NATIONAL ACCELERATOR LABORATORY

Sources and Properties of Synchrotron Radiation

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Beam Diagnostics Using Synchrotron Radiation:
Theory and Practice
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Power Radiated by a Charge in a Dipole

- A relativistic charge $q = Ze$ with mass $m = \mu m_e$, velocity $\beta c \approx c$, and energy $E = eE_{\mathrm{eV}} = \gamma mc^2$ travels through a dipole with a uniform field B_d.
  - Radius of curvature of the orbit: $\rho = \dfrac{\gamma\beta mc}{qB_d} \approx \dfrac{E_{\mathrm{eV}}}{ZcB_d}$
    - In the LHC, lead ions will sometimes be used in place of 7-TeV protons. Since ρ and B_d are unchanged, the maximum ion energy must be $Z = 82$ times higher—574 TeV, or 100 μJ *per ion*!
  - Power emitted in synchrotron radiation: $P_s = \dfrac{2}{3} Z^2 W_0 \gamma^4 \dfrac{c^2}{\rho^2}$
    - We define: $W_0 = \dfrac{e^2}{4\pi\varepsilon_0 c} = r_e m_e c = 7.6956 \times 10^{-37}\ \mathrm{J \cdot s}$
    - $r_e = 2.818$ fm is called the “classical electron radius”
    - The factor of γ^4 can make the radiated power substantial—except in the LHC, where $\rho = 6$ km.

Critical Energy E_c in the Dipole

- The “critical energy” for the charge in the dipole is:

$$E_c = \hbar\omega_c = \frac{3}{2}\gamma^3\hbar\frac{c}{\rho}$$

  - ω_c is the revolution frequency c/ρ, but scaled by γ^3.
- Half the total energy is below E_c, half above.
- For electrons (and positrons), the factor of γ^3 moves ω_c from the revolution frequency (~1 MHz) to x-rays.
  - Almost all power is emitted as hard photons.
  - Visible light is far into the tail.

Dipole: Angular Spectral Energy Density

- The charge passing through the magnet emits energy:

$$W_d = Z^2 \frac{3}{4\pi^2} W_0 \left(\frac{\omega}{\omega_c}\right)^2 \left[\left(1+\psi^2\right)^2 K_{2/3}^2(\zeta) + \left(1+\psi^2\right)\psi^2 K_{1/3}^2(\zeta)\right] d\omega\, d\xi\, d\psi$$

  - $d\omega$ is the frequency interval.
  - $d\xi d\psi = d\Omega / \gamma^2$ is the normalized solid angle:
    - ξ and ψ are the horizontal and vertical angles respectively, both divided by the characteristic $1/\gamma$ angle of the radiation.
      - Note that Hofmann does not normalize these angles.
  - $K_{2/3}$ and $K_{1/3}$ are modified Bessel functions, with argument:

$$\zeta = \frac{\omega}{2\omega_c}\left(1+\psi^2\right)^{3/2}$$

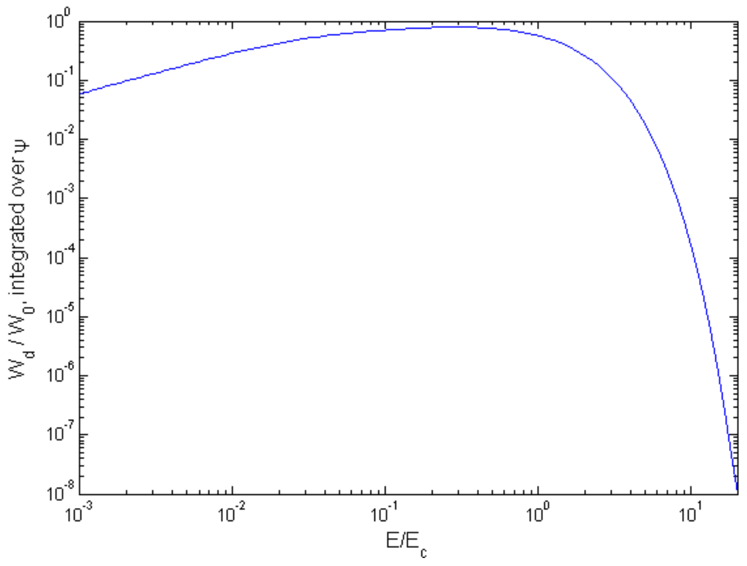
  - The Bessel terms describe the two polarization components:
    - $K_{2/3}$: In the plane of the bend (usually the horizontal plane)
    - $K_{1/3}$: Perpendicular to the bend plane (and so usually vertical)

Synchrotron Power in Various Rings

Ring	Particle Type	Beam Energy	Normalized Energy	Beam Current	Ring Circum-ference	Dipoles		Synchrotron Radiation		
						Radius of Curvature	Critical Wave-length	Power in Dipoles	Average Power	Per Particle Per Turn
		E	$\gamma = \frac{E}{mc^2}$	I	C	ρ	λ_c	$\frac{I}{ec}P_s$	$2\pi\rho\frac{I}{ec}P_s$	$\frac{2\pi\rho}{c}P_s$
		[GeV]		[A]	[m]	[m]	[nm]	[W/m]	[W]	[eV]
SPEAR-3	e^-	3.0	5,870	0.5	234	7.86	0.163	9,230	460,000	912,000
PEP-2 HER	e^-	9.0	17,610	2.0	2,200	165	0.127	6,790	7,040,000	3,520,000
PEP-2 LER	e^+	3.15	6,160	2.5	2,200	13.75	0.246	18,300	1,580,000	633,000
LHC	p	450	480	0.582	26,659	6,013	228,000	8.2E-07	0.0309	0.0531
LHC	p	7,000	7,460	0.582	26,659	6,013	60.7	0.048	1,810	3,110
LHC	$_{208}\mathrm{Pb}^{82+}$	574,000	2,960	0.0061	26,659	6,013	968	0.084	3,170	520,000

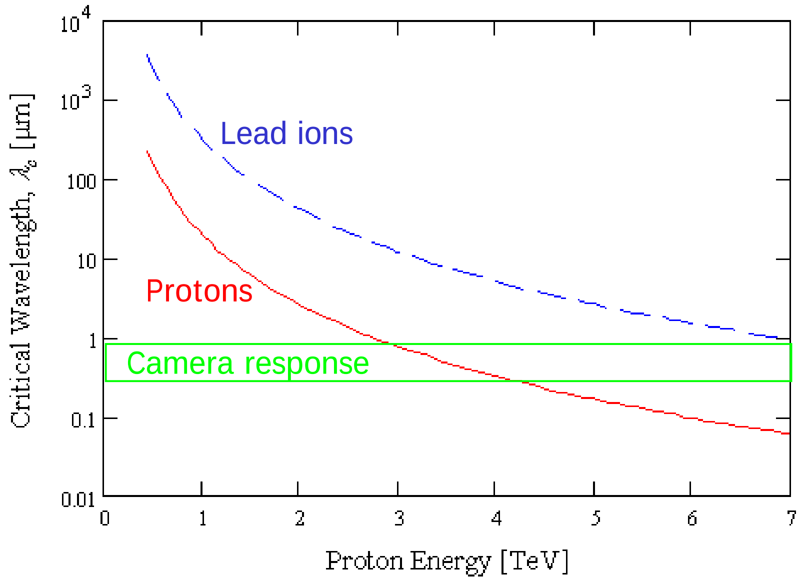
Spectrum and Critical Frequency

Spectrum near Critical Frequency



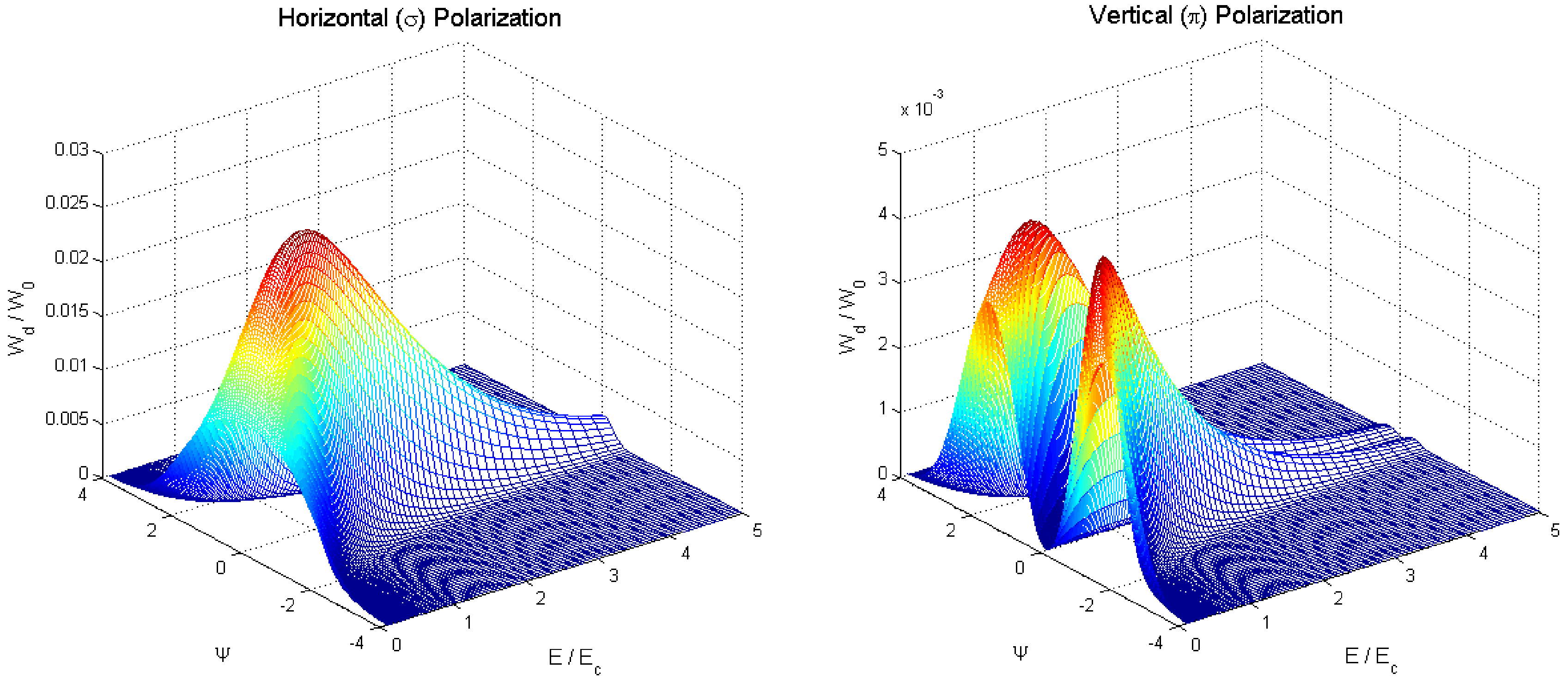
- Normalized dipole emission, intergrated over vertical angle ψ, versus energy E/E_c
- Long tail for $E < E_c$
  - Visible light << E_c for electron rings
- Rapid drop in emission for $E > 10E_c$
- Peak is below E_c

Critical Wavelength vs. LHC Beam Energy



- Cameras respond from near IR to near UV
- Proton emission wavelengths are too long to see below ~1.2 TeV
- Ion emission is too long below ~3 TeV
  - Ion energy given as "equivalent proton energy": Dipole set to the same field as for 3-TeV protons

Polarization

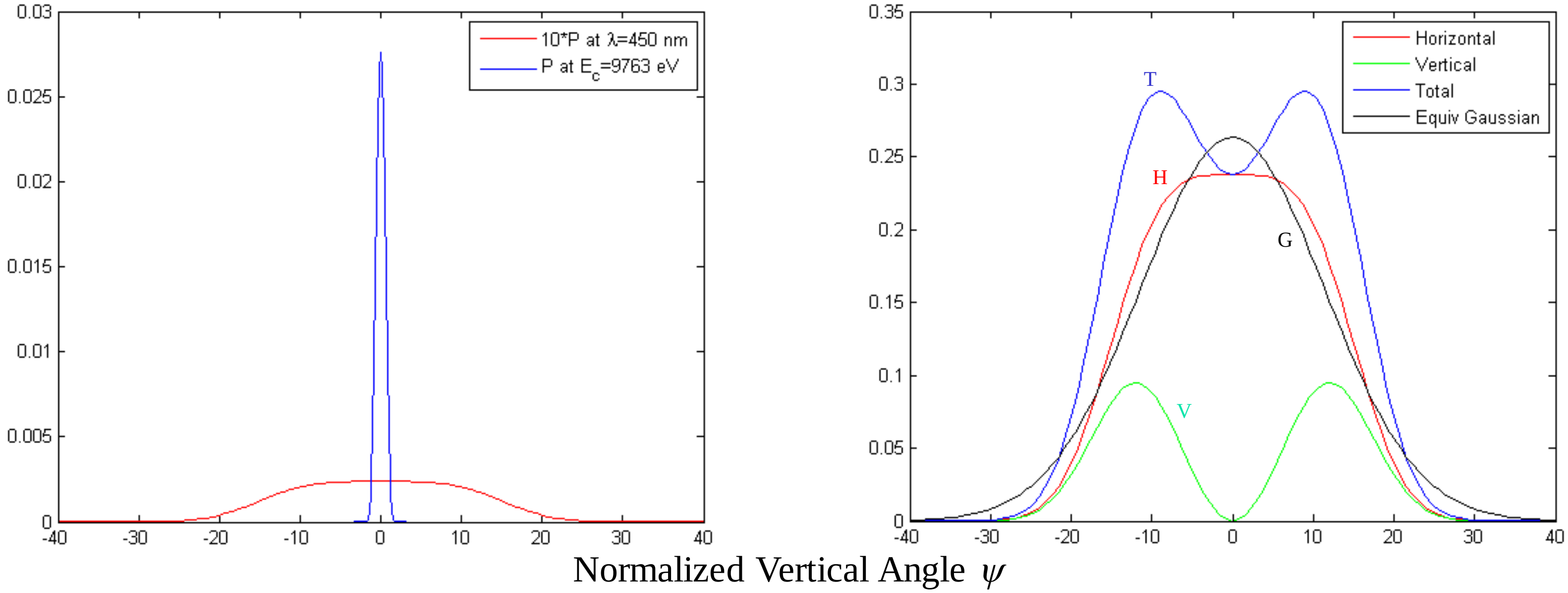


- The vertical component is zero on the midplane ($\psi = 0$).
  - Symmetry: No preference for upward versus downward field.
- 7 times more radiation is polarized horizontally than vertically.
- Low photon energies span a wider range of angles.

Visible Light versus Vertical Angle

Horizontally Polarized Power:
10×Power at 450 nm vs. Power at E_c

Horizontally and Vertically Polarized Power
at λ = 450 nm, and Best-Fit Gaussian



- Profile in visible light is much weaker and broader than with x rays.
- Profile of horizontally polarized light is roughly Gaussian, but with a flatter top and smaller tails.
- Plots here are for emission from the PEP-2 HER at λ = 450 nm.
- The area under the horizontal profile is ~3 times the area under the vertical.

Opening Angle of the Radiation

- Characteristic RMS angular size of the radiation:
  - A function of λ/λ_c
  - At the critical wavelength (x rays), the angular width is $0.6/\gamma$.
  - $1/\gamma$ is also typical of the full spectrum.
  - But for visible light (from electrons, with $\lambda >> \lambda_c$), the RMS width is almost *independent* of γ.

$$\sigma_\lambda = \frac{2.8}{\gamma}\sqrt{\frac{2}{\pi}}\cdot\begin{Bmatrix} \left(\dfrac{\lambda}{10\lambda_c}\right)^{0.354} & \text{for } \lambda > 10\lambda_c \\ \left(\dfrac{\lambda}{10\lambda_c}\right)^{0.549} & \text{for } \lambda < 10\lambda_c \end{Bmatrix} = 2.8\sqrt{\frac{2}{\pi}}\cdot\begin{Bmatrix} \gamma^{0.062}\left(\dfrac{3}{40\pi}\dfrac{\lambda}{\rho}\right)^{0.354} & \text{for } \lambda > 10\lambda_c \\ \gamma^{0.647}\left(\dfrac{3}{40\pi}\dfrac{\lambda}{\rho}\right)^{0.549} & \text{for } \lambda < 10\lambda_c \end{Bmatrix}$$

Radiation from a Long Dipole

- We've implicitly assumed that our distant observer receives the full $1/\gamma$ light cone of the particle as it bends through an angle θ in a constant dipole field.
  - The dipole must be long enough: $L_d = \rho\theta > \rho/\gamma$
- Some examples:

Ring	**Particle**	$\boldsymbol{E}$ [GeV]	$\boldsymbol{L_d}$ [m]	$\boldsymbol{\rho}$ [m]	$\boldsymbol{\rho/\gamma}$ [m]	**Comments**
PEP-2 LER	Positron	3.15	0.45	13.75	0.0022	Dipoles in $e^{\pm}$ rings are generally long
LHC	Proton	450	9.5	6013	12.5	Dipole emits no visible at this energy
		3000	9.5	6013	1.88	Begin to use light from dipole center
		7000	9.5	6013	0.81	
LHC	Pb Ion	7000	9.5	6013	2.03	

Short Dipoles and Edge Radiation

- If the dipole is short, the observer sees a faster “blip” of radiation, which pushes the spectrum to higher frequencies.
- Rapidly rising edge field of a (long) dipole has the same effect.

$$B(z) = \frac{B_d}{2}\left(1 + \frac{2}{\pi}\arctan\frac{z}{L_e}\right)$$

  - Edge radiation can be useful with protons, since the wavelengths from the central region of the dipole are often too long for the camera.

Edge: Angular Spectral Energy Density

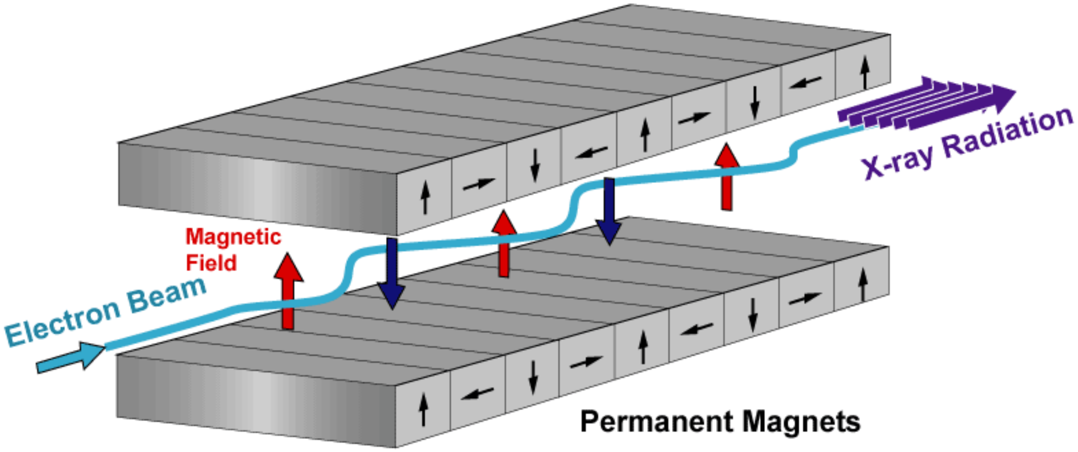
- A charge passing through a magnet edge emits energy:

$$W_e = Z^2 \frac{8}{9\pi^2} W_0 \left(\frac{\omega_c}{\omega} \right)^2 \frac{\left(1-\xi^2+\psi^2\right)^2 + \left(2\xi\psi\right)^2}{\left(1+\xi^2+\psi^2\right)^6} \exp\left[-\frac{\omega L_e}{\gamma^2 c} \left(1+\xi^2+\psi^2\right) \right] d\omega \, d\xi \, d\psi$$

- Note that this expression diverges at low frequencies. Why?
  - Time the observer receives the edge radiation: $(1/\beta - 1)L_e/c = L_e/(2\gamma^2 c)$
  - Characteristic time of radiation from the center of the magnet: $1/\omega_c$
  - To make a useful distinction between edge and central radiation, we assume that the edge radiation is faster: $L_e/(2\gamma^2 c) << 1/\omega_c$
    - But this is the same as the condition that the edge is short: $L_e << \rho/\gamma$
  - The derivation is only meaningful for $\omega >> \omega_c$
    - No divergence in the region of validity

Undulators

- Combine radiation from many consecutive bends.
  - Alternate bend directions to enter and exit on the same line.
- Periodic motion introduces new considerations:
  - A wavelength resonant with the “wiggle” motion.
  - The amplitude of the wiggle makes a difference.



- An undulator made with permanent magnets.
- Other designs use permanent magnets with iron poles (a “hybrid”), electromagnets, and superconductors.
- The light is horizontally polarized on the midplane.
- Helical undulators are also used. The light is circularly polarized on axis.

Sinusoidal Motion

- Sinusoidal vertical magetic field B_y, with period λ_u

$$B_y(z) = B_u \cos(k_u z)$$

$$\lambda_u = 2\pi / k_u$$

- Acceleration of charge *q*
  - *t'* is the time at *q*
  - The observer receives the radiation at a later time *t*
  - Significant because *q* moves toward the observer

$$\frac{d^2 x}{dt'^2} = -\frac{qB_u}{\gamma m}\frac{dz}{dt'}\cos\left[k_u z(t')\right]$$

- Velocity (*x* and *z*)

$$\frac{dx}{dt'} = -\frac{qB_u}{\gamma m k_u}\sin(k_u z) = -\frac{c}{\gamma}K_u \sin(k_u z)$$

$$\frac{dz}{dt'} = \sqrt{(\beta c)^2 - \left(\frac{dx}{dt'}\right)^2} = \beta c\sqrt{1 - \left(\frac{K_u}{\beta\gamma}\right)^2 \sin^2(k_u z)}$$

- Angle of wiggle to the *z* axis

$$\frac{dx}{dz} = \frac{dx/dt'}{dz/dt'} = -\frac{K_u}{\beta\gamma}\frac{\sin(k_u z)}{\sqrt{1 - \left[\frac{K_u}{\beta\gamma}\sin(k_u z)\right]^2}}$$

Undulator: $K_u < 1$

$$K_u = \frac{qB_u}{mck_u} = \frac{Z}{\mu}\cdot\frac{eB_u\lambda_u}{2\pi m_e c} = \frac{Z}{\mu}K_{ue} = 0.9337\frac{Z}{\mu}B_u[\mathrm{T}]\lambda_u[\mathrm{cm}]$$

- Since $\gamma >> 1$ and $\beta \approx 1$:
  - $dx/dt' << c$ and $dz/dt' \approx \beta c \approx c$
  - The maximum of the angle $dx/dz = K_u/(\beta\gamma) \approx K_u/\gamma << 1$
- K_u is the **ratio** of the **angle of the wiggle motion** to the **opening angle of the radiation**, $1/\gamma$.
  - As the particle passes through the undulator, the observer is always inside the radiation cone.
  - Each undulator pole (each half period) is a short magnet.
- Due to the large mass, protons or lead ions have $K_u << 1$

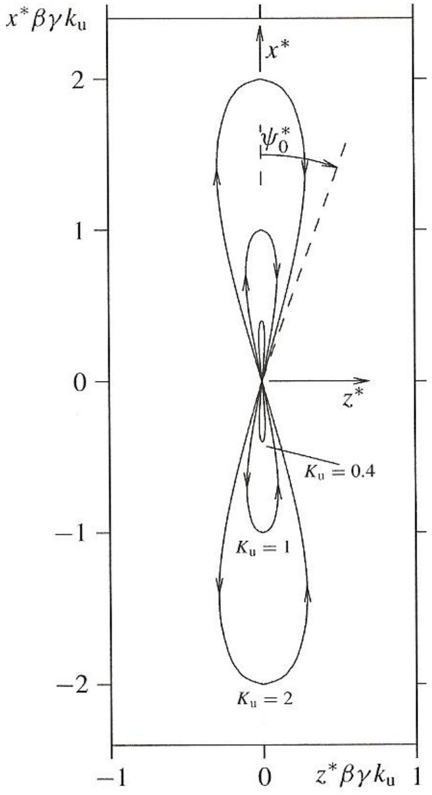
Wiggler: $K_u > 1$

- Angle remains small, but requires expanding to 2nd order in $K_u/\gamma << 1$.
- Can't neglect the modulation of the *z* velocity.
  - A magnetic field does not change the beam energy: γ and $\beta = \sqrt{1-1/\gamma^2}$ are still constants.
  - But this energy is shared between *x* and *z*.
  - The *z* motion has an average drift velocity $\beta^* c$:

$$\left\langle \frac{dz}{dt'} \right\rangle \equiv \beta^* c = \beta c \left(1 - \frac{K_u^2}{4\beta^2\gamma^2} \right) < \beta c$$

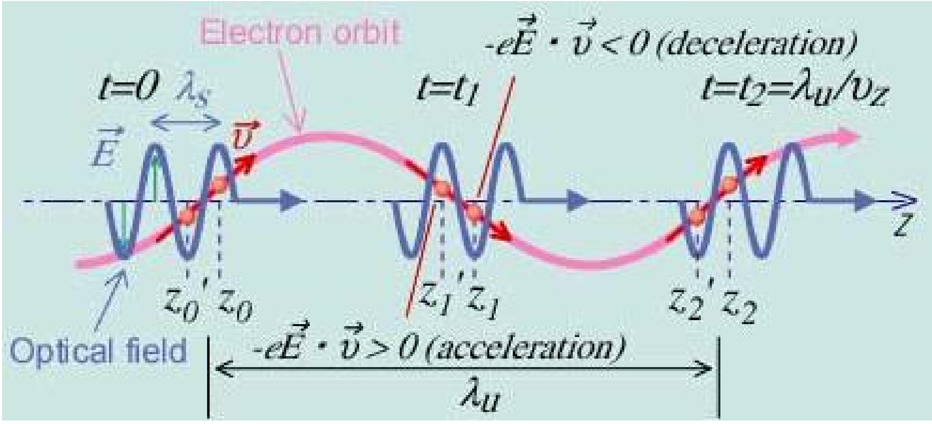
$$\gamma^* = \frac{1}{\sqrt{1-\beta^{*2}}} = \frac{\gamma^2}{1+K_u^2/2}$$

  - In a frame moving at this drift velocity, the charge moves in a figure 8.



Resonance Condition

- Charge moves along z axis by one undulator period in time $t = \lambda_u/(\beta^* c)$
- Wavefront of light at an angle $\theta \ll 1$ to z moves along z by $ct/\cos\theta > \lambda_u$
  - Charge slips behind phase of light
- At the resonant optical wavelength λ_1 (with n^{th} harmonic $\lambda_n = \lambda/n$):
  - Charge slips behind light by λ_1 per undulator period λ_u
  - Each period, kinetic energy is lost (gained): converted to (or taken from) light
  - Spontaneous emission from undulator has peaks at λ_n
  - In a free-electron laser (FEL), this gives gain at λ_1:
    - Charges form one bunch per wavelength λ_1, causing stronger emission at λ_1

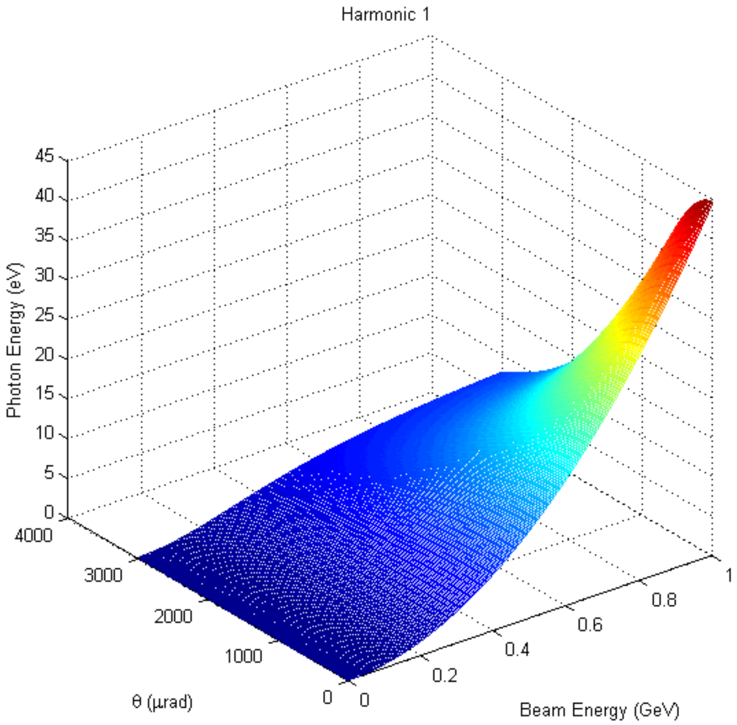


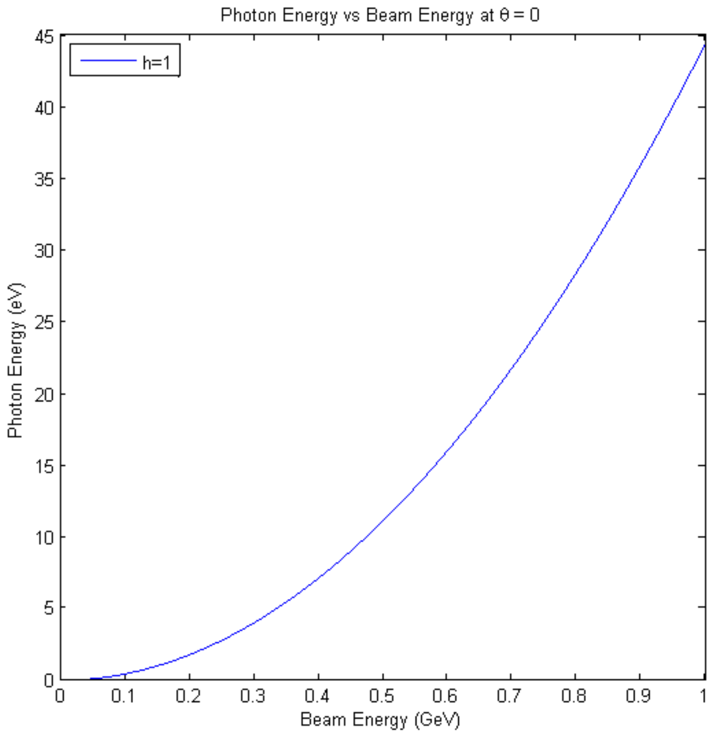
Slippage: $\dfrac{\lambda_u}{\beta^* \cos\theta} - \lambda_u = \lambda_1$

Gives resonance condition:

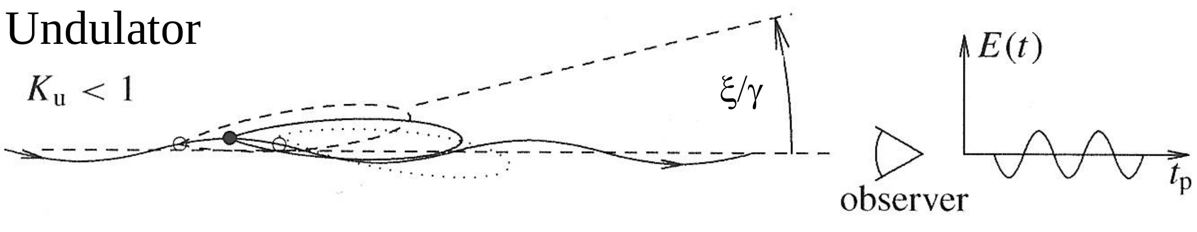
$$\frac{\lambda_1}{\lambda_u} = \frac{1+\gamma^2\theta^2 + K_u^2/2}{2\gamma^2}$$

Plots of Resonance Condition

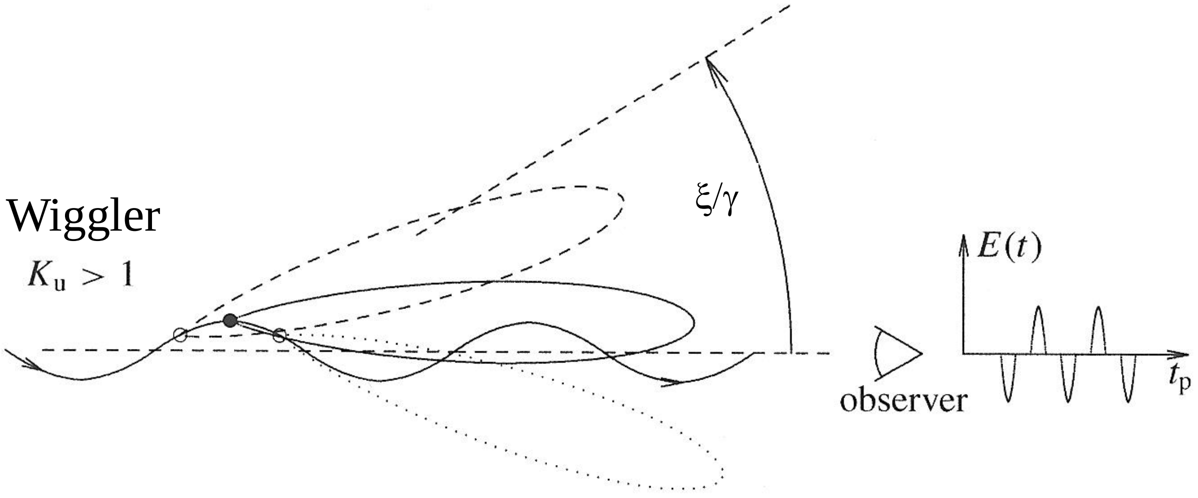




Undulator versus Wiggler



- Wiggle < Opening angle
- Observer always receives light, with modulation
- Broad spectrum, as with dipole radiation



- Wiggle > Opening angle
- Observer receives light in short bursts
- Spectrum has peaks at resonant wavelength and harmonics

Undulator: Angular Spectral Energy Density

$$W_u = \frac{Z^4}{\mu^2} W_0 N_u^2 K_{ue}^2 \frac{\left(1-\xi^2+\psi^2\right)^2 + \left(2\xi\psi\right)^2}{\left(1+\xi^2+\psi^2\right)^4} f_\omega d\omega d\xi d\psi$$

$$f_\omega = \left(\frac{2\omega}{\omega+\omega_1}\right)^2 \mathrm{sinc}^2\left(\pi N_u \frac{\omega-\omega_1}{\omega_1}\right)$$

- The sinc term (where sinc $x = (\sin x)/x$) causes the spectrum to peak at the first harmonic $\omega_1 = 2\pi c/\lambda_1$
- The peak is very narrow when the number of periods, N_u , is large.

Wiggler: Angular Spectral Energy Density

- A complicated sum over harmonics *h* and Bessel functions *l*.

$$W_w = \frac{Z^4}{\mu^2} W_0 N_u^2 K_{ue}^2 \sum_{h=1}^{\infty} h^2 F_h(\theta,\phi)\, f_{\omega h}\, d\omega\, \gamma^2 \theta\, d\theta\, d\phi$$

$$f_{\omega h} = \mathrm{sinc}^2\left(\pi N_u \frac{\omega - h\omega_1}{\omega_1}\right)$$

$$F_h = \frac{\left(2S_{h1}\gamma^* \theta \cos\phi - S_{h2} K_u^*\right)^2 + \left(2S_{h1}\gamma^* \theta \sin\phi\right)^2}{K_u^{*2}\left(1 + K_u^2/2\right)^2 \left(1 + \gamma^{*2}\theta^2\right)^2}$$

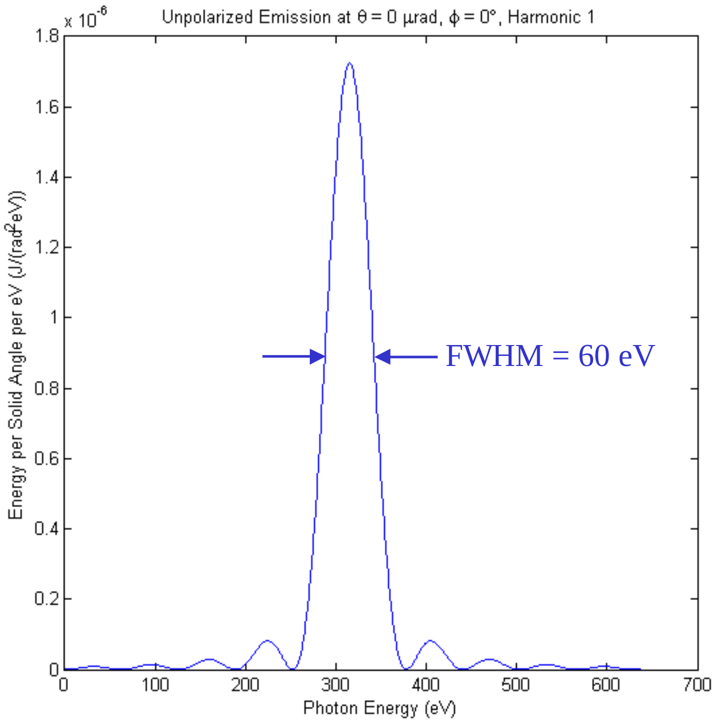
$$S_{h1} = \sum_{l=-\infty}^{\infty} J_l(ha_u) J_{2l+h}(hb_u) \qquad S_{h2} = \sum_{l=-\infty}^{\infty} J_l(ha_u)\left[J_{2l+h+1}(hb_u) + J_{2l+h-1}(hb_u)\right]$$

$$K_u^{*2} = \frac{K_u^2}{1 + K_u^2/2} \qquad a_u = \frac{K_u^{*2}}{4\left(1 + \gamma^{*2}\theta^2\right)} \qquad b_u = \frac{2K_u^* \gamma^* \theta \cos\phi}{1 + \gamma^{*2}\theta^2}$$

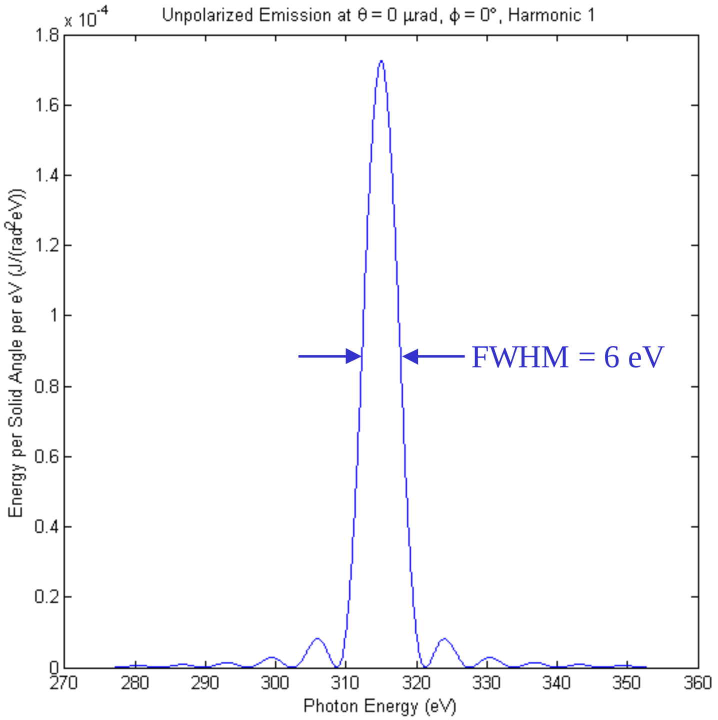
N_u and Bandwidth

First Harmonic

$N_u = 5$



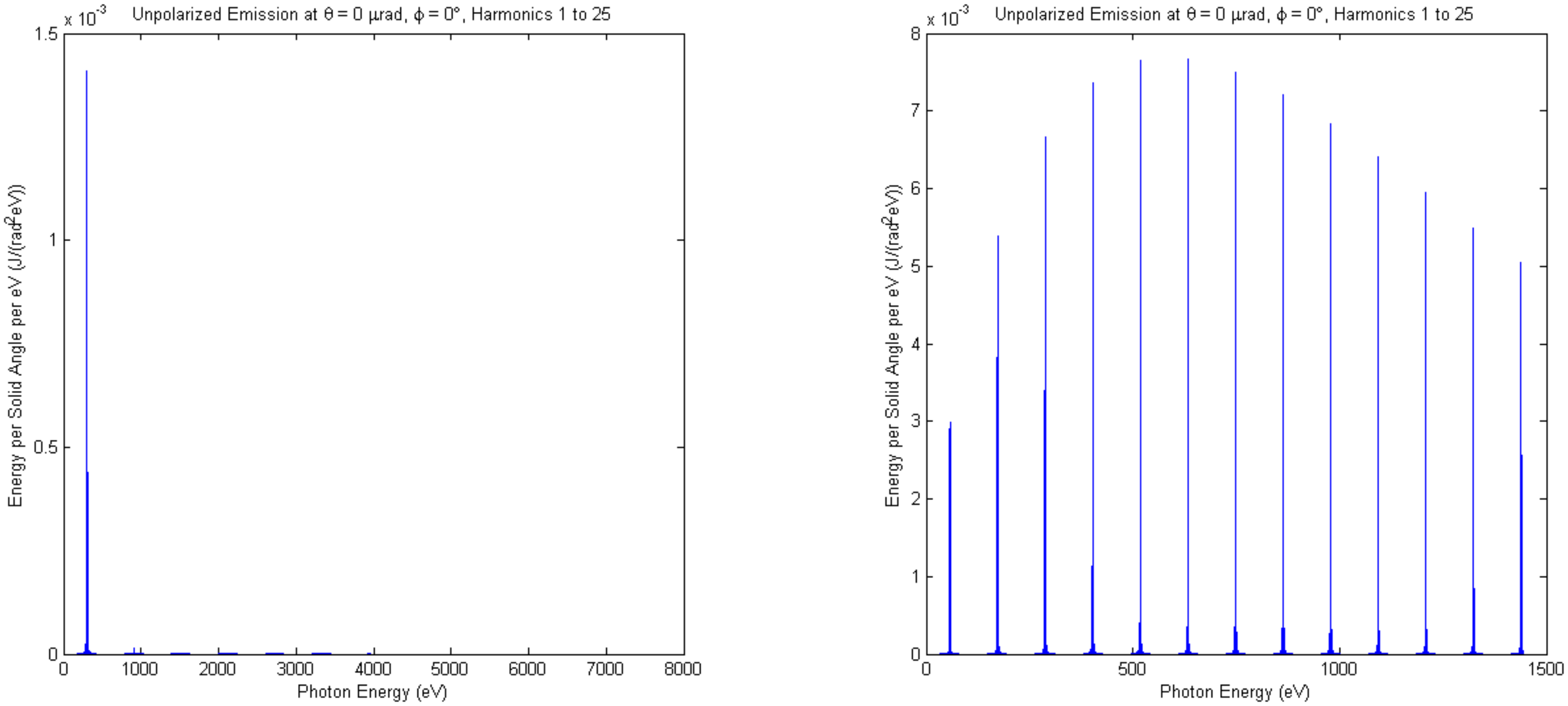
$N_u = 50$



- Same shape, but bandwidth is inversely proportional to the number of periods.

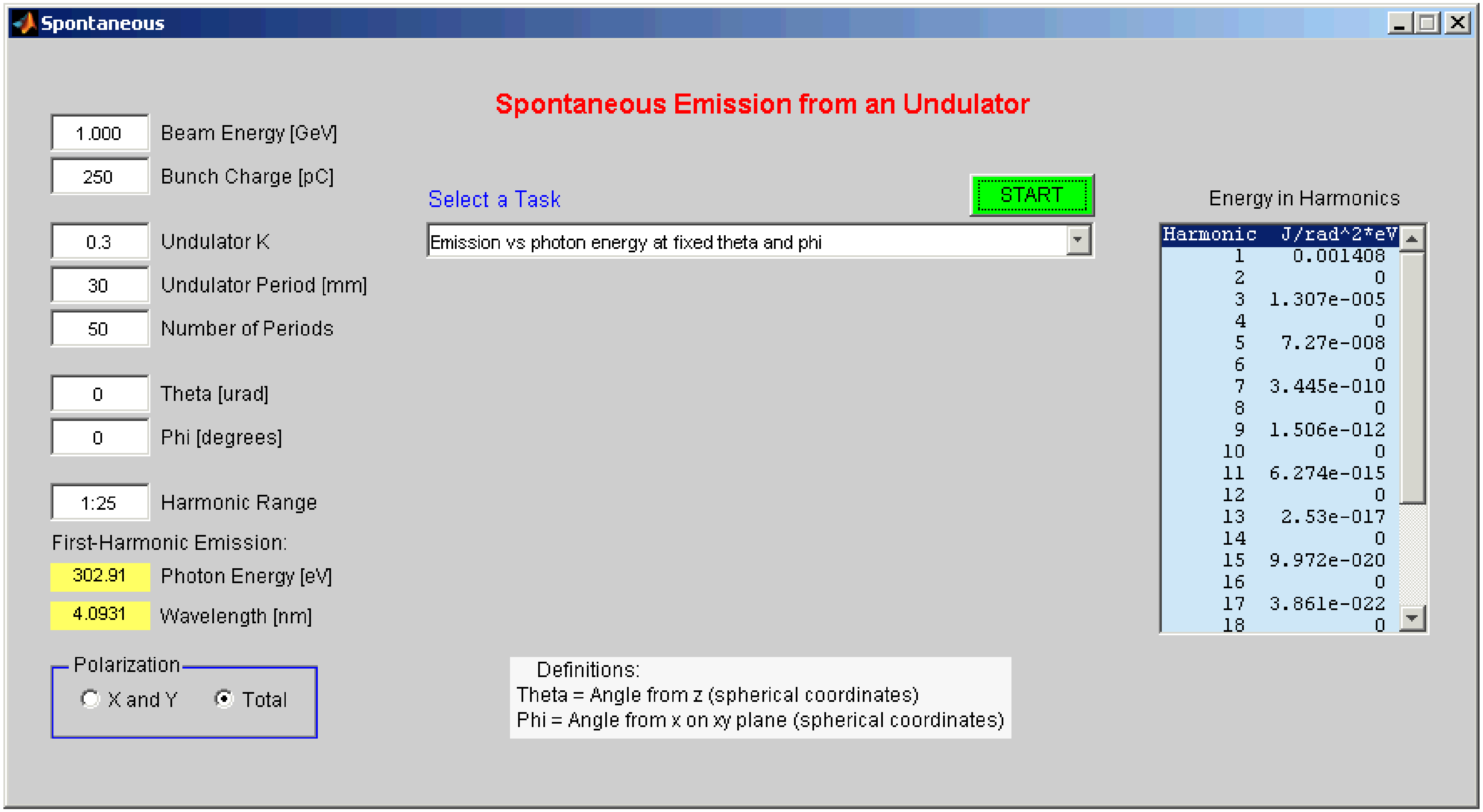
K_u and Harmonics

$K_u = 0.3$ Harmonics 1 to 25 $K_u = 3$



- At high K_u, there is more power in the odd harmonics than in the fundamental.
- Symmetry forbids power in even harmonics on axis ($\theta = 0$).
- Increasing K_u lowers the fundamental energy, following the resonance condition.

Matlab Tool for Undulators and Wigglers



Spontaneous Emission from an Undulator

1.000 Beam Energy [GeV]
250 Bunch Charge [pC]

0.3 Undulator K
30 Undulator Period [mm]
50 Number of Periods

0 Theta [urad]
0 Phi [degrees]

1:25 Harmonic Range

First-Harmonic Emission:
302.91 Photon Energy [eV]
4.0931 Wavelength [nm]

Polarization
○ X and Y ● Total

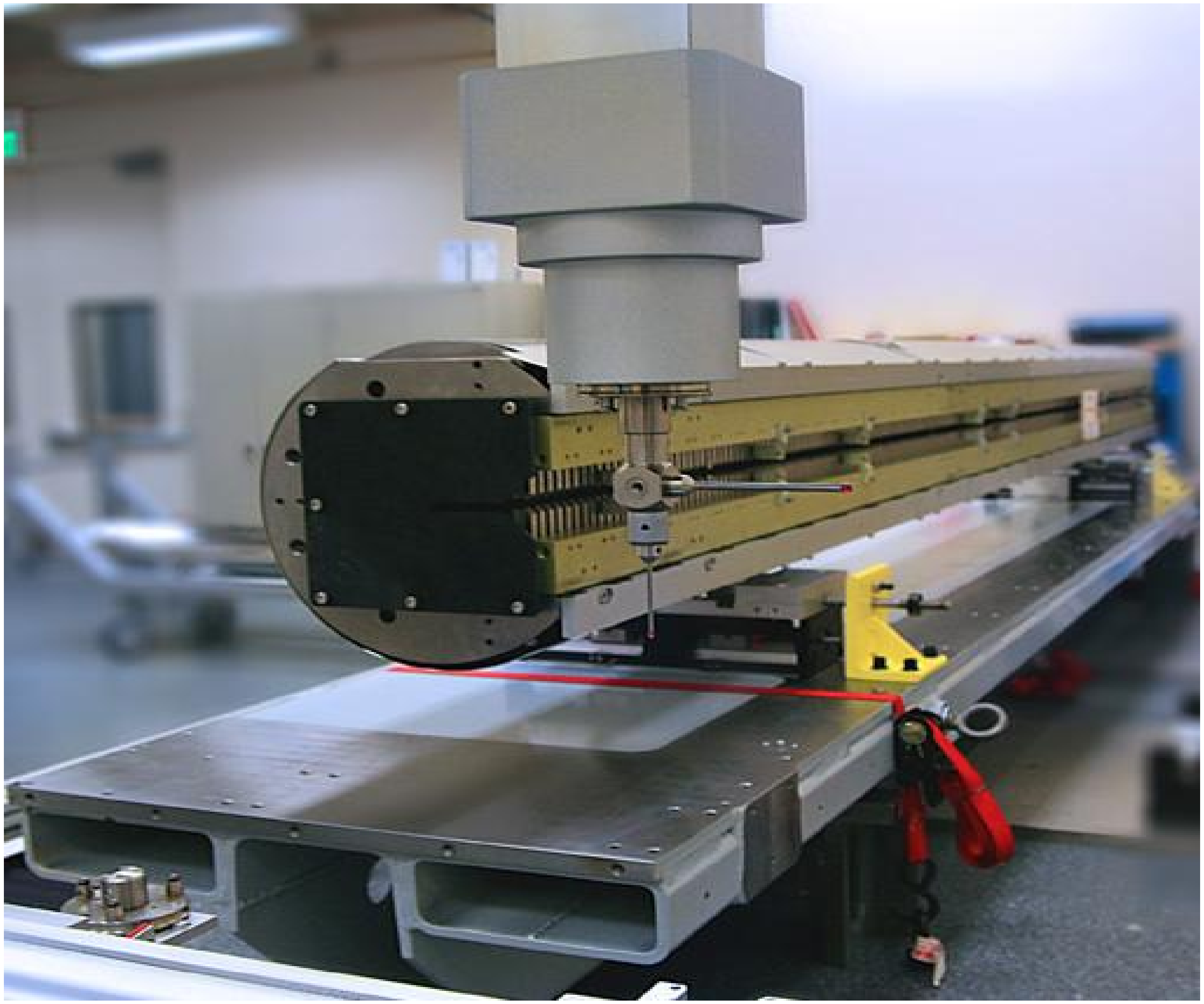
Select a Task
Emission vs photon energy at fixed theta and phi

Energy in Harmonics

```
Harmonic   J/rad^2*eV
       1     0.001408
       2            0
       3    1.307e-005
       4            0
       5     7.27e-008
       6            0
       7    3.445e-010
       8            0
       9    1.506e-012
      10            0
      11    6.274e-015
      12            0
      13     2.53e-017
      14            0
      15    9.972e-020
      16            0
      17    3.861e-022
      18            0
```

Definitions:
Theta = Angle from z (spherical coordinates)
Phi = Angle from x on xy plane (spherical coordinates)

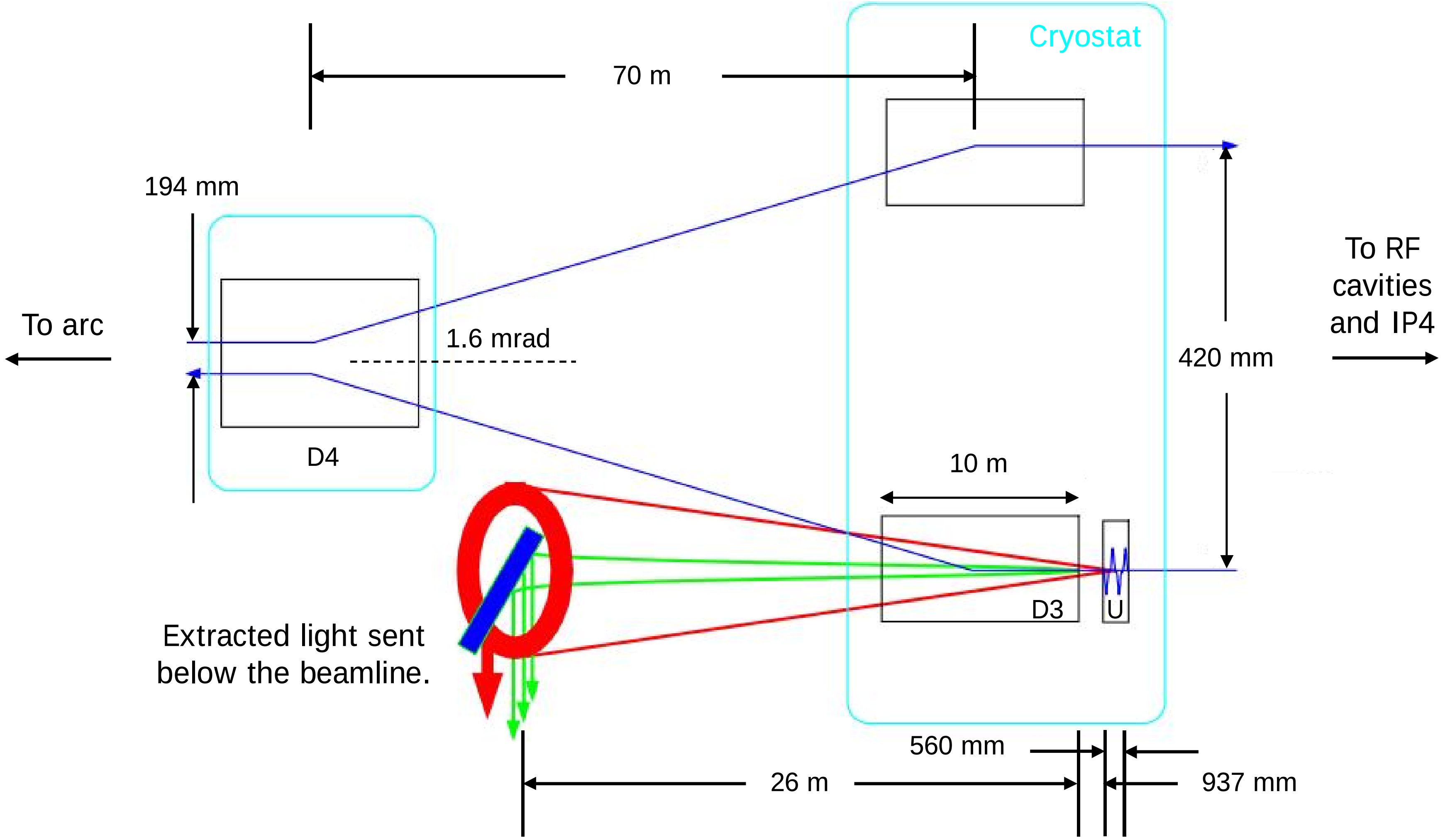
LCLS Wiggler Section on the Test Stand



- $N_u = 113$
- $\lambda_w = 30$ mm
- $L_u = 3.38$ m
- $K_u = 3.5$
  - A wiggler or an undulator?
- $B_u = 1.25$ T
- Gap = 6.8 mm with small taper to vary K_u
- Number of sections = 33
- Total undulator length = 132 m

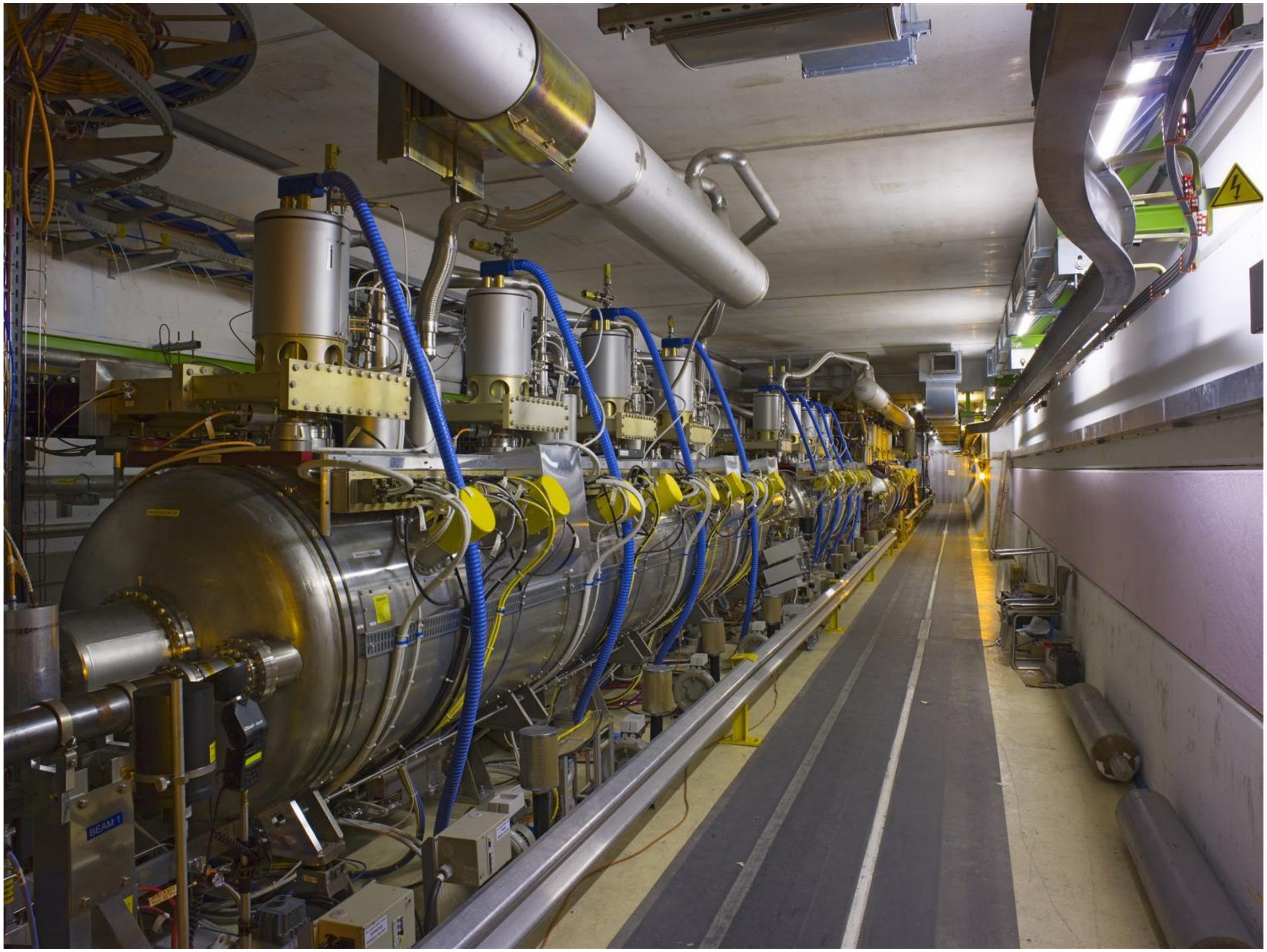
The LCLS Undulator Hall

Layout at the LHC: Dipole and Undulator

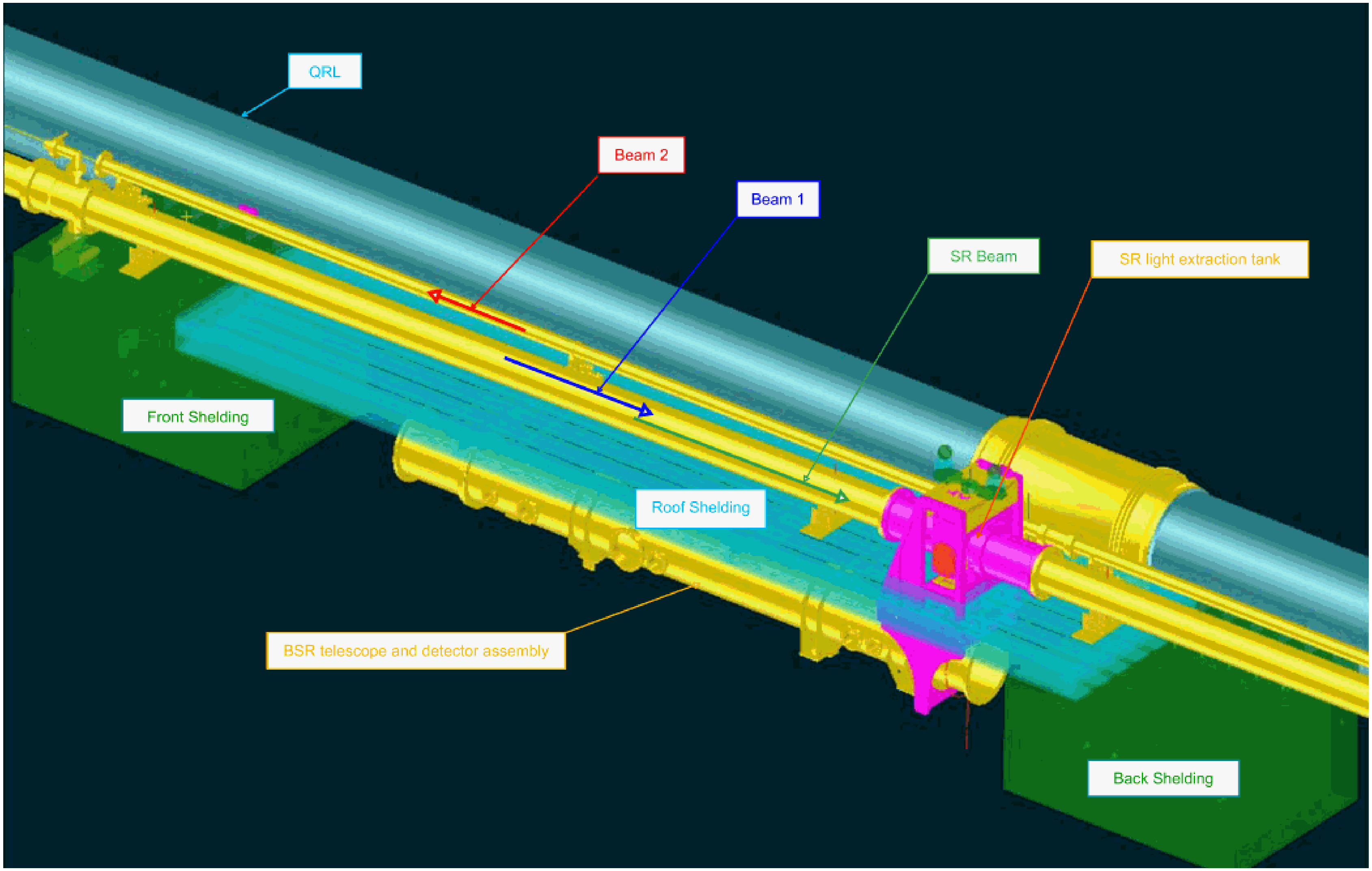


Extracted light sent below the beamline.

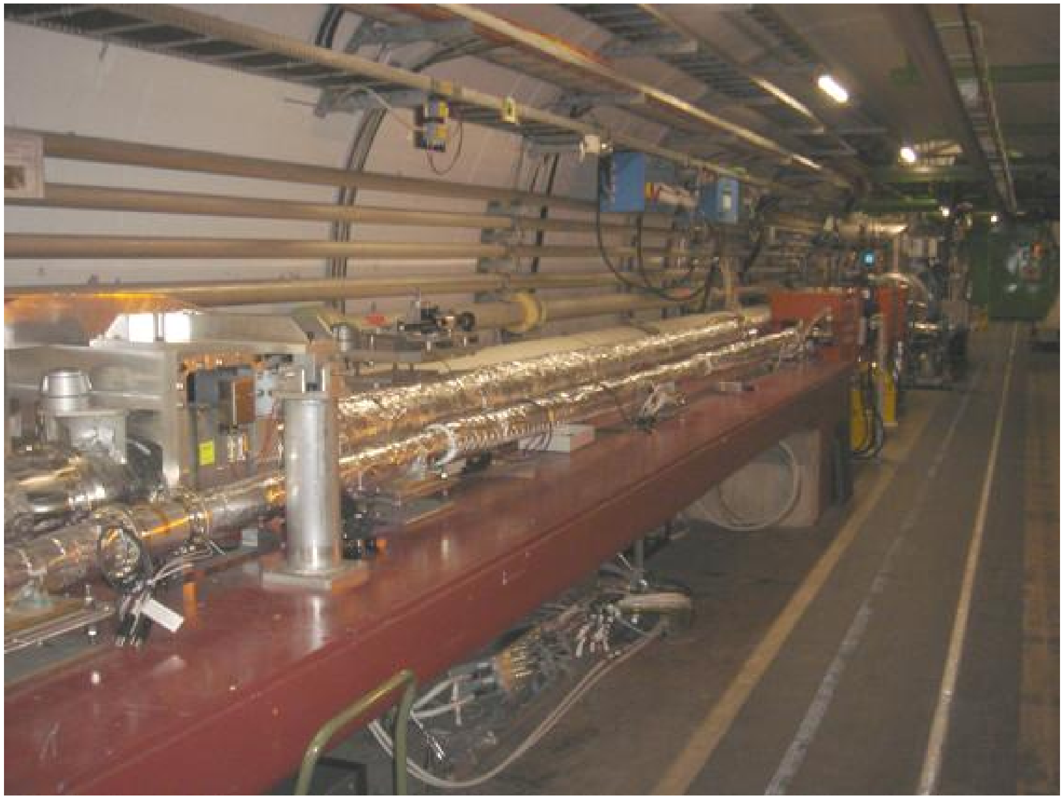
RF Cavities

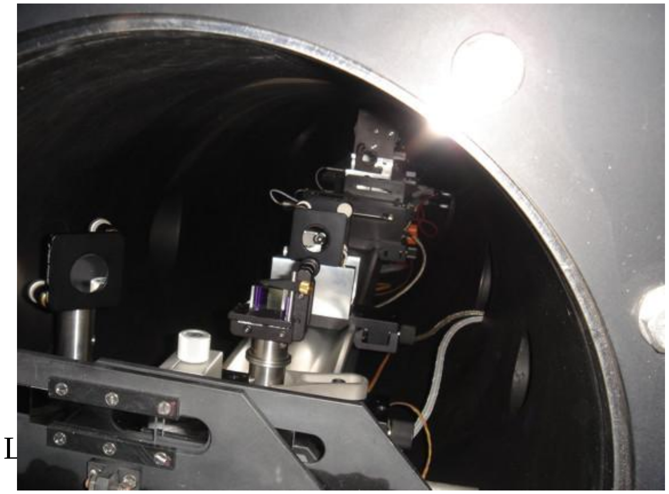


Original Optics Housed in a Tube

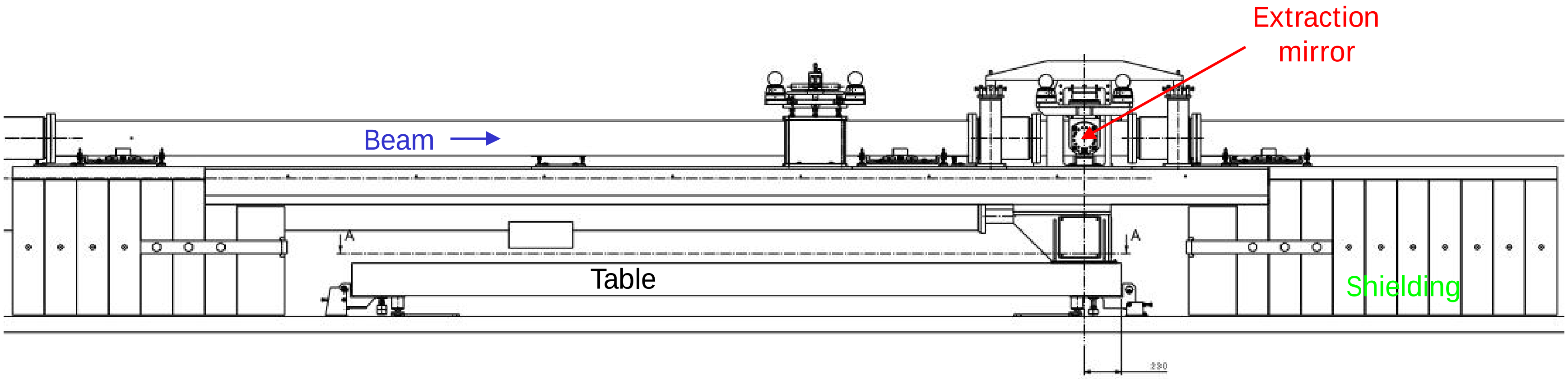


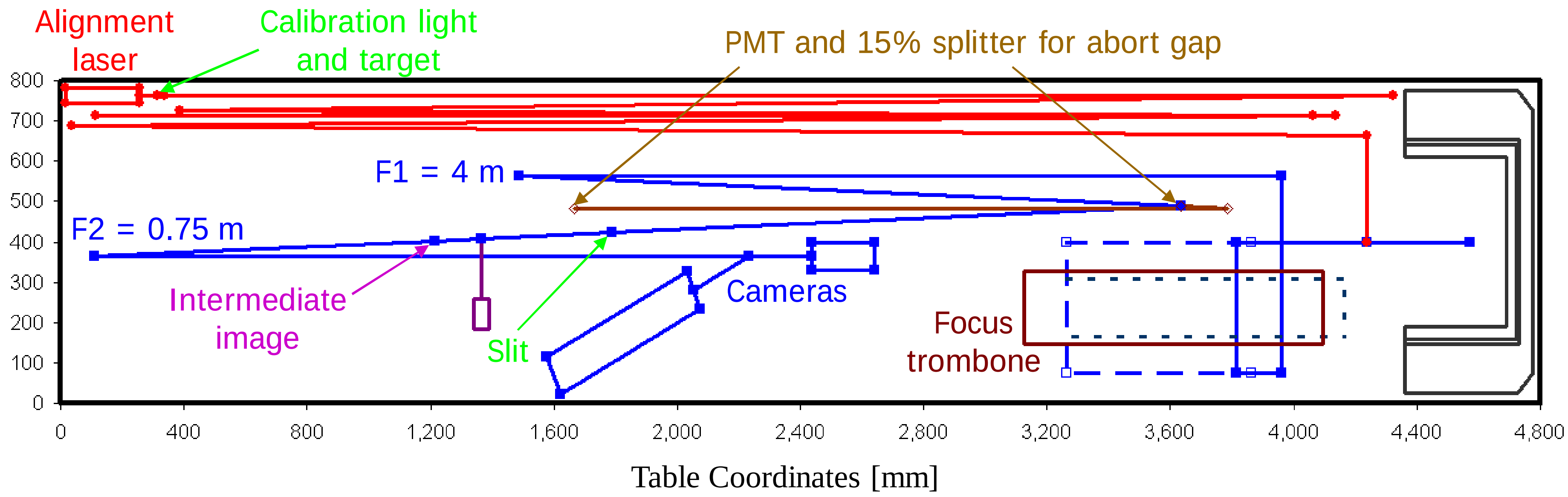
Poor Access with the Original Layout



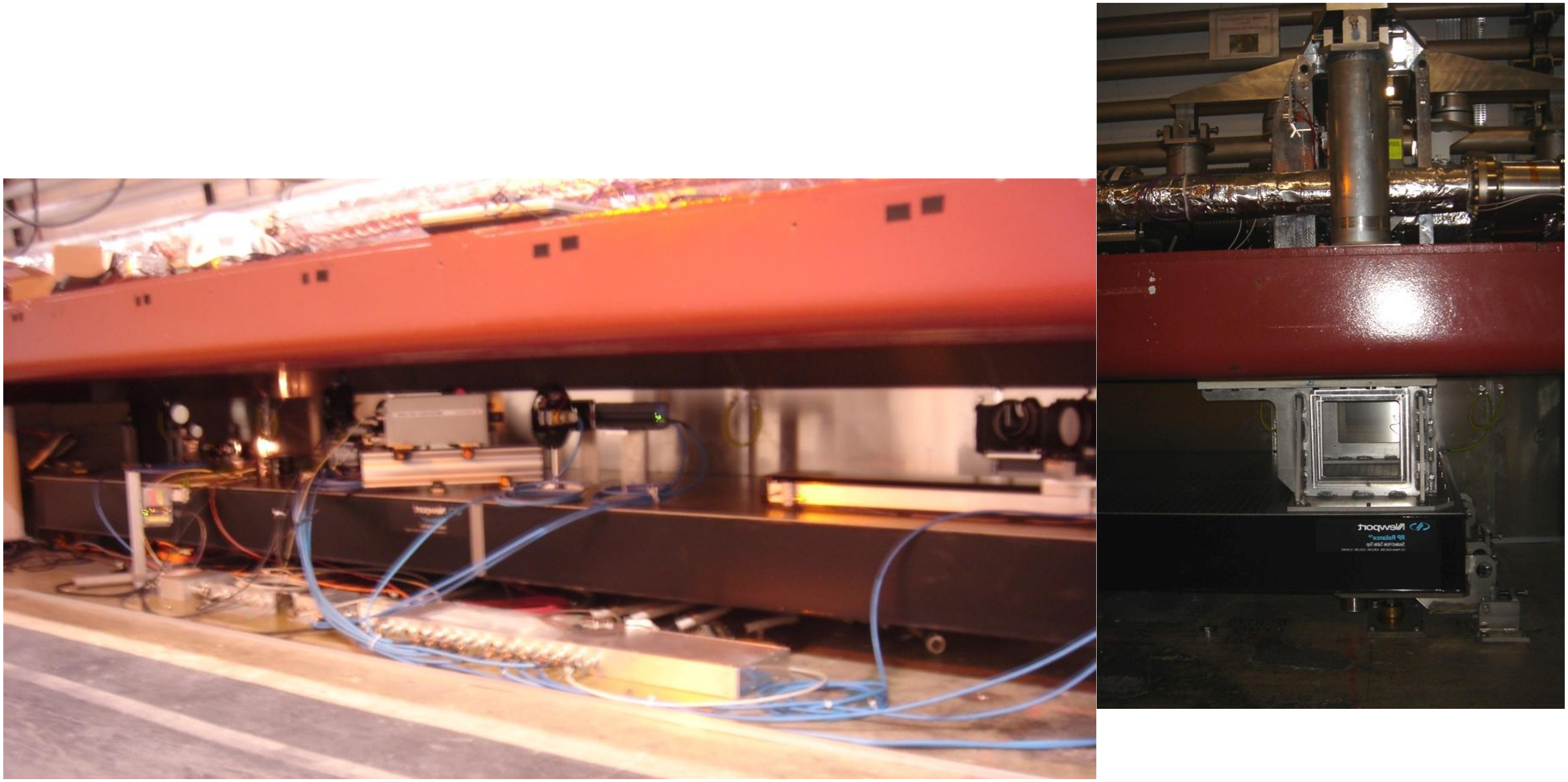


Layout with the New Optical Table



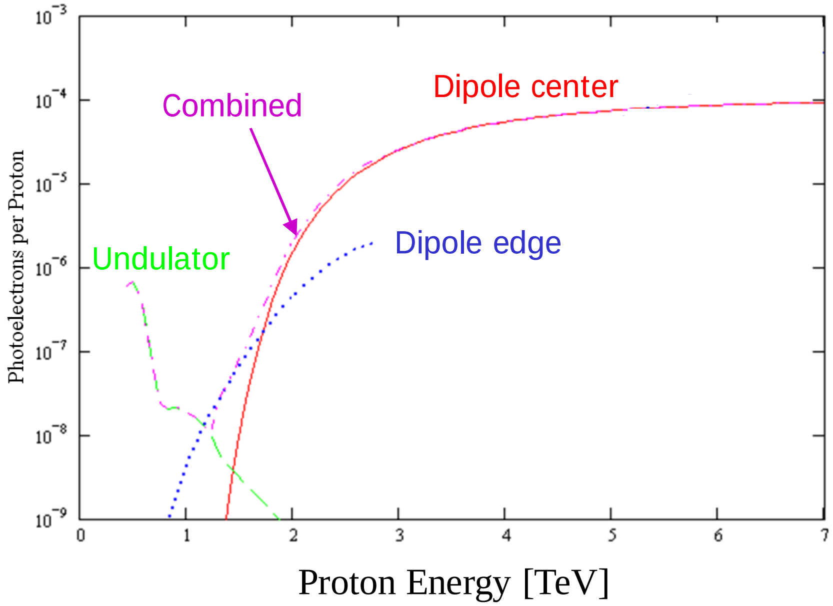


New Table Installed under the Beamline

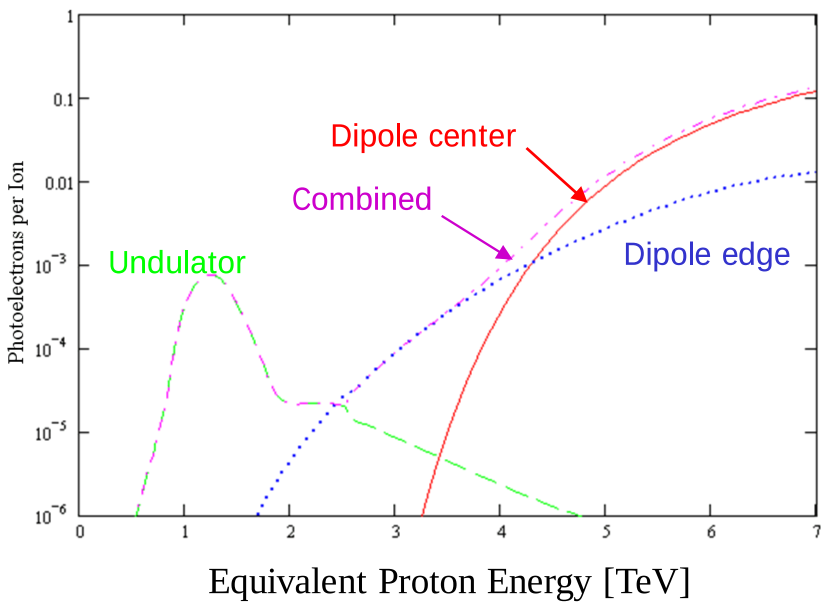


Photoelectrons per Particle at Camera

Protons



Lead Ions



- In the crossover region between undulator and dipole radiation:
  - Weak signal
  - Two comparable sources: poor focus over a narrow energy range
- Focus moves with energy: from undulator, to dipole edge, and then to dipole center.
- Dipole egde radiation is distinct from central radiation only for $\omega >> \omega_c$

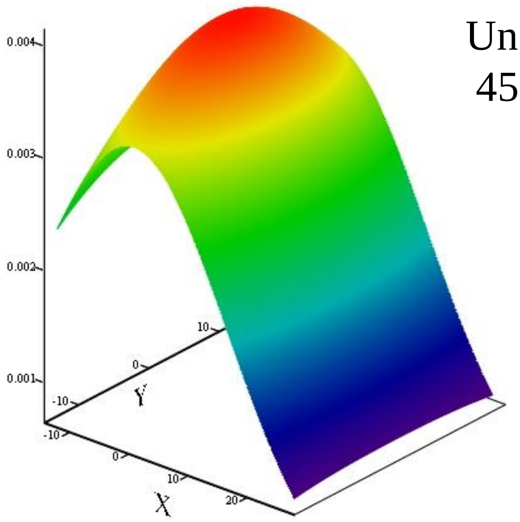
First Beam Image at the LHC

- 2008-09-09: First protons stored at 450 GeV
- 2008-09-19: Busbar failure and liquid-helium explosion
- No testing of the old synchrotron-light monitor
- 2008-11: Started redesign of SLM
- 2009-07: Installed two new SLMs in tunnel
- 2009-11: Restart of LHC. Protons stored at 450 GeV, and later ramped to 1.18 TeV.
- 2009-12-09: Current in undulator for Beam 2. No current in other undulator, pending a 12-hour test of helium level sensor.
- 2009-12-10: First image of 450-GeV protons in Beam 2, using undulator light
- 2009-12-15: Image of Beam 1 using dipole light

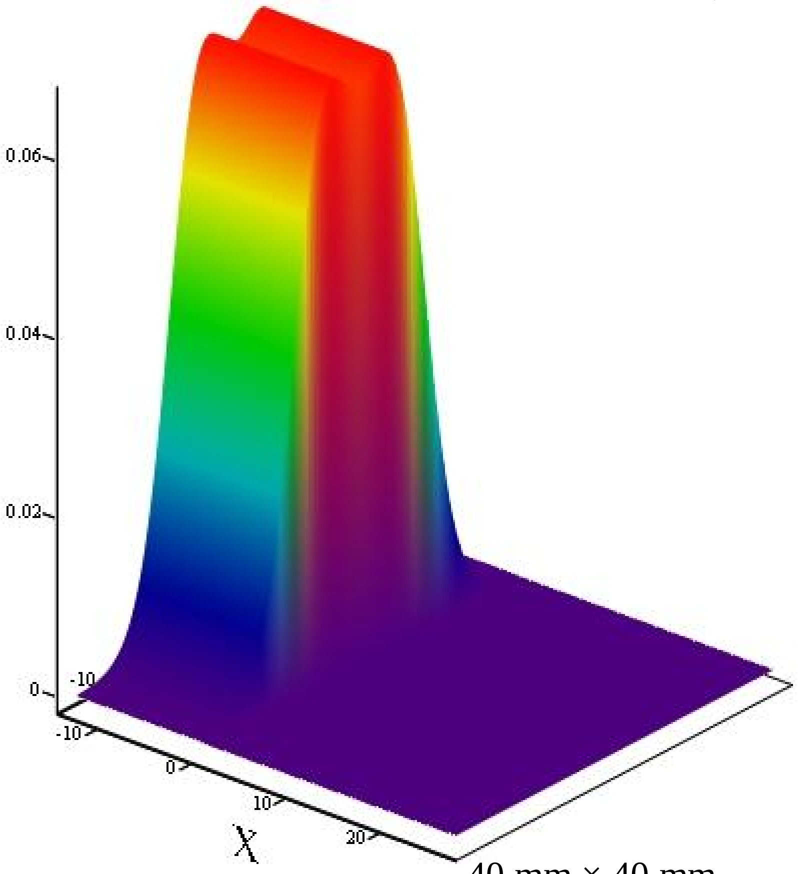
Peculiarities of Synchrotron-Light Imaging

- Imaging a typical source (ordinary photography):
  - An object reflects unpolarized incident light in all directions.
  - A lens collects some of this light over a range of angles.
  - The object’s transverse size is greater than its depth.
- Imaging synchrotron light:
  - The source of the light is the object’s own emission.
  - Light is radiated only in the forward direction.
  - Longitudinal profile is Gaussian at any instant.
    - But over the exposure time, the source goes around the ring many times.
  - Dipole light is emitted tangent to the beam’s instantaneous circular path.
    - Vertically: A narrow forward-directed cone.
    - Horizontally: A stripe of light painted along the midplane of the vacuum chamber.
    - Like a car rounding a bend in the dark with its bright headlights on.
    - The camera images a glowing, curved string along a tangent.
  - Undulator or wiggler light is emitted all along the axis.
    - A narrow forward-directed cone, both vertically and horizontally.
    - The camera images a glowing string head-on.
  - A telescope: We must image a ~100-µm object from a distance of ~10 m.

Light on the LHC Extraction Mirror

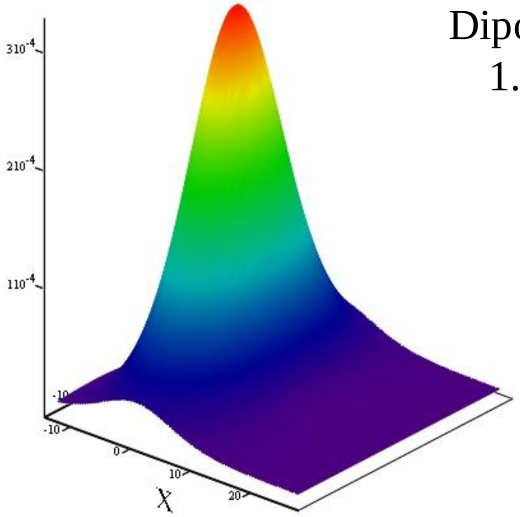


Undulator
450 GeV



Dipole Center
7 TeV

40 mm × 40 mm
mirror



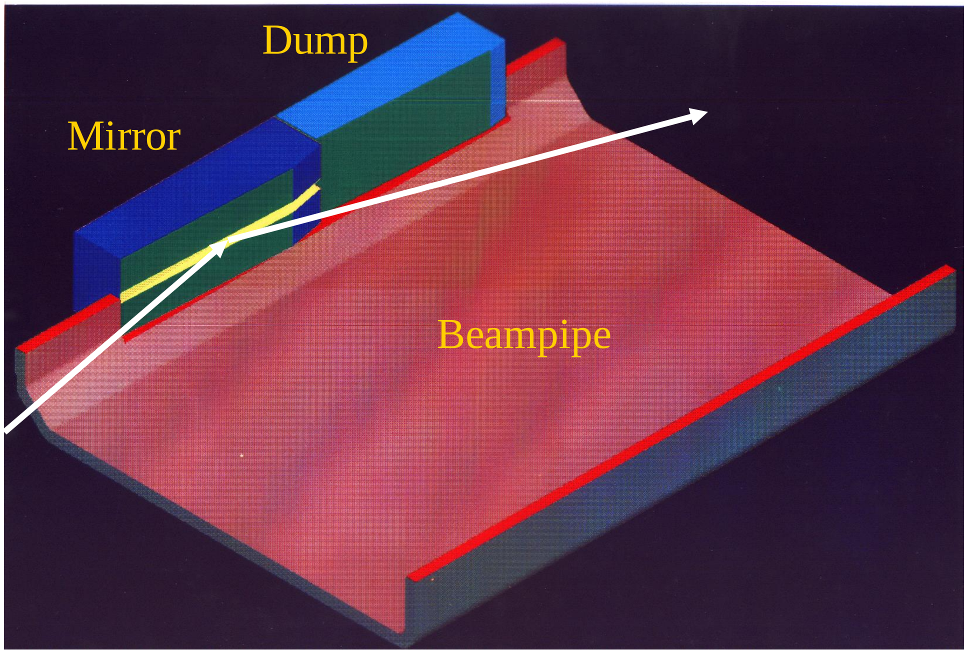
Dipole Edge
1.5 TeV

Imaging with Visible Synchrotron Light

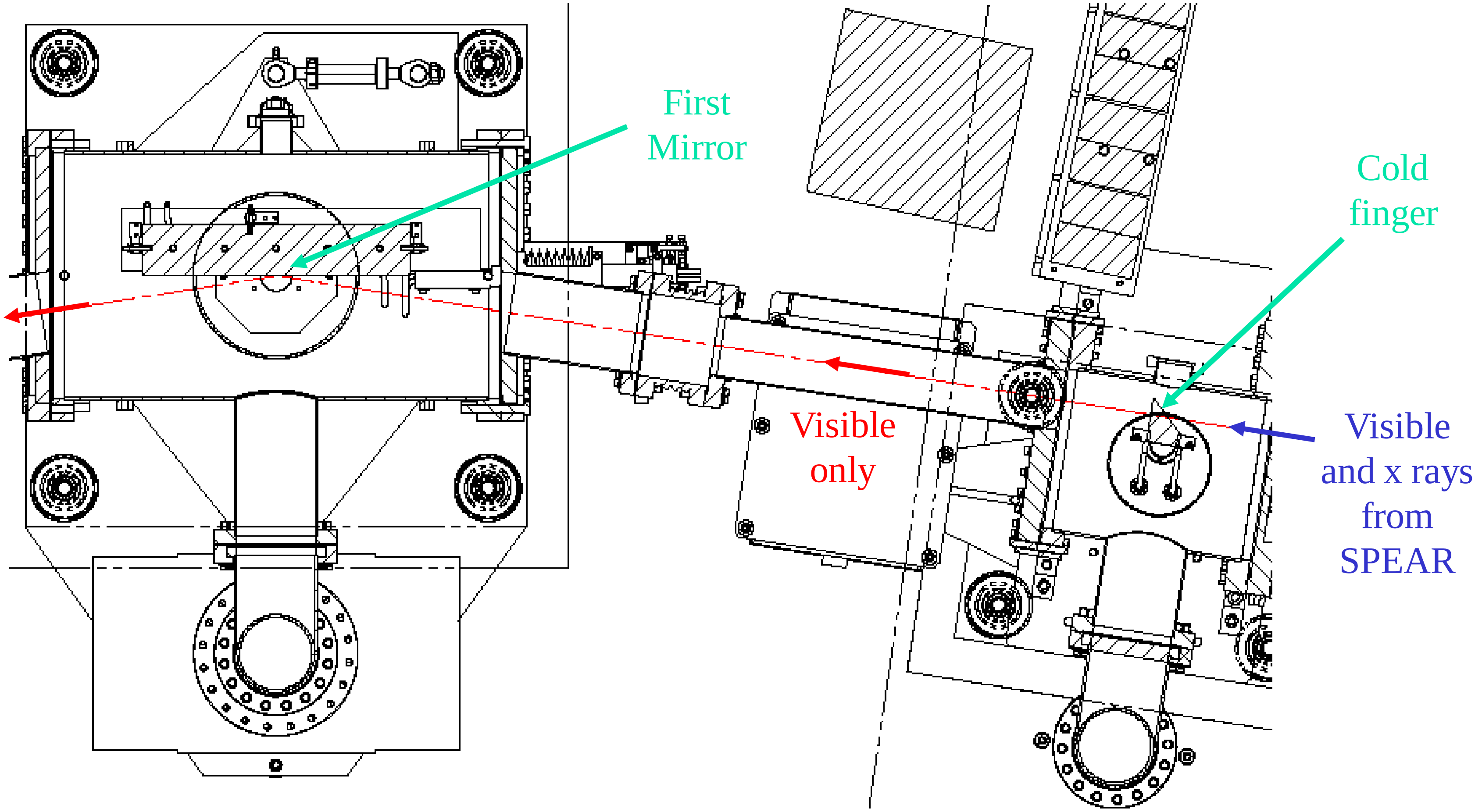
- Visible light has advantages (in addition to being easy to see):
  - You can use common parts like windows, lenses, mirrors, video cameras, and specialized instruments like streak cameras.
  - Want to avoid thermal distortion of the first mirror (M1), with little effect on the visible image
    - Recall that the opening angle for visible light is wider than at the critical wavelength
    - The narrow x-ray fan has most of the heat. Remove it using:
      - A “cold finger”: a narrow cooled mask on the midplane, upstream of M1, that blocks the x-ray fan while casting a thin shadow across the mirror.
      - A slot along the middle of M1, to let the x-rays bypass the mirror.
      - A thin, low-Z substrate (beryllium) for M1, to transmit most of the x-rays.
        - But beryllium is toxic and thin mirrors are hard to keep flat.

Slotted First Mirror in PEP-2

- Midplane slot allowed x-ray fan to bypass M1 and strike the thermal dump.
  - But slot was a mechanical weak point.
  - PEP's x-ray fan was very narrow and very hot: too difficult to cool a cold finger.



“Cold Finger” Mask in SPEAR-3

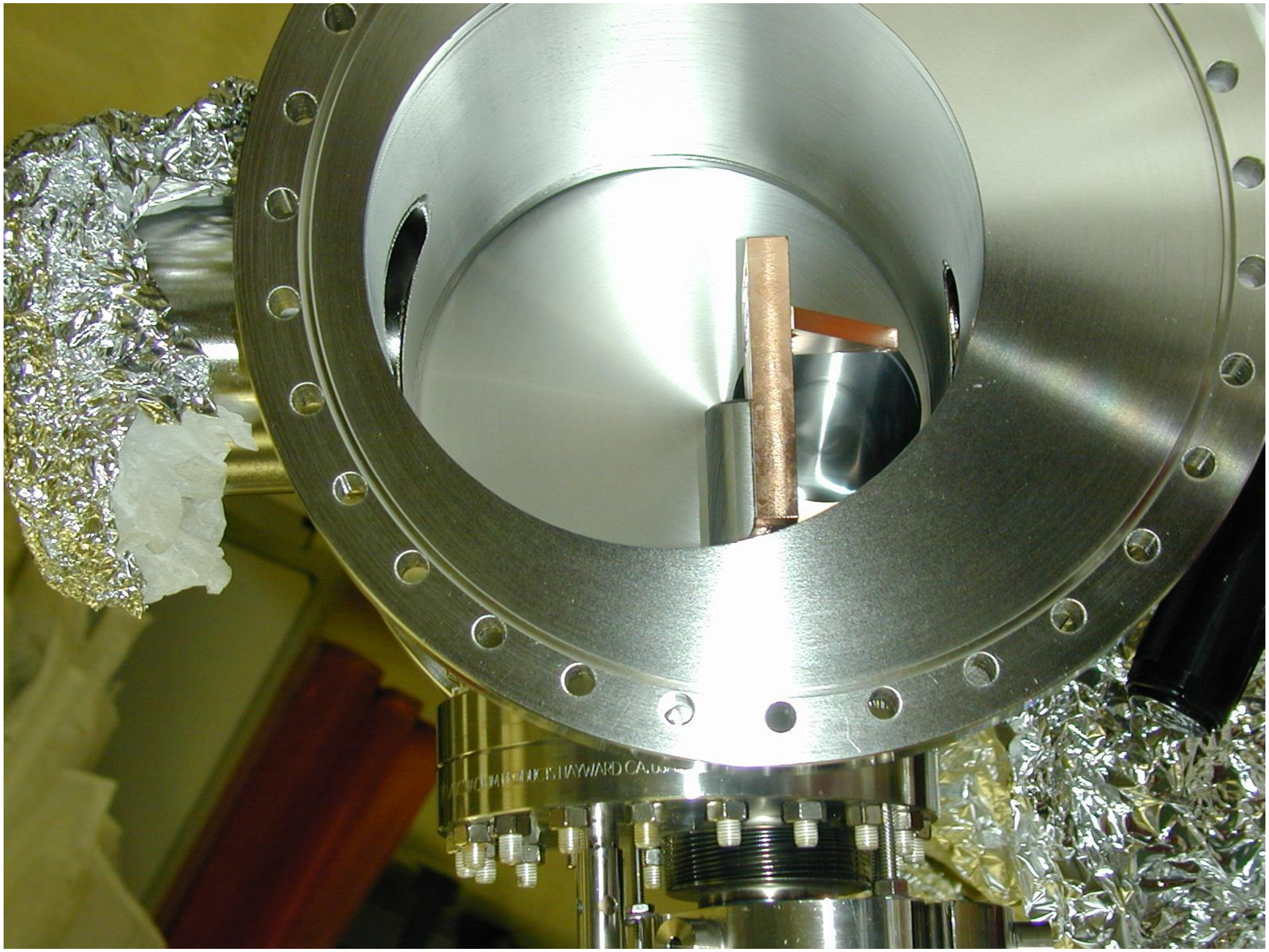

First Mirror

Cold finger

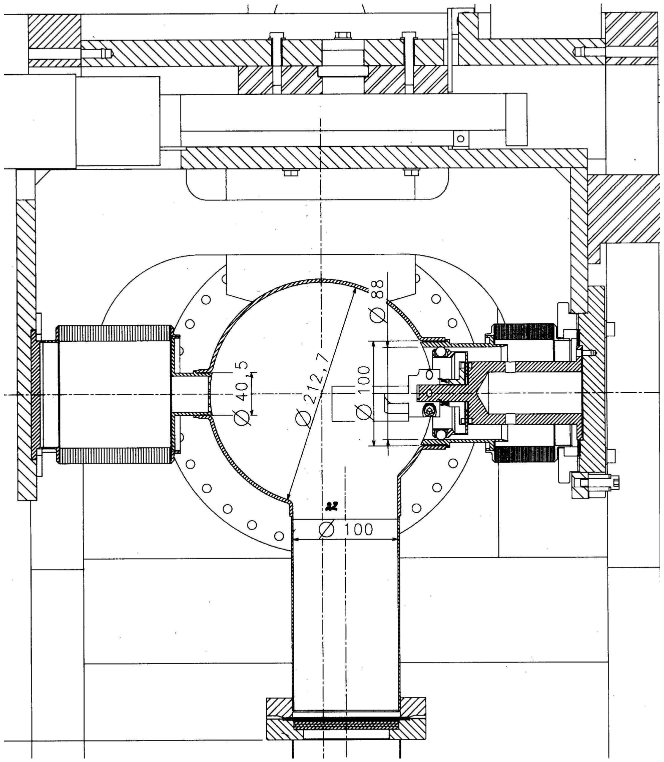
Visible only

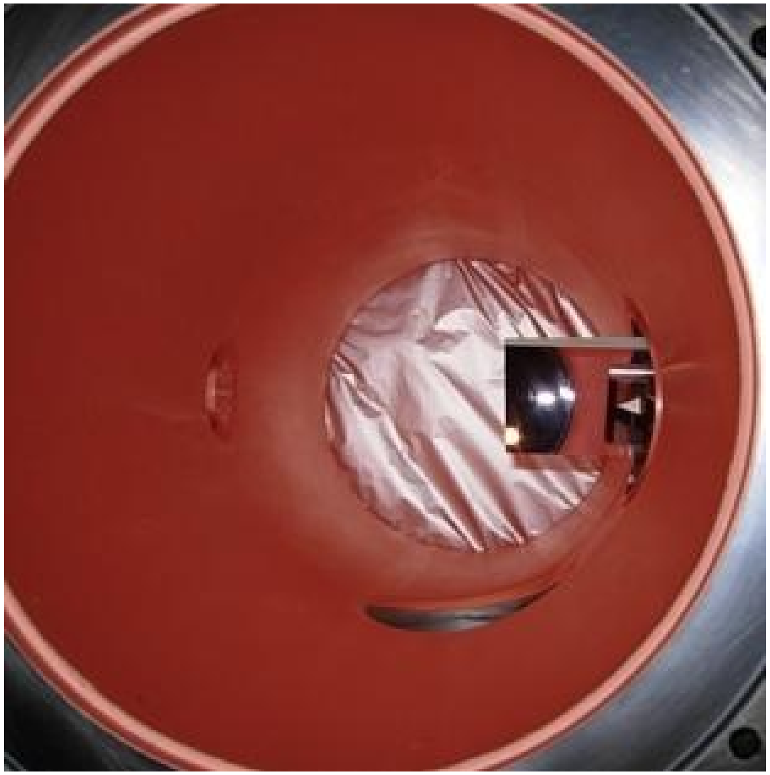
Visible and x rays from SPEAR

Photo of the Cold Finger in SPEAR-3



Extraction Mirror in the LHC





Protons—with their negligible heat load—can use a simple mirror.