USPAS
Cyclotrons
June 2011

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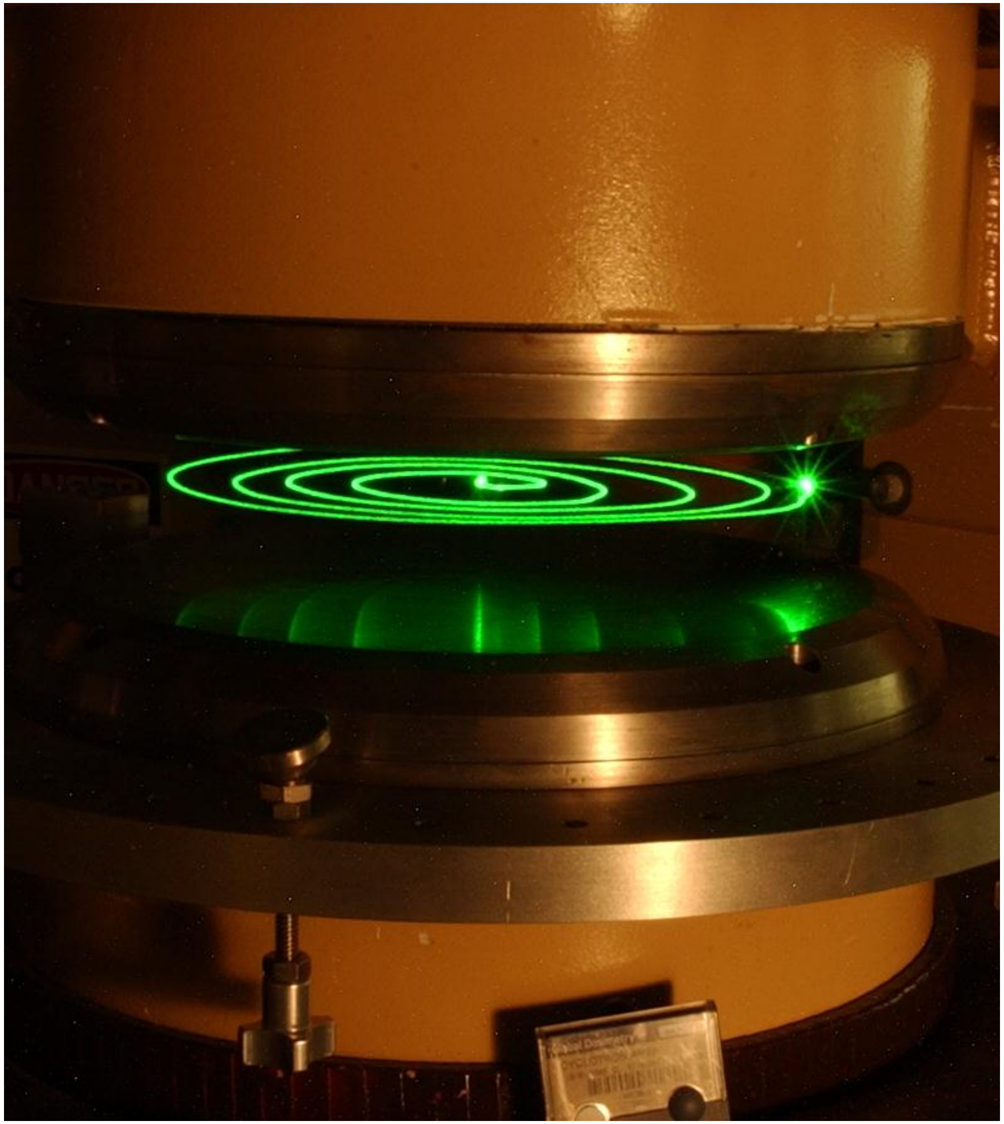


Thank you to:

Timothy Antaya, MIT

Timothy Ponter, IBA

Institute for Research in Electronics & Applied Physics

Rutgers The State University of New Jersey

Starting Notes

- This is the foundation for a week-long cyclotron course. Please, help me make it better:
  - Feed back during and after
  - What would you like to hear more about ?
- Trying to couple theory with real life examples
  - Practical and engineering questions ?

Cyclotrons - Outline

- Why Cyclotrons in 2011 ?
- Brief History
- How the cyclotron works
  - Theory
  - Components
  - Examples
- Types of cyclotrons & Examples
  - Classical "Lawrence Cyclotron"
  - Synchrocyclotron
  - Azimuthally Varying Field & Isochronous Cyclotrons
- "Secrets of the trade" as we go

WHY ?

- Many applications
  - High Energy Physics: Daedalus Neutrino Expt
    - 1 MW @ 1 GeV (state of the art)
  - Medical applications
    - Isotope production for treatment and diagnostics (Tc-99 problem)
    - Direct beam cancer therapy (95% success with 250 MeV proton eye treatment)
  - Power Applications
    - Accelerator driven systems
    - Transmutation of long lived waste
  - Security
    - NNSA active interrogation – looking for fissile materials
  - Industrial needs
    - 4 Billion $$$/year industry, > 50% is in the cyclotron business
- Cost & Efficiency – Cyclotrons are the best deal around !
  - State-of-the-art 9T (250 MeV) synchrocyclotron ~ 1M$ → 0.4 cents per MeV (to use an equivalent SC Linac would be about 1M$/MeV)
  - Total Power Requirement~ 100 kW
- Educational Value – priceless!

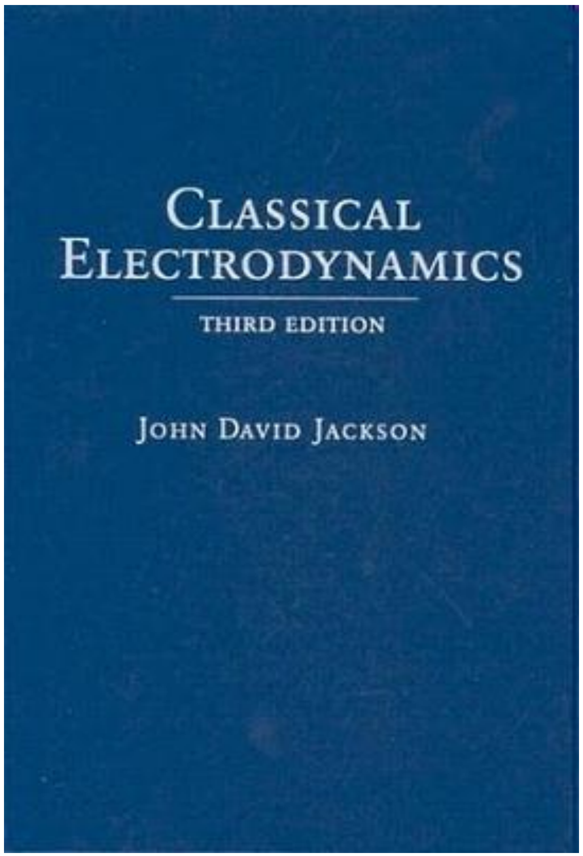
"They're the coolest type of particle accelerator"

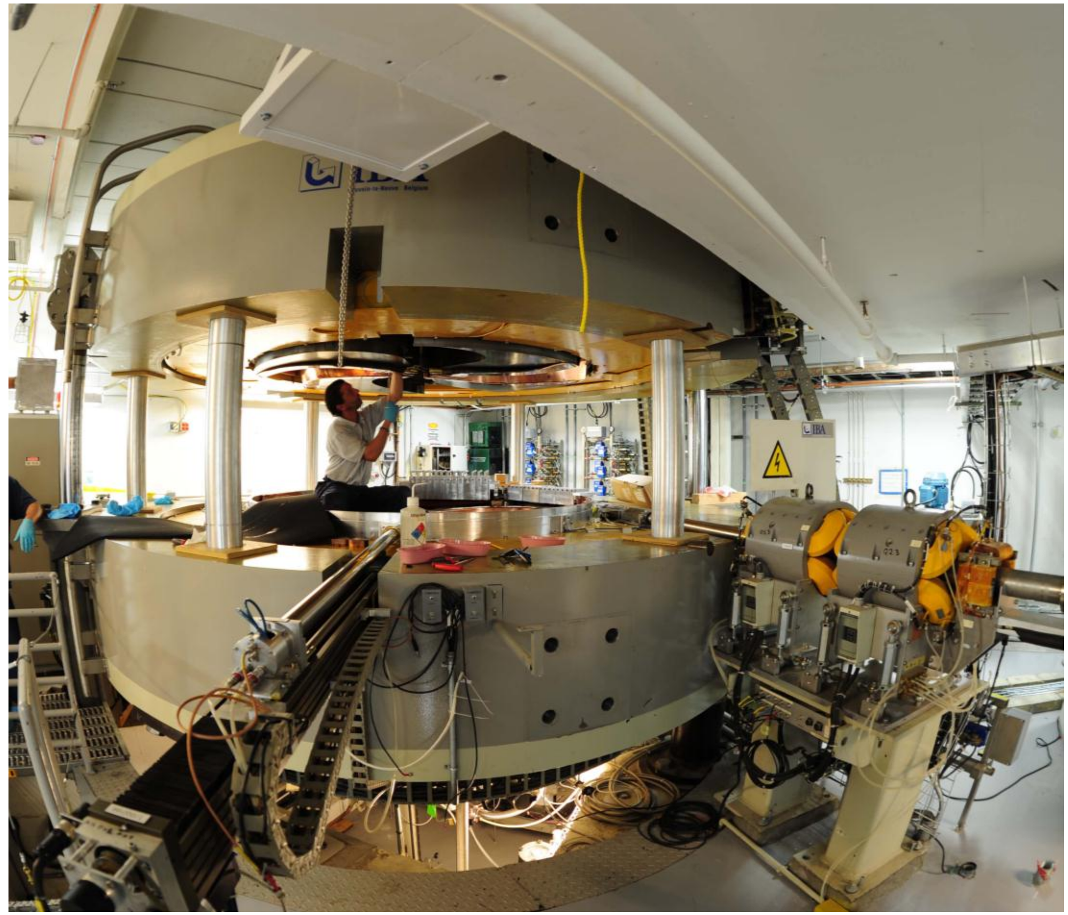
- Prof. Mark Yuly, Physics Chair of Houghton College

In Perspective

"Read this book and you will learn ***all*** that there is to accelerator physics"

– former prof.





"Build one of these, and you will experience [almost] all there is of accelerator physics"

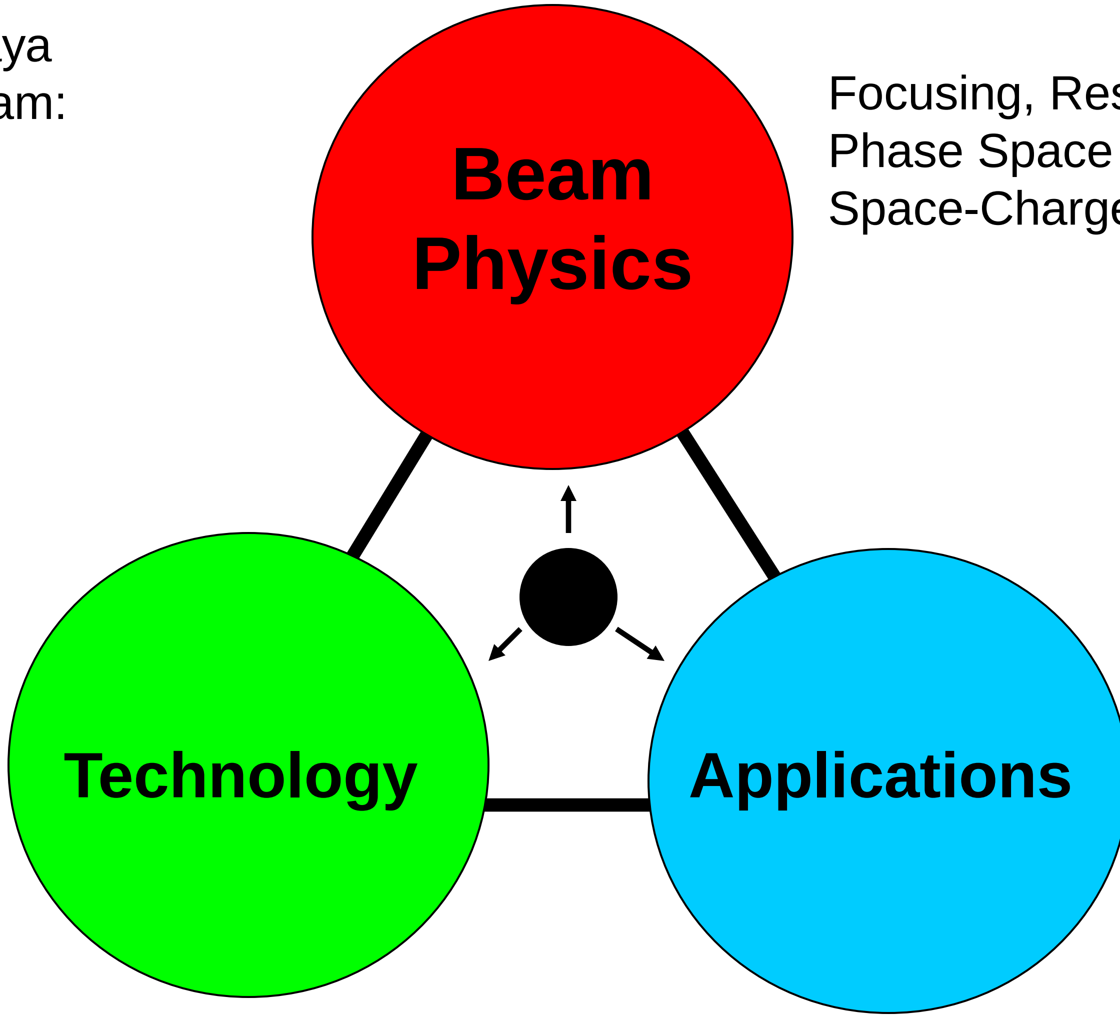
- TWK

SOMETHING FOR EVERYONE

Applicable to all of accelerator physics

Timothy Antaya
Phase Diagram:

Focusing, Resonances,
Phase Space Evolution,
Space-Charge Challenges



Magnets, RF, HV, Ion Sources

Basic Science, Medicine, Security…

"Start the Ball Rolling"

1927: Lord Rutherford requested a "copious supply" of projectiles more energetic than natural alpha and beta particles. At the opening of their High Tension Laboratory, Rutherford went on to reiterate the goal:

What we require is an apparatus to give us a potential of the order of 10 million volts which can be safely accommodated in a reasonably sized room and operated by a few kilowatts of power. We require too an exhausted tube capable of withstanding this voltage… I see no reason why such a requirement cannot be made practical.[1]

Many Failed Attempts

Just one example:

1928: Curt Urban, Arno Brasch, and Fritz Lange successfully achieved 15 MV by harnessing lightning in the Italian Alps !

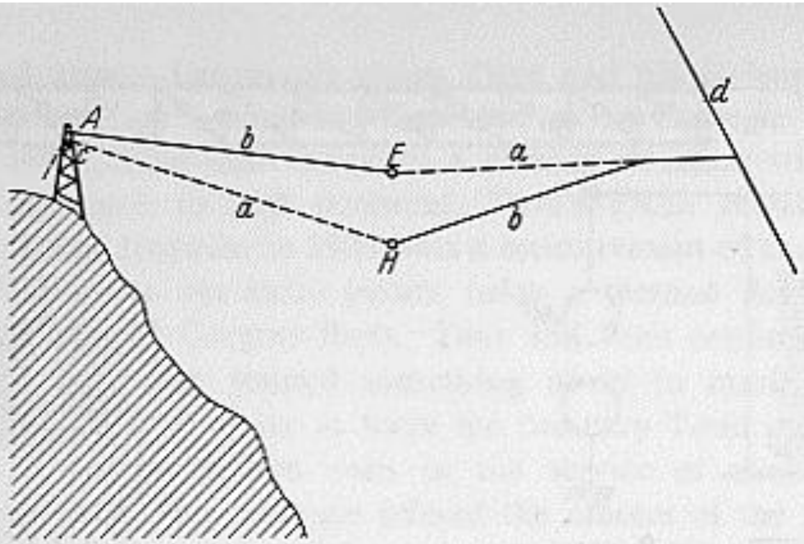


FIG. 2.1 Brasch and Lange's lightning catcher. E and H are the spheres between which the discharge occurs; AE, the antenna; a,a, insulators; b,b, conductors; d, a grounded wire. Brasch and Lange, *Zs. f. Phys.*, *70* (1931), 17.

The two who ***survived*** the experiment went on to design an accelerator tube capable of withstanding that voltage.

Wideroe Linac

1929: Rolf Wideroe

R. Wideroe proposed an accelerator by using an alternating voltage across several accelerating "gaps."

It was not without myriad of problems

- Focusing of the beam
- Vacuum leaks
- Oscillating high voltages
- Length
- Imagination

His professor refused any further work because it was "sure to fail."

Never the less, thankfully Wideroe still published his idea in *Archiv fur Electrotechnic*

Wideroe in the 1960's having the last laugh…

Ernest Orlando Lawrence

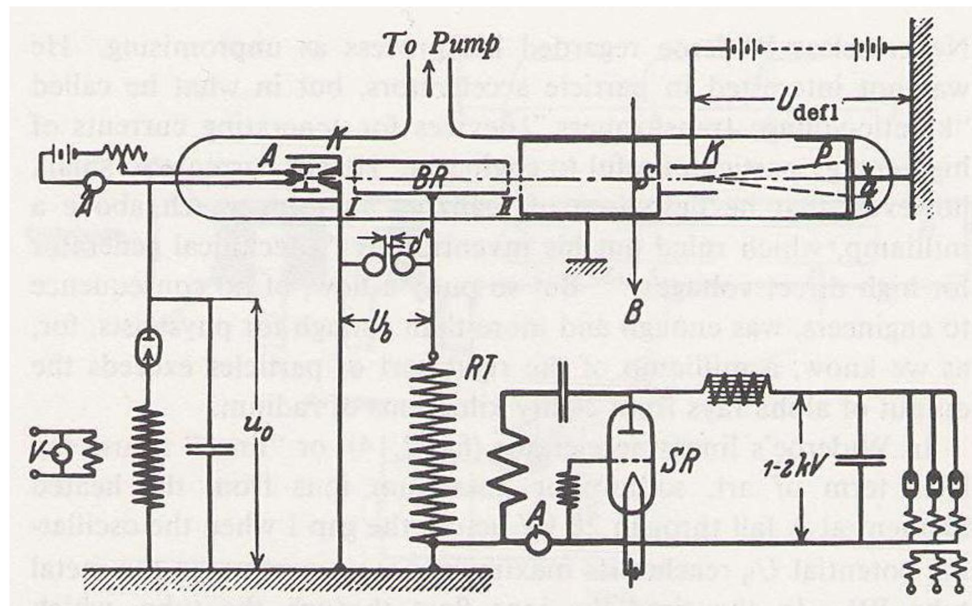
1909 – 1958

(2 hours shy of 17 years before I was born)

PLATE 1.4 Lawrence as a young associate professor. University Archives, TBL.

In April 1929, UC Berkley's youngest Physics professor happened across *Archiv fur Electrotechnic*.

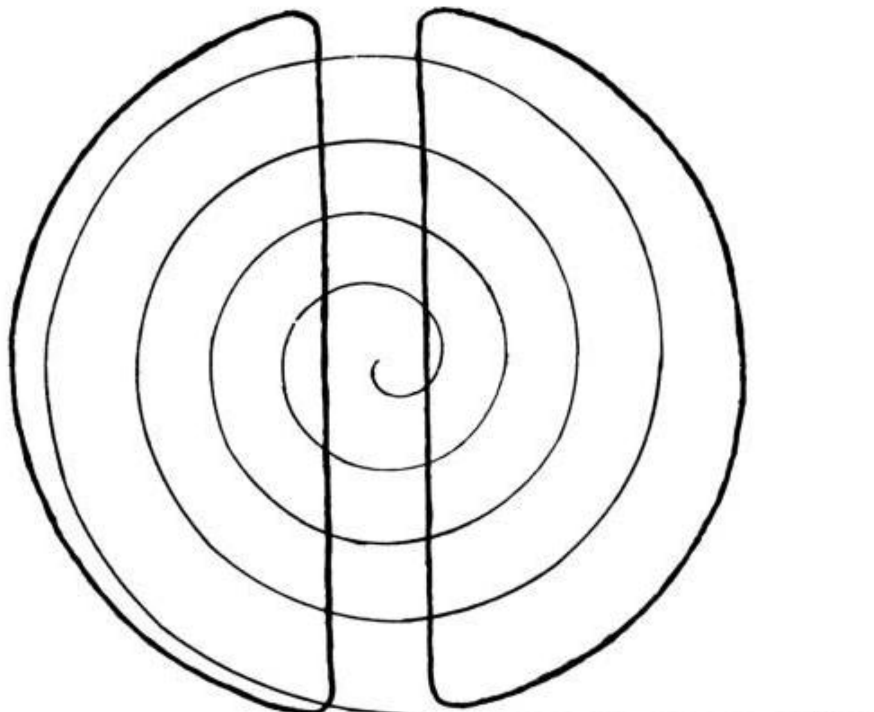
Not able to read German he just looked at the diagrams and pictures of the journal.



Immediately after seeing Wideroe's schematic, Ernest fully comprehended it's implications.

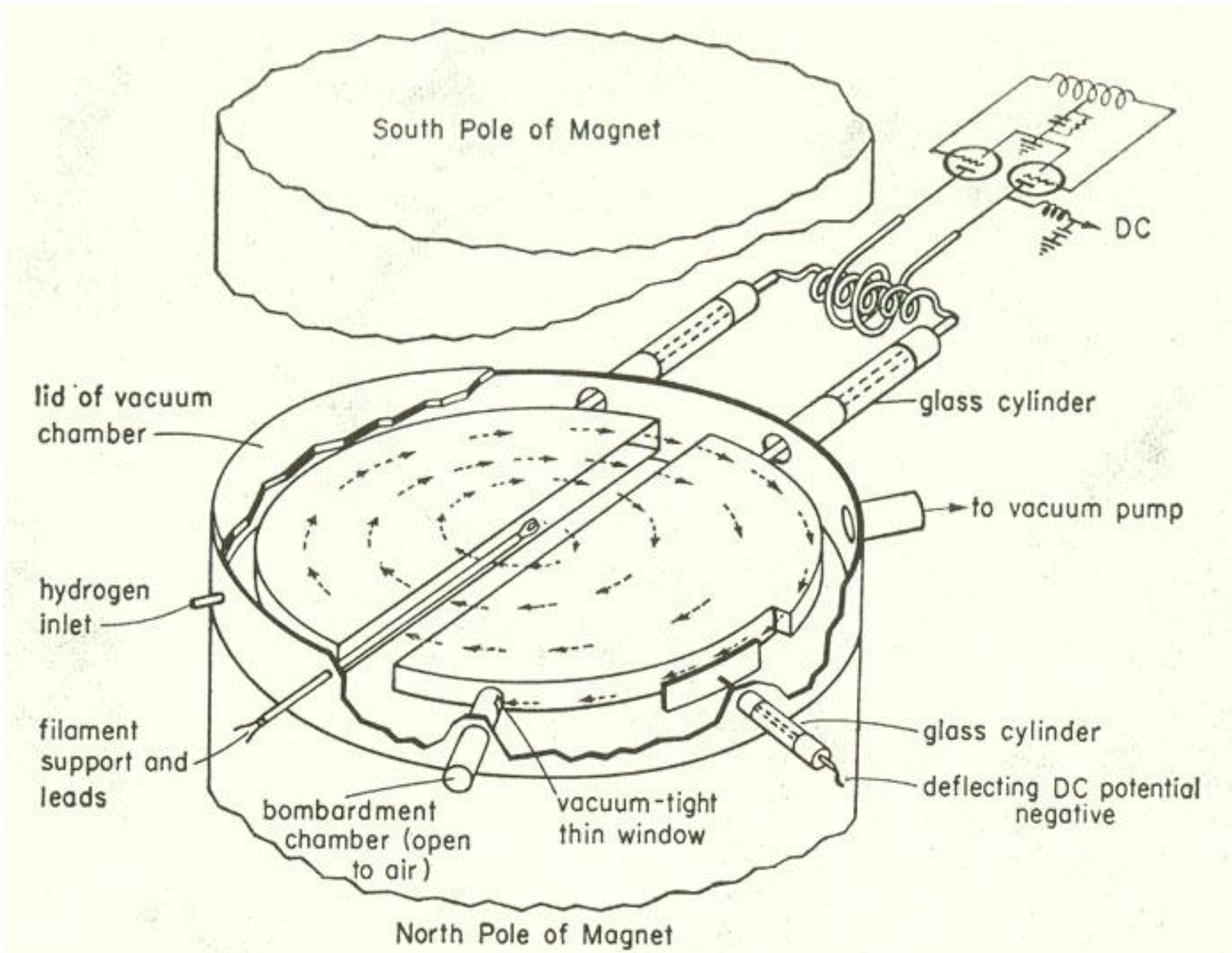
HOW DOES THE CYCLOTRON WORK ?

The cyclotron as seen by the…



. . . the inventor

HOW THE CYCLOTRON WORKS



Major Components of a Cyclotron

HOW THE CYCLOTRON WORKS

A Mechanical Analog to the Cyclotron:

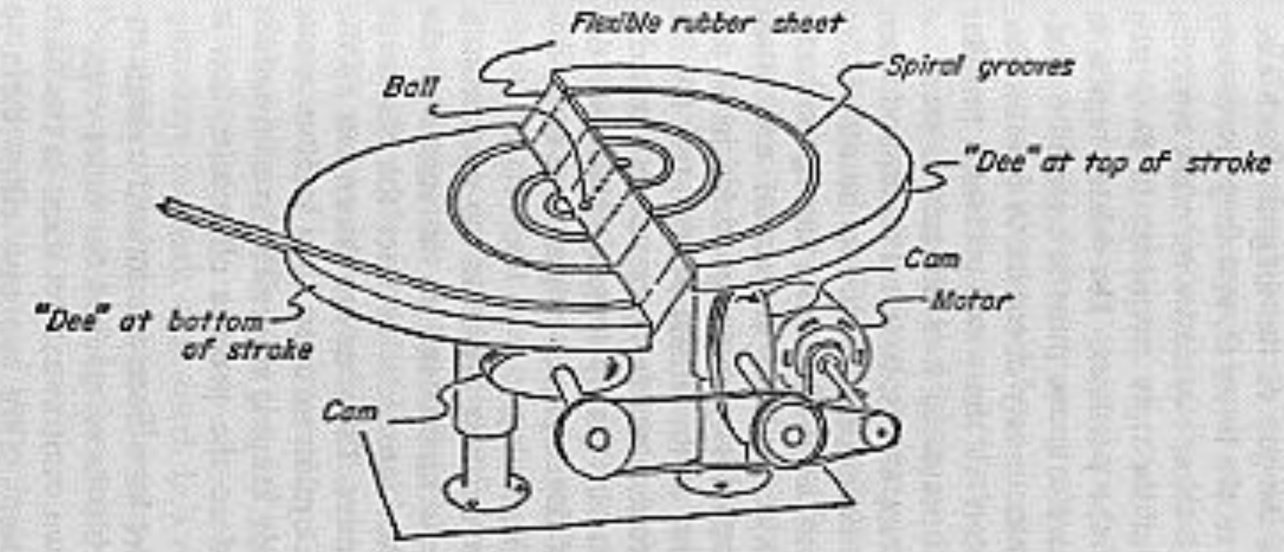


FIG. 24. *In this mechanical analog of the cyclotron the ball undergoes acceleration each time it rolls down the sloping section joining the two movable platforms, which correspond to the accelerating electrodes of the real machine. When correctly timed, the cam mechanism raises each platform as the ball traverses the spiral groove; thus the ball conserves its speed and makes a down-hill passage at the next crossing. The operation is quite similar to that of the movable bowling-alley track shown in Fig. 15,* B.

Derivation of Cyclotron Theory at the board.

Magnetic Resonance Accel.

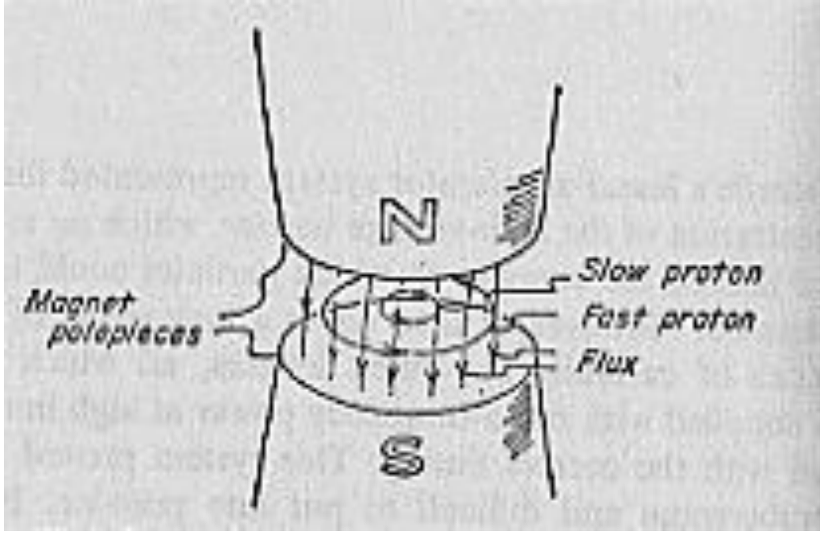
$$\vec{F}_{centriptal} = \frac{m\vec{v}^2}{r}\hat{r} \qquad \vec{F}_{magnetic} = q(\vec{v}\times\vec{B})$$

$$\vec{F}_{magnetic} = \vec{F}_{centriptal} = \frac{m\vec{v}^2}{r}\hat{r} = q(\vec{v}\times\vec{B})$$

SIMION DEMO

$$f_{cyc} = \frac{\omega_{cyc}}{2\pi} = \frac{qB}{2\pi m}$$

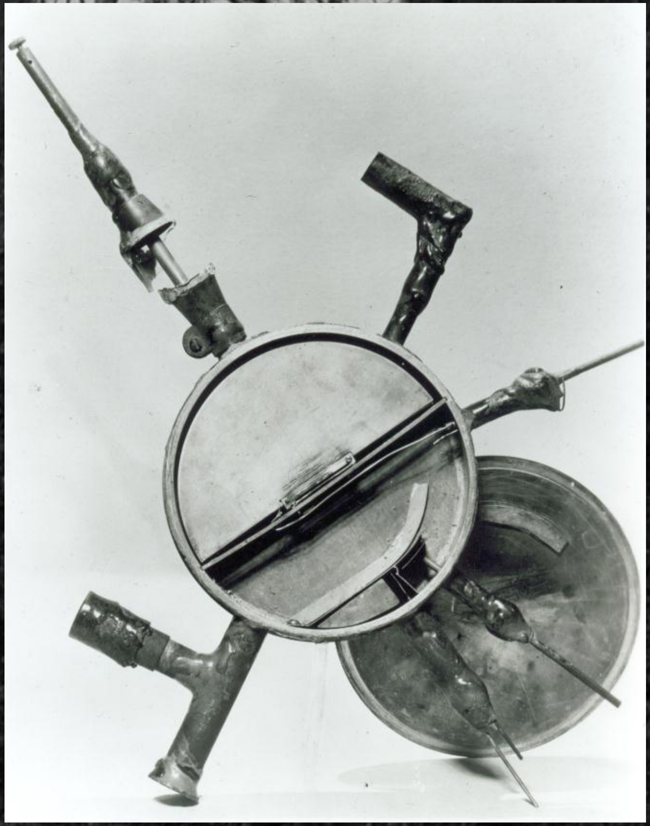
$$E(r) = \frac{q^2B^2r^2}{2m}$$



In the non-relativistic scenario, revolution frequency is independent of radius !

Energy scales ~ $\boldsymbol{B}^2$, r^2 → higher fields, larger pole diameter

Cyclotron Progression

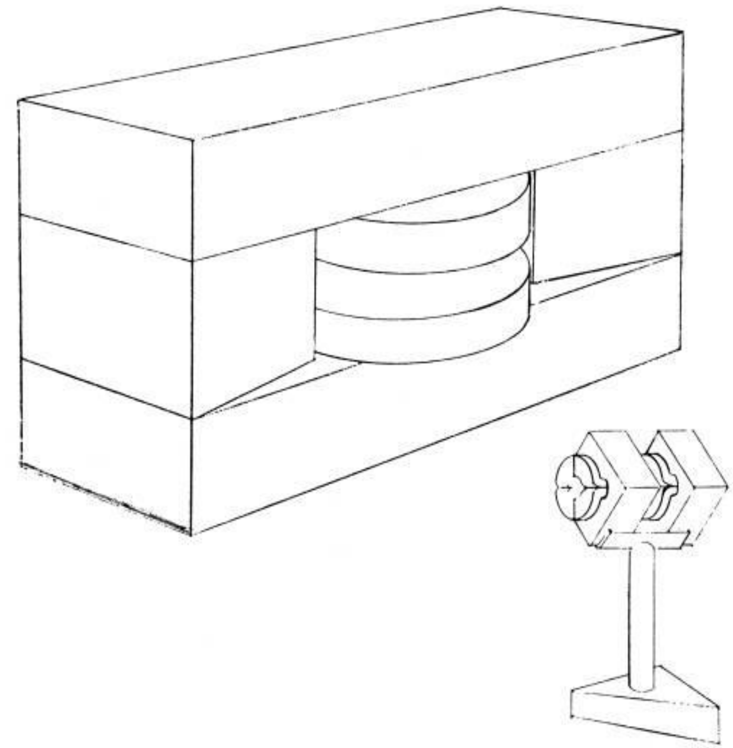


The Cyclotron evolved at Berkeley:

4”	80 keV protons	1931
11”	1 MeV protons	1932
27”	5.5 MeV d	1937
37”	8.0 MeV d	1938
60”	16 MeV d	1939
184”	>100 MeV p	1946

THE MAGNET !

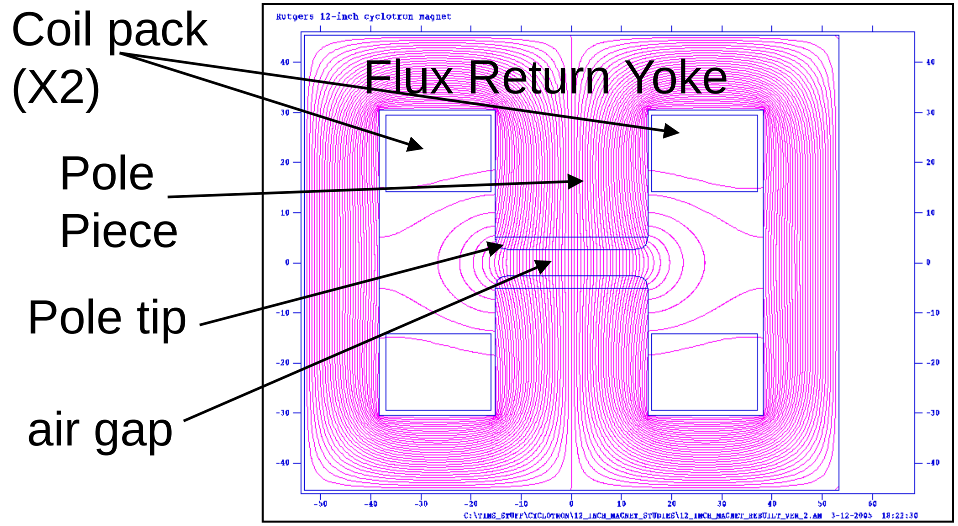
The cyclotron as seen by the…



. . . the mechanical engineer

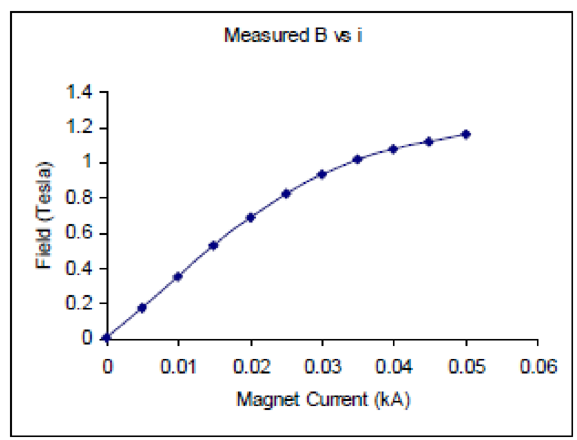
Magnetic Field of H-Magnet

Rutgers 19-inch Cyclotron Magnet
(coils not shown). 6 tons



Typical Iron-based magnet

Saturation 1.3 Tesla (13,000 Gauss)

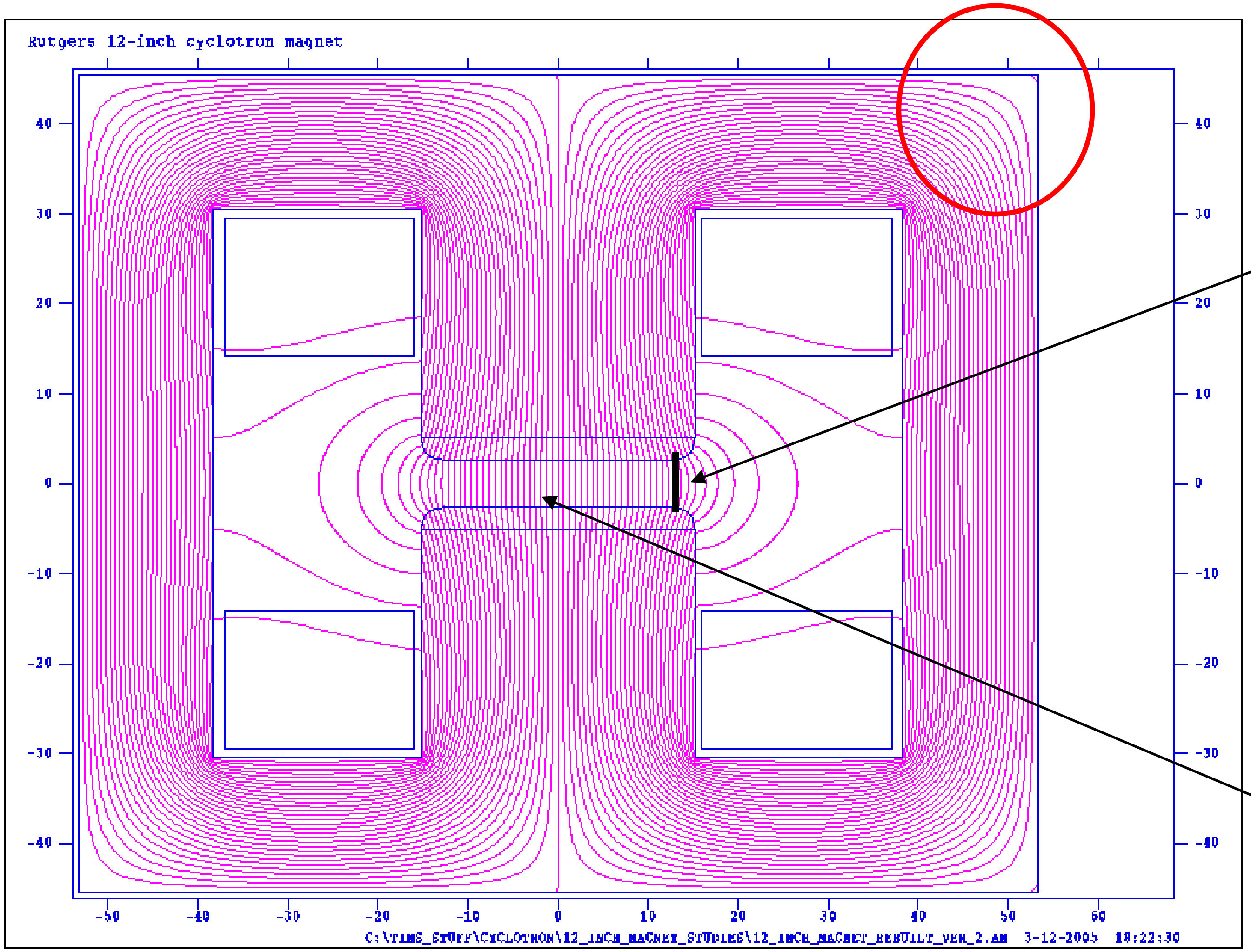


If field is maxed out, then we need a bigger magnet…

Magnetic Field of H-Magnet

Soft iron "contains" the field.

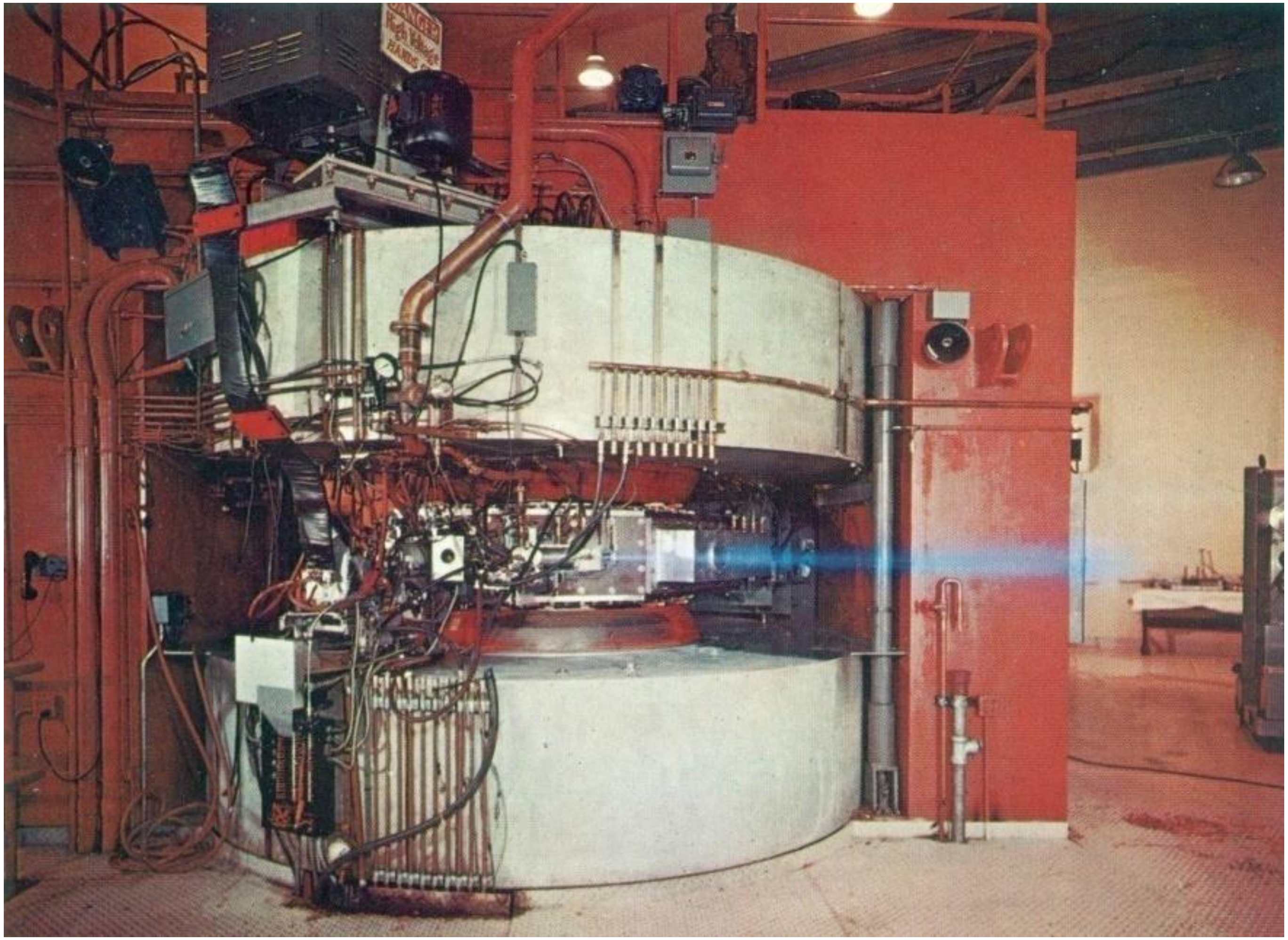
Note low flux density in corners.



Useful field typically ends a distance ~ g/2 in from pole edge

$B \sim 2\mu_o NI/g$

Argonne Nat'l Lab 60-inch - 1952



265 tons, 10.8 MeV protons (H_2), 21.6 MeV Deuterons

Shown extracting into air.   “Sounds like bacon sizzeling !” – Al Youngs

The Harvard 95-inch Cyclotron - 1949

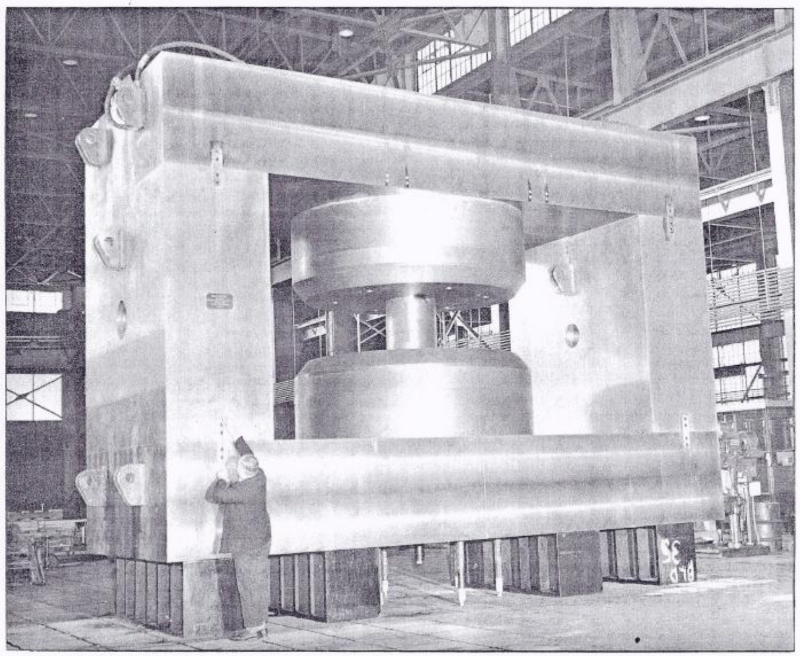
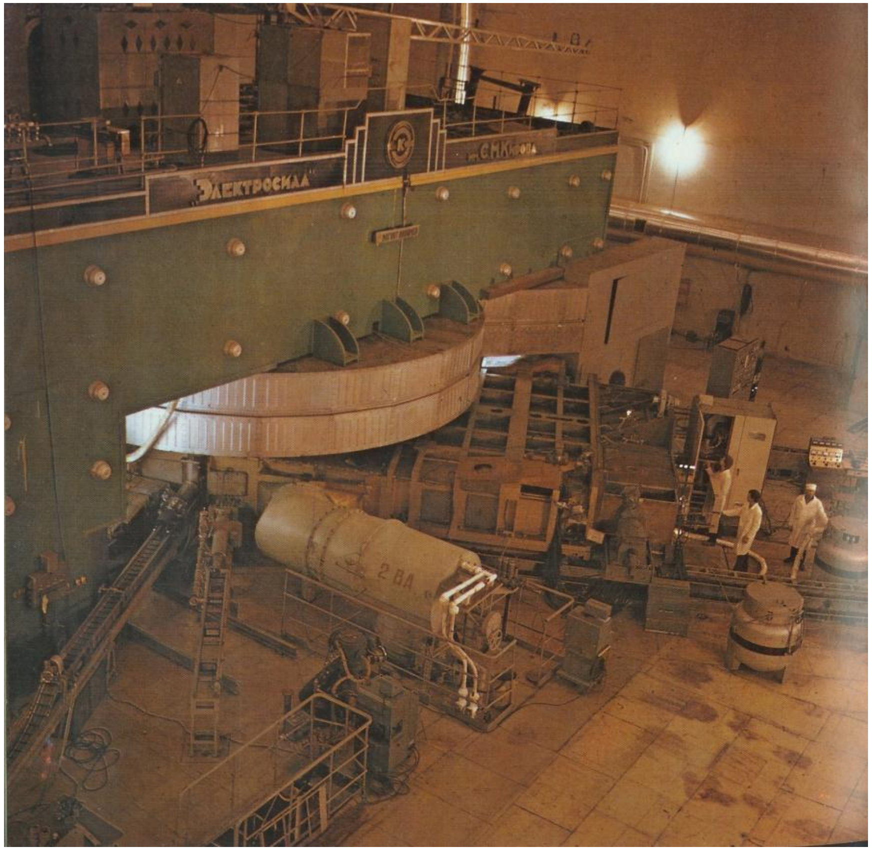


FIG. II-2 MAGNET ASSEMBLED AT WATERTOWN ARSENAL, MINUS COILS AND POLE TIPS. THE CENTER POST IS A TEMPORARY PIECE.

168 MeV, 641 tons

Fermi's 170-Inch Cyclotron - 1951

460 MeV proton, 2,200 tons



Dubna 700 MeV Phasotron - Synchrocyclotron, protons.

7,000 tons

1980, $18x10^6$ rubles

Gatchina Synchrocyclotron

1000 MeV Protons, 10,000 tons

How far can this madness go ?

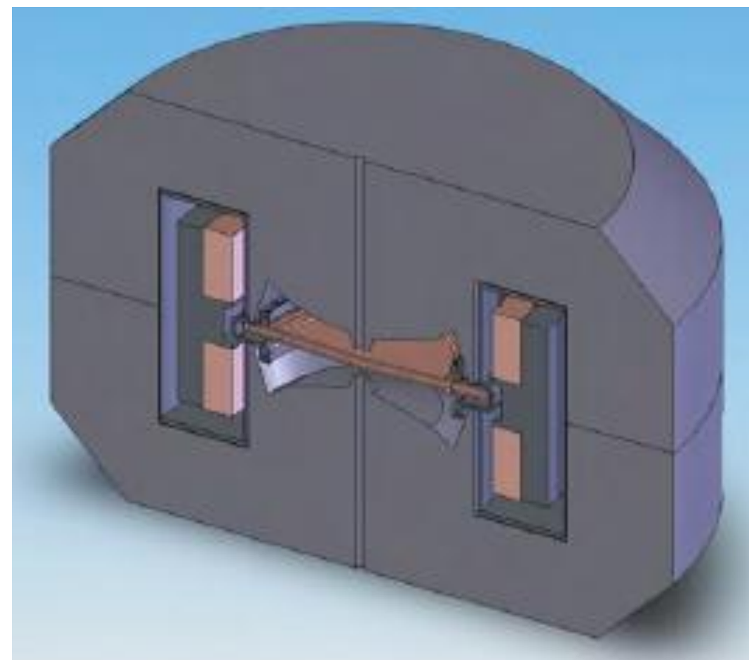
Superconducting Cyclotron

Developed by Timothy Antaya of MIT

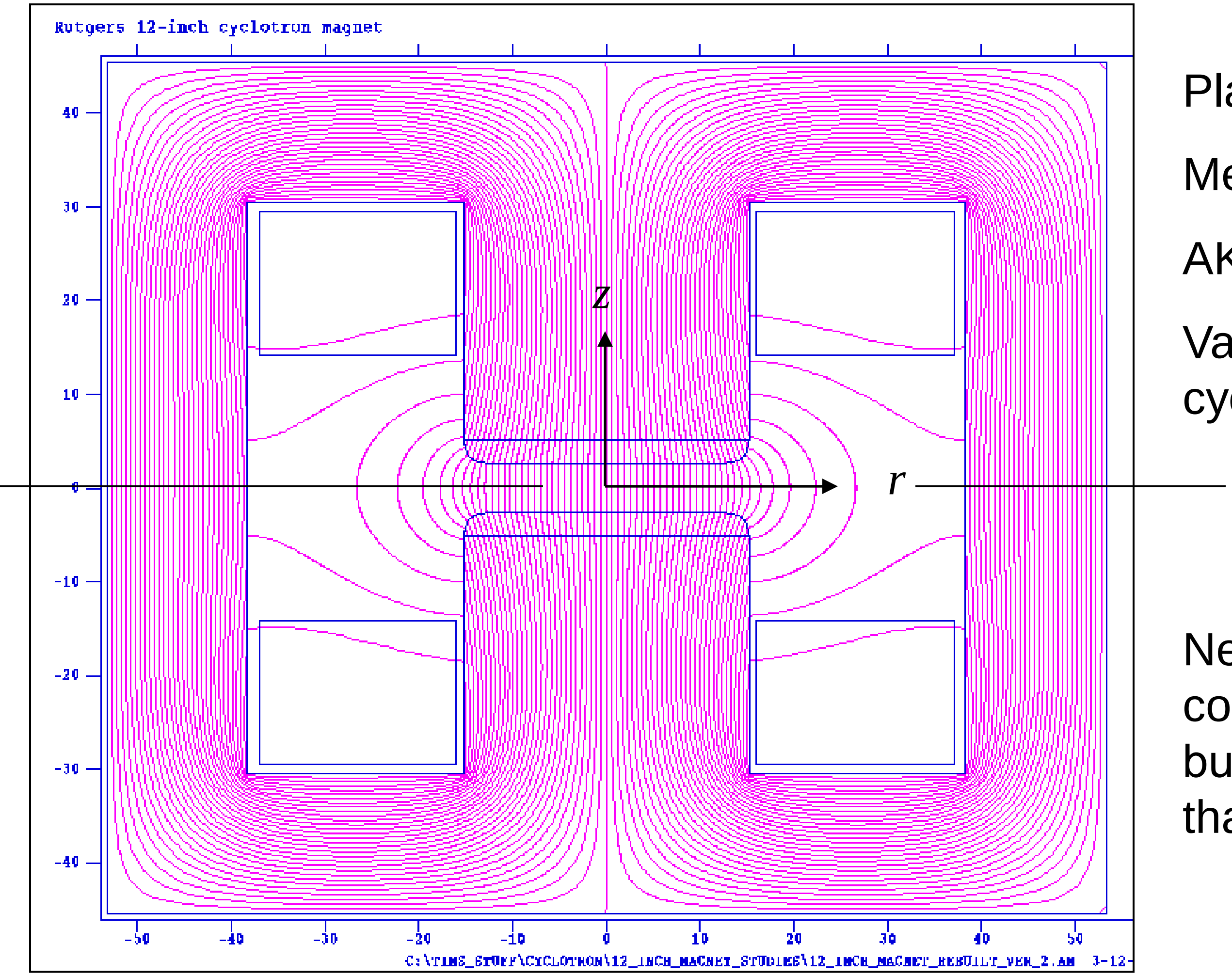
9 Tesla superconducting magnet

f_{cyc} ~ 140 MHz

250 MeV protons   2011



The Median Plane



Plane of vertical symmetry:

Median Plane $z = 0$

AKA Accelerating Plane

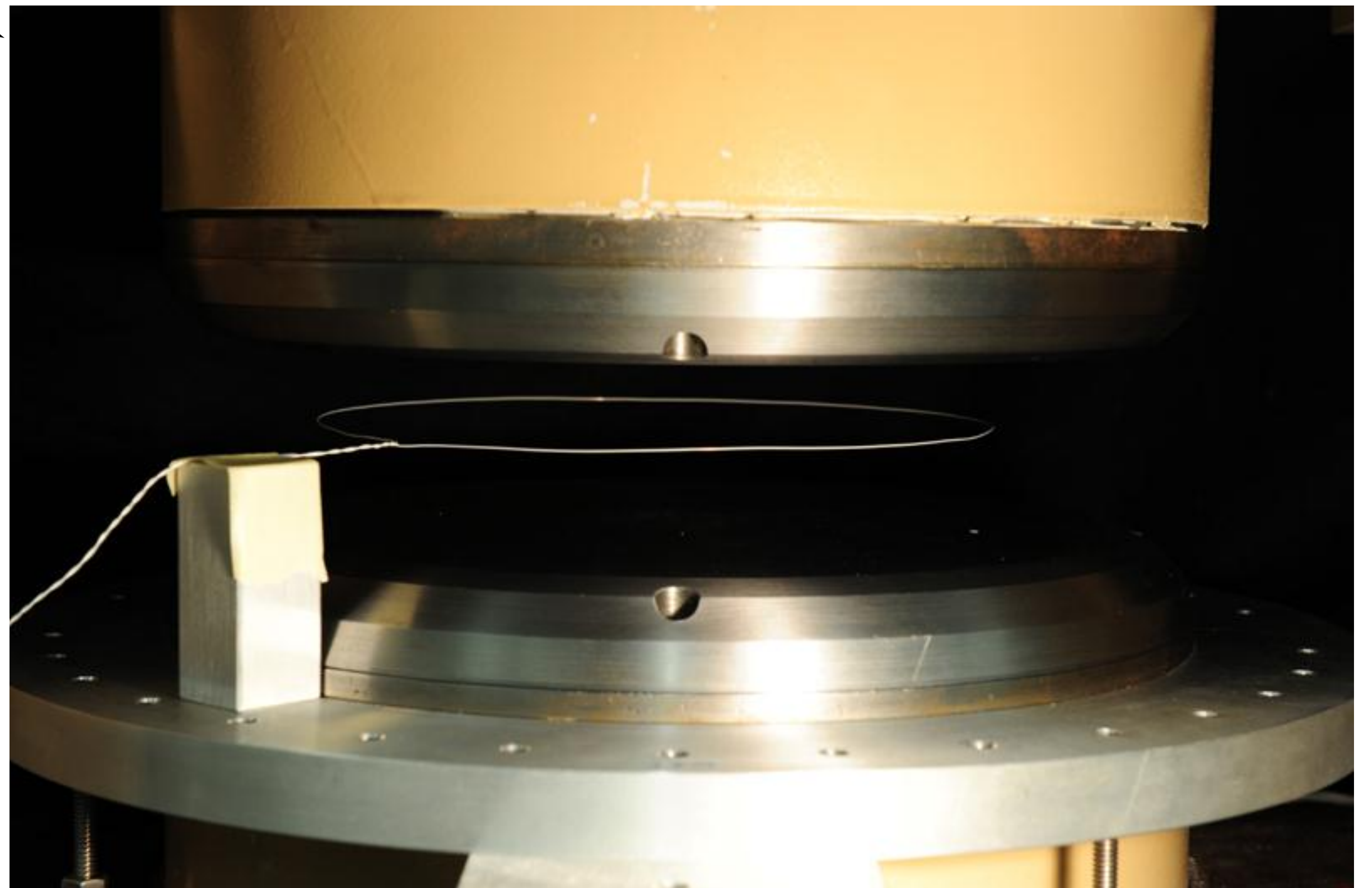
Valid for all types of cyclotrons

Need to verify MP after construction. **Beam** will do it, but you don’t want to get to that point !

LOCATING THE MEDIAN PLANE

TOP SECRET

Loop locates orbit of corresponding radius. Tension along the wire determines momentum of beam at the radius. HW problem

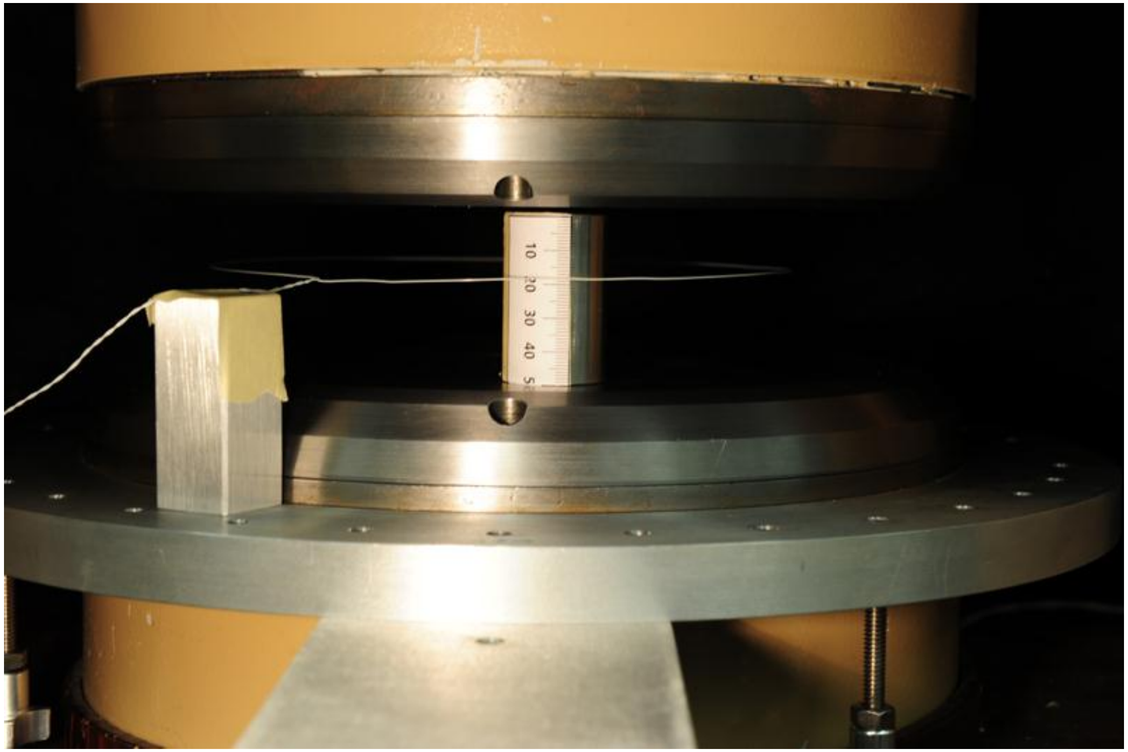


Adjusting the Median Plane

Intentional current imbalance in the excitation coils can raise/lower the median plane.

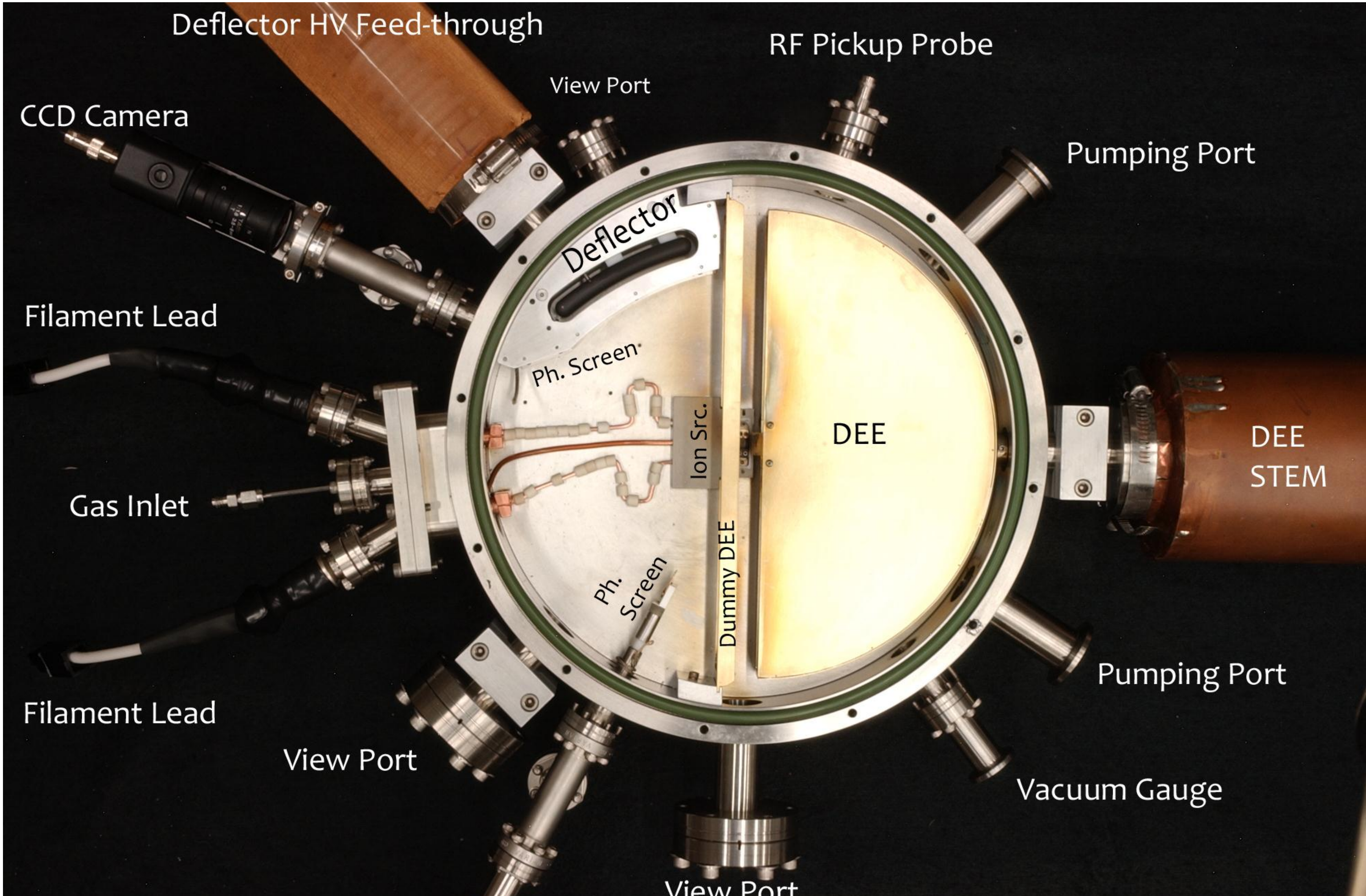
-Bring the median plane to the ion source

-Or, bring the median plane to the extraction channel height.



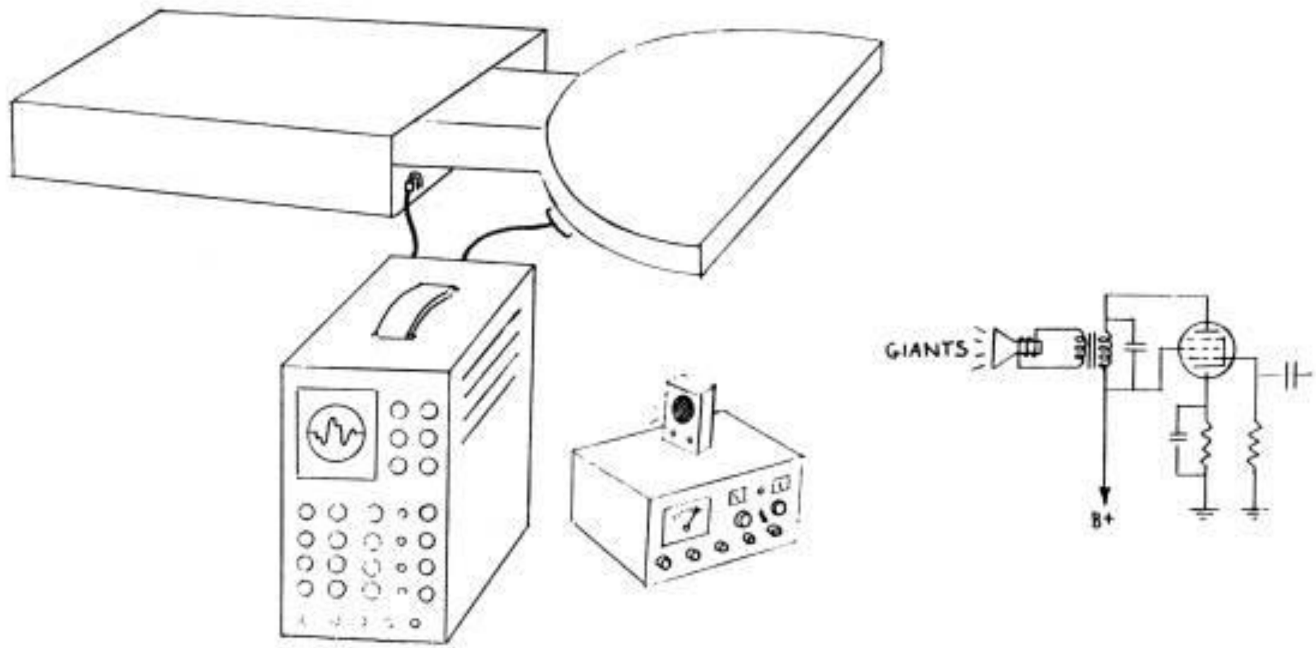
More about the magnetic field later …

Rutgers 12-inch Chamber



CYCLOTRON RF

The cyclotron as seen by the…



. . . the electrical engineer

This could be a full lecture topic

The Electric Field

The DEE-DEE or DEE-Dummy DEE gap is the region of acceleration.

In our discussion

- Acceleration is delta function (neglect transit time effects)
- Ignore focusing effects (in reality, they are only pertinent at the early stages)
- The accelerating voltage oscillates in the Radio Frequency (RF) regime
- Neglecting beam loading (You can't ! 10 MeV @ 1 mA = 10 kW of beam power)

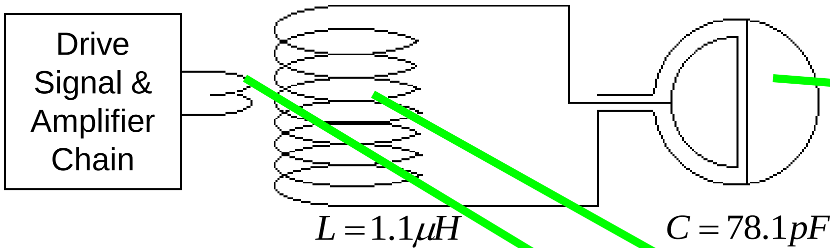
There is a practical limit to the maximum electric field (~100 kV/cm max!)

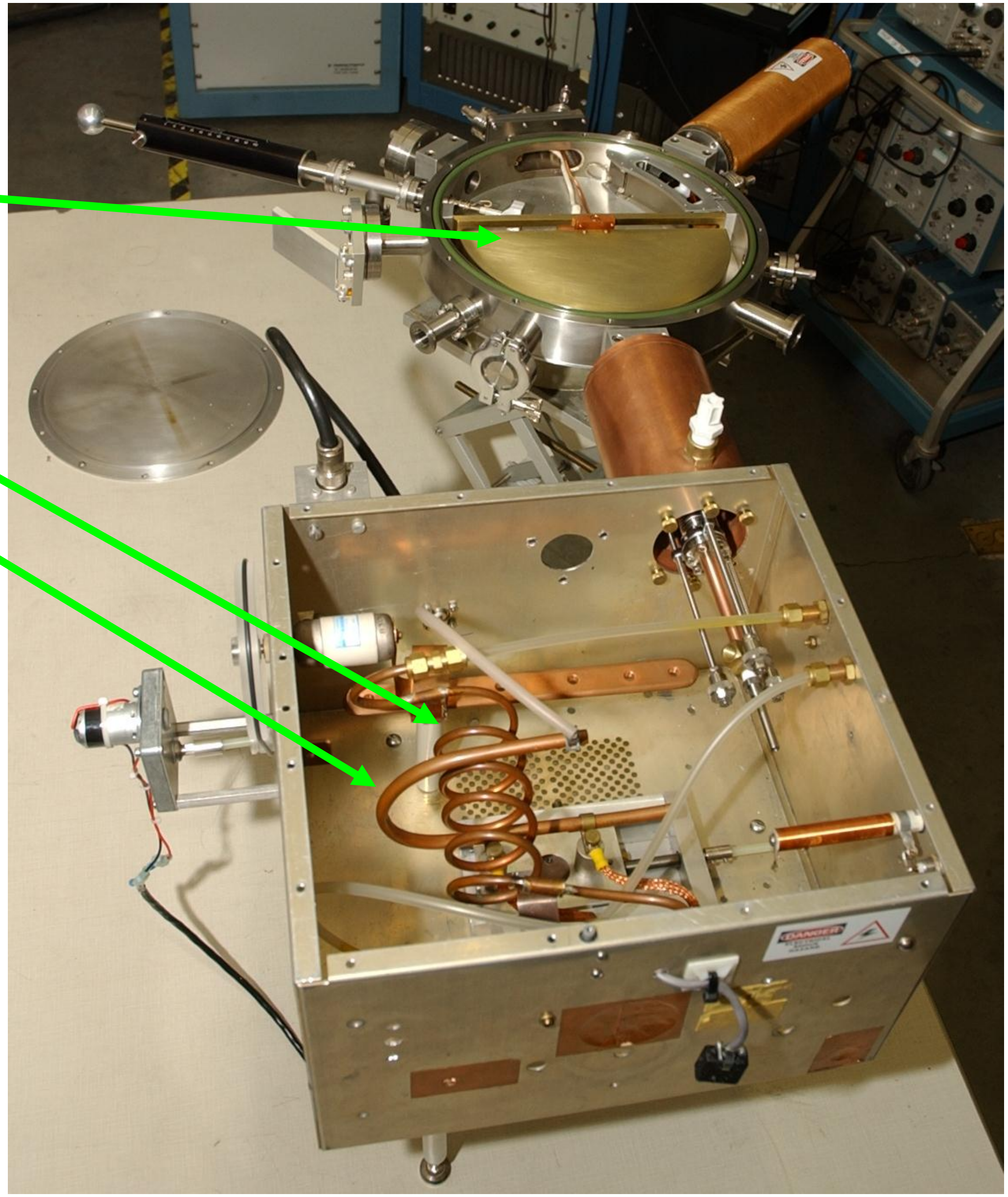
- smaller DEE → limited aperture
- larger vacuum chamber → bigger magnet

- There typically is a minimum DEE voltage requirement and/or a specific value

The Cyclotron RF System

$$f_{cyc} = \frac{qB}{2\pi m} = \sim 15MHz \qquad f_o = \frac{1}{2\pi\sqrt{LC}}$$



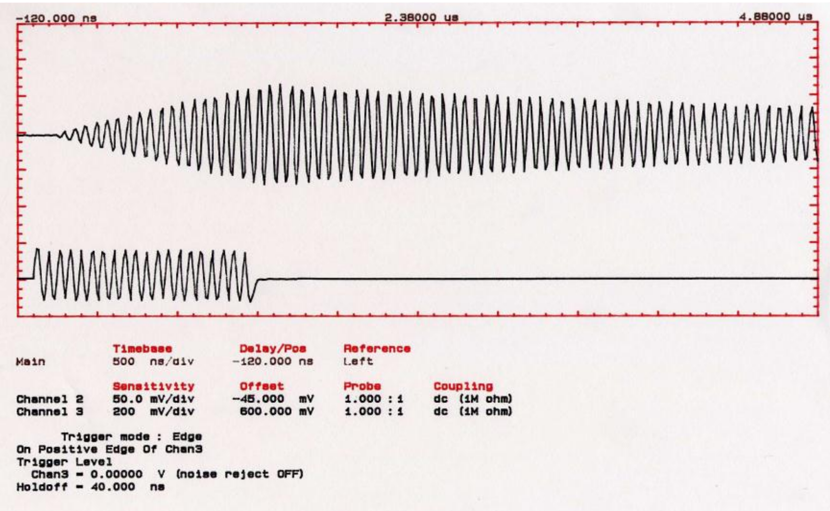


Why use a resonant circuit ?

$$Q = \frac{\omega L}{R_{AC}} = \frac{\omega U}{\langle P \rangle}$$

To develop high voltages with modest RF power.

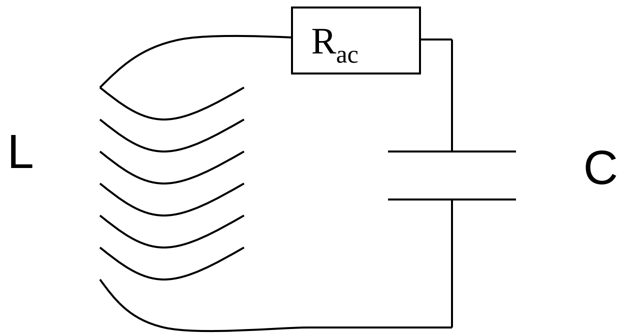
The highest voltage for given power occurs when: $Q_{loaded} = \frac{1}{2} Q_o$



RF Power Calculations

Ohm's law: $$P = I_{rms}^2 R = V_{rms}^2 / R \quad \longrightarrow \quad I_{rms} = \sqrt{\frac{P}{R}}$$

Definition of Resonance: $$\langle U_B \rangle = \langle U_E \rangle => \frac{1}{2} L I^2 = \frac{1}{2} C V^2 \qquad f = \frac{1}{2\pi\sqrt{LC}}$$



$$V_{rms} = I_{rms}\sqrt{\frac{L}{C}} = \sqrt{\frac{PL}{R_{ac}C}}$$

$$V_{peak} = \sqrt{\frac{2PL}{R_{ac}C}}$$

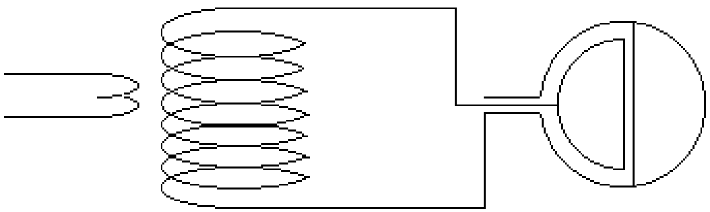
-Decrease capacitance

-Reduce R_{AC}

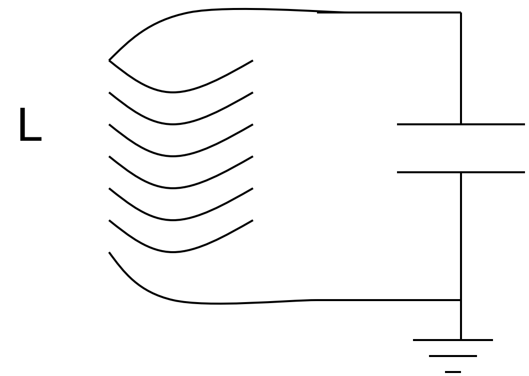
-Voltage goes as Sqrt of power

Argument for 2-DEES

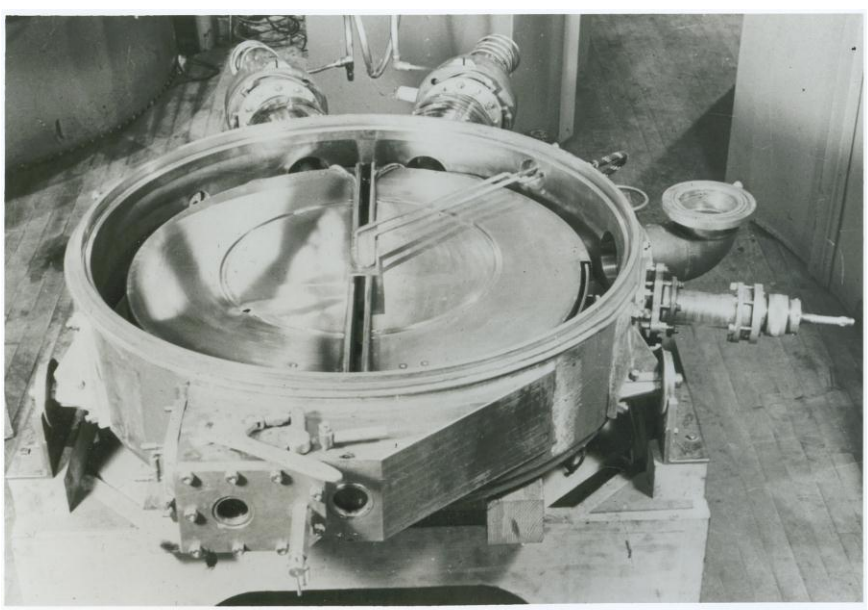
Single 'ended' system:

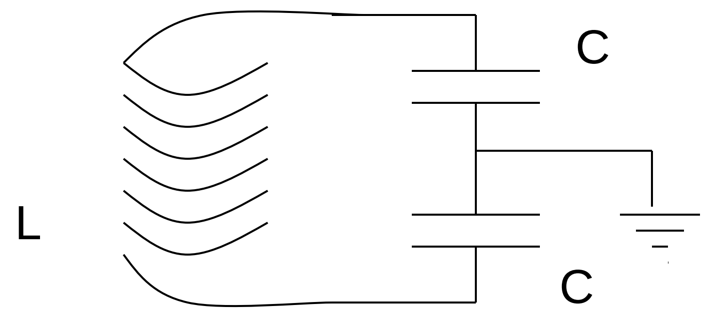


Electrical equivalent



2 DEE system:





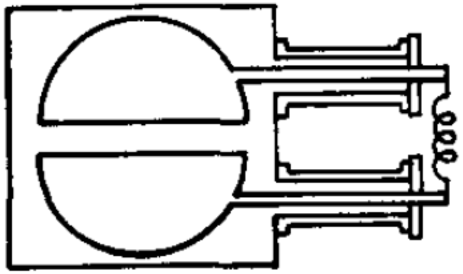
$$\frac{1}{C_{equiv}} = \frac{1}{C_1} + \frac{1}{C_2} \qquad C_1 = C_2 = C$$

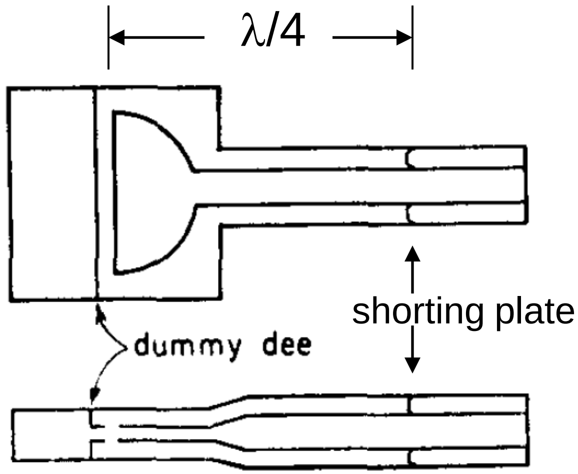
$$C_{equiv} = \frac{1}{2}C \qquad V_{peak} = \sqrt{\frac{2PL}{R_{ac}C}}$$

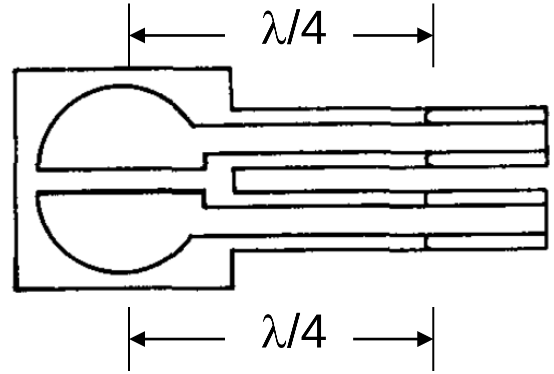
→ Halves the power requirement for a given DEE voltage.

Quarter Wave Resonators

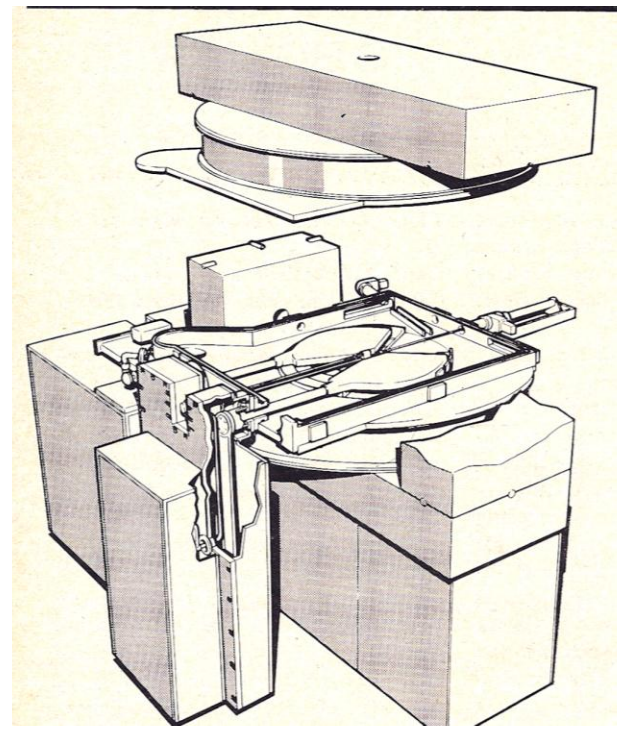
So far lumped values:







DESY **Zyklotron**:



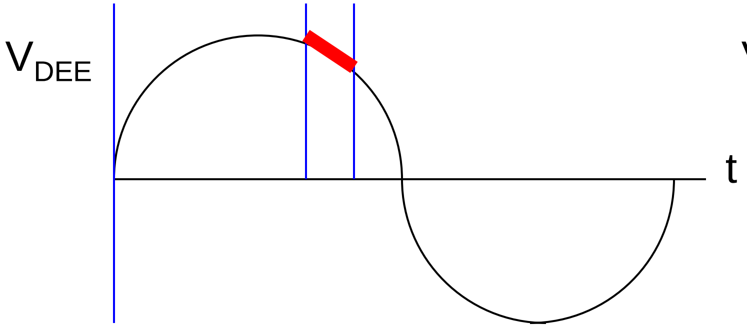
The Cyclotron Corporation

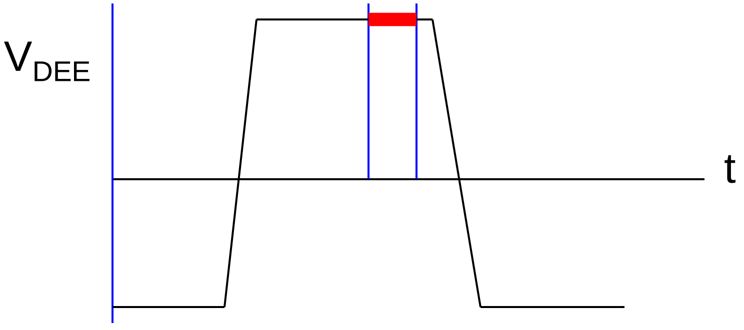
Harmonic Number

$$f_{cyc} = H \frac{\omega_{cyc}}{2\pi} = H \frac{qB}{2\pi m}$$

Where H is an odd integer, 1, 3, 5, …

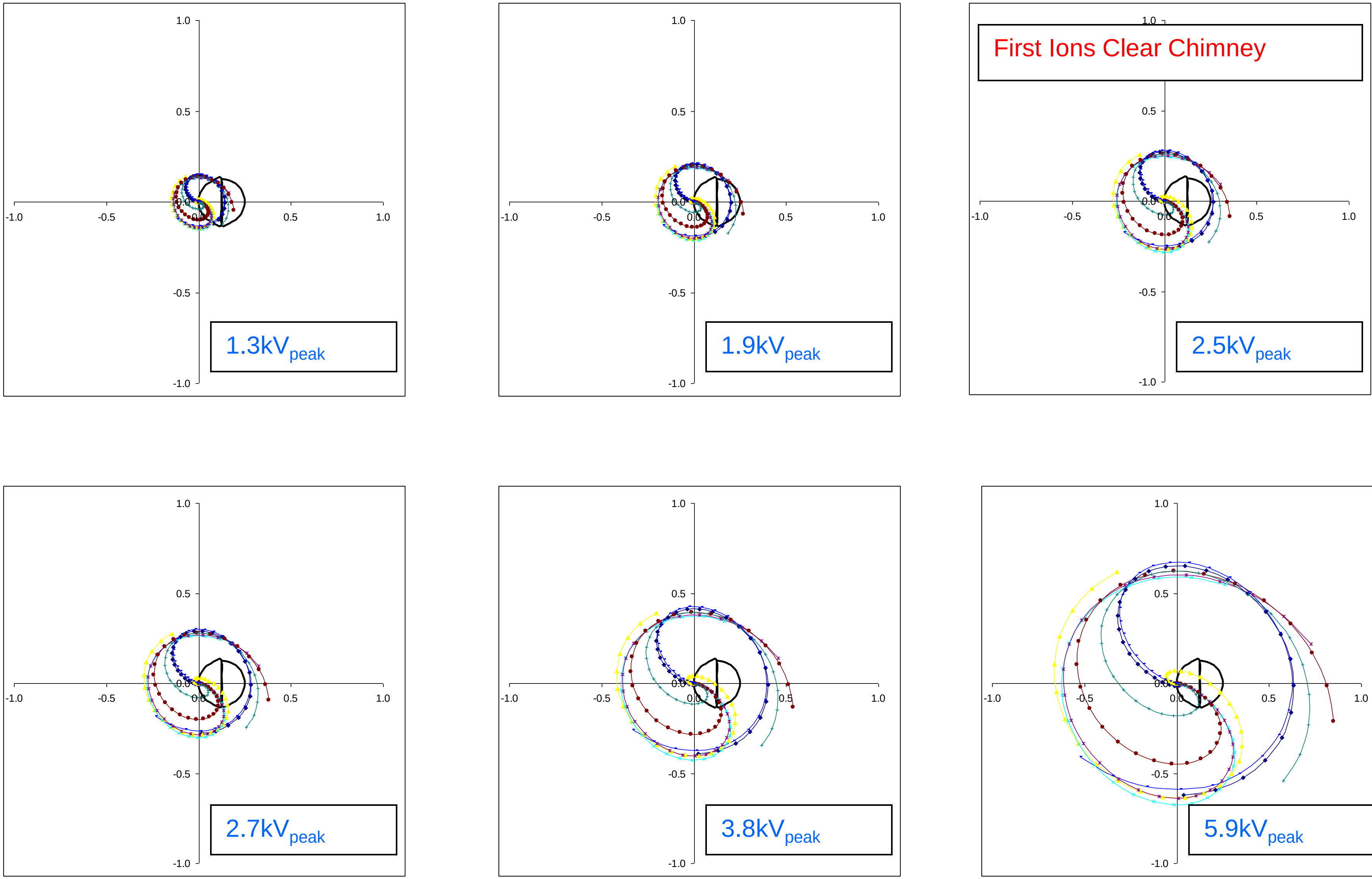
- Higher frequency
  - smaller RF components.
  - Land in Commercially available regime
  - However the RF bucket is smaller.
- Use multiple odd-harmonics: Fourier series to reduce energy spread.





What Dee Voltage is Needed ?

Scale : inches



Minimum DEE Voltage

In addition to clearing the chimney, there is another requirement placed on the DEE voltage.

There is a phase slippage associated with the reduced W.F. Field & relativity.

(this is another full lecture)

The phase slippage per turn can be written as

$$\frac{d\phi}{dN} = 2\pi\left(\frac{\omega_{RF} - \omega_{cyc}}{\omega_{cyc}}\right) = 2\pi\left(\frac{B_0 - B}{B} + \frac{B_0 T}{B E_0}\right)$$

The maximum phase slippage allowed $\pi/2$

Unless one intentionally chooses the RF frequency such that the ions are initially advanced by $\pi/2$ → extending permissible phase slippage to π ! This is an exploitation of phase stability which executes ½ synchrotron oscillation.

$$\int \frac{d\phi}{dN} = 2\pi\left(\frac{\omega_{RF} - \omega_{cyc}}{\omega_{cyc}}\right) < \pi$$

Minimum Dee Voltage

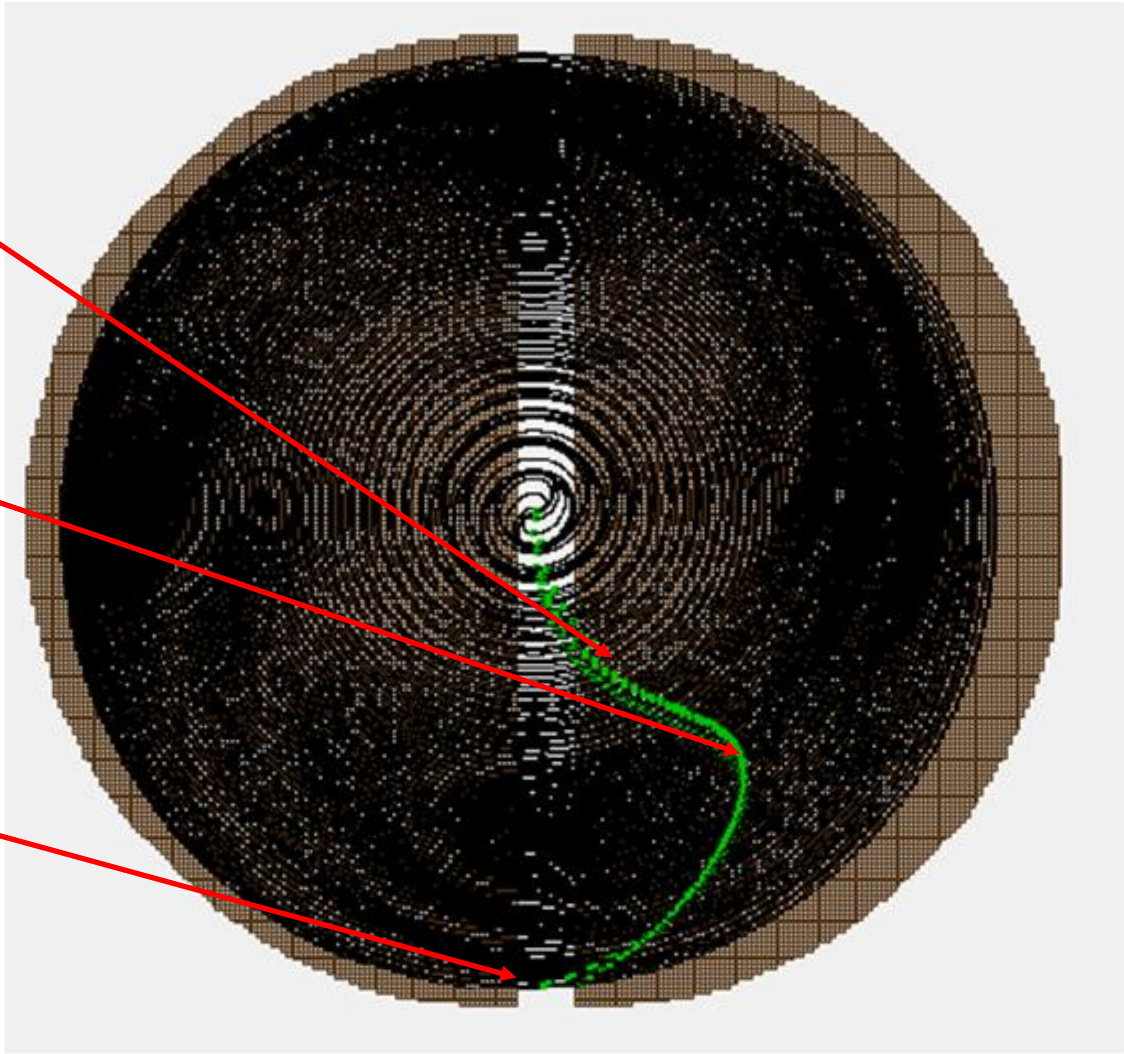
Green dots are a strobe at the RF frequency showing ion's locations once per cycle.

Initially ions are showing up to the gap early, receiving less and less kick.

Their revolution frequency briefly matches the RF frequency

They begin to show up later and later and ultimately:

$$\int \frac{d\phi}{dN} = 0$$



SIMION DEMO

Of Phase slippage

The Ion Source

- Internal Sources
  - Hot Filament → e- emission to ionization
    - Good for modulated beam
    - Poor relative lifetime
    - Useful for low Z
  - P.I.G. ‘cold cathode’ → e- emission
    - Long warm up time – minutes
    - Excellent lifetime
    - Useful for low to medium Z
    - High intensity
- External Sources
  - Electron Cyclotron Resonance
    - Good beam quality (low emittance)
    - Several sources – (i.e. backups)
    - The entire periodic table & high charge states (q/m) [also pure]
    - Suffers from space charge limitation

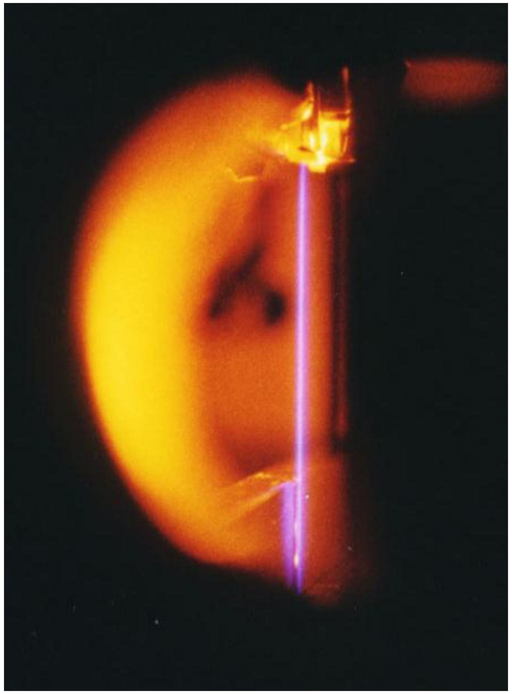
Ion Sources are an entire lecture topic

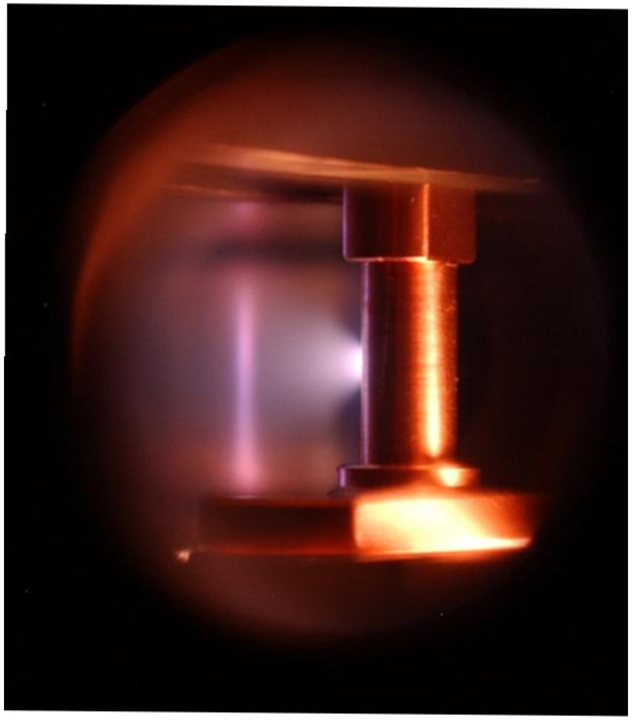
Hot Filament Examples

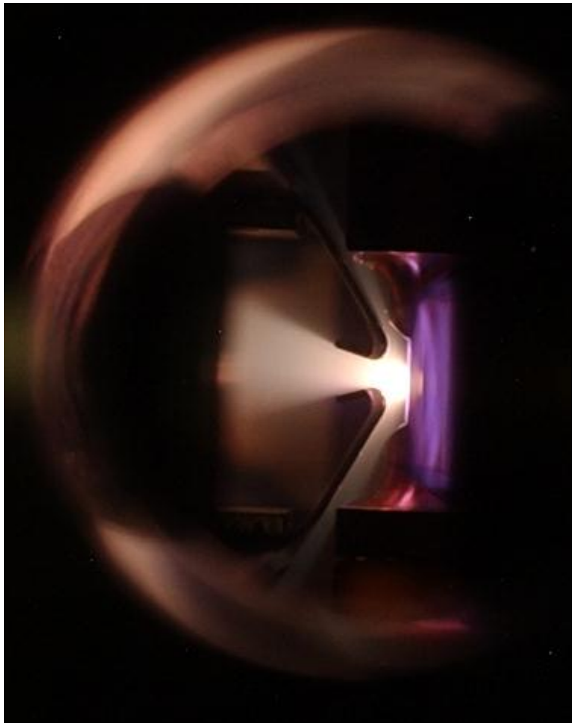
Hot filament at the bottom or top of chamber

Electrons follow the B-field lines

Ionize ambient or supplied gas along the way







Ion Source Parameters

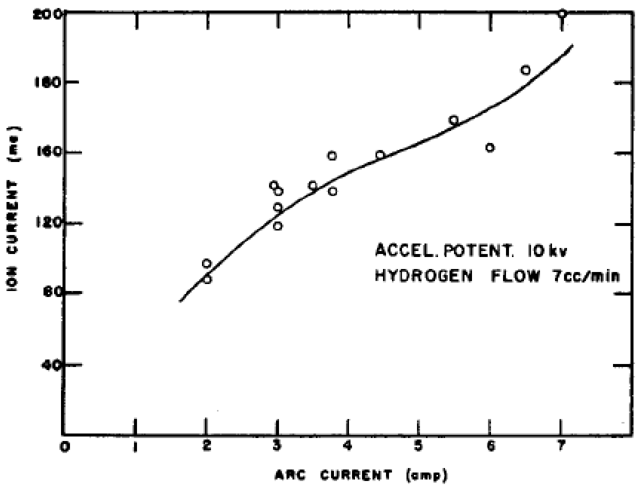


Fig. 6. Ion output as a function of arc current.

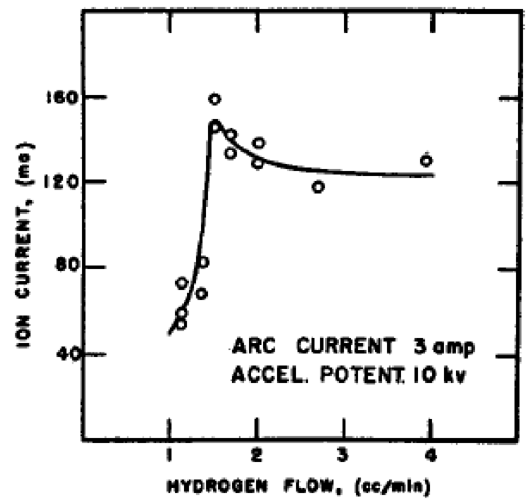


Fig. 5. The effect of hydrogen flow on ion output.

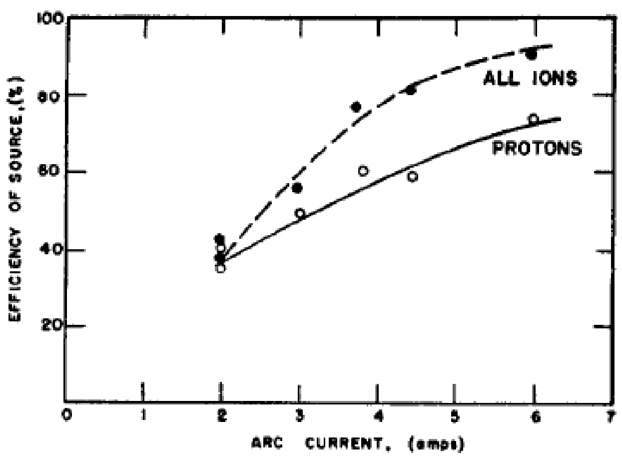


Fig. 4. Ion source efficiency as a function of arc current.

"knobs"

- arc current (in current limit)
- gas flow

Practical limits

- source destruction from heat
- space charge

P.I.G. Ion Source Construction

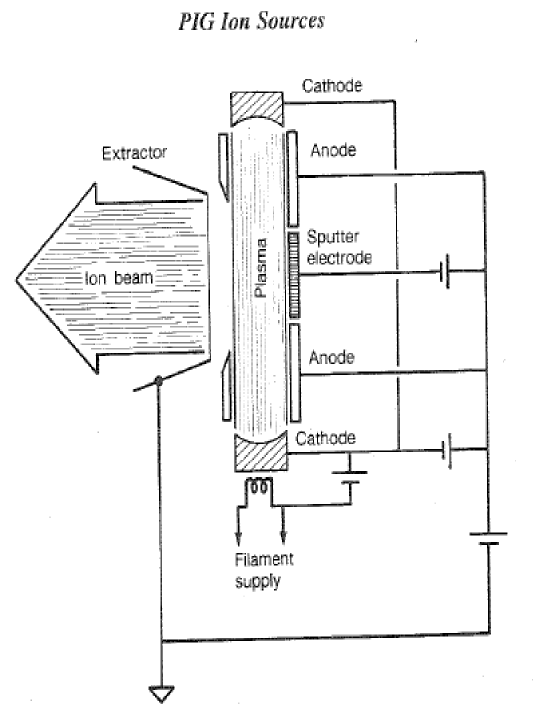
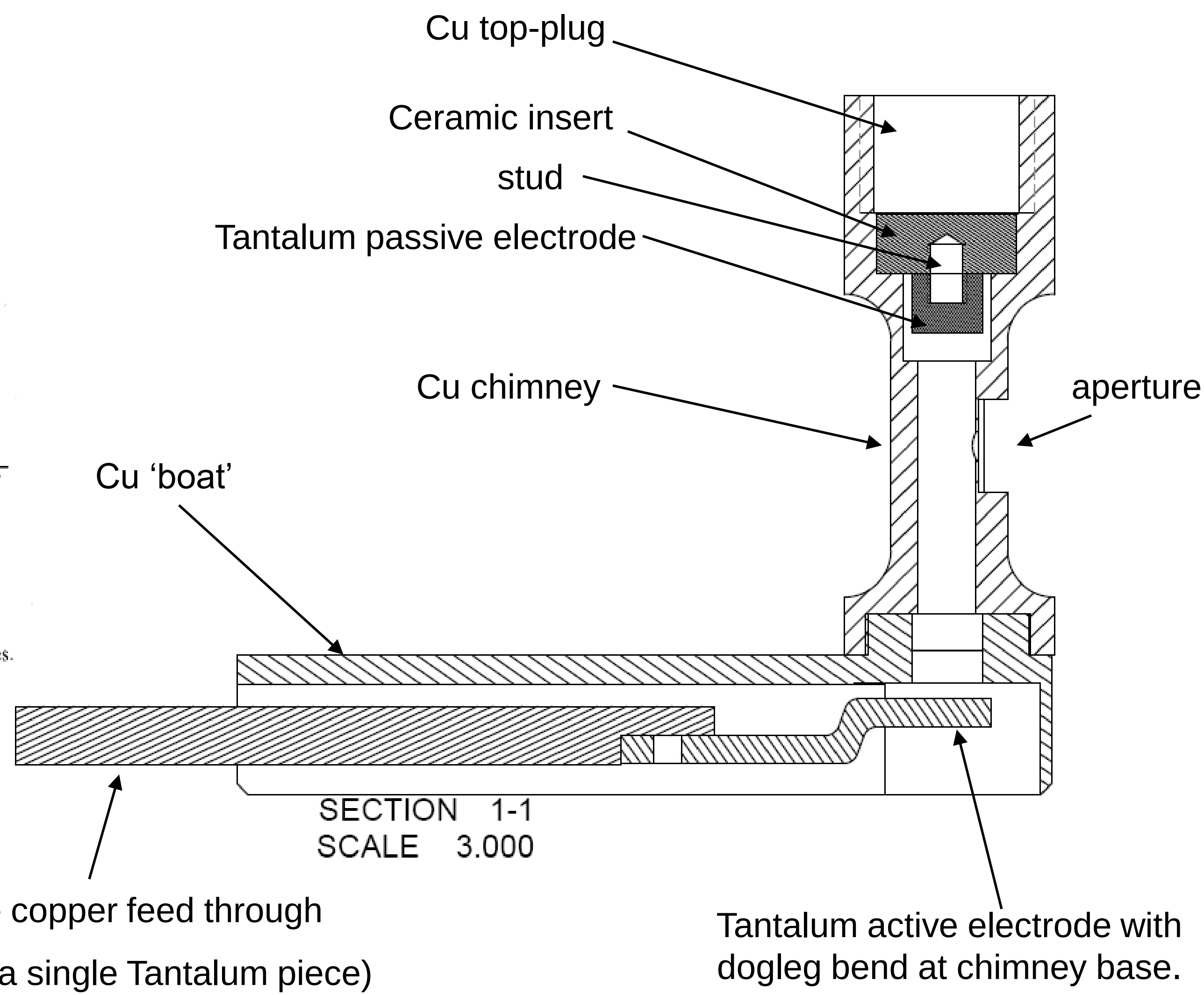


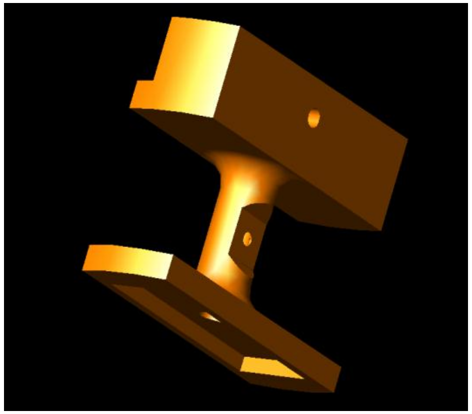
Figure 8.1 Schematic of the PIG source and its power supplies.



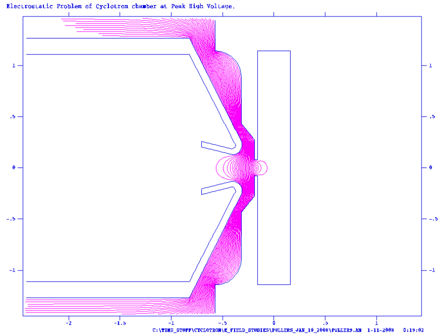
Symmetry of Chimney & Pullers

Initial conditions require attention !

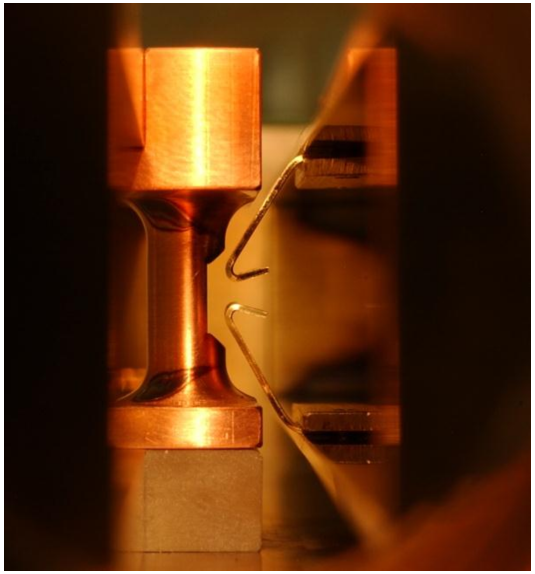
A vertically symmetric chimney is required !



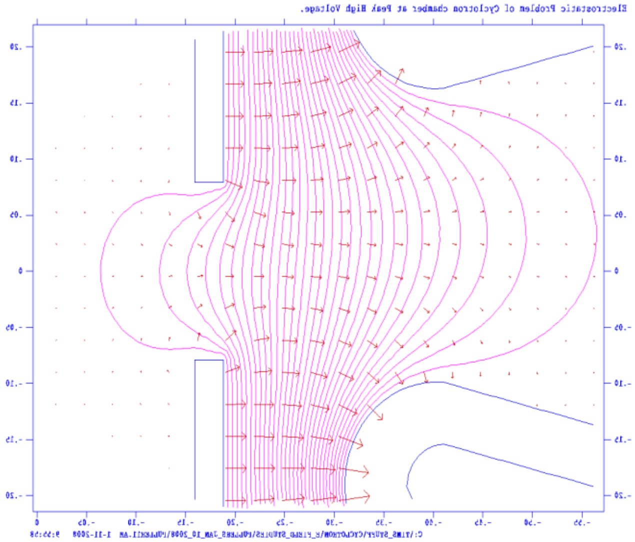
PSF modeling showing enhanced E-field at chimney.



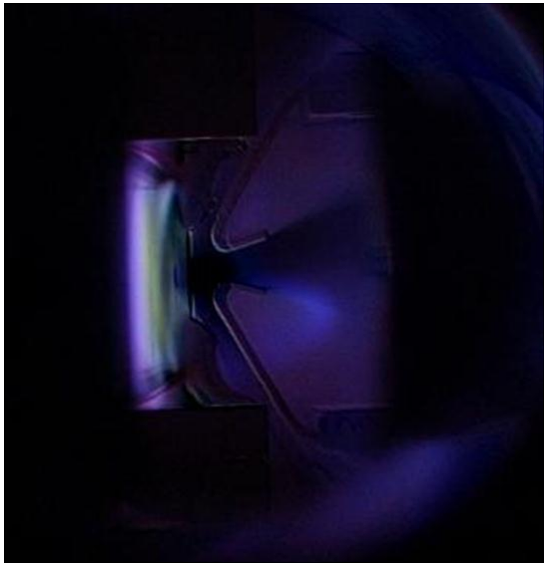
A vertical misalignment of the pullers w.r.t. to the chimney pulls the beam downward:



Side port view of pullers.

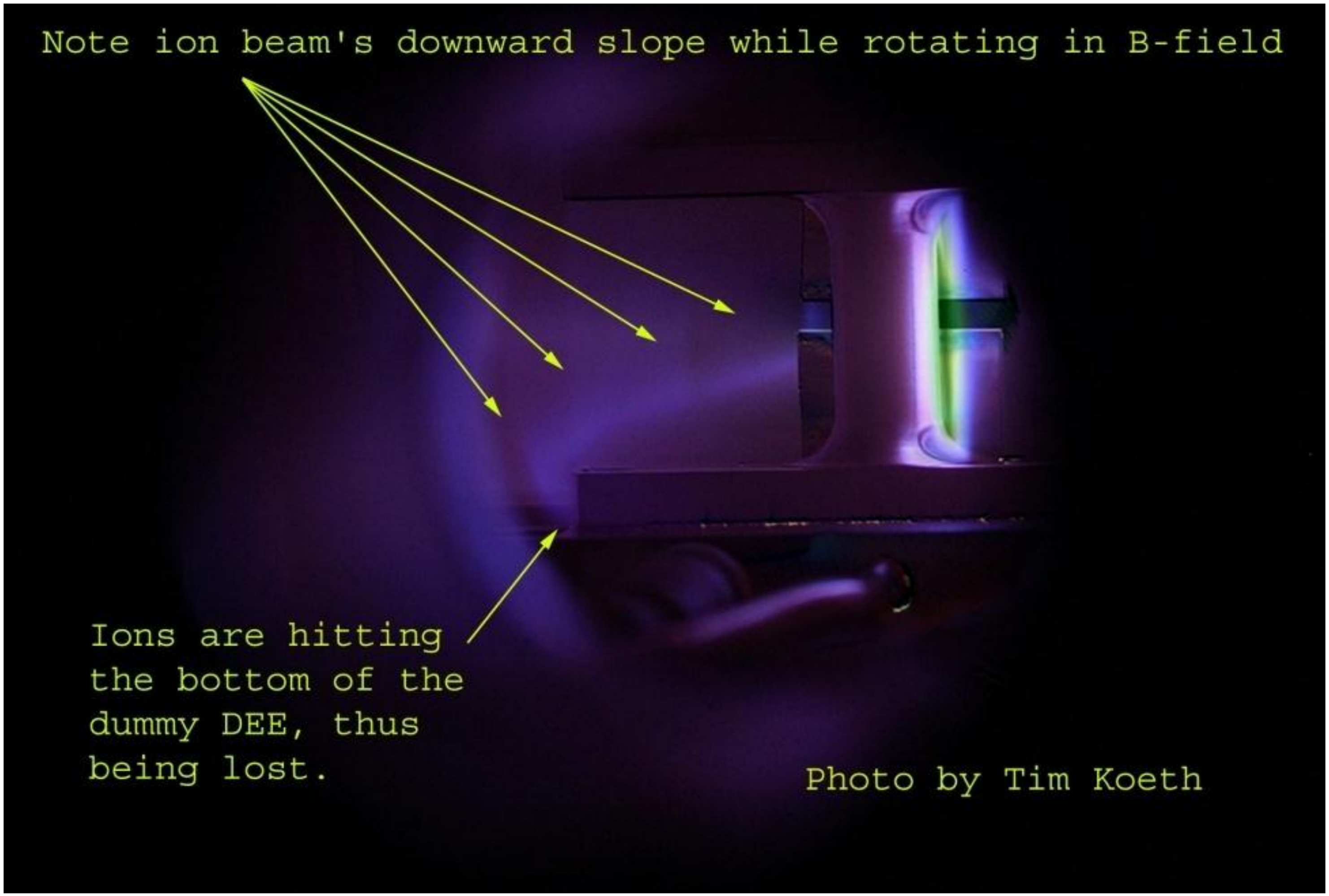


PSF model of misalignment.

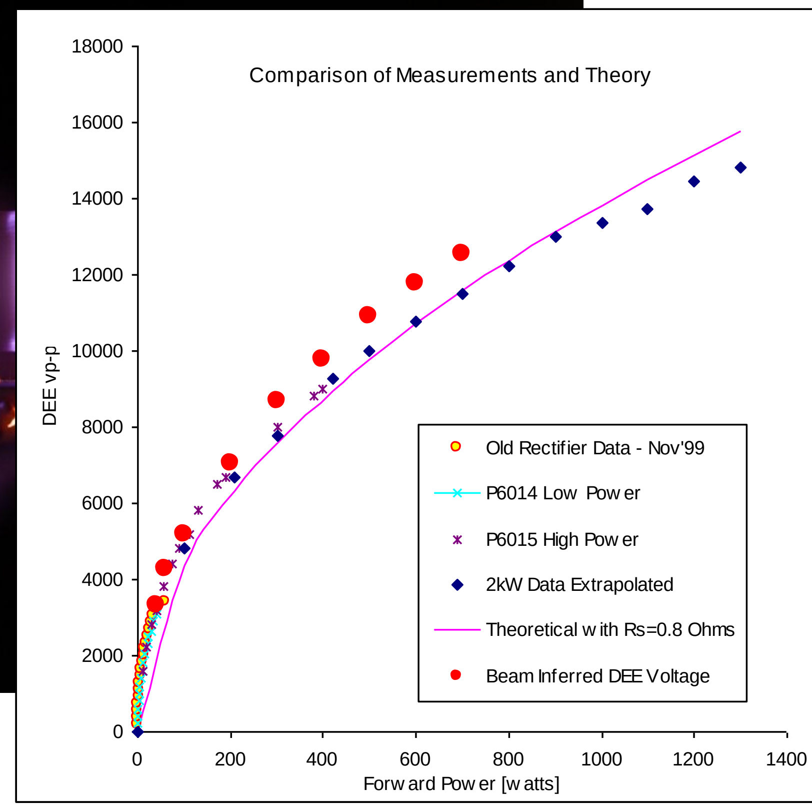


Same view during operation,

EXAMPLE WITH BEAM



DEE Voltage Verification



- In the 1st half revolution

$$E(r) = \frac{qB^2}{2m} r^2 = \frac{1}{2} V_{p-p}$$

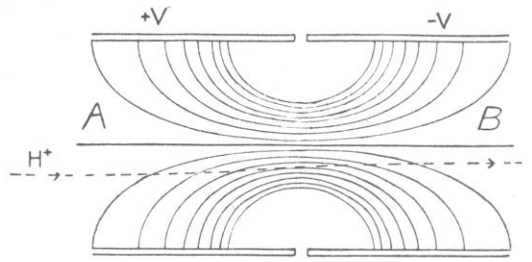
Orbit Stability

What happens to ions which leave the ion source with an angle, or above or below the median plane ?

EOL and MS Livingston got real lucky !

1931: Although the cyclotron worked, the beam intensity was very weak. Lawrence ☹
- Prof. Lawrence: must have wire grids, and shims for a uniform magnetic field !
-Student Livingston removed the grids while Lawrence was out of town ➔ beam intensity shot up.

Livingston took this remarkable finding to Lawrence. To which Lawrence responded: “It’s obvious what is happening…”



. Diagram indicating the focussing action of the electric field between the accelerating electrodes.

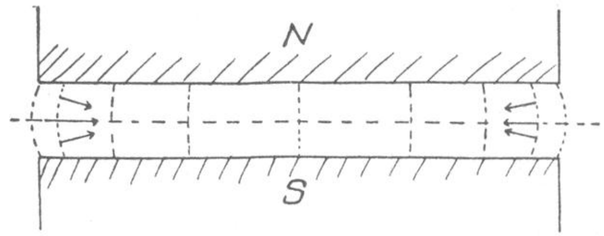
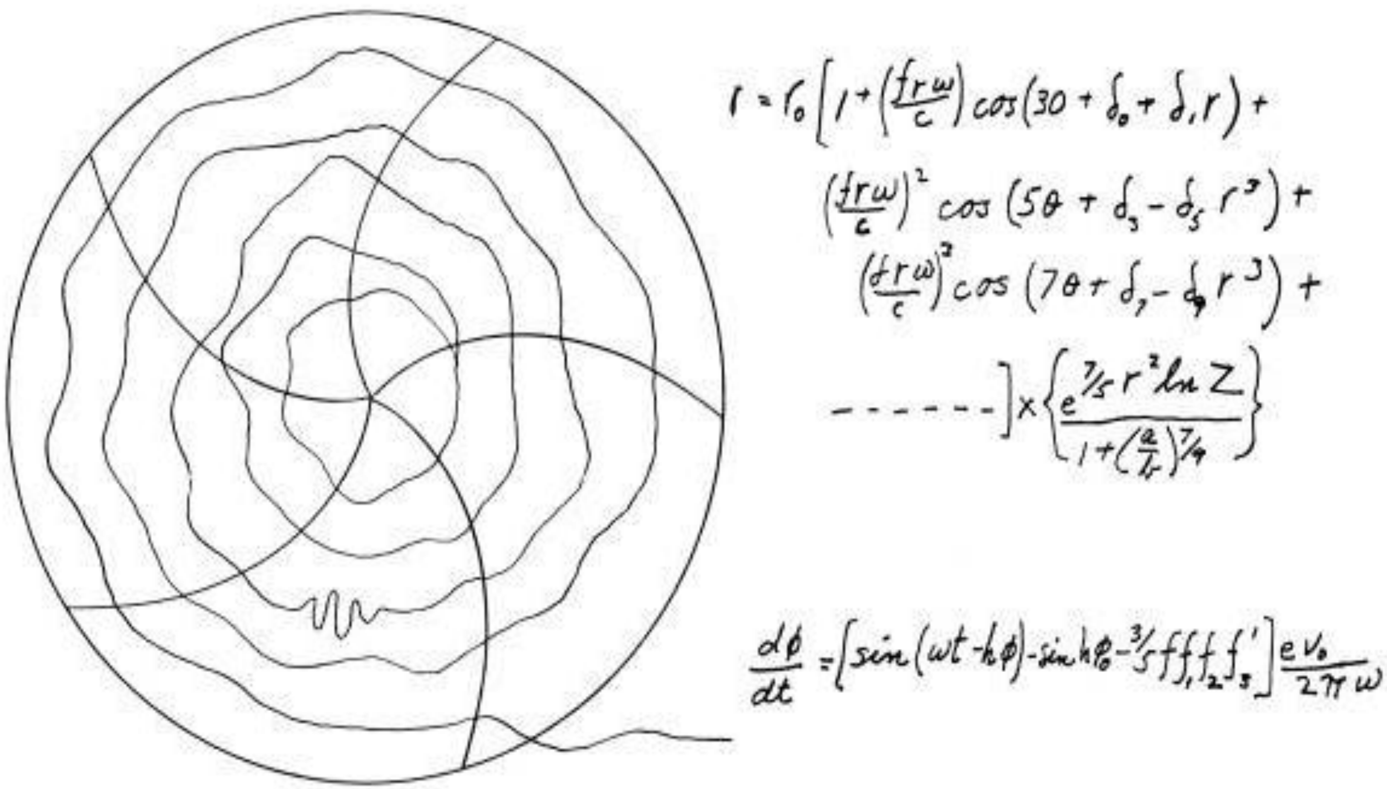


Diagram indicating focussing action of magnetic field.

Intentionally introduce radial B-field component at the cost of an vertical gradient: to be coined weak focusing.

The cyclotron as seen by the…



… the theoretical physicist

Derivation of Weak Focusing at the board.

What are Limits of Stability ?

How far displaced can one stray from the Equilibrium Orbit and maintain the beam ?

Use numerical tools to find the EO and limits of stability

- Radial first (find EO & radial limits)
- Axial second
- Additionally, identify resonances

How ? Launch many particles in a static field (shut off RF for now), let them propagate for a while and follow their evolution

If it is too cumbersome to record each time step of each particle, just record their behavior at one location in the machine & plot phase space (r,r' and z,z') – this provides a time-independent characterization of the beam

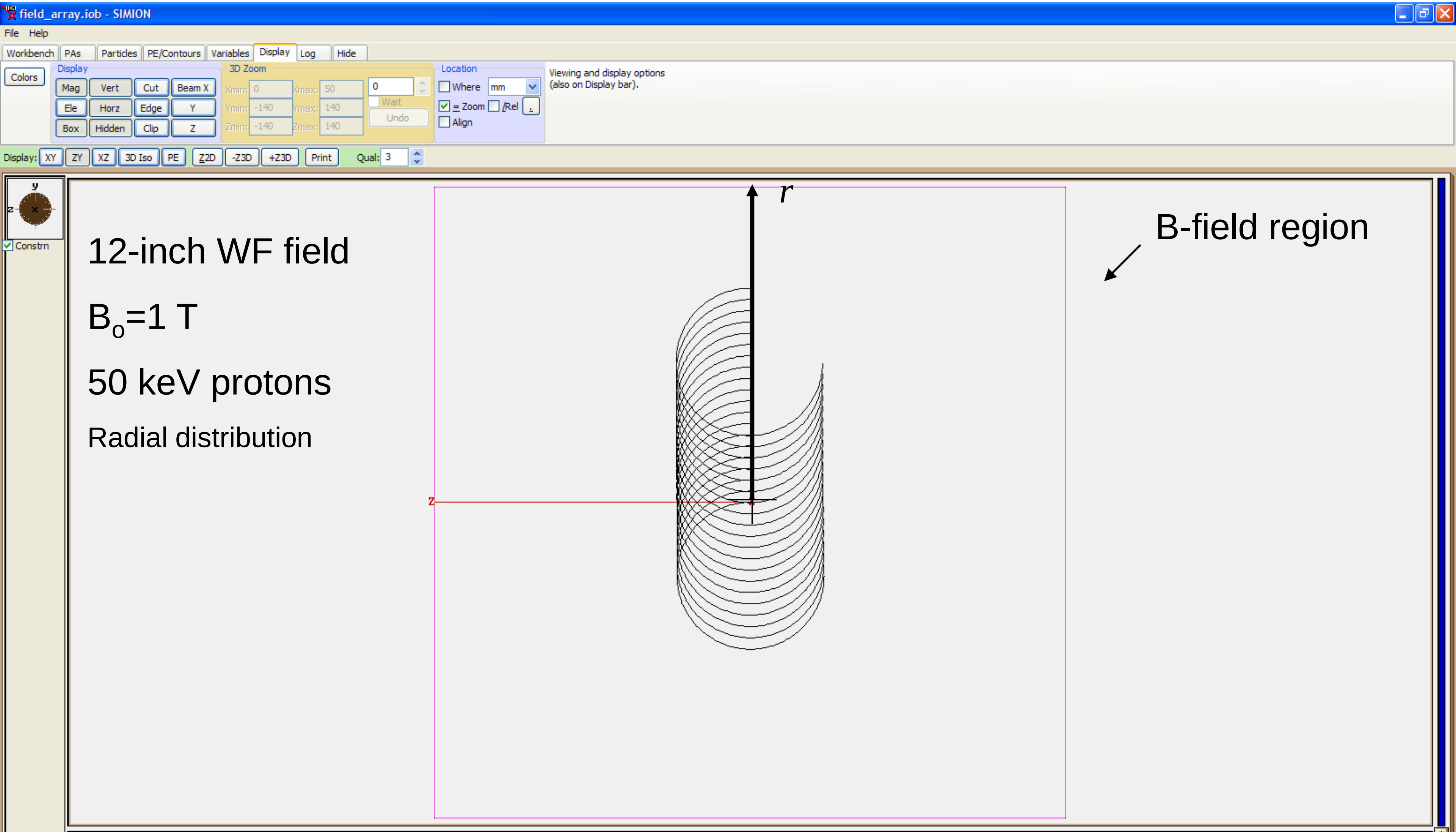
WARNING: Simulations are **Frienemies** !!!

Do not implicitly trust them ! (simulate with one eye open)

Check and double check

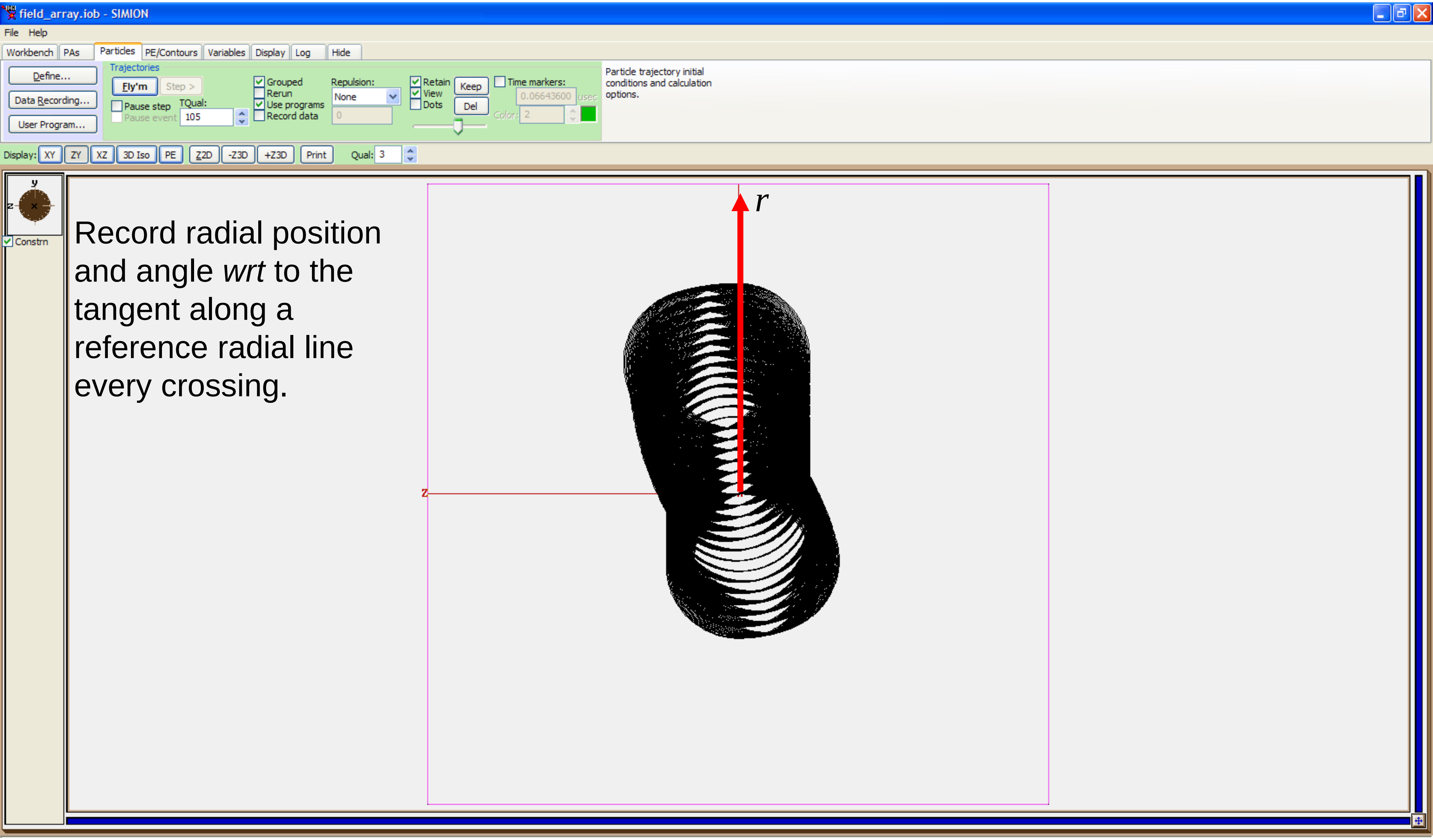
Benchmark them against the experiment !

To locate the EO, a radial distribution of a given energy was launched in the median plane. The following is a screen shot of the initial radial particle distribution; almost completing one revolution.



Same SIMION model some time later…

Note greater procession of larger radial offsets



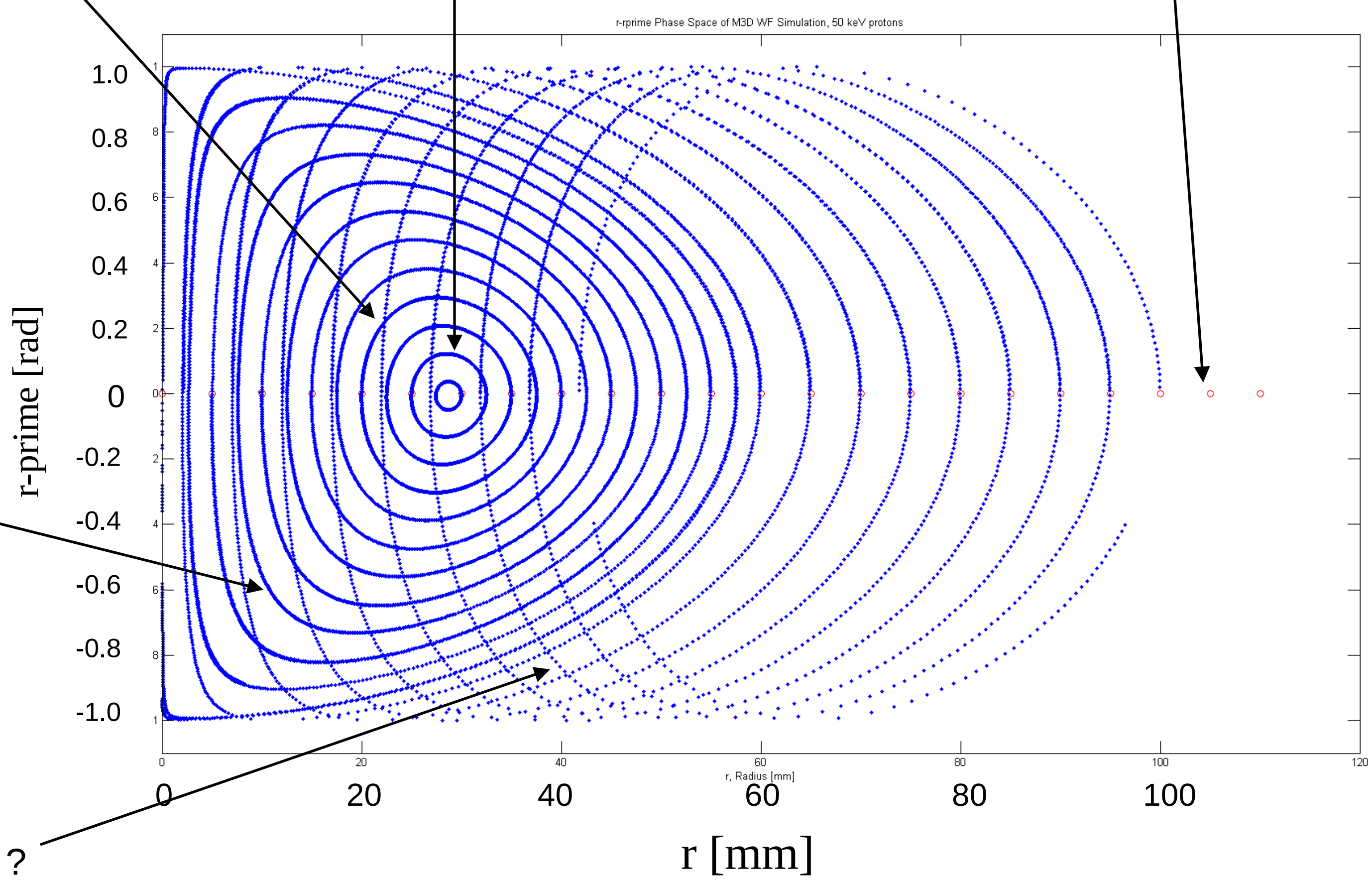
Plot Phase [Trace] Space

1st plot *r* vs. *r'*

EO Identified ~ 30 mm

Linear oscillations noted for offsets on order of ±15 mm

Red circles identify initial conditions

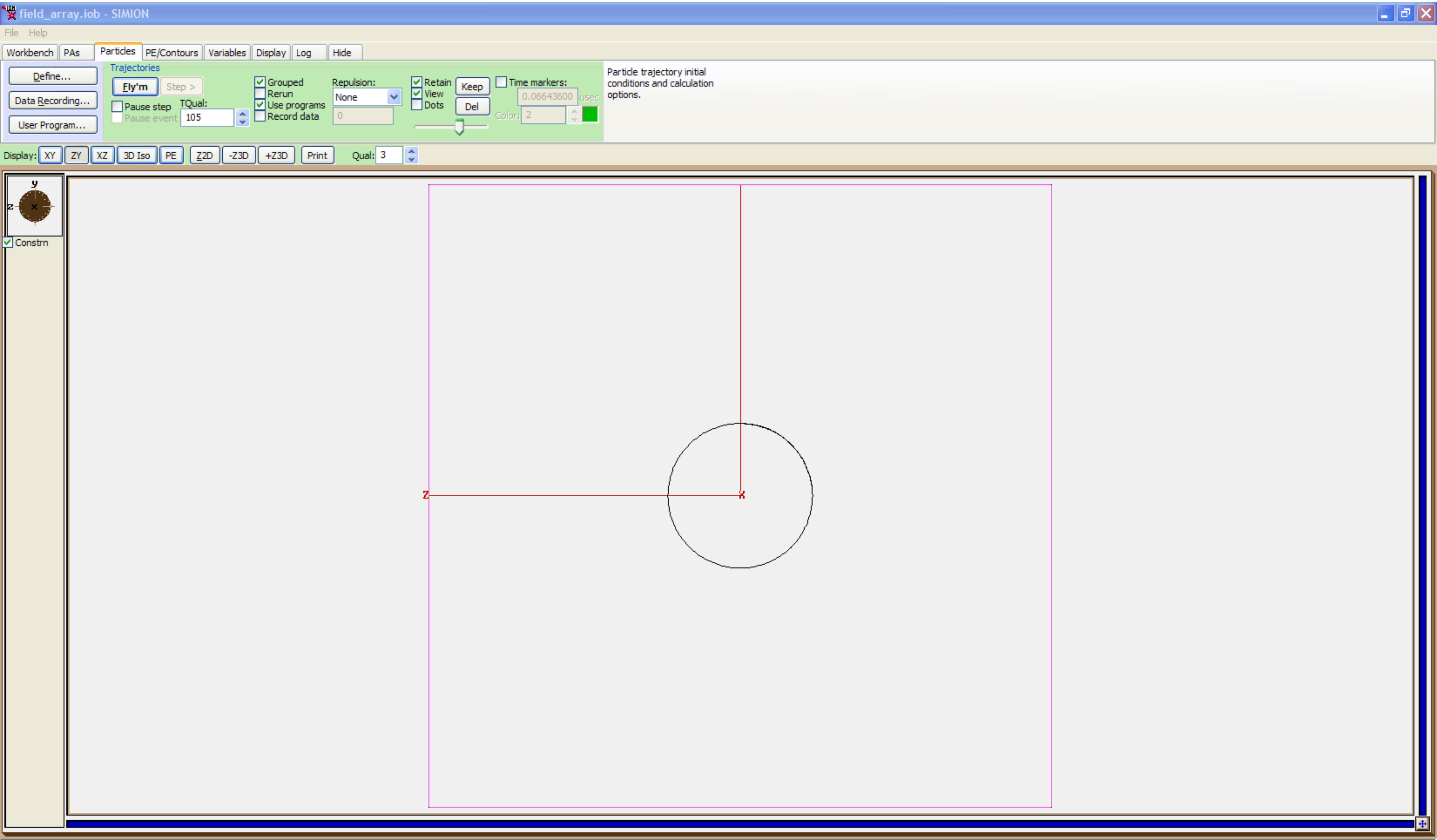


Strong non-linear behavior for large radial amplitudes, but stable !

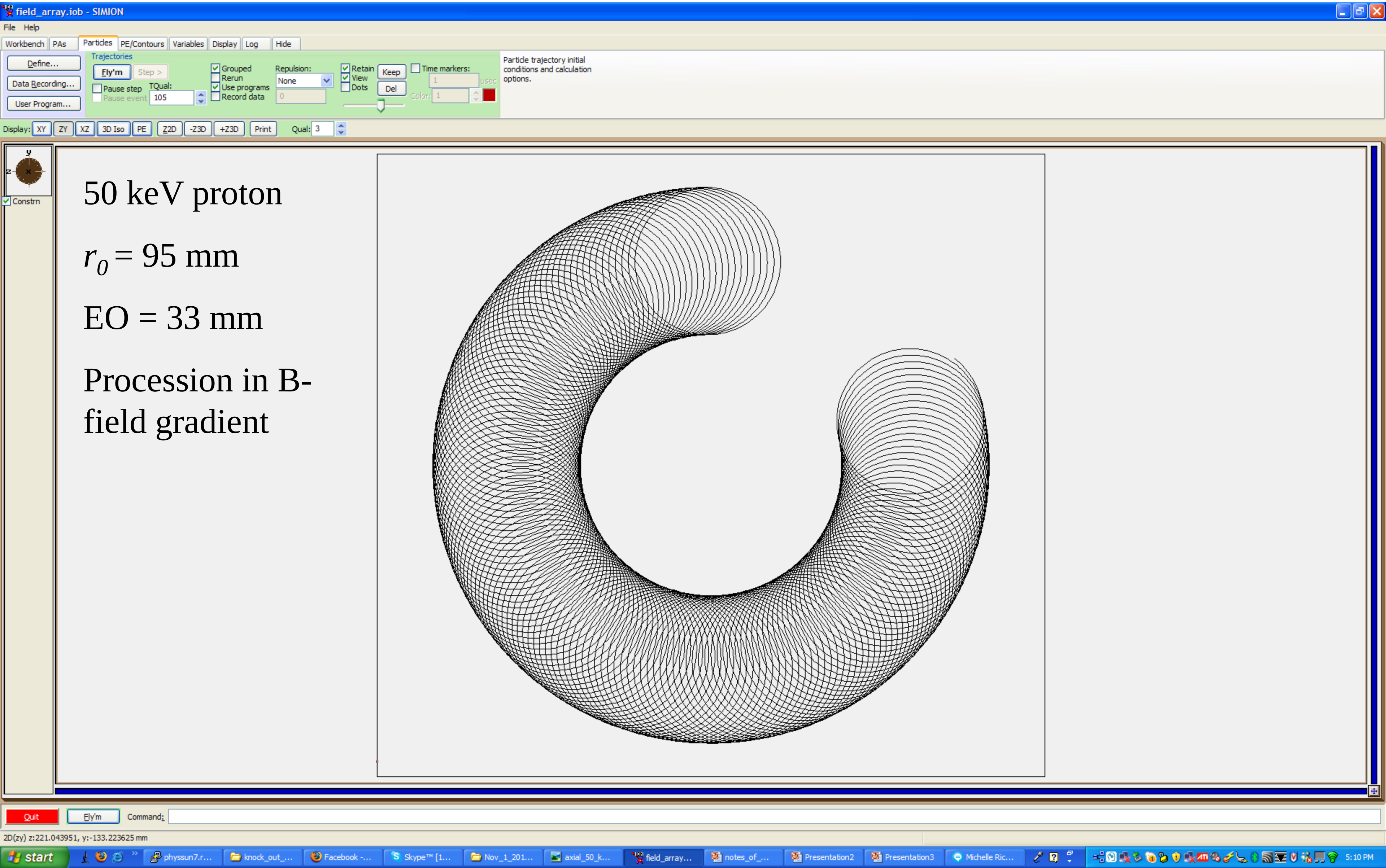
What are these ? ! ?

LOCATING EO

The 50 keV EO was found at r = 32.495 mm



Other Stable Orbit Found



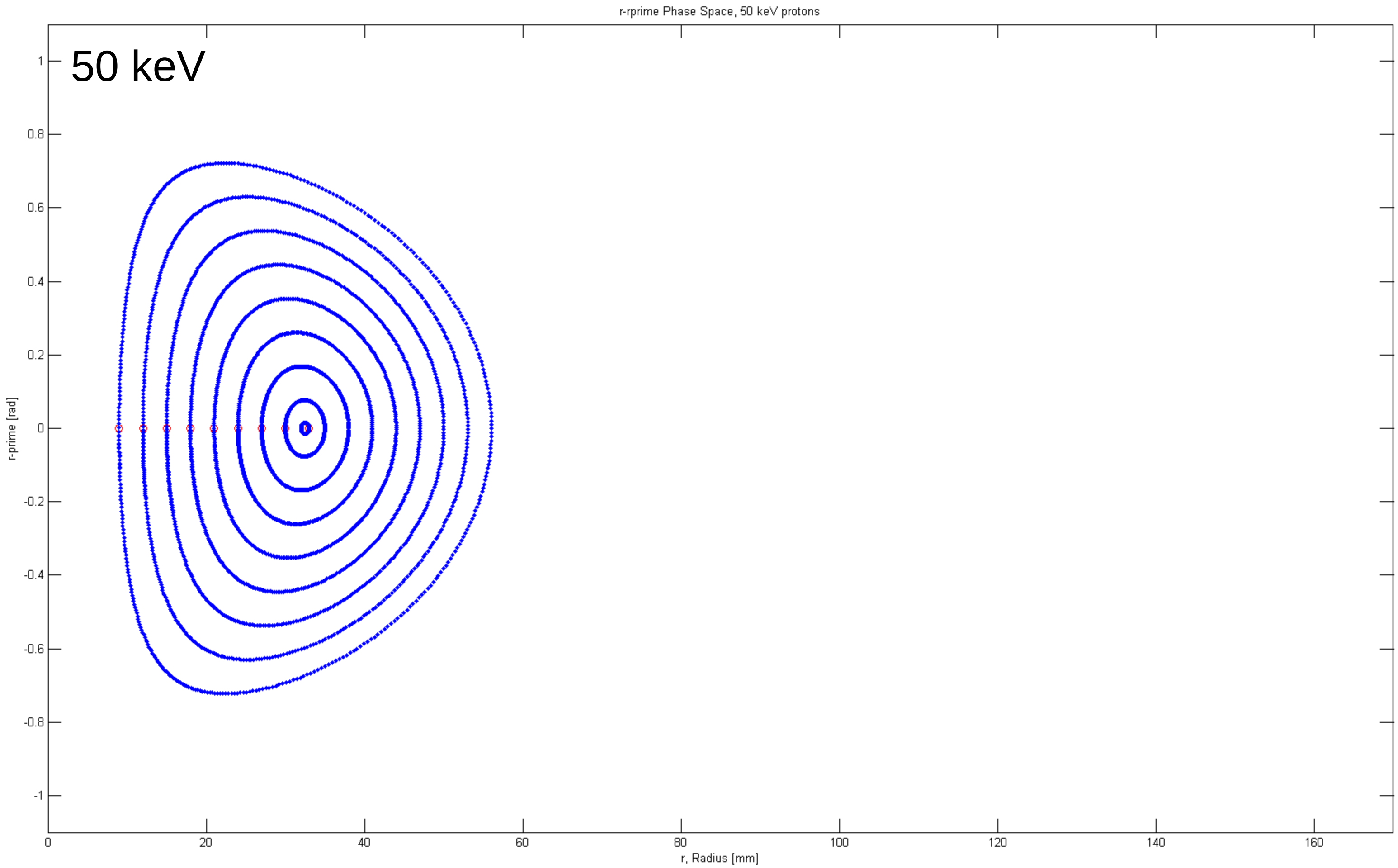
50 keV proton

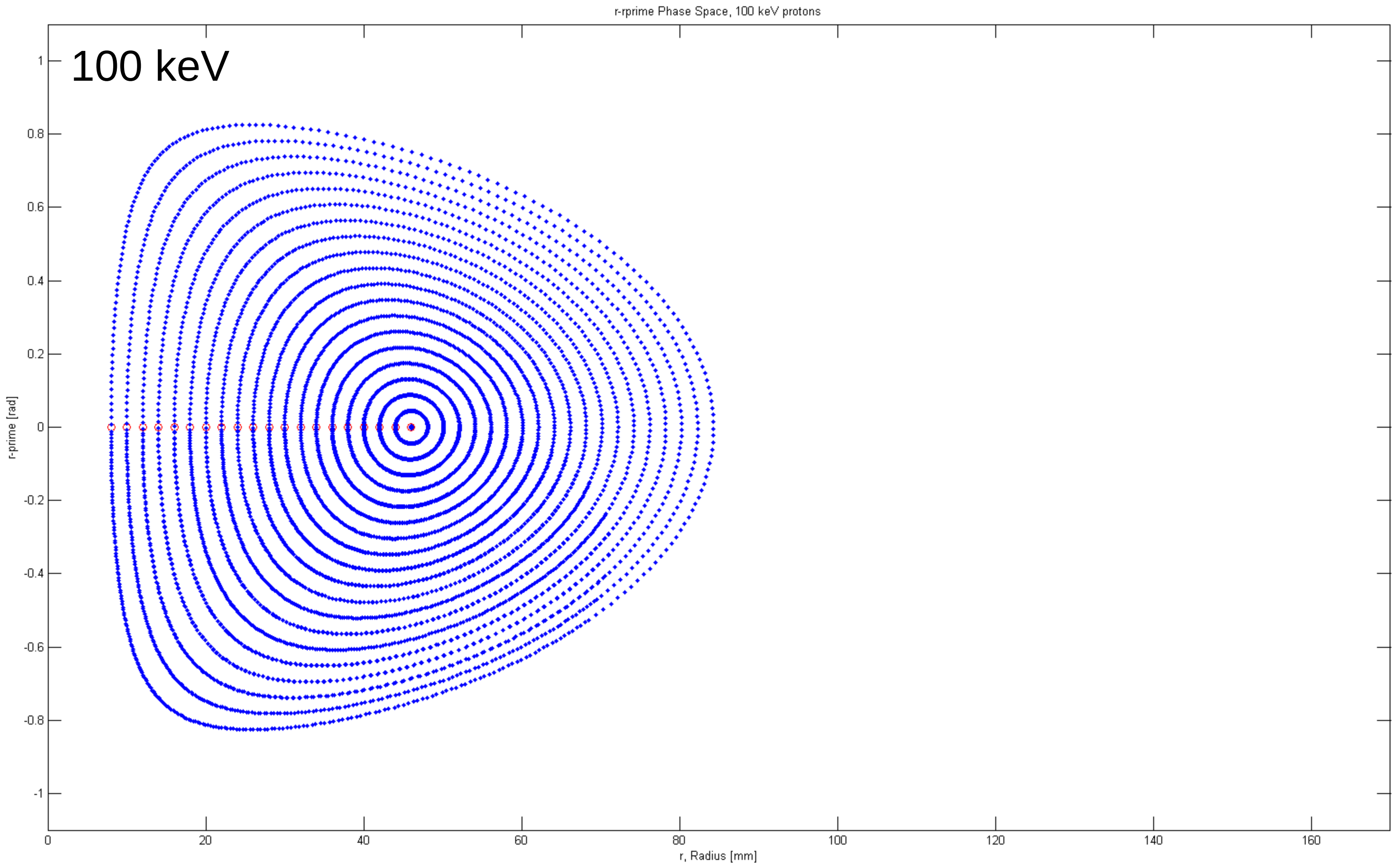
$r_0 = 95$ mm

EO = 33 mm

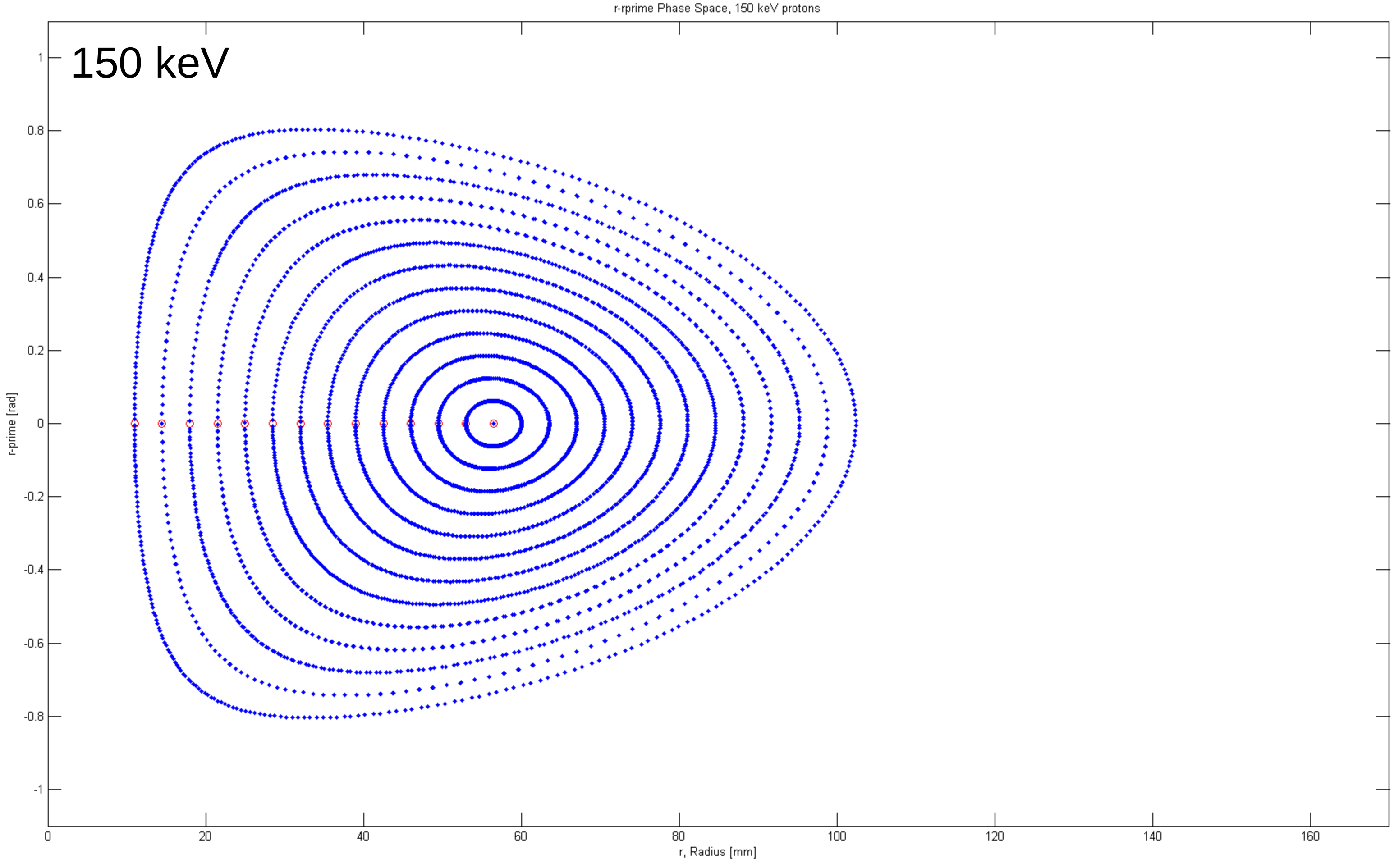
Procession in B-field gradient

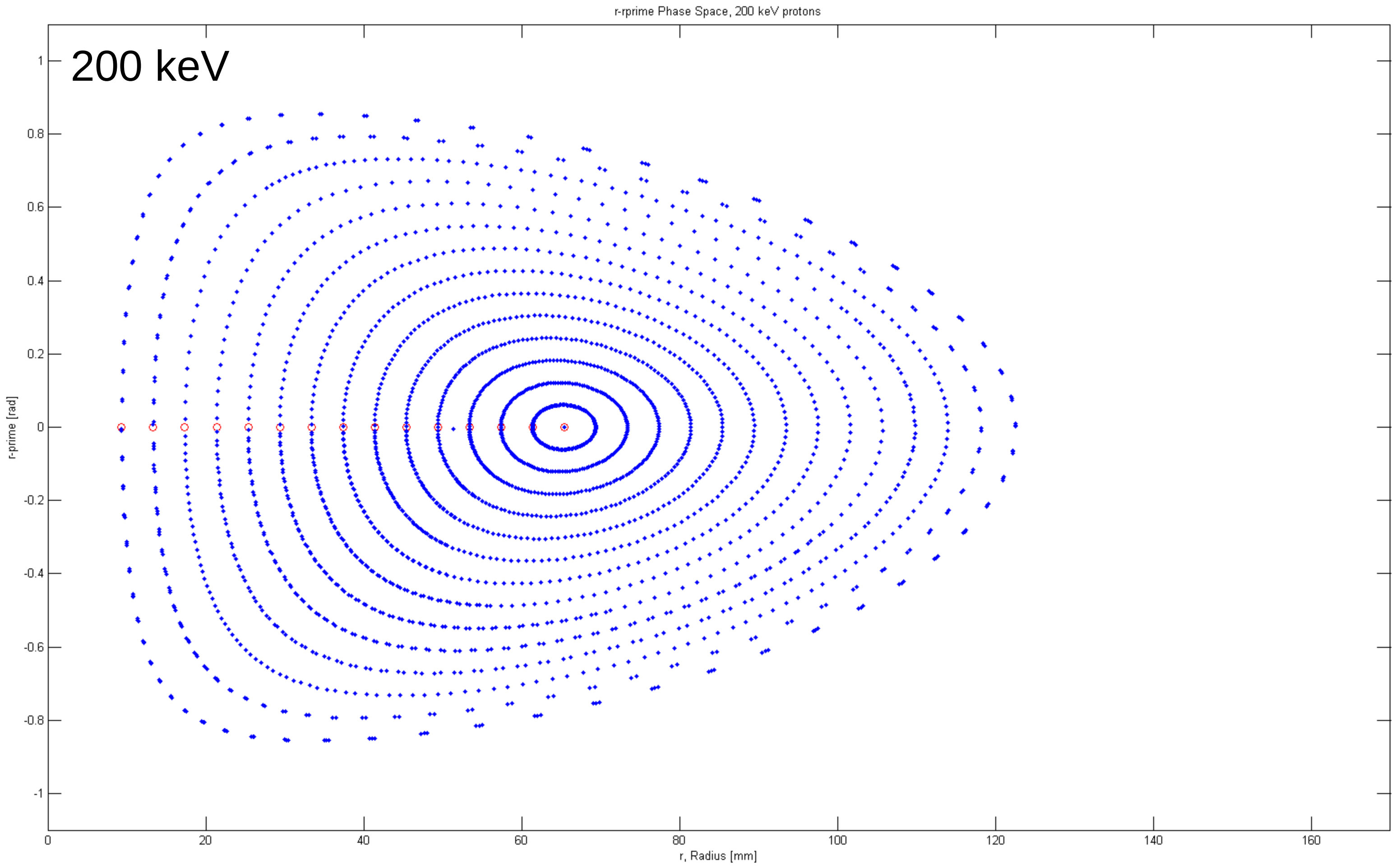
Radial Phase Space Plots

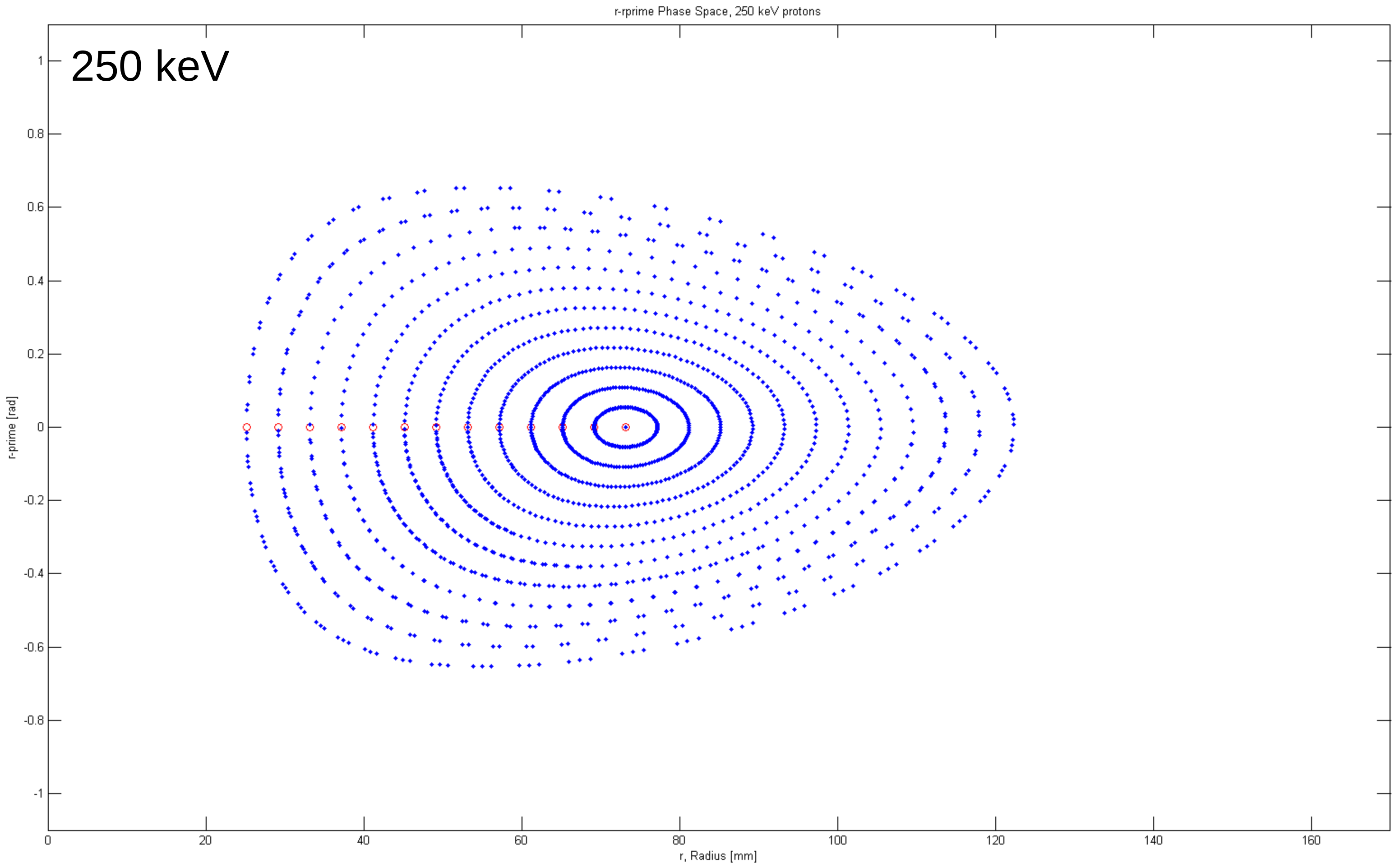

r-rprime Phase Space, 50 keV protons
50 keV
r-prime [rad]
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
0
20
40
60
80
100
120
140
160
r, Radius [mm]

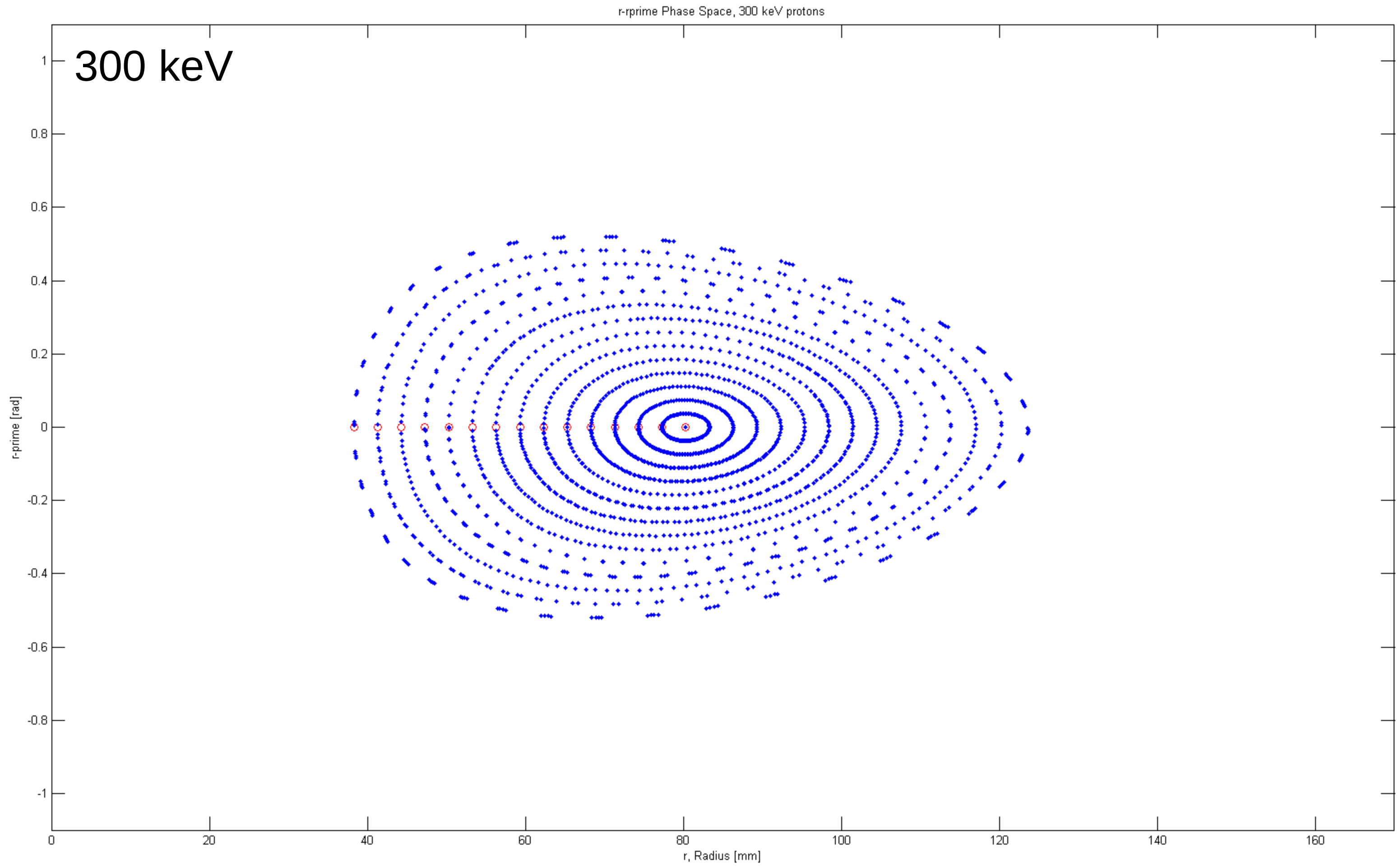


r-rprime Phase Space, 100 keV protons
100 keV
r-prime [rad]
r, Radius [mm]

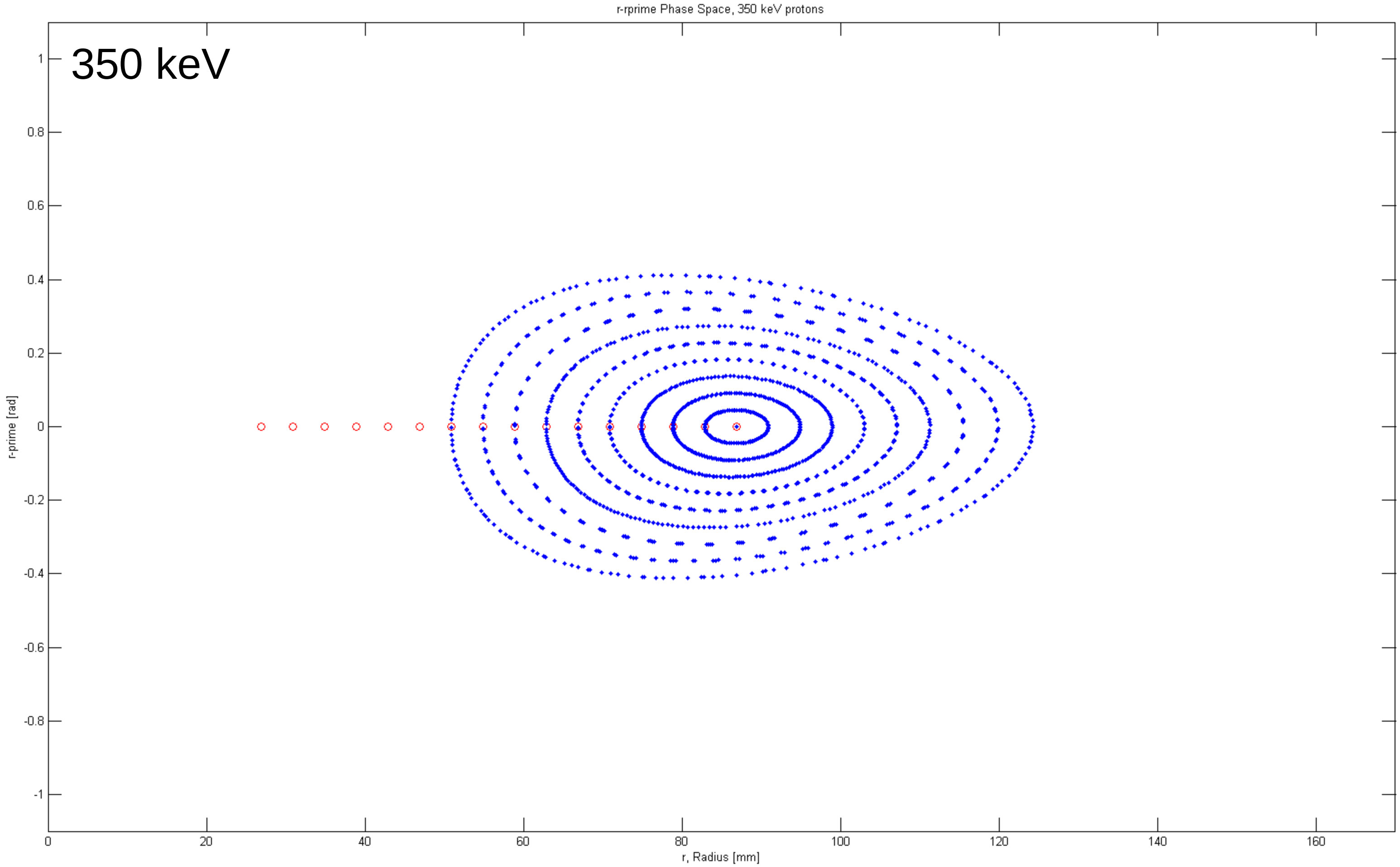

r-rprime Phase Space, 150 keV protons
150 keV
r-prime [rad]
r, Radius [mm]
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
0
20
40
60
80
100
120
140
160

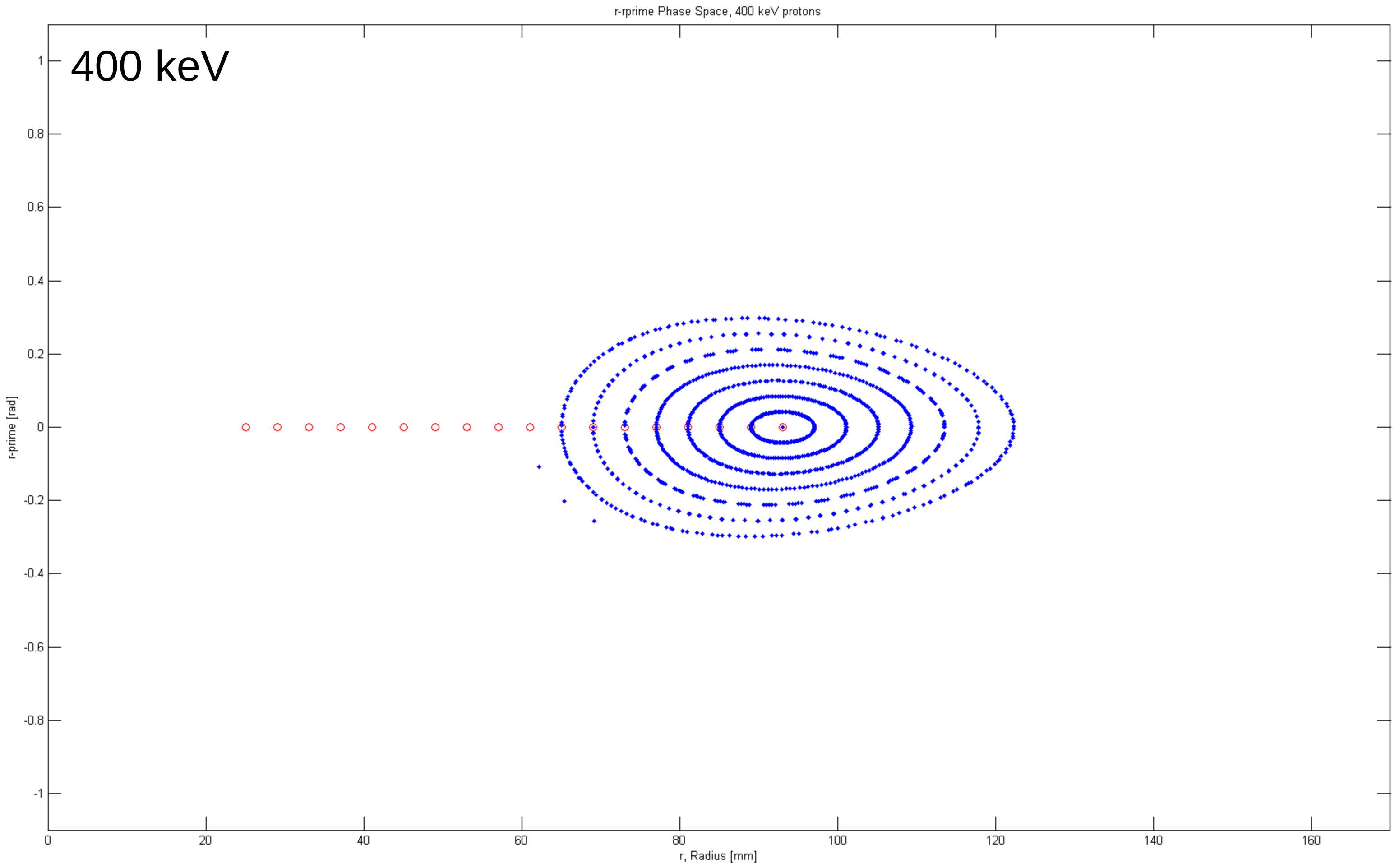

r-rprime Phase Space, 200 keV protons
200 keV
r-prime [rad]
r, Radius [mm]
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
0
20
40
60
80
100
120
140
160

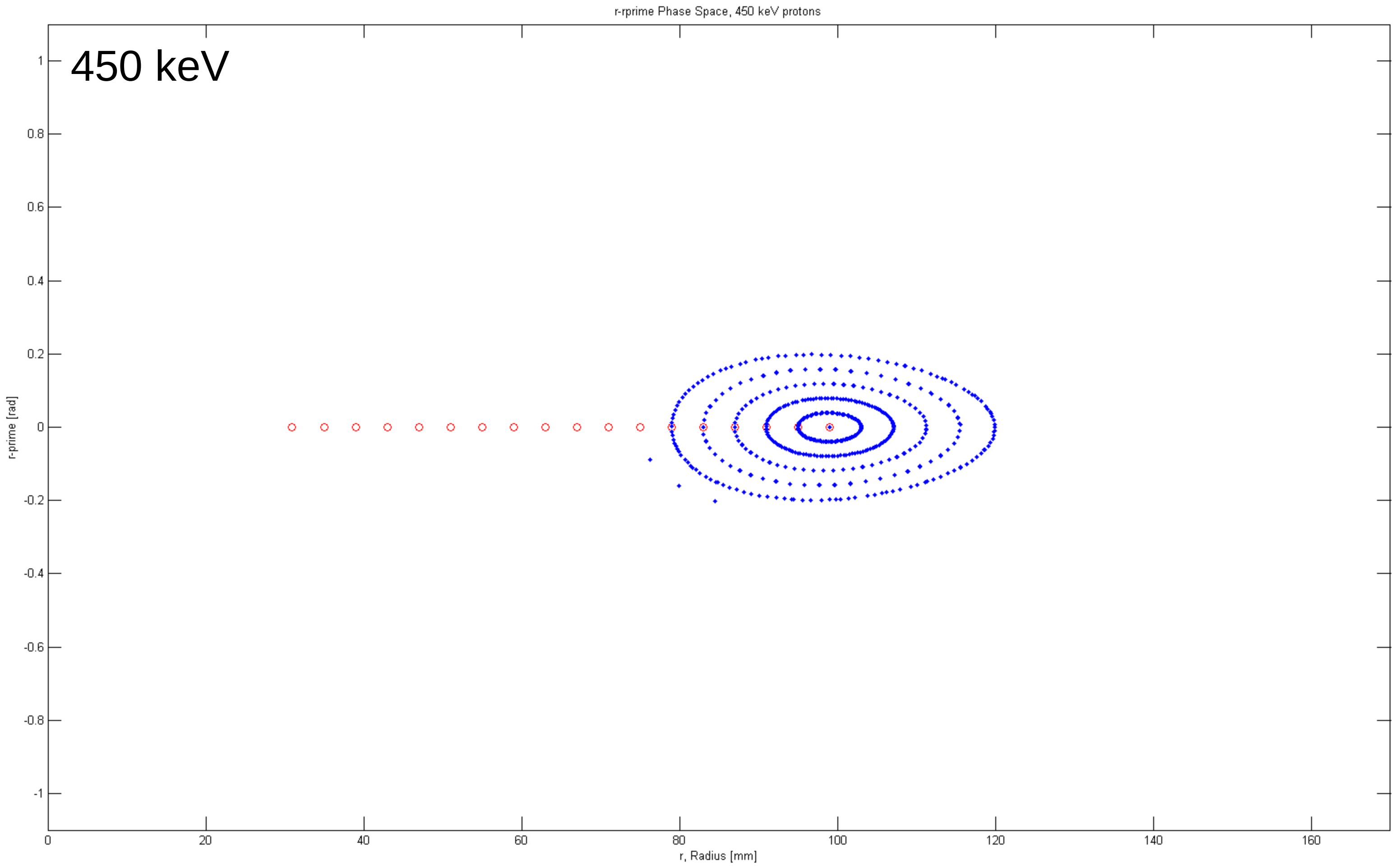

r-rprime Phase Space, 250 keV protons
250 keV
r-prime [rad]
r, Radius [mm]
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
0
20
40
60
80
100
120
140
160



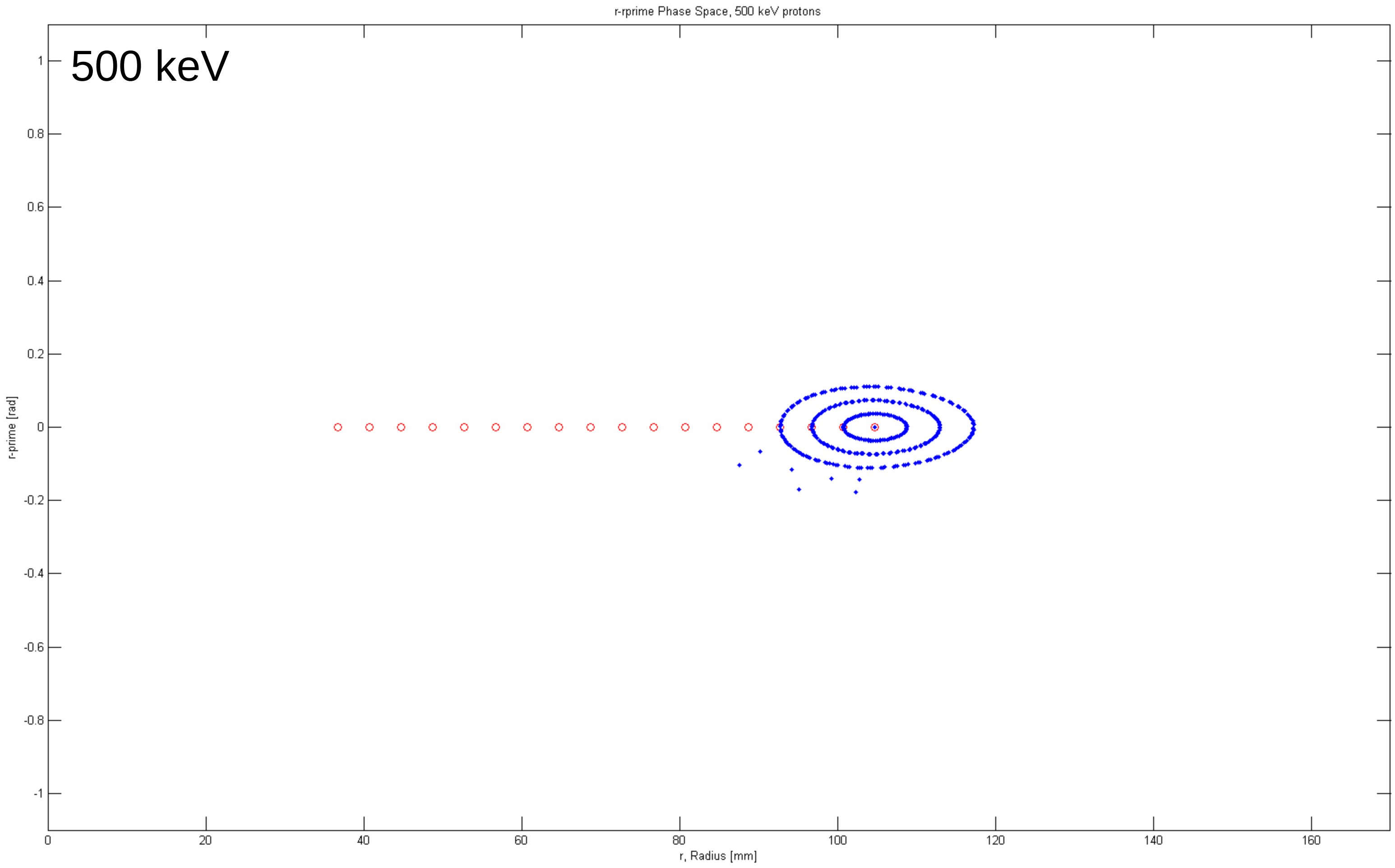
r-rprime Phase Space, 300 keV protons
300 keV
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
r-prime [rad]
0
20
40
60
80
100
120
140
160
r, Radius [mm]

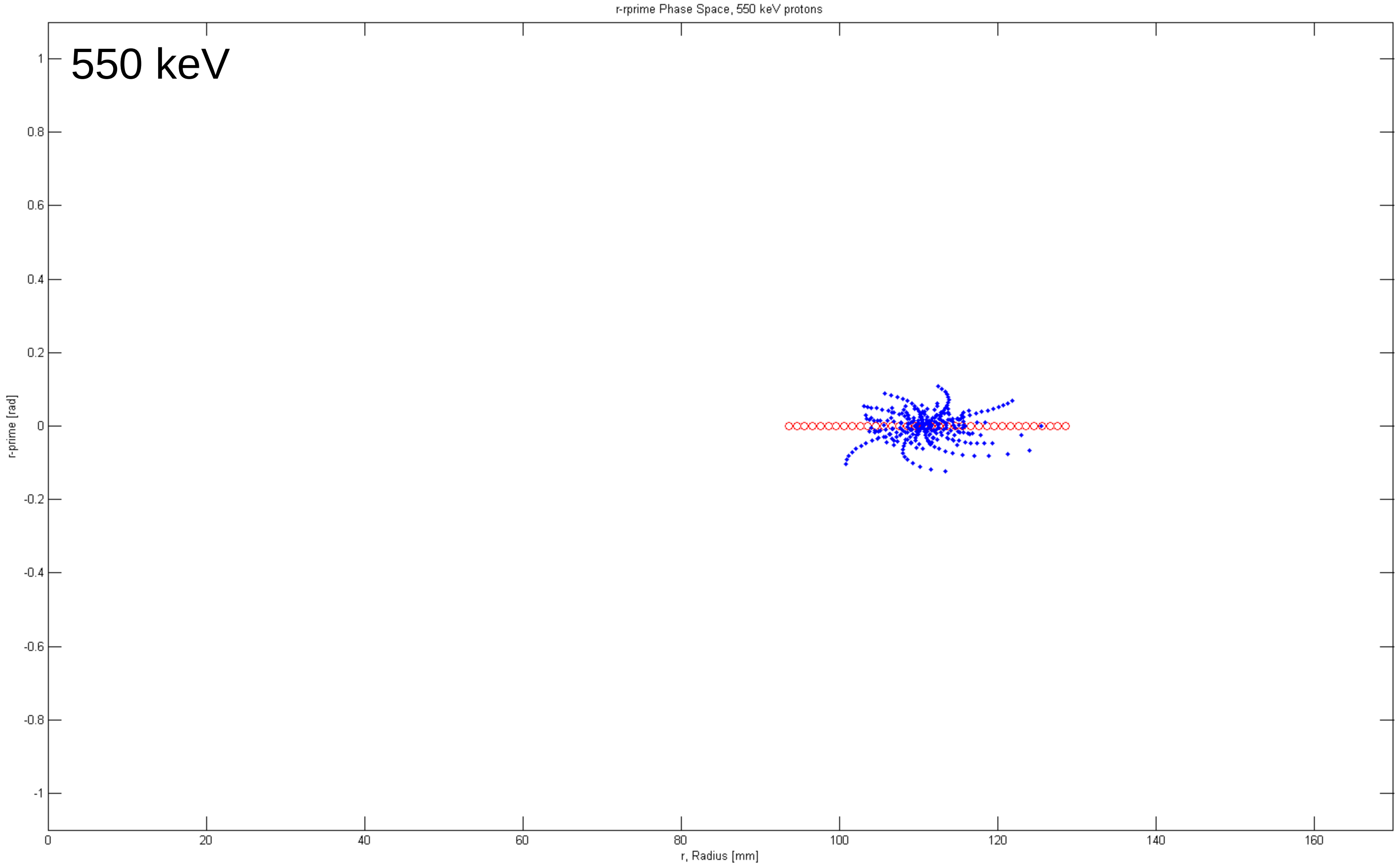

r-rprime Phase Space, 350 keV protons
350 keV
r-prime [rad]
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
0
20
40
60
80
100
120
140
160
r, Radius [mm]


r-rprime Phase Space, 400 keV protons
400 keV
r-prime [rad]
r, Radius [mm]
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
0
20
40
60
80
100
120
140
160

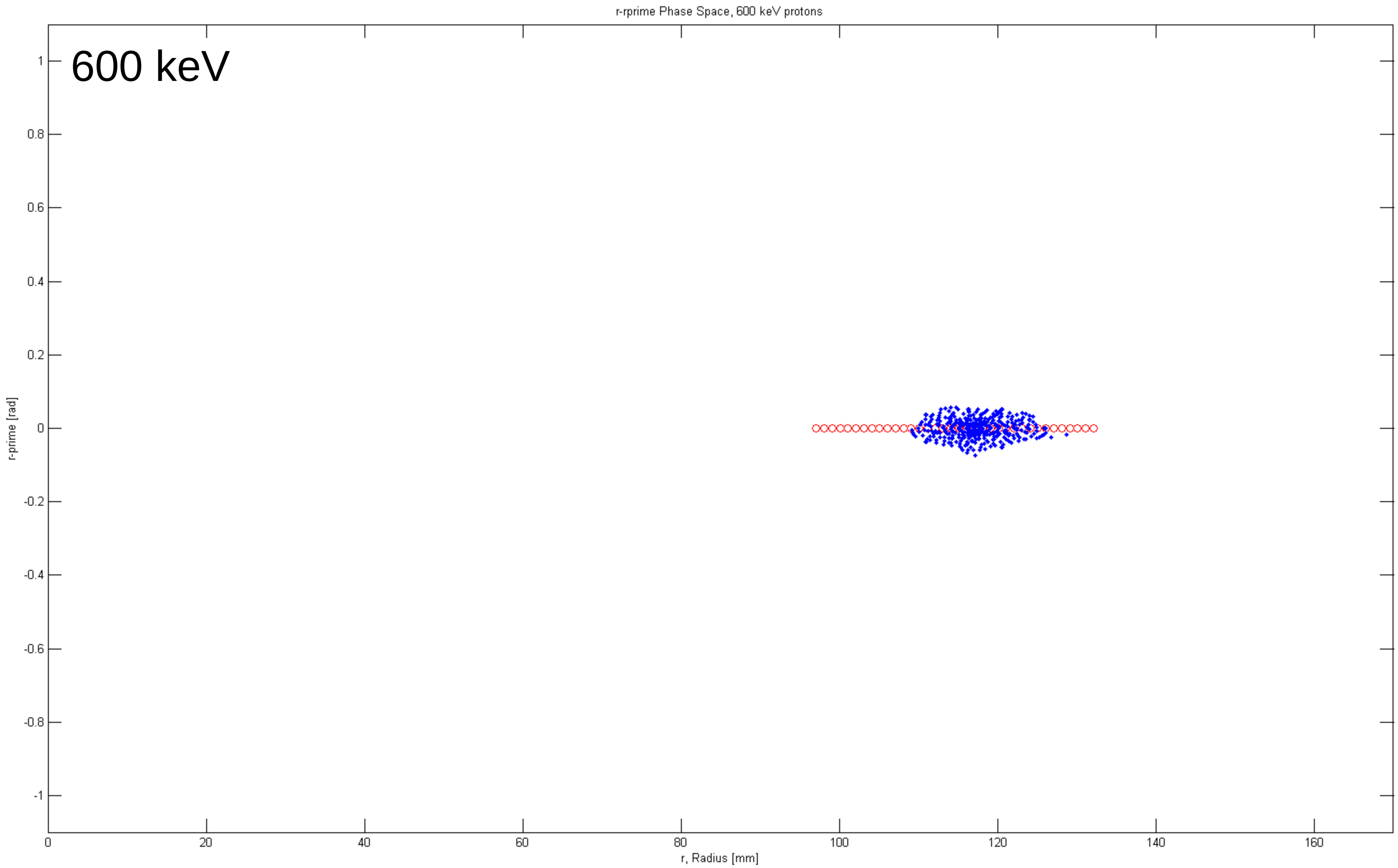


r-rprime Phase Space, 450 keV protons
450 keV
r-prime [rad]
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
0
20
40
60
80
100
120
140
160
r, Radius [mm]


r-rprime Phase Space, 500 keV protons
500 keV
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
r-prime [rad]
0
20
40
60
80
100
120
140
160
r, Radius [mm]

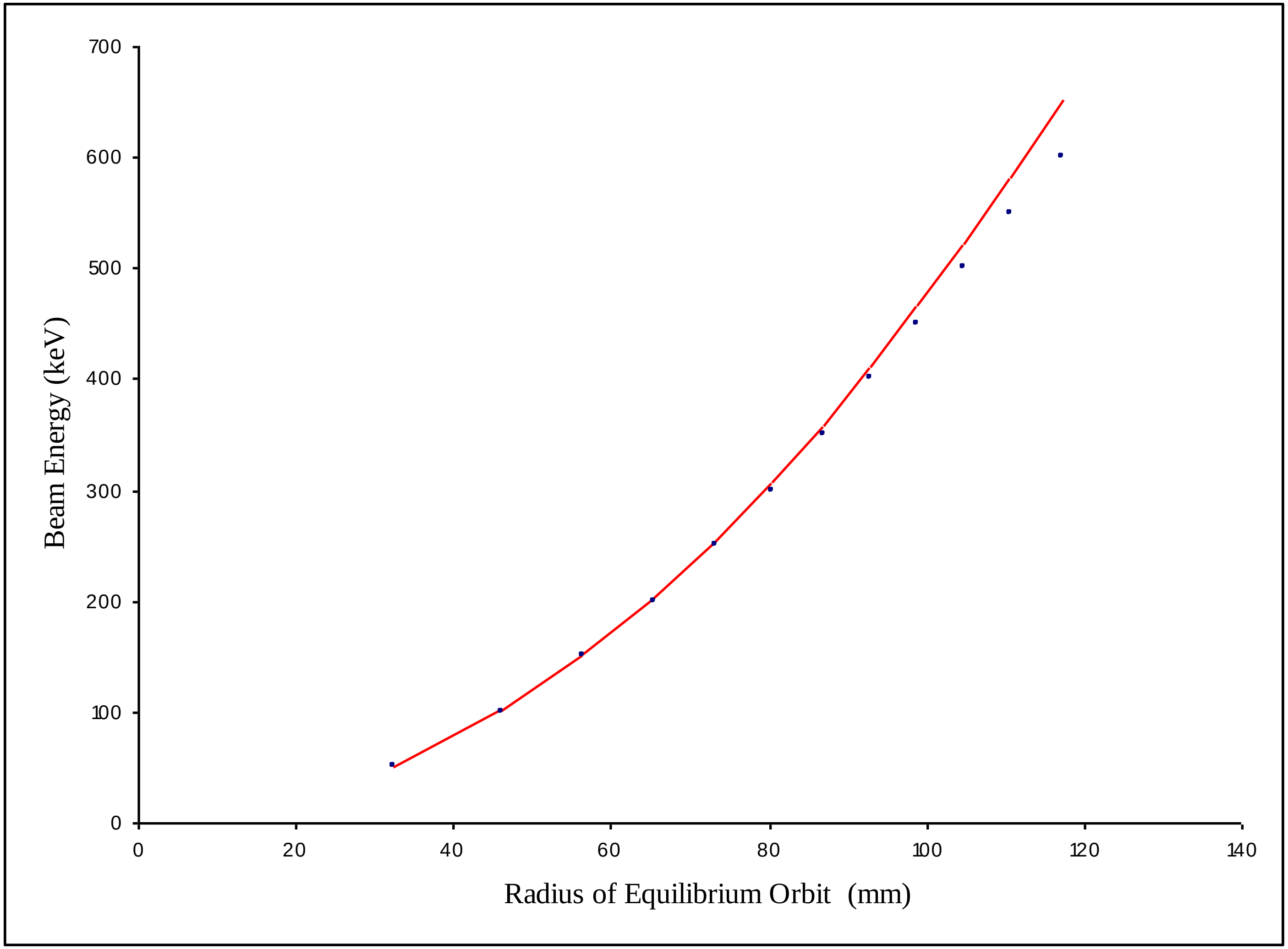


r-rprime Phase Space, 550 keV protons
550 keV
r-prime [rad]
r, Radius [mm]


r-rprime Phase Space, 600 keV protons
600 keV
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
r-prime [rad]
0
20
40
60
80
100
120
140
160
r, Radius [mm]

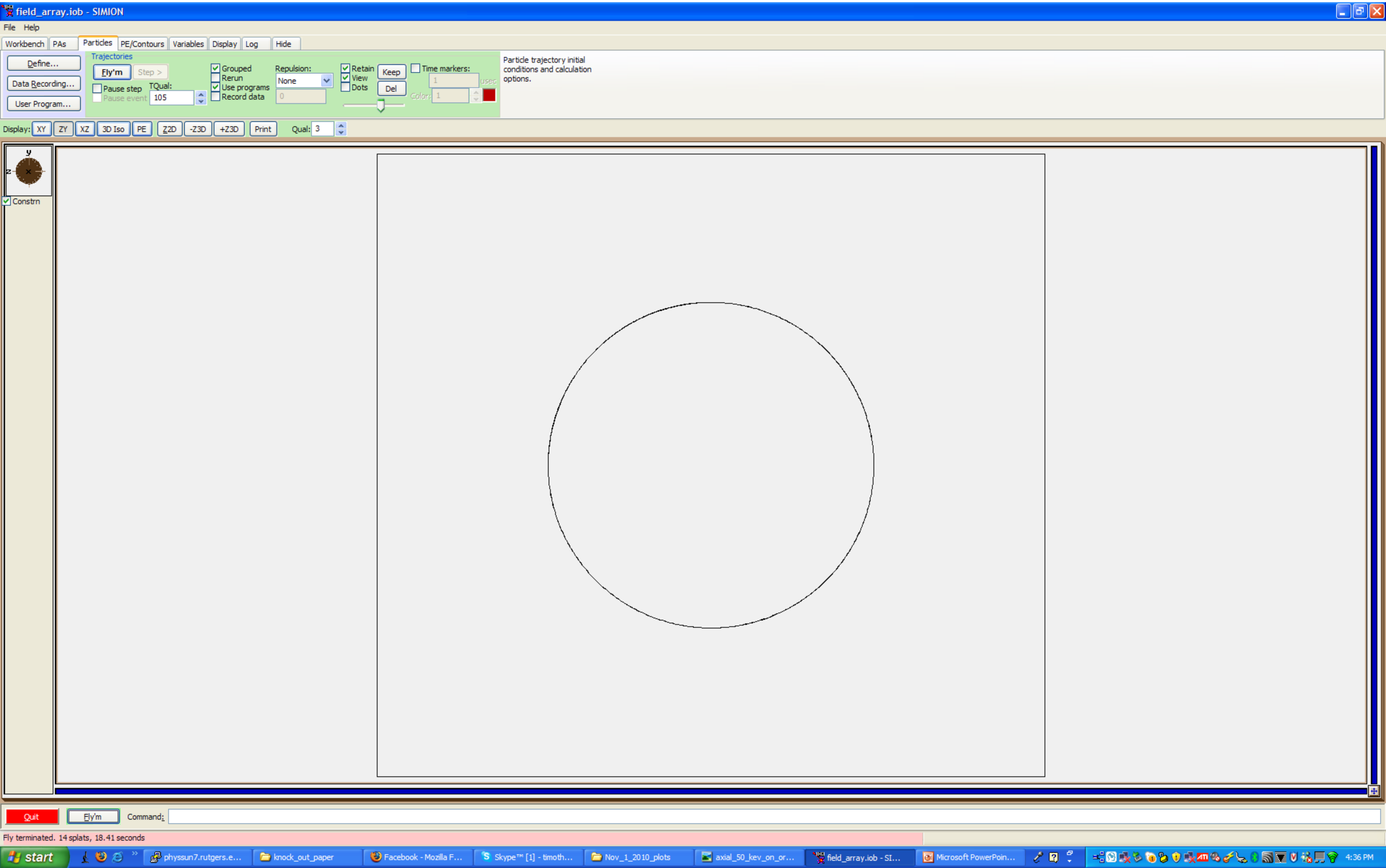
An Interesting Observation

Plot the weak focusing field's EO from PSF/SIMION (blue dots) against the EO of a uniform field (solid red line). Both fields have a peak central field of 0.997 Tesla.

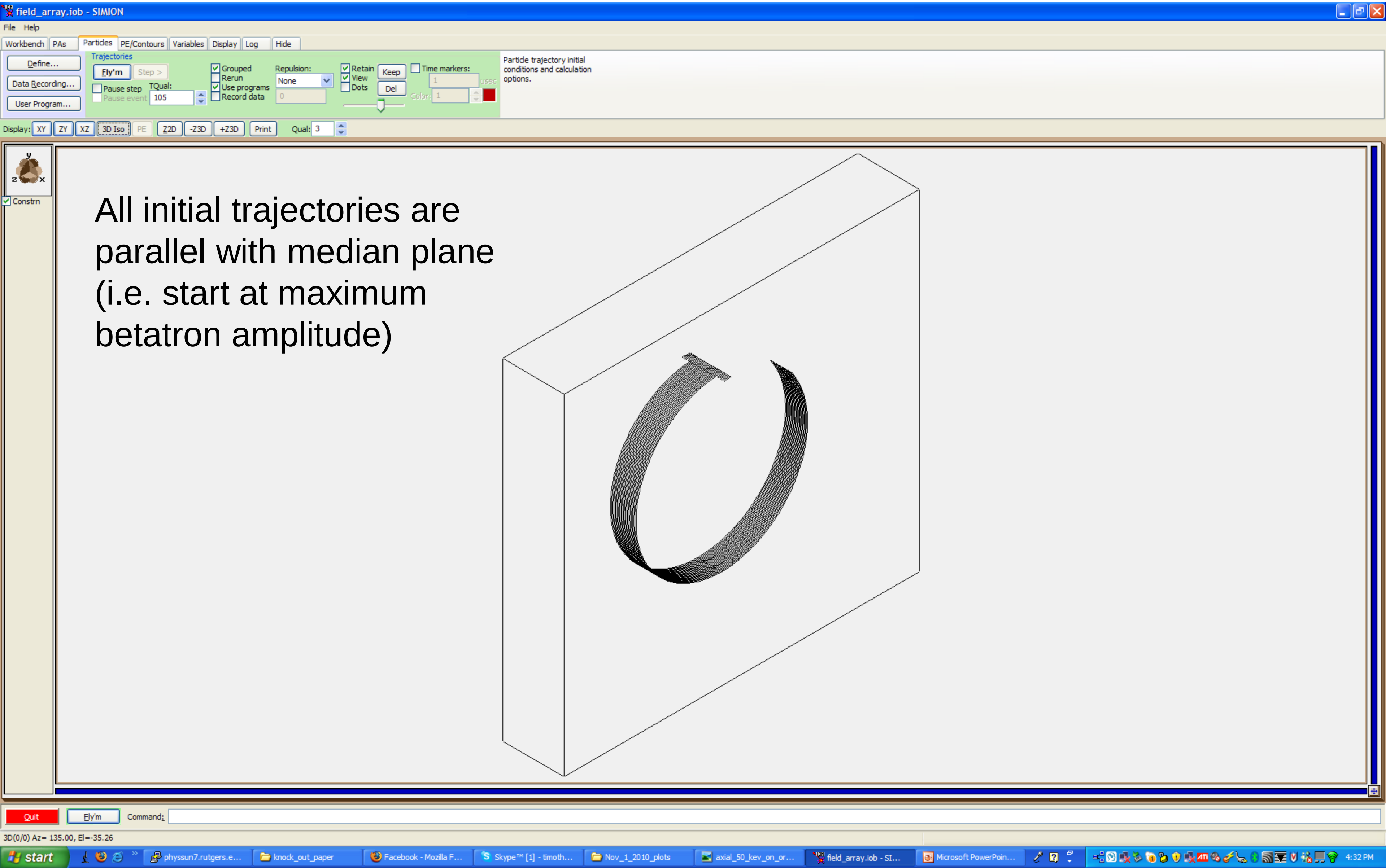


Vertical Phase Space Plots

Now that we know the E.O. we will launch a vertical sequence of mono-energetic ions at their equilbrium orbit. Shown here is the 1st turn of ~ 20 ions from a top view (again, RF is off)

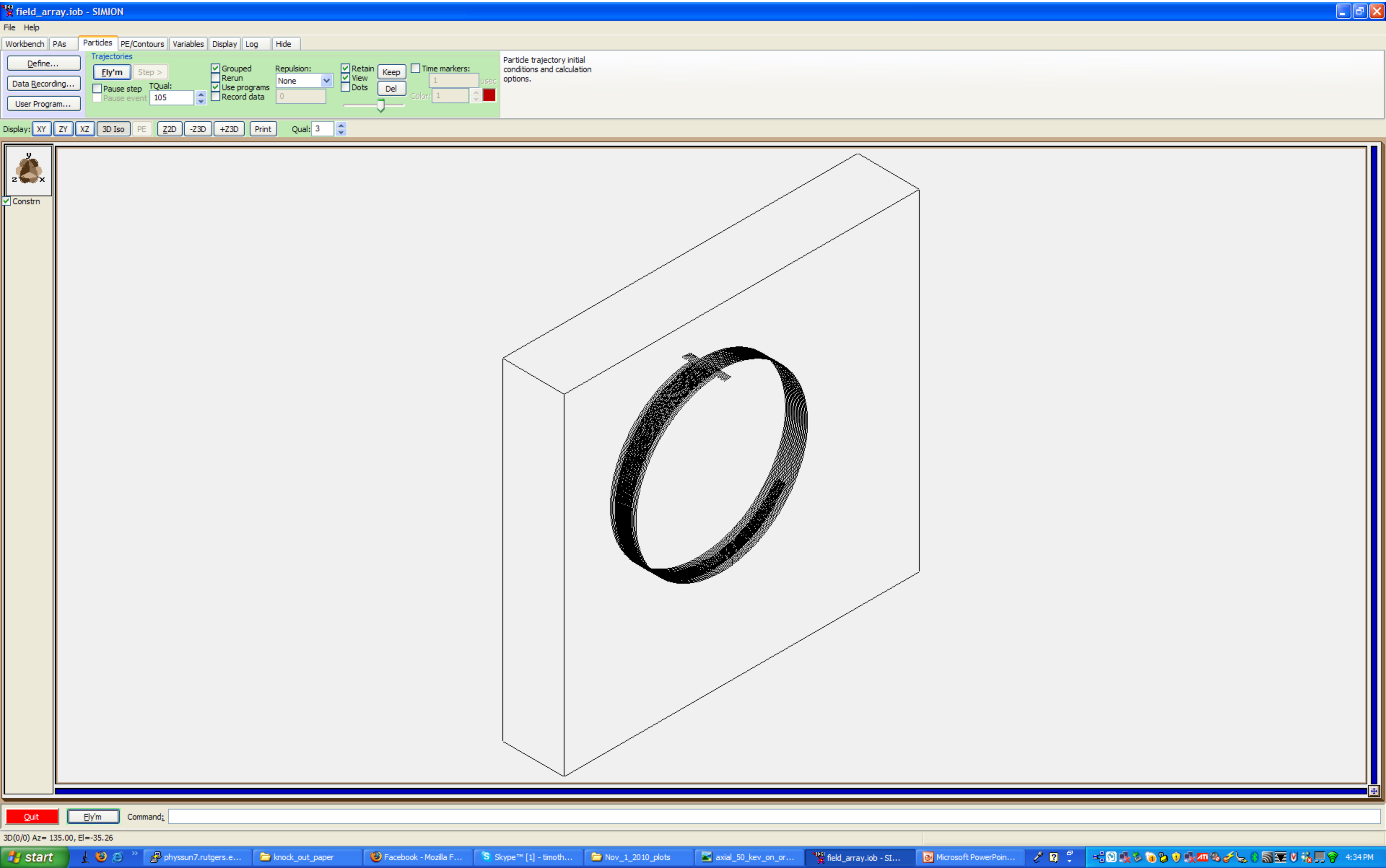


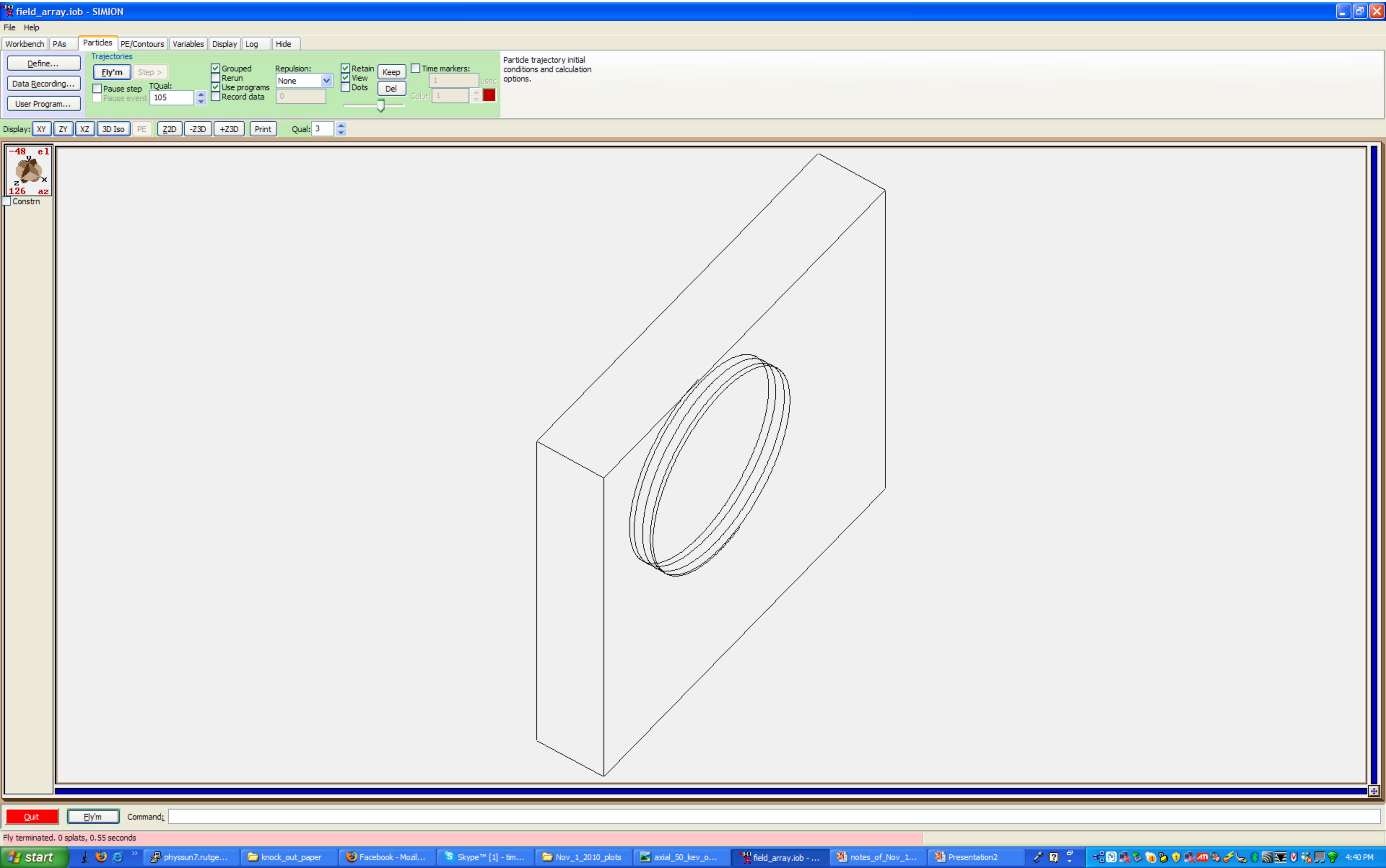
The 3D Iso view. Note the upper and lower extreme ions are lost on the Dummy DEE face.



Note that the ions are approaching a focus - ~1.5 revolutions complete.

Q: Imagine the ions come to a focus in 2 turns – what is field index n ?




field_array.iob - SIMION
File Help
Workbench PAs Particles PE/Contours Variables Display Log Hide
Trajectories
Define...
Data Recording...
User Program...
Fly'm Step >
Pause step
Pause event
TQual:
105
Grouped
Rerun
Use programs
Record data
Repulsion:
None
0
Retain
View
Dots
Keep
Del
Time markers:
1
usec
Color:
1
Particle trajectory initial conditions and calculation options.
Display: XY ZY XZ 3D Iso PE Z2D -Z3D +Z3D Print Qual: 3
-48 el
126 az
Constrn
Quit
Fly'm
Command:
Fly terminated. 0 splats, 0.55 seconds
start
physsun7.rutge...
knock_out_paper
Facebook - Mozil...
Skype™ [1] - tim...
Nov_1_2010_plots
axial_50_kev_o...
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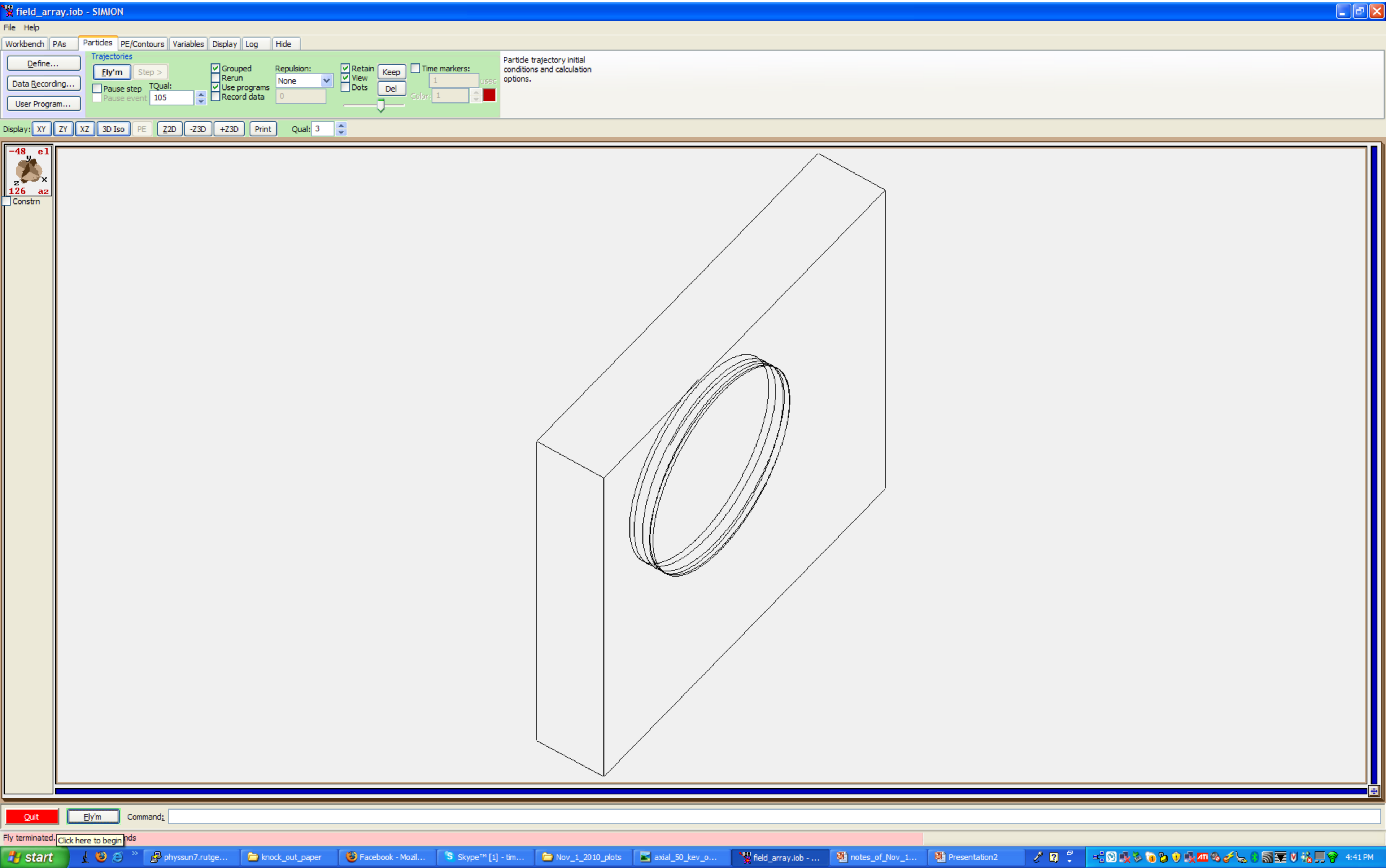
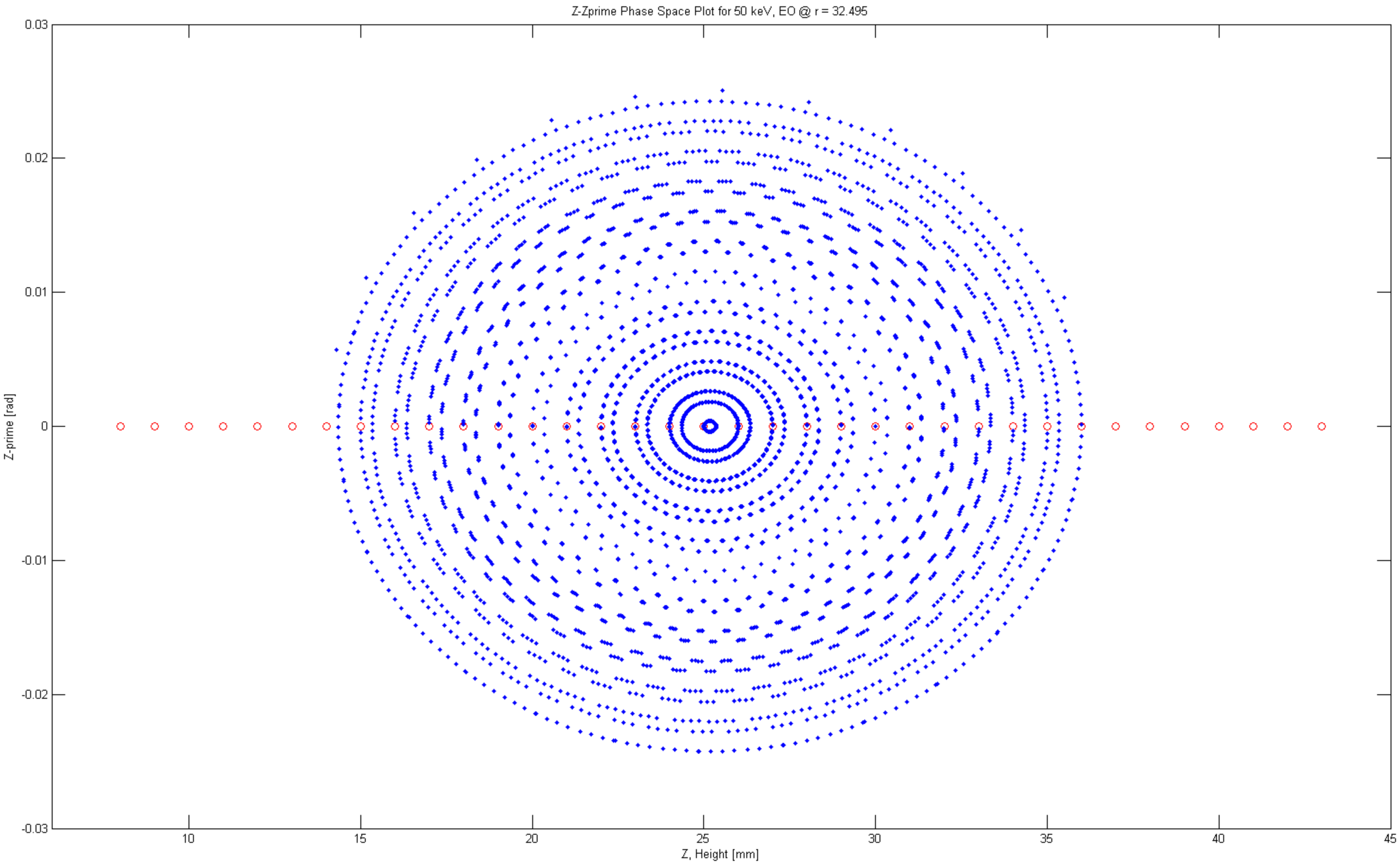
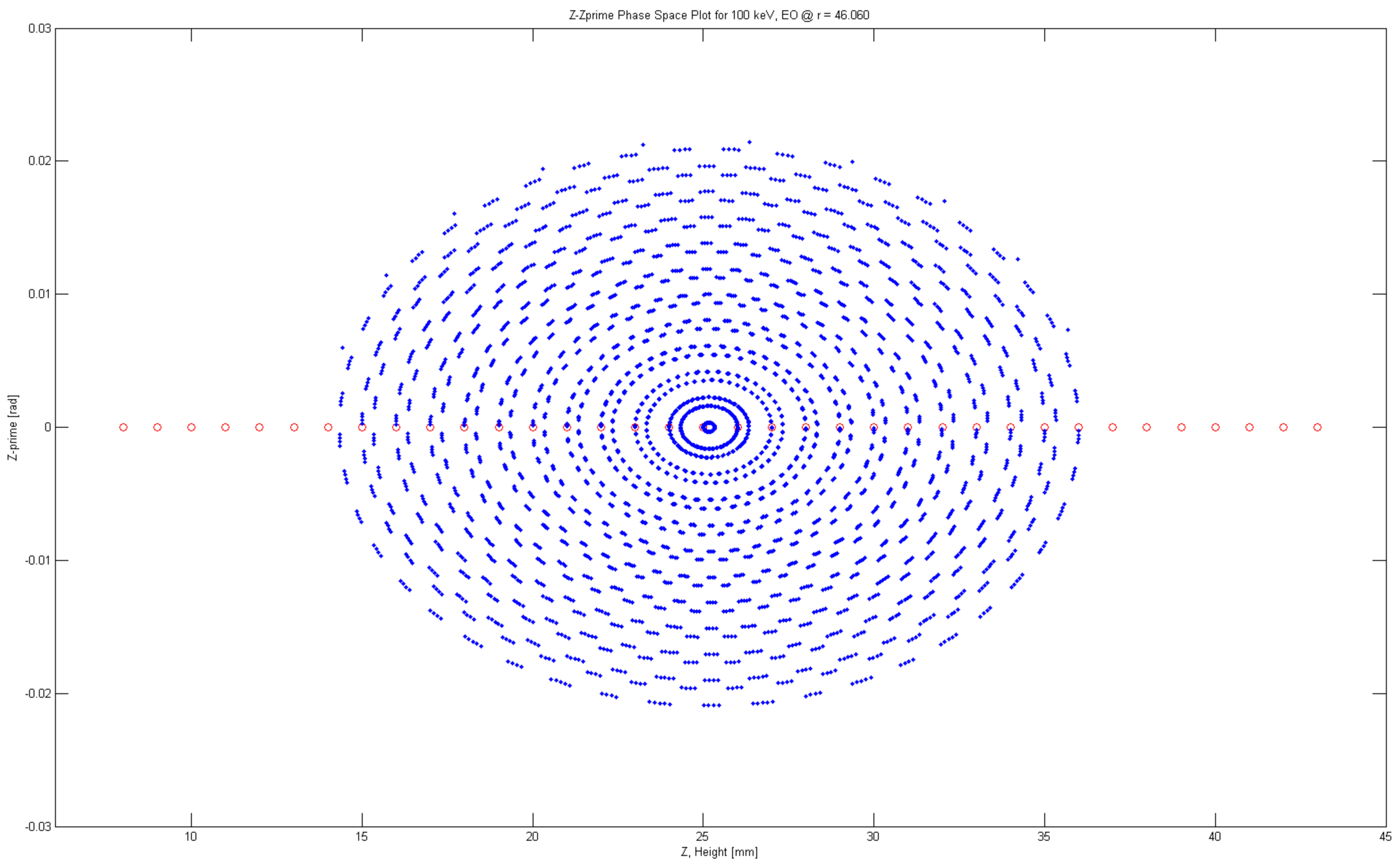
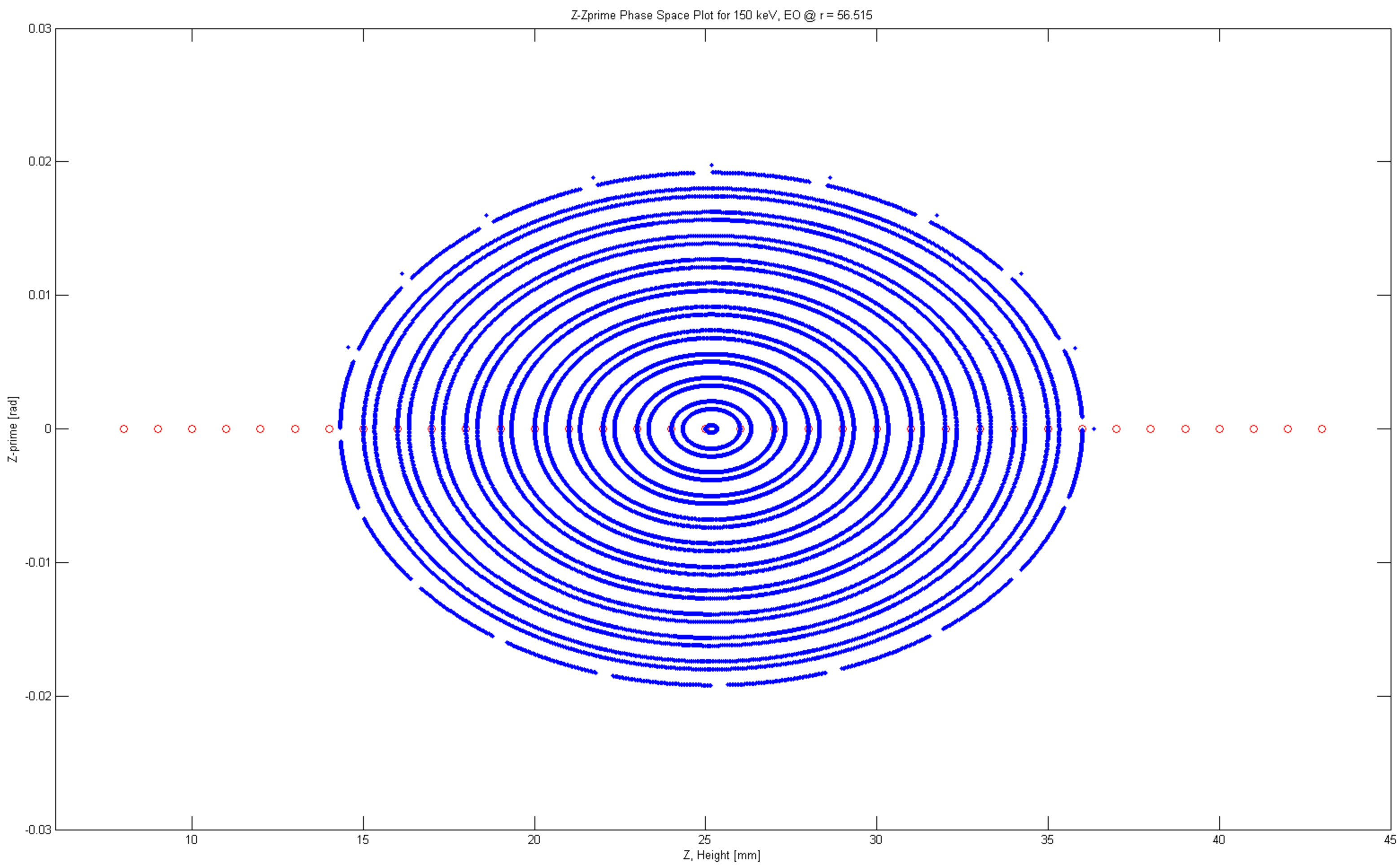
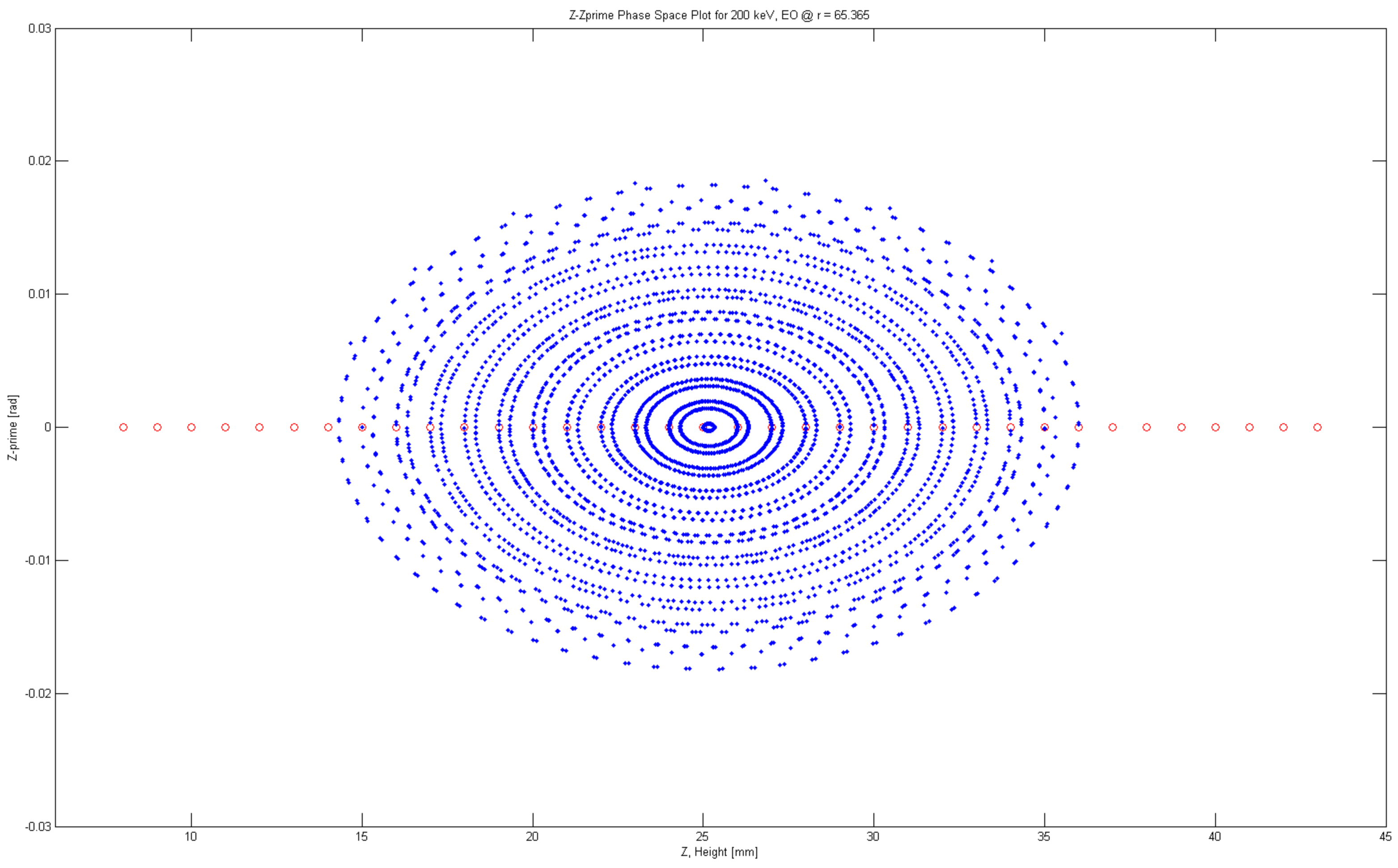
notes_of_Nov_1...
Presentation2
4:40 PM

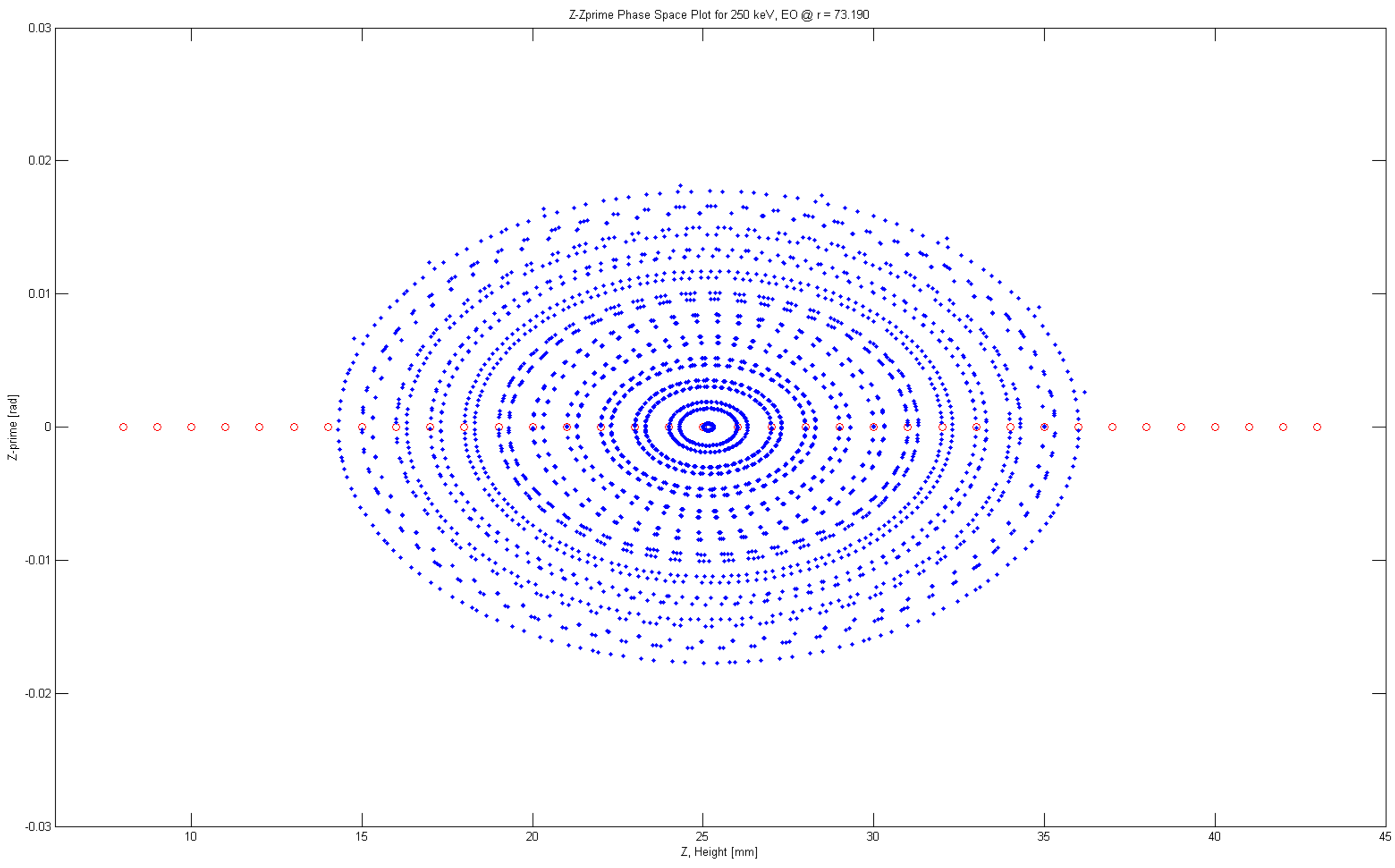

field_array.iob - SIMION
File Help
Workbench PAs Particles PE/Contours Variables Display Log Hide
Trajectories
Define...
Data Recording...
User Program...
Fly'm Step >
Pause step
Pause event
TQual:
105
Grouped
Rerun
Use programs
Record data
Repulsion:
None
0
Retain
View
Dots
Keep
Del
Time markers:
1
usec
Color: 1
Particle trajectory initial conditions and calculation options.
Display: XY ZY XZ 3D Iso PE Z2D -Z3D +Z3D Print Qual: 3
-48 el
126 az
Constrn
Quit
Fly'm
Command:
Fly terminated.
Click here to begin
start
physsun7.rutge...
knock_out_paper
Facebook - Mozil...
Skype™ [1] - tim...
Nov_1_2010_plots
axial_50_kev_o...
field_array.iob - ...
notes_of_Nov_1...
Presentation2
4:41 PM


Z-Zprime Phase Space Plot for 50 keV, EO @ r = 32.495
Z-prime [rad]
Z, Height [mm]
0.03
0.02
0.01
0
-0.01
-0.02
-0.03
10
15
20
25
30
35
40
45

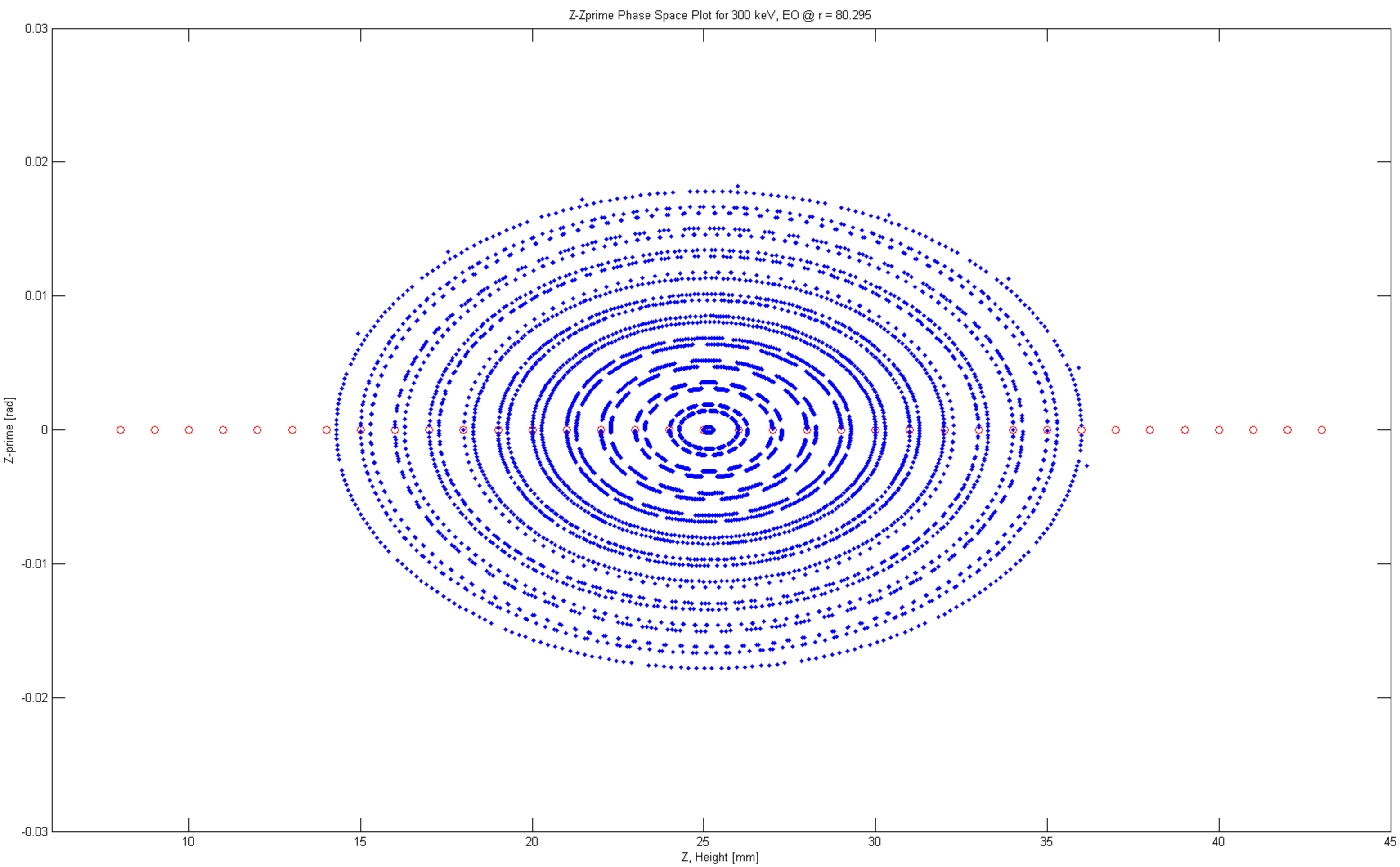

Z-Zprime Phase Space Plot for 100 keV, EO @ r = 46.060
0.03
0.02
0.01
0
-0.01
-0.02
-0.03
Z-prime [rad]
10
15
20
25
30
35
40
45
Z, Height [mm]

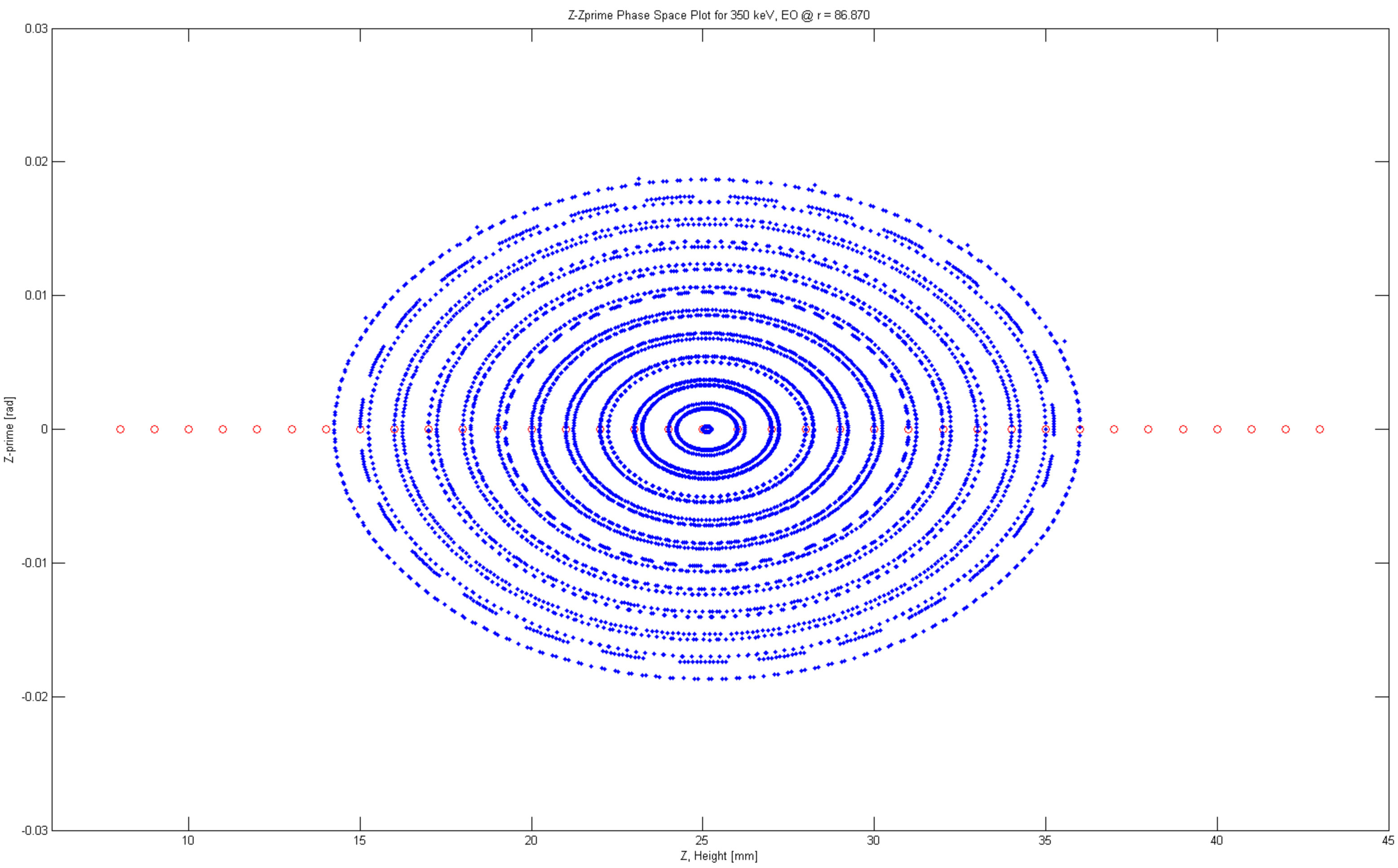

Z-Zprime Phase Space Plot for 150 keV, EO @ r = 56.515
0.03
0.02
0.01
0
-0.01
-0.02
-0.03
Z-prime [rad]
10
15
20
25
30
35
40
45
Z, Height [mm]

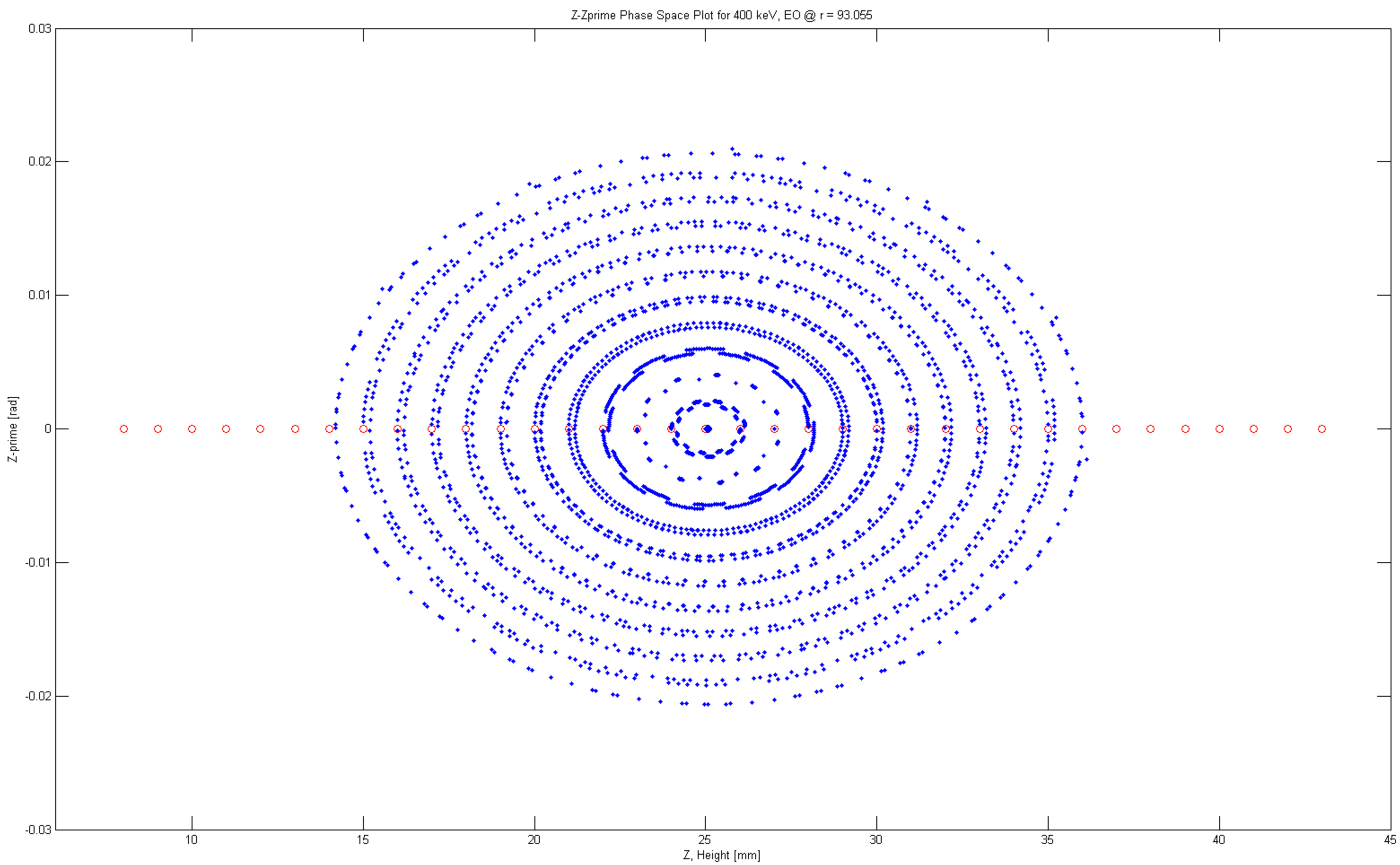

Z-Zprime Phase Space Plot for 200 keV, EO @ r = 65.365
0.03
0.02
0.01
0
-0.01
-0.02
-0.03
Z-prime [rad]
10
15
20
25
30
35
40
45
Z, Height [mm]



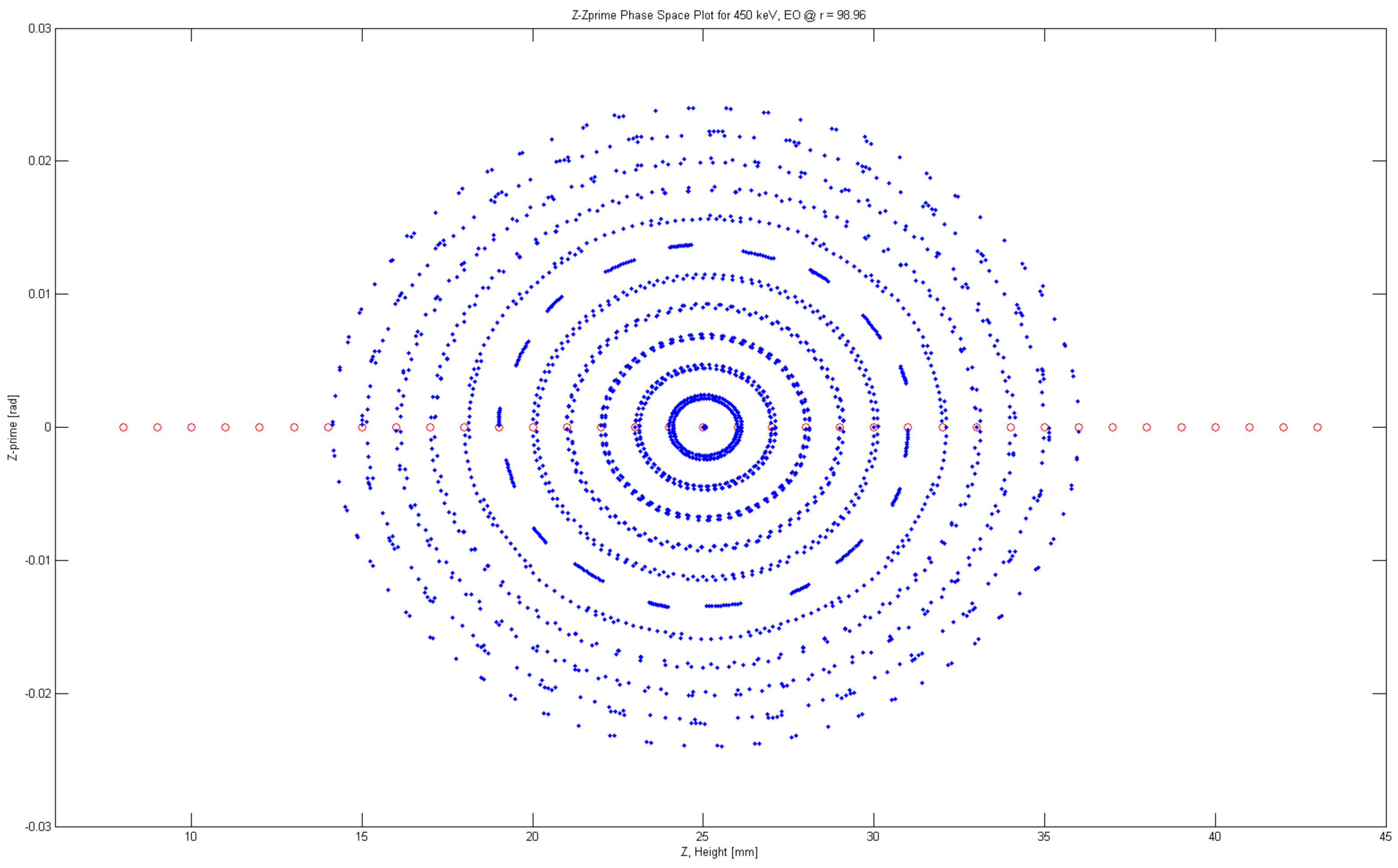
Z-Zprime Phase Space Plot for 250 keV, EO @ r = 73.190
0.03
0.02
0.01
0
-0.01
-0.02
-0.03
Z-prime [rad]
10
15
20
25
30
35
40
45
Z, Height [mm]

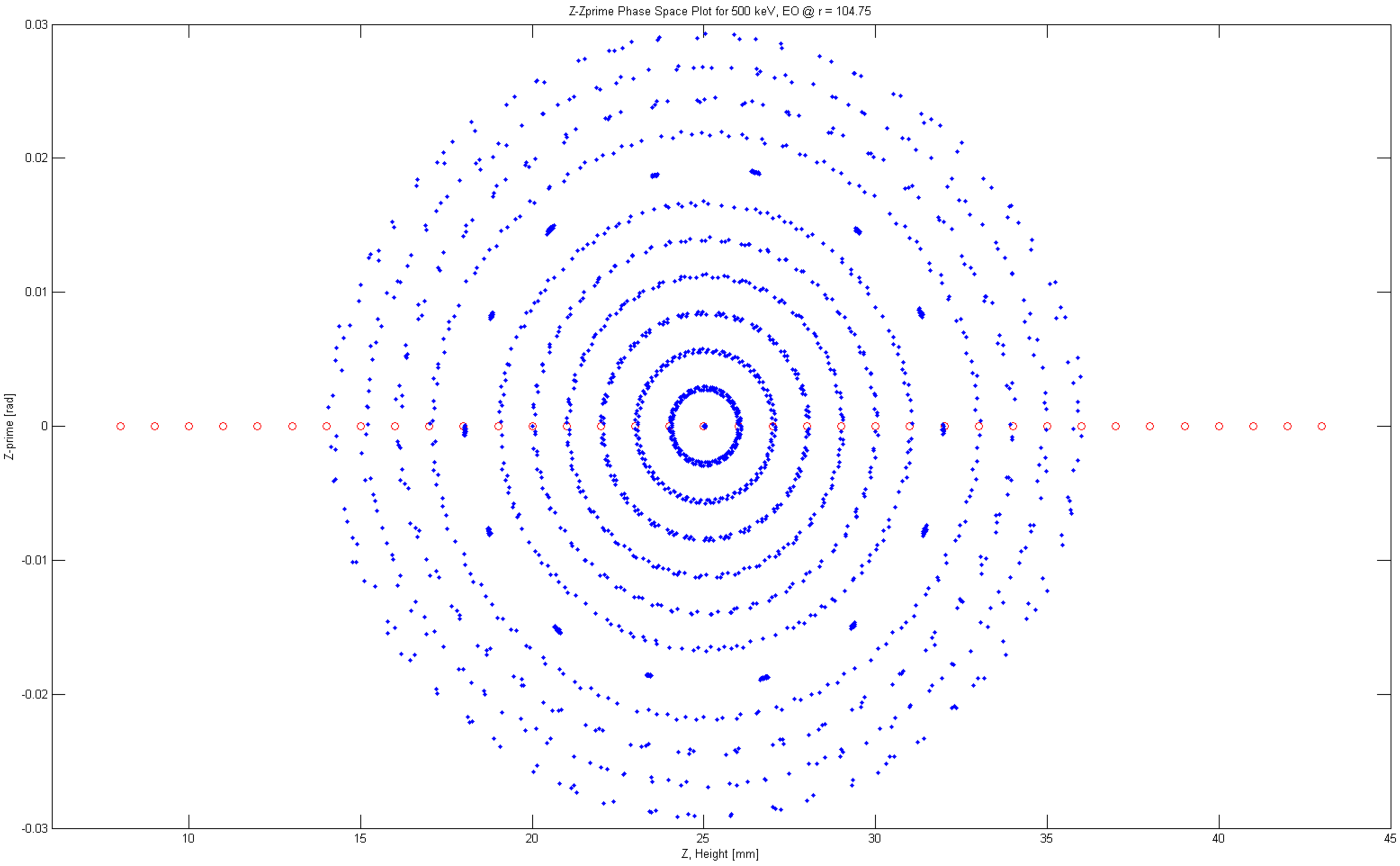

Z-Zprime Phase Space Plot for 300 keV, EO @ r = 80.295
0.03
0.02
0.01
0
-0.01
-0.02
-0.03
Z-prime [rad]
10
15
20
25
30
35
40
45
Z, Height [mm]


Z-Zprime Phase Space Plot for 350 keV, EO @ r = 86.870
0.03
0.02
0.01
0
-0.01
-0.02
-0.03
Z-prime [rad]
10
15
20
25
30
35
40
45
Z, Height [mm]

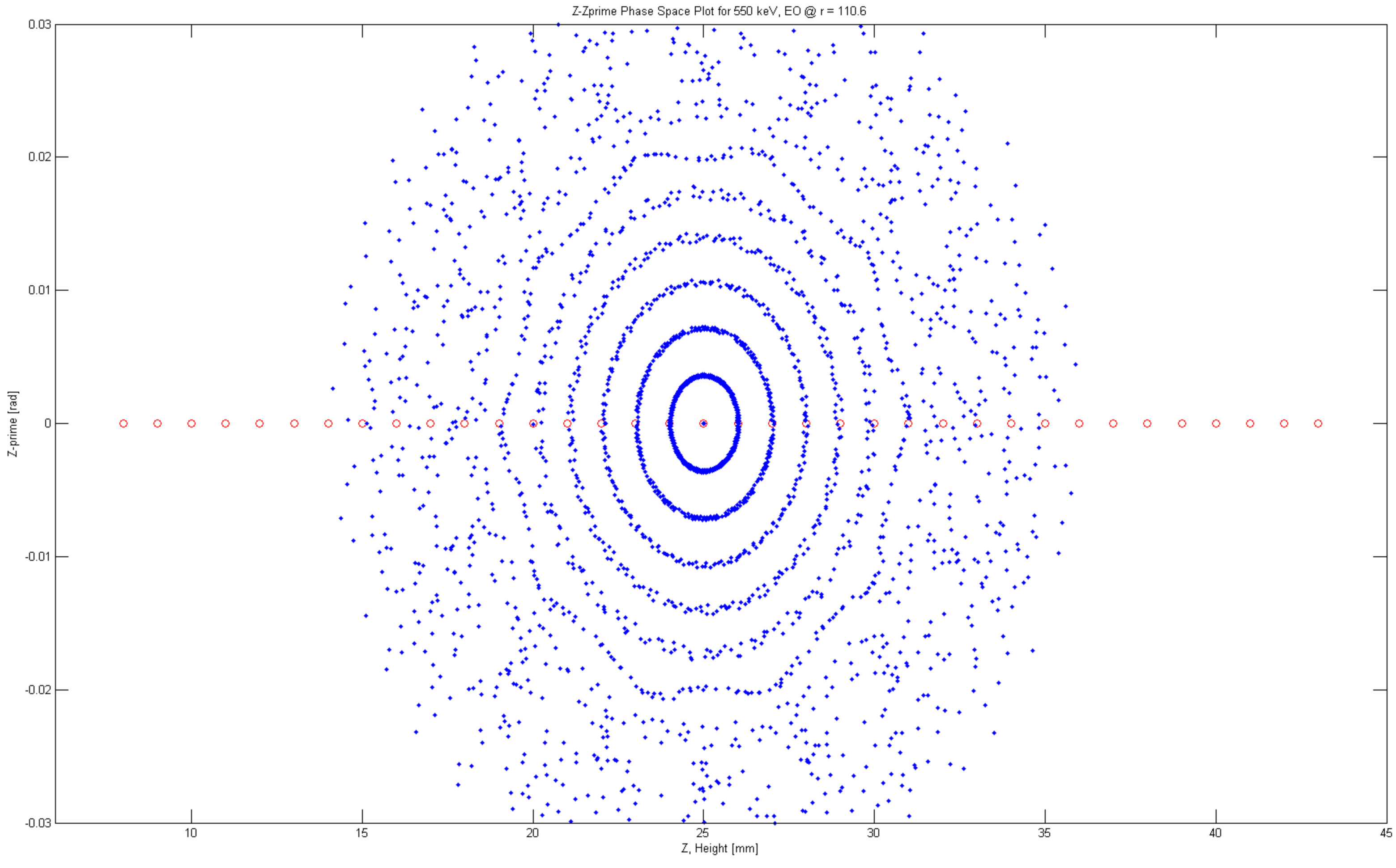


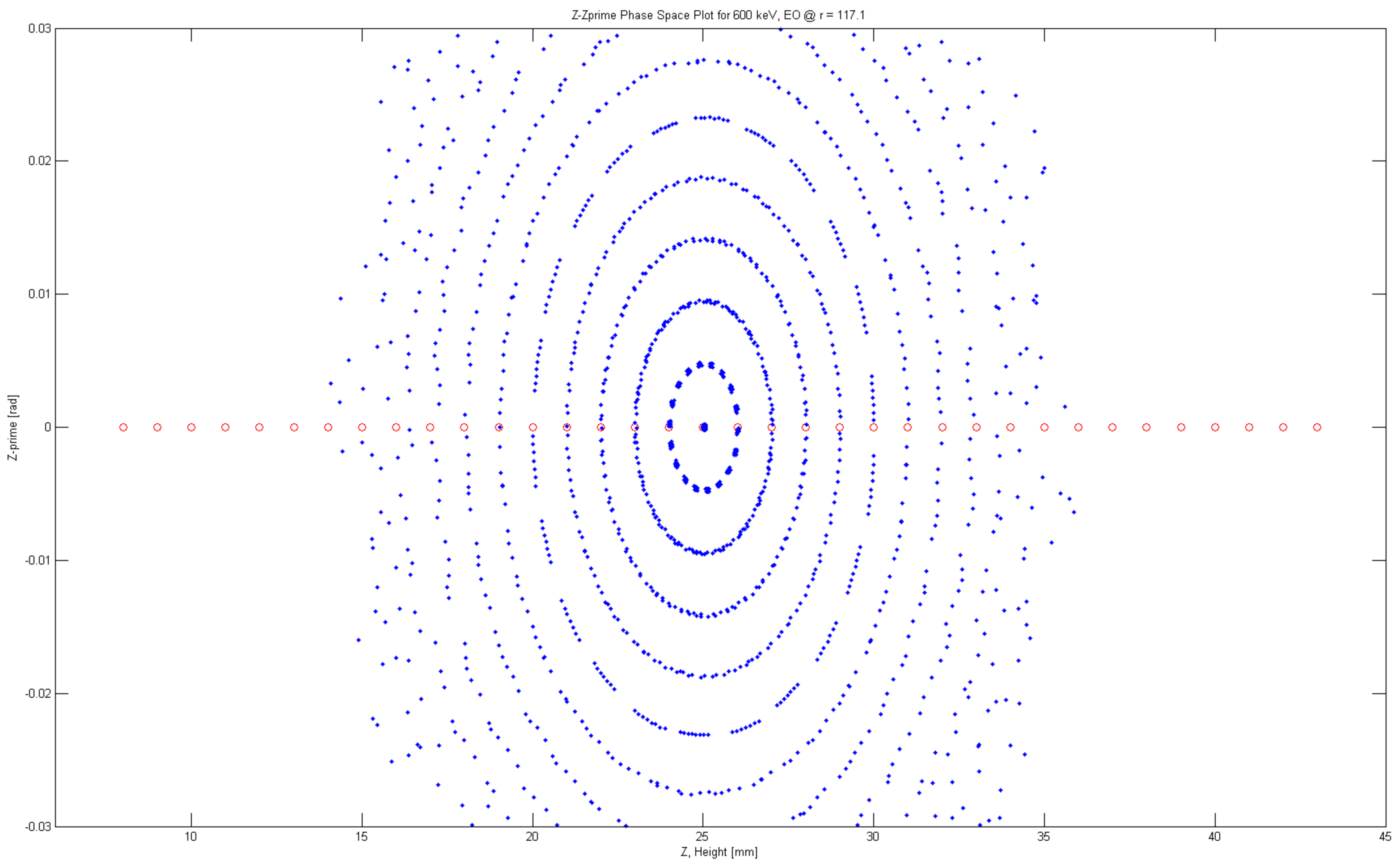
Z-Zprime Phase Space Plot for 400 keV, EO @ r = 93.055
0.03
0.02
0.01
0
-0.01
-0.02
-0.03
Z-prime [rad]
10
15
20
25
30
35
40
45
Z, Height [mm]


Z-Zprime Phase Space Plot for 450 keV, EO @ r = 98.96
Z-prime [rad]
Z, Height [mm]
0.03
0.02
0.01
0
-0.01
-0.02
-0.03
10
15
20
25
30
35
40
45



Z-Zprime Phase Space Plot for 500 keV, EO @ r = 104.75
0.03
0.02
0.01
0
-0.01
-0.02
-0.03
Z-prime [rad]
10
15
20
25
30
35
40
45
Z, Height [mm]


Z-Zprime Phase Space Plot for 550 keV, EO @ r = 110.6
0.03
0.02
0.01
0
-0.01
-0.02
-0.03
Z-prime [rad]
10
15
20
25
30
35
40
45
Z, Height [mm]



Z-Zprime Phase Space Plot for 600 keV, EO @ r = 117.1
0.03
0.02
0.01
0
-0.01
-0.02
-0.03
Z-prime [rad]
10
15
20
25
30
35
40
45
Z, Height [mm]

Weak Focusing Example

Intentionally introduce radial B-field component:

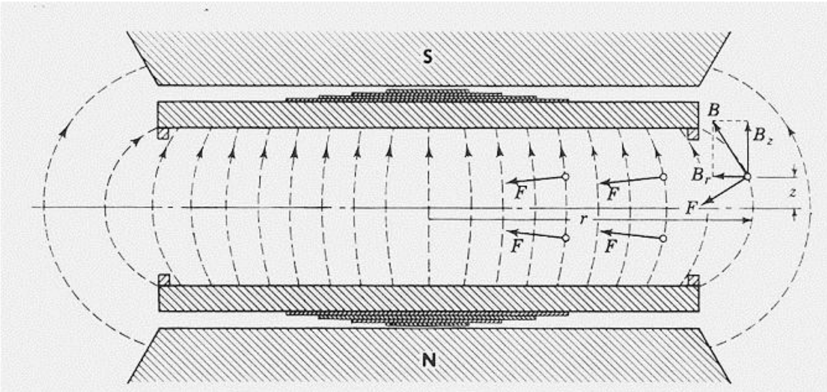
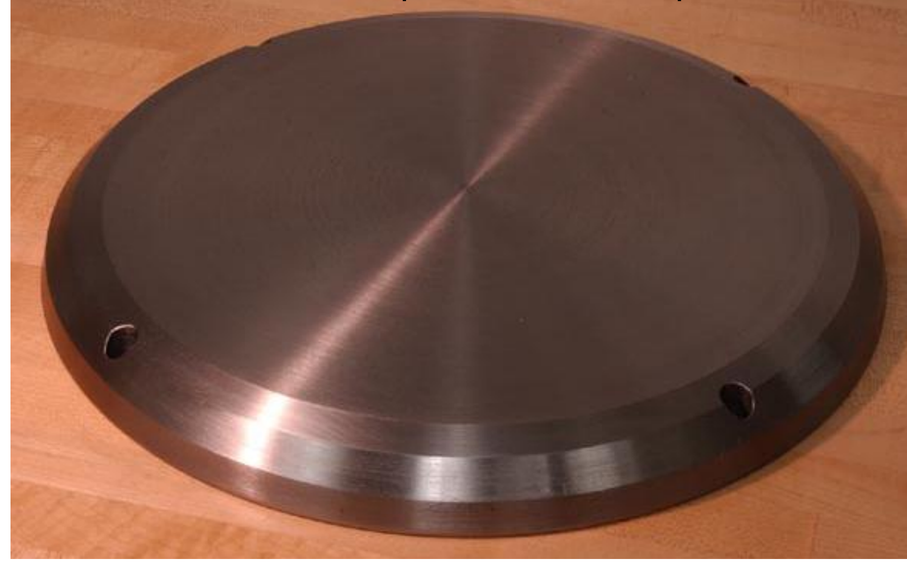
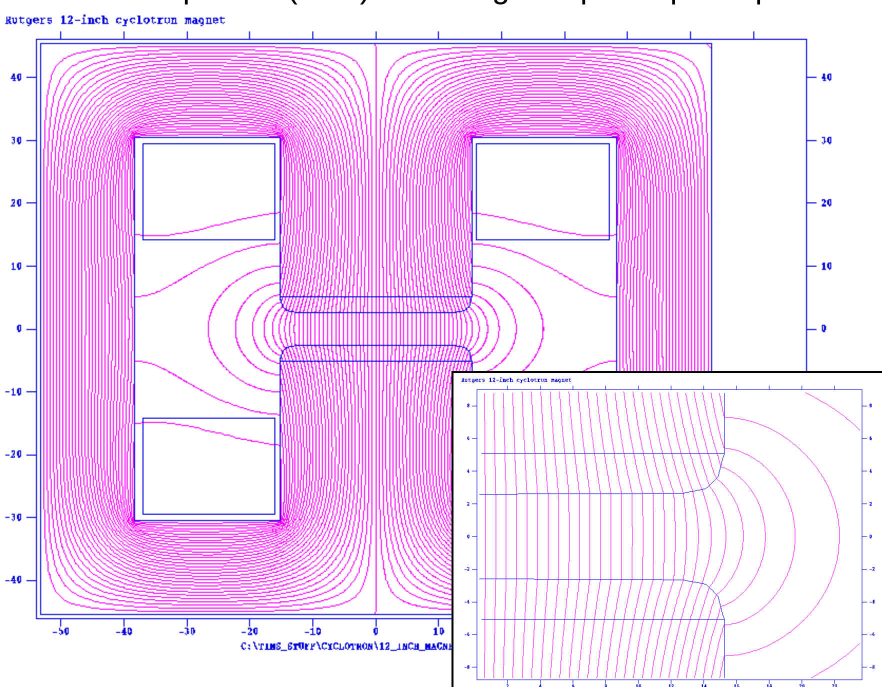


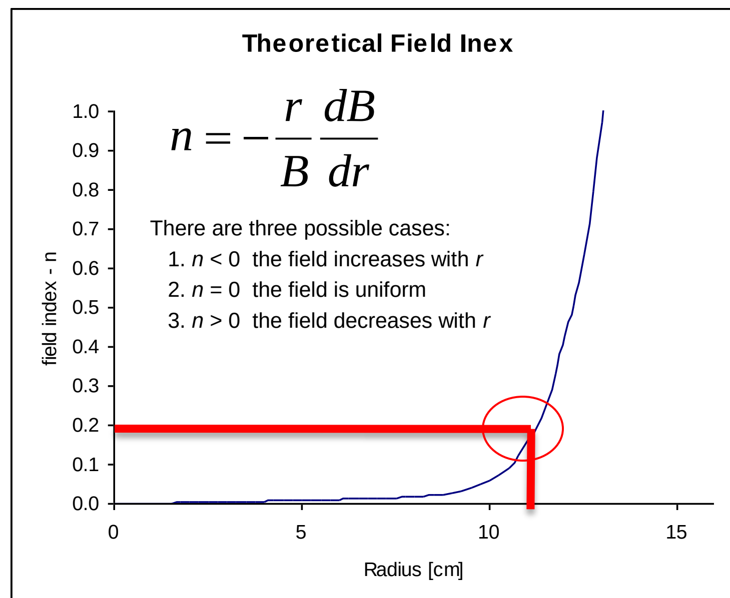
Fig. 6-7. Radially decreasing magnetic field between poles of a cyclotron magnet, showing shims for field correction.

Pole tips with radial slope

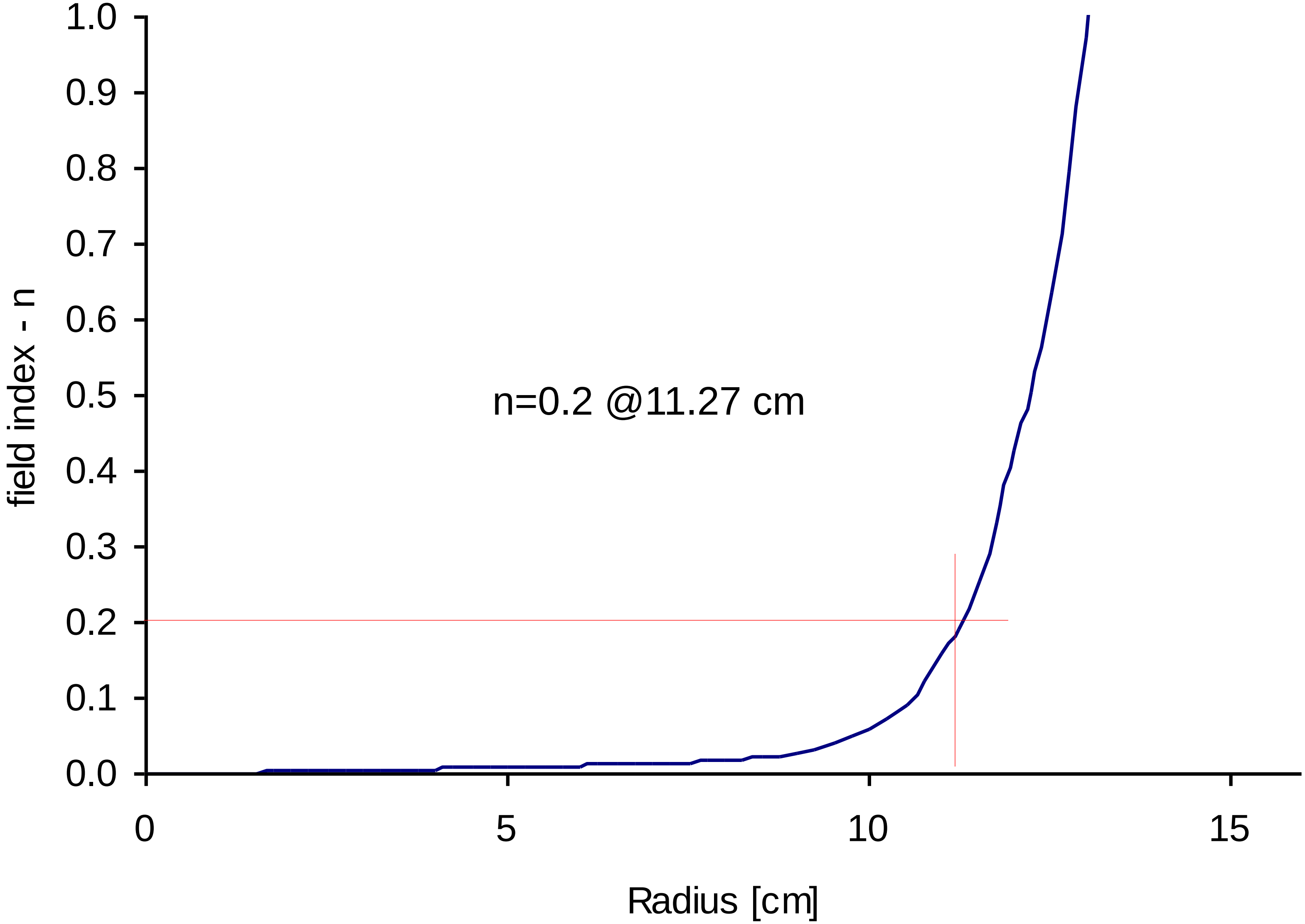


Poisson Superfish (PSF) modeling of tapered pole tips



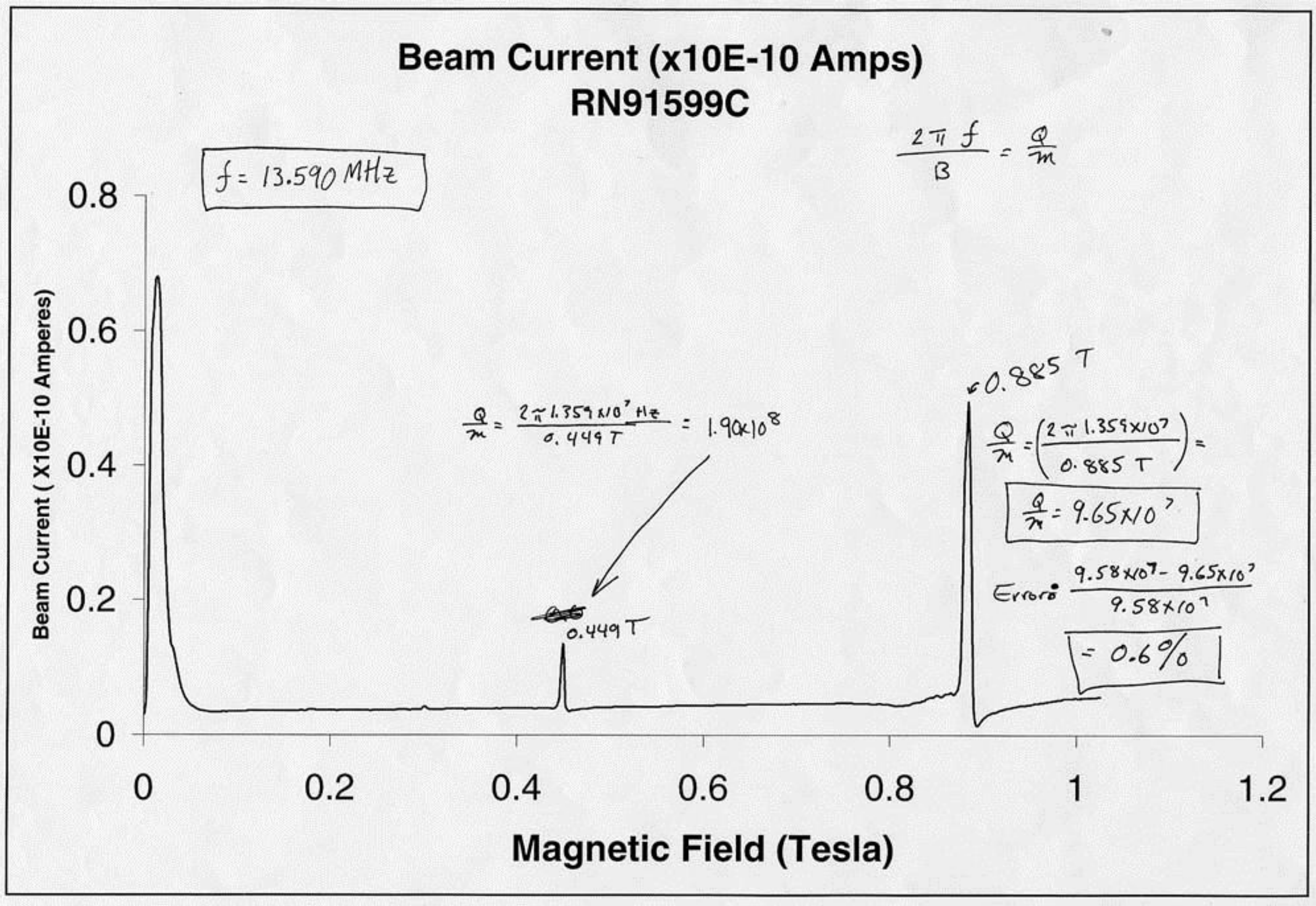


Weak Focusing Field Index



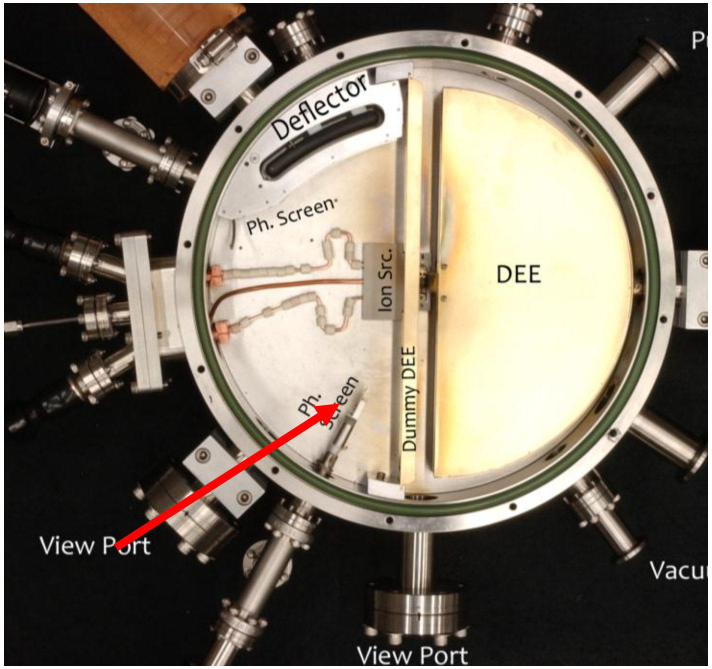
Example of Operation

1st successful operation was recorded by slowly sweeping B-field to locate resonance condition. September 16, 1999:



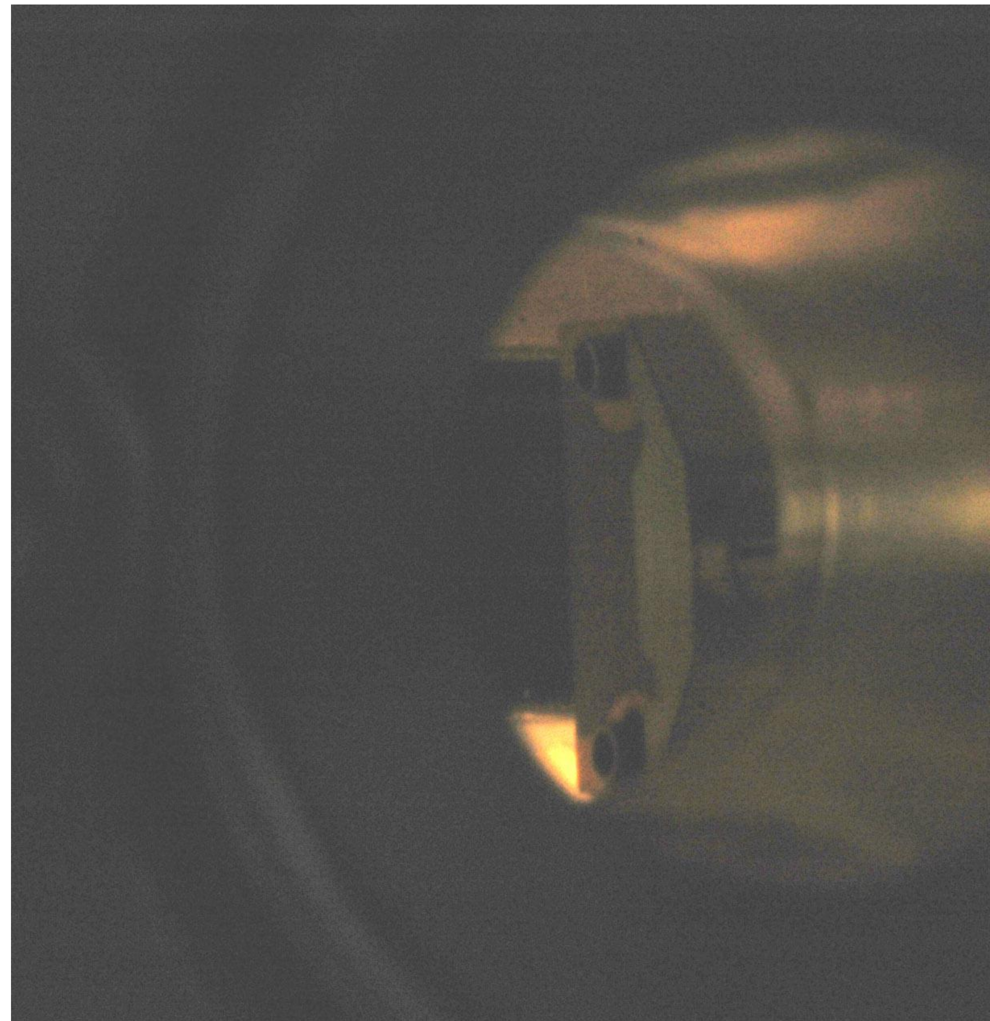
Faraday cup insertion set to intercept beam at 300keV.

Example of Beam

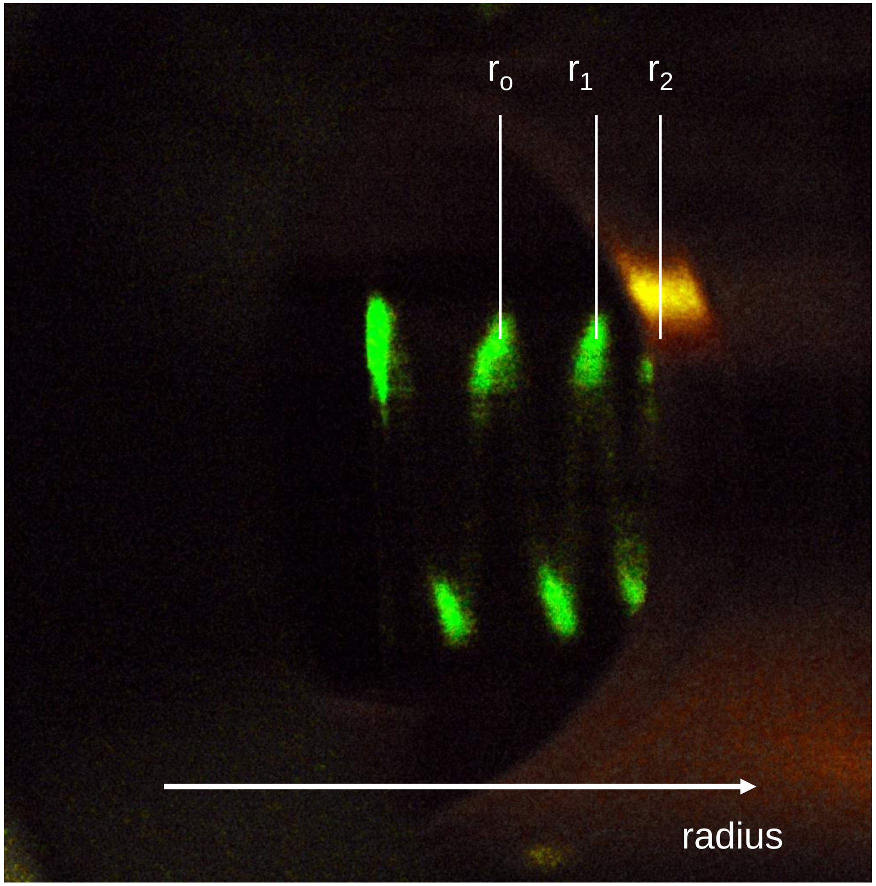


Transversely viewing the beam on a phosphor screen

Observed beam spot above median plane.



What is this ?



f = 14.864 MHz

B_o= 0.977 Tesla

RF Power: 300 W

(7,500 V_{p-p})

r_0=8.6 cm (338keV)

r_1=9.2 cm (387keV)

r_2=9.6 cm (421keV)

15 second exposure while positioner was slid in and out.

Expected Betatron Motion

Ion Energy [eV]: $E(r) = \frac{qB^2}{2m} r^2$

Take derivative: $\frac{\partial r(E)}{\partial E} = \frac{1}{\alpha}\frac{\partial}{\partial E}\sqrt{E} = \frac{1}{2\alpha\sqrt{E}}$ where $\alpha^2 = \frac{qB^2}{2m}$

Turns spacing: $\Delta r(E) = \frac{\Delta E}{2\alpha\sqrt{E}}$ Or since E(r): $\Delta r(r) = \frac{\Delta E m}{qB^2 r}$

where ΔE is energy gained per rev, or just DEE $V_{p\text{-}p}$

Using our operating values:

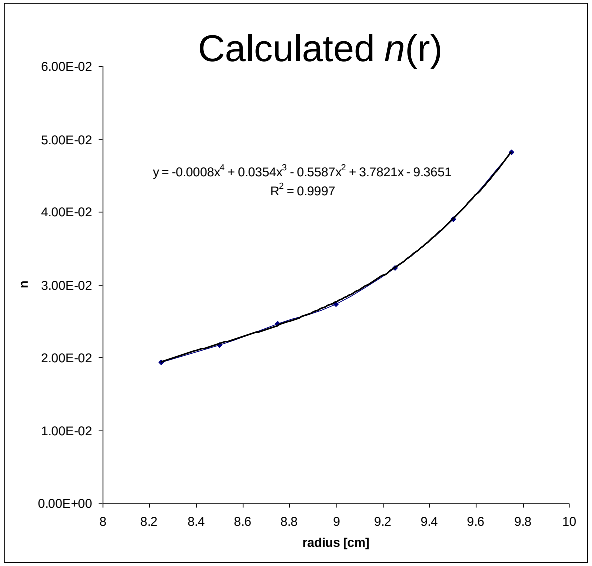
$$\Delta r(r) = \frac{(7500eV)(1.67E-27)}{(1.6E-27)(0.977)^2}\frac{1}{r} \quad \text{or} \quad \Delta r(r) = (8.2E-5)\frac{1}{r}$$

Vertical Betatron Relationship:

$$f_{\beta-vert} = \sqrt{n}\, f_0$$

$$T_{\beta-vert} = \frac{1}{\sqrt{n}} T_0$$

$\sqrt{n}$ phase advance in one ion revolution

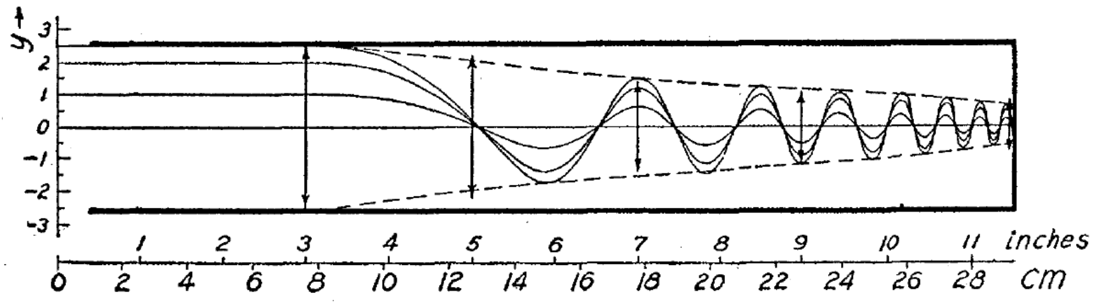


r [cm]	n(r)	Fraction of Betatron Period
8.6	0.025	0
8.7	0.026	0.2
8.8	0.026	0.3
8.9	0.027	0.5
9.0	0.028	0.6
9.1	0.030	0.8
9.2	0.031	1.0
9.2	0.033	1.2
9.3	0.034	1.3
9.4	0.037	1.5
9.5	0.039	1.7
9.6	0.041	1.9
9.7	0.042	2.1

Almost a Perfect Match !

1937: Robert R. Wilson (a diversion)

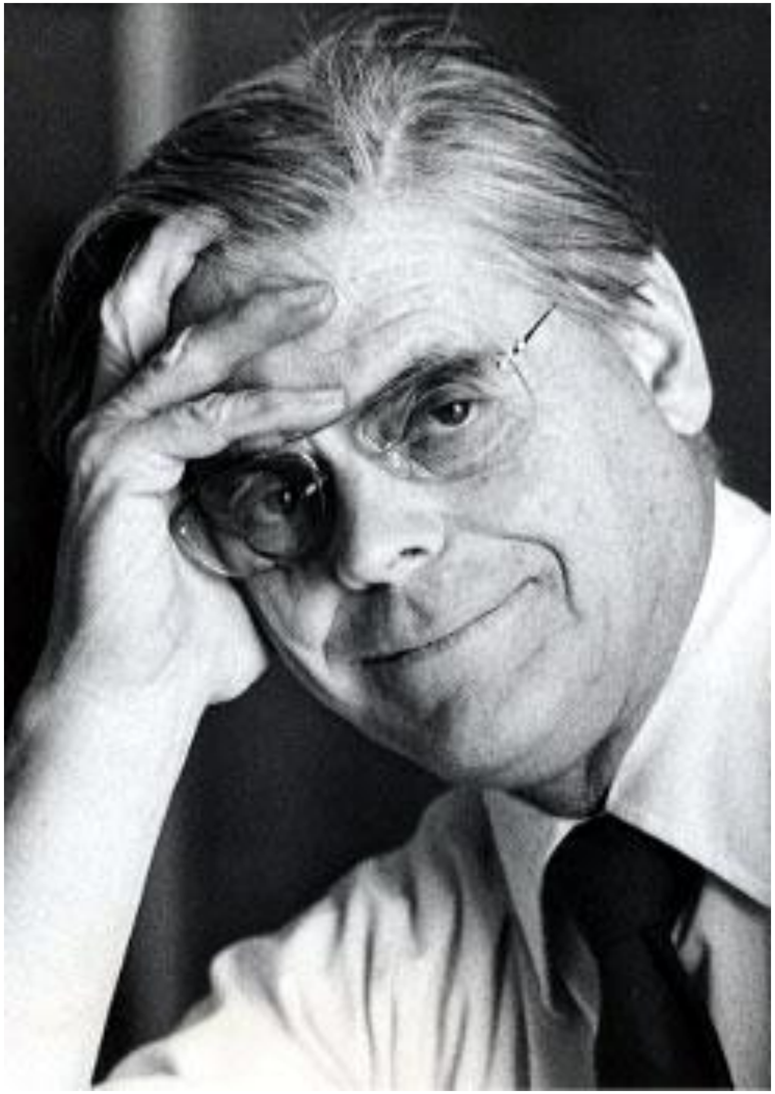
was a graduate student under E. O. Lawrence at Berkeley. As part of his Ph.D. work he calculated the effects of magnetic and electric focusing in the cyclotron. He even sketched the calculated trajectories for differing initial offsets:



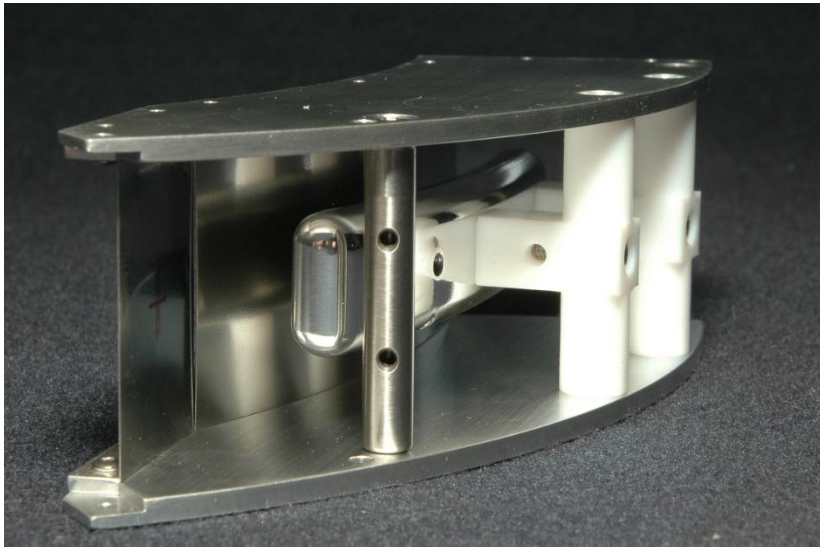
(*Phys. Rev.* 53. R. R. Wilson, March **1938**)

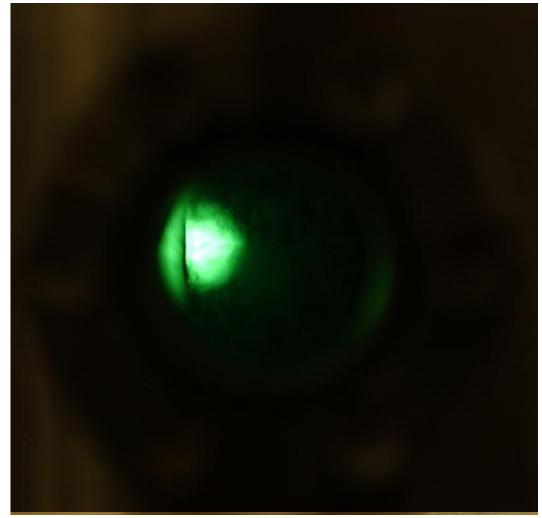
He never took a photo of it !

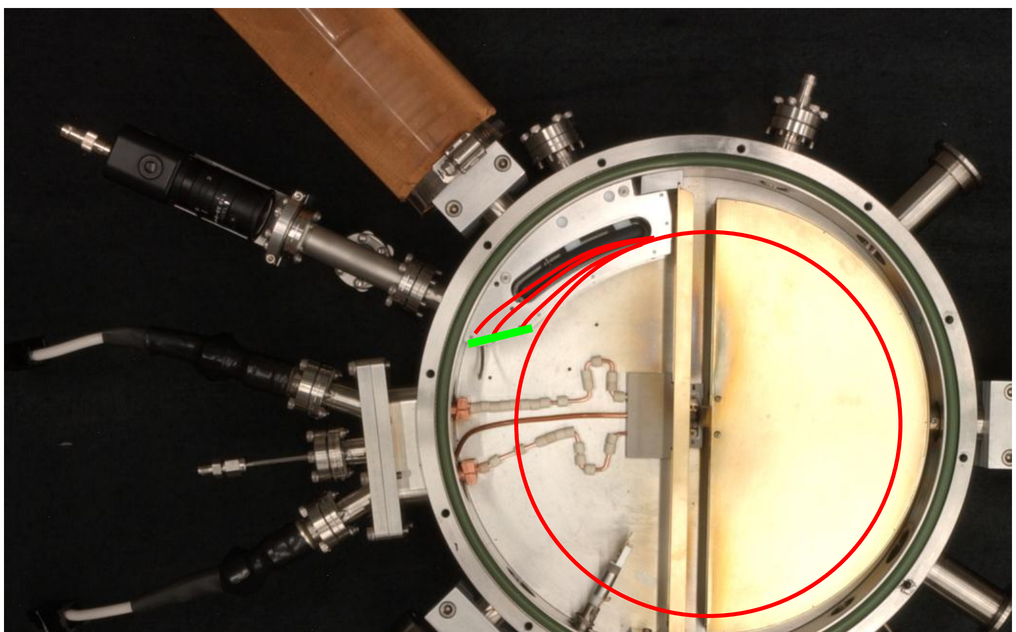
Oddly, betatron motion is named from Kerst's betatron studies first published in 1941



Extraction: Electrostatic Deflector







Home Work
Problem

RELATIVITY

A Limit to the Classical Cyclotron

Relativistic mass increase imposes an upper limit to the classical weak-focusing cyclotron - on the order of 10 MeV protons.

There are a few ways to over come this limitation

< read Rose & Bethe, and Lawrence's response >

I. The Synchrocyclotron: Let the RF frequency w decrease as the energy increases

- $\omega=\omega_0/\gamma$
- Utilize the same magnetic field as the weak focusing cyclotron
- Relies on Phase Stability ! (much lower DEE voltage)

The center is relativistically correct in the center $$\omega_0 = \frac{qB}{M_0}$$

During acceleration the RF frequency changes with the mass increase

$$\omega_{rf} = \frac{\omega_0}{\gamma} = \frac{qB}{\gamma M_0} = \omega$$

The frequency change is always **synchronously** matched to the mass increase

Synchrocyclotron

The RF frequency begins at the highest value and drops during acceleration

Consider 250 MeV Protons (γ=1.26) in a SC 5 Tesla W.F. magnet (Bfinal 90% B0)

$f_0=\omega_0/2\pi=15.4$ MHz·5T = 77 MHZ → 1 RF period = 12 ns

$f_{final}=\sim 0.9B_0 e/1.26M_0$ = 54 MHz → 1 RF period = 18 ns

Assume 10,000 turns to attain 250 MeV

one full acceleration cycle ~ 15 ns x 10,000 = 150 μs.

With RF recovery time: 1000 acceleration cycles per second → 15% duty cycle

Relativity cured, but at the cost of low beam intensity.

SC Syncrhocyclotron

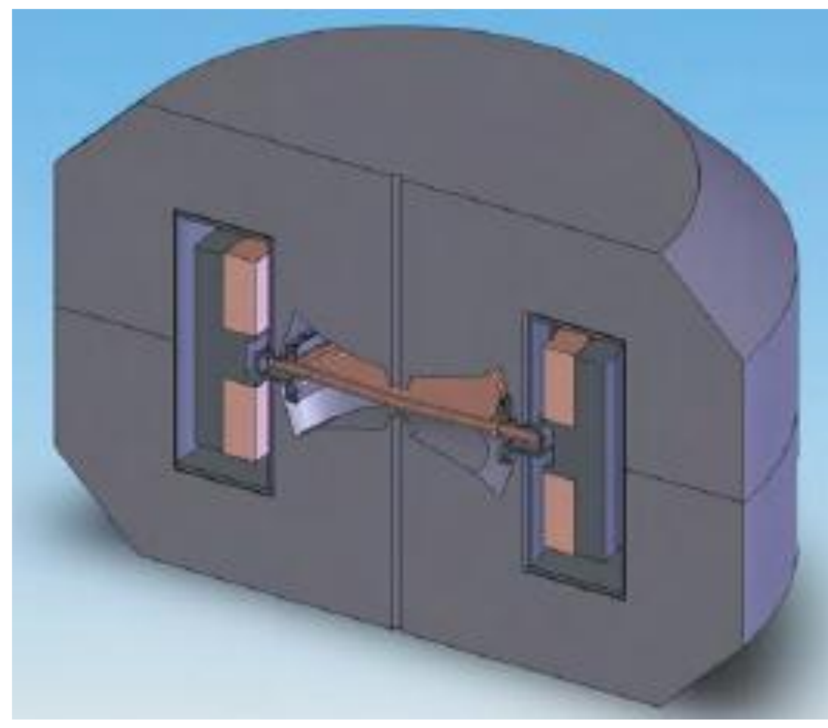
9 Tesla superconducting magnet

f_{cyc} ~ 140 MHz

250 MeV

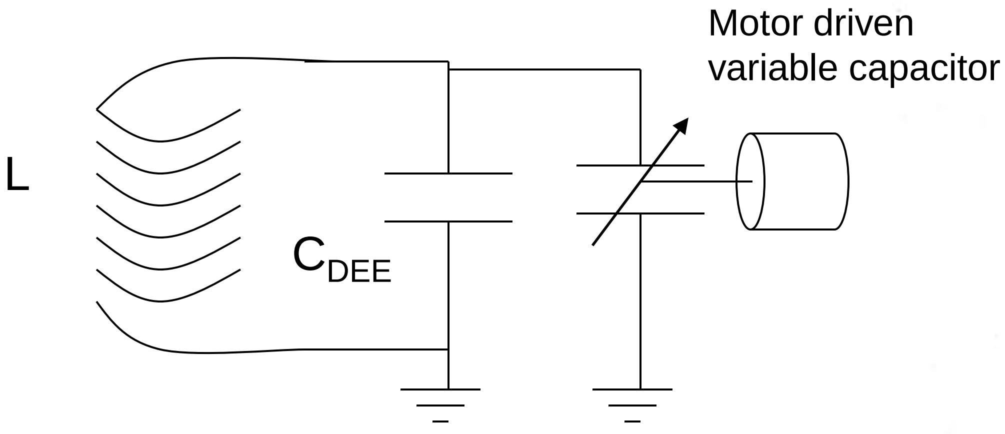
NOTE Weak Focusing Pole tips !!!

This is why we spent so much time reviving the old cyclotron spirits.

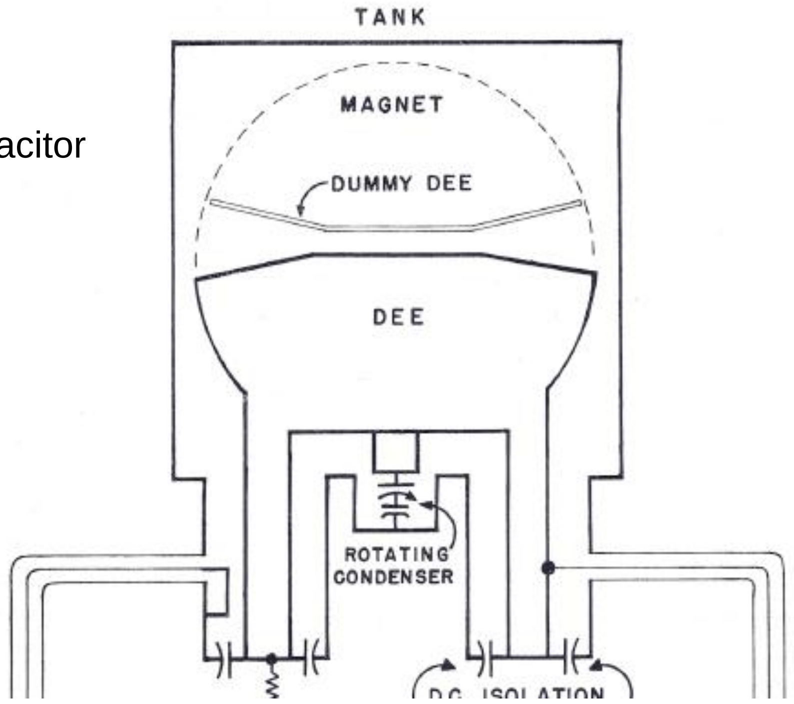


Synchrocyclotron RF (aka FM cyclotron)

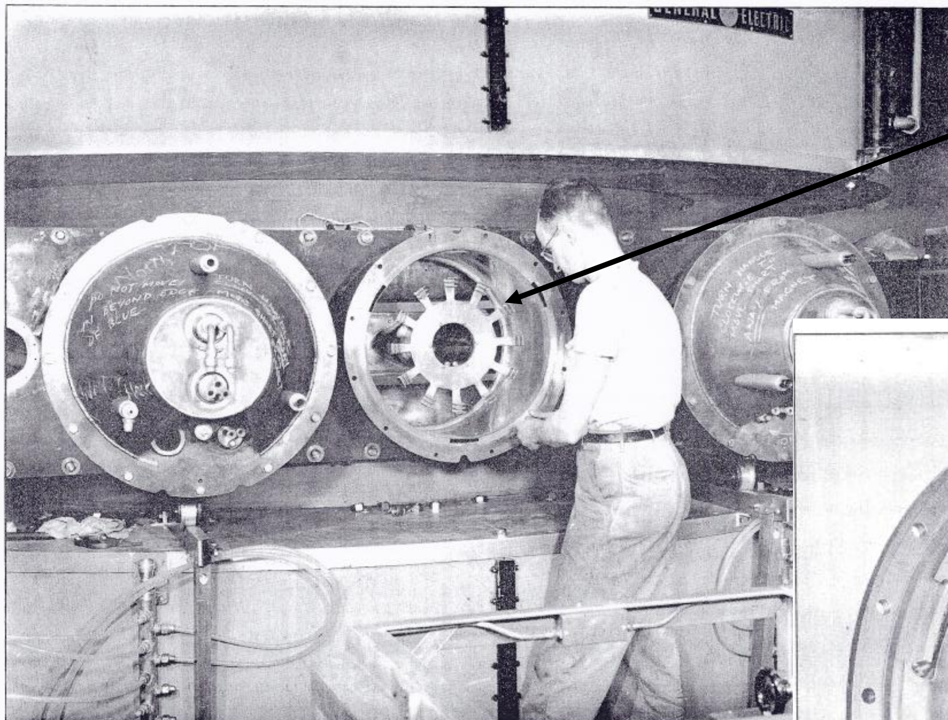
Electrical equivalent



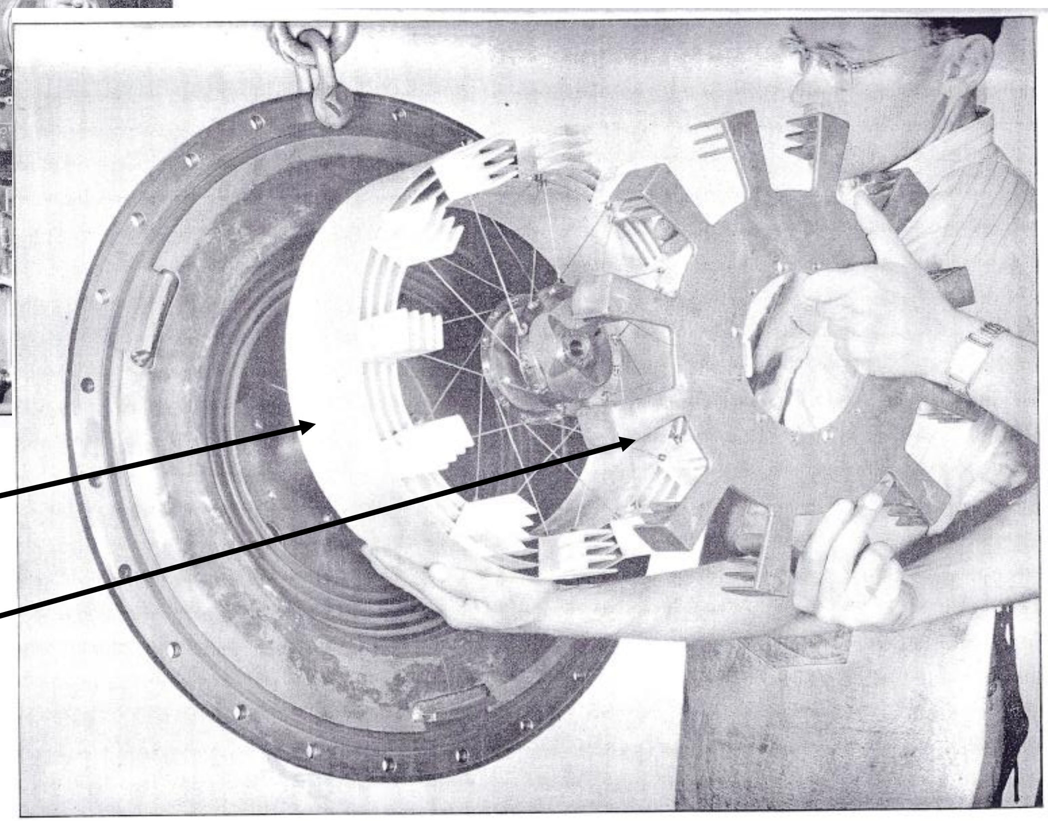
$$f(t) = \frac{1}{2\pi\sqrt{LC_{DEE} + C(t)}}$$



Synchrocyclotron RF



The rotating capacitor (condenser) of the Harvard 95-inch cyclotron.



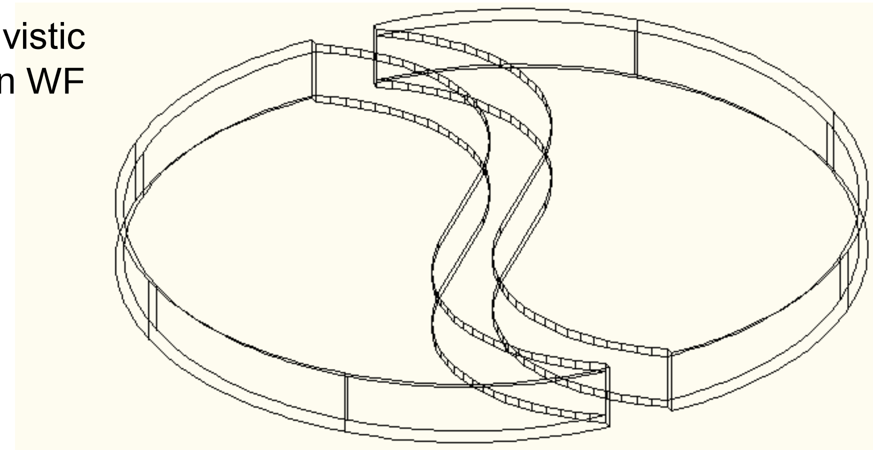
The rotor

The stator

II. Azimuthally Modualted DEE

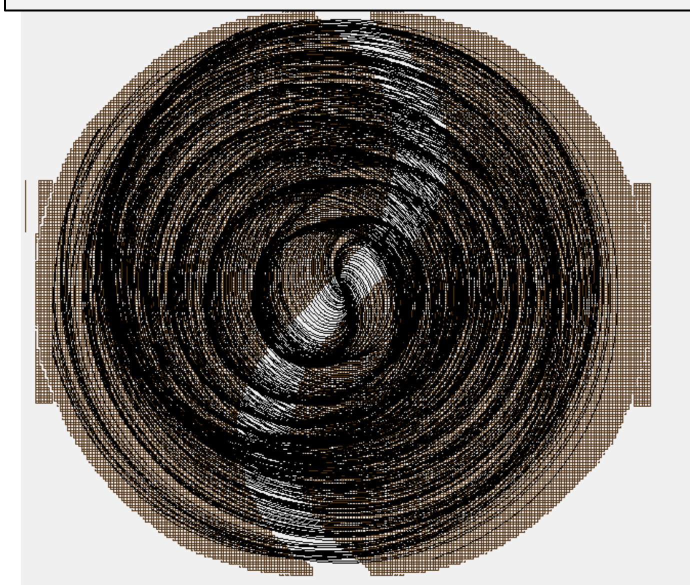
Patent Pending
61/282,537

A 'cure' for relativistic mass increase in WF cyclotrons.



B-K Electrode: In simple terms, if the ion won't come to the gap, then bring the gap to the ions. There are limits…

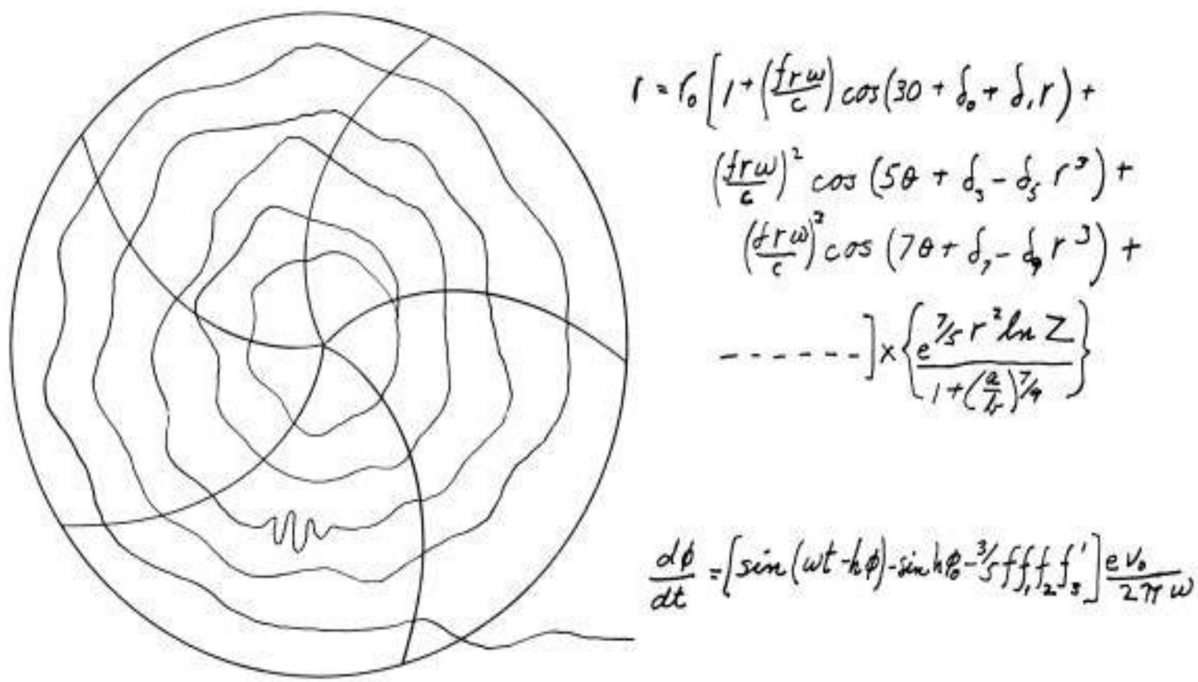
SIMION Tracks Using the Curved DEE



Heidi's Presentation at the NYCSeF – March 2010

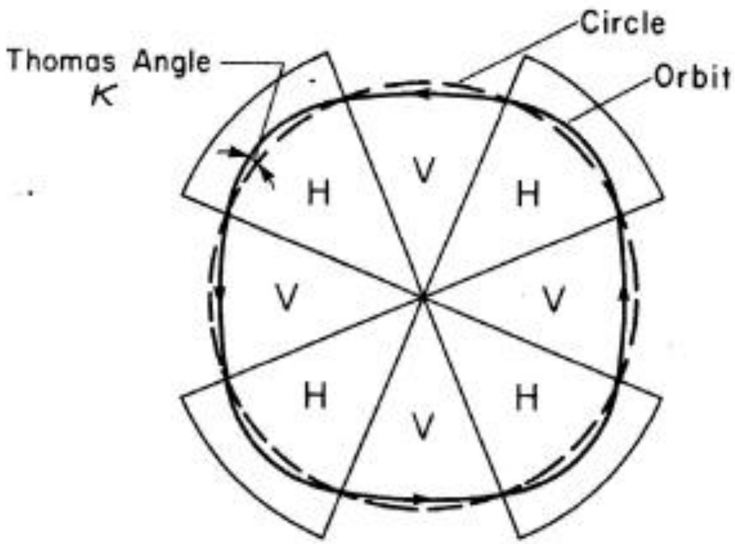
III. Azimuthally Varying Field

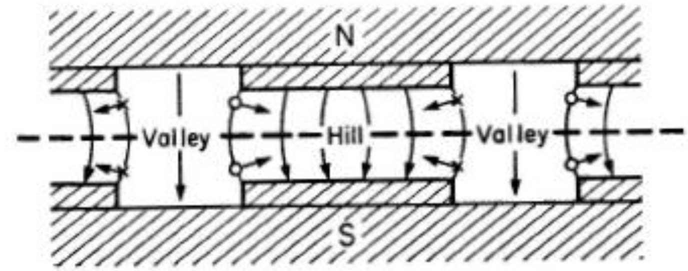
The cyclotron as seen by the…



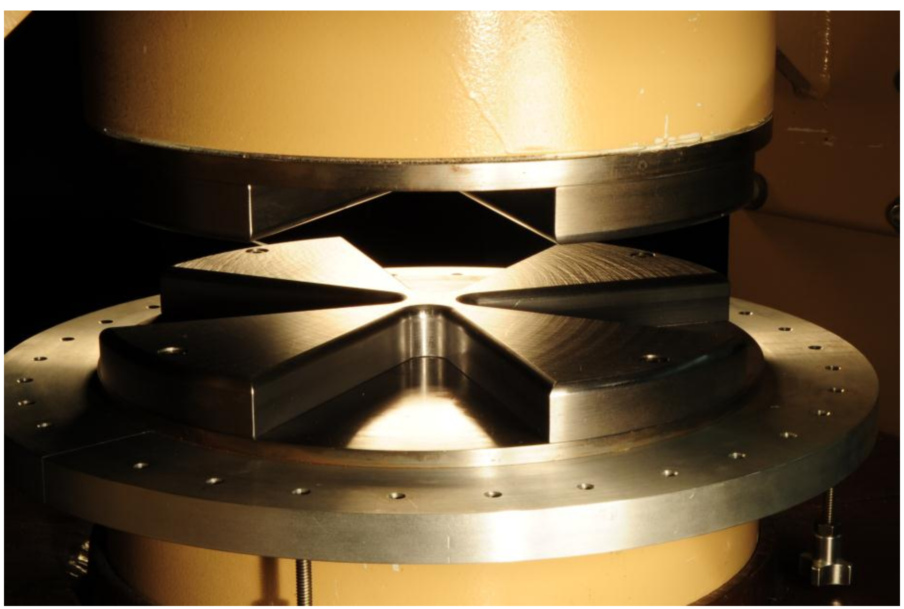
. . . the theoretical physicist

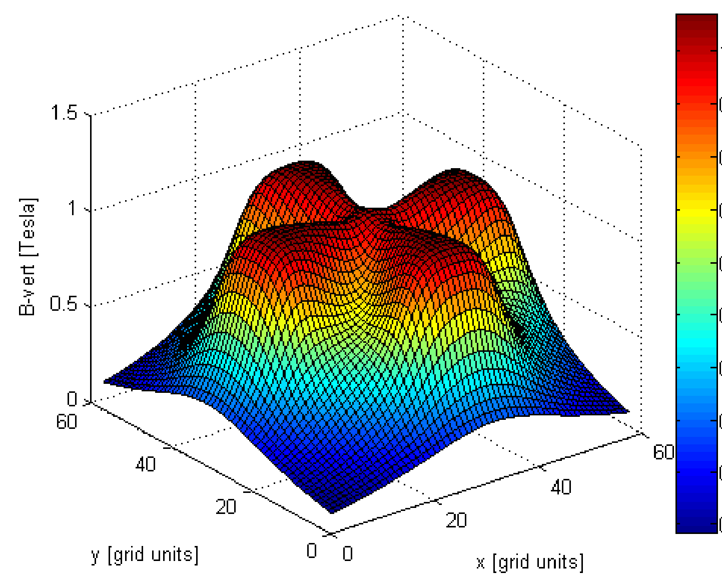
Thomas Cyclotron Example



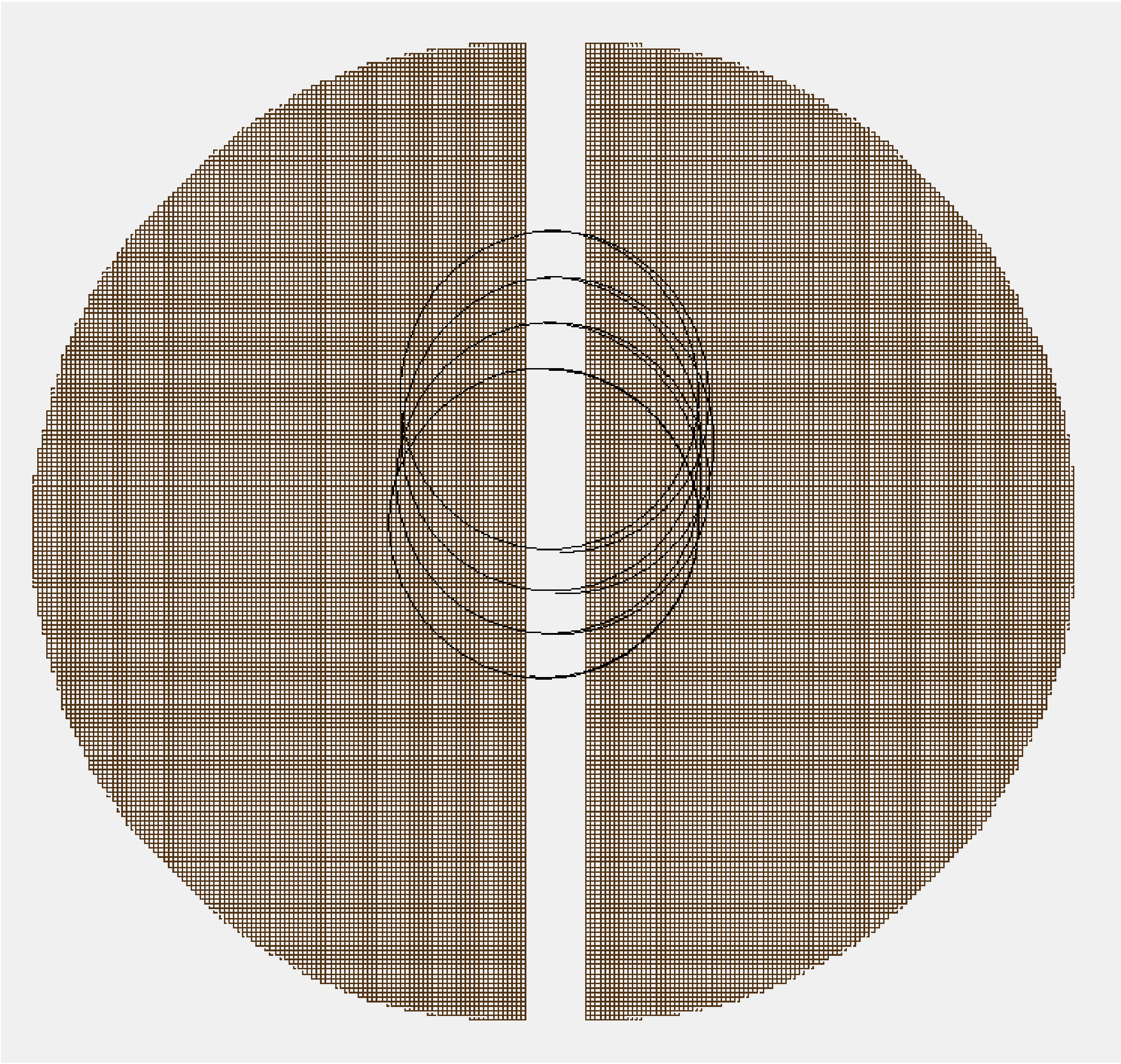


	Thomas cyclotron	FFAG / AGS
Periodic $B(\theta)$	Yes	Yes / No
Alternating $\partial B/\partial r$	Yes	Yes
Axial force	$q\,v_r B_\theta$	$q\,v_\theta B_r$
Lens pattern	FFFFFFFF	FDFDFDFD
Edge focusing	Dominant	Negligible
AG focusing	Negligible	Dominant





Finding the Equilibrium Orbit

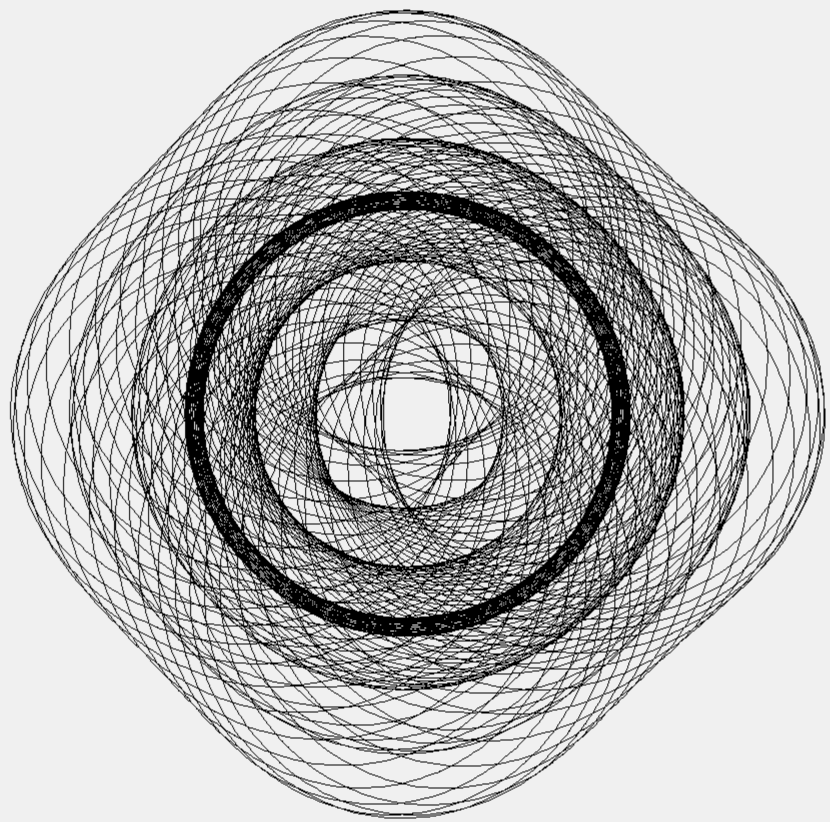


Finding the Equilibrium Orbit

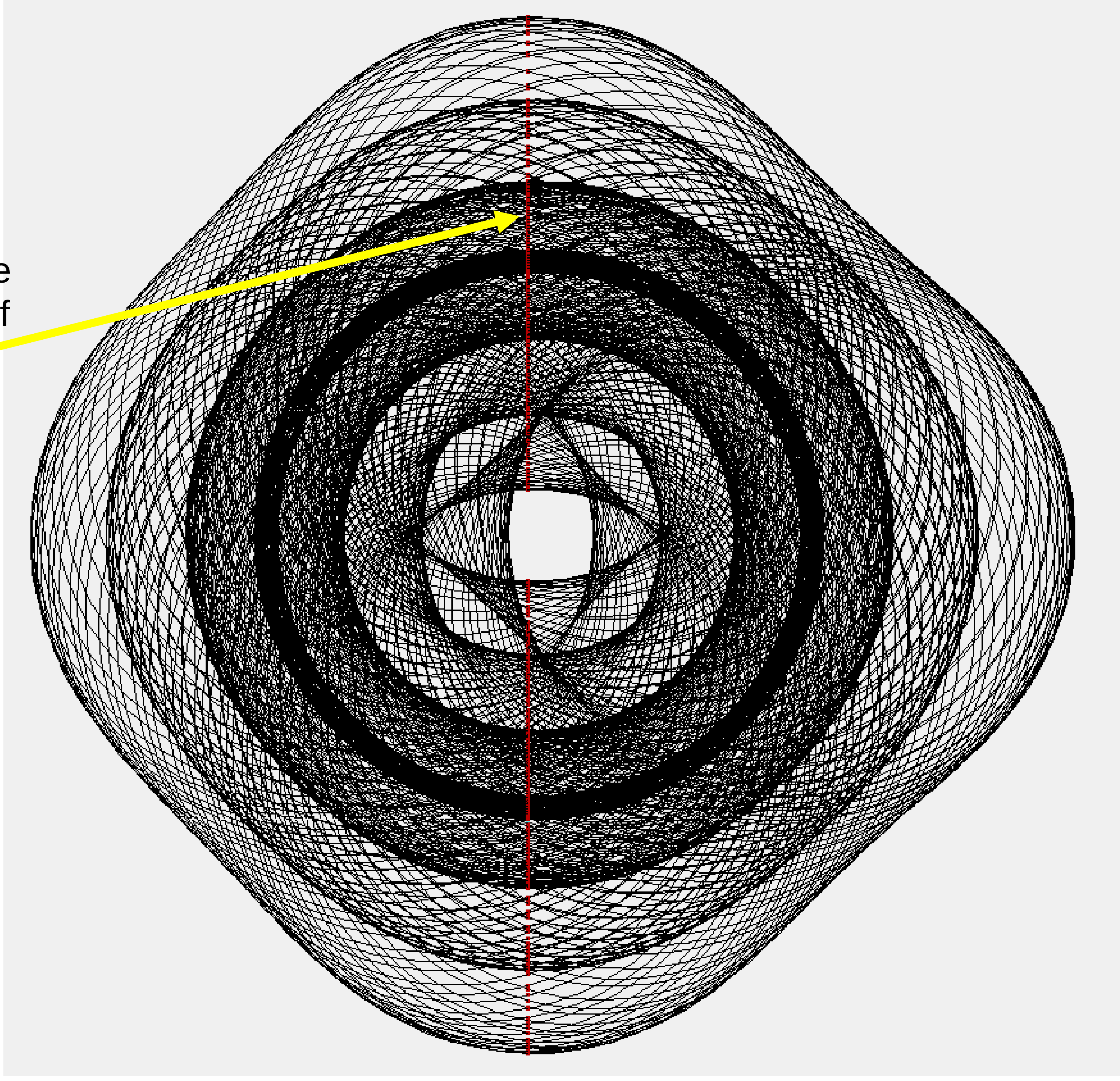
After the four ions have processed for a while, several things become evident. First, the field must be stable for the ions to be able to so wildly process.

Second, it is obvious to the eye that one of the ions was launched close to the EO. The darkest ‘ring’ is the ion trajectory closest to being on the EO.

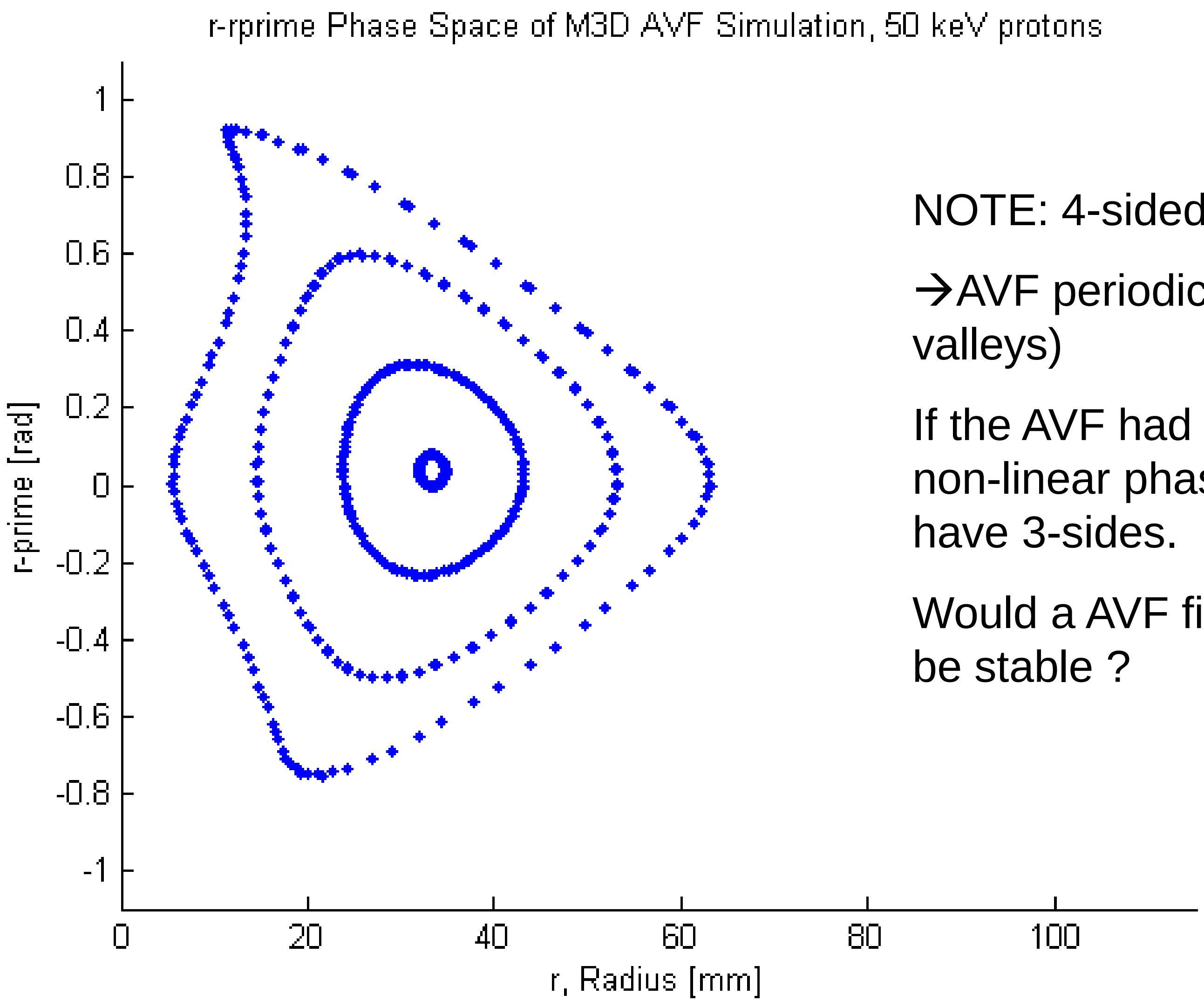
Third, the largest orbit shows the characteristic of the 4-sector AVF field with a square envelope.



Now, to plot the phase space, we record the position and angle of the ion as it crosses a radial plane. These are the locations of the red dots.



Thomas Pole Phase Space Map



NOTE: 4-sided distorted rectangle

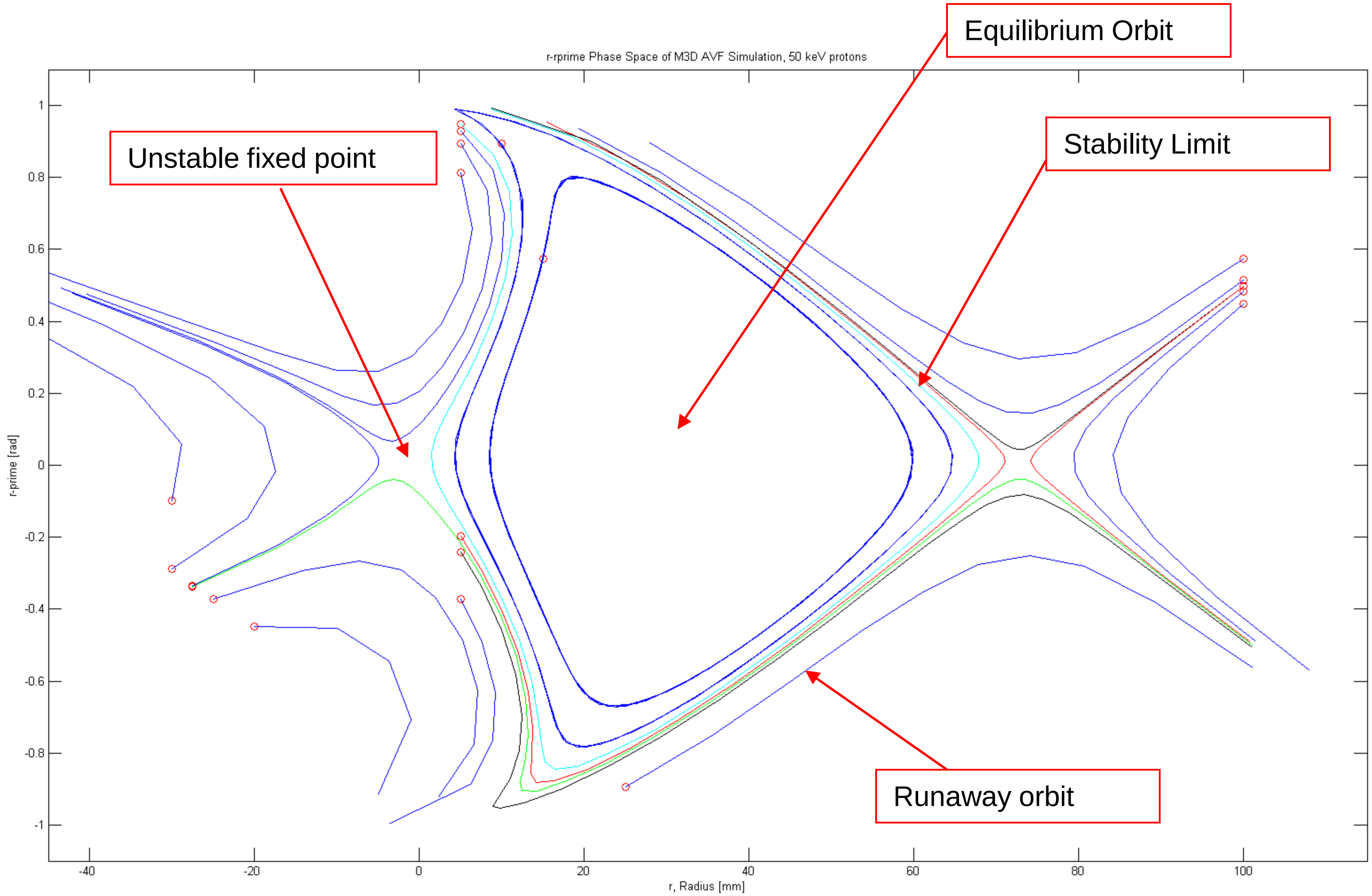
→AVF periodicity of 4 (i.e. 4 hills 4 valleys)

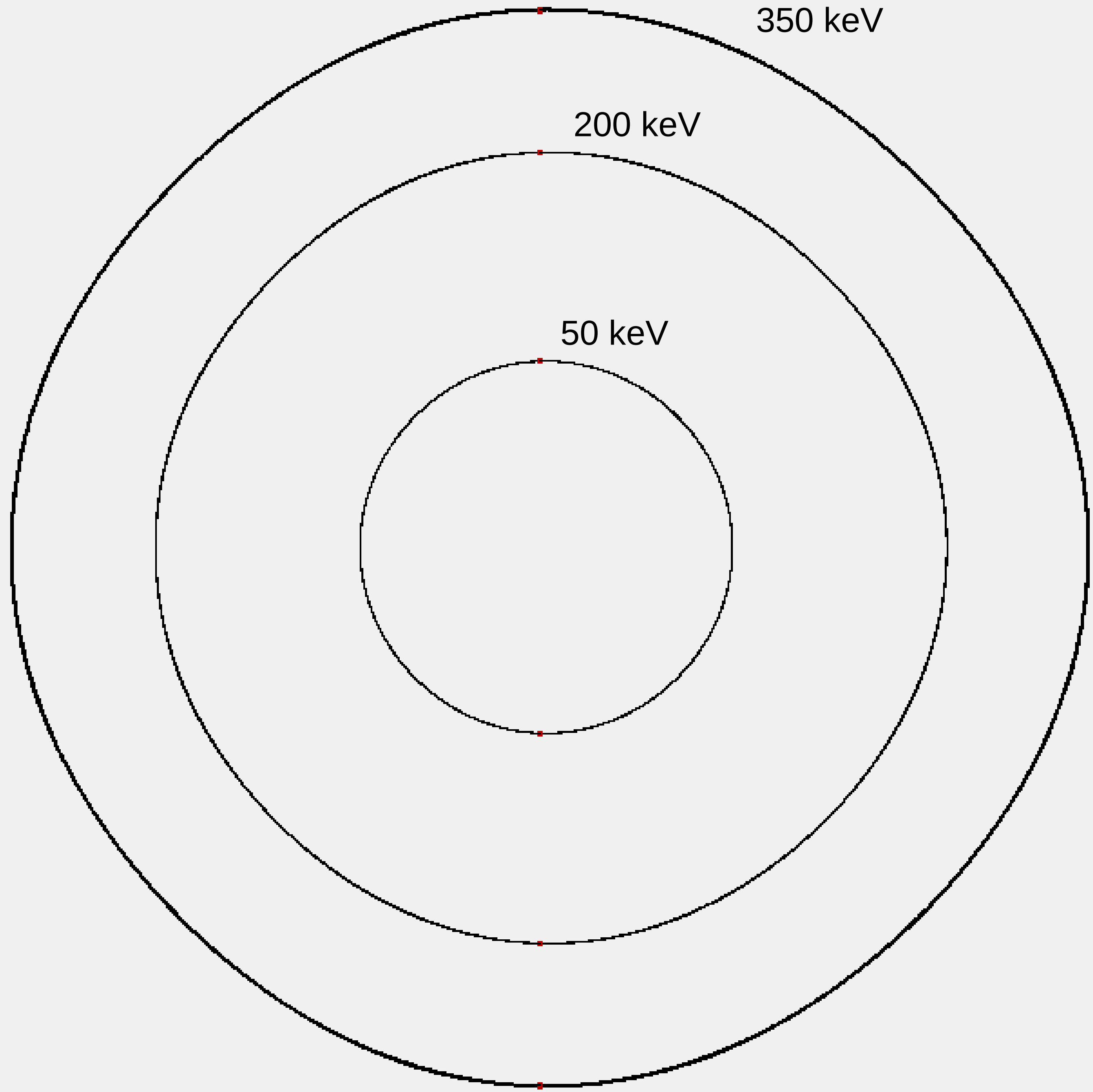
If the AVF had periodicity of 3, then non-linear phase space plots would have 3-sides.

Would a AVF field with periodicity 2 be stable ?

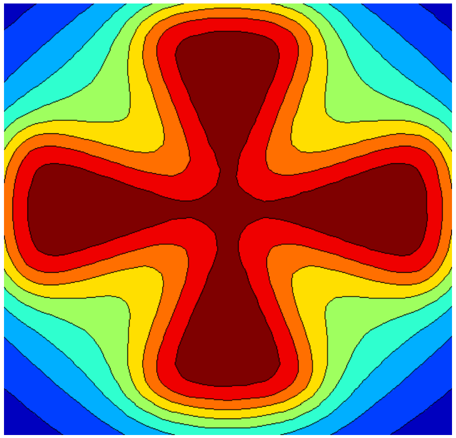
Find fixed points by careful searching:

connect the dots to follow 'flow' - red circles indicate starting location



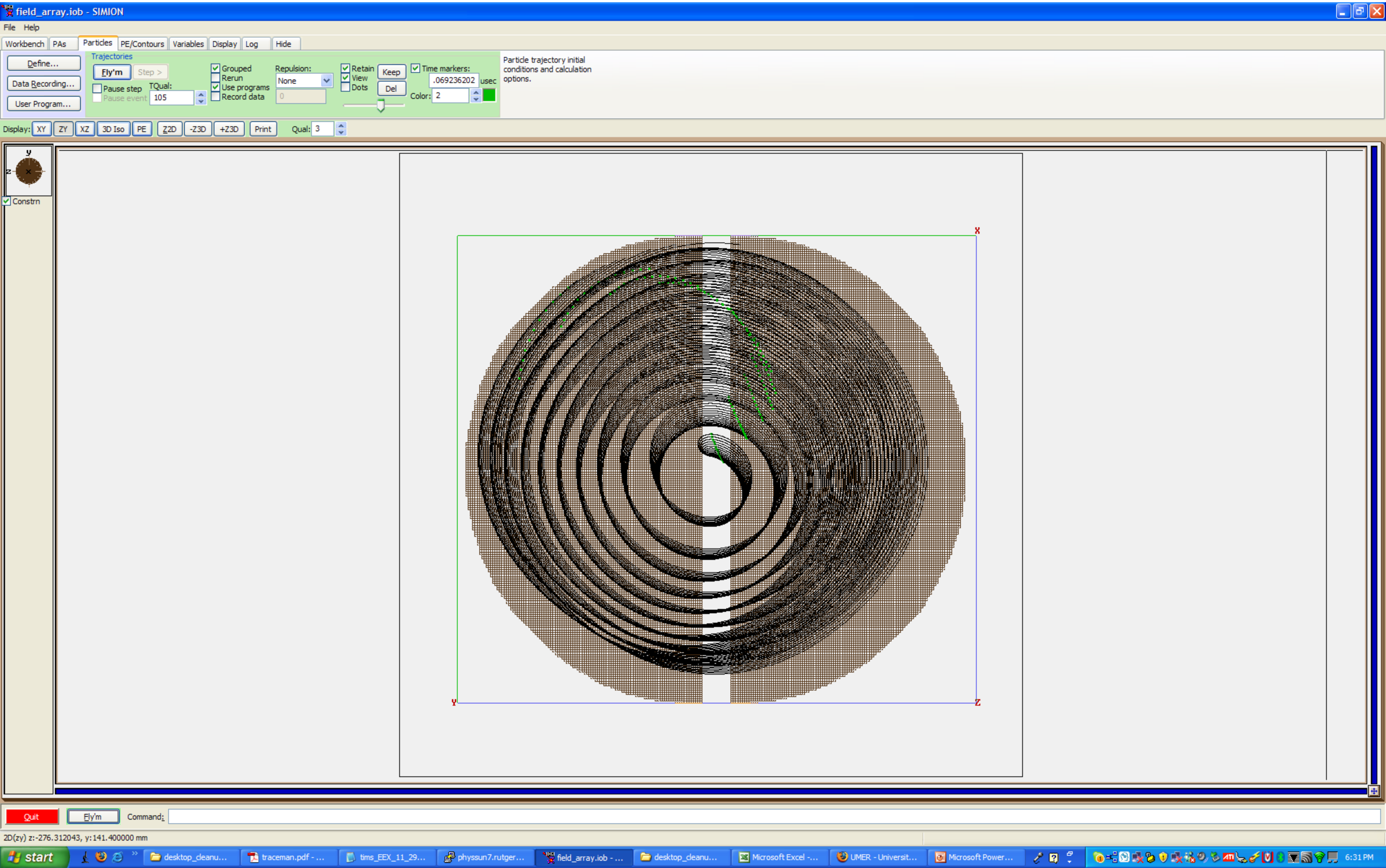


Note the scalloping of the larger orbits, showing the effect of the hills and valleys on the bending.



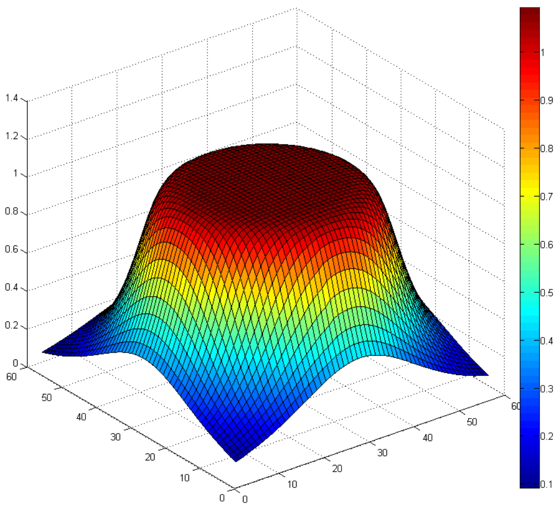

X
Y
Z
DEE view shut off for clarity

Showing that about 20 degrees of RF (out of 360) can successfully accelerate ions to the periphery. This does not include the ion source obstruction.

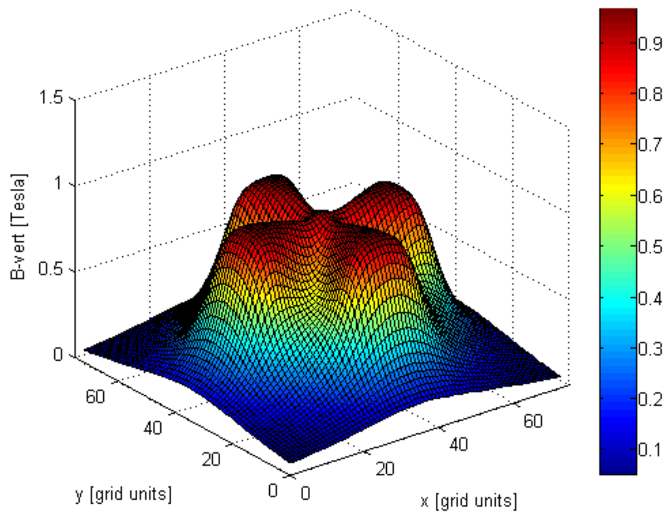


LETS COMPARE POLE TIPS:

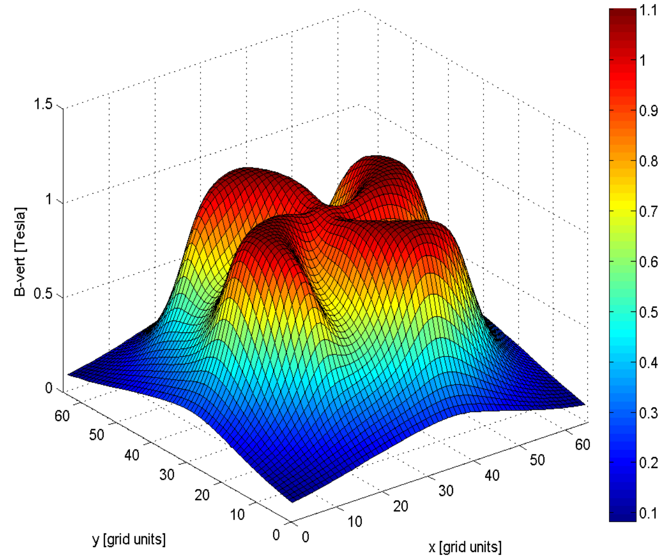
Weak Focusing



AVF



Spiral AVF



We’ve launched 50 keV protons at differing radii for the above B-field configurations.

-We’ll find the limits of stability

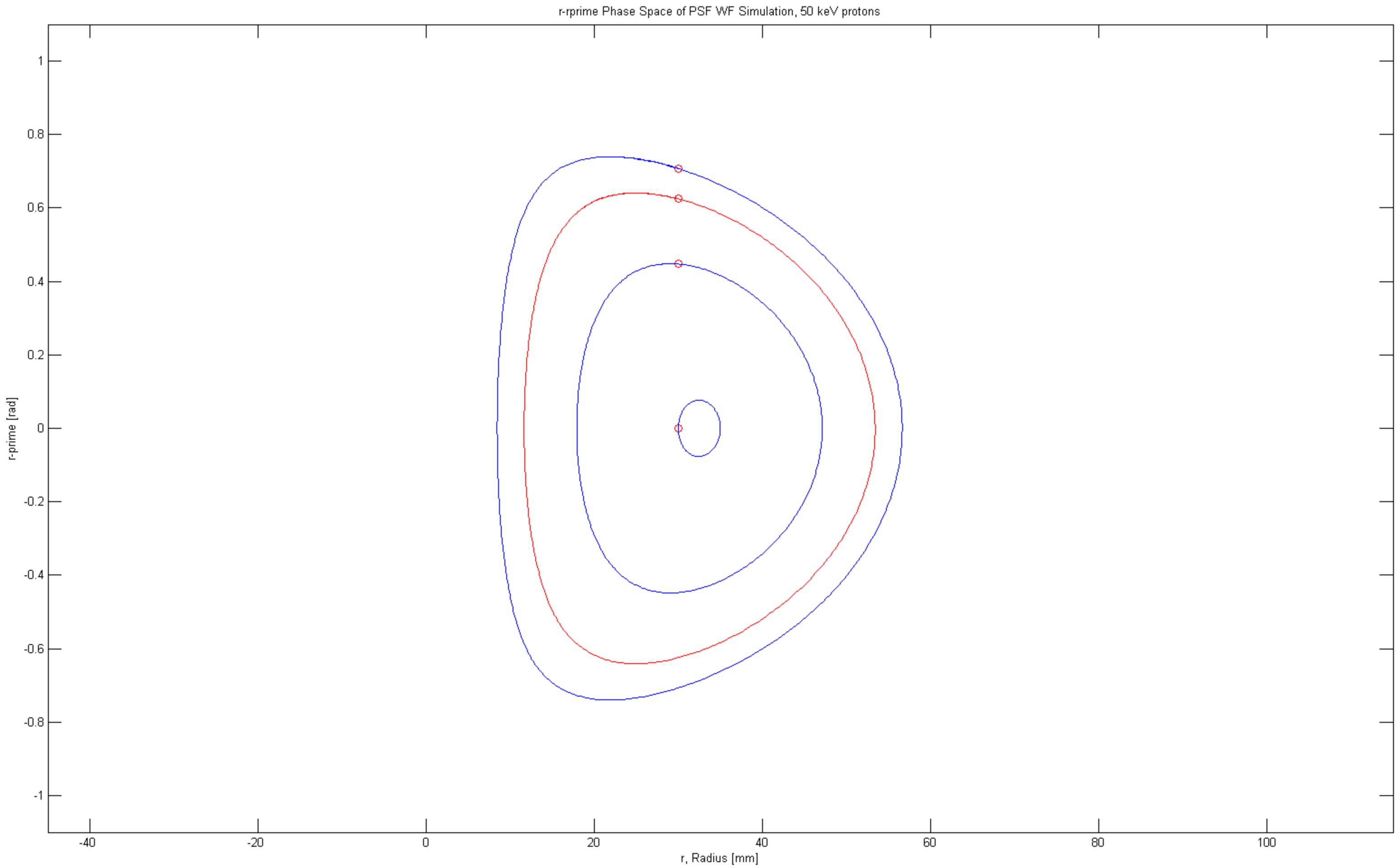
-We’ll identify the Equilibrium Orbits

-Launch a vertical distribution on the equilibrium orbit to find the vertical acceptance

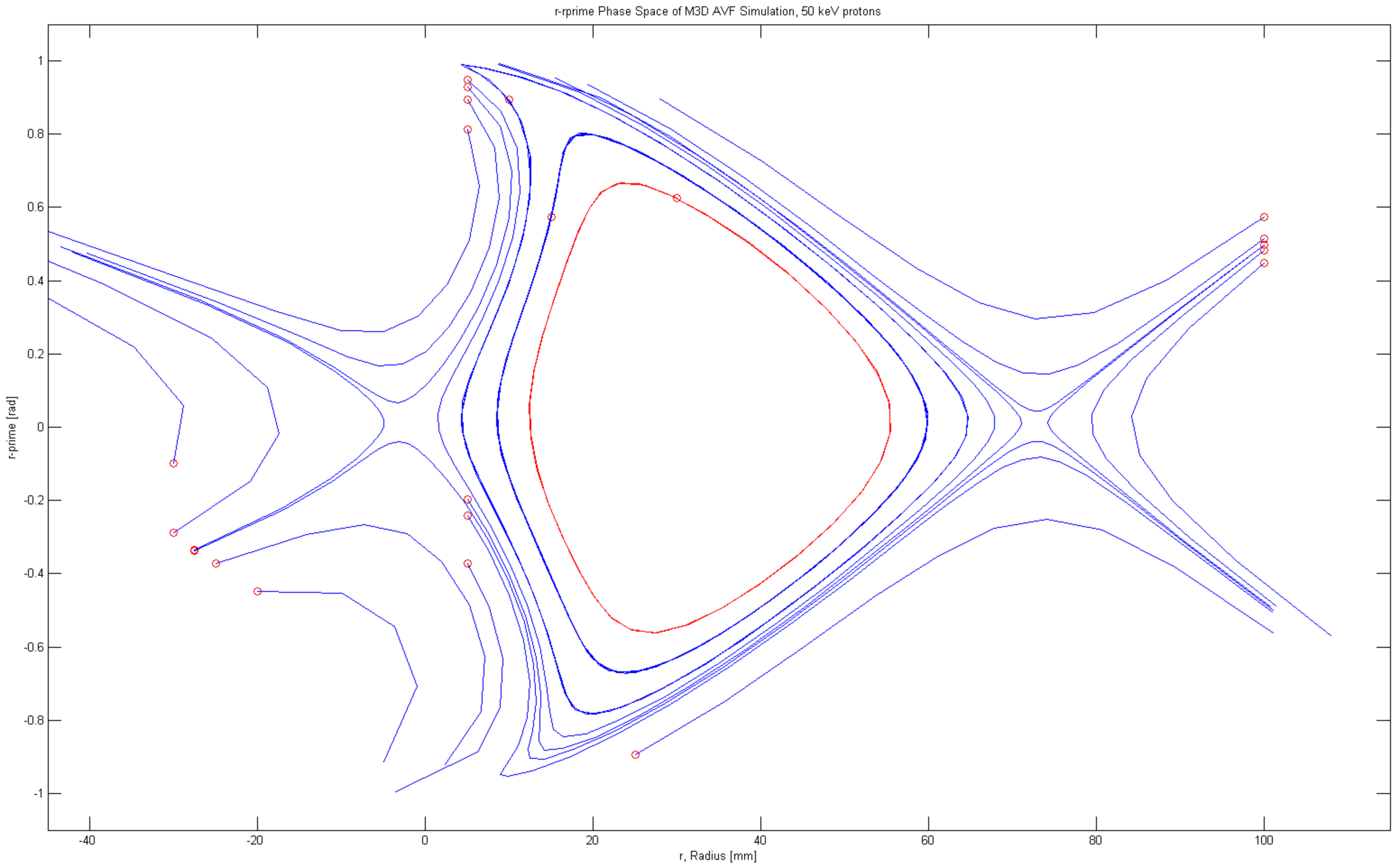
-Circles identify starting location indicating direction of flow

-Finally, in all three cases a single particle’s (with identical initial conditions) trace-space is identified by a red orbit, aiding the comparison.

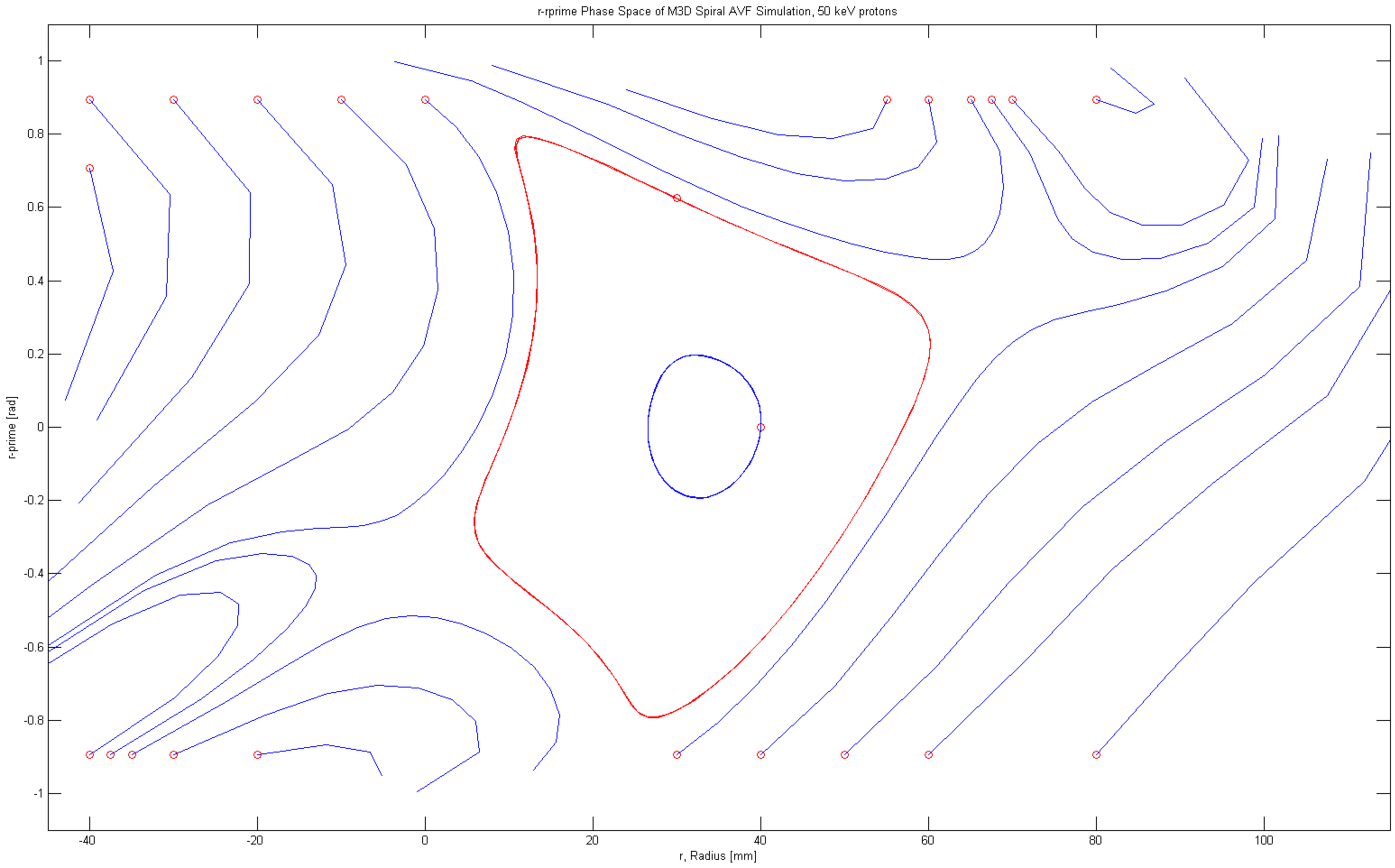
Weak Focusing Radial Phase Space



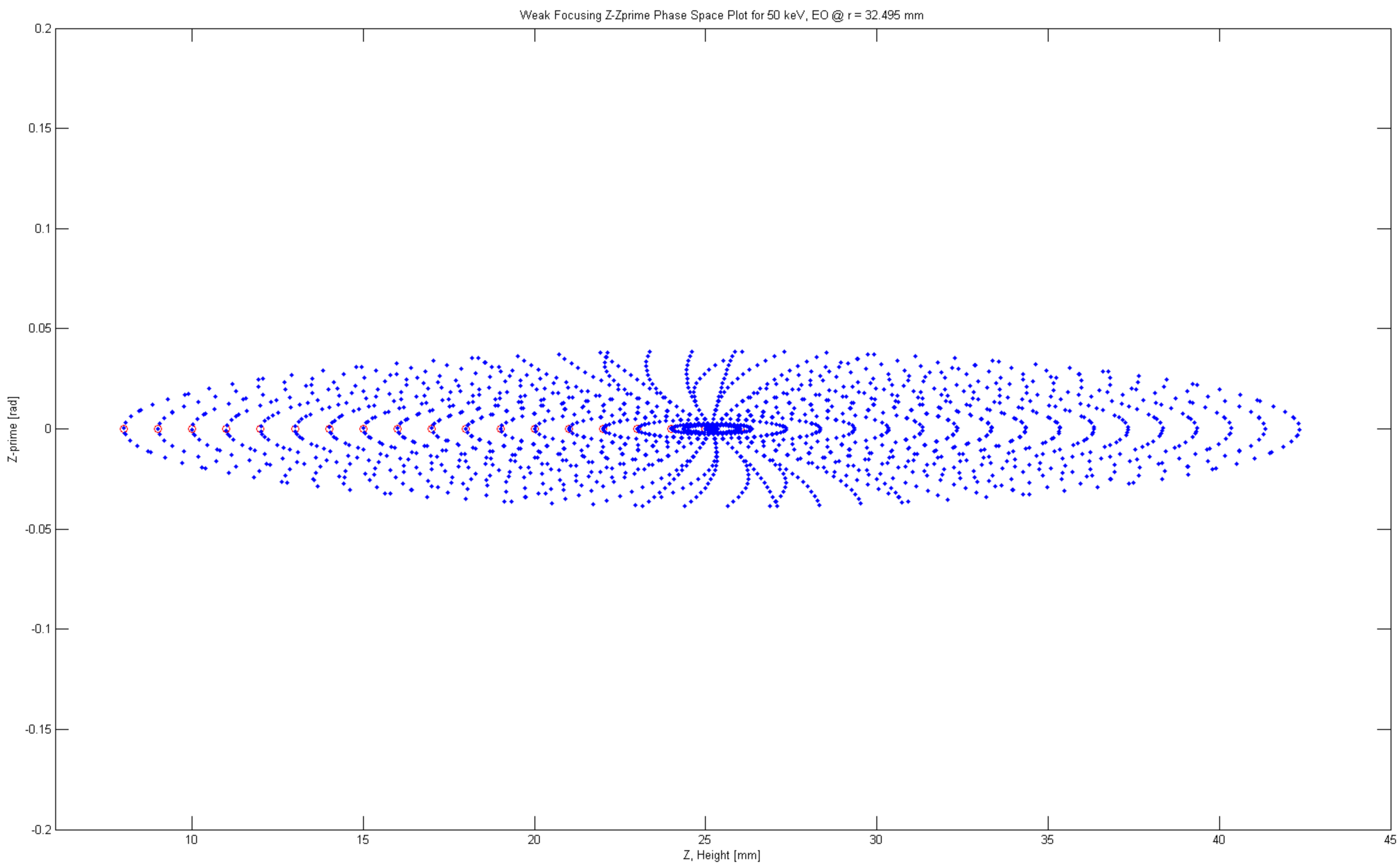
AVF Radial Phase Space



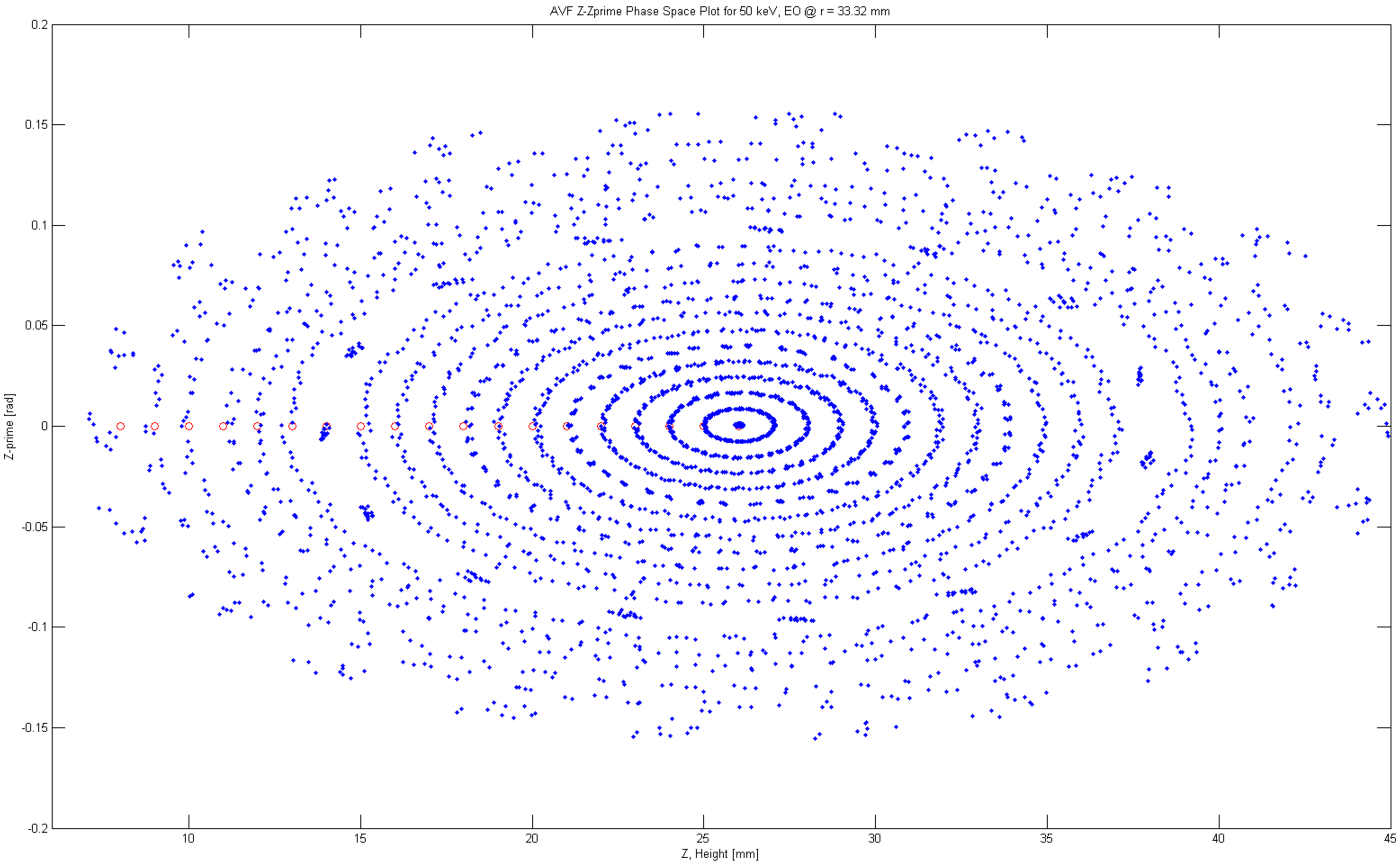
Spiral AVF Radial Phase Space



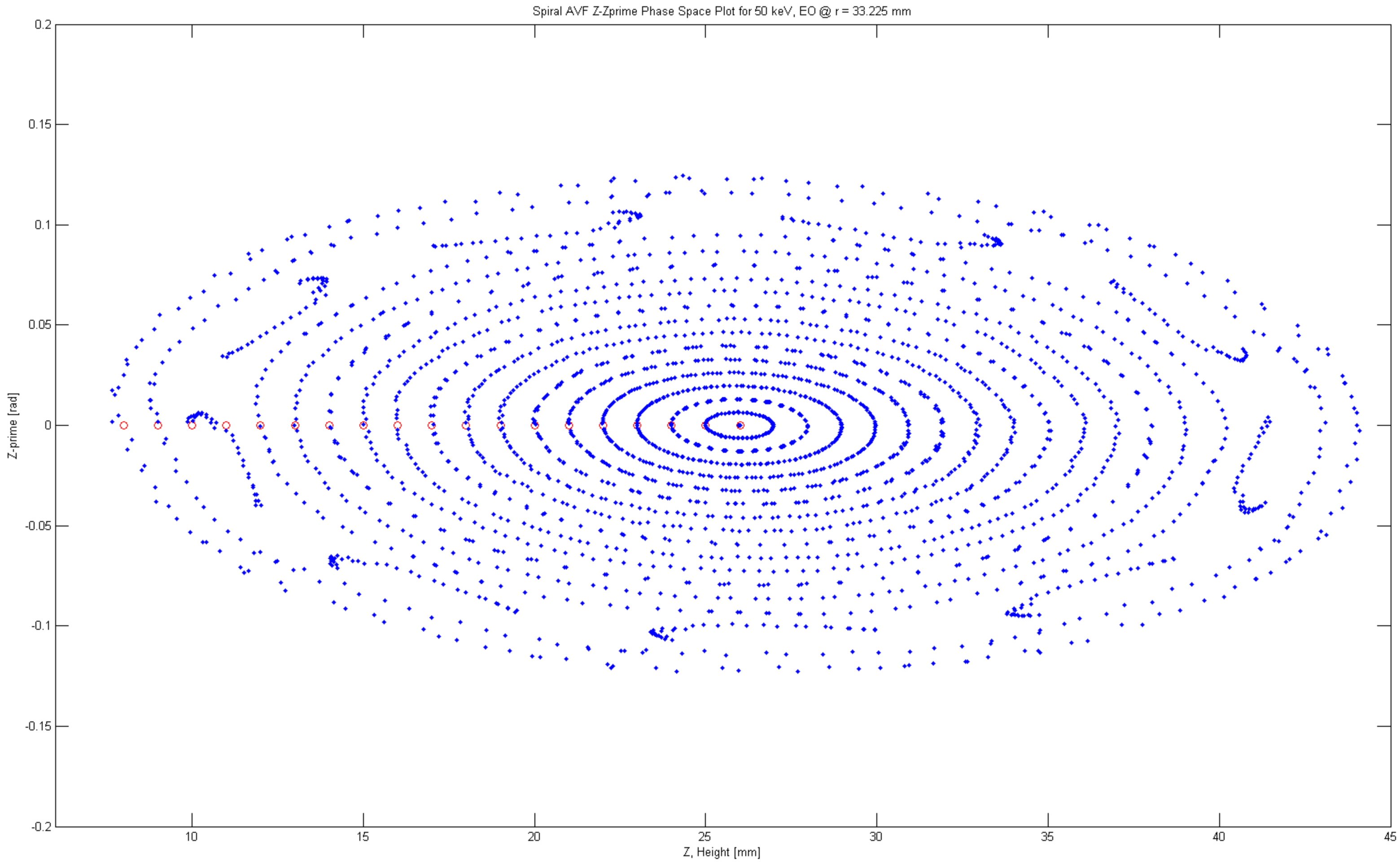
Weak Focusing Vertical Phase Space



AVF Vertical Phase Space

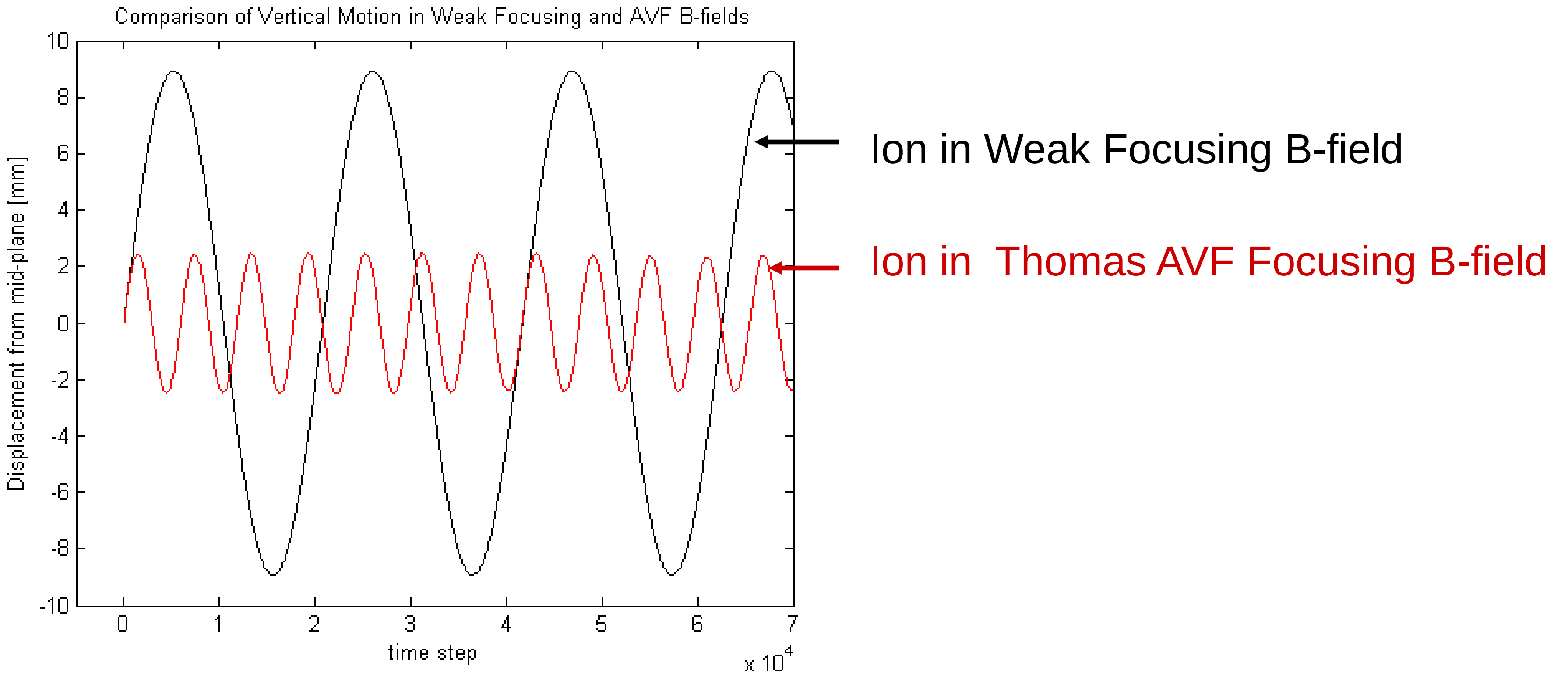


Spiral AVF Vertical Phase Space



What does this mean ?

Let's explain through an example. Two identical 50 keV ions are launched, one in the Weak Focusing field (black trace), the other in the AVF field (red trace) with the same angle (z'). We've recorded the vertical motion (z) as they circulate.



It is clear that the ions in the AVF field have a much smaller vertical excursion. This means the AVF field can accept a larger angular spread in ions, providing an overall greater beam intensity. Note that the frequency of oscillation is higher in the AVF field.

III-B Isochronous Cyclotron

The Isochronous cyclotron is a special case of the Spiral AVF configuration.

Raise the magnetic field with radius such that the relativistic mass increase is just cancelled.

- Make $B=\gamma B_o$ → magnetic field must increase with radius
- $\omega = qB/M = q\gamma B_o/\gamma M_o = qB_o/M_o$ = constant
- If the field increases with radius, magnetic structure must be different.

Use the energy-mass conservation of $E^2=p^2c^2+E_o^2$ and since $B_f=B_o(1+T_f/E_o)$ we can show that:

$$B = \frac{B_o}{\left[1-(Z/A)^2(r/a)^2\right]^{1/2}}$$

where $a=E_o/ecB_o$

To first order $B \sim B_o[1+(r/a)^2]$

Clearly $n < 0$, axially unstable !!!

Isochronous Cyclotron

A peculiar consequence of the isochronous cyclotron.

The momentum compaction factor have the value of $\alpha=1/\gamma^2$

Plug this into the relationship between revolution period change and momentum change we see that:

$$\frac{d\tau}{t} = \left(\frac{1}{\alpha} - \frac{1}{\gamma^2}\right)\frac{dp}{p} = 0$$

$d\tau/\tau=0$: the acceleration period is unrelated to the change in momentum and there is not phase stability !

- If the energy gain is wrong, there is no correction
- Isochronous cyclotrons must have a **designed** energy gain per turn
- Energy gain errors, such as RF-gap crossing time must be suppressed

The isochronous field and the energy gain per turn must match. The turn number is fixed as is the orbit shape for a given ion and final energy.

Isochronous Cyclotron

Define an average field index:

$$k = \frac{r}{\langle B \rangle} \frac{d\langle B \rangle}{dr} = -n$$

Flutter – the ‘hill’ to ‘valley ratio of the azimuthally varying field.

From Curl B = 0 in the magnet gap, we find that the azimuthal variation of B_z gives rise to a B_θ which results in a new axial force which is always restoring:

$$F_z = q v_r B_\theta$$

Isochronous Cyclotron

Pro:

- **CW beam (beam every RF cycle) – high average current**
- **Fixed RF frequency source**
- **Variable energy and ion species are possible**
- **small beam sizes and energy spread**

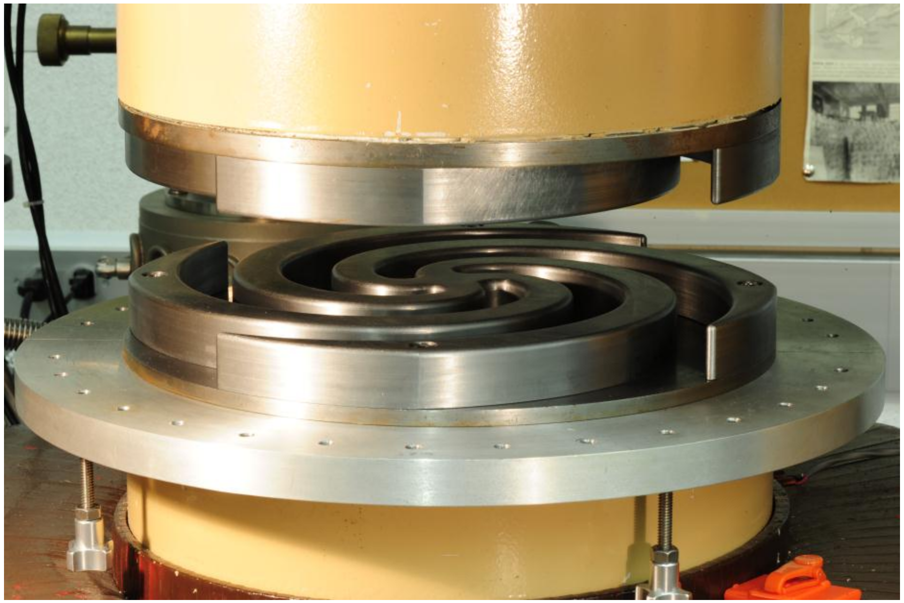
Con:

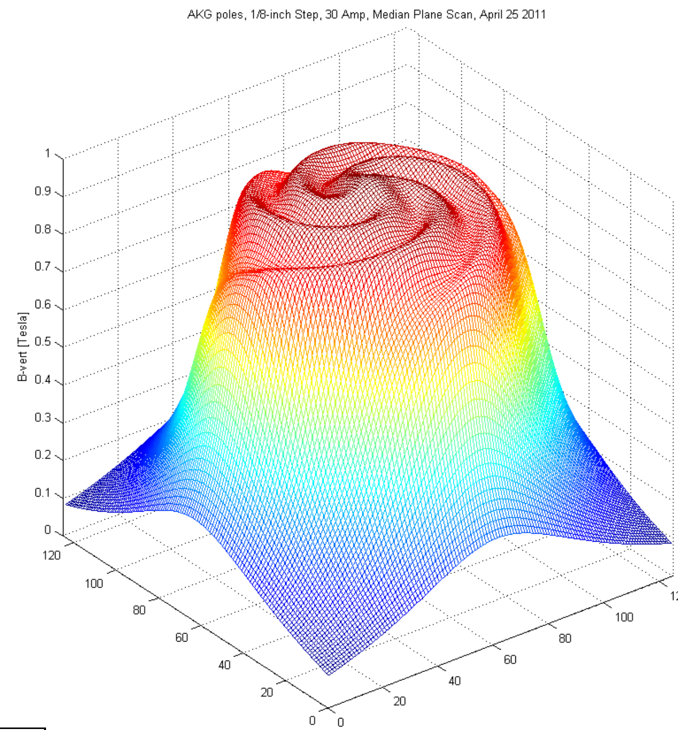
- High field precision required ! 1/10,000
- Complex magnetic field – requires substantial simulation
- More resonances

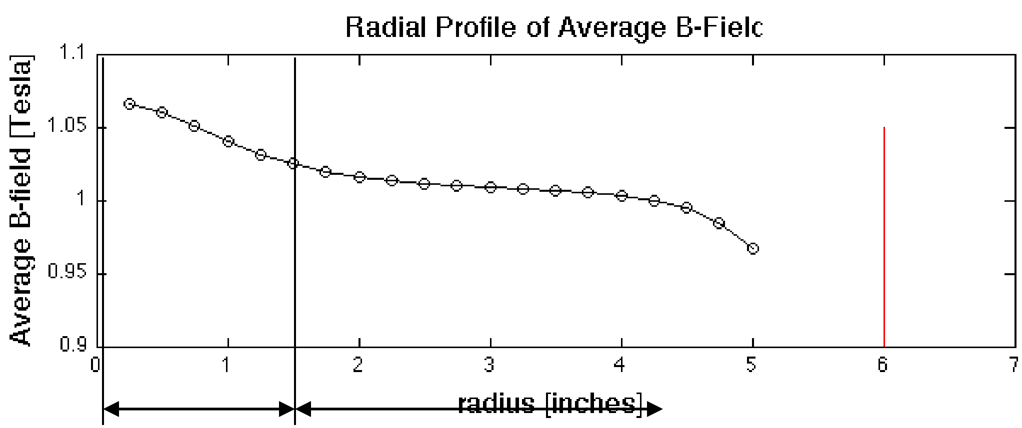
Interesting:

- “other” non-central orbits

An Isochronous Attempt



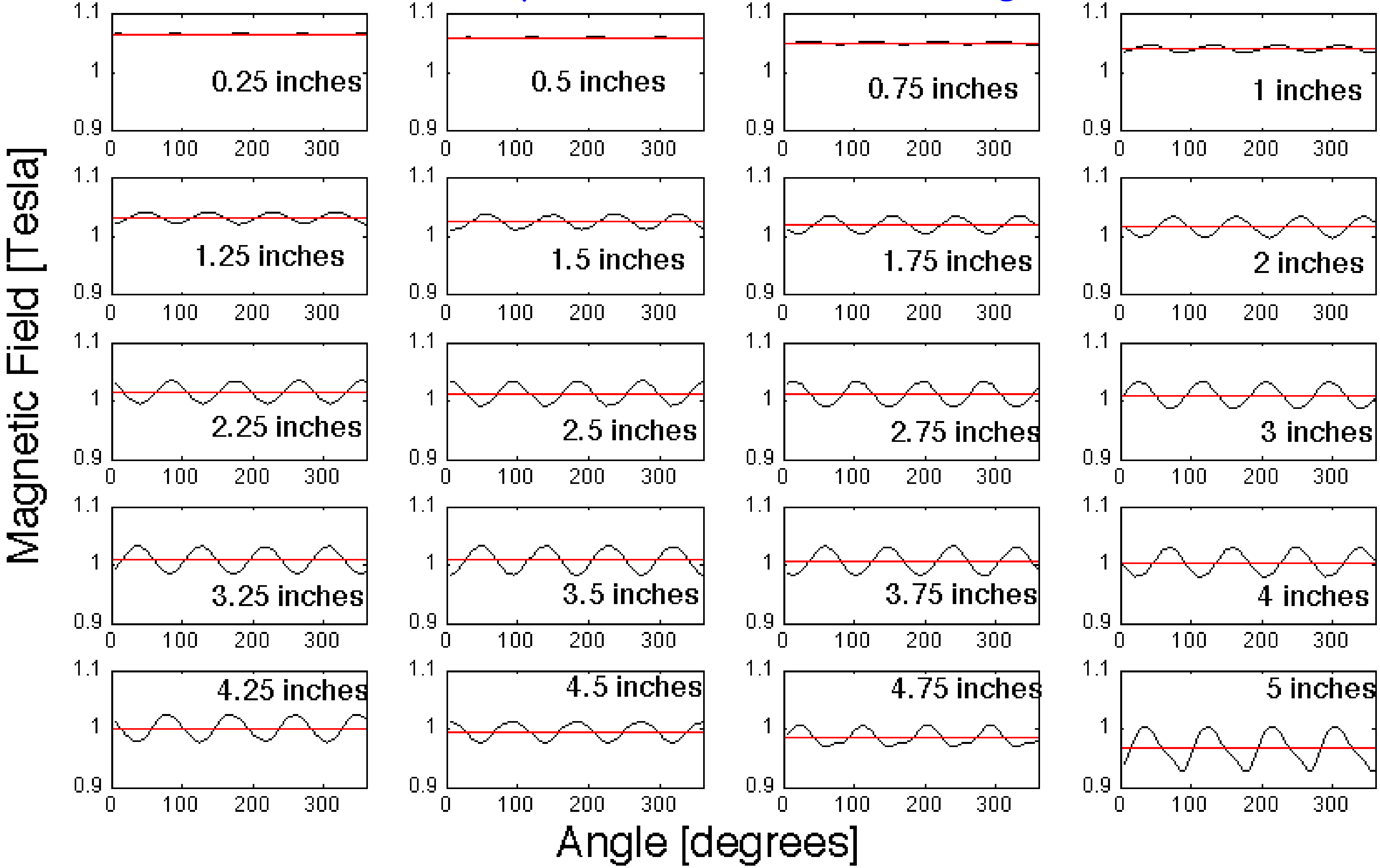




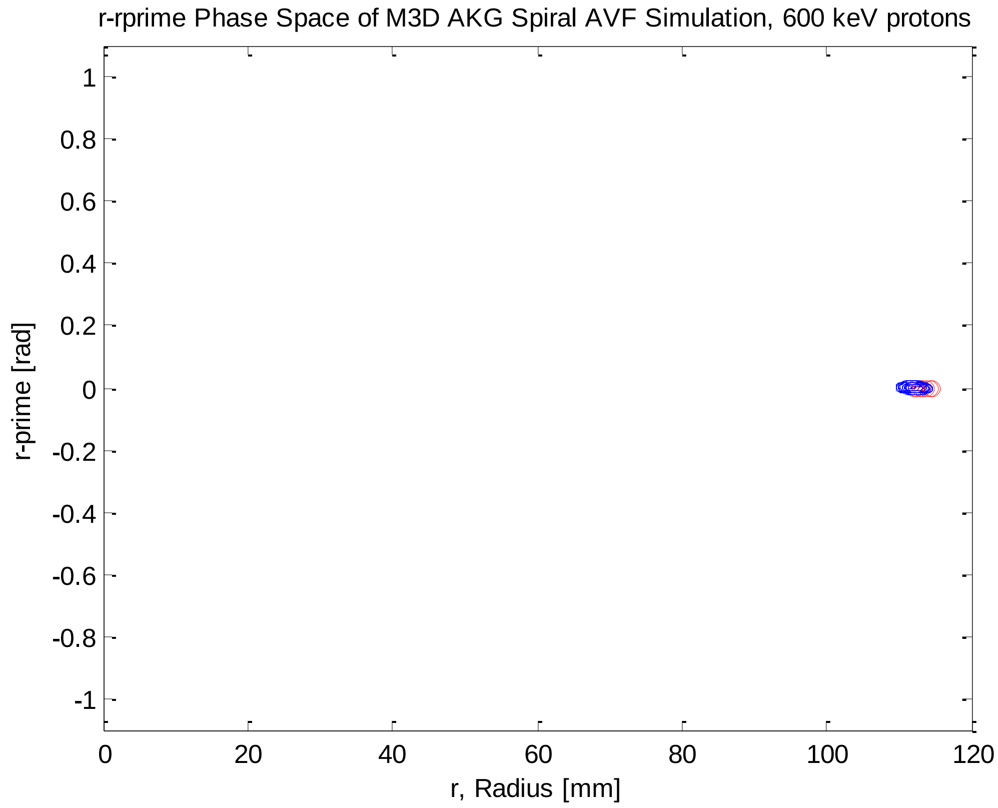
Weak Focusing AVF Spiral Focsing → <B> = constant

AVF Profile with Increasing Radii

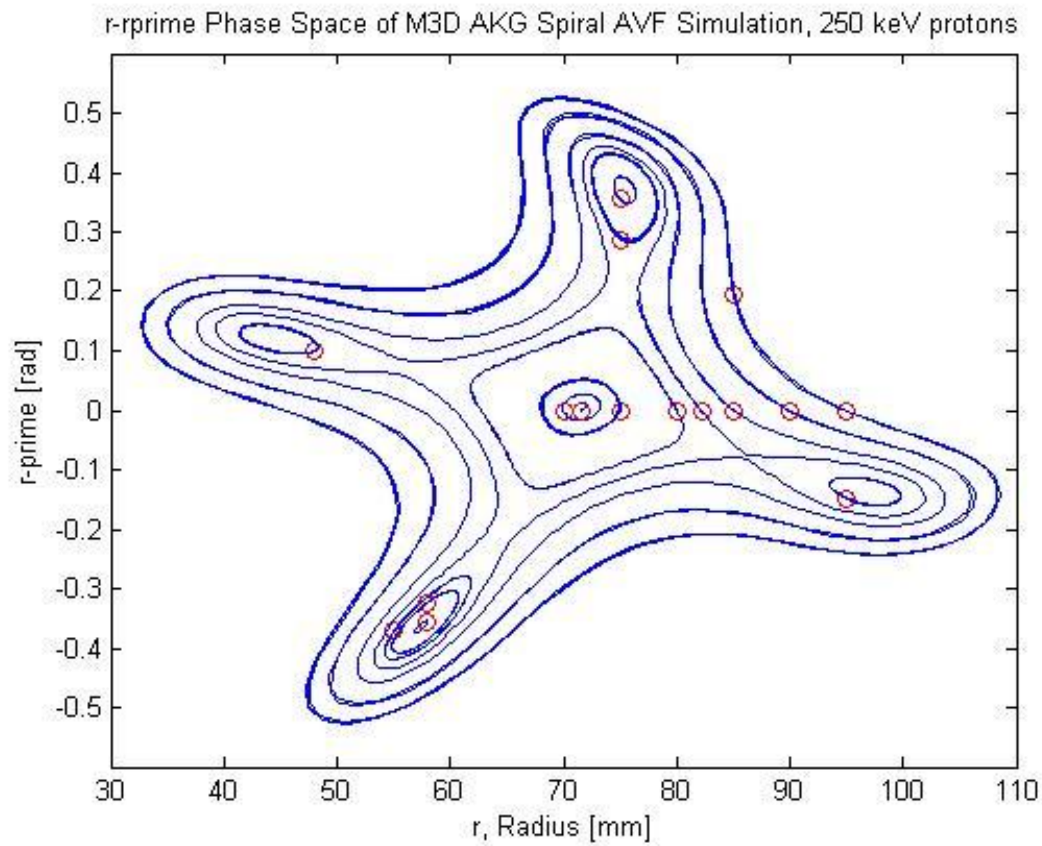
NOTE: very little flutter in central region.



AVF Field Radial Stability



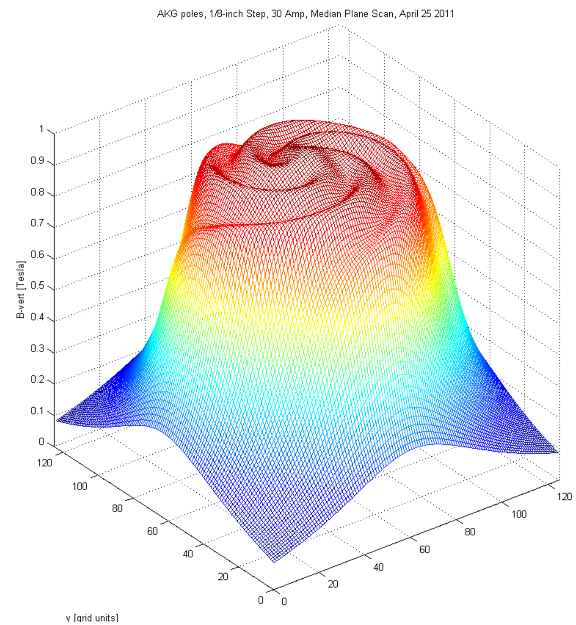
What is this at 250 keV ?



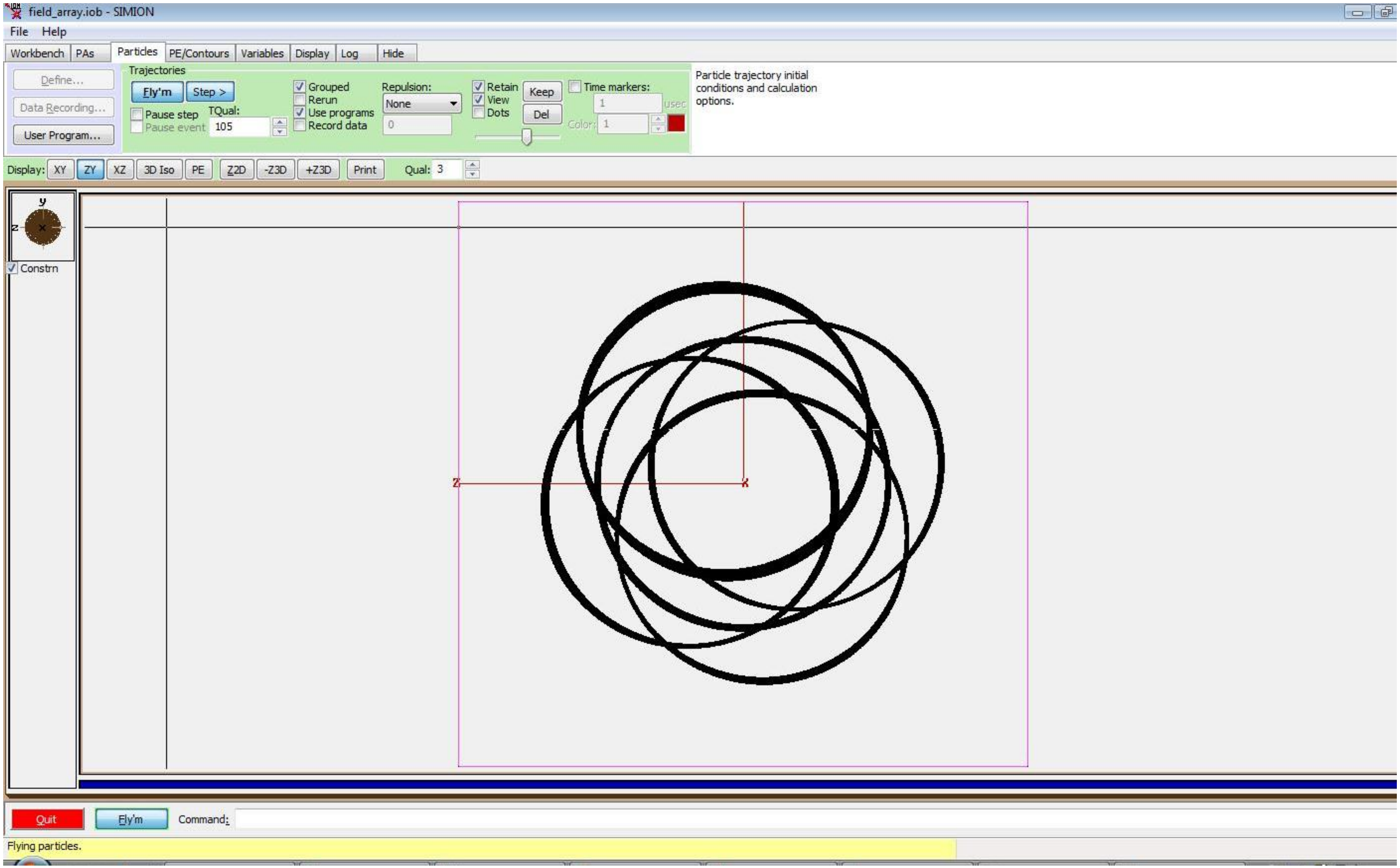
AVF makes the EO complicated

- We know that the orbits are no longer circular
- As a result of AVF many more orbits are allowed
- Lets generalize the orbit properties
  - Take ***v=p/m*** to be the average velocity of an ion orbit having momentum ***p*** and energy ***E***
  - Take τ to be the revolution period
  - Then define the EO properties as
    - EO circumference $C=vt$ also $C = \oint ds$
    - EO equivalent radius: $R=C/2\pi$
    - Cyclotron frequency $\omega=v/R$
    - Average B_z:

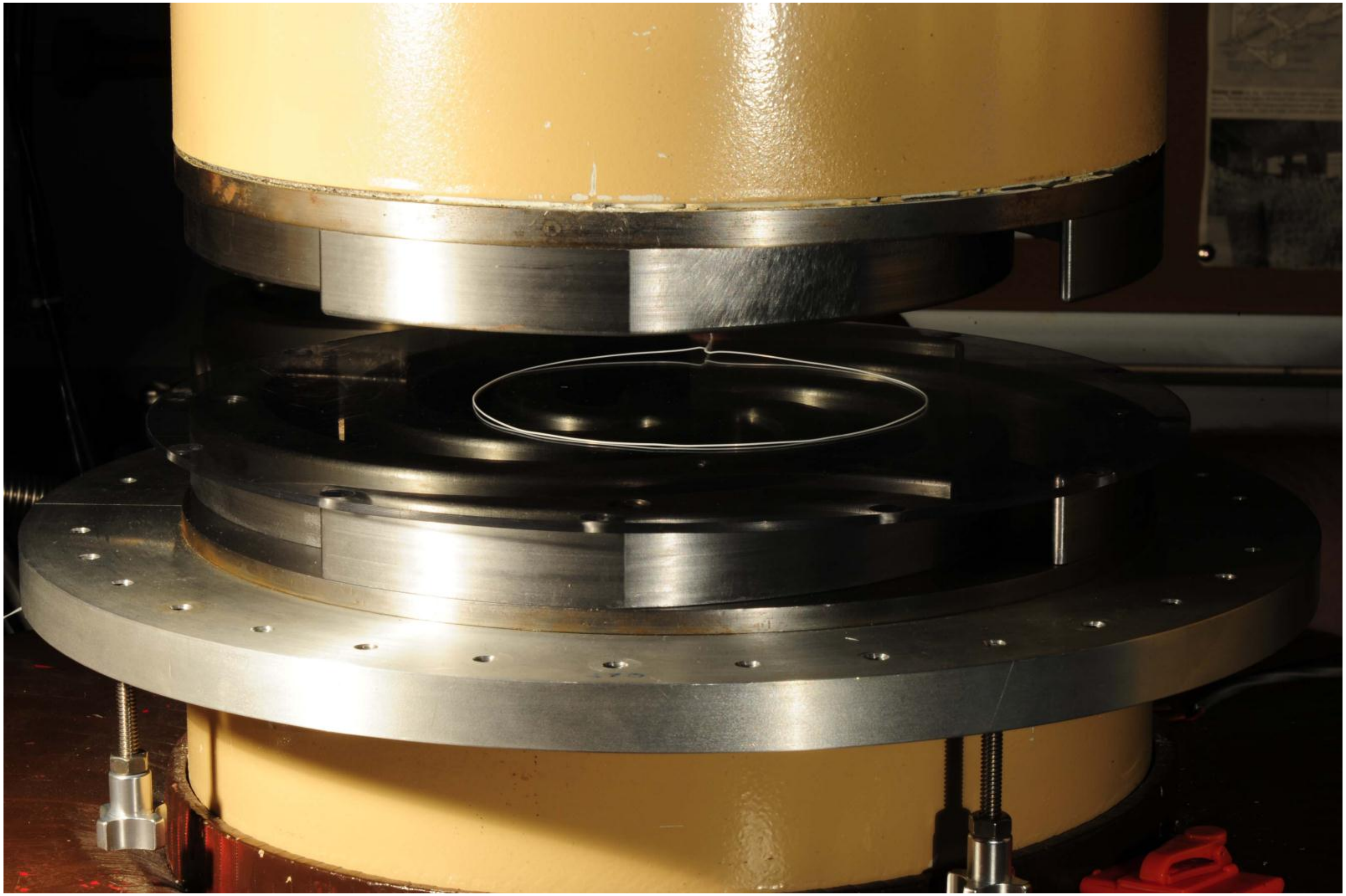
$$\langle B_Z \rangle = \frac{1}{C}\oint B_z ds = \frac{m\omega}{Cq}$$

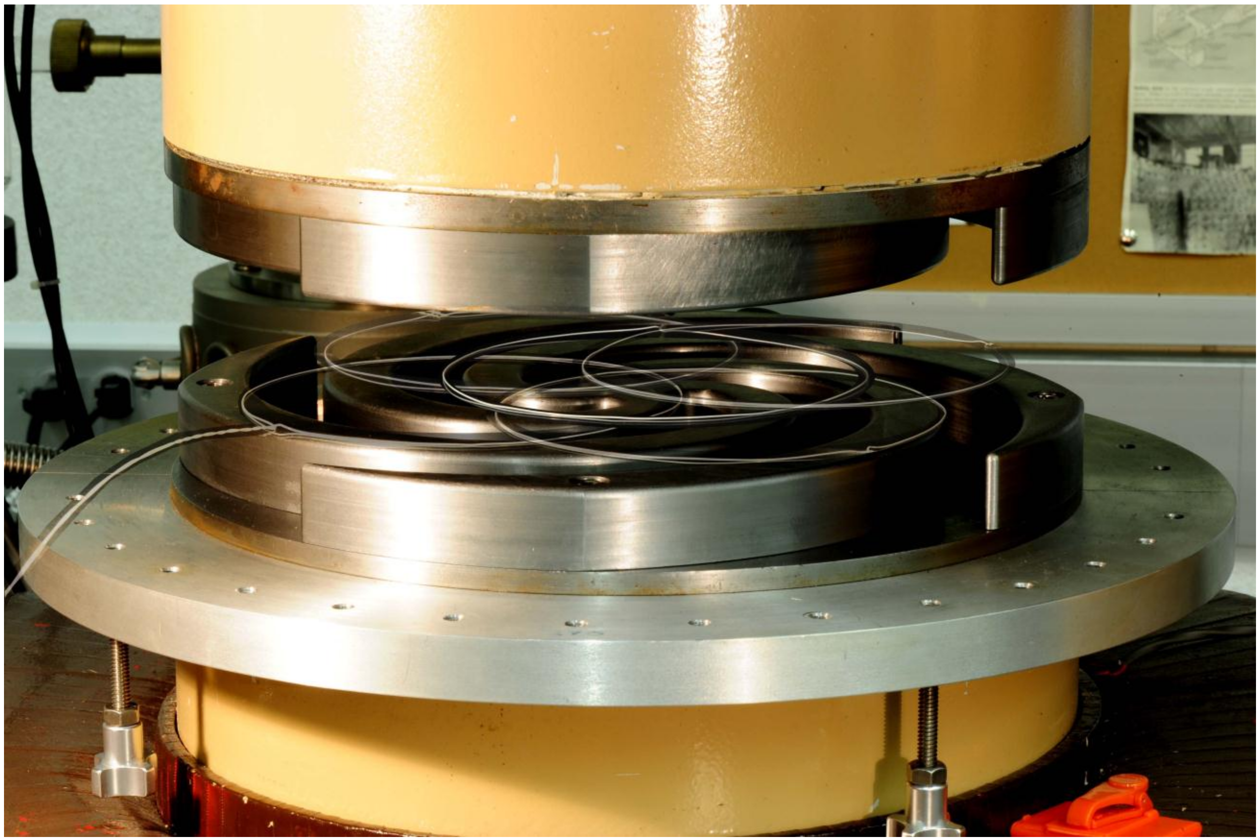


FIVE STABLE 250 KEV ORBITS!

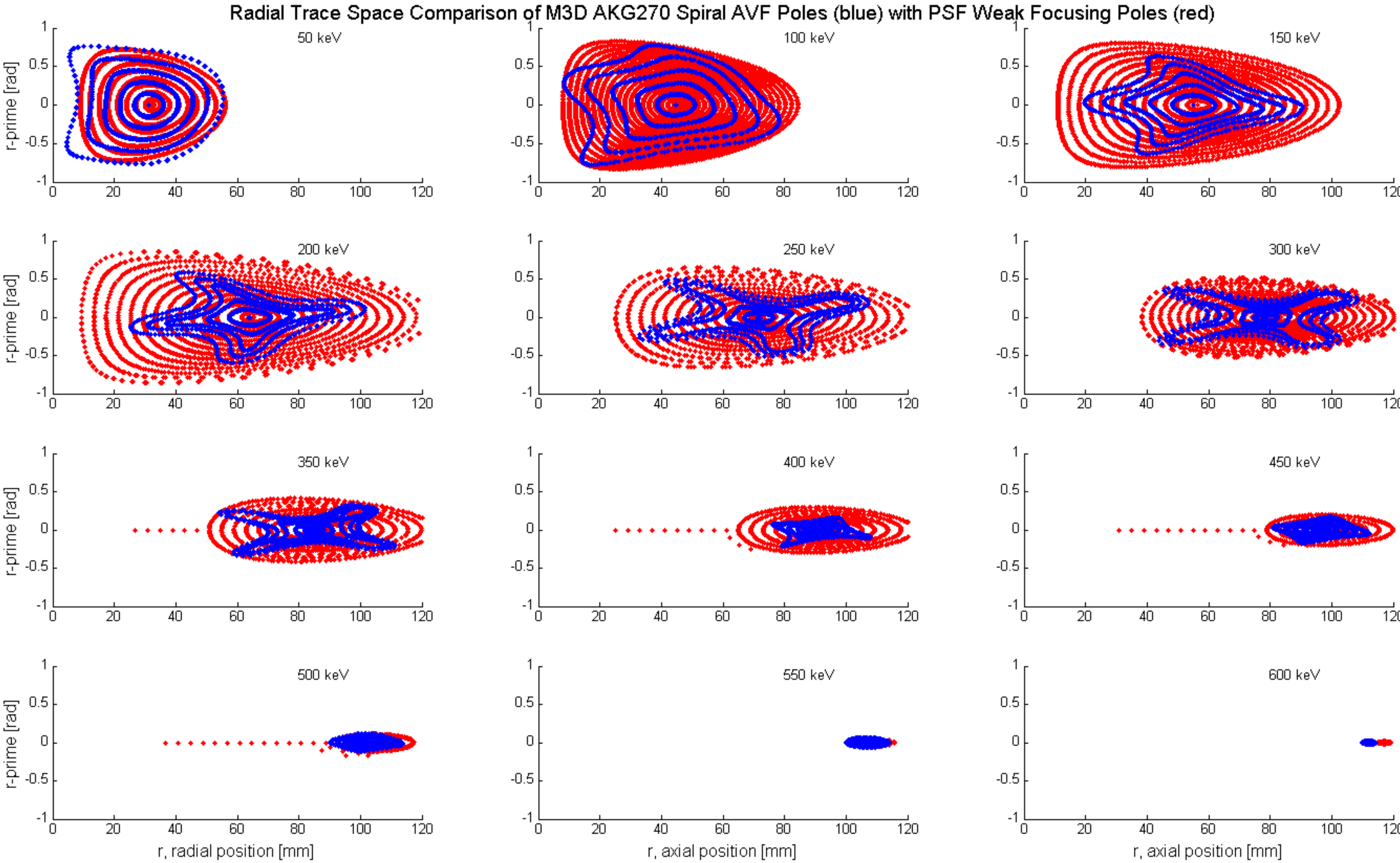


Found Five Stable Orbits

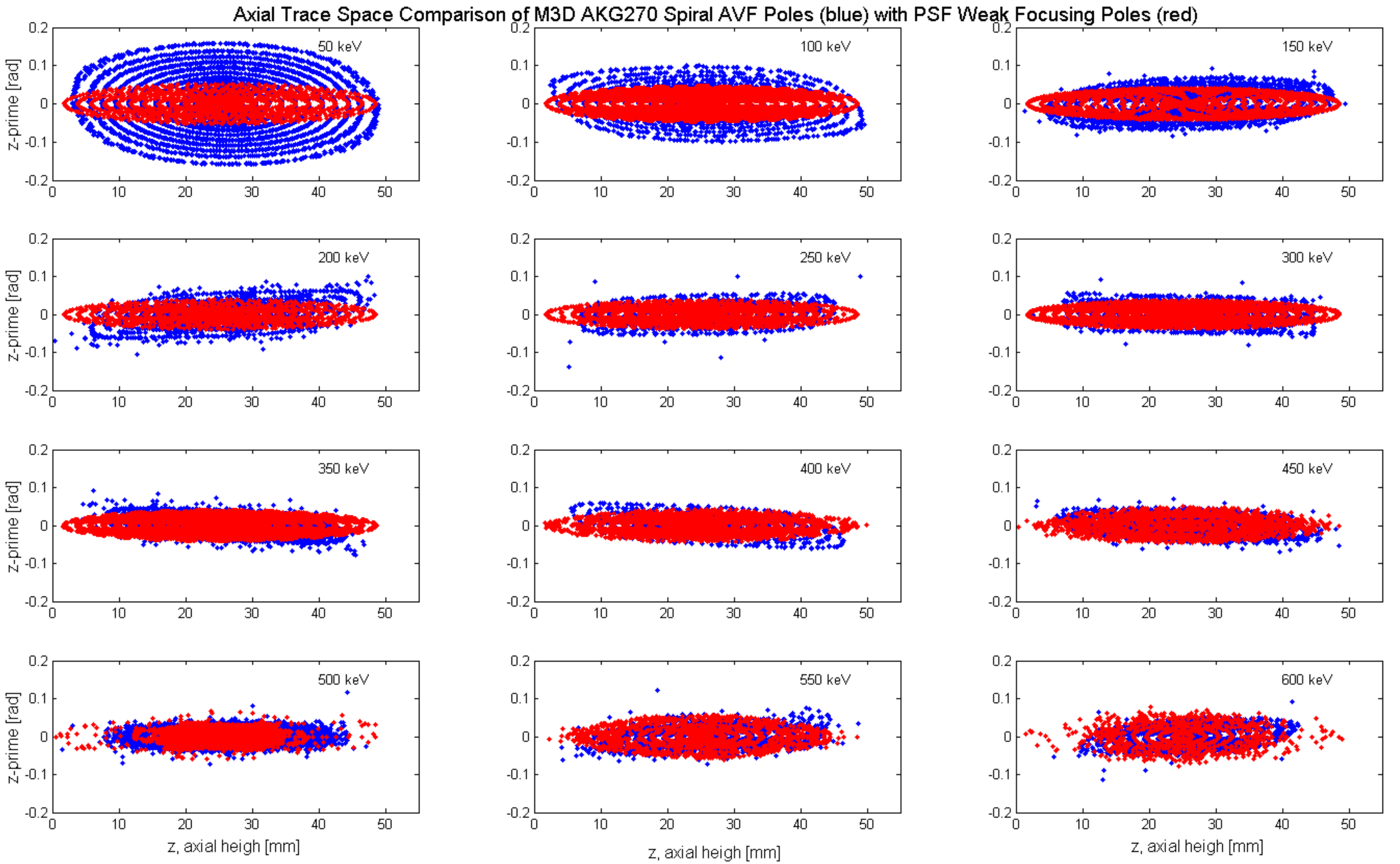




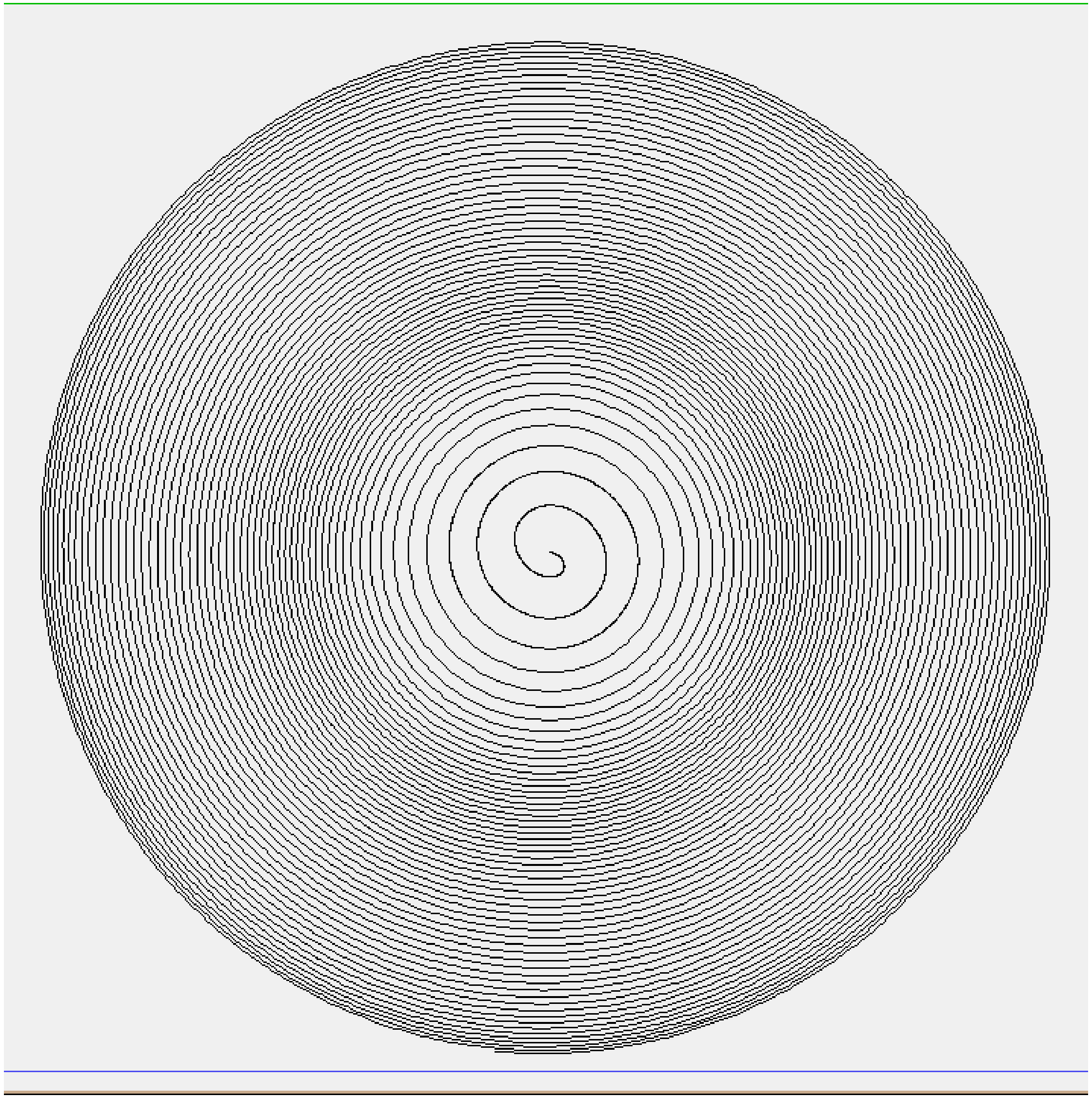
Spiral AVF vs WF (Radial)



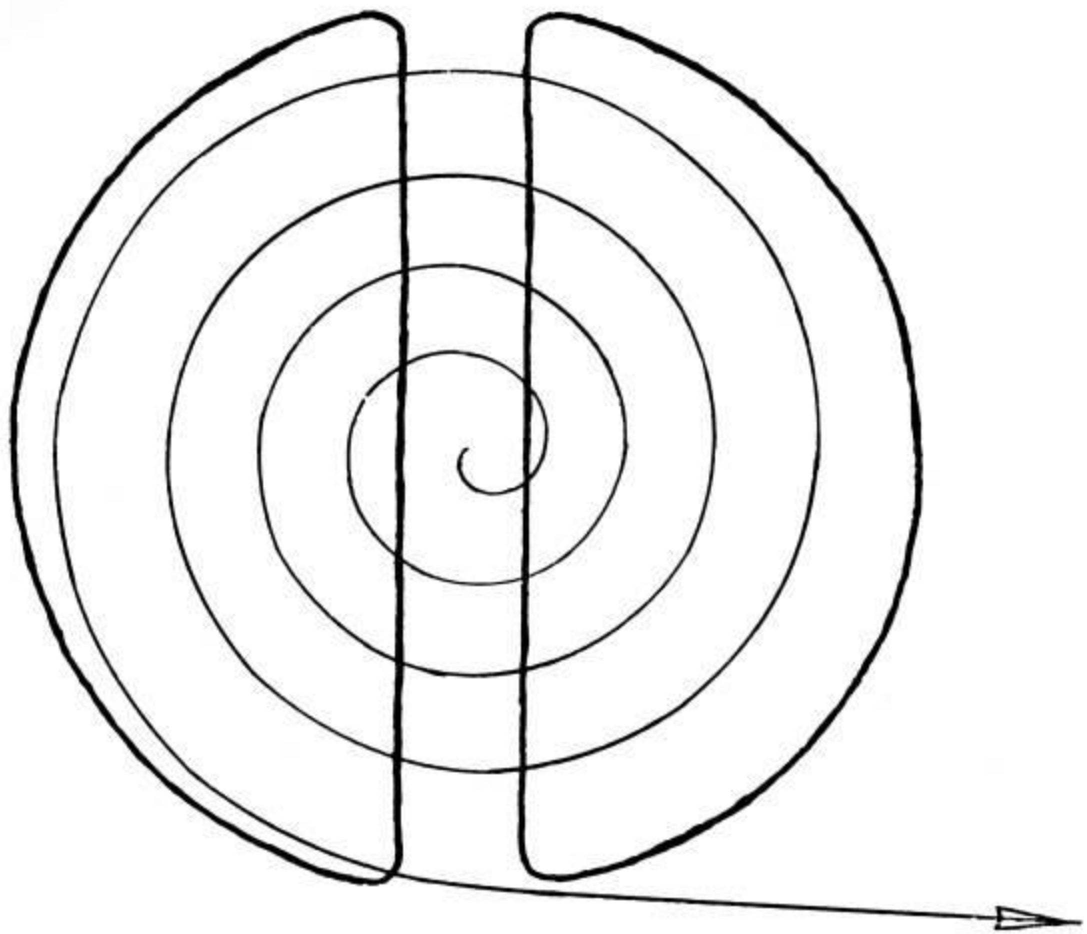
Spiral AVF vs WF (Axial)



Protons flown in SIMION



The cyclotron as seen by the…



. . . the student