What is Control Theory?

What is Feedback? How does it work? What is Stability?

John D. Fox, Claudio Rivetta

Stanford Linear Accelerator Center
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Outline

Origins

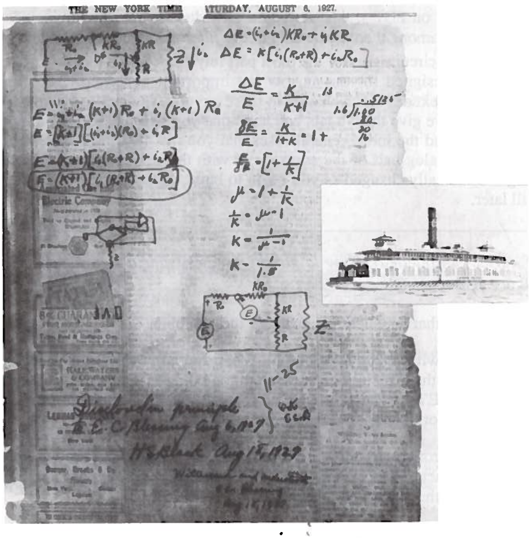


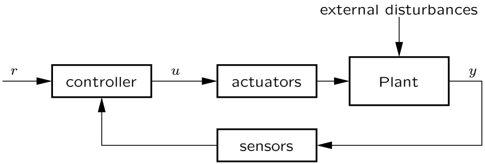
Figure OVR4.1: Harold Black's notes on the feedforward amplifier conceived as he rode the ferry from Staten Island to work, one summer morning in 1927. (Copyright 1977 IEEE. Reprinted, with permission, from Harold S. Black, "Inventing the Negative Feedback Amplifier," IEEE Spectrum, Dec. 1977)

Feedback Basics

The objective is to make the output y of a dynamic system (plant) behave in a desired way by manipulating input or inputs of the plant.

Regulator problem - keep y small or constant

Servomechanism problem - make y follow a reference signal r



Feedback controller acts to reject the external disturbances.

The error between y and the desired value is the measure of feedback system performance. There are many ways to define the numerical performance metric

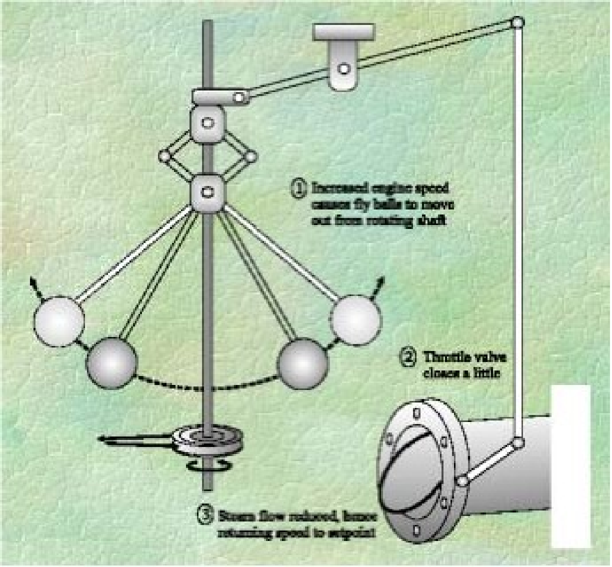
- RMS or maximum errors in steady-state operation
- Step response performance such as rise time, settling time, overshoot.

An additional measure of feedback performance is the average or peak actuator effort. Peak actuator effort is almost always important due to the finite actuator range.

Feedback system robustness - how does the performance change if the plant parameters or dynamics change? How do the changes in sensors and actuators affect the system?

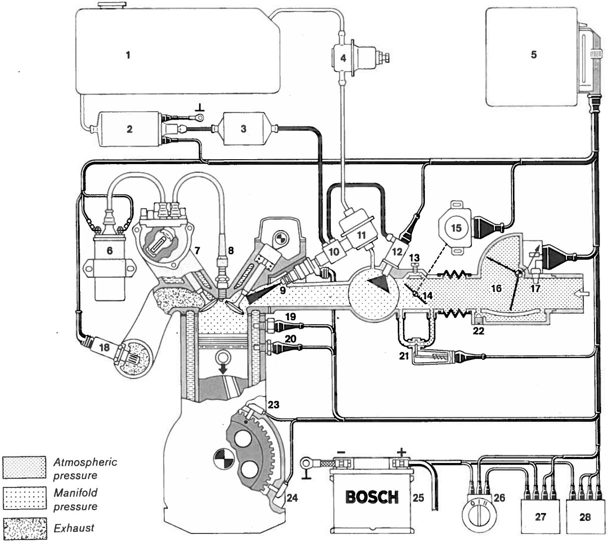
Feedback Basics - engine control via Watt's regulator

- Setpoint
- Controller
- Actuator
- Plant
- Disturbance
- Sensor



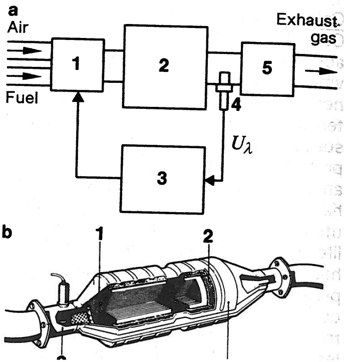
Feedback and the Motor City

The Motronic system
1 Fuel tank
2 Electric fuel pump
3 Fuel filter
4 Vibration damper
5 Control unit (ECU)
6 Ignition coil
7 High-voltage distributor
8 Spark plug
9 Fuel-injection valve
10 Fuel distributor
11 Pressure regulator
12 Cold-start valve
13 Idle-speed adjusting screw
14 Throttle valve
15 Throttle-valve switch
16 Air-flow sensor
17 Air-temperature sensor
18 Lambda sensor
19 Thermo-time switch
20 Engine-temperature sensor
21 Auxiliary-air device
22 Idle-mixture adjusting screw
23 Reference-mark sensor
24 Engine-speed sensor
25 Battery
26 Ignition-starting switch
27 Main relay
28 Pump relay

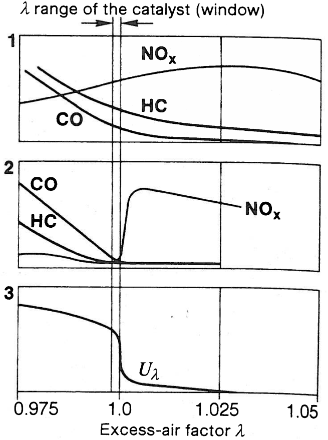


Oxygen sensor, feedback for mixture control

Exhaust gas emission control with the three-way catalyst and the lambda sensor.
a) Functional diagram. 1 Fuel-metering system, 2 Engine, 3 Control unit, 4 Lambda sensor, 5 Three-way catalyst, U_λ Sensor voltage.
b) Three-way catalyst. 1 Ceramic monolith, 2 Wire mesh, 3 Lambda sensor, 4 Housing



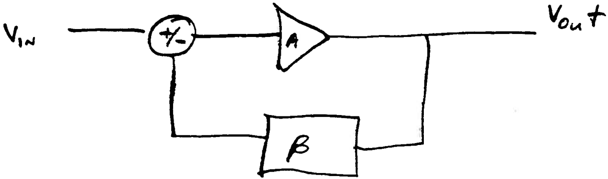
Catalyst action as a function of excess-air factor λ.
1 Exhaust emissions upstream of the three-way catalyst, 2 Exhaust emissions downstream of the three-way catalyst, 3 Electrical signal from the lambda sensor, U_λ Sensor voltage



Stability

OSCILLATOR $\Longleftrightarrow$ AMPLIFIER

CONSIDER THE SYSTEM



where $A(\omega)$, $\beta(\omega)$ are complex

THE CIRCUIT CAN OSCILLATE IF THE NET PHASE SHIFT AROUND THE LOOP $= 2n\pi$ AND THE LOOP GAIN ≥ 1

Stability

THE CIRCUIT CAN OSCILLATE IF THE NET PHASE SHIFT AROUND THE LOOP $= 2n\pi$ AND THE LOOP GAIN ≥ 1

Suppose $|\beta A| \equiv 1$, $\Delta\phi \equiv 2n\pi$

THEN V_{out} REPLICATES ITSELF (ie, Self sustaining oscillations)

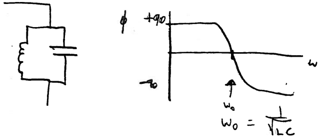
IF $A\beta > 1$, the output grows exponentially till Something Limits

Stability

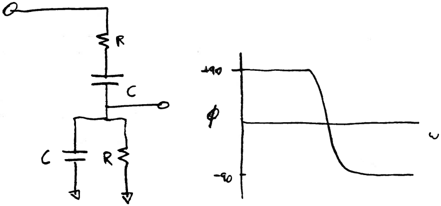
THIS CAN HAPPEN ACCIDENTALLY —

OR ONE Deliberately chooses β to have A KNOWN ϕ vs ω

LC OSCILLATOR (ALSO QUARTZ CRYSTAL)



WIEN-ROBINSON BRIDGE



The simplest controller

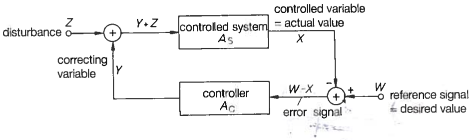


Fig. 22.1. Block diagram of a feedback control loop

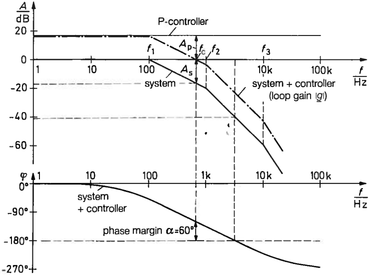


Fig. 22.2. Bode plot of a system with a P-controller

How do we pick the gain?

The simplest controller, gets complicated

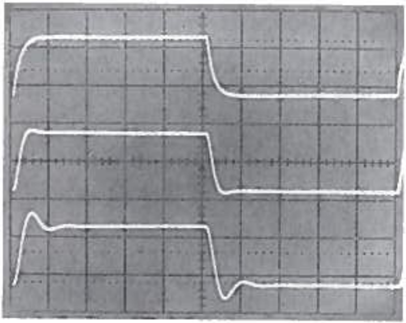


Fig. 22.3. Step response as a function of the phase margin, for the constant-gain crossover frequency f_c: $\alpha = 90°$ (*upper*), $\alpha = 60°$ (*middle*), and $\alpha = 45°$ (*lower*)

Can we do better?

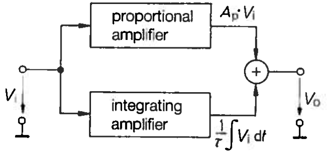


Fig. 22.5. Block diagram of the PI-controller

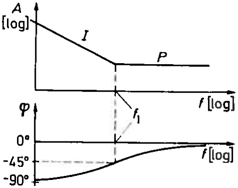


Fig. 22.6. Bode plot of the PI-controller

What should this do to the frequency response? What should this do to the time (Step) response?

we can do better, but how much?

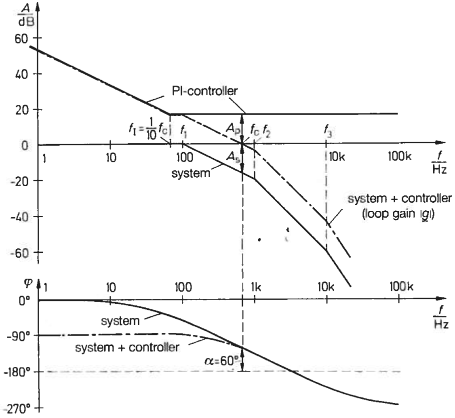


Fig. 22.4. Bode plot of a system with a PI-controller

What should this do to the frequency response? What should this do to the time (Step) response?

The PI controller

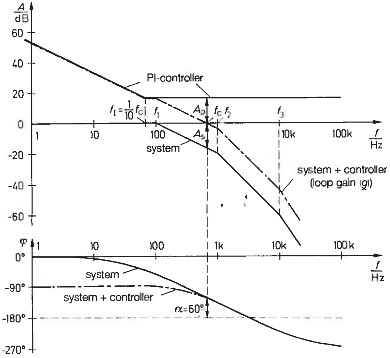


Fig. 22.4. Bode plot of a system with a PI-controller

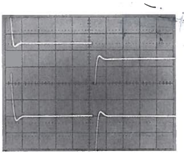


Fig. 22.8. Error signal for the P-controller (*upper*) and the PI-controller with an optimum value of f_I (*lower*)

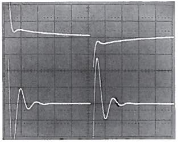


Fig. 22.9. Error signal of the PI-controller: f_I too small (*upper*) and f_I too large (*lower*)

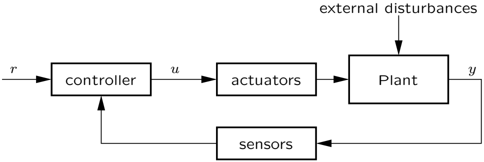
We want to learn how to design controllers, and understand stability. We want to understand how a controller changes the dynamics of a system.

Feedback, basically, it can get very complicated

The objective is to make the output y of a dynamic system (plant) behave in a desired way by manipulating input or inputs of the plant.

Regulator problem - keep y small or constant

Servomechanism problem - make y follow a reference signal r



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Time and Frequency Domains

Fourier transforms

A function f(x) may be Fourier transformed into a function F(s),

$$F(s) = \int_{-\infty}^{\infty} f(x) e^{-i2\pi xs} dx \tag{1}$$

and likewise a function F(s) can be transformed into a function f(x)

$$f(x) = \int_{-\infty}^{\infty} F(s) e^{i2\pi xs} ds \tag{2}$$

The Laplace transform is related to the Fourier Transform but involves an integral from 0 to infinity

Time and Frequency Domains

Discrete Fourier Transform

For systems involving discrete samples of data, such as from sampling circuits or from samples taken from circulating bunches, the discrete-time Fourier transform is similar

$$F(\nu) = \frac{1}{N} \sum_{\tau=0}^{N-1} f(\tau) e^{-i2\pi(\nu/N)\tau} \tag{3}$$

$$f(\tau) = \sum_{\nu=0}^{N-1} F(\nu) e^{i2\pi(\nu/N)\tau} \tag{4}$$

There is a related transform, the Z transform, which is the discrete-time equivalent of the Laplace transform

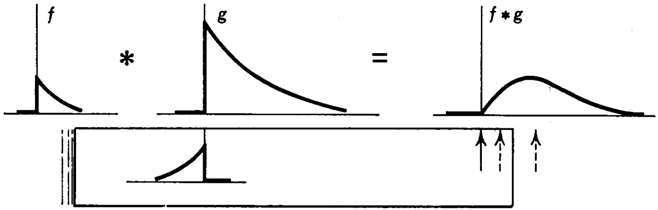
Time and Frequency Domains

Convolution of two functions

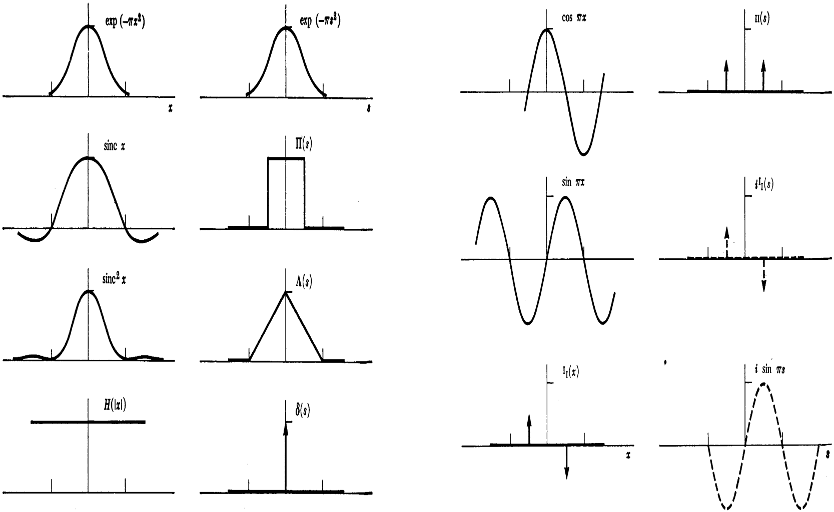
The convolution of two functions f(x) and g(x) is defined as $f(x) \star g(x)$

$$f(x) \star g(x) = \int_{-\infty}^{\infty} f(u)g(x-u)du \quad (5)$$

In pictorial form



Common Transform Pairs (from Bracewell)



Linear Time Invariant Systems

If a system converts an input $u(t)$ into an output $y(t)$

$$y(t) = L[u(t)] \tag{6}$$

the system is linear if for two constants a1 and a2

$$L[a_1 u_1 + a_2 u_2] = a_1 L[u_1(t)] + a_2 L[u_2(t)]. \tag{7}$$

The response of two inputs is the superposition of the individual outputs. If an input is only a single frequency ω, the output can only contain that single frequency ω.

Linear Time Invariant Systems

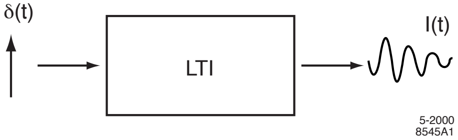
A system is time invariant if for a time delay δ the output has shift invariance, or that

$$L\left[u(t)\right] = y(t) \tag{8}$$

$$L\left[u(t-\delta)\right] = y(t-\delta) \tag{9}$$

Impulse response of LTI system

The impulse response $I(t)$ of a system is found by exciting the system with a δ-function in the time domain.

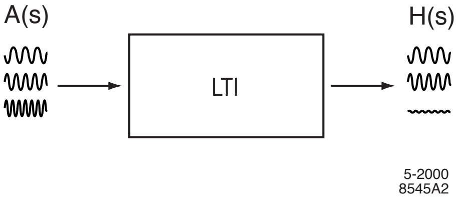


for a general input $u(t)$ the output is a convolution

$$y(t) = u(t) \star I(t) \tag{10}$$

Frequency Response of LTI system

Frequency response $H(s)$ is the transfer function in the frequency domain. Measured by network analyzer via magnitude and phase vs. frequency.



For a general input in the frequency domain $I(s)$ the output $O(s)$ is the product

$$O(s) = H(s)I(s) \quad (11)$$

Frequency Response and Time Response relationship

The time response is also the inverse transform of the product of the Fourier transform of the input $u(t)$ and the frequency response $H(s)$

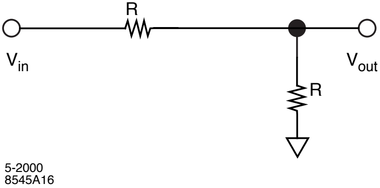
$$y(t) = u(t) * I(t) \tag{12}$$

$$y(t) = IFT\left[FT(u(t))H(s)\right] \tag{13}$$

For an LTI system, we can measure in either domain, and compute the response via appropriate convolutions, transforms or inverse transforms

A Quiz on LTI Systems

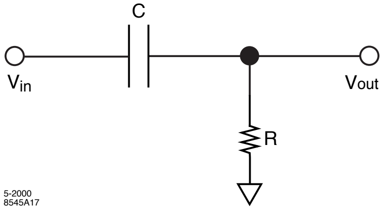
Consider this simple circuit - a voltage divider



Is this an LTI system? Is it ALWAYS an LTI system?

A Quiz on LTI Systems

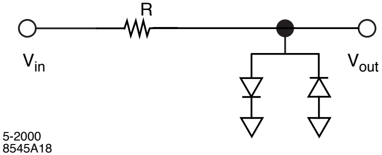
Consider this simple circuit - a high-pass filter



Here the output is frequency dependent. Is this an LTI system?

A Quiz on LTI Systems

Consider this simple circuit - a diode clipper (a limiter)



Is this an LTI system? When? What output frequencies are present for an input at ω?. Two signals ω_1 and ω_2? Does it have an Impulse Response $I(t)$?