}$$

$$Q_{\max} = \frac{2(\beta\gamma)^2}{\left[1 + \left(\frac{m}{M}\right)^2 + \left(\frac{2\gamma m}{M}\right)\right]} \to 2(\beta\gamma)^2 = 2\left(\frac{p_o}{M}\right)^2, \Phi = 0^0$$

$$Q / Q_{\max} \sim \cos^2\Phi$$

Algebra for a SR collision

Energy Transfer in a Collision

$$\Delta p_T \sim 2\alpha / bv, \; Z = 1$$
$$\Delta\varepsilon \sim \Delta p_T^2 / 2m$$
$$\Delta\varepsilon \sim 2\alpha^2 / b^2 v^2 m$$

$$d\sigma = d\vec{b} = bdbd\varphi = (d\sigma / dT)dT$$
$$\frac{d\sigma_\delta}{dT} = 2\pi b\left(\frac{db}{dT}\right) = \left[\frac{2\pi\alpha^2}{\beta^2 c^2 T^2 m}\right]$$

$$\frac{dN_\delta}{dTd(\rho x)} = 2\pi\left(\frac{N_o Z}{A}\right)\frac{\alpha^2 \lambda\!\!\!^{-}_e}{T^2}$$

$$dN_\delta / d(\rho x)\big|_{T>T_o} \sim \frac{7.8\%}{gm/cm^2}, T_o = 1MeV$$

If we are interested not in scattering angles (nucleus) but in significant energy transfer (recall billiards) we must consider scattering off the electrons (incoherent0) in the atom.

Note energy transfer ~ 1/m.

Change variables to recoil energy from impact parameter b.

The probability to eject an e (delta ray) with T > 1 MeV is a few % per gm/cm^2.

dE/dx and MIP

$$d\varepsilon / d\vec{b} \sim 2\alpha^2 / b^2 v^2 m$$

$$d\varepsilon \sim \int 2\pi b db (d\varepsilon / d\vec{b})$$

$$\sim 4\pi\alpha^2 / mv^2 [\ln(b_{\max} / b_{\min})]$$

Energy loss by projectile goes as inverse square of projectile velocity. As velocity -> c all singly charged particles deposit the same energy.

At NR velocity the specific energy loss goes as the inverse square of the velocity.

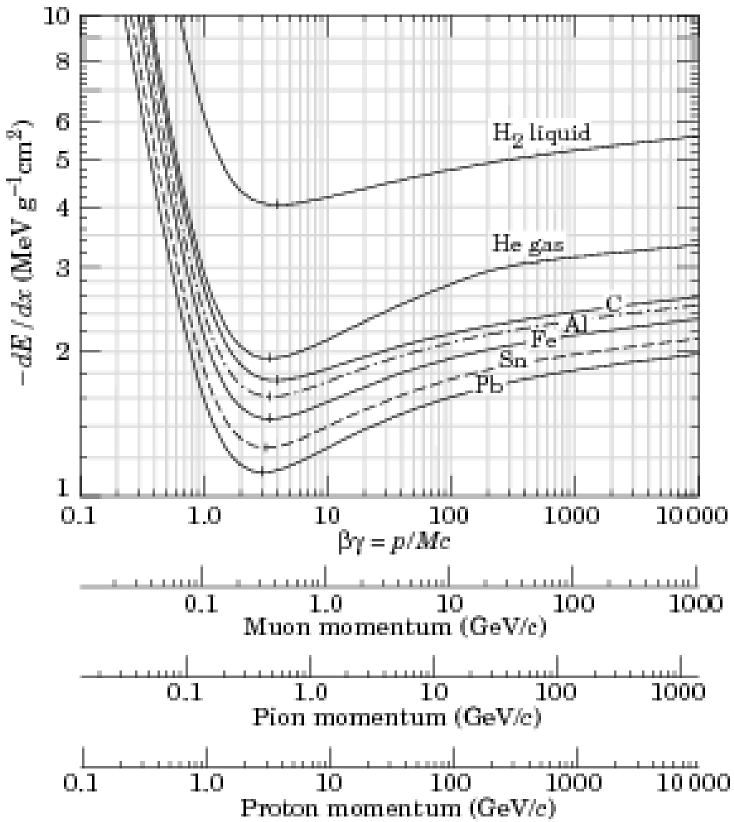
$$dE_I / d(\rho x) \sim (N_o Z / A) d\varepsilon$$

$$\sim 4\pi (N_o Z / A)(\alpha^2 \lambda\!\!\!^{-}_e)(1/\beta^2)[\ln()]$$

$$dE_I / d(\rho x)_{\min} = (1.5 MeV / gm / cm^2) = MIP$$

dE/dx and Range

1/v^2 MIP



$$T = p^2 / 2M$$

$$dE_I / dx \sim 1/ p^2 \sim 1/T \sim dT / dx$$

$$\int_{T_o}^{0} T dT \sim \int_{0}^{R} dx$$

$$R \sim T_o^2 \sim p_o^4$$

At some depth in material the particle loses all the energy and "ranges out". The range goes as the square of the incident energy, the fourth power of the incident momentum for NR particles.

Relativistic Rise

$$b_{\min}^{-1} \sim T_{\max} \sim \gamma M$$

$$b_{\max}^{-1} \sim \hbar < \omega_o > / \gamma$$

$$(b_{\max} / b_{\min})_{Bethe} \sim \gamma^2 M c^2 / \hbar < \omega_o >$$

The rise at very high energies is caused by the fact that the electric field extends very far in the transverse direction – so far that it can interact with more than one atom. This effect is clearly most important at high densities. It shows up in the log factor at high b

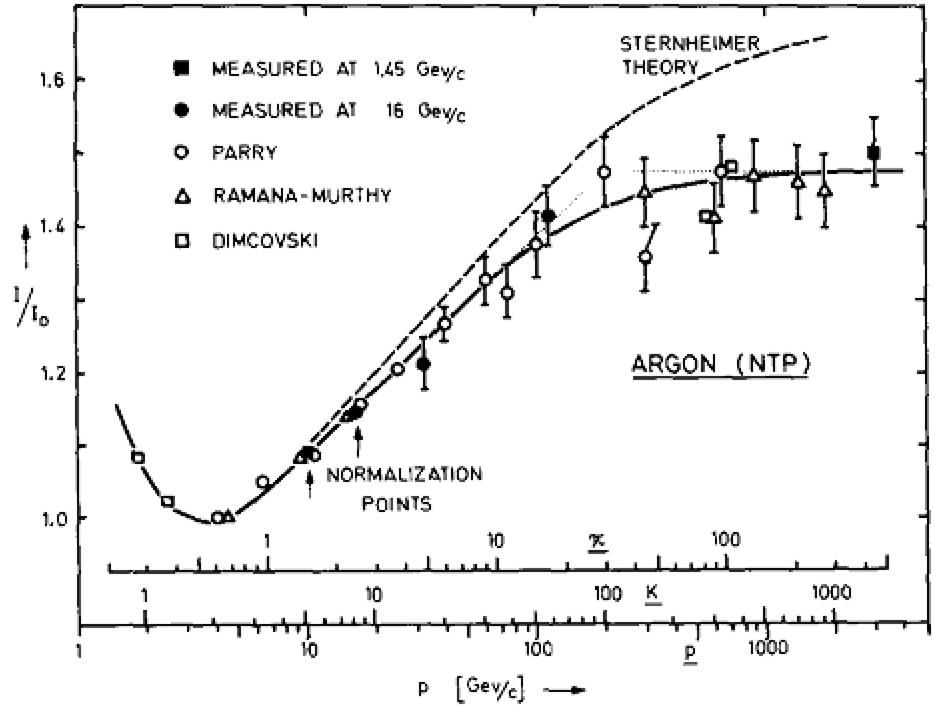
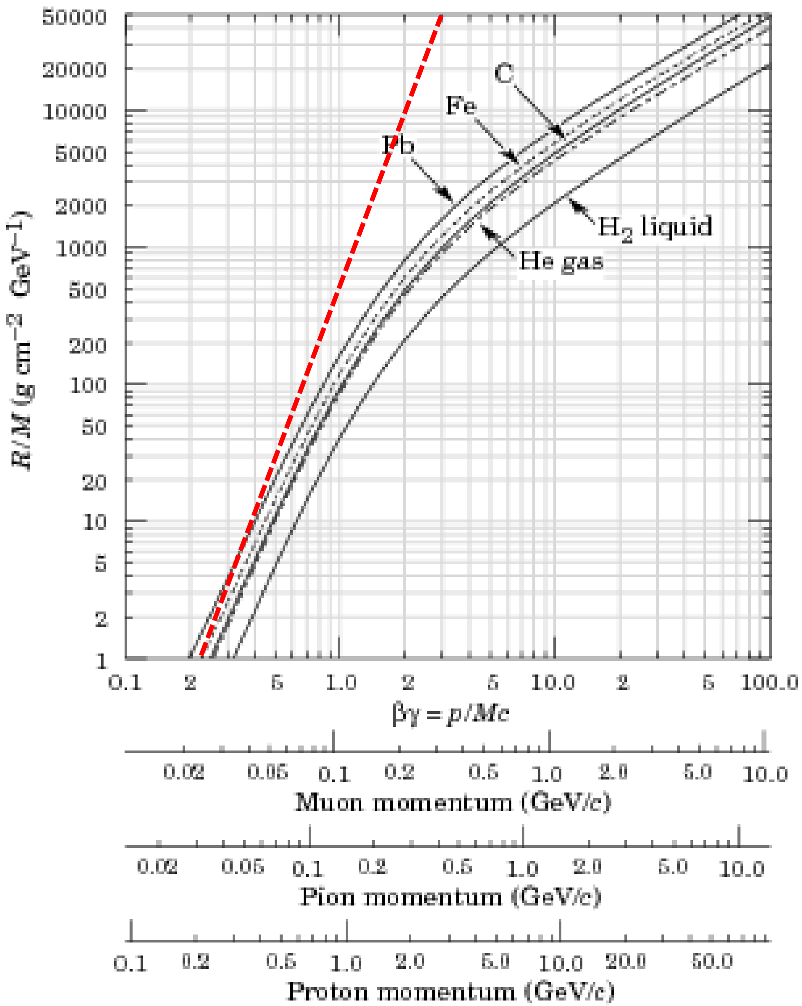


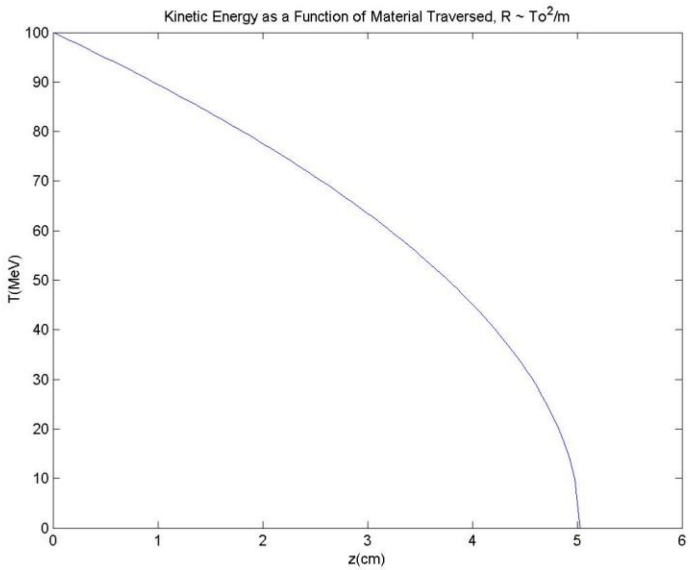
Fig. 1. Relativistic rise of the ionization as measured by proportional counters.

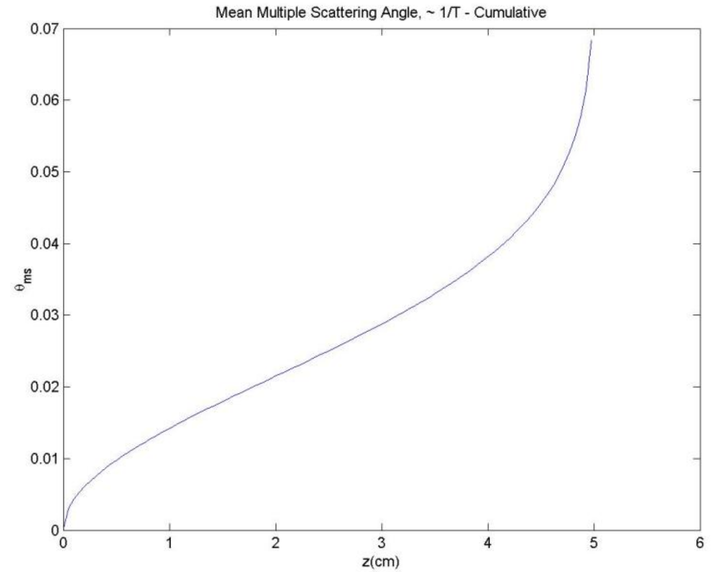
Range vs. Momentum

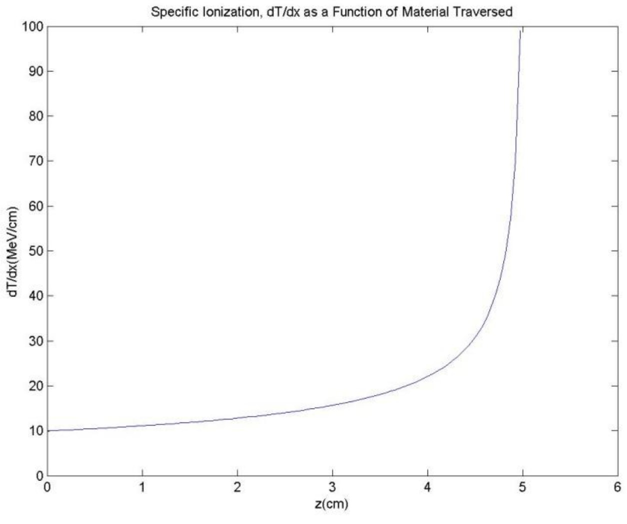


Plots taken from the Particle Data Group – review of Particle Properties.

Demo – Range, T







Proton, 100 MeV = To. Note that the deposited energy is largest at the end of the range. That is the idea behind p and ion cancer therapy.

dE/dx vs p for Particle ID

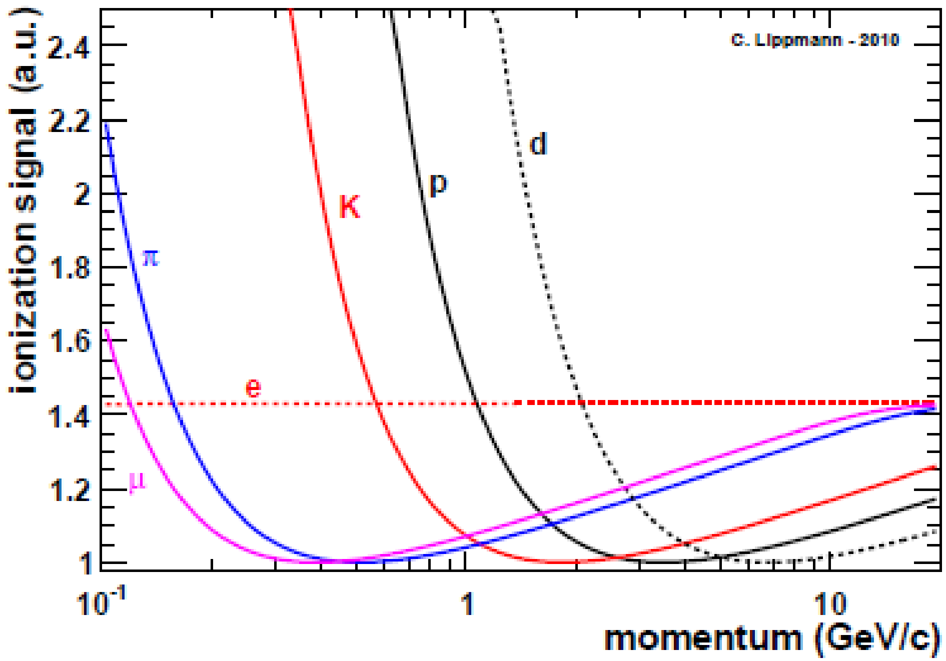
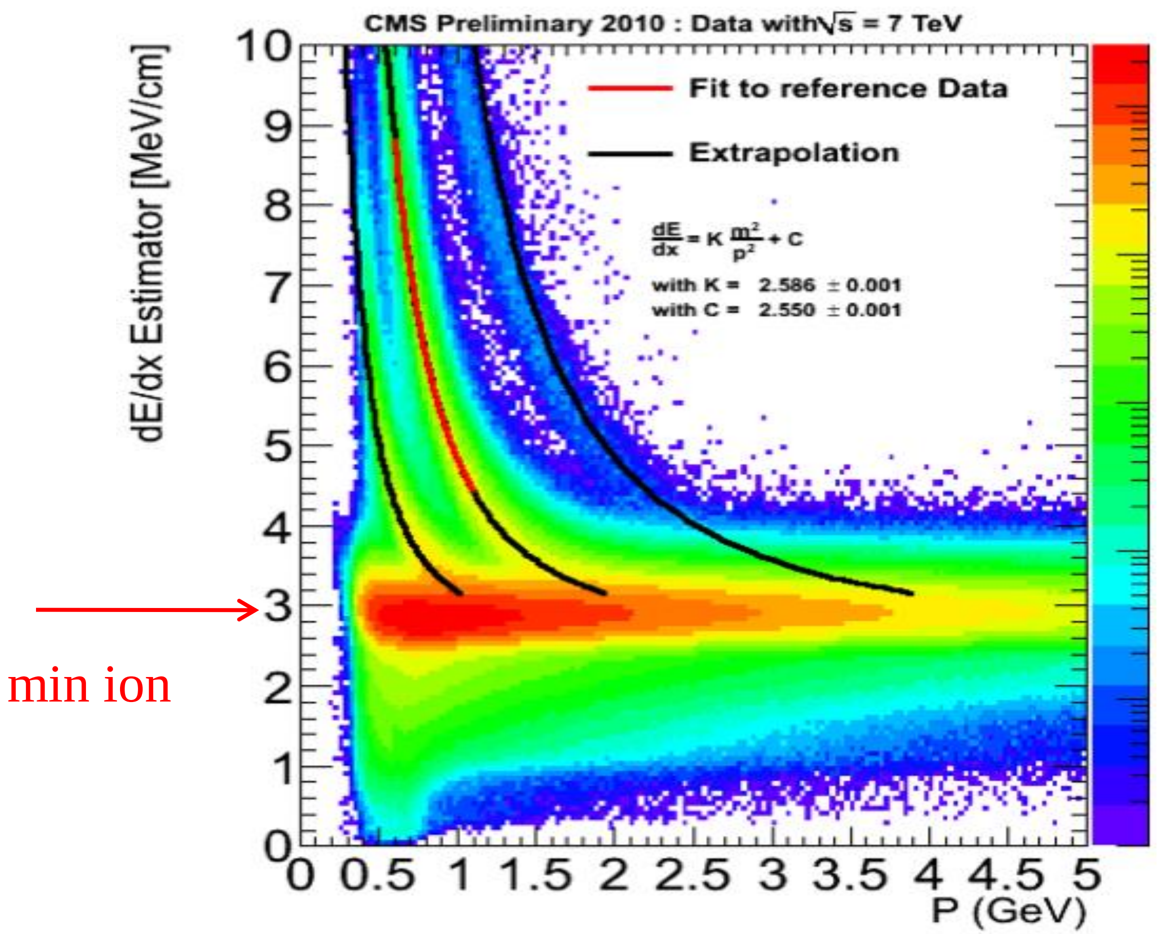


Figure 10: Typical curves of the ionization signal as a function of particle momentum for a number of known charged particles. A parameterization like the one suggested in Ref. [20] was used to calculate the curves.

If you measure the particle momentum (B field discussed later) and you measure dE/dx => mass, or "particle ID"

Particle ID – dE/dx in Tracker

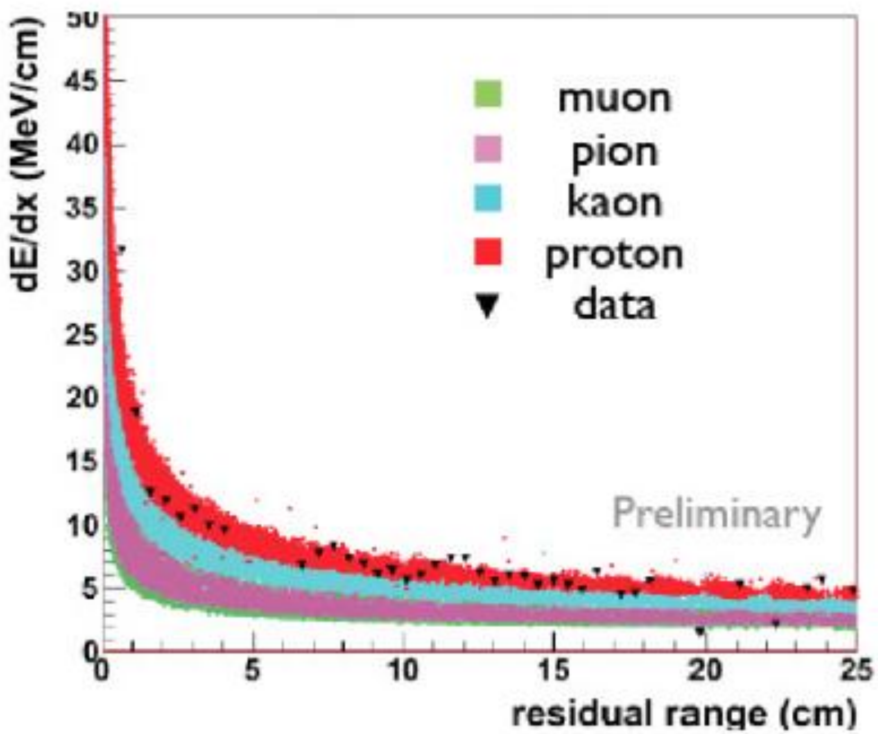


Use energy deposited in several Si strip layers (restricted energy loss) to measure dE/dx. Useful for particle id. Once commissioned, use in heavy stable particle searches.

$$dE/dx \sim 1/\beta^2 \sim M^2/P^2$$

-mass measurement using P and dE/dx from tracker

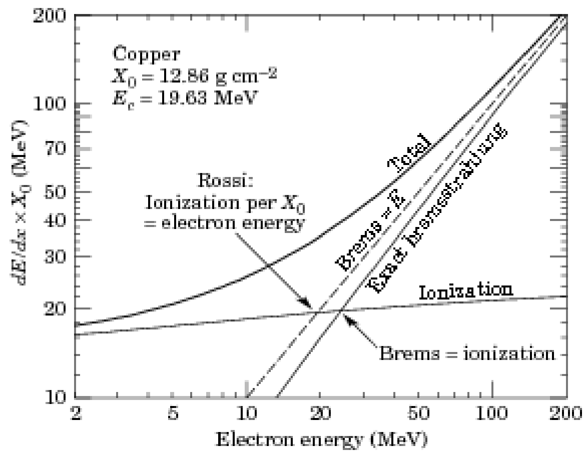
LAr – dE/dx vs. Range

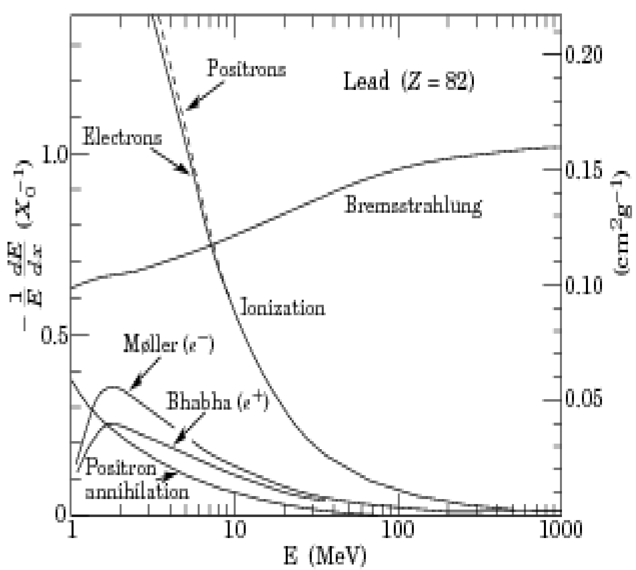


Use the neutrino target as the detector.

Look at remaining range vs. dE/dx.

dE/dx – e on Cu, Pb





At very high energies, electrons begin to radiate a substantial amount of energy. A critical energy, Ec, is defined when the radiative energy loss = the ionization energy loss. For e on Cu it is about 20 MeV (or E/m ~40). For Pb it is ~ 6 MeV.

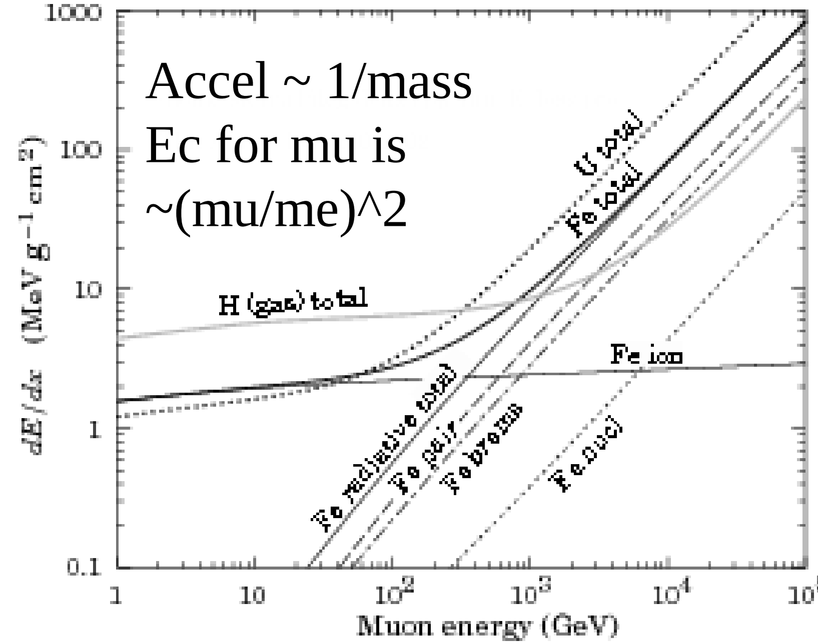
We discuss the radiation properties later.

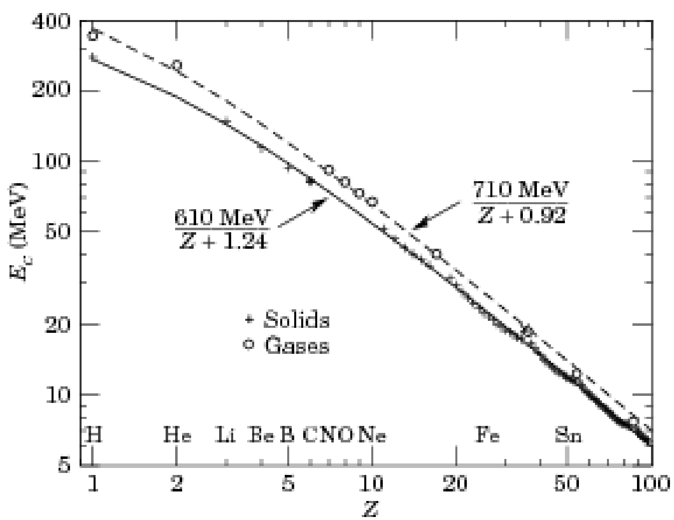
Critical Energy – Muons

$$[dE_B / d(\rho x)]/[dE_I / d(\rho x)] \sim$$
$$[16/3(N_o / A)(Z^2\alpha)(\alpha ƛ_p)^2 \ln()E]/$$
$$[4\pi(N_o / A)Z(\alpha^2 ƛ_T)\ln'()]$$

$$E \sim (3\pi/4)(m_p / m_T)[m_p c^2 / Z\alpha]$$
$$\sim 166\ MeV / Z$$

Accel ~ 1/mass
Ec for mu is
~(mu/me)^2





Bremmstrahlung is due to the strong electric field of the nucleus accelerating the electrons (projectile). It is coherent over the size of the nucleus. Ionization is due to electron recoil in the atom and is incoherent. Therefore Ec ~ 1/Z.