Fundamentals of Optical Imaging

for

Synchrotron Light Diagnostics

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Beam Diagnostics Using Synchrotron Radiation:
Theory and Practice
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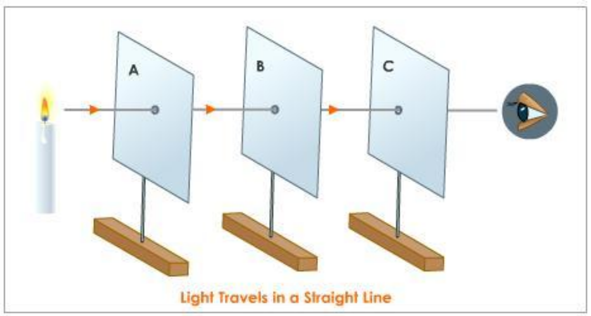
Objective

- Fundamentals
  - to understand where it comes from, hence we know the limitation
- Some practical knowledge in optics
  - about synchrotron light diagnostics.

The fundamental properties of light

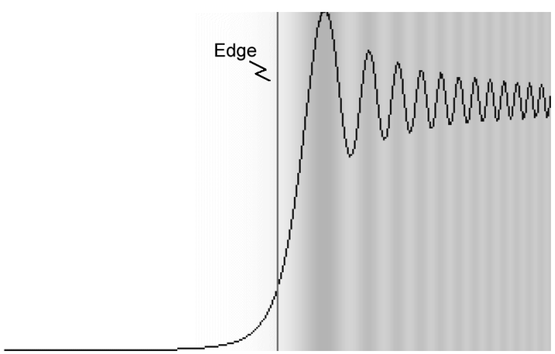
- Rectilinear propagation of light
- Wave property of light
- Quantum property of light
- Beauty is in the beholder's eye
- The property of light is detector specific, depending on the property of the equipment intercepting it in order to reveal its presence
- Dare to ask. Light travels in straight line. Really?

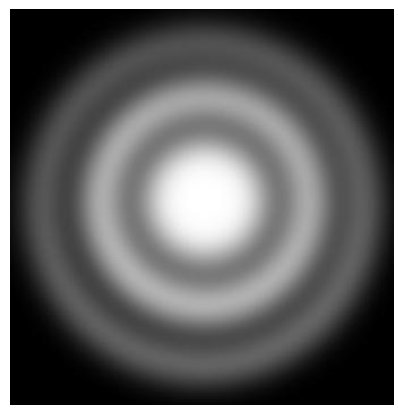
Wave properties of light

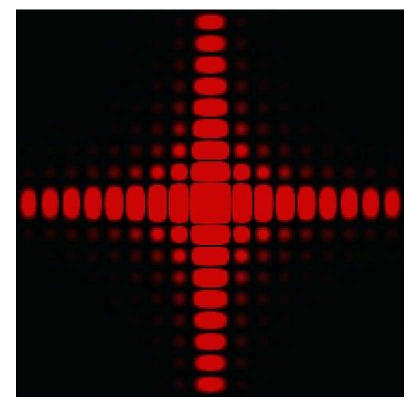


Class Experiment

Observation of diffraction of light between two fingers





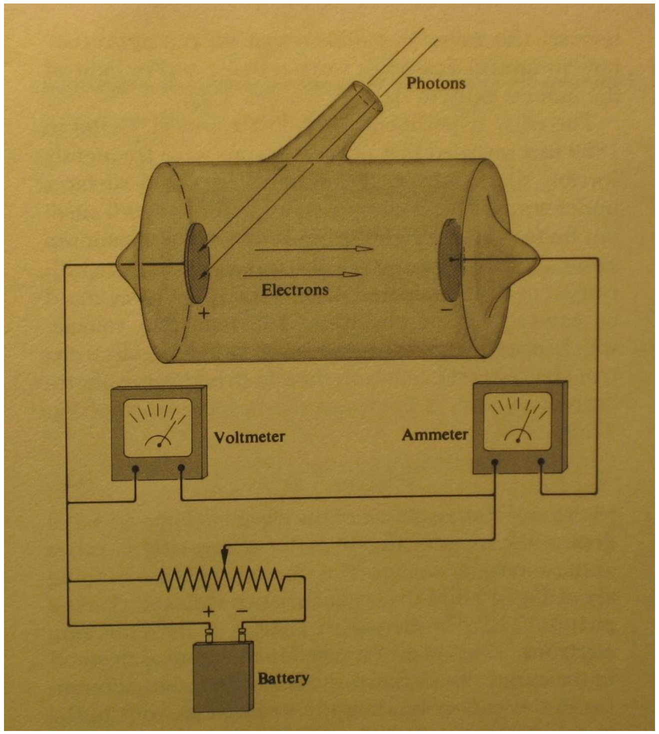


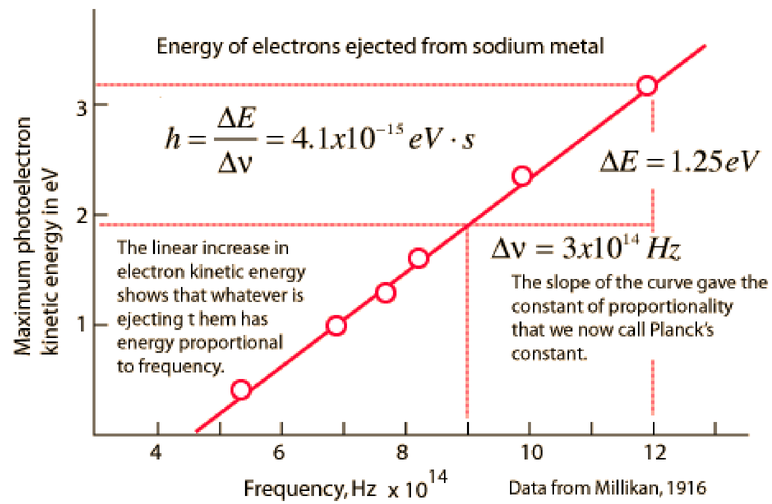
Fringes made by a straightedge obstacle in visible light beam Diffraction by circular & square aperture

Quantum Property of light

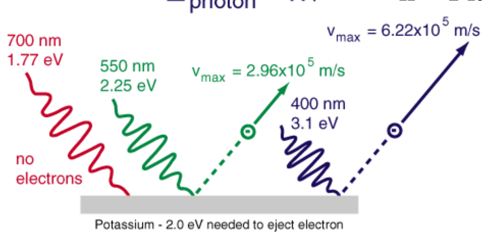
- Photon energy = h * frequency of light
- h = Planck's constant = 6.625×10^{-34} Js

The Quantum particle properties of light

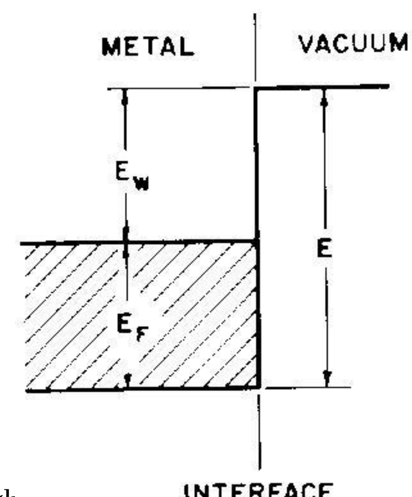




$E_{photon} = h\nu$ h = Planck's constant = 6.625 x 10 $^{-34}$ Js



Photoelectric effect



Physical representation of light

- Wave theory: Velocity, frequency, wave length, K vector
  - $U = U_0 \, e^{-i(\omega t - kx)}$

- Synchrotron light source
  - Energy (ev) x Wave length (nm) = 1239.842 (ev) (nm)
  - Wave length (nm) = 1239.842 / energy (ev)
  - Energy (ev) = 1239.842 / wavelength (nm)
  - 1 ev = 1.6 x 10^{-19} Joule

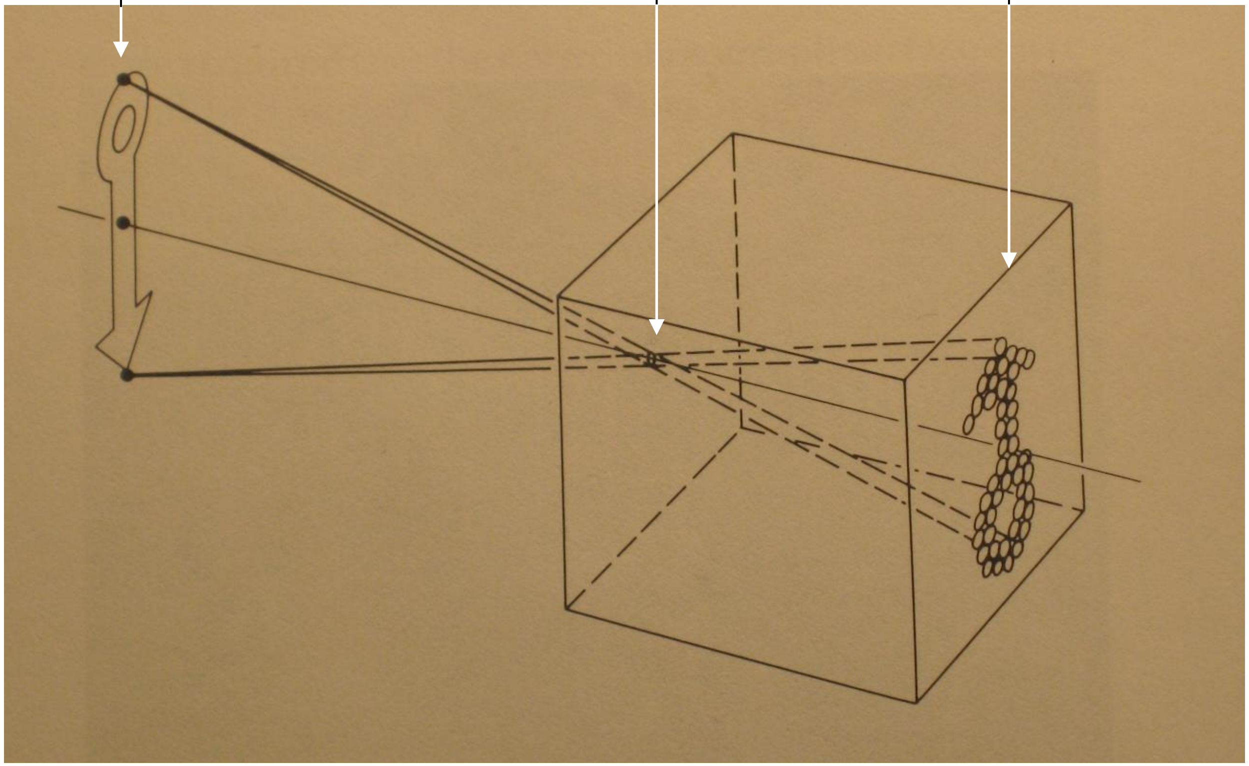
- Chemist
  - Wave number (cm^{-1}) = 1 / wave length = Velocity of light / Frequency
  - Wave length = 1/ wave number
  - Frequency = C x wave number =>
  - wave number directly proportional to Energy

Exploiting the various properties of light

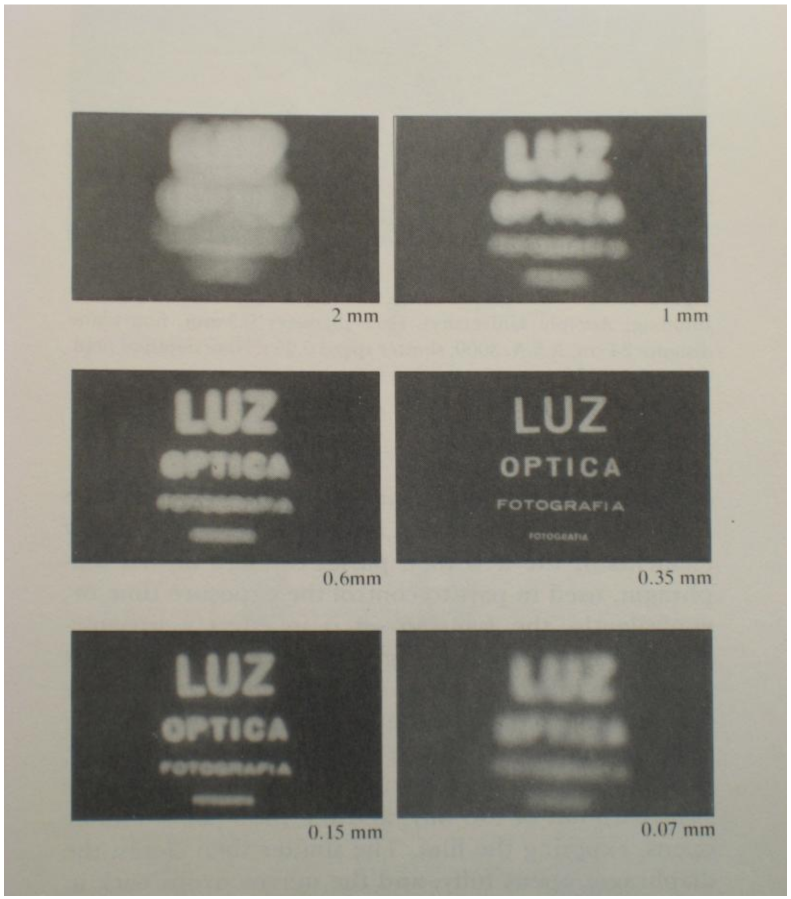
- Rectilinear propagation
  - Pinhole camera
  - Geometrical optics
- Wave
  - Measuring Lateral beam size by Stellar Interferometer
  - X-ray Imaging with Fresnel lens
- Quantized photon
  - Temporal beam size by photon statistics
  - Electronics image

How pinhole camera work

- Each point source is projected on the screen
  - Object Pinhole Focal plane



The effect of aperture size on image quality



$d = 1.9 \sqrt{f\lambda}$

where

d is diameter,

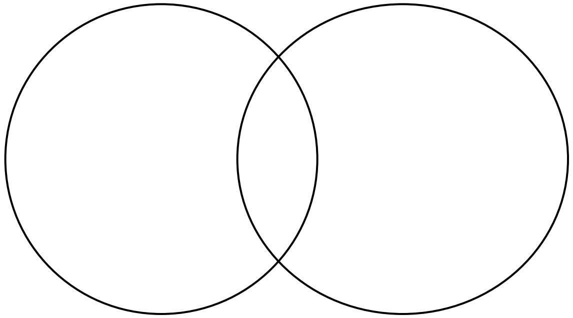
f is focal length (distance from

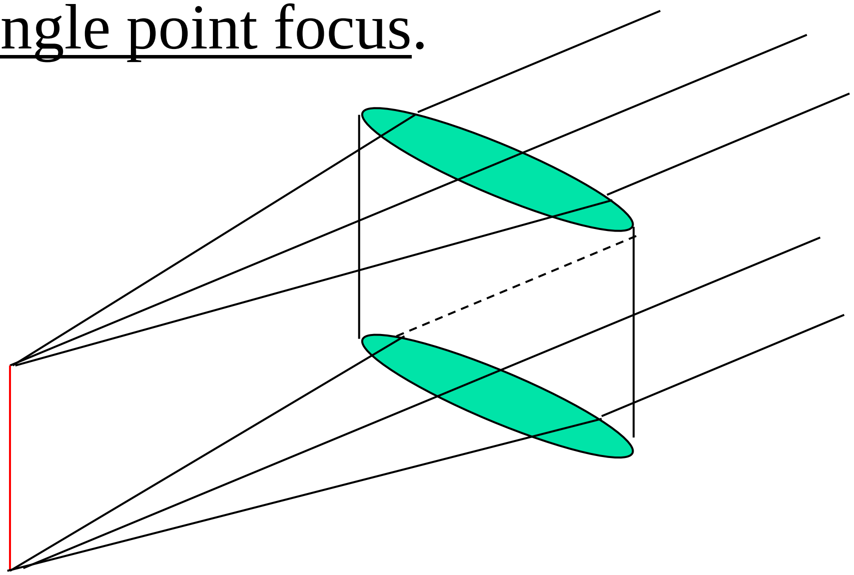
pinhole to focal plane) and

λ is the wavelength of light.

Lens used in imaging system

- Spherical lens :
  - Formed by the intersection of two intersecting spheres
  - Paraxial rays form a single point focus.

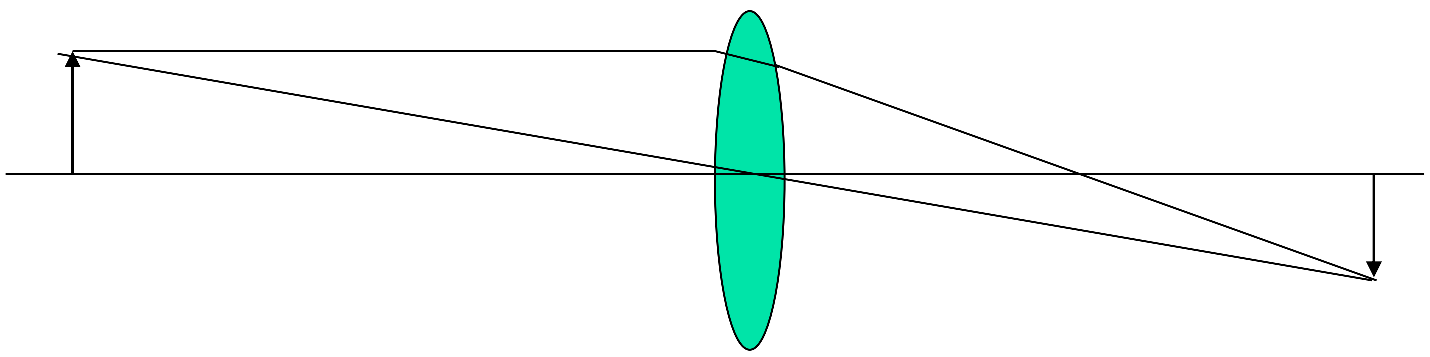




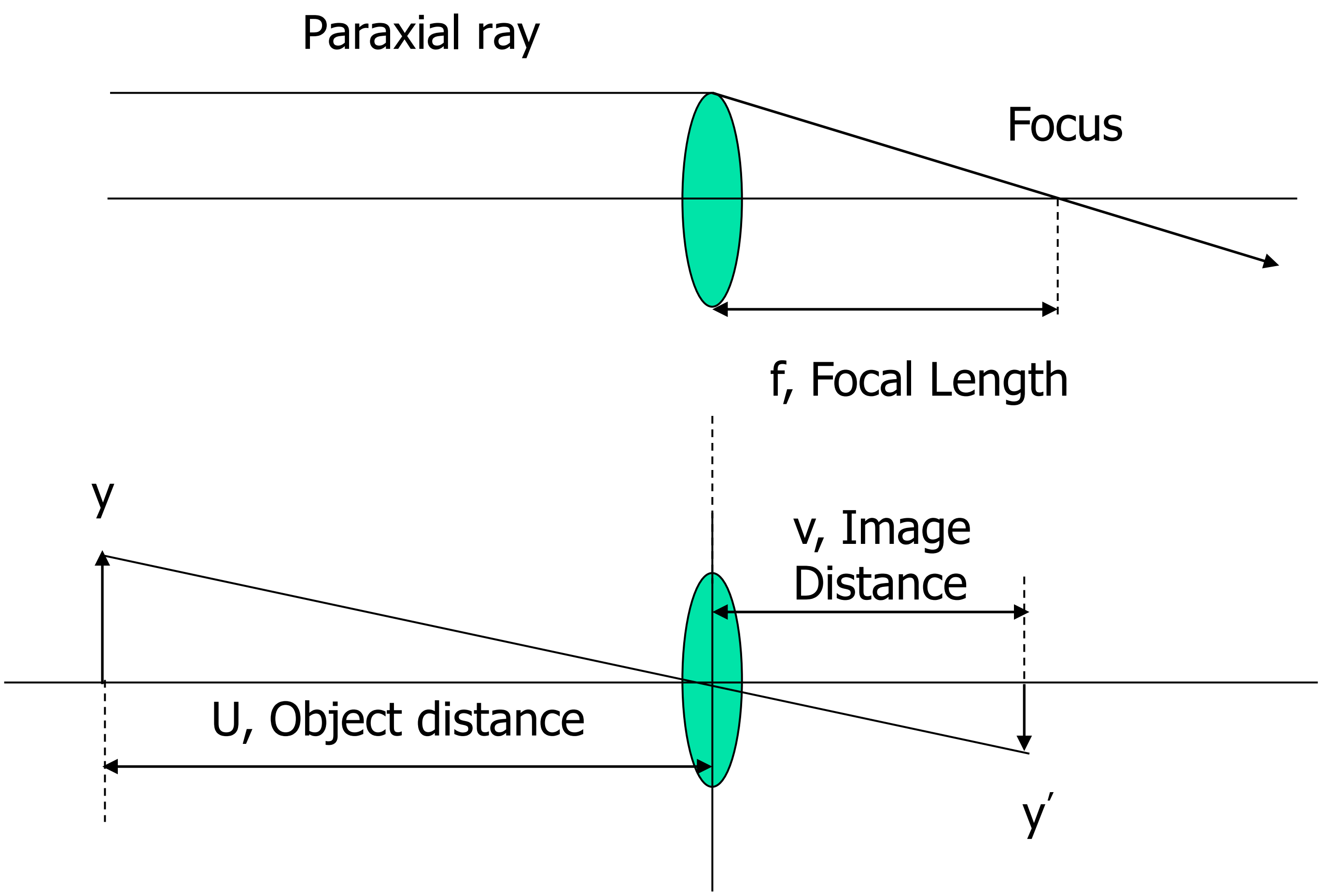
- Cylindrical lens :
  - Formed by the intersection of two intersecting spherical cylinders with their axis parallel to each other.
  - Paraxial rays form a single line focus, parallel to the axis of the cylinder

How image is formed in rectilinear theory?

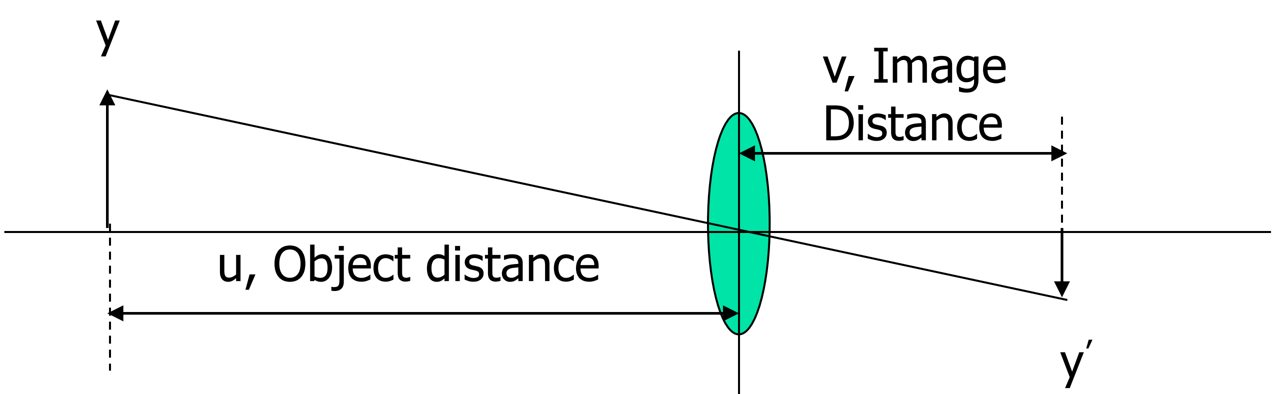
- An image is formed by a collection of points of equal optical path from the source
- optical path = nx = refractive index of the optical medium X free space distance)



Review of Paraxial Optics I



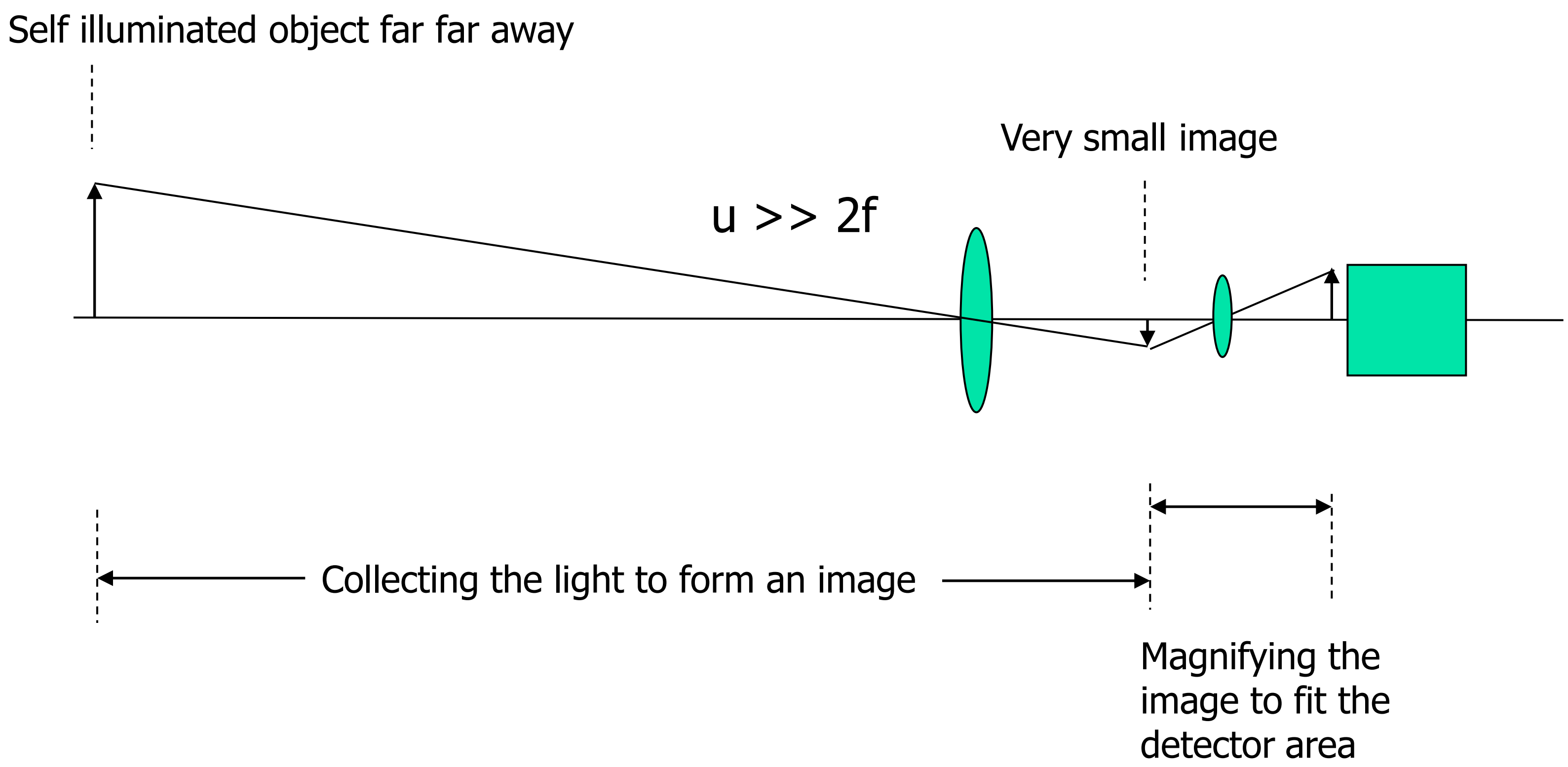
Imaging with a single lens



Magnification = M = y′ / y = v / u

For real image size	Magnification	Object distance
Larger than the object	M > 1,	f < u < 2 f
Equal to the object	M = 1,	u = 2 f
Smaller than the object	M < 1,	u > 2 f

Imaging with two lens



Imaging systems for synchrotron beam diagnostics

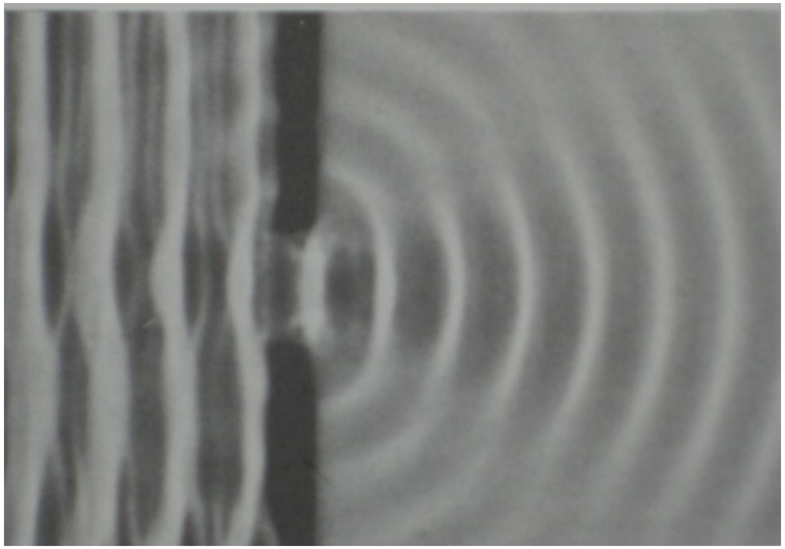
- Single lens imaging
  - Beam size measurement by stellar interferometer– imaging the light source to the camera
  - Capture the object image into a streak camera
- Two lens imaging
  - Imaging the light emitted by a single electron bunch with a digital camera (PRIMAX)
  - Relate the image into the streak camera
  - Relate the image into the PRIMAX camera with a spinning mirror

How image is formed, in wave theory ?

- Image is formed by a collection of point of equal phase.

Diffraction of surface wave on liquid

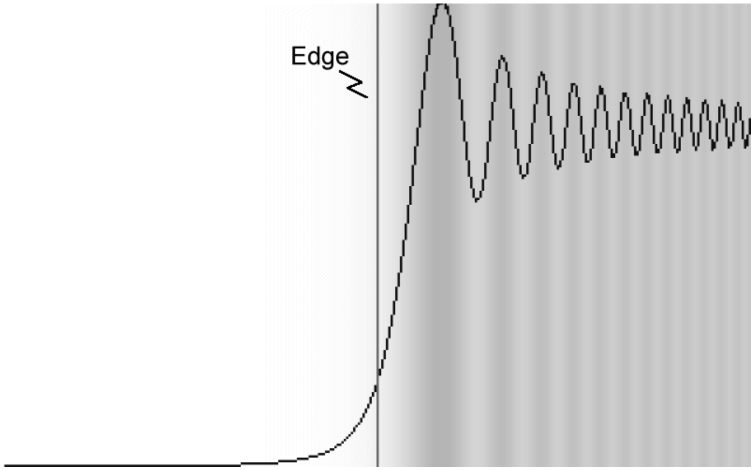
- Ripple tank, diffraction of surface wave on liquid

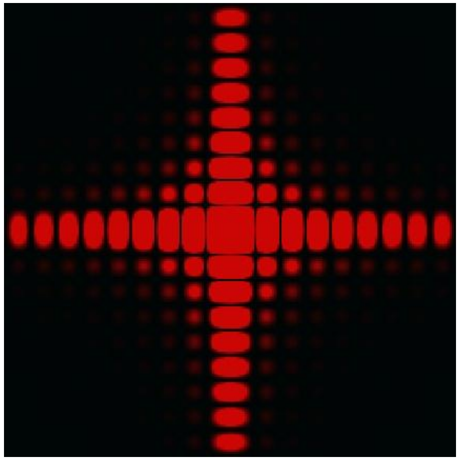


- When waves pass through an aperture or past the edge of an obstacle, they always spread to some extent into the region which is not directly exposed to the oncoming waves. This phenomena is called diffraction

Diffraction of Light

- Deviation of light from rectilinear propagation
  - Light passes by a sharp edge

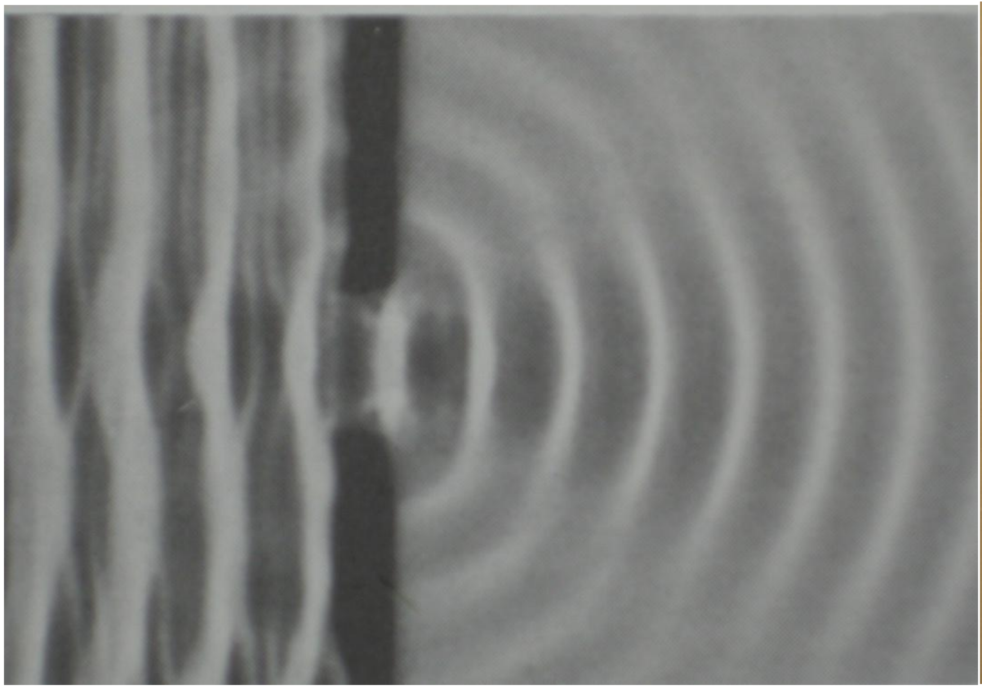


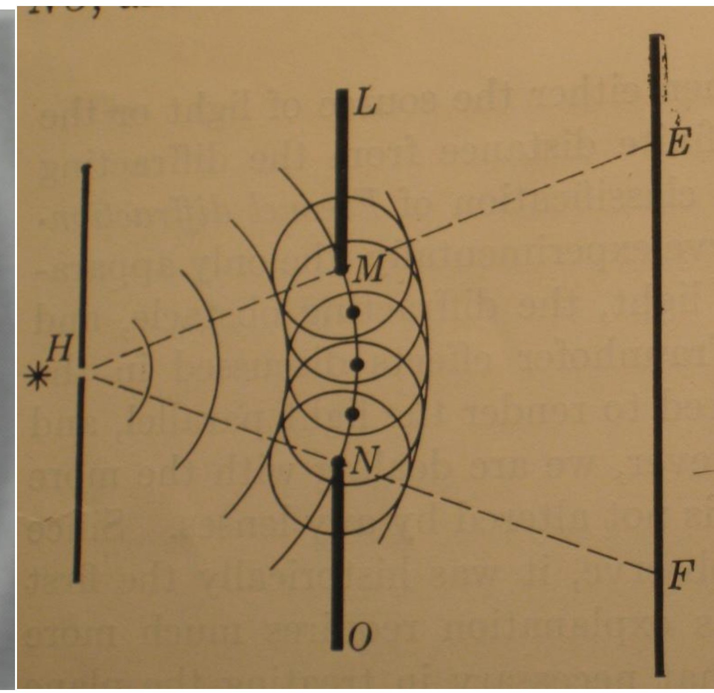


- Diffraction of light through a narrow slit
- Diffraction of light through a small square aperture

Diffraction

- Generation of a spherical wave
- Wave front construction over wavelets.

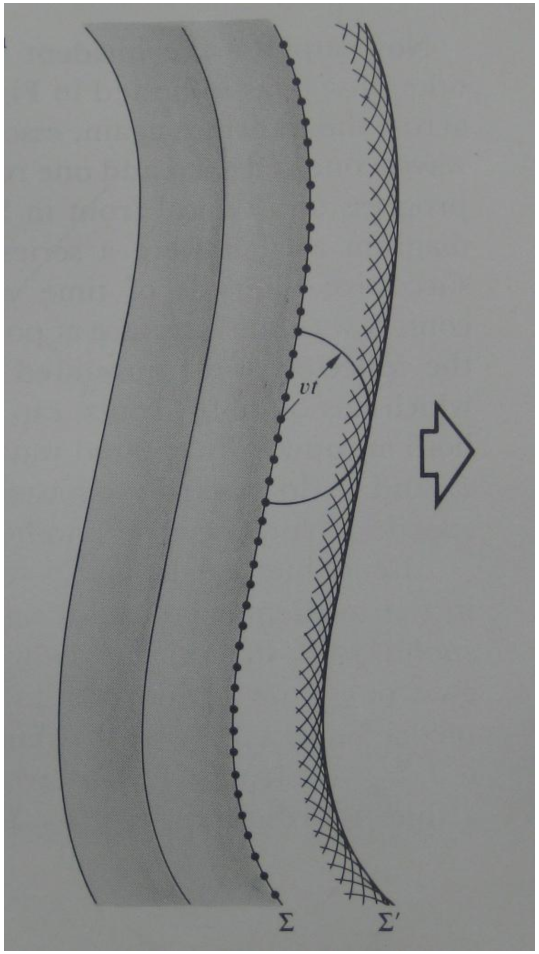




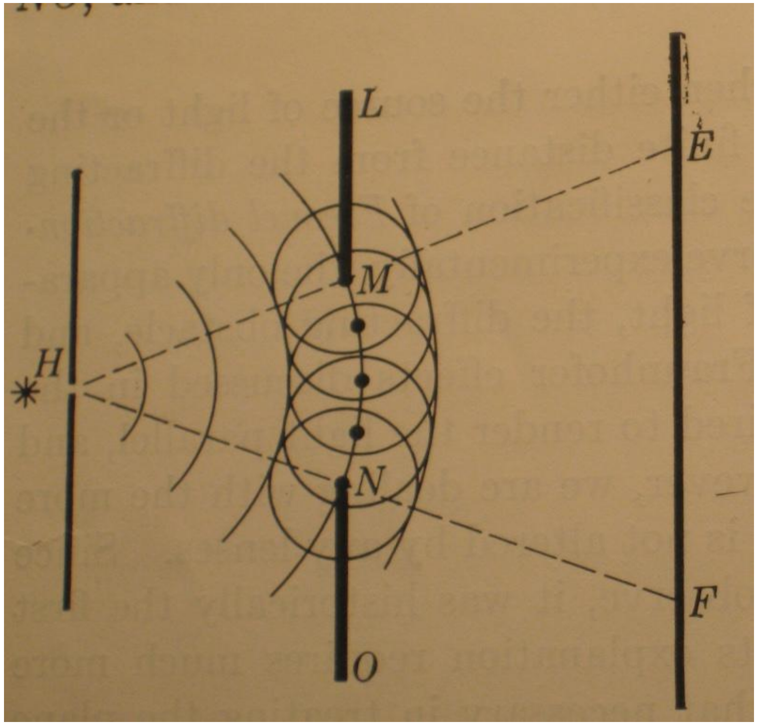
Huygens' Principle

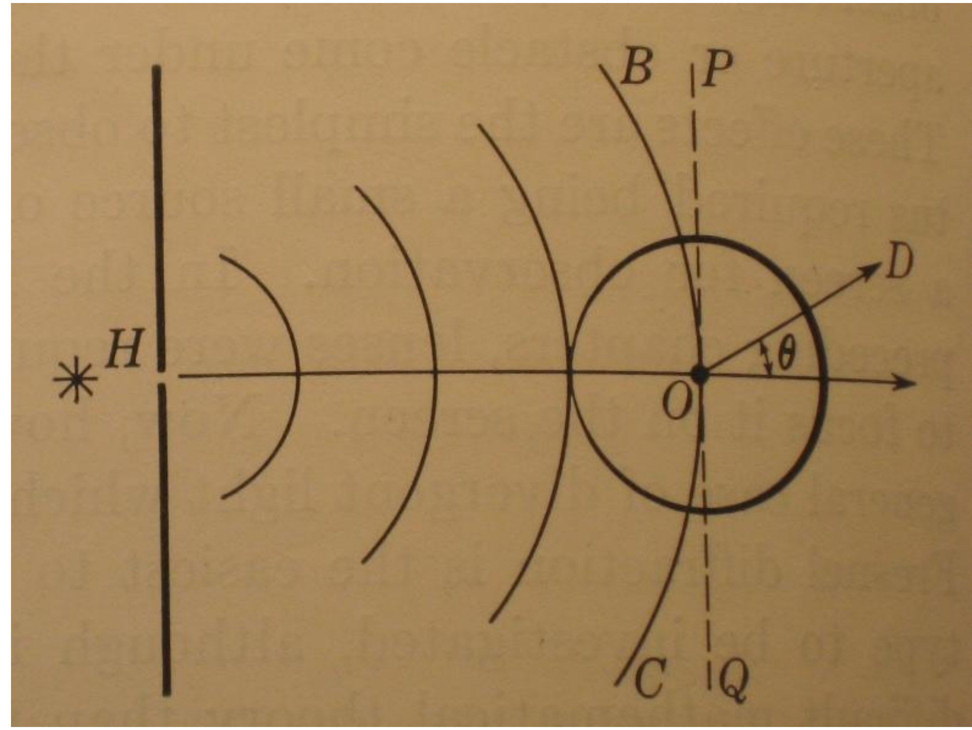
- In order to explain this bending of light, Huygens proposed: Each point on a wave front may be regarded as a new source of waves (as observed in the surface wave)
- Huygens-Fresnel principle:

  Every unobstructed point of a wave front, at a given instant in time, serves as a source of spherical secondary wavelets (with the same frequency as that of the primary wave). The amplitude of the optical filed at any point beyond is the superposition of all these wavelets (considering their amplitude and relative phase).



Huygens' principle, obliquity factor





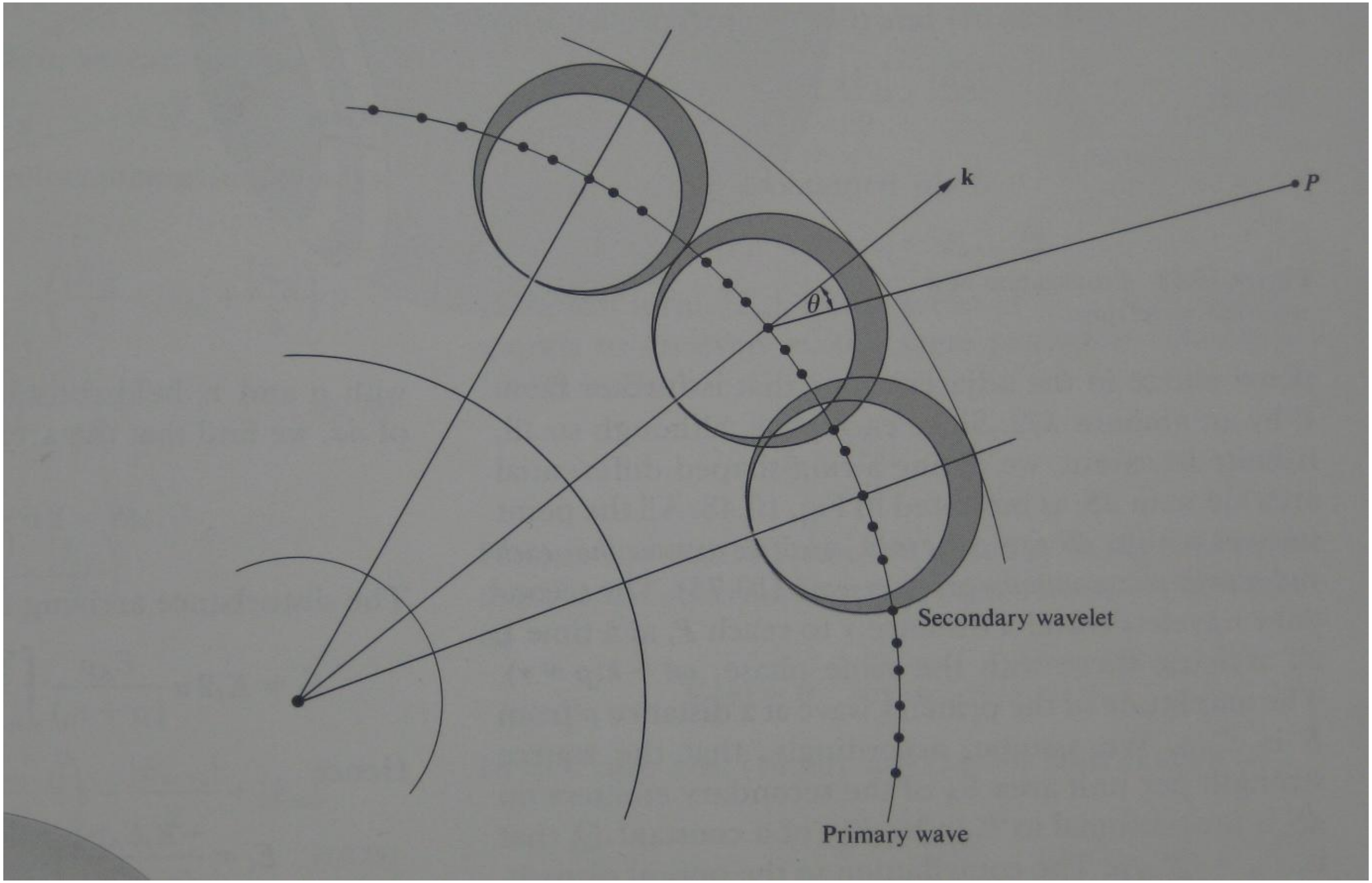
Backward wave ? Never observed

- $K(\theta) = ½ (1 + \cos\theta)$
- $K(0) = 1$, $K(\pi) = 0$

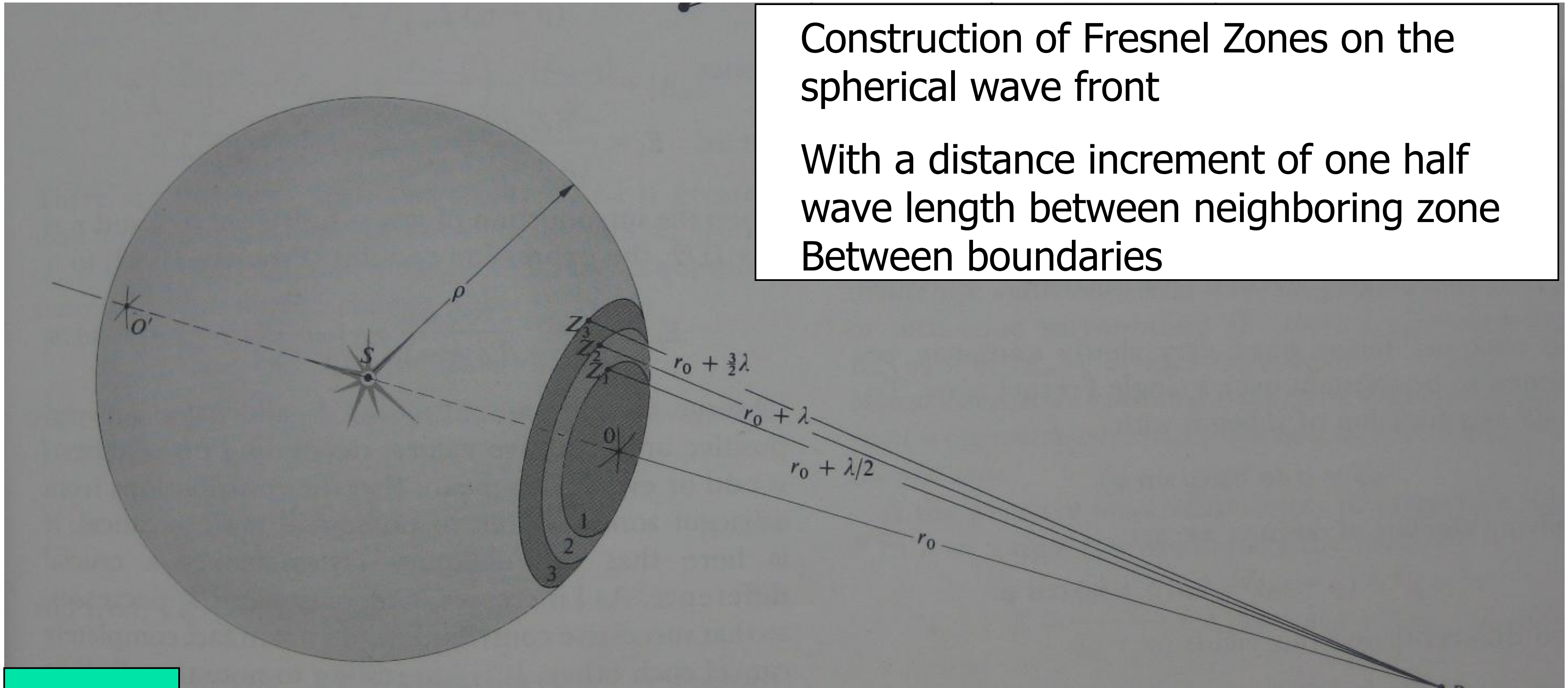
Obliquity or inclination factor – Huygen Fresnel Principle

- If each wavelet radiated uniformly in all directions, in addition to generating an ongoing wave, there would also be a reverse wave traveling back toward the source. No such wave is found experimentally, so we must modify the radiation pattern of the secondary emissions.
- In order to meet this condition, the amplitude of the wave generated from the point on the envelope of the spherical wavelet is modified by an obliquity or inclination factor
- $K(\theta) = ½ (1 + \cos \theta)$.
- Where θ is the angle between the normal of the combined wave front and the radial vector of the spherical wave of the secondary wavelet. $K(0) = 1$, $K(\pi) = 0$

Amplitude modified secondary spherical wavelets



FRENEL DIFFRACTION SCALAR THEORY
The free propagation of a Spherical Wave



Propagation of a spherical wavefront.

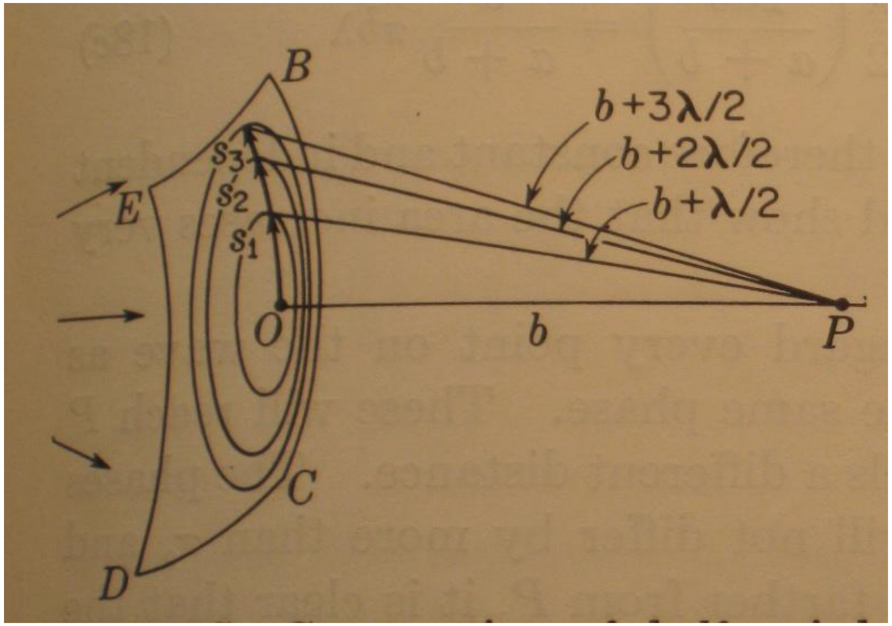
Construction of Fresnel Zones on the spherical wave front

With a distance increment of one half wave length between neighboring zone Between boundaries

FRENEL DIFFRACTION

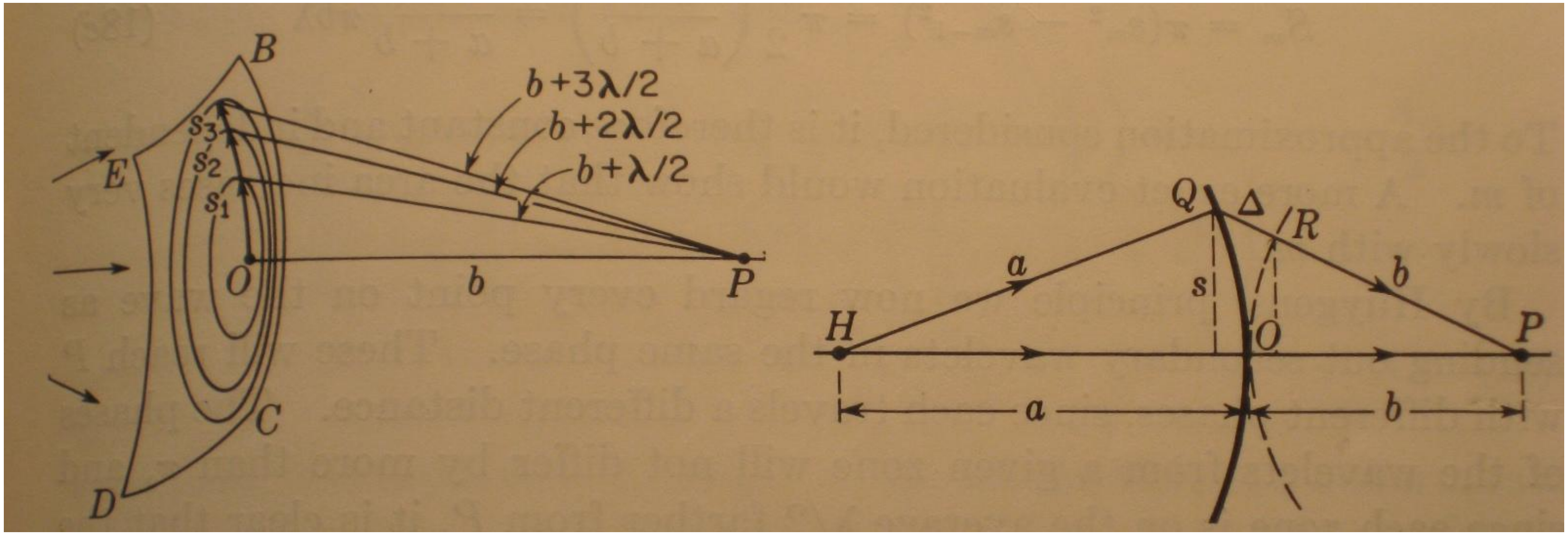
The free propagation of a Spherical Wave

- The wave front is divided into a number of annular regions.
- The boundaries of the various regions correspond to the intersections of the wave front with a series of spheres centered at P of radius r + λ/2, r+ λ , r + 3λ /2…. R + n λ /2

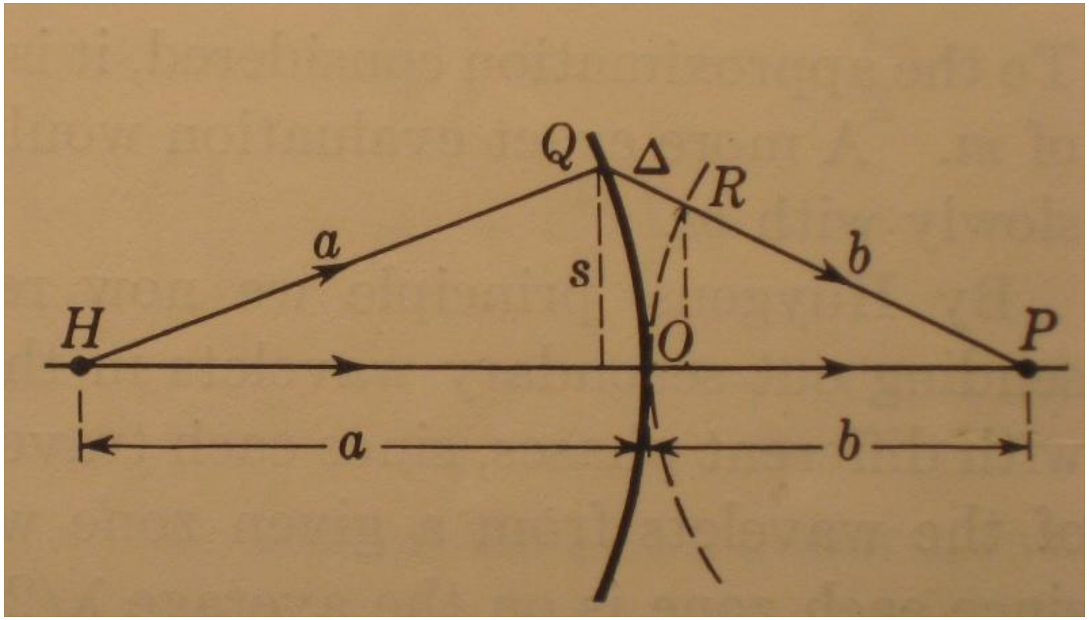


Fresnel Zones

- Construction of half-period zones on a spherical wave front.
- Path difference Δ at a distance s form the pole of a spherical wave.



Evaluation of the area of each Fresnel Zone



- The distance s is small compared with a and b, *s* << *a*,*b*
- Δ is approximated by the sum of the sagitta

$$\Delta = \frac{s^2}{2a} + \frac{s^2}{2b} = s^2 \frac{a+b}{2ab}$$

Approximation of the Area of Fresnel Zone

- The radii s of the Fresnel zones are such that

$$m\frac{\lambda}{2} = s_m{}^2\,\frac{a+b}{2ab}$$

- The area of any one zone becomes equal

$$S_m = \pi(s_m{}^2 - s_{m-1}{}^2) = \pi\frac{\lambda}{2}\left(\frac{2ab}{a+b}\right) = \frac{a}{a+b}\,\pi b\lambda$$

- A more exact evaluation would show that the area increases very slowly with *m*.

The phase relationship between successive Fresnel zones

- The phases of the wavelets from a given zone will not differ by more than π, and since each zone is on the average λ/2 farther from P, it is clear that the successive zones will produce resultants at P which differ by π.
- If we represent by A_m the resultant amplitude of the light from the m zone, the successive values of A_m will have alternating signs, because changing the phase by π means reversing the direction of the amplitude vector. The resultant amplitude due to the whole wave A,

$$A = A_1 - A_2 + A_3 - A_4 \ldots\ldots + (-1)^{m-1} - A_m$$

Evaluation of the m-zone amplitude – Decreasing with m

- Reasons
  - 1. The area of each zone determines the number of wavelets it contributes, the term should be approximately equal but should increase slowly.
  - 2. Inversely proportional to the distance $d_{m,}$, $1/d_m$
  - 3. Decreasing with increasing obliquity angle θ, according to the obliquity factor.
- A_m = constant . $(S_m /d_m) (1 + \cos\theta)$
  - where d_m is the *average distance to P*. $d_m = b + \Delta$
  - S_m is proportional to $b + \Delta$ instead of b with exact calculation.
  - S_m /d_m becomes constant, independent of m
- A_m *decreases slowly due to* $(1 + \cos\theta)$, and becomes nearly equal at large m.

Evaluation of the resulting term

- If m is odd, grouping the terms:

$$A = \frac{A_1}{2} + \left(\frac{A_1}{2} - A_2 + \frac{A_3}{2}\right) + \left(\frac{A_3}{2} - A_4 + \frac{A_5}{2}\right) + \cdots + \frac{A_m}{2}$$

$$= A_1 - \frac{A_2}{2} - \left(\frac{A_2}{2} - A_3 + \frac{A_4}{2}\right) - \left(\frac{A_4}{2} - A_5 + \frac{A_6}{2}\right) - \cdots$$

$$- \frac{A_{m-1}}{2} + A_m$$

- Terms within the parentheses are all positive, because the amplitude is decreasing
- A= $A_1/2 + A_m/2$
- A= $A_1/2 - A_m/2$ if m is even.

Application of the Fresnel diffraction theory in synchrotron light diagnostics

- Optimum size of the pinhole for a pinhole camera
- Fresnel lens for X-ray imaging

Resolution of a pinhole camera

- The best resolution in general is found, when the aperture as seen from the image includes about *nine-tenths of the first Frenel zone*, m=1 so that if *a* is the distance of the object, b that of the image from the screen and r the radius of the opening,

$$r^2 (a + b)/(ab) = 0.9 \lambda$$

- If a >> b, in the open air,

$$\text{Diameter} = 1.8972 \ (b \lambda)^{1/2}$$

Fresnel Lens

- $A = A_1 - A_2 + A_3 - A_4 \ldots\ldots.$
- *By covering the even zones*, $A = A_1 + A_3\ \ldots\ldots$
- By covering the odd zones, $A = -A_2 - A_3 - A_4 \ldots\ldots.$
- All the terms are in phase with each other and add up at P.