## US Particle Accelerator School

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## Topics

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# 2. Purpose, Goals and Intended Audience 

## Purpose

- Provide an overview of Accelerator Power Electronics Engineering with an emphasis on Power Conversion


## Goals

- Provide a historical overview of Accelerator Power Supplies from early designs, to presently employed technology, to some promising future developments now in incubation
- Give other, non-power conversion disciplines a glimpse of, and a better understanding of, Power Conversion Engineering
- Survey the most pertinent power supply topologies from the perspectives of load type and rating
- Define the information needed for the power supply designer to make appropriate choices for power supply type, design, and rating
- Civil, Mechanical Designers - interest in facility space, mounting, cooling

- Control Designers - an insight into some interface requirements

- Electrical Distribution System Designers - AC distribution requirements, address and reduce harmonics and EMI
- Maintenance Personnel - power system reliability and maintainability
- Magnet Designers - tradeoffs between power supply output voltage, current and stability limitations and the magnet design. The power supply role in magnet protection via cooling interlocks and ground fault detection and protection

- Operators - Power supply control and operating characteristics

- Physicists - Power system rating limitations, magnet configuration options vs. physics tradeoffs, long and short-term current stability limitations
- Power Conversion / Power Supply Designers - power systems from another point of view

- Project Engineers and Managers - Power conversion system costs

- Safety Engineers / Designers - Personnel and equipment safety in an electrical power environment. General power safety provisions


# 3. Typical Load Types 

* Resistive - Filament and Titanium Sublimation Pumps (TSPs)
- Capacitive Loads
* Direct Current (DC) Magnets
* Pulsed Magnets - Beam Separators, Deflectors, Electron Beam Guns, Kickers
* Radio Frequency (RF) loads - Klystrons, Thyratrons, Power Tubes
* Laser Drivers


## Electron Beam Guns (Filament) / Titanium Sublimation Pump Heaters

- High temperature $-1,500{ }^{\circ} \mathrm{C}$ not uncommon
- High current - 10 to l00s of amperes, low voltage, typically < 50 V
- Short thermal time-constants - 100s of milliseconds, power stability needed to keep temperature constant
- Resistive with (+) metal or (-) carbon temperature coefficient of resistance
- Power with constant voltage, current or power, depending upon circumstances
- Heat gradually to avoid thermally shocking and breaking brittle loads
- Usually linear V-I and R-T characteristics, but sometimes non-linear




## Electron Beam Gun Filaments／Titanium Sublimation Pump Heaters Ideal Characteristics

－Low potential barrier（work function）
－High melting point
－Chemical stability at high temperatures
－Long life
＂ペ＂。＂
Work function－the minimum energy which must be supplied to extract an electron from a solid；symbol $\phi$ ，units $J$（joule），or more often eV （electron－volt）．It is a measure of how tightly electrons are bound to a material．The work function of several metals is given below：

| Material | Work function（eV） |
| :---: | :---: |
| Sodium | 2.75 |
| Silver | 4.26 |
| Silicon | 4.60 |
| Gold | 5.31 |
| Graphite | 5.37 |
| Tungsten | 5.40 |

## Titanium Sublimation Pumps (TSP s)

- Titanium Sublimation Pumps (TSPs) are used to pump chemically reactive, getterable gases, such as $\mathrm{H}_{2}, \mathrm{H}_{2} \mathrm{O}, \mathrm{CO}, \mathrm{N}_{2}, \mathrm{O}_{2}, \mathrm{CO}_{2}$ from vacuum vessels. Titanium is effective, easily sublimed, and inexpensive.
- TSPs filaments are $85 \%$ titanium and $15 \%$ molybdenum, a combination which prevents premature filament "burnout" and have high pumping speeds, typically $10 \mathrm{l} / \mathrm{sec} / \mathrm{cm}^{2}$

(n)

Sublimate - To transform directly from the solid to the gaseous state or from the gaseous to the solid state without becoming a liquid.

## DC Magnet Loads - Characteristics

- Linear and inductive with long ( mS to sec) electrical time-constants ( $\tau=L / R$ )
- Families include dipole steering, quadrupole and sextupole focusing / defocusing, corrector/trims
- Driven by constant current and require high current stability ( $\triangle$ I in PPM)
- Correctors / trims frequently require current modulation for beam-based alignment / diagnostic systems, orbit correction and stabilization
- Air-cooled or water-cooled (temperature or flow interlocks to power supply)
- Occasionally series-connected in strings and powered from a common power supply to reduce power system cost


Representation

## DC Magnet Loads - Characteristics



Using Kirchoff's voltage law (KVL):
$-v(t)+\left(R_{\text {cable }}+R_{\text {magnet }}\right) i(t)+L \frac{d i(t)}{d t}=0$
$R i(t)+L \frac{d i(t)}{d t}=v(t)$
Converting to the s domain
$R I(s)+\operatorname{LsI}(s)-\operatorname{Li}(0)=V(s), \quad B u t i(0)=0$ and $V(s)=\frac{V}{s}$



Rearranging gives
$I(s) \frac{L}{R}\left(s+\frac{R}{L}\right)=\frac{V}{R} \frac{1}{s} \quad$ let $\frac{R}{L}=\alpha$ and $\frac{L}{R}=\frac{1}{\alpha}=\tau$
$I(s)=\frac{V}{R} \frac{\alpha}{s(s+\alpha)}$

$$
i(t)=\frac{V}{R}\left(1-e^{-\frac{t}{\tau}}\right)
$$



## DC Magnet Loads - Characteristics




DC V-I Characteristic

## DC Magnet Loads - Characteristics




String (series-connect) magnets for economy when magnets are not adjacent to insertion devices or no local optics tuning is involved. The current in each series-connected magnet is the same.

## Klystron Load

- Klystrons are used in RF and microwave systems to accelerate electron beams
- Their transfer function is called perveance ( $k$ ) which is the ratio between beam current and accelerating voltage. Perveance is usually expressed in $\mu$ p.
- In booster rings they are used in a pulsed mode to accelerate the electron beam
- In storage rings they are used as continuous-mode to supply make-up energy to the electron beam to compensate for the energy lost by synchrotron radiation


Klystron Load


- Electrons and positrons may be accelerated by injecting them into structures with traveling electromagnetic waves
- The microwaves from klystrons in the are fed into the accelerator structure via waveguides. This creates a pattern of electric and magnetic fields, which form an electromagnetic wave traveling down the accelerator. The beam energy is a function of the energy boost per klystron and the total number of klystrons.



## Electron Beam Gun Electrical Load Characteristics

- Electron gun exhibits non-linear V-I characteristics
- Capacitive loading
- High voltage, low DC current
- High peak pulsed current
- Subject to arcing
- Limited fault energy capability - arc protection (crowbar) needed


If work surface (anode) is difficult to insulate - put at ground potential. Float filament at $H V$.


If work surface (anode) is easy to insulate - float at HV. Put filament at ground potential.

## Pulsed Loads - Beam Separators and Deflectors

## Characteristics

- Capacitive loading
- High voltage, low DC current
- High peak pulsed current
- Subject to arcing
- Limited energy capability - arc protection (crowbar) needed


Fig. 2 Separator chamber.

- Kicker magnets interact with positively or negatively charged particle beams which, in most cases, are grouped into bunches
- The purpose of an injection kicker is to fully deflect (kick) bunches, without disturbance to the preceding or following bunches from a linear section (LINAC) into a storage ring
- An ejection kicker will do the inverse, that is,
 kick a particle beam into a linear line section from a storage ring
- Short time constants $(\tau=L / R)<1 m S$
- Characteristic impedance is like a transmission line
- High voltage, low impedance
- Fast pulse, match or terminating resistors
- Subject to reflection and breakdown

Fig. 5. SLAC-designed kicker magnet.


Top view


Fig. 6. SLAC-style kicker magnet.

## Flash Lamp Load Characteristics

- Pulsed load - a gas-filled flash lamp
- High voltage, high current load
- Highly non-linear load, subject to break-down
- Subject to aging, misfiring, arcing, and jitter
- Limited energy capability - arc protection (crowbar) needed


Fig. 2 Series and Coil based Trigger systems

# 4. Power Line Considerations 

## Power Line Considerations

- Fundamental Quantities
- The "Per Unit" Calculation System
- Harmonics and Fourier Series
- Harmonic Distortion and Causes
- Why Power Factor is Important
- Electromagnetic Interference (EMI) / Electromagnetic Compatibility (EMC)


## Characteristics of Sinusoidal Waves

- Generation of sine waves

- Plotting of sine waves

- Expression of sine waves

$$
\begin{aligned}
& v(t)=V_{\max } \sin (\omega t) \\
& \omega=2 \pi f
\end{aligned}
$$

- Harmonics between sine waves

- Average value:

$$
V_{a v e}=\frac{1}{T} \int_{0}^{T} v(t) d t
$$

for AC sine system

$$
v(t)=V_{m} \cos (\omega t) \text {,then } V_{\text {ave }}=\frac{1}{T} \int_{0}^{T} V_{m} \cos (\omega t) d t=0
$$

- RMS value:

$$
V_{r m s}=\sqrt{\frac{l}{T} \int_{O}^{T} v(t)^{2} d t}
$$

for AC sine system

$$
V_{r m s}=\sqrt{\frac{1}{T} \int_{0}^{T}\left(V_{m} \cos (\omega t)\right)^{2} d t}=V_{m} \sqrt{\frac{1}{T} \int_{0}^{T} \frac{1+\cos (2 \omega t)}{2} d t}=\frac{V_{m}}{\sqrt{2}}
$$

Some American "Standard" Commercial and Residential AC Voltages

| Class | Voltage | Type | Derivatives |
| :---: | :---: | :---: | :---: |
| High <br> Voltage | 138 kV | $3 \phi$ | None |
|  | 69 kV | $3 \phi$ | None |
|  | 13.8 kV | $3 \phi$ | None |
|  | 4.16 kV | $3 \phi$ | None |
| Low <br> Voltage | 480 V | $3 \phi$ | None |
|  | 1208 V | $3 \phi$ | $277 \mathrm{~V}, 1 \phi$ |
|  | $1 \phi$ | $120 \mathrm{~V}, 1 \phi$ |  |

$$
V_{L L}(R M S)=\sqrt{\frac{1}{T} \int_{0}^{T} v_{L L}{ }^{2}(t) d t}
$$

For $1 \phi$ AC input

$$
\begin{aligned}
& V_{\phi}=V_{L L} \\
& I_{\phi}=I_{L}
\end{aligned}
$$

where $V_{\phi}$ and $I_{\phi}$ are $R M S$ values


The Apparent, Real and Reactive "powers" are:
Apparent $\quad S_{I \phi}=V_{L L} I_{L}=P_{I \phi}+j Q_{I \phi} \quad V A$
Real $\quad P_{I \phi}=V_{L L} I_{L} \cos \left(\alpha_{I_{L}}-\beta_{V_{L L}}\right)$ Watt $\cos \left(\alpha_{I_{L}}-\beta_{V_{L L}}\right)=$ powerfactor
Reactive $\quad Q_{I \phi}=V_{L L} I_{L} \sin \left(\alpha_{I_{L}}-\beta_{V_{L L}}\right) \quad \operatorname{VAR}$
All "powers" are average "powers"
$S_{l \phi}=\sqrt{\frac{1}{T} \int_{0}^{T} v_{L L}^{2}(t) d t} * \sqrt{\frac{1}{T} \int_{0}^{T} i_{L}^{2}(t) d t}=\frac{1}{T} \int_{0}^{T} v_{L L}(t) i_{L}(t) d t$

Instantaneous power $p(t)$ is the product of the instantaneous voltage $v(t)$ and the instantaneous current i(t)
Derivation

$$
p(t)=v(t) i(t)
$$

let $\quad v(t)=\sqrt{2}|V| \sin (\omega t)$

$$
i(t)=\sqrt{2}|\boldsymbol{I}| \sin (\omega t+\phi)
$$

and

$$
p(t)=\sqrt{2}|V| \sin (\omega t) \sqrt{2}|I| \sin (\omega t+\phi)=2|V \| I| \sin (\omega t) \sin (\omega t+\phi)
$$

Identity

$$
\sin (a) \sin (b)=\frac{1}{2}(\cos (a-b)-\cos (a+b))
$$

Substituting

$$
\begin{aligned}
& p(t)=|V||I|[\cos (\omega t-\omega t-\phi)-\cos (\omega t+\omega t+\phi)] \\
& p(t)=|V||I|[\cos (-\phi)-\cos (2 \omega t+\phi)]
\end{aligned}
$$

recognizing that $\cos (-\phi)=\cos (\phi)$

$$
p(t)=|V||I| \cos (\phi)-|V||I| \cos (2 \omega t+\phi)
$$

It is seen that
$p(t)=D C$ component + AC component with twice the frequency of the voltage or current
The DC component is a max imum when the voltage and current are in phase
The power is the product of the RMS values of the line-line voltage and line current



- Voltage leads current by $60^{\circ}$ at 60 Hz
- Power is + (delivered to load) and - (returned to the AC line) and 120 Hz


## Three Phase Wye



$$
\begin{aligned}
& V_{L L}=\sqrt{3} V_{\phi} \text { and } I_{L}=I_{\phi} \\
& S_{3 \phi}=3 V_{\phi} \quad I_{\phi} \\
& S_{3 \phi}=\sqrt{3} V_{L L} \quad I_{L}
\end{aligned}
$$

Three Phase Quantities - Delta


$$
\begin{aligned}
& V_{L L}=V_{\phi} \quad \text { and } I_{L}=\sqrt{3} I_{\phi} \\
& S_{3 \phi}=3 V_{\phi} \quad I_{\phi} \\
& S_{3 \phi}=\sqrt{3} V_{L L} \quad I_{L}
\end{aligned}
$$

Three Phase Quantities


For Wye
For Delta
$S_{3 \phi}=\sqrt{3} V_{L L} I_{L} \quad S_{3 \phi}=\sqrt{3} V_{L L} I_{L}$
$V_{A-B}=\left|V_{A-B}\right| e^{j 0} V_{B-C}=\left|V_{B-C}\right| e^{-j 120} V_{C-A} \neq\left|V_{C-A}\right| e^{-j 240}$

Three Phase Quantitios


- 3 times the single phase power with only 3 conductors, not 6
- For balanced load, $p(t)$ is constant

Section 4 - Power Line Considerations

## Three Phase Quantities

This will show that three phase power is constant
$p(t)=v_{a b}(t)+v_{b c}(t)+v_{c a}(t)$
$\boldsymbol{p}(\boldsymbol{t})=\frac{1}{\sqrt{3}}\left[\sqrt{2}\left|\boldsymbol{V}_{\boldsymbol{a}}\right| \sin (\omega t) \sqrt{2}\left|\boldsymbol{I}_{\boldsymbol{a}}\right| \sin (\omega t-\phi)\right.$

$$
+\sqrt{2}\left|\boldsymbol{V}_{\boldsymbol{b}}\right| \sin \left(\omega t-120^{\circ}\right) \sqrt{2}\left|\boldsymbol{I}_{\boldsymbol{b}}\right| \sin \left(\omega t-120^{\circ}-\phi\right)
$$

$$
\left.+\sqrt{2}\left|V_{\boldsymbol{c}}\right| \sin \left(\omega t-240^{\circ}\right) \sqrt{2}\left|I_{\boldsymbol{c}}\right| \sin \left(\omega t-240^{\circ}-\phi\right)\right]
$$

But $\frac{\left|V_{\boldsymbol{a}}\right|}{\sqrt{3}}=\frac{\left|V_{\boldsymbol{b}}\right|}{\sqrt{3}}=\frac{\left|V_{\boldsymbol{c}}\right|}{\sqrt{3}}=\frac{|V|}{\sqrt{3}}$ and $\left|I_{\boldsymbol{a}}\right|=\left|I_{\boldsymbol{b}}\right|=\left|I_{\boldsymbol{c}}\right|=|I|$
Identity

$$
\sin (\alpha) \sin (\beta)=\frac{1}{2}(\cos (\alpha-\beta)-\cos (\alpha+\beta))
$$

Making the substitutions we have
$p(t)=\frac{1}{\sqrt{3}}\lceil|V||I|[\cos (\omega t-\omega t+\phi)-\cos (\omega t+\omega t-\phi)$
$+|\boldsymbol{V}||I|\left[\cos \left(\omega t-120^{\circ}-\omega t+120^{\circ}+\phi\right)-\boldsymbol{c o s}\left(\omega t-120^{\circ}+\omega t-120^{\circ}-\phi\right)\right.$
$+|V||I|\left[\cos \left(\omega t-240^{\circ}-\omega t+240^{\circ}+\phi\right)-\cos \left(\omega t-240^{\circ}+\omega t-240^{\circ}-\phi\right)\right]$
$p(t)=\frac{|V|}{\sqrt{3}}|I|\left[\cos (\phi)-\cos (2 \omega t-\phi)+\cos (\phi)-\cos \left(2 \omega t-240^{\circ}-\phi\right)+\cos (\phi)-\cos \left(2 \omega t-480^{\circ}-\phi\right)\right]$
Re cognizing that $\cos \left(-480^{\circ}\right)=\cos \left(-120^{\circ}\right)$ the following is obtained
$p(t)=\frac{|V|}{\sqrt{3}}|I|\left[\cos (\phi)-\cos (2 \omega t-\phi)+\cos (\phi)-\cos \left(2 \omega t-240^{\circ}-\phi\right)+\cos (\phi)-\cos \left(2 \omega t-120^{\circ}-\phi\right)\right]$
$p(t)=\frac{|V|}{\sqrt{3}}|I|\left[3 \cos (\phi)-\left(\cos (2 \omega t-\phi)+\cos \left(2 \omega t-120^{\circ}-\phi\right)+\cos \left(2 \omega t-240^{\circ}-\phi\right)\right)\right]$
Acknowledging that $\boldsymbol{\operatorname { c o s }}(2 \omega t-\phi)+\cos \left(2 \omega t-120^{\circ}-\phi\right)+\cos \left(2 \omega t-240^{\circ}-\phi\right)=0$ yields
$p(t)=3 \frac{|V|}{\sqrt{3}}|I| \cos (\phi)=\sqrt{3}|V||I| \cos (\phi)$
A constant power, with a maximum DC offset when $\phi=0^{\circ}$ and where $V$ and I are RMS values

Delta - Wye Configuration - The Preferred Choice For Power Systems



## Neutral Wire Size - Unbalanced and / or Non-linear Loads



For balanced linear or non-linear loads

$$
\left|I_{A}\right|=\left|I_{B}\right|=\left|I_{C}\right|=\left|I_{L}\right|
$$ $\left|I_{N}\right|=\sqrt{\left|I_{A}\right|^{2}+\left|I_{B}\right|^{2}+\left|I_{C}\right|^{2}}=\sqrt{3} I_{L}$

For unbalanced linear or non-linear loads $\quad\left|I_{A}\right| \neq\left|I_{B}\right| \neq\left|I_{C}\right|$ $\left|I_{N}\right|=\sqrt{\left|I_{A}\right|^{2}+\left|I_{B}\right|^{2}+\left|I_{C}\right|^{2}}$
The neutral conductor can safely be sized for $\sqrt{3} * \operatorname{MAX}\left(I_{A}, I_{B}, I_{C}\right)$

## Why Used

- To make quantities and values convenient and manageable
- To put quantities on a single per phase or 3-phase basis
- To avoid having to remember to correct for transformer turns ratios, reflected voltages, current and impedances
- No worries about delta or wye configurations


## Why Mentioned Here

- Because the power supplies will interface to the AC line
- Because all AC power equipment (generators, motors, transformers and chokes) impedances are expressed in \%
- Because line limitations (short-circuit currents, V droop, transients, harmonics) must be considered. These effects are usually calculated in the per unit system

The "Per Unit" or "Percent" Calculation System

| Establish Power, Voltage, Current and Impedance Bases |  |  |  |
| :---: | :---: | :---: | :---: |
| Base | Per $\phi$ Phase | 3 Phase | Notes |
| $S, P, Q$ | = Base kVA | $\begin{aligned} & =\text { Base } k V A \\ & =3^{*} \operatorname{per} \phi \text { Base } k V A \end{aligned}$ | One power base must be used throughout |
| V | $=$ Base kV $(L-N)$ | $=$ Base kV (L-L) | $V$ Base location dependent |
| I | = Base kVA / Base kV | = Base kVA / $\sqrt{3}$ Base kV | I Base location dependent |
| Z | $=\left(\right.$ Base kV) ${ }^{2} /$ Base kVA | $=\left(\right.$ Base kV) ${ }^{2} /$ Base kVA | Z Base location dependent Z Base phase independent per $\phi$ Z Base $=3 \phi$ Z Base |

- p.u. = actual value / Base value
- p.u. new $=$ p.u. ${ }_{\text {given }}\left(\frac{\text { Base } k V_{\text {given }}}{\text { Base } k V_{\text {new }}}\right)^{2} *\left(\frac{\text { Base } k V A_{\text {new }}}{\text { Base }^{k V A_{\text {given }}}}\right)$
- \% = p.u. * $100 \%$
- Choose the system and base that yield the most convenient numbers and calculations!



## Establish Bases

In Section A

Base $S=10,000 \mathrm{kVA}$
Base $V=13.8 \mathrm{kV}$
Base $I=\frac{S}{V}=\frac{10,000 \mathrm{kVA}}{13.8 \mathrm{kV}}=725 \mathrm{~A}$
Base $Z=\frac{V^{2}}{S}=\frac{(13.8 \mathrm{kV})^{2}}{10,000 \mathrm{kVA}}=19 \Omega$

Section B
$S=10,000 \mathrm{kVA}$
$V=138 k V$
$I=\frac{10,000 \mathrm{kVA}}{138 \mathrm{kV}}=72.5 \mathrm{~A}$
$Z=\frac{(138 \mathrm{kV})^{2}}{10,000 \mathrm{kVA}}=1900 \Omega$

Section C
$S=10,000 \mathrm{kVA}$
$V=69 \mathrm{kV}$
$I=\frac{10,000 \mathrm{kVA}}{69 \mathrm{kV}}=145 \mathrm{~A}$
$Z=\frac{(69 \mathrm{kV})^{2}}{10,000 \mathrm{kVA}}=476 \Omega$

Obtain pu values


Combine impedances - Solve for I


A transformer impedance of 5\% means:

- The short circuit current is 20X rated full load input / output
- The voltage drop across the transformer at full load is $5 \%$ of rated


Referring to the one-line diagram below, determine the line currents in the:
A. Generator
B. Transmission Line
C. M1
D. M2


- Non-sinusoidal waves are complex and are composed of sine and cosine harmonics
- The harmonics are integral multiples of the fundamental frequency ( $1^{\text {st }}$ harmonic) of the wave. The second harmonic is twice the fundamental frequency, the third harmonic is $3 X$ the fundamental frequency, etc.


Harmonics, Complex Waveforms and Fourier Series


Trigonometric form of the Fourier Series
$a_{0}=\frac{1}{T} \int_{0}^{T} f(t) d t$
$a_{k}=\frac{2}{T} \int_{0}^{T} f(t) \cos k \omega_{0} t d t$
$b_{k}=\frac{2}{T} \int_{0}^{T} f(t) \sin k \omega_{0} t d t$

Complex form from Euler $e^{j x}=\cos x+j \sin x$
$c_{k}=\frac{1}{T} \int_{0}^{T} f(t) e^{-j k \omega_{0} t} d t$

Fourier Coefficient Facilitators


| Even function <br> symmetry | $f(t)=f(-t)$ | No sine terms | Has DC component if no <br> half-wave symmetry |
| :--- | :---: | :---: | :---: |
| Odd function <br> symmetry | $f(t)=-f(-t)$ | No cosine terms | No DC component |
| Half-wave <br> symmetry | $f(t)=-f(t-1 / 2 T)$ | Have sines and cosines <br> but only odd harmonics | No DC component |

Example of Even Function Symmetry



- $f(t)=f(-t)$ even function
- No sine terms
- No half-wave symmetry
- DC component, $a_{o}$
- $a_{k}=\frac{2}{T} \int_{0}^{T} f(t) \cos k \omega_{o} t d t$
- No even or odd function symmetry
- Have sine and cosine terms
- No half-wave symmetry
- DC component, $a_{o}$
- $a_{o}, a_{k}, b_{k}$ terms

- $f(t)=-f(-t)$ odd function
- No cosine terms
- Half-wave symmetry
- No DC component
- $b_{k}=\frac{2}{T} \int_{0}^{T} f(t) \sin k \omega_{o} t d t$

- No even or odd function symmetry
- Have sine and cosine terms
- No half-wave symmetry
- DC component exists
- $a_{o}, a_{k}, b_{k}$


## Distorted (Complex) Waveforms



## Homework Problem \#2

A waveform $v(t)$ was analyzed and found to consist of 6 components as shown here.


## Homework Problem \#2 (Continued)

a. Write the Fourier series for $v(t)$ in terms of $\omega t$
b. Show the harmonic content graphically by plotting the frequency spectrum
c. Give the numerical result of
$b_{3}=\frac{2}{T} \int_{0}^{T} v(t) \sin 3 \omega t d t$
Help : $\int \sin ^{2}(3 \omega t) d t=\frac{1}{3} * \frac{\frac{1}{2} \cos (3 \omega t) \sin (3 \omega t)+\frac{3}{2} \omega t}{\omega}$
$b_{4}=\frac{2}{T} \int_{0}^{T} v(t) \sin 4 \omega t d t$
Help : $\int \cos (4 \omega t) \sin (4 \omega t) d t=\frac{-1}{8} \frac{\cos ^{2}(4 \omega t)}{\omega}$

Each waveform below can be written as a Fourier series. The result depends upon the choice of origin. For each of the 6 cases, state the type of symmetry present, non-zero coefficients and the expected harmonics.


Signal Total Harmonic Distortion (THD): The ratio of the square root of the summed squares of the amplitudes of all harmonic frequencies above the fundamental frequency to the fundamental frequency

$$
\begin{aligned}
& T H D_{V}=\frac{\left[\sum_{i=2}^{N} V_{i}^{2}\right]^{1 / 2}}{V_{l}} * 100 \% \\
& T H D_{I}=\frac{\left[\sum_{i=2}^{N} I_{i}^{2}\right]^{1 / 2}}{I_{1}} * 100 \%
\end{aligned}
$$

- Unbalanced 3-phase, non-linear loads
- SCR or diode commutation


State 1: $A-B(+) S C R$ s $1-5$ On


State 2 : C-A (-), 5 off, 4 on, SCR s 1 - 4 On



$$
\begin{array}{ll}
V_{P N}=V_{A N}-V_{L s} & V_{L S}=L_{S} \frac{d i_{A}}{d t} \\
V_{u}=\frac{q}{\omega T} \int_{\alpha}^{\alpha+\mu} V_{L S} d(\omega t)=\frac{q}{2 \pi} \omega L_{S} \int_{0}^{I_{d}} d i_{A}=\frac{q}{2 \pi} \omega L_{S} I_{d} \\
V_{d}=V_{d o}-V_{u} & \begin{array}{l}
\text { Commutation } \\
\text { voltage drop }
\end{array} \\
V_{d}=\frac{3 \sqrt{2}}{\pi} V_{L L} \cos \alpha-\frac{q \omega L_{S}}{2 \pi} I_{d} &
\end{array}
$$

$\alpha=$ gate trigger retard angle and $q=$ number of possible rectifier states

## Conclusions

-The current commutation takes a finite commutation interval u.
-During the commutation interval, three SCRs conduct.
-Commutation voltage drop and hence the line voltage distortion is directly proportional to the inductive reactance of the input AC line or transformer and the DC current flowing in the load

SCR / diode commutation line notches:

- Are a source of line voltage distortion
- If deep enough, they cause extra zero crossovers in the line voltage. In 3 phase systems, instead of 2 zero crossovers per cycle, 6 additional zero crossovers can be experienced
- The extra zero crossovers can upset equipment timing. This can cause SCRs to trigger at the wrong time, damaging the power supply or cause false turn-on and damage to other equipment.

line notch


## Reducing SCR commutation effects

- Commutation notches (voltage drops) are directly proportional to system Z and DC load current. To reduce commutation notch depth, use a stiff (large, low Z) line.


Reducing SCR commutation effects on other equipment

- Isolate other equipment by placing them on another line



## International Harmonic Distortion Standards

| Australia | AS 2279 - "Disturbances in Mains Supply Networks" |
| :--- | :--- |
| Britain | G5/3 - "Standard for Harmonic Control in Power <br> Systems" |
| Europe | Verband Deutscher Elektrotechniker (VDE) <br> IEC 555 Series for harmonic current distortion limits for <br> small devices |
| United States | IEEE 519 - 1992 "Standard Practices and Requirements for <br> Harmonic Control in Electrical Power Systems". Used as the <br> standard around the world |

## Table 10.2 Low Voltage System Classification And Distortion Limits

|  | Special <br> Applications ${ }^{1}$ | General <br> System | Dedicated <br> System $^{2}$ |
| :---: | :---: | :---: | :---: |
| THD (Voltage) | $3 \%$ | $5 \%$ | $10 \%$ |
| Notch Depth | $10 \%$ | $20 \%$ | $50 \%$ |
| Notch Area ${ }^{3}$ | $16,400 \mathrm{~V}-\mu \mathrm{S}$ | $22,800 \mathrm{~V}-\mu \mathrm{S}$ | $36,500 \mathrm{~V}-\mu \mathrm{S}$ |
| 1. Airports and hospitals <br> 2. Exclusive use converters <br> 3. Multiply by V/480 for other than 480 V systems |  |  |  |

## IEEE 519-1992 Current Distortion Limits

| General Distribution Systems - 120 V Through 69 kV |  |  |
| :---: | :---: | :---: |
|  | $I_{S C} / I_{L}$ | Maximum THD |
|  | < 20 | 5 |
|  | $20<50$ | 8 |
|  | $50<100$ | 12 |
|  | $100<1,000$ | 15 |
|  | > 1,000 | 20 |
|  | um short-circu <br> m load current <br> tem short-circ | mmon Coupling (PCC) <br> oad current ratio |



Single Phase

$$
\begin{aligned}
& S_{I \phi}=V_{\phi} I_{\phi}=P_{I \phi}+Q_{I \phi} \\
& Q_{I \phi}=V_{\phi} I_{\phi} \sin \left(\alpha_{I_{\phi}}-\beta_{V_{\phi}}\right) \\
& P_{I \phi}=V_{\phi} I_{\phi} \cos \left(\alpha_{I_{\phi}}-\beta_{V_{\phi}}\right) \\
& P F=\frac{P_{I \phi}}{S_{I \phi}}=\cos \left(\alpha_{I_{\phi}}-\beta_{V_{\phi}}\right) \\
& 0 \leq P F \leq 1, \text { leading or lagging, current is reference } \\
& \text { PF is not efficiency } \quad \text { Eff }=\frac{P_{o}}{P_{i}}
\end{aligned}
$$

Balanced three Phase

$$
S_{3 \phi}=3 V_{\phi} I_{\phi}=\sqrt{3} V_{L L} I_{L}
$$

$$
P_{3 \phi}=3 V_{\phi} I_{\phi} \cos \left(\alpha_{I \phi}-\beta_{V_{\phi}}\right)
$$

$$
P F_{3 \phi}=\frac{P_{3 \phi}}{S_{3 \phi}}=\cos \left(\alpha_{I_{\phi}}-\beta_{V_{\phi}}\right)
$$

Unbalanced three phase power
$S_{3 \phi}=V_{\phi A} I_{\phi A}+V_{\phi B} I_{\phi B}+V_{\phi C} I_{\phi C}$
$P_{3 \phi}=V_{\phi A} I_{\phi A} \cos \left(\alpha_{I_{\phi A}}-\beta_{V_{\phi} A}\right)+V_{\phi B} I_{\phi B}\left(\alpha_{I_{\phi B}}-\beta_{V_{\phi} B}\right)+V_{\phi C} I_{\phi C}\left(\alpha_{I_{\phi} C}-\beta_{V_{\phi} C}\right)$
$P F_{3 \phi}=\frac{P_{3 \phi}}{S_{3 \phi}}$
Power Factor is Important - Capital Equipment Cost

$S=\frac{P}{P F}=\frac{500 \mathrm{~kW}}{0.65}=769 \mathrm{kVA}$
$I=\frac{769 \mathrm{kVA}}{\sqrt{3} * 480 \mathrm{~V}}=925 \mathrm{~A}$
$I_{C B}=925 A * 1.25=1,156 A$, buy $1200 A$
Buy 1000kVA switchgear/transformer

$S=\frac{P}{P F}=\frac{500 \mathrm{~kW}}{0.9}=555 \mathrm{kVA}$
$I=\frac{555 \mathrm{kVA}}{\sqrt{3} * 480 \mathrm{~V}}=667 \mathrm{~A}$
$I_{C B}=667 A^{*} 1.25=834 A$, buy $1000 A$
Buy 750kVA switchgear/transformer

Power Factor is Important - Energy Cost

$S=\frac{P}{P F}=\frac{5 M W}{0.65}=7.7 M V A$
Electric rate $=\frac{\$ 0.06}{k W-H r}$
9 months $* \frac{30 \text { days }}{\text { month }} * \frac{24 \mathrm{hr}}{d a y}=\frac{6480 \mathrm{hr}}{y r}$
$7.7 M V A * \frac{\$ 0.06}{k W-H r} * \frac{6480 h r}{y r}=\frac{\$ 3.0 M}{y r}$

$$
\frac{\$ 3 M}{y r} * 20 y r=\$ 60 M
$$



Electric rate $=\frac{\$ 0.06}{k W-H r}$
9 months $* \frac{30 \text { days }}{\text { month }} * \frac{24 h r}{\text { day }}=\frac{6480 h r}{y r}$
$5.6 M V A * \frac{\$ 0.06}{k W-H r} * \frac{6480 h r}{y r}=\frac{\$ 2.2 M}{y r}$
$\frac{\$ 2.2 M}{y r} * 20 y r=\$ 44 M$

## Power Factor Improvement

Higher Power Factor Translates to:

- Lower apparent power consumption
- Lower equipment electrical losses
- Electrically/physically smaller equipment
- Less expensive equipment
- Lower electric bill
- Implies lower distortion of the line voltage and current

A $10 \mathrm{~kW}, 3$ p power supply has an efficiency of $90 \%$ and operates with a leading power factor of 0.8. Determine the size of the inductor needed to improve the power factor to leading 0.95 .


# Electromagnetic Interference (EMI) Electromagnetic Compatibility (EMC) 

Electromagnetic Interference (EMI) is any electromagnetic disturbance that interrupts, obstructs, or otherwise degrades or limits the effective performance of electronics/electrical devices, equipment or systems. Sometimes also referred to as radio frequency interference (RFI)

Electromagnetic Compatibility (EMC) describes how an electronic device will behave in a "real world" setting of EMI

Broadband Interference This type of interference usually exhibits energy over a wide frequency range and is generally a result of sudden changes in voltage or current. It is normally measured in decibels above one micro-volt (or micro-ampere) per megahertz $d B \mu V / M H z$ or $d B \mu A / M H z$

Narrowband Interference has its spectral energy confined to a specific frequency or frequencies. This type of interference is usually produced by a circuit which contains energy only at the frequency of oscillation and harmonics of that frequency. It is normally measured in "decibels above one micro-volt (or micro-ampere)", e.g., $d B \mu \operatorname{Vor} d B \mu A$.

## Glossary of Terms

## Five Types of EMI

- Conducted Emissions (CE) - the EMI emitted into lines and connections by an electronic device. Of particular interest is the EMI conducted onto the AC input power lines
- Conducted Susceptibility (CS) - the EMI present on lines and connections (e.g. power lines) and its effect on a connected electronic device.
- Radiated Emissions (RE) - the EMI radiated by an electronic device
- Radiated Susceptibility (RS) - radiated EMI effect on an electronic device
- Electromagnetic Pulse (EMP) - radiated EMI by lightning or atomic blast


## Culprits and Victims

- Culprits are devices, equipment or systems that emit EMI
- Victims are devices, equipment or systems that are susceptible to EMI


## EMI / EMC Standards

## USA

- MIL-STD-461E Emissions \& Susceptibility Standard for Defense Electronics This standard sets the Emissions \& Susceptibility (Immunity) noise limits and test levels for electrical / electronic and electromechanical equipment
- MIL-STD-462E is the companion standard that describes the methods and test procedures for certification under MIL-STD-461.
- The object of the standards is to maximize safety and reliability and to minimize downtime and breakdowns of equipment essential for defense.
- The worldwide defense electronics and aerospace community recognizes and generally accepts MIL-STD-461.


## EMI / EMC Standards

USA
Federal Communications Commission (FCC) under the Code of Federal Regulations CFR, Part 15, Sub-Part J, for Class A and B devices and equipment.

## Germany

Verband Deutscher Elektrotechniker (VDE) has developed VDE 0871 for Level A and Level B.

## European Community <br> EMC Directives of 1996

The FCC and VDE specifications are similar in that Class A and Level A describe industrial equipment, while Class B and Level B are applicable to consumer equipment.

## Differential Mode Noise



- Produced as a natural result of complex, high frequency switching of V and I
- $V_{1}=-V_{2}$

- Magnitudes are equal
- Phase difference is $180^{\circ}$
- $V_{S U M}=V_{1}+V_{2}=0$
- $V_{\text {DIFF }}=V_{1}-V_{2}$
- $I_{\text {DIFF }}=\left(V_{1}-V_{2}\right) / R_{\text {Load }}$
- Filter from line-to-line

- Produced as a result of circuit imbalances, currents produced by simultaneous high frequency voltages on (+) and (-) lines
- Capacitive coupled to ground
- $V_{1}=V_{2}=V_{C O M}$
- Magnitudes are equal

- Phase difference is $0^{\circ}$
- $I_{\text {COM }}=V_{\text {COM }} / 2 \pi f C_{P}$
- Common mode current flows to ground
- Filter from lines to ground


## Conducted Emissions

## Conducted emissions

- EMI conducted onto AC Lines by the power supply.
- Typically 10 kHz to 30 MHz
- Measured in $\mu V$ or $d B-\mu V$ (Reference: $1 \mu V=0 d B$ )

$$
d B=20 * \log _{10} \frac{\text { measured } \mu \mathrm{V}}{1 \mu \mathrm{~V}}
$$



Test equipment used - Spectrum analyzers with Line Impedance
Stabilization Networks (LISNs) that

- Filter and divert external AC line intrinsic noise from the EMI measurements
- Isolate and decouple the AC line high voltage and prevent line transients from damaging spectrum analyzers and other sensitive test equipment
- Present a known, fixed impedance at RF frequencies to the power supply undergoing test



## LISN considerations:

- Desired impedance (typically $50 \Omega$ )
- Bandwidth (typically victims are susceptible to 10 kHz to 30 MHz )
- Line type (DC, Single phase, 3 d delta, 3 phase wye)
- Line voltage (120 V, 208 V, 480 V, etc)
- Load current



# Noise Filters 

## Differential Mode Noise



- Produced as a natural result of complex, high frequency switching $V$ and I
- $V_{1}=-V_{2}$
- Magnitudes are equal
- Phase difference is $180^{\circ}$
- $V_{\text {DIFF }}=V_{1}-V_{2}=$ unwanted signal
- $I_{D}=\left(V_{1}-V_{2}\right) / R_{\text {Load }}$

- Current flow in opposite directions so that the magnetic field is contained within the spirals
- The tighter the cable twist the greater the containment and noise attenuation
- Shielding the pair (and tying the shield to ground in one or more places) will also increase noise attenuation

- Produced as a result of circuit imbalances, currents produced by simultaneous high frequency voltages on (+) and (-) capacitively coupled to ground
- $V_{1}=V_{2}=V_{C O M}$
- Magnitudes are equal
- Phase difference is $0^{O}$
- $V_{\text {DIFF }}=V_{1}-V_{2}=$ wanted signal
- $I_{D}=\left(V_{1}-V_{2}\right) / R_{\text {Load }}$
- $V_{S U M}=V_{1}+V_{2}=0$

- Common mode current generated by common mode voltages impressed across parasitic capacitances to ground
- Current flows are the same magnitude and in the same direction so that the spirals have no effect on containing the magnetic fields
- The pair must be shielded and the shield tied to ground in one or more places for noise attenuation

- Configurations C, L, Pi,T
- Attenuation 20 to 70 dB
- Filters both differential and common mode noise

- $L$ and $C$ are not good noise ( $f>f_{s w}$ ) filters
- L looks capacitive at $f>f_{s w}$, C looks inductive at $f>f_{s w}$
- $L_{d f}$ is a differential / common mode noise filter inductor and might be a real inductance or the intrinsic inductance of the bus
- $C_{d f}$ is a differential mode noise filter capacitor
- $C_{c f}$ are common mode noise filter capacitors

Output Line Feed-through Noise Filters


- C filters are the most common EMI filter, consisting of a 3 terminal feedthru capacitor, used to attenuate high frequency signals

- L filters consist of one inductive element and one capacitor. One disadvantage is that the inductor element in smaller filters consists of a ferrite bead that will saturate and lose effectiveness at larger load currents
-T filters consist of two inductive elements and one capacitor. This filter presents a high impedance to both the source and load of the circuit

- Pi filters consist of two capacitors and one inductor. They present a low impedance to both source and load. The additional capacitor element, provides better high frequency attenuation than the C or L filters




Differential mode currents flow in opposite directions. Magnetic fields cancel, choke presents low impedance, low attenuation to noise


Common Mode Choke


Common mode currents flow in same direction. Magnetic fields add, choke presents high impedance, high attenuation to noise

Main AC Bus


- Separate noisy power supplies from sensitive I \& C loads by Faradayshielded transformers to attenuate common mode noise


## Radiated Emissions

## Radiated emissions

- EMI radiated from cables, transformers, other components.
- Typically 30 MHz to > 1 GHz .30 MHz start because cables and other equipment are effective radiators of frequencies above 30 MHz
- Measured in $\mu V / m$ or $d B-\mu V / m$ (Reference: $1 \mu V / m=0 d B)$
- Measured 3 m (residential) or 30 m (industrial) from the emitting equipment. TVs located within 3 m of computers in the home and within 30 m in the industrial setting. Limits 100 to $200 \mu \mathrm{~V} / \mathrm{m}$ are $1 / 10$ of $T V$ reception signal
- Industrial FCC Class A limits of $200 \mu \mathrm{~V} / \mathrm{m}$ are higher (less severe) because it is assumed that there will be an intervening wall between culprit and victim that will provide some shielding


## Test equipment used

- Spectrum Analyzers, rotating tables, conical and/or log periodic antennas and anechoic chambers designed to minimize reflections and absorb external EMI



## Anechoic Chamber

Biconical < 200MHz Log Periodic > 200 MHz




## Radiated Emissions

Any component or cable > 1/20 wavelength ( $\lambda$ ) will be an efficient radiating or receiving antenna

Cable Lengths Vs Wavelength

| Frequency | $\boldsymbol{\lambda}$ | $1 / 4 \lambda$ | $1 / 20 \boldsymbol{\lambda}$ |
| :---: | :---: | :---: | :---: |
| 10 kHz | 30 km | 7500 m | $1500 \mathrm{~m}=59,000 \mathrm{in}$ |
| 100 kHz | 3 km | 750 m | $150 \mathrm{~m}=5,900 \mathrm{in}$ |
| 1 MHz | 300 m | 75 m | $15 \mathrm{~m}=590 \mathrm{in}$ |
| 10 MHz | 30 m | 7.5 m | $1.5 \mathrm{~m}=59 \mathrm{in}$ |
| 30 MHz | 10 m | 2.5 m | $50 \mathrm{~cm}=20 \mathrm{in}$ |
| 100 MHz | 3 m | 75 cm | $15 \mathrm{~cm}=6 \mathrm{in}$ |
| 1 GHz | 30 cm | 7.5 cm | $1.5 \mathrm{~cm}=0.6 \mathrm{in}$ |

- $B=T=10,000$ gauss
- $A=m^{2}$

Faraday's Induced Voltage Law
$\cdot(T / s) * m^{2}=V$
$V=\llbracket \in \square d l=-\frac{d \varphi}{d t}=-\frac{d B}{d t} A \quad$ Hint: Homework problem
$V \propto \frac{d B}{d t}$ the magnitude and rate of change of flux density with time
$V \propto A \quad$ the area of the loop cut by flux

Moral - minimize loop areas by:
running supply and return bus or cable conductors together
twisting cables whenever possible


Radiated Noise Reduction By PCB Ground Planes


## Use shielded cables

## Use shielded enclosures (if necessary for interior controls)



## Radiated Noise Reduction - Other Considerations

## Shielding

-Use ground planes extensively to minimize $E$ and $H$ fields

- If ribbon cable is used, employ and spread ground conductors throughout to minimize loop areas
- Avoid air gaps in transformer/inductor cores.
- Use toroid windings for air core inductors
- If shielding is impractical, then filter


## Filtering

- Use common mode chokes whenever practical
- Use EMI ferrites, not low-loss ferrites - useful frequency range 50 to 500 MHz . Be careful of DC or low-frequency current saturation
- Use capacitors and feed-through capacitors, separately or in conjunction with chokes/ferrites. Be mindful of capacitor ESR and inductance


## Homework Problem \# 5

A uniform magnetic field $B$ is normal to the plane of a circular ring 10 cm in diameter made of \#10 AWG copper wire having a diameter of 0.10 inches. At what rate must $B$ change with time if an induced current of $10 A$ is to appear in the ring? The resistivity of copper is about $1.67 \mu \Omega * c m$.

# Section 5. DC Power Supplies 

* Definition, Purpose and Scope
* The Recent Past
* The Present
- DC Power Supplies in Particle Accelerators


## Definition

- A "DC power supply" is a device or system that draws uncontrolled, unregulated input AC or DC power at one voltage level and converts it to controlled and precisely regulated DC power at its output in a form required by the load


## Purpose

- Change the output to a different level from the input (step-up or step-down)
- Rectify AC to DC
- Isolate the output from the input
- Provide for a means to vary the output
- Stabilize the output against input line, load, temperature and time (aging) changes


## Example

- 208 VAC is available. The load is a logic circuit in a personal computer that requires regulated 5V DC power. The power supply makes the 208 V AC power source and 5 V DC load compatible


Reliability

Power System Types - The DC Magnet Power System


Some characteristics of the power supplies most often used in particle or synchrotron accelerators are:

- They can be DC-DC converters
- They are voltage or current sources that use the AC mains (off-line) as their source of energy. The bipolar power supplies discussed later are typically used for small corrector magnets are DC-DC converters fed from a common off-line power supply
- They are not $A C$ controllers.
- They have a single output.
- The output voltage or current is not fixed (such as those used by the telephone and communications industry), but are adjustable from zero to the full rating
- They are not computer power supplies or printed circuit board converters
- The DC output power ratings range from a few watts to several megawatts
-Typical loads are magnets or capacitor banks


# The Recent Past Circa 1960-1990 

# 60 Hz Low Frequency Transformers 

- Transformers are inductors with linked flux $\varphi$ :

$$
\begin{array}{lll}
\text { Volts }=\# \text { of turns } * \text { time rate of change of flux in the coil } & V=N * d \varphi / d t \\
\text { or } & V=(\text { inductance }) *(\text { rate of change of current }) & V=L_{m} * d i / d t
\end{array}
$$



- $L_{m}$ is derived from the number of turns, the area of turn path, length of flux and $\mu$ of the iron. It is normally referred to as the magnetizing inductance
- Iron core transformers are used at frequencies below 1 kHz
- Magnetizing inductance for a low frequency transformer is large, typically requiring > $1 \%$ of the total current to produce the desired voltage

Equivalent Transformer Circuit

- The current required to magnetize the core with flux is called the magnetizing current and is made up of two parts:

1. The magnetizing inductance in which the current is out of phase with the voltage.
2. The magnetizing losses which are in phase with the current are due to eddy current losses and magnetizing current losses or hysteresis losses.

- The inductance is obtained by driving the transformer with the secondary open circuited $I_{2}=0$ and measuring the Primary voltage and current.

$$
\begin{aligned}
& L_{m}=\left(V_{1} / I_{l}\right) /\left(2 * \pi^{*} f\right) \text { out of phase } \\
& R_{m}=V_{1} / I_{1} \text { in phase }
\end{aligned}
$$




Turns cut by the same flux produce a voltage with the same volts per turn as the driving turns

$$
\frac{V_{1}}{N_{1}}=\frac{d \phi}{d t}=\frac{V_{2}}{N_{2}} \quad \text { or } \quad \frac{V_{1}}{N_{1}}=\frac{V_{2}}{N_{2}}
$$

Where $N_{1}$ and $N_{2}$ are the number of turns cut by the same flux
With a source and a load, the primary current equals the source current minus the magnetizing current. Ampere-turns in the primary and secondary must equate

$$
\begin{array}{lll}
N_{1}\left(I_{1}-I_{m}\right)=I_{2} N_{2} & \text { or } & \frac{N_{1}}{N_{2}}=\frac{I_{2}}{I_{1}-I_{m}} \\
\text { If } I_{m} \ll I_{1} \text { then } & \frac{N_{1}}{N_{2}}=\frac{I_{2}}{I_{1}}
\end{array}
$$

Impedance Ratios and Reflected Impedances


$$
\begin{aligned}
& \boldsymbol{R}_{1}=\frac{\boldsymbol{V}_{1}}{\boldsymbol{I}_{1}}=\frac{\frac{\boldsymbol{N}_{1}}{\boldsymbol{N}_{2}} \boldsymbol{V}_{2}}{\frac{\boldsymbol{N}_{2}}{\boldsymbol{N}_{l}} \boldsymbol{I}_{2}}=\frac{\boldsymbol{N}_{l} \boldsymbol{V}_{2}}{\boldsymbol{N}_{2}} * \frac{\boldsymbol{N}_{l}}{\boldsymbol{N}_{2} \boldsymbol{I}_{2}} \\
& \boldsymbol{R}_{1}=\frac{\boldsymbol{N}_{l}^{2}}{\boldsymbol{N}_{2}^{2}} * \frac{\boldsymbol{V}_{2}}{\boldsymbol{I}_{2}}=\frac{\boldsymbol{N}_{l}^{2}}{\boldsymbol{N}_{2}^{2}} * \boldsymbol{R}_{2} \\
& \frac{\boldsymbol{R}_{1}}{\boldsymbol{R}_{2}}=\left(\frac{\boldsymbol{N}_{1}}{\boldsymbol{N}_{2}}\right)^{2}
\end{aligned}
$$

## Example

 $\boldsymbol{R}_{2}=4 \Omega$, what is the value of the reflected resistance as seen on the primary side?

$$
\begin{aligned}
& \boldsymbol{R}_{1}=\frac{\boldsymbol{N}_{1}^{2}}{\boldsymbol{N}_{2}^{2}} * \boldsymbol{R}_{2}=\frac{27^{2}}{9^{2}} * 4 \Omega \\
& \boldsymbol{R}_{1}=9 * 4 \Omega=36 \Omega
\end{aligned}
$$

## Leakage Inductance - Equivalent Transformer Circuit

- Flux which does not couple both windings is called the leakage flux and acts like series inductors called the leakage inductance
- If the secondary is shorted and the magnetizing current is small $\left(I_{m} \ll I_{1}\right)$, then the leakage inductance is proportional to the primary voltage divided by the primary current (or secondary current referred to the primary side)


$$
\begin{aligned}
& I_{1}=\frac{V_{1}}{\left(X_{1}+X_{2}\right)}=\frac{V_{1}}{2 \pi f\left(L_{1}+L_{2}\right)} \\
& L_{1}+L_{2}=\frac{V_{1}}{2 \pi f I_{1}}
\end{aligned}
$$

- If the secondary is shorted the "percent impedance" is the drive voltage divided by the rated input voltage with rated load current flowing

$$
\frac{V_{1}}{V_{\text {rated }}} \text { with } I_{\text {rated }} X 100 \%=\% \text { impedance }
$$

## Homework Problem \#6

A 1000kVA, 12.47 kV to $480 \mathrm{~V}, 60 \mathrm{~Hz}$ three-phase transformer has an impedance of $5 \%$. Calculate:
a. The actual impedance and leakage inductance referred to the primary winding
b. The actual impedance and leakage inductance referred to the secondary winding
c. The magnetizing inductance referred to the primary winding


An air gap is undesirable in a transformer because:

- a large $L_{m}$ is desired to reduce the magnetizing and inrush current
- a small $L_{l}$ is desired to lower energy and switching losses


## Transformer Example



$$
V_{\text {pripeak }}=100 \mathrm{~V} \quad V_{o}=24 \mathrm{~V} \quad V_{d}=0.8 \mathrm{~V} \quad V_{\text {secpeak }}=24.8 \mathrm{~V} \quad I_{o}=2.5 \mathrm{~A} \quad D=0.5
$$

Vpri


$$
I_{\text {secrms }}=I_{\text {secavg }} * \sqrt{D}=I_{o} * \sqrt{D}=2.5 * \sqrt{0.5}=1.77 \text { A rms on each side where } D \text { is duty ratio }
$$

IdI

transformer turns ratio - each secondary
$I_{\text {prirms }}=I_{\text {secrms }} / N=1.77 A / 4.03=0.439 A \quad$ Contribution from each secondary

Id 2

$I_{\text {totalprirms }}=0.439 A * 2=0.878 A$

$$
P_{\text {totalpriavg }}=V_{\text {prirms }} * I_{\text {totalprirms }}=100 \mathrm{~V} * \sqrt{0.5} * 0.878 \mathrm{~A}=62 \mathrm{~W}
$$

$\eta=P_{o} / P_{\text {totalpriavg }}=60 \mathrm{~W} / 62 \mathrm{~W}=96.8 \%$

Calculate the output voltage in the circuit shown below.


- Low frequency, 60 Hz , transformers almost always use iron cores to direct the flux to tightly couple the windings to reduce the size, cost, leakage reactance and magnetizing current (large magnetizing inductance) of a transformer
- For low power applications < 2.5 kW single phase transformer are used to eliminate the need for costly 3 phase input power lines.
- 3 phase lines and transformers are used to reduce the cost of higher power systems (usually >2.5 kW)
- 3 phase lines allow the use of phase shifting transformers to generate any number of output phases


Delta -wye, extended delta, polygon and zig-zag produce 6 phase outputs


## Three phase Transformer most common type

Three phase Transformers

- A three phase transformer can be constructed with 1 core or 2 or 3 independent cores
- Independent core transformers are more expensive ( use more steel) and can result in line imbalances


Wye-Wye Transformer


Delta - Delta Transformer



Core Delta -Delta Transformer

open Wye

## Balanced Bridge Harmonics - Trigonometric Identities

Addition formulae

$$
\begin{aligned}
& \sin (A+B)=\sin A \cos B+\sin B \cos A \\
& \sin (A-B)=\sin A \cos B-\sin B \cos A
\end{aligned}
$$

Therefore

$$
\begin{aligned}
& \sin (A+B)+\sin (A-B)=2 \sin A \cos B \\
& \sin (A+B)-\sin (A-B)=2 \sin B \cos A \\
& \text { and }
\end{aligned}
$$

$$
\sin a+\sin b=2 \sin \frac{a+b}{2} \cos \frac{a-b}{2}
$$

$$
\sin a-\sin b=2 \sin \frac{a-b}{2} \cos \frac{a+b}{2}
$$

Similarly, since

$$
\begin{aligned}
& \cos (A+B)=\cos A \cos B-\sin A \sin B \\
& \cos a+\cos b=2 \cos \frac{a+b}{2} \cos \frac{a-b}{2} \\
& \cos a-\cos b=-2 \sin \frac{a+b}{2} \sin \frac{a-b}{2}
\end{aligned}
$$

Three Phase Wye-Wye

Vdo
For a transformer ratio, $N_{Y Y}$

$$
\begin{aligned}
V_{s} & =N_{Y Y} V_{p} ; I_{s}=I_{p} / N_{Y Y} \\
V_{A B Y s} & =\sqrt{3} N_{Y Y} V_{L N_{p}} \sin (\omega t+\pi / 6) \\
V_{B C Y s} & =\sqrt{3} N_{Y Y} V_{L N p} \sin (\omega t-\pi / 2) \\
V_{C A Y s} & =\sqrt{3} N_{Y Y} V_{L N p} \sin (\omega t-7 \pi / 6) \\
I_{A B Y s} & =\left(\sqrt{3} I_{L N p} / N_{Y Y}\right) \sin \left(\omega t+\pi / 6+\phi_{Z}\right) \\
I_{B C Y s} & =\left(\sqrt{3} I_{L N p} / N_{Y Y}\right) \sin \left(\omega t-\pi / 2+\phi_{Z}\right) \\
I_{C A Y s} & =\left(\sqrt{3} I_{L N_{p}} / N_{Y Y}\right) \sin \left(\omega t-7 \pi / 6+\phi_{Z}\right)
\end{aligned}
$$

## Spectrum of Wye-Wye

Assume full conduction into a large inductive load

The load current, $I_{L}$, is then constant

The current out of the A leg of the transformer is

$$
=\frac{2 I_{L}}{T}\left[\int_{T / 12}^{5 T / 12} \sin \frac{2 \pi n t}{T} d t-\int_{7 T / 12}^{11 T / 12} \sin \frac{2 \pi n t}{T} d t\right]
$$

$$
\begin{aligned}
I_{\text {ANYS }}(t) & =0 & & 0 \leq t \leq T / 12 \\
& =I_{L} & & T / 12 \leq t \leq 5 T / 12 \\
& =0 & & 5 T / 12 \leq t \leq 7 T / 12 \\
& =-I_{L} & & 7 T / 12 \leq t \leq 11 T / 12 \\
& =0 & & 11 T / 12 \leq t \leq T
\end{aligned}
$$

$$
b_{n}=\frac{2}{T} \int_{0}^{T} I_{A N Y s}(t) \sin \frac{2 \pi n t}{T} d t
$$

$$
=\frac{4 I_{L}}{T} \int_{T / 12}^{5 T / 12} \sin \frac{2 \pi n t}{T} d t
$$

$$
=-\left.\frac{2 I_{L}}{n \pi} \cos \frac{2 \pi n t}{T}\right|_{T / 12} ^{5 T / 12}
$$

The Fourier series expansion is
$I_{A N Y S}(t)=a_{0}+\sum_{n=1}^{\infty} a_{n} \cos \frac{2 \pi n t}{T}+b_{n} \sin \frac{2 \pi n t}{T}$

$$
=-\frac{2 I_{L}}{n \pi}[\cos (5 n \pi / 6)-\cos (n \pi / 6)]
$$

From the symmetry of the waveform,
$a_{0}=a_{n}=0$

$$
b_{n}=\frac{4 I_{L}}{n \pi} \sin \frac{n \pi}{2} \sin \frac{n \pi}{3}
$$

## Wye-Wye Primary Current



Note that the first term eliminated all of the even harmonics and the second eliminated all multiples of the third harmonic.

The current on the primary leg of the transformer, due to the $Y Y$ winding is
$I_{A N Y P}(t)=N_{Y Y} \frac{4 I_{L}}{n \pi} \sum_{n=1}^{\infty} \sin \frac{n \pi}{2} \sin \frac{n \pi}{3} \sin \frac{2 \pi n t}{T}$

Wye-Wye Primary Current




| Harmonic | 1 | 3 | 5 | 7 | 9 | 11 | 13 | 15 | 17 | 19 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Frequency | 60 | 180 | 300 | 420 | 540 | 660 | 780 | 900 | 1020 | 1140 |
| Amplitude | 1.103 | 0 | -0.221 | -0.158 | 0 | 0.100 | 0.085 | 0 | -0.065 | -0.058 |

## Three Phase Wye-Delta



In order to have balanced current on the primary
$I_{A \Delta s}+I_{B \Delta s}+I_{C \Delta s}=0$
When two delta leg A switches conduct
$I_{B \Delta s}=I_{C \Delta s}$
so that

$$
I_{A \Delta s}+2 I_{B \Delta s}=0
$$

The current through the switch is then

$$
\begin{aligned}
& I_{L}=I_{A \Delta s}-I_{B \Delta s} \\
& I_{L}=I_{A \Delta s}+\frac{1}{2} I_{A \Delta s} \\
& I_{L}=\frac{3}{2} I_{A \Delta s} \\
& I_{A \Delta s}=\frac{2}{3} I_{L}
\end{aligned}
$$

For a transformer ratio $N_{Y \Delta}$
$V_{A B \Delta s}=N_{Y \Delta} V_{L N p} \sin (\omega t)$
$V_{B C \Delta s}=N_{Y \Delta} V_{L N p} \sin (\omega t-2 \pi / 3)$
$V_{C A \Delta s}=N_{Y \Delta} V_{L N p} \sin (\omega t-4 \pi / 3)$
$I_{A B \Delta s}=\left(V_{L N p} / N_{Y \Delta}\right) \sin \left(\omega t+\phi_{Z}\right)$
$I_{B C \Delta s}=\left(V_{L N p} / N_{Y \Delta}\right) \sin \left(\omega t-2 \pi / 3+\phi_{Z}\right)$
$I_{C A \Delta s}=\left(V_{L N p} / N_{Y \Delta}\right) \sin \left(\omega t-4 \pi / 3+\phi_{Z}\right)$
For equal secondary voltages $N_{Y \Delta}=\sqrt{3} N_{Y Y}$


$$
\begin{array}{rlrl}
I_{A A s}(t) & =I_{L} / 3 & & 0 \leq t \leq T / 6 \\
& =2 I_{L} / 3 & & T / 6 \leq t \leq T / 3 \\
& =I_{L} / 3 & & T / 3 \leq t \leq T / 2 \\
& =-I_{L} / 3 & & T / 2 \leq t \leq 2 T / 3 \\
& =-2 I_{L} / 3 & 2 T / 3 \leq t \leq 5 T / 6 \\
& =-I_{L} / 3 & 5 T / 6 \leq t \leq T
\end{array}
$$

$$
a_{0}=a_{n}=0
$$

Again, by symmetry, only the $b_{n}$ terms are non-zero

$$
=\frac{2 I_{L}}{3 n \pi}\left[\left(\cos 0+\cos \frac{\pi n}{3}\right)-\left(\cos \frac{2 \pi n}{3}+\cos \pi n\right)\right]
$$

$$
\begin{aligned}
& =\frac{4 I_{L}}{3 n \pi}\left(\cos \frac{n \pi}{6} \cos \frac{n \pi}{6}-\cos \frac{5 n \pi}{6} \cos \frac{n \pi}{6}\right) \\
& =\frac{4 I_{L}}{3 n \pi} \cos \frac{n \pi}{6}\left(\cos \frac{n \pi}{6}-\cos \frac{5 n \pi}{6}\right) \\
& =\frac{8 I_{L}}{3 n \pi} \cos \frac{n \pi}{6} \sin \frac{n \pi}{2} \sin \frac{n \pi}{3}
\end{aligned}
$$


$I_{\text {AAs }}(t)=\frac{8 I_{L}}{3 n \pi} \sum_{n=1}^{\infty} \cos \frac{n \pi}{6} \sin \frac{n \pi}{2} \sin \frac{n \pi}{3} \sin \frac{2 \pi n t}{T}$
Note that multiples of the $2^{\text {nd }}$ and $3^{\text {rd }}$ harmonics are also suppressed.
The $\cos \frac{n \pi}{6}$ term does not introduce any extra zeros, but it
does contribute to the sign of the terms.
The non-vanishing terms are $n=1,5,7,11, \cdots$, for which the magnitude is $\sqrt{3} / 2$.
Referred back to the primary, the current is
$I_{A \Delta p}(t)=N_{Y \Delta} \frac{8 I_{L}}{3 n \pi} \sum_{n=1}^{\infty} \cos \frac{n \pi}{6} \sin \frac{n \pi}{2} \sin \frac{n \pi}{3} \sin \frac{2 \pi n t}{T}$
$I_{A \Delta p}(t)=N_{Y Y} \frac{8 \sqrt{3} I_{L}}{3 n \pi} \sum_{n=1}^{\infty} \cos \frac{n \pi}{6} \sin \frac{n \pi}{2} \sin \frac{n \pi}{3} \sin \frac{2 \pi n t}{T}$

## Primary Current in the Wye-Delta





| Harmonic | 1 | 3 | 5 | 7 | 9 | 11 | 13 | 15 | 17 | 19 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Frequency | 60 | 180 | 300 | 420 | 540 | 660 | 780 | 900 | 1020 | 1140 |
| Amplitude | 1.103 | 0 | 0.221 | 0.158 | 0 | 0.100 | 0.085 | 0 | 0.065 | 0.058 |

Total Current (Primary Wye Current) in Wye-Wye-Delta


## Total Primary Current in Wye-Wye-Delta

The total current in the A leg of the primary is the sum of these two terms.
$I_{A p}(t)=I_{A N Y_{p}}(t)+I_{A \Delta p}(t)$

The only non-vanishing terms in both of these series are $n=1,5,7,11$
and all other values of $n$ which have the same phase

The values of $\cos \frac{n \pi}{6}$ for these $n$ are $\frac{\sqrt{3}}{2},-\frac{\sqrt{3}}{2},-\frac{\sqrt{3}}{2}, \frac{\sqrt{3}}{2}$

The surviving terms in each series have the same magnitude, but half have different signs so that the only remaining harmonics in the total balanced 12 pulse bridge are $\quad n=1,11,13,23,25,35, \cdots$ with coefficient $N_{Y Y} \frac{4 I_{L}}{n \pi}$

Total Primary Current in Wye-Wye-Delta




| Harmonic | 1 | 3 | 5 | 7 | 9 | 11 | 13 | 15 | 17 | 19 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Frequency | 60 | 180 | 300 | 420 | 540 | 660 | 780 | 900 | 1020 | 1140 |
| Amplitude | 2.205 | 0 | 0 | 0 | 0 | 0.200 | 0.085 | 0.170 | 0 | 0 |

Total Primary Current in Wye-Wye-Delta



Three Phase Phase Shifting Transformer Phase shifting transformer for 12 Pulse operation


## Three Phase Phase Shifting Transformer

Extended Delta Phase shifting transformer
EXTENDED DELTA 13.8kV to $480 \mathrm{~V} 7.5^{0}$


Extended Delta-Wye Transformer 3 core


PRIMARY VOLTAGE RELATIONSHIP


SECONDARY VOLTAGE RELATIONSHIP

Two transformers Phase rotation change on input produces a $30^{\circ}$ phase shift Leakage inductance is the same on all phases

## Standards for Power Rectifier Transformer

1) Pool-Cathode Mercury-Arc Rectifier Transformer ASA C57.18-1964
2) Practice for Semiconductor Power Rectifiers ANSI C34.2-1973
3) IEEE standards for Transformer \& Inductors for Electronic power conversion Equipment ANSI/IEEE std 388-1992

Insulation Class Recommended for Rectifier Transformer.

1) Oil $65^{\circ} \mathrm{C}$ rise over ambient ( paper oil insulation)
2) Dry type Class $B 80^{\circ} \mathrm{C}$ rise over ambient, (Paper, varnish)
3) Class H $150^{\circ} \mathrm{C}$ over ambient ( Fiberglass, epoxy)

## Phase Relationship and labeling

1) General requirements for distribution power and regulating transformers ANSI C57.12.00-1973

Low Frequency Transformers have been around a long time and designs are well established. There are a few problems related to rectifier operation that should be considered when using transformers;

1) Harmonic currents in the core and coils can result in excessive losses.
2) Presence of DC and/or second harmonic currents/ voltage can saturate the core resulting in more harmonics and excessive core hysteresis loss.
3) Short circuits are common in rectifiers resulting in high forces on the coils and the coil bracing resulting in coil faults.
4) Connection to the center of a wye can generate excessive third harmonic current resulting in voltage distortion and overheating.
5) The fast switching voltages of rectifiers under commutation can produce nonuniform voltage distribution on coil windings resulting in insulation failure.

## Rectifiers

Introduction to power supplies, diodes, SCRs and rectifiers

## Four Types of Converters

- Rectifier converting an ac voltage to a dc voltage
- Inverter converting a dc voltage to an ac voltage
- Chopper or a switch-mode power supply that converts a dc voltage to another dc voltage
- Cycloconverter and cycloinverter converting an ac voltage to another ac voltage

Multiple Quadrant Operation


Diode Characteristics


In the reverse direction, there is a small leakage current up until the reverse breakdown voltage is reached

Forward voltage drop, $V_{f}$ : a small current conduct in forward direction up to a threshold voltage, 0.3 V for germanium and 0.7 V for silicon

## Diode Considerations



Schematic representation

- Forward voltage drop, $V_{F}$ or $V_{F(A V)}$
- Forward current, $I_{F}$ or $I_{F(A V)}$
- Maximum reverse (blocking) voltage, $V_{R}$
- Average reverse (leakage) current, $I_{R(A V)}$
- Reverse recovery time, $t_{r r}$
- Peak surge current, $I_{\text {surge }}$
- Cooling (air, water, oil, other)
- Package style


## Thyristors - Silicon Controlled Rectifier (SCR)



Schematic representation

## SCR properties

- It is simply a conventional rectifier controlled by a gate signal
- It is controlled from the off to on states by a signal applied to the gate
- It has a low forward resistance and a high reverse resistance
- It remains on once it is turned on even after removal of the gate signal
- The anode-cathode current must drop below the "holding" value in order to turn it off

SCR Characteristics


## SCR Considerations

- Maximum forward current
- Reverse breakdown voltage
- Gate trigger voltage and current
- Minimum holding current, $I_{h}$
- Power dissipation
- Maximum reverse $d v / d t$
- Peak forward voltage
- A rectifier converts ac voltage to dc voltage
- Classifications

Uncontrolled rectifiers (diodes)
Fully Controlled rectifiers (all SCRs)
Semi-controlled rectifiers (SCRs and diodes)

$1 \phi$ Full Wave ( $q=2$ Pulse) Rectifier


- $q \equiv$ the number of possible rectifier states
- SCR s are electronic switches
- Freewheeling diode normally does nothing, but dissipates energy stored in an inductive load if power supply is suddenly turned off
- For inductive load freewheeling diode conducts if SCR retard is large and if load VI phase angle is large


State 1: SCR s 1-3 On


State 2: SCR s 2-4 On
$1 \phi$ Full Wave ( $q=2$ Pulse) Rectifier

$1 \phi$ Full Wave ( $q=2$ Pulse) Rectifier

$V_{d o}=\frac{1}{T} \int_{t}^{T} v_{L L}(t) d t=\frac{1}{T} \int_{t}^{T} \sqrt{2} V_{L L} \sin \omega t d t=\frac{1}{\omega T} \int_{\alpha}^{\omega T} \sqrt{2} V_{L L} \sin \omega t d \omega t$
the SCR gate trigger retard angle range is $0 \leq \alpha \leq \pi$
$V_{d o}=\frac{\sqrt{2} V_{L L}}{\pi}(1+\cos \alpha)$ for resistive load

## $1 \phi$, Full Wave ( $q=2$ Pulse) Rectifier Summary

- 2 pulse rectifier - low input power factor, high output ripple
- Freewheeling diode needed whenever filter or load V-I angle is large
- Ripple frequency is 120 Hz (if input is 60 Hz )
- Large filter needed
- Limited in use to power supplies < 2.5 kW
$V_{d o}=\frac{1}{T} \int_{t}^{T} v_{L L}(t) d t=\frac{1}{T} \int_{t}^{T} \sqrt{2} V_{L L} \sin \omega t d t=\frac{1}{\omega T} \int_{\alpha}^{\omega T} \sqrt{2} V_{L L} \sin \omega t d \omega t$
the SCR gate trigger retard angle range is $0 \leq \alpha \leq \pi$
$V_{d o}=\frac{\sqrt{2} V_{L L}}{\pi}(1+\cos \alpha)$ for resistive load


Assuming the American standard phase rotation of

$$
\begin{aligned}
& V_{A-B}=|V| e^{j 0} \quad V_{B-C}=|V| e^{-j 120} \quad V_{C-A}=|V| e^{-j 240} \\
& \text { The thyristor firing sequence is: } \\
& 1-5,1-4,2-4,2-6,3-6,3-5
\end{aligned}
$$

## $3 \phi, q=6$ Pulse Rectifier



State 1: $A-B(+) S C R$ s $1-5$ On
Note: Phase SCRs from full retard to full forward slowly to bring the rectifier output voltage up slowly and reduce the capacitor inrush current

## $3 \phi, q=6$ Pulse Rectifier



State 2 : C-A (-), 5 off, SCR s 1-4 On

## $3 \phi, q=6$ Pulse Rectifier



State 3: B-C(+), 1 off, SCR s 2-4 On

## $3 \phi, q=6$ Pulse Rectifier



State 4 : A-B (-), 4 off, SCR s $2-6$ On

## $3 \phi, q=6$ Pulse Rectifier



State 5 : C-A (+), 2 off, SCR s 3-6 On

## $3 \phi, q=6$ Pulse Rectifier



State 6 : B-C (-), 6 off, SCR s 3 - 5 On


For $0 \leq \alpha \leq \frac{\pi}{3}$ where $\alpha$ is the gate trigger retard angle and conduction is continuous
$V_{d o}=\frac{3 \sqrt{2}}{\pi} V_{L L} \cos \alpha$
For $\frac{\pi}{3} \leq \alpha \leq \frac{2 \pi}{3}$ where conduction can be discontinuous
$V_{d o}=\frac{3 \sqrt{2}}{\pi} V_{L L}\left(1+\cos \left(\alpha+\frac{\pi}{3}\right)\right)$ for resistive load


For $0 \leq \alpha \leq \frac{\pi}{3}$ where $\alpha$ is the gate trigger retard angle and conduction is continuous
$V_{d o}=\frac{3 \sqrt{2}}{\pi} V_{L L} \cos \alpha$
For $\frac{\pi}{3} \leq \alpha \leq \frac{2 \pi}{3}$ where conduction can be discontinuous
$V_{d o}=\frac{3 \sqrt{2}}{\pi} V_{L L}\left(1+\cos \left(\alpha+\frac{\pi}{3}\right)\right)$ for resistive load


For $0 \leq \alpha \leq \frac{\pi}{3}$ where $\alpha$ is the gate trigger retard angle and conduction is continuous
$V_{d o}=\frac{3 \sqrt{2}}{\pi} V_{L L} \cos \alpha$
For $\frac{\pi}{3} \leq \alpha \leq \frac{2 \pi}{3}$ where conduction can be discontinuous
$V_{d o}=\frac{3 \sqrt{2}}{\pi} V_{L L}\left(1+\cos \left(\alpha+\frac{\pi}{3}\right)\right)$ for resistive load

- 6 pulse - high input PF $\rightarrow 0.95$
- Use soft-start to limit filter capacitor inrush current.
- Freewheeling diode for inductive loads where VI angle is large
- Output ripple frequency is 360 Hz for 60 Hz input
- Relatively low output ripple and easy to filter with small LC
- Limited to loads < 350 kW
- Diodes or SCRs are air or water-cooled depending upon load current


SCR sequence for $30^{\circ}$ lagging wye secondary
1-5, 7-11, 1-4, 7-10, 2-4, 8-10, 2-6, 8-12, 3-6, 9-12, 3-5, 9-11
$6 \phi, q=12$ Pulse By Parallel Bridges


Transformer phases and SCR firing sequence are the same as shown for the series-connected bridges

## For Both Series And Parallel-Connected Bridges

- Input transformer $\Delta$ primary, $\Delta-Y$ secondaries for 6 AC phases
- $\Delta$ - Y secondaries are phase shifted $30^{\circ}$
- $5^{\text {th }}$ and $7^{\text {th }}$ harmonics virtually non-existent in input line, $\ll 5 \%$ THD of line voltage $<20$ \% THD of line current
- Very high input PF to 0.97
- Output ripple frequency is 720 Hz for 60 Hz input
- Use soft-start to limit filter capacitor inrush current
- Freewheeling diode for inductive loads where VI angle is large
- For loads $\geq 350$ kW


## Series-connected bridges

- For high-voltage, low-current loads


## Parallel-connected bridges

- For high-current, low-voltage loads $\geq 350 \mathrm{~kW}$.
- Inter-phase transformer needed for current sharing
$12 \phi, q=24$ Pulse Rectifier

TRANSFORMER 1


- Input transformer polygon primary to $+7.5^{\circ} \Delta-Y$ secondaries for $-30^{\circ}$ shift
- Input transformer polygon primary to - $7.5^{\circ} \Delta-Y$ secondaries for $+30^{\circ}$ shift
- $15^{\circ}$ shift between the 4 sets of bridges
- For loads $\geq 1$ MW DC or Pulsed


Primary Controlled Rectifier - In Line SCRs

secondary to primary voltage ratio



Advantage Compared To In Line SCRs

- $\frac{1}{\sqrt{3}}$ lower SCR current and power (SCR on-voltage is constant)

Disadvantage

- Transformer wiring more complex

Similarities

- Other characteristics similar to In Line SCR controller

- Primary SCR in open wye with filter inductor in lower voltage primary
- High voltage secondary with diodes and filter capacitor isolated from main load
- Protected against secondary faults. High output impedance, capacitor bank isolated from load
- Secondary uses diodes only.



## KLYSTRON POWER SUPPLY

-Large Joule. under Load with Fault from filter Capacitor an Inductor


- Large Joules under Load
Fault from filter Capacitor
-Low Joules under Fault
- Filter Loss $V_{\text {max }} * I_{\text {ripple }}$ or $\sim 5 \%$ of Load


## KLYSTRON ARC VOLTAGE/CURRENT


^C CURRENT WITH KLYSTRON ARC




Assume ideal components in the phase-controlled circuit above. For a purely resistive load:
A. Explain how the circuit operates
B. Draw the load voltage waveform and determine the boundary conditions of the delay angle $\alpha$
C. Calculate the average load voltage and average load current as a function of $\alpha$
D. Find the RMS value of the load current.


Assume ideal components in the phase-controlled circuit above. For a purely inductive load:
A. Explain how the circuit operates.
B. Draw the load voltage and load current waveforms and determine the boundary conditions of the delay angle $\alpha$
C. Calculate the average load voltage and average load current as a function of $\alpha$
D. Find the RMS value of the load current.

## High Voltage Low Current DC supplies

Voltage Multipliers, Cockroft Walton or Cascade Supplies


- Voltage multipliers or cascaded supplies

- Electron beam gun supplies and deflector supplies
- Half-wave, full-wave, three-phase, or six phase
- Capacitor / rectifier arranged to couple AC voltage through the capacitor after being charged by $D C$ from the rectifier
- 20kV to $1,000 \mathrm{kV}, 0$ to 10 mA DC
- AC high frequency drive $\sim 5 \mathrm{kHz}$ to 50 kHz
- Advantages - simple, reliable, inexpensive
- Disadvantages- low output power, poor regulation high output ripple, high output Z, 1st stage draws high current


## High Voltage Multiplier DC supplies

## Voltage multiple 20kHz 0.5 uFd 1 kv stage 10 mA Load


— load - no-load - Ripple
-Disadvantages:

$$
\begin{array}{ll}
\text { Poor regulation } & E_{\text {drop }}=I_{\text {load }} /\left(f^{*} C\right) *\left(2 / 3 * n^{\wedge} 3+n^{\wedge} 2 / 2-n / 6\right) \\
\text { Large ripple } & E_{\text {ripple }}=I_{\text {load }} /(f * C)^{*} n^{*}(n+1) / 2
\end{array}
$$

# The Present - Switchmode Power Supplies Circa 1990 - Present 

| Topology | Disadvantages |
| :--- | :--- |
| - SCRs for rectification and <br> regulation | • Low power factor |
|  | • High AC line harmonic distortion |
|  | - Narrow bandwidth |
|  | - Slow transient response |
| - SCRs for rectification and <br> gross regulation <br> Fine regulation by post linear <br> transistors | - High AC line harmonic distortion power factor when line $\mathrm{V} \neq$ load $V$ |

## The Present Popular Solution

| Topology | Advantages |
| :---: | :---: |
| - SCRs (or diodes) for rectification <br> - High speed switches (switch-mode inverters) for regulation | - Rectifier SCRs or diodes are full on - hence high power factor (> 0.9) possible <br> - High PF means low AC line harmonic distortion ( $<5 \% \mathrm{~V},<25 \% \mathrm{I}$ ) <br> - Fast ( 10 kHz to 100 kHz ) switching means wide bandwidth (> 100 s of Hz ), fast transient response (microseconds) <br> - Fast switching means more corrections per unit time - better output stability |

- Simple control loops compared to SCR rectifier/post-regulator combination
- Fast switching, high frequency operation for electrically and physically smaller transformers and filter components

The Present Popular Solution (Continued)

| Topology | Disadvantages |
| :--- | :--- |
| - SCRs (or diodes) for |  |
| rectification |  | \(\left.\begin{array}{l}- High speed, fast-edge switching can generate <br>

conducted and radiated electromagnetic <br>

interference (EMI)\end{array}\right\}\)| High speed switches |
| :--- |
| (switch-mode inverters) for |
| regulation |$\quad$.

## SCR Regulation Vs Switchmode Regulation

|  | SCR | Switchmode |
| :--- | :--- | :--- |
| Efficiency | Low at low load, high at <br> full load | High, whether low or high load |
| Operating frequency | 60 Hz | 10 kHz to 1,000 kHz |
| Transient Response | Tens of milliseconds | Tens of microseconds |
| Short-term-stability | 100s of ppm | 10s of ppm |
| Input filter | Large | Smaller, HF regulator provides <br> supplemental filtering |
| Isolation/Line-matching <br> transformer | Large and upstream of <br> the rectifiers | Smaller because of high frequency. <br> Downstream of the regulator |
| Output filter | None | High frequency ripple = smaller size |
| Power factor | Low when output is low | Always high |
| Line distortion | High when output is low | Always low |
| EMI | High when output is low | High, but higher frequency, easier to <br> filter |

## Linear Vs. Switchmode-Advantage Summary

| Linear | Switchmode |
| :---: | :---: |
| Output current/voltage is adjusted by <br> varying pass transistor resistance | Output current/voltage is adjusted by <br> varying switch duty factor |
| Transistor voltage and current are in <br> phase so transistor power loss is high | Switch voltage and current are out of <br> phase so switch power is low |
| Efficiency is dependent upon the output <br> operating point and is maximum at 100 <br> \% load | Efficiency is high and relatively constant |

## Line Commutated Switches

- Typically thyristor (4 - element) family devices SCRs, Triacs
- Employ natural current zero occurs each 1/2 cycle for turnoff
- Slow, tied to 60 Hz line and no turnoff control
- Not suitable as fast switch


## Force Commutated

- Typically SCRs, Triacs



## Self Commutated

- Devices have the ability to turn on or turn off by the application of a forward or reverse bias to the control elements (gate - emitter)
- Typically Bipolar Junction Transistors (BJTs), Metal Oxide Semiconductor Field Effect Transistors (MOSFETs) or Insulated Gate Bipolar Junction Transistors (IGBTs)
- Only self-commutated switches used in modern switchmode power supplies


| Self-Commutated Device | Bipolar Junction <br> Transistor (BJT) | Metal Oxide Field Effect Transistor <br> (MOSFET) | Insulated Gate Bipolar Transistor (IGBT) |
| :---: | :---: | :---: | :---: |
| Symbol |  |  |  |
| Available Ratings | $\begin{aligned} & 600 \mathrm{~V}, 10 \rightarrow 100 \mathrm{~A} \\ & 1000 \mathrm{~V}, 10 \rightarrow 100 \mathrm{~A} \end{aligned}$ | $\begin{aligned} 150 \mathrm{~V}, 10 & \rightarrow 600 \mathrm{~A} \\ 600 \mathrm{~V}, 10 & \rightarrow 100 \mathrm{~A} \\ 1200 \mathrm{~V}, 10 & \rightarrow 100 \mathrm{~A} \end{aligned}$ | $600 \mathrm{~V}, 10 \rightarrow 800 \mathrm{~A}$ <br> $1200 \mathrm{~V}, 10 \rightarrow 2400 \mathrm{~A}$ <br> $1700 \mathrm{~V}, 50 \rightarrow 2400 \mathrm{~A}$ <br> $3300 \mathrm{~V}, 200 \rightarrow 1500 \mathrm{~A}$ <br> $6500 \mathrm{~V}, 200 \rightarrow 800 \mathrm{~A}$ |
| Switching Speed | $D C \leq f s \leq 2 \mathrm{kHz}$ | $D C \leq f_{s} \leq 1,000 \mathrm{kHz}$ | $D C \leq f_{s} \leq 20 \mathrm{kHz}$ |
| $\begin{aligned} & \text { Vce or Vds } \\ & \text { f(Vge/Vgs, Ic/Id) } \end{aligned}$ | $0.5 \mathrm{~V} \rightarrow 1.5 \mathrm{~V}$ | $1.5 \mathrm{~V} \rightarrow 6 \mathrm{~V}$ | $1.0 \rightarrow 3.0 \mathrm{~V}$ |
| Conduction Loss <br> (Vce*Ic) or <br> (Vds*Id) | Lowest | Highest | Reasonable |
| Control Mode | Current | Voltage | Voltage |

- Used in vast majority of switchmode power supplies, except MOSFETs for corrector/ trim bipolars
- Voltage controlled device - faster than BJT
- MOSFET faster, but $V_{D S}$ too large
- 20 kHz for $P W M$
- Robust, failure rate < 50 FITs
- Commercially available since 1990


IGBT

| IGBT Availability |  |
| :--- | :--- |
| 600 V | $10 \rightarrow 800 \mathrm{~A}$ |
| 1200 V | $10 \rightarrow 2400 \mathrm{~A}$ |
| $1600 / 1700 \mathrm{~V}$ | $50 \rightarrow 2400 \mathrm{~A}$ |
| $2500 / 3300 \mathrm{~V}$ | $200 \rightarrow 1500 \mathrm{~A}$ |
| $4500 / 6500 \mathrm{~V}$ | $200 \rightarrow 800 \mathrm{~A}$ |



Available as 6-pack, half-bridge, single switch

## Manufacturers of IGBTs and IGBT Gate Drivers

| $A B B$ | http://www.abbsem.com/english/igbt.htm |
| :---: | :---: |
| Concept Technology | http://www.igbt-driver.com/ |
| Collmer Semiconductor | http://www.collmer.com/ |
| Eupec | http://www.eupec.com |
| International Rectifier | http://www.irf.com |
| Intersil | http://www.intersil.com/ |
| IXYS | http://www.ixys.net |
| Mitsubishi | http://www.mitsubishichips.com/ |
| Powerex | http://www.pwrx.com |
| Semikron | http://www.semikron.com |
| Toshiba | http://www.toshiba.com/ |
| Westcode | http://www.westcode.com/ |

- There are many topologies, but most are combinations of the types that will be discussed here.
- Each topology contains a unique set of design trade-offs

Voltage stresses on the switches
Chopped versus smooth input and output currents
Utilization of the transformer windings

- Choosing the best topology requires a study of

Input and output voltage ranges
Current ranges
Cost versus performance, size and weight

## Two Broad Categories

## Flyback Converters

- The line-to-load matching/isolation transformer doubles as the output filter choke
- Advantage - reduction of one major component
- Disadvantage - constrained to low power applications. Not employed in accelerator power supplies


## Forward Converters

- The line-to-load matching/isolation transformer is separate from the output filter choke
- May be used in low and high power systems. Used in the vast majority of accelerator power supplies
- Disadvantage - the increased cost and space associated with a separate transformer and choke

Typical Forward Converters Listed In Order Of Increasing Use

- Half-bridge Converter
- Boost Regulator
- Buck Regulator
- Full-bridge Converter

Typical Forward Converters Listed in Order of Increasing Complexity

- Buck Regulator
- Boost Regulator
- Half-bridge Converter
- Full-bridge Converter

Basic switchmode tool kit


Most fundamental switchmode converter topologies are constructed by rearranging the three components

Switchmode Topologies


## Definition of the Pulse Width Modulated (PWM) Waveform



$$
\text { Duty Cycle }=\text { Duty Ratio }=D=\frac{T_{o n}}{T_{o n}+T_{o f f}}=\frac{T_{o n}}{T_{s}}
$$

$$
D^{\prime}=1-D=\frac{T_{o f f}}{T_{o n}+T_{o f f}}=\frac{T_{o f f}}{T_{s}}
$$



- Used in the majority of switchmode power supplies
- Bucks the input voltage down to a lower voltage
- Perhaps the simplest of all
- Input current discontinuous (chopped) - output current smooth

- Switching device Q1 turned on by square wave drive circuit with controlled on-to-off ratio (duty factor, $D$ )
- $V_{i n}-V_{o}$ impressed across $L$
- Current in Lincreases linearly
- Capacitor C charges to Vo

- Switching device Q1 turns off
- Voltage across $L$ reverses: - Vo impressed across $L$
- Diode D turns on
- Current in L decreases linearly
- C discharges into the Load

Buck Converter Waveforms


## Buck Converter Conduction

Buck converter inductor current can be continuous, critically continuous or discontinuous


Discontinuous current is caused by:

- Too light a load
- Too small an inductor
- Too small filter capacitor
- Discontinuous difficult to control output and output $\neq D^{*}$ Vin


## Buck Converter

## Summary

- Output polarity is the same as the input polarity
- In steady-state L volt-seconds with Q1 on = volt-seconds with Q1 off

$$
\begin{aligned}
& \left(V_{\text {In }}-V_{O}\right) * t_{o n}=\left(V_{O} * t_{\text {off }}\right) \\
& V_{O}=V_{\text {In }} * t_{o n} /\left(t_{o n}+t_{\text {off }}\right)=V_{\text {In }} * D
\end{aligned}
$$

- Output voltage is always less than the input voltage because $D \leq 1$
- Switch duty factor (D) range 0 to 0.95
- Output voltage is not related to load current so output impedance is very low (approximates a true voltage source)


## Buck Converter Vs Other Topologies

## An Advantage

- Few components, 1 switch - simple circuit, high reliability if not overstressed


## Disadvantages

- Output is DC and unipolar so no chance of high-frequency transformer or bipolar output
- Low frequency transformer must be used in front of the Buck for isolation and to match the line voltage to the load voltage


## Application

- Used very widely in accelerator power systems, typically for large power supplies (perhaps $\geq 350 \mathrm{~kW}$ and used in conjunction with a 12-pulse rectifier with 6-phase transformer)


## Boost Converter



- Boosts the input voltage to a higher output voltage
- Input current is smooth - output current is discontinuous (chopped)


State 1 - Power Transfer

- Switching device Q1 turned on by square wave drive circuit with controlled on-to-off ratio (duty factor, D)
- $V_{i n}$ impressed across $L$
- Current in L increases linearly in forward direction
- Diode D is reversed biased (open)
- Capacitor C discharges into the load


State 2 - Regulation

- Q1 turned off. L polarity reverses.
- $V_{O}=V_{I n}+V_{L}, V_{L}=V_{O}-V_{I n}$
- $V_{O}>V_{I n}$, $L$ current decreases linearly
- Diode D is forward biased (closed)
- Capacitor C is recharged

Boost Converter Waveforms


## Boost Converter

## Summary

- Output polarity is the same as the input polarity
- In steady-state, L volt-seconds with Q1 on $=$ volt-seconds with Q1 off

$$
\begin{aligned}
& V_{I n} * t_{o n}=\left(V_{O}-V_{I n}\right) * t_{o f f} \\
& V_{O}=V_{I n} *\left(t_{o n}+t_{\text {off }}\right) / t_{\text {off }} \\
& V_{O}=V_{I n} /(1-D)
\end{aligned}
$$

- Output voltage is always greater than the input voltage because $D \leq 1$
- IGBT duty factor (D) range 0 to 0.95
- Limitation of D yielding greater output voltage is the limitation on the input current through the choke and diode
- Output voltage is not related to load current so output impedance is very low (approximates a true voltage source).


## Some Advantages

- Few components, 1 switch - simple circuit, high reliability if not overstressed
- Input current is always continuous, so smaller input filter capacitor needed


## Some Disadvantages

- Capacitor C current is always discontinuous so a much larger output capacitor is needed for same output ripple voltage
- Output is DC and unipolar so no chance of high-frequency transformer or bipolar output
- Low frequency transformer must be used in front of the Boost for isolation and to match the line voltage to the load voltage
- Minimum output voltage equal input voltage

- Full wave rectifier, output ripple is multiples of the input frequency
- Equal in popularity to buck topology for high-power converters
- Used when line and load voltages are not matched
- Voltage stress on switches $=$ input voltage
- Good transformer utilization, power is transmitted on both half-cycles


State 1 - Power

- Power is derived from the input rectifier and slugs of energy from $C_{i n}$
- Q1 and Q3 are closed. Current flows through Q1 and the primary winding of $T$ and Q3
- A voltage $\left(V_{i n}\right)$ is developed across the primary winding of T. A similar voltage is $\left(V_{\text {in }}{ }^{*} N\right)$ is developed across the secondary winding of $T$
- The secondary voltage causes rectifiers D5 and D7 to conduct current

Full-Bridge Converter Switching - Q1, Q2, Q3 and Q4 Off


State 2 - Power Off

- Q1 and Q3 are turned off. All switches are off
- $C_{\text {in }}$ recharges
- The transformer primary current flows in the same direction but the voltage reverses polarity. This causes D2 and D4 to conduct. Stored leakage inductance energy is returned to the input filter capacitor. The transformer current decays to zero.
- The secondary rectifiers D5, D6, D7 and D8 are all off


State 3 - Power

- Power is derived from the input rectifier and slugs of energy from $C_{\text {in }}$
- Q2 and Q4 are closed and current flows through Q2, the primary winding of $T$ and Q4
- A voltage $\left(V_{i n}\right)$ is developed across the primary winding of T. A similar voltage $(\operatorname{Vin} * N)$ is developed across the secondary winding of $T$
- The secondary voltage causes rectifiers D6 and D8 to conduct current


State 4 - Power Off

- Q2 and Q4 are turned off. All switches are off
- $C_{\text {in }}$ recharges
- The current in the transformer primary flows in the same direction but the voltage reverses polarity. This causes D1 and D3 to conduct. Stored leakage inductance energy is returned to the input filter capacitor. The transformer current goes to zero.
- The secondary rectifiers D5, D6, D7 and D8 all turn off

Full Bridge Converter - IGBT Switching

$I_{C Q 2}, I_{C Q 4}$ Switching delayed by $180^{\circ}$


## Full Bridge Waveforms



- Some inductive energy can be recovered to recharge input filter $C_{i n}$
- Same pulses applied to Q1 \& Q3 and the same, but $180^{\circ}$ delayed, pulses are applied to Q2 \& Q4
- Switching sequence is Q1 \& Q3 are turned on, then turned off after providing the required ON time
- After delay (to account for finite switch turn off and turn on), Q2 \& Q4 are turned on. After providing the required ON time, Q2 \& Q4 are turned off.
- Sequence repeats
- Q1 and Q4 or Q2 and Q3 are never turned on together
- Only the leading edge (or trailing) edge of the gating and current pulse move
- Symmetrical +/- pulse obtained. Must be rectified to provide a DC output
- The output ripple is twice the switching frequency


## Full Bridge Converter

## Advantages

- Simple primary winding needed for the main transformer, driven to the full supply voltage in both directions
- Power switches operate under extremely well-defined conditions. The maximum stress voltage will not exceed the supply line voltage under any conditions.
- Positive clamping by 4 energy recovery diodes suppresses voltage transients that normally would have been generated by the leakage inductances.
- The input filter capacitor $C_{\text {in }}$ is relatively small
- Modest part count for high reliability.
- Can be used with or without line-to-load matching transformer
- Transformer matches the load to the input line.
- With transformer unipolar output, without transformer, used for bipolar operation
- Capable of high power output (500 kW)


## Full Bridge Converter

## Disadvantage

- Four (4) switches are required, and since 2 switches operate in series, the effective saturated on-state power loss is somewhat greater than in the 2 switch, half-bridge case. In high voltage, off-line switching systems, these losses are acceptably small.

Summary of 3 Forward Converters

| Converter <br> Type | Topology | $V_{o}$ | $\boldsymbol{P}_{o}$ | Transformer | Output <br> Type |
| :--- | :--- | :--- | :--- | :--- | :--- |
| Buck | 1 switch | $V_{o}=V_{\text {in }} * D$ | Any | Not possible | Unipolar |
| Boost | 1 switch | $V_{o}=V_{\text {in }} /(1-D)$ | $I_{\text {in }}$ limits <br> Po | Not possible | Unipolar |
| Full Bridge | 4 switches <br> Minor switch <br> losses | $V_{o}=V_{\text {in }} * D * n$ | Any | Possible | Unipolar/ <br> bipolar |

## Pulse Width Modulation (PWM) Techniques

Pulse Width Modulation


| $\boldsymbol{V}_{\text {Ref }}$ | $V_{\text {Ref }}-V_{\text {Ramp }}=V_{Q 1}$ pulse width | $V_{O}$ |
| :---: | :---: | :---: |
| $V_{\text {Ref }} \downarrow$ | $V_{\text {Ref }}-V_{\text {Ramp }}=V_{Q 1}$ pulse width $\downarrow$ | $V_{O} \downarrow$ |

Bipolar Bridge


## Bipolar Bridge

## Generalities

- Diagonal switching
- Two PWMs are usually employed
- Switches Q1 and Q3 are the + output leg

- Switches Q2 and Q4 are the - output leg
- An output rectifier is not required
- Since the output desired DC, but contains + and - components, a nonpolarized output filter capacitor must be used
- 2 and 4 quadrant operation is possible


## Two types of PWM

- Sign/magnitude in which the sign of the reference signal determines which pair of switches to turn on and the magnitude determines the pulse duration/duty factor
- "50/50" scheme in which there are 2 separate, complimentary PWM signals

Bipolar Bridge - Sign / Magnitude PWM

| Reference <br> Signal | Q1/Q3 D | Q2/Q4 D |
| :---: | :---: | :---: |
| 0 | Off | Off |
| $+25 \%$ | 0.25 | Off |
| $+50 \%$ | 0.50 | Off |
| $+75 \%$ | 0.75 | Off |
| $+100 \%$ | 1.00 | Off |
| $-25 \%$ | Off | 0.25 |
| $-50 \%$ | Off | 0.50 |
| $-75 \%$ | Off | 0.75 |
| $-100 \%$ | Off | 1.00 |



- Switch only one leg at a time
- The 2 switches in the active leg switch on and off together

Bipolar Bridge - Sign / Magnitude PWM - (+) Output


Q1/Q3 closed


Q1/Q3 opened

Bipolar Bridge - Sign / Magnitude PWM - (-) Output


Q2/Q4 closed

Bipolar Bridge - Sign / Magnitude PWM - Waveforms


## "50/50" Bipolar PWM

| Desired Output <br> Reference <br> Signal | Q1/Q3D | $Q 2 / Q 4 \mathrm{D}$ |
| :---: | :---: | :---: |
| $-100 \%$ | $0.0 \%$ | $100.0 \%$ |
| $-75 \%$ | $12.5 \%$ | $87.5 \%$ |
| $-50 \%$ | $25.0 \%$ | $75.0 \%$ |
| $-25 \%$ | $37.5 \%$ | $62.5 \%$ |
| $0 \%$ | $50.0 \%$ | $50.0 \%$ |
| $25 \%$ | $62.5 \%$ | $37.5 \%$ |
| $50 \%$ | $75.0 \%$ | $25.0 \%$ |
| $75 \%$ | $87.5 \%$ | $12.5 \%$ |
| $100 \%$ | $100.0 \%$ | $0.0 \%$ |



- Both bridge legs are always active
- Q1/Q3 (+) bridge
- Q2/Q4 (-) bridge
- Q1/Q3 $180{ }^{\circ}$ phase shifted
- Q2/Q4 $180^{\circ}$ phase shifted
- Q1 is complement of Q4
- Q2 is complement of Q3
"50/50" Bipolar Switching For - 4 V Output

Q1/Q3 on 30\% Q2/Q4 on 70\%


Q4 / D4 Q3/D3

|  | Q1 | Q3 | $Q 2$ | $Q 4$ | $V_{O C}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | On | Off | On | Off | 0 V |
| 2 | Off | Off | On | On | -10 V |
| 3 | Off | On | Off | On | 0 V |
| 4 | Off | Off | On | On | -10 V |


"50/50" Bipolar Switching For 0 V Output

"50/50" Bipolar Switching For + 4 V Output


## Bipolar PS PWM Strategies Compared

| PWM Type | Advantages | Disadvantages |
| :--- | :--- | :--- |
| Sign/Magnitude | Zero crossing transitions are <br> discontinuous |  |
| "50/50" | Output voltage pulse 2X the switching <br> frequency. Easier to filter <br> Smoothest transitions through zero. |  |

# Conducting and Switching Losses 

${ }^{14} \mid V_{c e}$
IGBT Loss Categories


Turn-on losses

Conduction losses

$$
P_{C o n d}=\frac{1}{t_{5}-t_{1}} * \int_{t_{2}}^{t_{3}} V_{C E} * I_{C} * d t
$$

Turnoff losses

$$
P_{S w O n}=\frac{1}{t_{5}-t_{1}} * \int_{t_{1}}^{t_{2}} v_{C E}(t) * i_{C}(t) * d t
$$

$$
P_{S w o f f}=\frac{1}{t_{5}-t_{1}} * \int_{t_{3}}^{t_{4}} v_{C E}(t) * i_{C}(t) * d t
$$

Reduce losses for greater efficiency and:

- Smaller AC distribution system
- Less heat load into cooling water system
- Less heat into buildings and building HVAC
- Reduce IGBT dissipation

Reducing Turn On Losses By Varying $\boldsymbol{R}_{G}$



Reducing Turn On Losses By Varying $\boldsymbol{R}_{G}$


Reducing Turn On Losses By Varying $\boldsymbol{R}_{G}$



Reducing Turn On Losses By Varying $\boldsymbol{R}_{G}$

| Case | $\boldsymbol{R}_{G}$ | $d V_{C E} / d t$ | $E_{O n}$ |
| :---: | :---: | :---: | :---: |
| 1 | $8.2 \Omega$ | - $0.6 \mathrm{kV} / \mu \mathrm{S}$ | 6.4 J |
| 2 | $3.3 \Omega$ | - $1.0 \mathrm{kV} / \mu \mathrm{S}$ | 4.1 J |
| 3 | $1.0 \Omega$ | - $2.8 \mathrm{kV} / \mu \mathrm{S}$ | 2.8 J |
| $\xrightarrow[\mathrm{C}_{\mathrm{G}}=]{\mathbf{R}_{\mathbf{G}}}$ |  | - $P_{\text {Diss }} \propto^{-1}-d V_{C E} / d t$ <br> - $d V_{C E} / d t$ is controlled via $R_{G}$ <br> - Lower losses but possibly increased EMI because of faster $d V_{C E} / d t$ |  |



- If the current rating of a single switch is insufficient (conduction loss is too great), add another switch in parallel.
- There are then 2 ways to switch Q1 and Q2, switch them ON and OFF together or stagger their On and OFF times

Conduction Loss Reduction By Simultaneous Switching of Q1 and Q2

$I_{\text {RMS } 2 S W-E a S w}=\frac{1}{2} * I_{R M S-1 S w} \quad V_{\text {RMS } 2 S w-E a S w}=V_{R M S-1 S w}$
$P_{\text {AvelSw }}=V_{\text {RMSISw }} * I_{\text {RMSISw }}$
$P_{\text {Ave2Sw-EaSw }}=V_{R M S 1 S w} * \frac{1}{2} I_{R M S I S w}=\frac{1}{2} * P_{\text {AvelSw }}$
The composite frequency is the same as in Q1 and Q2

Conducted Loss Reduction By Staggered Switching of Q1 and Q2


- Duty factor is each switch is halved
- $P_{\text {ave }}$ in each switch is $1 / 2$ that of the single switch case
- The composite frequency is twice that of Q1 and Q2

Conducted Loss Reduction By Paralleled Buck Regulators


Features:

- A second switch Q1 is added.
- Q1 and Q2 are staggered switched
- D2 is added, L2 is added
- Current in D1, D2 is $1 / 2$ the load current
- Current in L1, L2 is 1/2 the load current
-L1, L2 energy $1 / 4$ that of single inductor since $E=1 / 2 * L * I^{2}$



$$
P_{S w O f f}=\frac{1}{T} \int_{0}^{t_{S w}} v_{C E}(t) i_{C}(t) d t
$$



- To increase the rate of decay of $I_{C}$ during turnoff
- To decrease the rate of $V_{C E}$ build up during turnoff
- To realize goal, add a resistor $R$, capacitor $C$, diode $D$ snubber network

- When the IGBT turns off, current commutates out of the IGBT into the capacitor, $C$ via the diode $D$
- This aids fast $I_{C}$ current decay
- C becomes linearly charged to the bus voltage
- $d V_{C E} / d t$ inversely proportional to $C$ - this slows $V_{C E}$ recovery

- When the IGBT turns on, the capacitor $C$, discharges through $R$ and the IGBT

- Small C $=$ fast $d V_{C E} / d t$, V appears with current still in the IGBT, have IGBT loss
- Large C means slow $d V_{C E} / d t$, current gone before voltage buildup but the resistor losses are high
- When the snubber circuit is optimized, the IGBT turnoff loss with snubber + snubber loss < IGBT loss w/o snubber!


## Design criteria

- $R$ must limit discharge I through IGBT to < IGBT rating
- $P_{R} \geq E_{C} / T=1 / 2 C V^{2} f$
- C ripple current rating $\geq \Sigma$ (ave charge + ave discharge currents)
- C must appreciably discharge each cycle, so $R C<$ minimum expected IGBT on time
- D has to be rated to hold off the bus voltage and carry peak capacitor charging current

Note: Turn-on losses in the latest IGBTs have been reduced so that snubber circuits are no longer required in most applications

## High Frequency Inductors and Transformers

## Low and High Frequency Transformers Compared

|  | Low frequency | High frequency |
| :--- | :--- | :--- |
| Standards | Well defined by ANSI, IEEE, NEMA and <br> UL | Not as well defined <br> Insulation standard followed |
| Operation | 60 Hz <br> Sine wave <br> 3 phase | 10 kHz to 100 kHz <br> Square wave - transformers Triangular <br> wave - inductors <br> Single phase |
| Core <br> material | 3 to l00 mil laminations of steel or Fe | 0.5 to 3 mil laminations of Fe or Si-Fe <br> Powdered Fe <br> Powdered ferrites, Ni-Zn, Mn-Zn |
| Winding <br> material | Single-strand Cu wire <br> Layer or bobbin-wound | Multi-strand Cu Litz wire <br> Cu foil, layer wound |

## Low and High Frequency Transformers Compared

The power rating of a transformer is dependent upon the kollowing factors $V * A=K_{1} * K_{2} * f * A_{C}{ }^{*} A_{E} * J * B_{M}$
where
$V^{*} A=$ power rating of the transformer $\left(V^{*} A\right)$
$K_{1}=$ waveshape factor (sine or square wave)
$K_{2}=$ copper fill factor (0 to 1)
$f \quad=$ excitation frequency $(\mathrm{Hz})$
$A_{C}=\operatorname{core} \operatorname{area}\left(\mathrm{m}^{2}\right)$
$A_{E}=$ winding area $\left(m^{2}\right)$
$J=$ conductor current density $\left(\frac{A}{m^{2}}\right)$
$B_{M}=$ peakflux density $\left(\frac{W b}{m^{2}}\right) \quad$ where a Weber $=1 *$ volt $*$ sec
The transformer area product $=A_{C} * A_{E} \propto \frac{V^{*} A}{B_{M} * f * J}$

An example of a 10kVA, 480V : 208V Transformer
At 60 Hz the volume and weight would be

| $f$ | $f$ ratio <br> to 60 Hz | Volume (in ${ }^{3}$ ) | Volume ratio to 60 Hz . | Weight <br> ( lb ) | Weight ratio to 60 Hz |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 60 Hz | 1 | $\begin{gathered} 18 \times 18 \times 18= \\ 5832\left(\text { in }^{3}\right) \end{gathered}$ | 1 | 100 | 1 |
| 20 kHz | 333 | $\begin{gathered} 6 H X 5.25 W X 3.37 D \\ 118\left(\mathrm{in}^{3}\right) \end{gathered}$ | $1 / 50$ | 5 | $1 / 20$ |

- Inductance
- Ripple current frequency
- Peak current
- RMS value of AC current
- DC current
- Saturation DC current
- Resonant frequency ( an order of magnitude > ripple frequency)


# Ripple Filters 

Low Frequency Filter


High Frequency Filter


| Low Frequency | High Frequency |
| :--- | :--- |
| Pass DC - reject f $>60 \mathrm{~Hz}$ | Pass DC - reject $f>$ switching frequency |
| Large L1 to reduce On inrush \& high PF | Large L2 to reduce inrush and <br> prevent discontinuous current |
| R2 C2 for "critical" damping | R4 C4 for "critical" damping |

Time Domain $y(t)=f(t) \otimes x(t)$ where $\otimes$ implies the convolution operation

- Difficult computations, particularly transient calculations, requires solution of differential or difference equations

Frequency Domain $Y(f)=F(f) * X(f)$ where *implies multiplication

- Easier computations, all calculations for steady-state or transient conditions that look algebraic in nature.


## Transfer Function

- Relates the output response of a circuit/system to the input stimulus
- Form is $T(f)=Y(f) / X(f)$ where $X(f)$ is the input stimulus and $Y(f)$ is the output response $\quad Y(f)=X(f) * T(f)$


## The "s" Operator

- $s$ is used in the frequency domain and in La Place analysis
- $s=j \omega=j 2 \pi f \quad j=\sqrt{-1}$


## Poles and Zeros

- Zero $=0 \quad$ Pole $=\infty$
- Zeros occur at frequencies that cause the transfer function to go to zero. Transfer function $=0$ is caused by a zero in the numerator and or a pole in the denominator $T(s)=0 / X(s)=0$ or $T(s)=Y(s) / \infty=0$
- Poles occur at frequencies that cause the transfer function to become infinite. Transfer function $=\infty$ is caused by a pole in the numerator or a zero in the denominator $T(s)=\infty / X(s)=\infty \quad$ or $\quad T(s)=Y(s) / 0=\infty$

A Simple Second Order Low Pass Ripple Filter


By voltage divider law

$$
V_{o}=V_{i} * \frac{\frac{1}{s C}}{\frac{1}{s C}+s L} \quad T=\frac{1}{s^{2} L C+1}
$$

Pole $s^{2} L C+1=0$
$\left(j 2 \pi f_{p}\right)^{2} L C+1=0$
Resonant frequency (pole)

$$
f_{p}=\frac{1}{2 \pi \sqrt{L C}}
$$

Zero occurs at
$\left(j 2 \pi f_{p}\right)^{2} L C+1=\infty$
Zero frequency at

$$
f_{z}=\infty
$$

A Simple Low Pass Filter


- Resonant frequency (pole) at $f_{p}$ will cause problems !
- At $f=\infty$, the output goes asymptotically to zero

The Praeg Low Pass Ripple Filter


Why important:

- Used as low and high frequency filters in virtually every power supply
- Provides the filtering of the previous $2^{\text {nd }}$ order filter
- Essentially critical damped
- No DC current in $R, C 2$


## Component Selection Criteria

- L and C1 must be chosen to yield the desired breakpoint frequency (1/10 of the ripple frequency for 40 dB attenuation)
- C1 and C2 must rated for the rectifier working and surge voltages
- C1 and C2 must be rated to carry the ripple current at the rectifier output frequency and at the switching frequency
- L must be large enough to offset the leading PF introduced by main filter capacitor, C1
- L must be large enough to limit the inrush current caused by rapid charge of C1 during power supply turn-on to an acceptable level
- L must be rated to carry the DC load current without overheating or saturating
- $C 2 \geq 5 * C 1$
- $R=(L / C)^{1 / 2}$


The Praeg Low Pass Ripple Filter


$$
T=\frac{s R C_{2}+1}{s^{3} R L C_{1} C_{2}+s^{2} L\left(C_{1}+C_{2}\right)+s R C_{2}+1}
$$

$C_{2} \geq 5^{*} C_{1}$

$$
R=\sqrt{\frac{L}{C_{l}}}
$$



$$
\begin{aligned}
& f:=1 \cdot H z, 2 \cdot H z . .1000 \cdot H z \quad s(f):=j \cdot 2 \cdot \pi \cdot f \quad L:=1.5 \cdot 10^{-3} \cdot H \quad f_{r}:=36 \cdot H z \quad C_{1}:=\frac{1}{4 \pi^{2} \cdot L \cdot f_{r}^{2}} \quad C_{1}=0.0130 F \\
& R:=\sqrt{\frac{L}{C_{1}}} \quad R=0.34 \Omega \quad C_{2}:=5 \cdot C_{1} \quad C_{2}=0.065 F \\
& T_{N}(f):=\frac{s(f) \cdot R \cdot C_{2}+1}{s(f)^{3} \cdot R \cdot L \cdot C_{1} \cdot C_{2}+s(f)^{2} \cdot L \cdot\left(C_{1}+C_{2}\right)+s(f) \cdot R \cdot C_{2}+1} \quad M(f):=20 \cdot \log (|T(f)|) \quad A R(f):=\arg (T(f)) \\
&
\end{aligned}
$$




36 kHz Praeg Filter
$f:=10 \cdot H z, 20 \cdot H z . .100000 \cdot H z \quad s(f):=j \cdot 2 \cdot \pi \cdot f \quad L:=1.5 \cdot 10^{-5} \cdot H \quad C_{I}:=0.00013 \cdot F \quad f_{r}:=\frac{1}{2 \cdot \pi \cdot \sqrt{L \cdot C_{1}}}$

$$
\begin{array}{lll}
R:=\sqrt{\frac{L}{C_{1}}} \quad R=0.34 \Omega \quad C_{2}:=5 \cdot C_{1} \quad C_{2}=6.5 \times 10^{-4} F & f_{r}=3604 \mathrm{~Hz} \\
T(f):=\frac{s(f) \cdot R \cdot C_{2}+1}{s(f)^{3} \cdot R \cdot L \cdot C_{1} \cdot C_{2}+s(f)^{2} \cdot L \cdot\left(C_{1}+C_{2}\right)+s(f) \cdot R \cdot C_{2}+1} & M(f):=20 \cdot \log (|T(f)|) & A R(f):=\arg (T(f)) \\
& A D(f):=\operatorname{AR}(f) \cdot 57.3
\end{array}
$$



$$
\begin{aligned}
& f_{r 1}=\frac{1}{2 \pi \sqrt{L C}} \\
& \text { Let }_{r 2}=n f_{r l}=\frac{n}{2 \pi \sqrt{L C}} \\
& n f_{r 1}=\frac{1}{2 \pi \sqrt{\frac{L}{n} \frac{C}{n}}}
\end{aligned}
$$

$L$ is smaller by the factor $n$
$C$ is smaller by the factor $n$

## Homework Problem \# 10

Given the circuit below:


Sketch $|H(j \omega)|$ versus $\omega$

# Other Design Considerations And Power Supply Costs 

## Heat Loading Into Building Air

- AC equipment $=\sum P_{\text {switchgear }}+P_{\text {transformer }}+P_{\text {cables }}$

Switchgear effiency $\geq 98 \% \quad$ Switchgear losses $=P_{O} *\left(\frac{1-\text { Eff }}{\text { Eff }}\right)$
Transformer efficiency $\geq 97 \%$ Transformer losses $=P_{O} *\left(\frac{1-\text { Eff }}{\text { Eff }}\right)$
$P_{\text {ACcables }}=\sum_{j} i_{\text {RMS }}^{2} * \frac{R}{f t} *$ Length

- Power supply losses $=\sum_{j}\left(P_{\text {in } j}-P_{\text {out } j}\right)$
- $P_{D C}$ output cable $=\sum_{i} i_{D C}^{2} * \frac{R}{f t} *$ Length


## Rack Cooling

- Thermal radiation from rack surface
- Electronics - maximum 50C inside rack
- Max rise in rack $=50 \mathrm{C}-T_{\text {ambient max }}$
- Size openings, back pressure drops $B p=(C F M /(k * \text { Opening Area }))^{2}$
- Fan vs load curve - junction is operating flow point


Power supply heat loss to water $=\sum$ electrical losses of all water-cooled components Heat lost (dissipated) by PS water cooled components $=$ Heat gained by cooling water system $Q=M * c^{*} \Delta T \quad$ cal $=g m * \frac{\mathrm{cal}}{\mathrm{gm} *{ }^{O} C} *\left({ }^{O} C_{\text {Outlet }}-{ }^{O} C_{\text {Inlet }}\right)$
$q=m * c * \Delta T \quad$ watt $=g p m * \frac{264 \text { watt }}{g p m *{ }^{O} C} *\left({ }^{O} C_{\text {Outlet }}-{ }^{O} C_{\text {Inlet }}\right)$
The system pressure drop is $\Delta P=\sum_{i} P_{i}$
Usually the power loss and the inlet and maximum allowable outlet temperatures are known. The mechanical group will usually ask for an estimate of the water flow requirements.
So solving for the flow yields

$$
m=\frac{q}{c^{*} \Delta T}=\frac{\text { watt }}{\frac{264 \text { watt }}{\text { gpm } *{ }^{O} C} *\left({ }^{o} C_{\text {Outlet }}-{ }^{o} C_{\text {Inlet }}\right)}
$$



$Q=$ Power that can be removed by the air or cooling water $(W)$
$T_{j}=$ Device junction temperature $\left({ }^{\circ} \mathrm{C}\right)$
$T_{c}=$ Device case temperature ( ${ }^{\circ} \mathrm{C}$ )
$T_{s}=$ Heatsink temperature ( ${ }^{\circ} C$ )

$T_{a}=$ Ambient air or cooling water inlet temperature ${ }^{\rho} C$ ) $\theta_{j c}=j u n c t i o n ~ t o ~ c a s e ~ t h e r m a l ~ r e s i s t a n c e ~(~ © ~ / ~ W) ~$
$\theta_{c s}=$ case to heatsink thermal resistance $\left.{ }^{\rho} \mathrm{C} / \mathrm{W}\right)$
$\theta_{s a}=$ Heatsink to ambient air or cooling water thermal resistance ( ${ }^{\circ} \mathrm{C} / \mathrm{W}$ )

then all of the device dissipation will be removed by the
 air or water

Calculate the actual air or water temperature rise from $q=m^{*} c^{*} \Delta T$
$\Delta T=\frac{q}{m^{*} c}=\frac{\mathrm{watts}}{\mathrm{gpm} * \frac{264 \text { watt }}{g p *^{\circ} C}}$
$\Delta T \leq$ the maximum allowable temperature rise

|  |  | Input | $C$ (V) |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Power Output | $\begin{gathered} 1 \phi \\ 120 \end{gathered}$ | $\begin{gathered} 3 \phi \\ 208 \end{gathered}$ | $\begin{aligned} & 3 \phi \\ & 480 \end{aligned}$ | $\begin{gathered} 3 \phi \\ 4160 \end{gathered}$ | RM | FS | $A C$ | WC |
| <2kW | X |  |  |  | $X$ |  | $X$ |  |
| $2 \mathrm{~kW} \rightarrow 5 \mathrm{~kW}$ |  | $X$ |  |  | $X$ |  | $X$ |  |
| $>5 \mathrm{~kW} \rightarrow 40 \mathrm{~kW}$ |  |  | $X$ |  | $X$ |  | $X$ |  |
| $>40 \mathrm{~kW} \rightarrow 100 \mathrm{~kW}$ |  |  | $X$ |  |  | X | $X$ |  |
| $>100 \mathrm{~kW} \rightarrow 1 \mathrm{MW}$ |  |  | $X$ |  |  | X | $X$ | $X$ |
| > 1 MW |  |  |  | $X$ |  | X | $X$ | X |
| $\begin{aligned} R M & =\text { Rack } \text { mounted } \\ A C & =\text { Air-cooled } \end{aligned}$ |  |  | FS $=$ Freestanding <br> WC = Water-cooled |  |  |  |  |  |



Cost Of Switchmode Power Supplies


## Homework Problem \# 11

A 100 kW power supply is $80 \%$ efficient. Approximately $50 \%$ of the power supply heat loss is removed by cooling water.

- How much heat is dissipated to building air and how much heat is removed by the water system.
- Calculate the water flow rate needed to limit the water temperature rise to $8^{\circ} \mathrm{C}$ maximum.


## PEP-II and SPEAR3 Dipole Power Supplies

- 1200 VDC, 800 Amperes, 960 KW
- Powers largest magnet string at Spear3, 36 ring bend magnets in series
- Requires 50 PPM (full scale) current regulation, $0.1 \%$ voltage regulation
- Requires 600 VAC, 6-Phase AC Input



## Storage Ring of the Diamond Project

- The power converter comprises of 8 paralleled modules
- Each module is a non-isolated step down PWM switching regulator operating at a fixed frequency of 2 kHz
- IGBT devices are used as the switching element
- The 8 PWM drives are phase shifted by $360 / 8^{\circ}$ to achieve a 16 kHz output ripple frequency
- 1 quadrant operation


Figure 1: Dipole Converter Topology.

## DC Power Supplies in Particle Accelerators

## Diamond Booster Magnet Power Converters

- Booster operates at 5 Hz to accelerate the electrons: 100 MeV to 3 GeV.
- Power converters produce an off-set sine wave current with high repeatability at 5 Hz
- To avoid disturbance on the ac distribution network the dipole and quadrupole power converters were designed to present a constant load despite having high circulating energy: 2 MVA in the case of the dipole
- Redundancy was introduced wherever this was economically feasible.
- Plug-in modules are used to simplify and speed up repairs.
- Component standardization and de-rating across all power converters was an additional design goal


## Diamond Booster Dipole Power Converter

- Booster dipole PC is rated at peaks of 1000A and 2000 V
- Three units are sufficient to produce the required output. The fourth is redundant
- Each unit is made up of a boost circuit and a 2-quadrant output regulator that produces the required offset sine wave current.
- The boost circuit regulates the voltage on the main energy storage capacitor and is controlled to draw constant power from the ac network.
- Displaced 4 kHz switching frequency


Figure 4: First few cycles after turn on.


Figure 1: Booster dipole power circuit.

## DC Power Supplies in Particle Accelerators

## THE 3HZ POWER SUPPLIES OF THE SOLEIL BOOSTER

Table 1: Major booster parameters

| Injection energy | 110 | MeV |
| :--- | :--- | :--- |
| Extraction energy | 2.75 | GeV |
|  |  |  |
| Number of dipoles | 36 | m |
| Dipole magnetic length | 2.16 | mm |
| Dipole gap | 22 | T |
| Dipole field @2.75GeV | 0.74 | A |
| Dipoles inj. current | 19.7 | A |
| Dipoles ext.current | 541 | $\mathrm{~m} \Omega$ |
| Dipoles load resistance | 400 | mH |
| Dipoles load inductance | 156 |  |

Positive master branch



Figure 4: dipoles PS main schematics

DC Power Supplies in Particle Accelerators
Power Supplies for the ATF2



Figure 2: Topology of CNAO synchrotron power supply.


Figure 1.3. MCOR12 Block Diagram.

Bipolar Power Supplies at SPEAR3 and LCLS


Figure 1.1. A typical MCOR installation

DC Power Supplies in Particle Accelerators
NEW MAGNET POWER SUPPLY FOR PAL LINAC

Table 1: Development specifications of MPS

|  | Bipolar | Unipolar |  |
| :---: | :---: | :---: | :---: |
| Size <br> $(W \times y ~ x ~ D) ~$ | $435 \times 135 \times 450$ | $435 \times 178 \times 450$ | mm |
| Input | $1 \phi 220 \mathrm{~V}$ | $3 \phi 30 \mathrm{~V}$ | V |
| Output | $\pm 10 / 20$ | $50 / 50$ | $\mathrm{~A} / \mathrm{V}$ |
| Output <br> stability | $\pm 50 \mathrm{ppm}$ | $\pm 20 \mathrm{ppm}$ | $<1$ hour |
|  | $\pm 100 \mathrm{ppm}$ | $\pm 50 \mathrm{ppm}$ | $>10$ hours |
| Output <br> resolution | 16 |  |  |
| Topology | Full-Bridge $4-\mathrm{Q}$ DC/DC converter |  |  |
| Switch freq. | 50 |  | kHz |
| Output Filter <br> Cut-off freq. | $<5$ |  |  |



Figure 1: Bipolar MPS operation of full-bridge fourquadrant $\mathrm{DC} / \mathrm{DC}$ converter.


Figure 4: Circuit diagram of bipolar MPS.

## PEP-II Large Power Supplies

Table 1: LGPS ratings.

| LGPS | V | I | P (kW) | Qty |
| :---: | :---: | :---: | :---: | :---: |
| BV1/2 | 80 | 900 | 72 | 1 |
| QF2L/R | 80 | 1250 | 100 | 2 |
| QF5L/R | 253 | 750 | 190 | 2 |
| QD4L/R | 200 | 1350 | 270 | 2 |



- Line-isolated
- 32 kHz Switch-output ripple
- High efficiency
- Fast output response
- Stability better than $\pm 10$ ppm
- 100A to 225A
- 70kW to 135 kW
- Low cost: US\$ 0.26-0.39/W



# Section 6. Pulsed Power Supplies 

- A transmission line is a "controlled impedance" device, usually consisting of two conductors.
- Its geometry and dielectric determine the electric and magnetic field distributions between the conductors.
- The voltage between the conductors is determined by the integral of the electric field between them.
- The current along the conductors is determined by the integral of the magnetic field on the conductor surfaces.
- Transmission lines support the propagation of fixed velocity waves in both directions along the line.
- Transmission lines guide transverse electro magnetic (TEM) waves, TE or TM waves are guided by waveguides


## Transmission Line Types

- Coaxial transmission lines
- Voltage between two coaxial conductors
- Currents of equal magnitude and opposite sign are carried on the conductors
- Conductors separated by air or dielectric
- Transverse electromagnetic (TEM) transmission line media
- Non-dispersive (propagates all frequency components equally), no cutoff frequency
- No external electric or magnetic fields

Transmission Line Types

- Coaxial transmission lines and cables



## Transmission Line Types

- Planar transmission line - Microstrip line consists of a single strip on dielectric separated from a ground plane

when $\left(\frac{W}{H}\right)<1 \quad$ Effective Dielectric Constant $\quad \varepsilon_{e}=\frac{\varepsilon_{r}+1}{2}+\frac{\varepsilon_{r}-1}{2}\left[\left(1+12\left(\frac{H}{W}\right)\right)^{-1 / 2}+0.04\left(1-\left(\frac{W}{H}\right)\right)^{2}\right]$
Characteristic Impedance $\quad Z_{O}=\frac{60}{\sqrt{\varepsilon_{e}}} \ln \left(8 \frac{H}{W}+0.25 \frac{W}{H}\right) \quad$ ohms
when $\left(\frac{W}{H}\right) \geq 1 \quad$ Effective Dielectric Constant $\quad \varepsilon_{e}=\frac{\varepsilon_{r}+1}{2}+\frac{\varepsilon_{r}-1}{2}\left[\left(1+12\left(\frac{H}{W}\right)\right)^{-1 / 2}\right]$
Characteristic Impedance

$$
Z_{O}=\frac{120 \pi}{\sqrt{\varepsilon_{e}}\left[\frac{W}{H}+1.393+\frac{2}{3} \ln \left(\frac{W}{H}+1.444\right)\right]}
$$

## Transmission Line Types

- Planar transmission line - Stripline consists of a single strip buried in a dielectric separated from two or more ground planes


Characteristic Impedance

$$
Z_{O}=\frac{60}{\sqrt{\varepsilon_{r}}} \ln \left[\frac{4 H}{0.67 \pi W\left(0.8+\frac{T}{D}\right)}\right] \text { ohms }
$$

## Transmission Line Types

- Lumped element transmission lines
- Combination of series inductors, shunt capacitors
- Single inductor-capacitor combination is a resonant circuit
- Series of an infinite combination of series L, shunt C turns into an ideal transmission line
- Electric fields of lines stored in capacitors
- Magnetic fields of lines stored in series inductors

$E=\hat{y} E_{y} \quad H=\hat{x} H_{x} \quad Z_{0}=\sqrt{\frac{L}{C}}$ Characteristic impedance - 377 ohms for air (free space) for air (and most dielectrics) $\mu_{r}=1$, for air $\varepsilon_{r}=1$ (most other dielectrics $\varepsilon_{r}>1$
$Z_{0}=\frac{\ln b / a}{2 \pi} \sqrt{\frac{\mu}{\varepsilon}}$ For coaxial line, $50 \Omega \leq Z_{0} \leq 80 \Omega$
$v=\frac{1}{\sqrt{\mu_{0} \mu_{r} \varepsilon_{0} \varepsilon_{r}}}=$ wave velocity wavelength $\lambda=\frac{v}{f} \quad$ time delay $=t_{d}=\sqrt{L C}$


The general situation at an interface between two transmission lines of impedance $Z_{1}$ and $Z_{2}$ is

- A source generates an incident voltage and current, $\left(V_{l}{ }^{+}, I_{1}{ }^{+}\right)$moving forward on Line 1, with $V_{1}{ }^{+}=Z_{1} I_{1}{ }^{+}$
- $\left(V_{1}{ }^{+}, I_{1}{ }^{+}\right)$at the interface causes a transmitted voltage and current, $\left(V_{2}{ }^{+}\right.$, $I_{2}{ }^{+}$), moving forward on Line 2, with $V_{2}{ }^{+}=Z_{2} I_{2}{ }^{+}$
- $\left(V_{1}{ }^{+}, I_{1}{ }^{+}\right)$at the interface also causes a reflected voltage and current, $\left(V_{1}^{-}, I_{1}^{-}\right)$, moving backward on Line 1, with $V_{1}^{-}=Z_{1} I_{1}^{-}$



## Equations at an Interface

The voltages on each side of the interface must be equal.
$V_{1}^{+}+V_{1}^{-} \quad=V_{2}^{+}$
Current must be conserved at the interface.

$$
I_{1}^{+} \quad=I_{2}^{+}+I_{1}^{-}
$$

Expressing the second equation in terms of the voltages
and impedances yields the Reflection Coefficient, Gamma
$\frac{V_{1}^{+}}{Z_{1}}$
$=\frac{V_{2}^{+}}{Z_{2}}+\frac{V_{1}^{-}}{Z_{1}}=\frac{V_{1}^{+}}{Z_{2}}+\frac{V_{1}^{-}}{Z_{2}}+\frac{V_{1}^{-}}{Z_{1}}$
$\frac{V_{1}^{-}}{V_{1}^{+}}$

$$
=\frac{\frac{1}{Z_{1}}-\frac{1}{Z_{2}}}{\frac{1}{Z_{1}}+\frac{1}{Z_{2}}}
$$

$$
=\frac{Z_{2}-Z_{1}}{Z_{2}+Z_{1}} \quad=\Gamma
$$

The transmission coefficient, $T$, is defined as

$T \quad \square \frac{V_{2}^{+}}{V_{1}^{+}}=\frac{\left(V_{1}^{+}+V_{1}^{-}\right)}{V_{1}^{+}}$
$=1+\Gamma$
$=\frac{Z_{2}+Z_{1}+Z_{2}-Z_{1}}{Z_{2}+Z_{1}}=\frac{2 Z_{2}}{Z_{2}+Z_{1}}$

- Join two transmission lines together
- If the impedances of both lines are the same, the electric and magnetic fields (voltage and current) can propagate without interruption.
- If not, the boundary conditions on the fields force a reflection of part of the signal


## Power Conservation

The flow of energy (power) is conserved at the interface.

$$
\begin{aligned}
P_{I N} & =V_{1}^{+} I_{1}^{+} \text {(assume all voltages and impedances are real) } \\
& =\frac{\left(V_{1}^{+}\right)^{2}}{Z_{1}} \\
& =\frac{\left(T V_{1}^{+}\right)^{2}}{Z_{2}}=\frac{\left(4 Z_{2}\right)}{\left(Z_{2}+Z_{1}\right)^{2}}\left(V_{1}^{+}\right)^{2} \\
P_{T} & =\frac{\left(\Gamma V_{1}^{+}\right)^{2}}{Z_{1}}=\frac{\left(Z_{2}-Z_{1}\right)^{2}}{Z_{1}\left(Z_{2}+Z_{1}\right)^{2}}\left(V_{1}^{+}\right)^{2} \\
P_{R} & =\frac{\left(4 Z_{2} Z_{1}+Z_{2}^{2}-2 Z_{2} Z_{1}+Z_{1}^{2}\right)}{Z_{1}\left(Z_{2}+Z_{1}\right)^{2}}\left(V_{1}^{+}\right)^{2}=\frac{\left(V_{1}^{+}\right)^{2}}{Z_{1}} \\
P_{T}+P_{R} & =P_{I N}
\end{aligned}
$$

## Open Line

- $Z_{1}=Z_{0}, Z_{2}$ infinite
- $\Gamma=1$
- $I_{2}=0$

- Voltage totally reflected without inversion

Shorted Line

- $Z_{2}$ zero
- $\Gamma=-1$
- $V_{2}=0$

- Voltage totally reflected with inversion


## More Complicated Example

$$
\left(\mathrm{V}_{1}^{+}, \mathrm{l}_{1}^{+}\right)
$$

- Pulse sent down line on controlled impedance

- First interface is with higher impedance device $\left(Z_{2}>Z_{1}\right)$
- Transmitted pulse
- Reflected pulse

- Reflected transmitted pulse reaches first interface
- Transmitted pulse down original line
- Reflected pulse on second line



## Homework Problem \#12

A. An artificial transmission line can be formed using lumped Ls and Cs. Calculate the delay of an artificial line composed of 8 sections of inductances $L=4 m H$ per section and capacitance $C=40 p F$ per section.
B. The frequency of a signal applied to a two-wire transmission cable is $3 G H z$. What is the signal wavelength if the cable dielectric is air?
C. What is the signal wavelength if the cable dielectric has a relative permittivity of 3.6?

Hint - relative permittivity of air is 1

## Homework Problem \#13

For the transmission line shown below, calculate the Reflection Coefficient $\Gamma$, the reflected voltage and the voltage and current along the line versus time.


Kicker Pulser


## The Pulse Forming Network (PFN)

Flatness is directly proportional to the number of LC meshes Rise-time is determined by the LC of the mesh closest to the load Pulse width $T$ is twice the one way transit time tof the wave in the PFN The one-way transit time is

$$
t=\sqrt{L^{*} C}
$$

and the pulse width $T$ is

$$
T=2 * \sqrt{L^{*} C}
$$

The load impedance and pulse width are usually specified. From these two parameters the PFN LC can be specified. The nominal $L$ and $C$ in each mesh is the total $L$ and $C$ divided by the number of meshes.

$$
\begin{aligned}
Z & =\sqrt{\frac{L}{C}} \\
T & =2 * Z * C \\
C & =\frac{T}{2 * Z} \\
L & =\frac{T * Z}{2}
\end{aligned}
$$

Since the PFN impedance is matched to the load impedance, all the PFN stored energy is dissipated in the load

## The Pulse Forming Network (PFN)

The PFN is typically tuned to the impedance of the load in order to reduce voltage and current reflections. The effective output voltage at the load obeys the voltage divider law and is effectively
$V_{l o a d}=V_{p f n} * \frac{Z_{\text {load }}}{Z_{\text {load }}+Z_{\text {pfn }}}$
$V_{\text {pfn }}=V_{\text {load }} * \frac{Z_{\text {load }}+Z_{\text {pfn }}}{Z_{\text {load }}}$

Because typically the PFN has the same impedance as the load, $V_{p f n}=2 * V_{\text {load }}$

Therefore the PFN must be charged to twice the desired load voltage.

The Pulse Forming Network (PFN)



Charging Supply


Klystron perveance $=\frac{I_{\text {klystron }}}{\left(V_{\text {beam voltage }}\right)^{3 / 2}}$
The perveance of 5045 klystron is 2 micropervs
The peak RF power from a 5045 is 65 MW , the beam volatge is 350 kV
$I_{\text {klystron }}=P *\left(V_{\text {beam voltage }}\right)^{3 / 2}=2 * 10^{-6} *(350 \mathrm{kV})^{3 / 2}=414 \mathrm{~A}$
The power needed to achieve $65 M W$ of $R F=V_{\text {beam voltage }}$ * I klystron

$$
=350 \mathrm{kV} * 414 \mathrm{~A}=144.0 \mathrm{MW}!
$$

Pulsed power is the right approach
Smaller power source
Less cooling required (klystron efficiency is 45\%)
Average power $=$ peak power $*$ duty cycle $($ on-time $* P R R)$
Average power $=144.9 \mathrm{MW} * 5 \mu S * 60 \mathrm{~Hz}=42.4 \mathrm{~kW}$ much lower power

- Open transmission lines are often used for Pulse Forming Networks (PFNs).
- They are typically charged up from a high impedance source
- Their open end is connected to a normally open switch that closes to connect the PFN to the load
- This situation can be viewed as a traveling wave reflecting back and forth off of two open ends
- Total voltage on the line is the sum of the incident and reflected waves $\left(V_{P F N}=2 V_{L O A D}\right)$
- Pulse has length $2 l / v$, since the tail of the pulse must reflect off of the other open end before it reaches the load


## Present Klystron Modulator Power Supply



- Primary VVT, with diode rectifier
- High voltage secondary with diodes and filter capacitor
- Protected against secondary faults



## Kicker Magnet Calculation

A 3 GeV electron moves in a magnetic field as shown below. Calculate the B field and the magnet current needed to provide a 2 milliradian deflection in the electron path. The magnet is 1 meter in length.

magnet aperature $60 \mathrm{~mm} \times 34 \mathrm{~mm}$
Some definitions and constants needed for conversion
$\mathrm{q}:=1.610^{-19} \mathrm{C} \quad$ Electron charge $\quad \mathrm{v}_{\mathrm{x}}:=3 \cdot 10^{8} \cdot \frac{\mathrm{~m}}{\mathrm{~s}} \quad$ electron velocity $\mathrm{eV}:=1.610^{-19} \cdot \mathrm{~J} \quad$ Electron energy electronmass :=9.109510 ${ }^{-31} \cdot \mathrm{~kg}$ electron rest mass $\quad n_{t}:=2 \begin{aligned} & \text { magnet turns (2 plates producing field in sam } \epsilon \\ & \text { direction) }\end{aligned}$ $L_{\text {mag }}:=1 \cdot \mathrm{~m}$ magnet length $\theta:=2 \cdot 10^{-3}$ rad $\quad$ This is the deflection anglend $\mu_{i}=4 \cdot \pi \cdot 10^{-7} \cdot \frac{\text { newton }}{\mathrm{amp}^{2}} \quad \mu_{\mathrm{r}}:=1$ $E_{x}:=3 \cdot 10^{9} \cdot e V \quad E_{y}:=E_{x} \cdot \tan (\theta) \quad E_{y}=9.6 \times 10^{-13} J \quad E_{y}=6 \times 10^{6} e^{2} \quad F_{y}:=\frac{E_{y}}{L_{m a g}} \quad F_{y}=9.6 \times 10^{-13} N$
$\mathrm{L}_{\text {field }}:=2 \cdot(60 \mathrm{~mm}+15 \cdot \mathrm{~mm}) \quad \mathrm{L}_{\text {field }}=0.15 \mathrm{~m} \quad \mathrm{~B}_{\mathrm{z}}:=\frac{\mathrm{F}_{\mathrm{y}}}{\mathrm{q} \cdot \mathrm{v}_{\mathrm{x}}} \quad \mathrm{B}_{\mathrm{z}}=0.02 \mathrm{~T}$

$$
\mathrm{I}:=\frac{\mathrm{B}_{\mathrm{z}} \cdot \mathrm{~L}_{\text {field }}}{\mu_{0} \cdot \mu_{\mathrm{r}} \cdot \mathrm{n}_{\mathrm{t}}} \quad \mathrm{I}=1194 \mathrm{~A}
$$

## Kicker or Fast Modulator

- Improve the rise time of modulator pulse using Cable PFN
- In line Switch with PFN
- Blumline with Shunt Switch


## Kicker Modulator

- Conventional Inline Kicker Modulator
- Thyratron for switches
- Improve the rise time of modulator pulse using Cable PFN



## Cable Transformer Modulator

- Cable Pulse transformer connecting the input of a cable in Parallel and the Output In Series and if the pulse is shorter than twice the electrical length of the cable and driving a matched load cable transformer works.
- Fast rise time with simple transformer
- Disadvantage stray capacitance and floating cable return limits transformer usage




## Solid-State Pulsers

# A Solid-State Turn-On Pulser 



- All pulse capacitors are pre-charged simultaneously
- IGBTs are all switched on together
- Capacitors are then simultaneously discharged producing sinusoidal V and I pulses in the pulse transformer and magnet. The secondary winding voltages are additive
- At the end of the pulse the IGBT is turned off. The magnet current decay causes a voltage reversal at the free-wheeling diode
- The freewheeling diodes conduct and the magnet current decays exponentially to zero

Comparison of Thyratron and Solid-State Pulser Parameters

| Parameter | Thyratron | Solid-state |
| :---: | :---: | :---: |
| Control turn-on | Yes | Yes |
| Control turn-off | No | Yes |
| Pulse Shaping | PFN | IGBT |
| Output Voltage | $1 / 2$ PFN voltage | Same as device voltage |

## Ongoing Developments

- Replace Thyratron with solid-state switch SCR, IGBT, MOSFET, etc
- Having a high enough di/dt capability is the problem
- IGBTs without PFNs are being used at the present time


Figure 1 The hybrid modulator block diagram shows the high voltage power supply, the solid state modulator with energy storage capacitor, and the pulse transformer.

KLYSTRON



Fig. 11) Switch Assembly SPR-08F45-6-WC $A B B$ Semiconductor $A G$

- Marx Generator charges capacitors in parallel for quickness, discharges them in series for high output voltage. For long pulses, advantage is to avoid the need for large iron core transformers based on volt-second product

- If the load is a magnet, the charging inductors are not required

- Another implementation, using solid-state switches in place of the charging inductors for smaller size and less diversion of capacitor current from load


- Fractional turn pulse transformer
-Similar to a induction accelerator
-Multiple primaries driven in parallel
-The secondary connected in series
- Solid-state driver consists of
- A solid state switch that turns on and off
- DC capacitor per primary winding


Solid State Induction Klystron Modulator




- Solid-state 10-stack installed alongside Gallery line-type PFN unit
- $22 \mathrm{kV}=>330 \mathrm{kV}$ via 15:1 xfmr
- Prototype currently at 255 kV @ $2.2 \mu \mathrm{sec}$ @ 120 PPS

Solid - State Induction Klystron Modulator modulator pulse



## Induction Kicker Modulator

2 ea. 10 kV 2.5 kA 0.6 meters 1.5 mrad



A controlled impedance transmission line often drives a kicker. The kicker is usually well modeled as an inductor. A matching circuit can be built around the kicker and its inductance so that this circuit, including the kicker magnet, has constant, frequency independent, impedance which is matched to the transmission line.

Assuming that the transmission line impedance is $Z_{0}$ and the kicker inductance is $L_{\text {Kicker }}$ derive the values of R1, R2, and C necessary to make a frequency independent (constant) impedance $Z_{0}$.


## Homework Problem \# 15

A. What is the significance of the value $\sqrt{\frac{\mu_{0}}{\varepsilon_{0}}}$ ?
$B$. What is the significance of the values $\frac{1}{\sqrt{\mu_{0} \varepsilon_{o}}}$ and $\sqrt{L^{*} C}$ ?
C. Calculate the speed of light in mediums with dielectric constants of:

$$
\varepsilon_{r}=1 \quad \varepsilon_{r}=2 \quad \varepsilon_{r}=4 \quad \varepsilon_{r}=8 \quad \varepsilon_{r}=16
$$

## 7. Magnetics

- Magnetic circuits are analogous to electric circuits and are important for the analysis of magnetic devices. The equations for both electric and magnetic circuits show strong similarities
- Various magnetic types, such as transformers and filter inductors, play a key role in many of the components used in power supplies
- Magnets are also extensively used in accelerators to guide, direct, steer and focus beams


## Field Due to a Current



Right Hand Rule:

- Thumb $=$ Current


Flux $\Phi$ Direction

- Fingers Point in Direction of Magnetic Field

Magnetic Units Including Turns

| Symbol | Description | SI units | cgs units |
| :---: | :---: | :---: | :---: |
| $N$ | Winding turns | turn $(t)$ | $t$ |
| $H$ | Field intensity | $(A \cdot t) / m$ | Oersted $($ Oe $)$ |
| $B$ | Flux density | tesla $(T)$ | gauss $(G)$ |
| $\mu$ | Permeability | $T \cdot m / A \cdot$ | G/Oe |
| $F$ | Magnetomotive force | $A \cdot t$ | gilbert $(G b)$ |
| $\Phi$ | Flux | weber/t $(W b / t)$ | maxwell |
| $R$ | Reluctance | $A \cdot t / W b$ |  |
| $P$ | Permeance | henry/t | $H / t$ |
| $I$ | Current | ampere $(A)$ | $A$ |
| $L$ | Inductance | henry $(H)$ | $H$ |

- U-U, U-I cores
- E-E, E-I, ETD cores
- POT cores
- RM cores
- $P Q$ and $P M$ cores
- EP, EFD and ER cores
- Toroid


Pot cores


## Permeability Definitions

- $\mu_{0}=$ permeability of vacuum $=4 * \pi * 10^{-7} \mathrm{H} / \mathrm{m}$
- $\mu_{r}=$ relative permeability
- $\mu_{m}=$ material permeability $=B / H$ at any given point
- $\mu_{m}=\mu_{0} * \mu_{r}$
- Permeability is an important core parameter
- Ferromagnetic materials used in transformer and inductor cores because of their high permeability

| Core Materials |
| :--- |
| Air |
| Alloys |
| Amorphous |
| Iron Powder |
| Manganese-Zinc Ferrite |
| Molybdenum Permalloy <br> Powder <br> Nickel-Zinc Ferrite <br> Sendust <br> Silicon Steel |

## Material Characterization





Core Material Guidelines

| Material | Frequency Range | $\boldsymbol{B}_{\text {sat }}$ | Cost |
| :---: | :---: | :---: | :---: |
| Ferrites | Good to <br> microwaves | 0.2 T | Low |
| MPP (Moly <br> Permalloy Powder) | 200 kHz | 0.2 to 0.55 T | High |
| Powdered Fe | 1 MHz | 0.4 to 1 T | Low |
| Laminated Si-Fe | 2 kHz | $1 T$ | Low |
| Laminated <br> Electrical Steel | 2 kHz | 0.5 to 1.8 T | Low |
| Ni-Fe Alloys | 100 kHz | 0.5 to 1.8 T | High |

Effect of permeability magnitude on transformer current

## Effect of permeability nonlinearity on

 transformer current


Relationship Between $v(t)$ and $\Phi(t)$

$$
\begin{aligned}
v(t) & =-\frac{d \Phi(t)}{d t}=V_{\max } \cos 2 \pi f t \\
v_{t}(t) & =\frac{v(t)}{N_{p}}=\frac{V_{\max } \cos 2 \pi f t}{N_{p}} \\
\Phi(t) & =-\int v_{t}(t) d t \\
\Phi(t) & =\int B(t) \bullet d a_{c}
\end{aligned}
$$


$\int_{s} B(t) \cdot d a_{c}=-\int v_{t}(t) d t=-\int \frac{V_{\max } \cos 2 \pi f t}{N_{p}} d t$ $B(t) A_{C}=\frac{V_{\text {max }} \sin 2 \pi f t}{2 \pi f N_{p}}$
$B(t)=\frac{V_{\max } \sin 2 \pi f t}{2 \pi f N_{p} A_{c}}$
$B_{\text {max }}$ occurs when $\sin 2 \pi f t=1$ and $V_{\max }=\sqrt{2} V_{r m s}$
$B_{\max }=\frac{\sqrt{2} V_{r m s}}{2 \pi f N_{p} A_{c}}=\frac{V_{r m s}}{4.44 f N_{p} A_{c}}$


## Transformer Design - Ensure Sufficient Core Crossection

$B_{\text {max }}=\frac{V_{\text {rms }}}{4.44 * f * A_{c} * N_{p} * 10^{-8}}$
where
$B_{\max }=$ maximum allowable flux density in gauss
$V_{r m s}=$ voltage applied to the primary in volts
$4.44=\frac{\sqrt{2}}{2 \pi}$ converts peak AC to rms and $\omega$ to $f$
$f=$ frequency of the applied voltage in hertz
$A_{c} \quad=$ Corecrossectional area in $\mathrm{cm}^{2}$
$N_{p} \quad=$ Number of primary winding turns
$10^{-8}=$ conversion from engineering to SLunits

Example for a $480 \mathrm{~V}, 600 \mathrm{kVA}$, laminated electrical steel core
$B_{\max }=\frac{480 \mathrm{~V} * 1.05(\text { voltage safety factor ) }}{4.44 * 60 \mathrm{~Hz} * 300 \mathrm{~cm}^{2} * 60 \text { turns } * 10^{-8}}=10,510$ gauss


For square wave or rectangular wave excitation
$B_{\max }=\frac{D * V_{\text {peak }}}{4 * f * A_{c} * N_{p} * 10^{-8}}$
where
$D \quad=$ duty cycle the wave
$V_{\text {peak }}=$ peak applied voltage

- Four quadrant B-H curves are known as hysteresis curves. Note that the curve is open in the middle. This is a consequence of the magnetic microstructure.
- Remanence is defined as the absolute value of the magnetic field when the applied voltage is removed. The remnant field can cause inrush current problems when the transformer is re-energized
- Coercive Force - The amount of reverse magnetic field which must be
 applied to a magnetic material to make the magnetic flux return to zero.


## Transformer Design Issues - Inrush Current

For the $480 \mathrm{~V}, 600 \mathrm{kVA}$ transformer
$i_{\max }=\frac{10^{3} * h * A_{c} *\left(\left(B_{r}+2 * B_{\max }\right)-130\right)}{3.2 * N_{p} * A_{s}}$
$i_{\max }=$ maximum instantaneous current in amperes
$h \quad=$ the length of the coil in inches
$A_{c} \quad=$ the crossectional area of the core in sq inches
$B_{\max }=$ Maximumfluxdensity $=10,500 G=1.05 T=71$ kilolines per square inch
$B_{r} \quad=$ residual flux density in kilolines (Maxwells) per square inch
$=60 \%$ of 1.05T, expressed as 43 kilolines per square inch
$N_{p} \quad=$ number of primary turns
$A_{s} \quad=$ effective square inches of the air-core magnetic field
Example $I_{f l}=\frac{600 \mathrm{kVA}}{\sqrt{3} * 480 \mathrm{~V}}=722 \mathrm{~A}$, the inrush current is
$i_{\text {max }}=\frac{10^{3} * 40 * 46.5 *((43+2 * 71)-130)}{3.2 * 60 * 69.4}=7.68 \mathrm{kA}$
This is about 11X the transformer full load (operating) current
Reduce the inrush current by increasing the number of primary turns and/or increasing the effective area of the air-core magnetic field

There are always energy losses in transformers. These energy losses generate heat in the form of core losses and winding losses. The losses are from the following sources:

1. Hysteresis loss from sweeping of flux from positive to negative and the area enclosed by the loop is the loss. Hysteresis loss is due to the energy used to align and re-align the magnetic domains. The smaller the loop area, the smaller the energy loss per cycle
2. Eddy current loss from the circulating currents within the cores due to flux generated voltages.
3. Copper or winding loss. This is also dependent on the wire size, switching frequency, etc. Skin effect and proximity effect will contribute to this loss.

- As the frequency of a given ac current in a conductor is increased, the power dissipation increases
- We ascribe this to an increase in ac resistance of the conductor but in actuality it is due to a rearrangement of the current distribution within the conductor
- The increase in loss is due to a tendency for the current to concentrate on the perimeter of the conductor rather than being uniform over the conductor area as it would be at dc
- This effect becomes more severe as frequency is increased
- This is called "skin effect"

$$
\delta=\frac{1}{\sqrt{\pi f \mu \sigma}} \quad \text { meters }
$$

This is the depth where the fields or current in a conductor have decreased to $37 \%$ of their nominal value.
In other words $63 \%$ of the current is carried in this depth.

## Proximity Effect

- A current carrying conductor will generate a magnetic field
- This field can induce eddy currents in nearby conductors, increasing losses in addition to any skin effect. The eddy currents obey Lenz's Law. They flow in a direction that reduces the flux in the conductor
- This is referred to as "proximity effect"
- In a transformer or inductor, the inner windings operate in a field created by the outer windings
- This can also limit the conductor size
- As a general rule the wire diameter or the layer thickness is usually less than twice the skin depth at the operating frequency. For multi-layer windings wire diameters of less than 0.5 skin depth may be required.

Proximity Effect


Current Concentrates At One Side

$\uparrow$ Proximity Effect - Multiple Parallel Wires

## Effect of Air Gap



## Why Do We Use Air Gaps?

- They are unavoidable in many cores
- In an inductor they permit increased energy storage for a given $B$ by reducing the effective permeability
- Air gaps also stabilize the inductance value for both bias and manufacturing variations
- In general gaps are undesired in transformers but very useful in inductors
- An air gap may be discrete or distributed
$L=\frac{\mu_{0} \mu_{r} N^{2} A_{c}}{\mu_{r} l_{g}+l_{c}}$
where
$N=$ the number of winding turns (dimensionless)
$A_{c}=$ the core cross sectional area in $m^{2}$
$l_{c}=$ the length of the magnetic path in the core in $m$
$l_{g}=$ the effective length of the air gap in $m$
$\mu_{r}=$ core material permeability under the operating conditions
$\mu_{r}$ is dimensionless
$\mu_{0}=\frac{4 \pi * 10^{-7} \mathrm{H}}{m}$

Purposes

- Used as filters for smoothing power supply ripple
- Used as fault current limiting reactors in AC power currents
- Used to limit di/dt in certain pulsed circuits

Requirements

- Must carry high DC current
- Must select core size that is able to store the required magnetic energy (volt-seconds)
- An air gap is sometimes employed to extend DC current capability without saturating. Iron and Ferrites are manufactured with distributed air gaps.


## 8. Controls

- Purposes
- Voltage and Current Mode Control
- V - I Automatic Crossover Network
- Power Supply Controllers


## Mathematical Preliminaries - Differential Equations

Differential equations describe systems that change with time
For a system with time varying quantities, $u(t), y(t)$, that satisfy
the differential equation $\quad \frac{d y(t)}{d t}=a u(t)$
$y(t)$ depends on its past values as well as those of $u(t)$
$\frac{d y(t)}{d t} \equiv \lim _{\Delta t \rightarrow 0} \frac{y\left(t_{0}+\Delta t\right)-y\left(t_{0}\right)}{\Delta t}=a u(t)$
$y\left(t_{0}+\Delta t\right) \cong y\left(t_{0}\right)+\Delta t a u\left(t_{0}\right)$
Continuing this to construct $y(t)$ at later times
$y\left(t_{0}+2 \Delta t\right) \cong y\left(t_{0}+\Delta t\right)+\Delta t a u\left(t_{0}+\Delta t\right) \cong y\left(t_{0}\right)+\Delta t a u\left(t_{0}\right)+\Delta t a u\left(t_{0}+\Delta t\right)$
$y\left(t_{0}+N \Delta t\right) \cong y\left(t_{0}\right)+a \sum_{n=0}^{N-1} \Delta t u\left(t_{0}+n \Delta t\right)$
Resulting in the integral equation
$y(t) \quad=y\left(t_{0}\right)+a \int_{t_{0}}^{t} d \tau u(\tau)$

## Linear Systems

A linear system, $h[x]$, is defined such that for functions $x_{1}$ and $x_{2}$ if
$y_{1}=h\left[x_{1}\right]$ and $y_{2}=h\left[x_{2}\right]$
then $a y_{1}+b y_{2}=h\left[a x_{1}+b x_{2}\right]$ This is the principle of superposition
Examples of Linear Systems
Constant gain system

$$
h_{1}[x]=A_{1} x
$$

Sum of two constant gain systems

Integrals

$$
h_{2}[x]=A_{2} x+A_{3} x
$$

$$
h_{3}[x]=\int_{t_{0}}^{t} x(\tau) d \tau
$$

Derivatives

$$
h_{4}[x]=\frac{d x}{d t}
$$

We are interested in linear systems because there are many mathematical tools available for use on linear systems and because many common physical systems and components are linear: Resistors, Inductors, Capacitors

# Example of a Nonlinear System 

$$
\begin{aligned}
& h[x]=e^{x} \quad \text { This is a nonlinear system } \\
& \text { Proof: } \\
& e^{(a x+b x)}=e^{a x} e^{b x} \neq a e^{x}+b e^{x}
\end{aligned}
$$

We note that non-linear systems can often be approximated by linear systems

## Impulse Function

- The problems we investigate involve a control signal acting on a system
- We simplify the solution by representing the control signal as a sequence of elementary functions
- Then we need to characterize the response of our system to these elementary functions
- Finally, we use the properties of linear systems to obtain the response of the system with the control signal acting on it
- Two such commonly used elementary functions are the impulse function and the step function


## Impulse Functions - Discrete and Continuous

Continuous Dirac delta function, $\delta(t)$ properties
$\delta(t)=0, \quad t \neq 0 \quad \delta(t)=\infty, \quad t=0$
$\int_{-\infty}^{\infty} \delta(t) d t=1$


Discrete impulse function properties
$\delta[n]=0, \quad n \neq 0 \quad \delta[n]=1, \quad n=0$
Functional representation
$y[n]=\sum_{k=-\infty}^{\infty} x[k] \delta[k-n]$



## Continuous Step Function

Properties
Height $=1$

$$
\begin{aligned}
& U(t)=0, \quad t<0 \\
& U(t)=1, \quad t>0
\end{aligned}
$$



Relation to impulse

$$
\delta(t)=\frac{d U(t)}{d t}
$$

Functional representation

$$
\begin{aligned}
y\left(t_{0}\right) & =-\int_{-\infty}^{\infty} x(t) \frac{d U\left(t_{0}-t\right)}{d t} d t \\
& =-\left.x(t) U\left(t_{0}-t\right)\right|_{t=-\infty} ^{t=\infty}+\int_{-\infty}^{\infty} \frac{d x(t)}{d t} U\left(t_{0}-t\right) d t \\
& =\int_{-\infty}^{\infty} U\left(t_{0}-t\right) \frac{d x(t)}{d t} d t+x(-\infty)
\end{aligned}
$$

## Discrete Step Function

Heaviside step function

Properties
$U[n]=0, \quad n<0$
$U[n]=1, \quad n \geq 0$
Relation to impulse
$\delta[n]=U[n]-U[n-1]$
Functional representation
$y[n]=\sum_{k=-\infty}^{\infty} x[k] \delta[n-k]$
$y[n]=\sum_{k=-\infty}^{\infty} x[k](U[n-k]-U[n-k-1])$
$y[n]=\sum_{k=-\infty}^{\infty}(x[k]-x[k-1]) U[n-k]$




- Introducing the complex exponential according to the Euler formula

$$
\begin{aligned}
& e^{j \omega t}=\cos \omega t+j \sin \omega t \\
& \omega=2 * \pi^{*} f \\
& j=\sqrt{-1}
\end{aligned}
$$

- Since the magnitude of the complex exponential is always 1 , this function gives us a steady state eigenfunction of the constant, differential and integral operators we will need to analyze circuits
- If the input to a system consists of a single frequency, the output will consist of just that same frequency, although with a different amplitude and phase than the input.
- The phasor amplitude is the eigenvalue of that frequency Eigenvalue = proper value or characteristic roots

For example the roots of the quadratic formula $y=(-b+/-S Q R T$ bsquared-4ac)/2a=eigenvalues or the roots of the equation

Kirchoff's current law - sum of all current into a node is 0 Kirchoff's voltage law - sum of all voltages around a loop is 0 Voltage-current relations across passive elements
$V=R^{*} I \quad V=L^{*} \frac{d i}{d t} \quad I=C \frac{d v}{d t}$
$v=R i+L \frac{d i}{d t}$
Represent the current $i$ as a complex exponential
$i(t)=I_{0} e^{j \omega t}$ then the equation for $v$ becomes
$v=R I_{0} e^{j \omega t}+L j \omega I_{0} e^{j \omega t}=(R+j \omega L) I_{0} e^{j \omega t}$
$I_{0} e^{j \omega t}$ is the eigenfunction
$(R+j \omega L)$ is the eigenvalue, which, is the impedance, $Z(\omega)$

- Fourier transforms represent some of the Eigen functions as combinations (sums/integrals) of complex exponentials.
- The standard Fourier transform pair for continuous functions is

$$
f(t)=\int_{-\infty}^{\infty} F(\omega) e^{j \omega t} \frac{d \omega}{2 \pi} \quad F(\omega)=\int_{-\infty}^{\infty} f(t) e^{-j \omega t} d t
$$

- For periodic systems, with a period, T, the only complex eigenvectors that can be used to represent the signals are those whose frequencies are multiples of the "fundamental harmonic", $\omega=2 \pi / T$.
- Periodic functions are represented by the infinite sums of the appropriately weighted harmonics. In this case the Fourier transform pairs are

$$
f(t)=\sum_{n=-\infty}^{\infty} F_{n} e^{j \frac{2 \pi n}{T} t} \quad F_{n}=\frac{1}{T} \int_{-T / 2}^{T / 2} f(t) e^{-j \frac{2 \pi n}{T} t} d t
$$

## Fourier Series

Using Euler's formula, we can also represent these relations as

$$
\begin{aligned}
& f(t)=\frac{1}{2} a_{0}+\sum_{n=1}^{\infty} a_{n} \cos \frac{2 \pi n}{T} t+\sum_{n=1}^{\infty} b_{n} \sin \frac{2 \pi n}{T} t \\
& a_{0}=\frac{1}{T} \int_{0}^{T} f(t) d t \\
& a_{n}=\frac{2}{T} \int_{-T / 2}^{T / 2} f(t) \cos \frac{2 \pi n}{T} t d t \\
& b_{n}=\frac{2}{T} \int_{-T / 2}^{T / 2} f(t) \sin \frac{2 \pi n}{T} t d t
\end{aligned}
$$

- When working with linear, time-invariant systems, there are several advantages to moving from the time domain to the frequency domain.

If $x_{1} \rightarrow y_{1}, \quad x_{2} \rightarrow y_{2}$ and if $a x_{1} \rightarrow a y_{1}, \quad b x_{2} \rightarrow b y_{2}$ and if $a x_{1}+b x_{2} \rightarrow a y_{1}+b y_{2}$, then system is linear

If $x(t) \rightarrow y(t)$ and if $x\left(t-t_{0}\right) \rightarrow y\left(t-t_{0}\right)$, then system is time-invariant

- Each frequency corresponds to a unique eigenfunction of the system and the system response for each frequency can be calculated independently.

Given an input, $x(t)$, a system, $h(t)$, and an output, $y(t)$, the Transfer Function is the Fourier Transform of $h(t)$

$$
\begin{aligned}
& H(\omega)=\int_{-\infty}^{\infty} h(t) e^{-j \omega t} d t \\
& H(\omega)=\frac{Y(\omega)}{X(\omega)} \\
& \text { where }
\end{aligned}
$$

$$
\begin{aligned}
& Y(\omega)=\int_{-\infty}^{\infty} y(t) e^{-j \omega t} d t \\
& X(\omega)=\int_{-\infty}^{\infty} x(t) e^{-j \omega t} d t
\end{aligned}
$$

- There is another transform often used in system analysis, the Laplace transform.
- It is closely related to the Fourier transform in that it is also based on system eigenfunctions.
- In addition to "real" frequencies, it also uses complex frequencies that allow it to also study decaying solutions.
- As with Fourier transform, integral must converge in order for transform to exist.
- It is convenient to use Laplace transforms for the study of solutions to problems with initial conditions.
- The variable used in Laplace transforms is often

$$
s=j \omega
$$

## Laplace Transforms

The definition of the Laplace transform is $F(s)=\int_{0}^{\infty} f(t) e^{-s t} d t$
Some simple transforms

$$
\begin{array}{ll}
f(t)=\delta\left(t-t_{0}\right) t_{0}>0 & F(s)=1 \\
f(t)=U(t) & F(s)=\frac{1}{s}
\end{array}
$$

Unit step with delay
$f(t)=U\left(t-t_{0}\right) \quad t \geq 0 \quad F(s)=\frac{e^{-s t} 0}{s}$
Exponentially decreasing function, starting at $t=0$

$$
f(t)=e^{-a t} U(t) a \geq 0 \quad F(s)=\frac{1}{s+a}
$$

Laplace transform of a derivative
let $\quad f(t)=\frac{d g(t)}{d t}$

$$
F(s)=s G(s)-g(0)
$$

- Laplace Transforms simplify the calculations of system behavior, but these calculations are performed in the s domain.
- In order to return to a time domain function, the s domain function must be inverted.
- Inversion of these functions can be performed via complex variable techniques.
- Much more commonly, one uses readily available tables of functions and their Laplace transform pairs
- There also exist such transform tables for Fourier transforms.
http://www.vibrationdata.com/Laplace.htm
http://mathworld.wolfram.com/FourierTransform.html


## Circuit Analysis Using Calculus

$K V L \quad A(t)=R i(t)+v_{c}(t) \quad B u t i(t)=C \frac{d v_{c}(t)}{d t}$
System equation
$R C \frac{d v_{c}(t)}{d t}+v_{c}(t)=A(t) \quad$ Let $R C=\tau$
Solution

$$
v_{c}(t)=v_{c}(0) e^{-\frac{t}{\tau}}+\frac{1}{R C} e^{-\frac{t}{\tau}} \int_{0}^{t} A(t) e^{\frac{t}{\tau}} d t
$$

For the case when A is constant

$v_{c}(t)=\left[v_{c}(0) e^{-\frac{t}{\tau}}+A\left(1-e^{-\frac{t}{\tau}}\right)\right]$
This is now in the form of an initial value multiplied by an eigenfunction and an input multiplied by the same eigenfunction

## Circuit Analysis Using Transforms

Repeat the same problem using Laplace transforms

$$
C \frac{d v_{c}(t)}{d t} \quad=\frac{\left[A(t)-v_{c}(t)\right]}{R}
$$

Transform both sides

$$
\begin{array}{ll}
C\left[s V_{c}(s)-v_{c}(0)\right] & =-\frac{1}{R} V_{c}(s)+\frac{1}{R} A(s) \\
\left(s C+\frac{1}{R}\right) V_{c}(s) & =C v_{c}(0)+\frac{1}{R} A(s) \\
V_{c}(s) & =\frac{1}{s+\tau^{-1}} v_{c}(0)+\frac{\tau^{-1}}{s+\tau^{-1}} A(s) \text { let } \tau^{-1}=\alpha
\end{array}
$$

For the case when $A$ is constant

$$
=\frac{1}{s+\alpha} v_{l}(0)+A \frac{1}{s} \frac{\alpha}{s+\alpha}
$$

Take the inverse transform
$v_{c}(t)$

$$
=v_{c}(0) e^{-\alpha t}+A\left(1-e^{-\alpha t}\right)
$$

$v_{c}(t)$
$=v_{c}(0) e^{\frac{-t}{\tau}}+A\left(1-e^{\frac{-t}{\tau}}\right)$

Same result as on the previous page

Take the inverse transform to obtain
$v(t)=V e^{-\frac{t}{\tau}}+\left(1-e^{-\frac{t}{\tau}}\right)$ A, as before
From the transform equation
$V(s)=\frac{1}{s+\tau^{-1}} v(0)+\frac{\tau^{-1}}{s+\tau^{-1}} A(s)$
we can immediately read off the system transfer function
as the ratio of $\frac{V(s)}{A(s)}$ when the initial conditions are zero.
We also see that both the transfer function and the response to the initial conditions have the same poles and therefore similar frequency characteristics

## One Pole Low-Pass Systems

- Dynamics are determined by the numerator and denominator of transfer function
- The values of s for which the numerator or denominator vanishes are called "zeroes" and "poles"
- One pole circuits all have the same shape response and depend only on the time constant, $\tau=R C$ or $L / R$
- A one pole circuit rises to $63 \%$ or decays to $37 \%$ of its final value at $t=\tau$

$$
H(s)=\frac{\tau^{-1}}{s+\tau^{-1}}
$$



## One Pole Low Pass Frequency Response

- Since we will analyze our systems primarily in the frequency domain, it is important to understand the properties of a one pole system as a function of frequency.
- We can calculate the transfer function using algebra on the system impedances

$$
\begin{aligned}
H(j \omega) & =\frac{\frac{1}{j \omega C}}{R+\frac{l}{j \omega C}} \\
& =\frac{\tau^{-1}}{j \omega+\tau^{-1}} \\
& =\frac{1}{1+j \omega \tau}
\end{aligned}
$$

## One Pole LP Frequency Response

Magnitude

$$
\begin{aligned}
|H(j \omega)| & =\frac{1}{\sqrt{1+(\omega \tau)^{2}}} \\
|H(j \omega)|_{d B} & =20 \log _{10}|H(j \omega)| \\
& =-10 \log _{10}\left[1+(\omega \tau)^{2}\right] \\
& \cong 0 \quad \text { for } \omega \tau \ll 1
\end{aligned}
$$

3 dB (half-power) point $\quad=-10 \log _{10} 2$ for $\omega \tau=1$
20 dB per decade attenuation $\cong-20 \log _{10} \omega-20 \log _{10} \tau$ for $\omega \tau \gg 1$
Phase $\quad \angle H(j \omega)=-\arctan (\omega \tau)$

$$
\begin{array}{ll}
\cong 0 & \omega \tau \ll 1 \\
=-45^{0} & \omega \tau=1 \\
\cong-90^{0} & \omega \tau \gg 1
\end{array}
$$

One Pole LP Frequency Response
Bode Diagram


Two Pole Low Pass Frequency Response


## Two Pole Systems

Find transfer function of voltage divider

$$
\begin{aligned}
H(j \omega) & =\frac{\frac{R_{L} / j \omega C}{R_{L}+1 / j \omega C}}{R_{S}+j \omega L+\frac{R_{L} / j \omega C}{R_{L}+1 / j \omega C}}=\frac{R_{L}}{-R_{L} L C \omega^{2}+j\left(R_{L} R_{S} C+L\right) \omega+\left(R_{S}+R_{L}\right)} \\
& =\frac{1}{L C} \frac{1}{-\omega^{2}+j\left(R_{S} / L+1 / R_{L} C\right) \omega+\left(1+R_{S} / R_{L}\right)(1 / L C)} \quad \text { let } \omega_{0}^{2}=1 / L C \\
& =\frac{\omega_{0}^{2}}{-\omega^{2}+j\left(R_{S} / L+1 / R_{L} C\right) \omega+\left(1+R_{S} / R_{L}\right) \omega_{0}^{2}}
\end{aligned}
$$

This has the form

$$
\begin{array}{ll}
H(s)=\frac{a_{0}}{s^{2}+a_{1} s+a_{0}} & =\frac{a_{0}}{\left(s-s_{1}\right)\left(s-s_{2}\right)} \\
s_{1}=-\frac{a_{1}}{2}+\sqrt{\left(\frac{a_{1}}{2}\right)^{2}-a_{0}} & s_{2}=-\frac{a_{1}}{2}-\sqrt{\left(\frac{a_{1}}{2}\right)^{2}-a_{0}} \\
\text { January } 2010 & \text { Section } 8 \text {-Controls }
\end{array}
$$

- Two pole circuits have two degrees of freedom. One degree sets the system time scale. One degree sets the stability parameter
- For a given time scale, the more stable the system, the slower its response. Two pole systems can be separated into three categories
- Over-damped system $a_{1}^{2} / a_{0}>4$
- Both poles are real
- No oscillation in step response
- Critically damped system $a_{1}^{2} / a_{0}=4$
- Both poles are real and identical
- Fastest step response with no oscillation
- Under-damped system $a_{1}^{2} / a_{0}<4$
- Poles are complex conjugates of each other
- Step response is faster than the other two, but has overshoot

Two Pole System Step Response


Two Pole System Frequency Response
Bode Diagram


## Summarizing

- Low and high frequency behavior is almost independent of $a_{1}$
- At low frequencies the magnitude is constant and the phase approaches $0^{0}$
- At high frequencies the magnitude decreases $40 \mathrm{~dB} /$ decade ( $20 \mathrm{~dB} / \mathrm{pole}$ ) and the Phase approaches $-180^{\circ}$ (-90 \%/pole)
- At $\omega_{0} a_{1}$ determines attenuation and phase slope
- Increased rise time and overshoot are the result of additional response near $\omega_{0}$
- A resonant circuit is a lossless $\left(R_{S}=0\right.$ and $R_{L}=\infty$ in diagram) second order circuit often encountered in pulsed-power systems. Real systems have loss (and damping), but can be well approximated by resonant circuits
- The resonant frequency is $f=\frac{1}{2 \pi \sqrt{L C}}$


## Bode Plots

- Bode plots are a standard way to present properties of feedback systems
- Each pole
- Corresponds to a 6 dB/octave ( $20 \mathrm{~dB} /$ decade) roll-off in amplitude above the pole
- Represent magnitude on log-log plot with a straight line that has a 6 dB/octave kink at the pole
- Corresponds to a 90 degree phase shift at high frequencies
- 0 angle shift at $f_{c} / 10$
- -45 degree shift at $f_{c}$
- -90 degree shift at $10^{*} f_{c}$


## Bode Plots

- Complex conjugate poles are slightly more complex

Far from the poles they have the same behavior as two real poles

- 12 dB /octave
- 180 degree phase shift

Near the pole frequency, their behavior depends on the damping factor of the complex pole pair

- Similar rules exist for zeros

6 dB/octave increase in gain above zero
+45 degree phase shift at the zero

- Purpose of a power supply is to provide stable power
- Use feedback circuits to
- Regulate a system, that is, keep the output fixed at a desired constant value
- Control a system, that is, force the output to follow a variable control input

$I_{o}=K\left(V_{i}-\beta Z I_{o}\right)$ rearranging gives

$$
\frac{I_{o}}{V_{i}}=A_{C L}=\frac{K}{1+\beta Z K}
$$

- $A_{C L}$ is called the closed loop gain, $\beta$ is the feedback factor
- $\beta$ Z K is called the open loop gain
- For $\beta$ Z K >> 1

$$
A_{C L}=\frac{1}{\beta Z}
$$

- Power supply characteristics ( $K$ ) unimportant, gain dependent upon $\beta$ Z

Stability - Introduction


The feedback loop acts as an active null detector so that the output always follows the input

| $\boldsymbol{I}_{o}=\boldsymbol{K}\left(\boldsymbol{V}_{i}-\boldsymbol{\beta} \mathbf{Z} \boldsymbol{I}_{o}\right)$ |  |  |
| :---: | :---: | :---: |
| $V_{i}$ | $V_{i}-\beta Z I_{o}$ | $I_{o}$ |
| $V_{i} \downarrow$ | $V_{i}-\beta Z I_{o} \downarrow$ | $I_{o} \downarrow$ |
| $I_{o}$ | $V_{i}-\beta Z I_{o} \downarrow$ | $I_{o} \downarrow$ |
| $I_{o} \downarrow$ | $V_{i}-\beta Z I_{o}$ | $I_{o}$ |

## Factors That Affect Power Supply Stability

## Three Types of Stability

- Stability against oscillation
- Stability against short and long-term output voltage or current drift
- Stability (Regulation) against rapid, short changes in line voltage or load characteristics


All the elements of the transfer function, gain, or in this case, the transconductance, are all functions of frequency $s=j \omega=j 2 \pi f$


Very simply : $1+\beta(s) Z(s) K(s)$ must not $=0$ or approach 0 $\beta(s) Z(s) K(s)$ must not $=-1$ in order to avoid oscillations
$|\beta| e^{j \alpha}|Z| e^{j \beta}|K| e^{j \varphi}=|\beta||Z||K| e^{j(\alpha+\beta+\varphi)}$
$|\beta||Z||K| \neq 1 \quad$ when $\quad \alpha+\beta+\varphi= \pm 180^{\circ}$

Stability Against Oscillation

$\frac{B(s) Z(s) I_{o}(s)}{V_{i}(s)}=\frac{B(s) Z(s) K(s) V_{i}(s)}{V_{i}(s)}=T(s)=\beta(s) Z(s) K(s)$

Very simply :
$|\beta\|Z\|| K \mid \neq 1$ when $\alpha+\beta+\varphi= \pm 180^{\circ}$


- For stability, the phase shift must be $<180^{\circ}$ when the $\mid$ gain $\mid=1$
- For stability, the |gain must be <1 when the phase shift is $180^{\circ}$

Short-Term (24 hour) Stability - essentially stability against cyclic or diurnal temperature changes.

$$
\begin{aligned}
& \frac{I_{o}}{V_{i}}=A_{C L}=\frac{K}{1+\beta Z K} \\
& \text { Since } \beta Z K \gg 1, \quad \frac{I_{o}}{V_{i}}=A_{C L}=\frac{1}{\beta Z} \quad K \text { is unimportant, }
\end{aligned}
$$ behavior dependent on load Z, feedback factor $\beta$ and upon $V_{i}$ stability



The part that has the greatest influence short-term stability is the feedback signal $\beta$ because the feedback acts as an active null detector.
$V_{i}$ is sometimes temperature stabilized

## Factors Affecting Short-Term (24 hour) Power Supply Drift Stability

- The diurnal temperature cycle can be as much as $40^{\circ} \mathrm{F}\left(22^{\circ} \mathrm{C}\right)$. This globally affects the internal parts as well as the external setpoint
- All parts (resistors, capacitors, semiconductors, op-amps, etc) are temperature dependent.
- The load is also temperature dependent and is subject to the same diurnal changes
- The input line voltage will change during the course of the day as more load is consumed or shed


## Ensuring Short-Term Drift Stability

## General

- Use low-temperature coefficient parts or balance (+) coefficient parts with (-) coefficient parts
- Enclose the power supply in a controlled environment where temperature change is held to a minimum
- 10 to 50 ppm attainable w/o temperature control (5 to 10 ppm ) with temperature control

For the read-back signal, use:

- Precision, low-temperature coefficient current transductors ( $0.3 \mathrm{ppm} /{ }^{\circ} \mathrm{C}$ ) with metal film burden resistor $\left(0.9 \mathrm{ppm} /{ }^{\circ} \mathrm{C}\right) \cong 1.2 \mathrm{ppm} /{ }^{\circ} \mathrm{C}$
- Precision, low-temperature coefficient resistors for current shunt or voltage read-back ( $10 \mathrm{ppm} /{ }^{\circ} \mathrm{C}$ )


Hitec (in Europe)
Danfysik Model 866
http://www.danfysik.com
$0- \pm 600 \mathrm{~A}$
$\pm 400 \mathrm{~mA}$ out
$0.3 \mathrm{ppm} /{ }^{\circ} \mathrm{C}$
DC-100 kHz $10 \mathrm{kA} / \mathrm{mS}$
Separate burden resistor


Danfysik Model 860 Series http://www.danfysik.com $0- \pm 1000$ A, $\pm 2000$ A, $\pm$ 3000 A
$\pm 10$ V out
$0.3 \mathrm{ppm} /{ }^{\circ} \mathrm{C}$
DC-100 kHz
$10 \mathrm{kA} / \mathrm{mS}$


## Long-Term Stability

- All parts are subject to aging.
- Resistors increase or decrease in value
- Capacitor dielectrics breakdown
- Capacitor electrolytes dry out or evaporate and leak
- Semiconductor bias points change
- Op-amp scale, linearity, monotonicity, gain and offsets change with time


## Stability Enhancement

- Accelerate initial aging components prior to intended use by baking at elevated temperatures
- Accelerate aging by exposure to electron beam
- Two types of Regulation - Load and Line
- Classic definition of Load Regulation ( 0\% is best)

$$
\% V_{R}=\frac{V_{N L}-V_{F L}}{V_{F L}} * 100 \% \quad \% I_{R}=\frac{I_{N L}-I_{F L}}{I_{F L}} * 100 \%
$$

- Classic definition employing $V_{N L}$ is usually not applicable. A limited version uses "decreased load or increased load" instead of a no-load condition

$$
\% V_{R}=\frac{V_{D L}-V_{F L}}{V_{F L}} * 100 \%
$$

$$
\% I_{R}=\frac{I_{D L}-I_{F L}}{I_{F L}} * 100 \%
$$

- In addition, the recovery time for the power supply output voltage or current to return the original condition is also specified
"The power supply shall have a voltage regulation of $0.5 \%$ for load changes of $\pm 5 \%$ from nominal with voltage recovery in $\leq 2$ milliseconds"
- Line Regulation - Definition (HL= output voltage under high line, $N L=$ output voltage under nominal line, $L L=$ output voltage under low line)

$$
\begin{array}{ll}
\% V_{R}=\frac{V_{H L}-V_{N L}}{V_{N L}} * 100 \% & \% I_{R}=\frac{I_{H L}-I_{N L}}{I_{N L}} * 100 \% \\
\% V_{R}=\frac{V_{N L}-V_{L L}}{V_{N L}} * 100 \% & \% I_{R}=\frac{I_{N L}-I_{L L}}{I_{N L}} * 100 \%
\end{array}
$$

- In addition, the recovery time for the power supply output voltage or current to return the original condition is also specified
"The power supply shall have a voltage/current regulation of $0.5 \%$ for line changes of $\pm 5 \%$ from nominal with voltage/current recovery in $\leq 2 \mathrm{mS}$ "


## Factors that Affect Stability (Regulation) Against Transient Effects

The ability of a power supply to respond to a transient condition depends upon the speed, depth and duration of the transient. The transient can be mitigated by the use of:

- Large filter capacitors and inductors in the input and output filters to maintain the input and output load voltage and current against line voltage changes and load changes..
- Employ fast regulating circuits. Regulating speed should be at least as fast as the fastest expected transient.


## PID Loops - Proportional Control

- Earliest controllers proportional only
- Proportional control consists of just a gain
- It has good response to instantaneous changes in the process or other cause of error
- Control effort is the product of the error and a finite gain Kp
- Eventually effort is too small to reduce error to zero
- There is always an error - it can never be eliminated



## PID Loops - Integral Control

- Integral control consists of a pure integrator
- The control effort is now $\int \boldsymbol{e}(\boldsymbol{t}) \boldsymbol{d} \boldsymbol{t}$
- Eliminates DC errors
- Limits high frequency response
- Introduces a phase delay that can cause sluggishness or oscillation



## PID Loops - Derivative Control

- Responds to the change of the error signal
- Control effort increases with frequency of error signal $s K_{d}$
- Useful either to cancel a pole or to predict periodic behavior
- Can emphasize high frequency noise



## PID Loops - Summary

- PID stands for Proportional, Integral, and Derivative control
- Standard, general purpose classical control element
- $K_{p}$ general cancelling of error signals
- $K_{i}$ eliminates DC error
- $K_{d}$ compensates for changes in the error signal or process



## Stability and Feedback Summary

- The transfer function is the relation between the input, $x$, and the output, $y$
- y more closely approaches the desired output by increasing feedback gain
- The efficiency of feedback for a dynamic (time-varying) system involves not only the gains, but also the speed of the system response
- Bandwidth is the frequency range over which the feedback achieves (close) to its nominal gain (3 dB point)
- DC Response is a measure of how closely the system tracks a constant input. Improve the DC Response by increasing the loop gain
- Step Response is The action of the system in response to an input step
- Settling Time is how long it takes to settle to within a certain fraction of its final value
- Overshoot is any ringing occurs as the system achieves its final value
- Ramp response is a measure of how well the system follows an input ramp command


## Controls

## Purposes

- Set the output voltage or current to a desired value
- Regulate the output voltage or current to the desired value in the presence of line, load and temperature changes
- Monitor load and power supply actual versus desired performance



$V_{0} \quad V_{\text {Error }}=V_{\text {Ref }}-V_{O}$. A Pulse Oscillator switches Q1 on with every pulse. $L$ current is converted to a voltage by a sense resistor. The $L$ current builds up to the threshold set by the error voltage which then turns off Q1 in order to keep the output voltage or current constant.



## Summary

- Typically 2 control loops - voltage and current
- The outer loop defines the source type - voltage or current stabilized
- The outer loop has lower BW and corrects for drift due to slow temperature changes and aging effects
- The inner loop has higher BW and compensates for fast transients, AC line changes



## Independent



- Provides a constant output voltage regardless of the output current
- Fixed DC output voltage


## Dependent



- Provides a constant output voltage regardless of the output current
- Continuously adjustable
- $V_{o}$ dependent on $V_{\text {Prog }}\left(V_{\text {Ref }}\right)$


## Norton Current Sources



- Provides a constant output current regardless of the output voltage
- Fixed DC output current


## Dependent



- Provides a constant output current regardless of the output voltage
- Continuously adjustable
- $I_{o}$ dependent on $V_{\text {Prog }}\left(V_{\text {Ref }}\right)$


## Voltage Sources

## Independent


$V_{O}$ does not vary, regardless of $I_{O}$

$I_{O}$ does not vary, regardless of $V_{O}$


Power Supply Front Panel

Automatic Voltage/Current Crossover - Example 1


Constant Voltage Mode. The power supply will operate in this mode whenever the current demanded by the load is less than that defined by the front panel current control. The output voltage is set by the front panel voltage control. The output current is set by the load resistance and the Vset.

Automatic Voltage/Current Crossover - Example 2


Constant Current Mode. The power supply will operate in this mode whenever the voltage demanded by the load is less than that defined by the front panel voltage control. The output current is set by the front panel current control. The output voltage is set by the load resistance and the I set.

## All-Analog Power Supply Controllers - Circa 1970s to 1980s




14 Hybrid Analog/Digital Power Supply Controllers - Circa 1980s to Present


14 Hybrid Analog/Digital Power Supply Controllers - Circa 1980s to Present


## All Digital Power Supply Controllers - Circa the Future



All Digital Power Supply Controllers - Circa the Future


## Daisy - Chaining of Power Supply Controls



| Controls Type | Characteristics |
| :---: | :---: |
| All analog controls | - Long, expensive multi-conductor cable <br> - Cables subject to noise pickup, ground loops, losses in signal strength <br> - Installation rigid, difficult to modify |
| Hybrid analog/digital controls | - PLCs, ADCs / DACs subject to noise pickup, ground loops, must keep out of power supply <br> - Serial data cable can be daisy-chained <br> - Installation rigid, difficult to modify |
| All digital controls | - Integrated high level digital signals exhibit greater immunity to noise pickup, ground loops <br> - Serial data cable can be daisy-chained <br> - Installation flexible, control system can be modified in software or firmware <br> - Will require novel implementation of interlocks, voltage and current transductors |

## Some Communication Busses

| Bus <br> Type | Single / <br> Differential | Protocol | Data Rate | Length | Connector | Comments |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| RS232 | $\begin{gathered} -12 \rightarrow+12 \mathrm{~V} \\ S E \end{gathered}$ | Serial | $115 \mathrm{~kb} / \mathrm{s}$ | $5 m$ | $\begin{aligned} & 25 / 15 / 9 p \text { in } \\ & \operatorname{sub} D \end{aligned}$ | Inexpensive wiring |
| BitBus <br> IEEE 1118 | $0-5 \mathrm{~V}$ <br> Differential | Serial | $375 \mathrm{~kb} / \mathrm{s}$ | $300 m$ | 9 pin sub D | Inexpensive wiring |
| $\begin{aligned} & I E E E 488 \\ & G P I B \end{aligned}$ |  | Parallel | 8Mb/s | $20 m$ | 24 pin | Measurement <br> Equipment |
| Ethernet | Optical/SE <br> Differential | Serial | $1 \mathrm{~Gb} / \mathrm{s}$ |  | RJ8, RJ45 <br> Optical | Move lots of data packets |
| USB 2.0 |  | Serial | $12 \mathrm{Mb} / \mathrm{s}$ | $5 m$ | 4 pin USB | Hot-swappable |
| Firewire IEEE1394 | $3.3 \mathrm{~V}$ <br> Differential | Serial | 800Mb/s | $46 m$ | $\begin{gathered} 4 \text { pin } / 6 \text { pin } \\ \text { Optical } \end{gathered}$ | Hot-swappable |
| SCSI | $\begin{gathered} \text { 3.3V Diff/ } \\ \text { Optical } \end{gathered}$ | Parallel | $1.28 \mathrm{~Gb} / \mathrm{s}$ | $12 m$ | $\begin{aligned} & 68 \text { pin } \\ & 80 \text { pin } \end{aligned}$ |  |
| eSATA |  | Serial | $3 G b / s$ |  |  | Hot-swappable |

# 9. Safety and Interlocks 

## Safety

National Fire Protection Association (NFPA)

- NFPA 70 - 2008, National Electrical Code
- NFPA 70E - 2009, Standard for Electrical Safety in the Workplace


## National Electrical Code

## National Electrical Code NFPA 70

- Deals with hardware design, inspection and installation
- Most Articles do not pertain directly to power systems, but some examples that do are:

1. Sizing of raceways and conduits to carry power and control cables. Some relevant Articles are
2. Sizing of power cables for ampacity. Some relevant Articles are
3. Discharge of stored energy in capacitors

Example of cable ampacity sizing
A power supply provides 375A to a magnet via cables. The ambient temperature is $45 C$ (104F), maximum and the cables are installed in cable tray. The cable tray fill conforms to the requirements of NECArticle 392.

Use NEC Table 310-17 for single conductor cables in free air at 30C. The derating for the 45C ambient is 0.87. The derating for the single conductor in a cable tray is 0.65 if placed touching other cables in the cable tray. The required amapcity is Ampacity $=\frac{I P S}{\text { deratings }}=\frac{375 \mathrm{~A}}{0.87 * 0.65}=663 \mathrm{~A}$

From Table 310-17 the basic amapcity of 500kcmil cable is 700A $>663 \mathrm{~A}$.

Use 2-1/C500kcmil cables

## National Electrical Code

Example of conduit sizing - Size a rigid metal conduit system to enclose 2-1/C4/0AWG cables with a 1-1/C\#2AWG ground conductor. Conductors are THHN 600V rated. Conduit size and fill are covered by Article 344.22 and Table 1 of Chapter 9

| Table 1-Maximum \% of Cross Section of Conduit and Tubing for |  |
| :---: | :---: |
| Conductors |  |$|$| Number of Conductors | All Conduit types |
| :---: | :---: |
| 1 | $53 \%$ |
| 2 | $31 \%$ |
| Over 2 | $40 \%$ |

Chapter 9 Table 5 area of $1 / C \# 4 / 0=0.32 \mathrm{in}^{2}$ and $1 / C \# 2 A W G=0.12 \mathrm{in}^{2}$. Total cable area $=0.76 \mathrm{in}^{2}$
$0.76 \mathrm{in}^{2 /} 0.4=1.9 \mathrm{in}^{2}$ From Chapter 9, Table 4 a 1-1/2 diameter conduit has an internal area of $2.071 \mathrm{in}^{2}$. Use this size

## National Electrical Code

Example of raceway sizing

| Cable | Diameter (in) | Area $\left(\right.$ in $^{2}$ ) | Quantity | Total Area |
| :---: | :---: | :---: | :---: | :---: |
| 3/C\#8AWG | 0.600 | 0.283 | 3 | 0.848 |
| 3/C\#6AWG | 0.684 | 0.367 | 4 | 1.470 |
| 3/C\#4AWG | 0.876 | 0.603 | 4 | 2.411 |
| 3/C\#2AWG | 1.005 | 0.793 | 5 | 3.966 |
| 3/C\#1/0AWG | 1.231 | 1.190 | 5 | 5.951 |
| 3/C\#2/0AWG | 1.328 | 1.385 | 4 | 5.540 |
| 3/C\#4/0AWG | 1.556 | 1.902 | 2 | 3.803 |
|  |  |  |  | 23.990 |


| Cable | Diameter (in) |  | Quantity | Sd (in) |
| :---: | :---: | :---: | :---: | :---: |
| 1/C\#4/0AWG | 0.642 |  | 6 | 3.852 |

Example of raceway sizing

- Article 392.9(A)(3) mixture of multiconductor cables and l/C\#4/0AWG cables
- Article 392.9, Column 2 sum of single conductor diameters
- 6-1/C4/0 (ea 0.642" diameter) cables placed in a single layer and no other cables on top of them
- 24 " wide tray. The allowable area reserved for multiconductor cables per Column 2, 28-1.2Sd $=28-6^{*} 0.642^{\prime \prime}=28-3.852 "=24.148$ in $^{2}$
- The total crossectional area of all the multiconductor cables is 23.990 in $^{2}$ which is less than the cable tray area reserved for the multiconductor cables - OK


24 " wide
-Example of capacitor bleeder resistor sizing per NEC Article 460. Code requires permanent fixed energy discharge devices on capacitors operating at
> 50 V working voltage

- < 600 V , discharge to 50 V or less in 1 minute
-> 600 V , discharge to 50 V or less in 5 minutes
- Redundant bleeder resistors recommended


$$
\begin{aligned}
& V_{f}=V_{i} e^{\frac{-t}{R C}} \\
& R=\frac{-t}{C \ln \left(V_{f} / V_{i}\right)}=\frac{-60 * \mathrm{sec}}{500 \mu \mathrm{~F} \ln (50 \mathrm{~V} / 500 \mathrm{~V})} \\
& R=50 \mathrm{kohm} \\
& P_{R}=\frac{V_{i}^{2}}{R}=\frac{(500 \mathrm{~V})^{2}}{50 \mathrm{k} \Omega}=5 \mathrm{~W} \\
& \text { Use two } 5 \mathrm{~W}, 100 \mathrm{k} \Omega \text { resistors in parallel }
\end{aligned}
$$

## NFPA 70E

NFPA 70E-2004-Standard for Electrical Safety in the Workplace

- Addresses employer and employee in the workplace
- Focus is on procedures, personnel protective equipment
- Attempts to mitigate effects of three major electrical hazard types - shock, arc flash and arc blast

- Limited approach boundary is the distance from an exposed live part within which a shock hazard exists
- Restricted approach boundary is the distance from an exposed live part within which there is an increased risk of shock, due to electrical arc over for personnel working in proximity to the live part
- Prohibited approach boundary is the distance from an exposed live part within which work is considered the same as making contact with the live part

Safety
NFPA 70E-2009-Approach boundaries - deal with the voltage hazard

| NFPA 70E, Table 130.2(C) |  |  |  |
| :---: | :---: | :---: | :---: |
| AC and DC <br> Voltage Range <br> Line - Line | Limited Approach <br> Boundary <br> Distance - No PPE | Restricted Approach <br> Boundary Distance <br> No PPE | Prohibited Approach <br> Boundary Distance <br> PPE Needed |
| $<50 \mathrm{~V}$ | None, if conditions <br> permit | None, if conditions <br> permit | None, if conditions <br> permit |
| $50-300 \mathrm{~V}$ | 3.5 ft | Avoid contact | Avoid contact |
| $>300-750 \mathrm{~V}$ | 3.5 ft | 1 ft | 1 inch |
| $>750-15 \mathrm{kV}$ | 5.0 ft | 2 ft 2 inches | 7 inches |
| $>15 \mathrm{kV}-36 \mathrm{kV}$ | 6.0 ft | 2 ft 7 inches | 10 inches |
| $>36 \mathrm{kV}$ | See NFPA 70E | See NFPA 70E | See NFPA 70E |

## Mitigating Voltage Hazard - Rubber Electrical Insulating Gloves

- They are marked with the class appropriate for the voltage, and should be subject to periodic electrical tests
- Leather protective gloves should be worn outside the rubber gloves to provide protection from cuts, abrasions, or punctures.
- Before each use, check for signs of damage or color change. Replace if contamination or sustain any physical damage is evident.
- Gloves should be stored in a closed, dry container.


Mitigating the Voltage Hazard

| Insulating Glove <br> Class | Rated Use Voltage <br> $A C^{1} / D C^{2,3}$ | Proof Test Voltage <br> $A C^{1} / D C^{2,3}$ |
| :---: | :---: | :---: |
| 00 | $500 / 750$ | $2,500 / 10,000$ |
| 0 | $1,000 / 1,500$ | $5,000 / 20,000$ |
| 1 | $7,500 / 11,250$ | $10,000 / 40,000$ |
| 2 | $17,000 / 25,500$ | $20,000 / 50,000$ |
| 3 | $36,500 / 39,750$ | $30,000 / 60,000$ |
| 4 | $40,000 / 54,000$ | 40,000 |

1. American Society for Testing and Materials (ASTM) Standard D-120
2. International Electrotechnical Commission (IEC) Standard 60903
3. DC applications: when gloves do not have an IEC DC voltage rating, use the $A C$ rating as the DC rating

The possibility of residual voltage on capacitors is high. Use one or more ground stick to remove the voltage (stored energy)


## What is Arc Flash?



- Short circuit through air
- Caused when circuit insulation or isolation is compromised
- A burn and explosion hazard, not an electrocution hazard
- Temperature can greatly exceed 5000 F
- Instantaneous, almost too fast for the eye to comprehend
- Arc flashes occur 5 - 10 times a day in electric equipment in US alone.
- Tool inserted or dropped into a breaker or service area
- Equipment cover removal causes a short
- Loose connections on bus work
- Improper bus work fabrication
- Insulation breakdown due to environmental factors or equipment aging
- Failure to ensure equipment is de-energized before work
- Primarily applications above 208 VAC


## Injuries Associated with Arc Flash

- Third Degree Burns, Blindness, Hearing Loss, Nerve Damage, Cardiac Arrest, Concussion, Death
- Arc flash hazard - a dangerous condition associated with the release of electrical energy caused by an electrical arc. Typically due to the molten plasma formed by the melting of conductors during an electrical short circuit
- Arc flash protection boundary - The distance form exposed live parts within which a person could receive a second degree (curable) burn $\left(1.2 \mathrm{cal} / \mathrm{cm}^{2}=\right.$ $5 \mathrm{~J} / \mathrm{cm}^{2}$ )
- Flash protection boundaries and energies are calculated using NFPA 70E [example Table 130.7(C)(9)(a)] and IEEE1584
- The calculations entail knowing the voltage class of the equipment, some details about its manufacture, the available short circuit and the opening times of the protective circuit breaker(s)
- An arc generates power that radiates out from a fault

$$
P_{a r c}=V_{a r c} * I_{a r c}
$$

- The total energy is the product of the arc power and duration of the arc

$$
E_{a r c}=P_{a r c} * t
$$

- The energy density decreases with distance from the arc
- An arc-flash hazard occurs when the energy density on the torso or face exceeds $1.2 \mathrm{cal} / \mathrm{cm}^{2}$, the energy density at which a second degree burn occurs.

Note: This is comparable to holding the flame from a cigarette lighter on your skin for 1 second

## Early Model

In 1982, the first calculations concerning arc-flash were published by Ralph Lee

- Calculated the maximum available power from an arc
- Assumed the power density dropped off with the inverse square of the distance from the arc
- Related the energy to biological studies that determined the second-degree burn threshold
- Formula for working distance to avoid second-degree burns reproduced in 130.3(A) of NFPA 70E

$$
D_{c}=\sqrt{2.65 M V A_{b f} t}
$$

## First Empirical Models

- Mid 1990's, other teams from DuPont, Doughty, et. al., and an IEEE Working Group IEEE1584, performed a set of experiments to measure the heat generated by an arc.
- Wanted empirical data to quantify arc flash hazards

- Measured dependence on arcing current
- Measured dependence on distance from arc
- Measured dependence on some enclosure configurations
- Measured voltage dependence
- Detailed configuration dependencies
- Found that enclosures focus the pressure blast from the arc and focus the energy
- Flash protection boundary for enclosures more conservative than Lee's
- Formulae reproduced in Annex D. 6 of NFPA 70E


## IEEE 1584208 VAC Exemption

- Arc needs to sustain itself in order to generate enough heat to cause burns
- Empirical evidence from studies showed that it was extremely difficult to sustain an arc from a 208VAC system
- Statement from IEEE 1584 "The arc-flash hazard need only be considered for large 208 V systems; systems fed by transformers smaller than 125 kVA should not be a concern."
- NFPA Technical Committee also addressed the arc-flash hazard issue from a parallel point of view
- Used common industrial experience to determine the level of protection required to prevent second-degree burns for certain well-defined operations
- Table 130.7(C)(9)(a) summarizes this experience
- Distinguishes between types of equipment
- Switchgear, MCCs, panelboards, and switchboards
- In order for table to be applicable for most low voltage systems, the fault current must be below a maximum allowed current and the fault clearing time must be less than a maximum allowed time
- Nothing in the panel being operated, including the main breaker, will protect us in case of an arc-flash during the operation of any part of that panel
- Main breaker could fail on line side during operation
- Arc-flash originating at operated breaker could spread to line side of main breaker
- The fronts of the panelboards and switchboards provide no protection against arc-flash
- Independent of the clearing time of the upstream protective device, the worker will move away from exposure in less than 2 seconds, so that "two seconds is a reasonable maximum time for calculations"


## Bolted Fault Current Calculation

- Power from a three-phase Wye or Delta transformer is
- The full load line current is

$$
\begin{aligned}
& S_{3 P}=\frac{V_{L L} * I_{L}}{\sqrt{3}} \\
& I_{F L}=\frac{M V A * 10^{6}}{\sqrt{3} * V_{L L}}
\end{aligned}
$$

- Available short circuit current using the transformer rating and circuit impedances (mainly leakage inductance of transformer)

$$
\begin{aligned}
& I_{B F}=\frac{M V A * 10^{6} * 100}{\sqrt{3} * V_{L L} * Z(\%)} \\
& I_{B F}=\frac{M V A * 10^{6} * 1}{\sqrt{3} * V_{L L} * Z(p u)}
\end{aligned}
$$

- Empirical formula from IEEE 1584
- Flash hazard is due to total energy absorbed by skin
- Bolted fault current ( $I_{b f}$ in $k A$ )
- Must calculate arcing current ( $I_{a}$ in $k A$ ) at which the flash energy is generated

$$
I_{a}=10^{K+0.662 \log I_{b f}+0.0966 V+0.000526 G+0.5588 V \log I_{b f}-0.00304 G \log I_{b f}}
$$

- $I_{a}$ lower than $I_{b f}$ therefore may last a longer time than $I_{b f}$ since it may take longer for the protective device to clear
- $K$ is the value for the arc ambient ( 0.153 for open air, 0.097 for boxes)
- $V$ is the system voltage in $k V$
- $G$ is the conductor gap in $m m$ (25mm for 480VAC MCCs and panels)
- Use $I_{a}$ and $0.85 I_{a}$ for clearing times from breaker or fuse time-current curves


## Clearing Times

- Once the arcing currents are determined, the clearing times must be determined
- The upstream fuse or breaker that will clear the fault must be identified
- The manufacturer's time-current curves for the appropriate device are identified
- The arcing current is found on the $x$ axis and the clearing time is read from the $y$ axis


## Protective Devices

- Protective devices act to clear fault currents
- Fuses (fixed time-current curves)
- Molded Case Circuit Breakers (MCCB)
- Thermal
- Magnetic (adjustable)
- Electronic (adjustable)
- Low Voltage Power Circuit Breakers (adjustable)
- Vacuum Contactors with Protective Relays (adjustable)

- Panelboard with 4 800A MCCBs
- Thermal-magnetic
- Adjustable with front panel rotary switches

- Motor Control Center (MCC) main breaker (2000A)
- Separate Frame and Trip units
- Adjustable with front panel rotary switches
- Electronic Trip Control LSIG
- Long Time
- Short Time
- Instantaneous
- Ground Current

- First breaker downstream of transformer
- Separate frame and trip unit
- Rated up to 4000 A
- Trip units are adjustable via front panel controls
- Typically inside a substation

- Protects primary side (12.47kV) of transformer
- Breaker has tripping mechanism but no sensor
- Protective relay
- Connected to current transformers on bus
- Activate breaker tripping mechanism
- Adjustable via DIP switches

- Each protective device has a time-current curve
- Current on $x$-axis
- Time on $y$-axis
- Currents below the trip value will not open the device
- Currents slightly above the trip value will trip after a long time to prevent fires from heating in the wires
- Currents very much larger than the trip setting signal a fault and trip the device as quickly as possible
- Adjustments set these values and, if applicable, the intermediate behavior of the breaker

Typical medium voltage fuse time-current curve (Ferraz-Shawmut)

## A155F1DORO - 65E to 100E Aaxl A155F2DORO - 125E to 200E



- Non-adjustable
- Top of y-axis is 1000 seconds and fuses not yet open on $2 X$ overcurrents
- These fuses take more than 0.1 second to open on 10X overcurrent
- Different classes of fuses for different voltages and currents

Types EHD, FDB, FD and HFD 90 Amperes


- Thermal breaker (90A)
- Small overcurrent takes > 30 minutes to trip
- Lines of constant $I^{2} t$ (constant energy) protect against thermal damage
- 17X (1530A) overcurrent trips in 30 msec
- Very high currents trip in 16 msec
- Note that curve gives Min, Max tolerance band



## Adjustable breaker can set various

 parameters- Long time moves left hand side vertical bar
- Short time pickup sets intermediate vertical bar
- Short time delay sets intermediate pedestal
- Instantaneous setting determines current at which instantaneous trip occurs

[^0]Once the system configuration and arcing currents are known, the "normalized" and actual arcing energies can be calculated (IEEE 1584 formulas)

$$
\begin{aligned}
& E_{n}=10^{K_{l}+K_{2}+1.081 \log _{a}+0.0011 \mathrm{G}} \\
& E_{85 n}=10^{K_{1}+K_{2}+1.081 \log _{85 a}+0.0011 \mathrm{G}} \\
& E_{a}=C_{f} E_{n}\left(\frac{t_{a}}{0.2}\right)\left(\frac{610}{D}\right)^{x} \\
& E_{85 a}=C_{f} E_{85 n}\left(\frac{t_{85 a}}{0.2}\right)\left(\frac{610}{D}\right)^{x}
\end{aligned}
$$

- $K_{1}=-0.555$ for arcs in a box, $K_{2}=-0.113$ for a grounded system
- En, $E_{85 n}, E_{\alpha}, E_{85 a}$ are expressed in cal/cm2
- $C_{f}=1.5$ (voltages below 1 kV and $=1.0$ for voltages above 1 kV )
- $t_{a}$ and $t_{85 a}$ are arcing time in seconds, obtained from the appropriate timecurrent curve
- $D$ is working distance in mm
- $x$ is a system-dependent distance factor from Table 4 of IEEE 1584 or Table D.7.2 of NFPA 70E


## Working Distance

- The "safe" working distance is defined as the minimum distance at which the worker's body and torso receives a second degree burn ( $E$ or $E a=1.2 \mathrm{cal} / \mathrm{cm}^{2}$ )
- Invert the above formulas to obtain the Flash Protection Boundary distances

$$
\begin{aligned}
& D=\left[C_{f}\left(\frac{E_{n}}{1.2}\right)\left(\frac{t_{a}}{0.2}\right)\right]^{1 / x}(610) \\
& D_{85}=\left[C_{f}\left(\frac{E_{85 n}}{1.2}\right)\left(\frac{t_{85 a}}{0.2}\right)\right]^{1 / x}(610)
\end{aligned}
$$

- Take the larger of the arcing energies and distances for use on the arc flash hazard labels


## Hazard/Risk Category

- The hazard/risk category is determined by selecting the row for which $E_{\min } \leq E<E_{\max }$ at the working distance.

| $\boldsymbol{E}_{\min }$ | $\boldsymbol{E}_{\max }$ | Hazard/Risk Category |
| :---: | :---: | :---: |
| 0 | 1.2 | 0 |
| 1.2 | 4 | 1 |
| 4 | 8 | 2 |
| 8 | 25 | 3 |
| 25 | 40 | 4 |

- The appropriate Personal Protective Equipment (PPE) required is then determined from Table 130.7(C)(10) and Table 130.7(C)(11) of NFPA 70E

- Calculate two arc-flash energies
- Operate feeder breaker in MCC
- Operate breaker downstream of "clean" transformer
- Main xfinr
- 1 MVA 12470V/480V
$-\quad Z=5.57 \%$
- "Clean power" xfmr
- $112.5 \mathrm{kVA} 480 \mathrm{~V} / 480 \mathrm{~V}$
$-\quad Z=5.60 \%$


## MCC Feeder Breaker Calculation

Types RD, CRD, RDC, CRDC Equipped With Digitrip PMS a 10 Trip Units. Typical Long


- $I_{f l}=1.2 \mathrm{kA}$
- $Z=5.57 \%$
- Fault currents

$$
\begin{aligned}
& -I_{b f}=21.6 \mathrm{kA} \\
& -I_{a}=12.7 \mathrm{kA} \\
& -0.85 I_{a}=10.8 \mathrm{kA}
\end{aligned}
$$

- Clearing times from C-H AD 29-167R
Curve SC-5630-93
- $t_{b f}=0.040 \mathrm{~s}$
- $t_{a}=0.051 \mathrm{~s}$
- $t_{0.85 a}=0.051 \mathrm{~s}$
- Times are short because breaker is set at its fastest setting


## MCC Feeder Breaker Calculation

- Working distance is 30 " (because breaker is recessed from handle)
- $E_{a}=0.94 \mathrm{cal} / \mathrm{cm}^{2}$
- $E_{0.85 a}=0.79 \mathrm{cal} / \mathrm{cm}^{2}$
- Maximum energy ( $0.94 \mathrm{cal} / \mathrm{cm}^{2}$ ) less than $1.2 \mathrm{cal} / \mathrm{cm}^{2}$ so operation of this circuit breaker is Hazard/Risk Category 0
- Table 130.7(C)(9)(a) not applicable because
- $I_{b f}<65 \mathrm{kA}$
- $t_{\text {clear }}>0.030$ seconds


## Types FD and HFD 150 Amperes



- $I_{f l}=135 \mathrm{~A}$
- $Z=6.23 \%$
- Fault currents
$-I_{b f}=2.2 \mathrm{kA}$
$-I_{a}=1.8 \mathrm{kA}$
$-0.85 I_{a}=1.5 \mathrm{kA}$
- Clearing times from C-H AD 29-167F
Curve SC-4438-88A
$-t_{b f}=0.017 \mathrm{~s}$
- $t_{a}=0.024 \mathrm{~s}$
$-t_{0.85 a}=1.6 \mathrm{~s}$
- Time is large because system is not properly coordinated


## Clean Power Breaker Calculation

- Working distance is 18" (NFPA70E standard)
- $E_{a}=0.12 \mathrm{cal} / \mathrm{cm}^{2}$
- $E_{0.85 a}=6.9 \mathrm{cal} / \mathrm{cm}^{2}$
- Maximum energy ( $6.9 \mathrm{cal} / \mathrm{cm}^{2}$ ) between 4 and $8 \mathrm{cal} / \mathrm{cm}^{2}$ so operation of this circuit breaker is Hazard/Risk Category 2
- Table 130.7(C)(9)(a) not applicable
$-I_{b f}<25 \mathrm{kA}$
- $t_{\text {clear }}>0.030$ seconds
- 112.5 kVA transformer in circuit led to long clearing times of upstream breaker
- Reduce arc-flash by
- Using faster clearing time upstream breaker
- Adding separate downstream breaker or fuse

Minimum Protective Clothing - Abridged NFPA 70E Table 130.7(C)(11)

| Hazard <br> Risk <br> Category | Clothing Description | Required Minimum <br> Arc Rating of PPE <br> [J/cm2 (cal/cm2)] |
| :---: | :--- | :---: |
| 0 | Long sleeve shirt, long pants, safety glasses | $5 \quad$ (1.2) |
| 1 | Long pants, FR long sleeve shirt, hard hat, safety <br> glasses | $16.74 \quad$ (4) |

- Decrease available energy by using smaller upstream transformer (208V transformers less than 125 kVA pose no arc-flash hazard)
- Decrease clearing time
- Size breaker trip units more aggressively
- Choose breakers for instantaneous trip times (smaller frame sizes generally trip faster than larger frame sizes)
- Choose breakers with adjustable trip units including adjustments for instantaneous trips
- Protective devices upstream of transformers need to allow "inrush" current when transformer is energized. Using only upstream sensors, it is difficult to be as aggressive as desirable for arc-flash protection downstream of transformer. Add overcurrent devices on transformer secondary
- Insert fast acting breakers or fuses in separate enclosures between the transformer and the equipment that needs to be operated. In general, separate the enclosures contain arc-flash generated in that enclosure
- Increase distance between worker and source of arc-flash
- Use remote controls to operate high arc-flash hazard devices
- Use extension handles on breakers to increase working distance of operation
- Install meters to use for verification that system is de-energized if work is required on system
- Install IR view-ports on panels that need to be monitored for overtemperature
- Install protective devices that sense arcs and not just overcurrent

Arc flash protection boundary determination from energy stored in capacitors Developed by Jim Sebek of SLAC and based on R. H. Lee in NFPA70E $D_{c}=\sqrt{2.65 M V_{\text {source }} A_{b f} t}$ where $D_{c}$ is the arc flash boundary in feet Recall that $E_{\text {stored }}=\frac{1}{2} C V^{2}$ is the stored capacitor energy in joules According to Lee the maximum energy dissipated in a fault occurs when the fault voltage is $\frac{\sqrt{2}}{2}$ of the supply voltage and the arc current is $\frac{\sqrt{2}}{2}$ of the available fault current (the arc is resistive). Therefore, $P_{\max }=\frac{1}{2} M V_{\text {source }} A_{b f}$ or, to keep $D_{c}$ inviolate, $2 P_{\max }=M V_{\text {source }} A_{b f}$ substituting yields $D_{c}=\sqrt{5.3 M V_{\text {source }} A_{b f} t}=\sqrt{5.3 P_{\max }(M W) t}=\sqrt{5.3 * 10^{-6} P_{\max }(W) t}$ Recognizing that watts**ime $=$ energy in joules yields

$$
D_{c}=\sqrt{5.3 * 10^{-6} E_{\text {stored }}}
$$

## More Information

More information

- http://ieeexplore.ieee.org/servlet/opac?punumber=8088
- NFPA 70E 2009 Edition
- http://www.mt-online.com/articles/0204arcflash.cfm
- http://www.eaton.com/ecm/idcplg?IdcService=GET FILE \&dID=12075
- http://www.eaton.com/ecm/idcplg?IdcService=GET_FILE\&dID=118182
- http://ecatalog.squared.com/pubs/Circuit\ Protection/0100DB0402.pdf


## Mitigating the Voltage and Arc Flash Hazards

| A MARNM |  |
| :---: | :---: |
| Arc Flash Hazard with covers or doors closed Appropriate PPE Required |  |
|  | Category __ arc flash hazard when operating controls. |
| Equipment D: |  |

The WARNING label is for operating enclosed circuit breakers. No shock hazard listed. Note the requirement for identification of the equipment to which it will be affixed

| ADA <br> When enc Flash \& Sh Appropriate | sure open ock Hazard PE Required |
| :---: | :---: |
| Flash Protection <br> Flash Hazard Category $\qquad$ 0 <br> Min. Arc Rating (cal/cm ${ }^{2}$ ) $\qquad$ NA <br> Flash Protection Boundary $\qquad$ 18" <br> PPE: Long-sleeve shirt and long pants (natural fibers), safety glasses. | Shock Protection <br> 480VAC Shock Hazard When: <br> Doors open, bus exposed <br> Limited Approach Boundary 42 <br> Restricted Approach Boundary $\qquad$ <br> Prohibited Approach Boundary $\qquad$ <br> PPE: Class $\qquad$ 500VAC |
| Equipment ID: LGPS01 BaBar Solenoid P.S. |  |
| Source 2: 120V 1ph from 2P02C1A, bkr\# 2, on South wall. |  |
| Load: BaBar Solenoid | Date: 12/09/04 |

The DANGER label is for exposed, energized work. Shock and arc flash hazards listed. Arc flash hazard category typically higher than for the Warning Label. Note the requirement for identification of the equipment to which it will be affixed

## 4 Types

- Personnel protection - Personnel Protection System (PPS)
- Personnel protection - Lock and Tag (LOTO)
- Load protection - Machine or Magnet Protection System (MPS)
- Power supply protection - Power supply internal interlocks


## Personnel Protection System (PPS) at SLAC

- Protection from hazards external to power supply (example accelerator housing door opened)
- Hazards are defined as voltages > 50 V , currents > 5 m A energy storage > 10 J .
- Must be hardwired (recently SLAC introduced PLC-based PPS)
- Two (2) PPS permissives are needed for power supply turn-on
-Two (2) separate and different read-backs are required
- Permissives and read-backs are usually 24 VDC systems
- Permissives and read-backs must be fail-safe
- If PPS is not practical, then energized equipment must be enclosed or live terminals covered


## PPS Example



Many variations of this example

## Lock \& Tag for Personnel Safety During Maintenance

- Procedures and requirements for servicing and maintaining machines and equipment in which unexpected energization, startup or release of energy could cause injury to personnel
- Provision for locking off source power, the discharge of stored energy prior and the total de-energization of equipment before working on exposed electrical circuits or other hazardous equipment


## Required by

- Occupational Safety and Health Administration (OSHA) under 29CFR1910.147


## Applicability

- All types of equipment containing electrical, mechanical, hydraulic, pneumatic, chemical and/or thermal active or stored energy
- For working on exposed electrical circuits that would expose personnel to any electrical hazard operating at > 50 V , >50A, >10J


## Items Locked Out (Off) - Tagged Out (Off)

The power source or power device
Application by
Authorized employee trained in LOTO and qualified to lock-off the equipment

## Types

General Lock and Tag Procedure (GLP) - used when there is only one source of energy and there is no stored energy

Equipment Specific Procedures - used for systems or equipment with more than one energy source or equipment with stored energy (e.g.; capacitor banks)

## Interlocks As LOTO

Interlocks are not used as a substitute for lock and tag

1. Preparation and notification
2. Location and isolation of energy sources
3. Shutdown
4. Lock and tag application
5. Verification of isolation
a. Attempt to start the locked-out equipment
b. Remove all stored energy
c. Use appropriate test equipment (e.g. voltmeter) to verify that the circuit elements and parts have been de-energized

Notes:

- Use only easily recognizable, dedicated, facility-approved locks and tags
- Never attempt to operate equipment that has been locked out
- Do not attempt to remove another person's lock


## For Locking and Tagging

- Padlocks, usually red-colored for personal use. Yellowcolored for administrative lock-out
- Tags
- Specialty locks (Kirk-Key Locks) for complex systems
- Master lock boxes
- Capacitor bleeder resistors per 2008 NEC, Article 460


## Removal of Energy Storage



- Working voltage $\leq 600 \mathrm{~V}$, discharge to 50 V or less in 1 minute
- Working voltage . 600 V , discharge to 50 V or less in 5 minutes
- Redundant resistors recommended
- Grounding hooks
- Clear or green-colored insulation
- Resistor to keep discharge current to safe value
- Electrical equipment grounding per 2008 NEC, Article 250





## Machine Protection Systems (MPS)

Machine (or magnet) protection systems protect loads from damage.

## Magnet Cooling Water Temperature / Flow Sensors

- Usually employ a simple normally closed (NC) contact that opens when a predetermined temperature has been reached.
- Water flow monitoring switches open when flow drops below a pre-established safe value
- Temperature / Flow switches are wired to the source power supply. If the water temperature is too high or if the flow drops the contacts open and turn the power supply off


## Vacuum Interlock System

- Sensors are similar to that described in the magnet cooling water system


## Orbit Interlock System

- Sensors consist of Beam Position Monitors and switches. Function is essentially the same in the magnet cooling water system
- Thermal switches - Klixons (a trade name) are NC contact bimetal switches mounted on the load cooling water outlet line. Their contacts open when temperature exceeds a pre-established safe value
- Multiple-winding, multiple water path magnets employ simple series connected Klixons.
- Klixons are wired to the source power supply. If the load overheats, the contacts open and turn off the power supply


Outlet water temperature monitored via Klixon switch

## Machine Protection Systems (MPS)



## Ground Fault Detection / Protection Systems

- Loads are usually located in crowded, dense areas with a multitude of other equipment. This makes them vulnerable to ground faults
- Power supplies are usually isolated from ground so that a single ground fault does not cause load-catastrophic ground fault current. Fix first fault before the second fault occurs



Internal interlocks protect the power supply itself

- Low input supply voltage
- Phase loss detection
- Output DC over-current
- Low frequency filter inductor temperature
- Heat-sink temperature or heat-sink cooling water flow
- IGBT temperature
- IGBT over-current
- Ground Fault current
- Output over-voltage
- Cabinet or chassis over-temperature


Manufacturers are many

- Allen-Bradley
-Rockwell International (AB)
- Siemens
- General Electric
- IDEC

Programming logic

- Ladder logic
- C language
- LabView
- Functional block diagrams
- Structured text


## PLC Uses and Networks



## Ladder Logic

PLC execution model


Source: Control Engineering and National Instruments

Ladder diagrams evolved in the 1960s when the automobile industry needed a more flexible and selfdocumenting alternative to relay and timing cabinets. A microprocessor was added and software designed to mimic the relay panels.

Left rail is the "power bus". The right rail is the "ground bus". Power flows through NO or NC contacts to power coils.

Each contact and coil is linked to a Boolean memory location.

Series contacts look like "AND" and parallel contacts look like "OR"

Execution is left to right and top to bottom

## Ladder Logic

## PLC execution model



Inherent in Ladder Logic is an unseen scanning engine and memory management stack.

Physical inputs are read and stored in a memory table

The ladder logic is run by reading and writing from the memory table for inputs, outputs and intermediary values.

At the end of the logic cycle, all the physical outputs are updated with the values from the memory table. More complex tasks (math, timers, etc) are handled by calls to built-in functions on the ladder rungs

Source: Control Engineering and National Instruments

## Ladder Logic

Most widely used application to program PLCs

## Strengths

- Intuitive and self-documenting. Can be learned very quickly by personnel with little or no software training
- Excellent debugging tools, include animation showing live "power flow". This makes the logic easy to understand and to debug faults
- Efficient representation for discrete logic


## Weaknesses

- Hierarchical data and logic flow. Is supplemented by functions, function blocks and subroutines
- Poor data structure. Rungs are executed in a left-to-right, top-to-bottom order. Timing is defined by the PLC processor speed
- Limited execution control
- Arithmetic operations are limited


## Programmable Logic Controllers

PLCs implement specific functions such as:

| I/O control | Timing | Report generation | Arithmetic |
| :--- | :--- | :--- | :--- |
| Logic | Communication | Data file manipulation | Counting |

## PLC Versus Programmable Automation Controllers (PAC)

Consider a PAC upgrade if your application requires:

- advanced control algorithms
- extensive database manipulation
- HMI functionality in one platform
- Integrated custom control routines
- complex process simulation
- very fast CPU processing
- memory requirements that exceed PLC specifications


# 10. Reliability, Availability and Maintainability 

## Reliability Overview

- Definition and Importance
- Glossary of Terms
- Calculation Standards
- Calculations - Power Supply/Power System
- Improvements by Redundancy - Examples
- Fault Modes And Effects Criticality Analysis (FMECA)
- The Reliability Process
- Maintainability - Cold-Swap, Warm-Swap and Hot-swap


## Reliability

According to IEEE Standard 90, reliability is the ability of a system or component to perform its required functions under stated conditions for a specified period of time

## Availability

The degree to which a system, subsystem, or equipment is operable and in a committable state during a mission (accelerator operation).

The ratio of the time a unit is functional during a given interval to the length of the interval.

## Reliability and Availability Importance

## Importance

Reliability is important because accelerators are expected to perform like industrial factories; i.e., to be on-line at all times. In particular, accelerator power supplies are expected to be available when needed, day after day, year after year. Reliability must be considered when subsystems are complex (contain large part count) or when a system is composed of a large number of subsystems or the accelerator simply will not function.

Failures lead to annoyance, inconvenience and a lasting user dissatisfaction that can play havoc with the accelerator's reputation. Frequent failure occurrences can have a devastating effect on project performance and funding.

Glossary of Terms and Definitions

| Availability | Ratio of operating time to operating + downtime $A=M T B F /(M T B F+M T T R)$. This is a dimensionless number |
| :---: | :---: |
| MTBF | Mean time between failures in hours |
| $\mathrm{MTBF}_{O}$ | The increased MTBF in hours that considers equipment operation at lower than rated power levels |
| $M T B F_{R}$ | The rated MTBF in hours |
| MTTR | The mean time to repair and recover beam in hours |
| $R(t)$ | Reliability or probability of success over the mission time (Typically 9 months $=6600 \mathrm{hours}$ ) |
| $\lambda, \lambda_{o}, \lambda_{R}$ | Failure rates in $\mathrm{hr}^{-1}$. These are the reciprocals of the MTBFs |
| 1/1 | One full rated power supply. Rated power $=$ delivered power |
| 1/2 | One out of two redundant power module configuration |
| 2/3 | Two out of three redundant power module configuration |
| $3 / 4$ | Three out of four redundant power module configuration |
| 4/5 | Four out of five redundant power module configuration |


| Failure rate is constant | $\lambda$ | $\left(h^{-1}\right)$ |
| :--- | :--- | :--- |
| Mission time | $t$ | $(h r)$ |
| Probability Density Function (PDF) | $f(t)=\lambda e^{-\lambda t}$ | (dimensionless) |
| Cumulative Density Function (CDF) | $F(t)=1-e^{-\lambda t}$ | (dimensionless) |
| Reliability (Success probability) | $R(t)=e^{-\lambda t}$ | (dimensionless) |
| Expected time to failure (MTBF) | $E(T)=\int_{-\infty}^{\infty} t f(t) d t=\frac{l}{\lambda}$ | $(h r)$ |

## Glossary - Math Expressions

Failure rate of $N$ series critical components

Re liability of $N$ series components $\quad R_{S}(t) \quad=\prod_{i=1}^{N} e^{-\lambda_{i} t}$

$$
\begin{equation*}
R_{S}(t) \quad=\prod_{i=1}^{N} R_{i}(t) \tag{dimensionless}
\end{equation*}
$$

Re liability of $N$ parallel components

$$
R_{p}(t) \quad=1-\prod_{i=1}^{N}\left(1-e^{-\lambda_{i} t}\right)
$$

Failure $N$ series components

$$
Q_{S}(t) \quad=\prod_{i=1}^{N} Q_{i}(t)
$$

Probability of failure of parallel $\quad Q_{p}(t) \quad=1-R(t)=\prod_{i=1}^{N}\left(1-e^{-\lambda_{i} t}\right)$ (dimensionless )

## Homework Problem \# 16

Homework Problem:
A. At least 2 of 4 parallel power supplies in an accelerator must continue to operate for the system to be successful. Let $R_{i}=0.9$. Find the probability of success.
B. Solve for three out of four for success
C. Solve for four out of four for success

## Glossary - Math Expressions

$$
\begin{array}{lll}
\text { MTBF of series critical components } & M T B F=1 / \lambda_{\text {composite }} & \text { ( } \mathrm{hr} \text { ) } \\
\text { MTBF of } N \text { series identical components } & M T B F_{\text {composite }}=M T B F_{i} / N & (\mathrm{hr}) \\
\text { Mean time to repair or recover is } & M T T R & (\mathrm{hr}) \\
\text { Availability is } & A=\frac{M T B F}{M T B F+M T T R} & \text { (dimensionless) } \\
\text { Availabilty of series components } & A_{\text {composite }}=\prod_{i=1}^{N} A_{i} & \text { (dimensionless) } \\
\text { Availbilty of identical components } & A_{\text {composite }}=A^{N} & \text { (dimensionless) }
\end{array}
$$

Glossary - Failure Rate Curve


- Infant mortality manufacturing defects, dirt, impurities. Infant mortality reduced for customer by burn-in and stress-screening
- Stable wear-out statistics, manufacturing anomalies, out-of tolerance conditions
- Wear-out failure dry electrolytic capacitors, aged and cracked cable insulation


## Reliability Calculation Standards

| $\begin{aligned} & \begin{array}{l} \text { MIL-HDBK-217F } \\ \text { (USA) } \end{array} \end{aligned}$ | - Internationally used <br> - Parts count <br> - Parts stress <br> - Broad in scope <br> - Pessimistic |
| :---: | :---: |
| Telcordia (Bellcore) (USA) | - National use <br> - Parts count <br> - Parts stress <br> - Narrow scope (telecommunications) <br> - Optimistic |
| CNET 93 <br> (France) | - Limited to France <br> - Parts count <br> - Parts stress <br> - Broad in scope |
| $\begin{aligned} & \begin{array}{l} H R D 5 \\ (U K) \end{array} \end{aligned}$ | - Limited to UK <br> - Parts count <br> - Parts stress <br> - Broad in scope |

## Parts Count and Parts Stress

## Parts Count

- Appropriate failure rate is assigned to each part in the subsystem (power supply) that is mission critical
- Failure rates are functions of environment (Ground fixed $\Pi_{G F}$ /Ground benign $\Pi_{G B}$ /Ground mobile, $\Pi_{G F}$ ) and ambient temperature ( $\Pi_{T}$ )
- The parts count method is simple and used early in system design when detailed information is unknown
- Failure rates are summed and the following information is obtained

$$
M T B F=\frac{1}{\sum \lambda}
$$

$$
R(t)=e^{-\sum \lambda t}
$$

## Parts Count and Parts Stress

Parts Stress - Same as the Parts Count method, except it takes into account more detailed information about the components and their operating stresses. The detailed information is implemented via additional $\Pi$ reliability factors, such as:

$$
\begin{array}{ll}
\Pi_{G B}=\text { ground benign } & 0<\Pi_{G B}<\infty \\
\Pi_{T}=\text { ambient temperature } & 0<\Pi_{T}<\infty \\
\Pi_{M Q}=\text { manufacturing quality } & 0<\Pi_{M Q}<\infty \\
\Pi_{V S}=\text { voltage stress factor } & 0<\Pi_{V S}<\infty \\
\Pi_{I S}=\text { current stress factor } & 0<\Pi_{I S}<\infty \\
\Pi_{P S}=\text { power stress factor } & 0<\Pi_{P S}<\infty \\
\lambda_{\text {resultant }}=\lambda_{\text {initial }} * \Pi_{G B} * \Pi_{T} * \Pi_{M Q} * \Pi_{V S} * \Pi_{I S} * \Pi_{P S}
\end{array}
$$

Example of Reliability Calculation - Power Supply


Example of Reliability Calculation - Power Supply

| Component Description | Qty | $\lambda$ | $\boldsymbol{\pi}{ }_{\boldsymbol{G B}}$ | $\pi_{T}$ | $\pi \times Q$ | $\boldsymbol{\pi}$ VS | $\pi_{\text {IS }}$ | $\pi$ PS | Mission <br> Loss | Total Rate $\lambda_{T} 10^{-6}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Circuit Breaker/Contactor/Fuse | 5 | 0.42 | 1.00 | 1.10 | 1.00 | 1.01 | 1.05 | 1.10 | Yes | 2.695 |
| 3 Phase Transformer | 1 | 0.05 | 1.00 | 1.10 | 1.00 | 1.50 | 1.50 | 1.50 | Yes | 0.186 |
| Input/Output Filter Choke | 2 | 0.02 | 1.00 | 1.10 | 1.10 | 1.42 | 1.60 | 1.75 | Yes | 0.144 |
| Secondary/DC Link Fuse | 2 | 0.08 | 1.00 | 1.10 | 1.89 | 1.02 | 0.95 | 0.90 | Yes | 0.291 |
| Main Filter Capacitor | 8 | 0.23 | 1.00 | 1.12 | 1.50 | 1.25 | 1.25 | 1.05 | Yes | 5.057 |
| Damping Capacitors/Resistor | 15 | 0.02 | 1.00 | 1.10 | 1.00 | 1.00 | 1.00 | 1.00 | No | 0.000 |
| IGBT/Diode | 8 | 0.03 | 1.00 | 1.10 | 1.50 | 1.00 | 1.00 | 1.00 | Yes | 0.330 |
| Heatsink Assembly | 1 | 0.01 | 1.00 | 1.10 | 1.00 | 1.00 | 1.00 | 1.00 | Yes | 0.011 |
| Gate Driver/PWM | 2 | 0.50 | 1.00 | 1.10 | 1.00 | 1.10 | 1.10 | 1.15 | Yes | 1.524 |
| Logic Board | 1 | 3.50 | 1.00 | 1.10 | 1.00 | 1.00 | 1.00 | 1.00 | Yes | 3.850 |
| Output Filter Capacitor | 6 | 0.25 | 1.00 | 1.10 | 1.00 | 1.25 | 1.25 | 1.00 | Yes | 2.578 |
| MTBF and Total Failure Rate |  |  |  |  |  |  |  | 60,000 |  | 16.667 |

Calculate the MTBF of a "typically commercial" 5 kW , switchmode power supply with EMI filter and appropriate electromechanical safety features amounting to $10 \%$ of the total number of components. The power supply operates at 50C ambient temperature. The power supply consists of the following components with the listed failure rates.:

- 2 each ICs, plastic linear, $\lambda=3.64$ failures per million hours each
- 1 each opto-isolator, $\lambda=1.32$ failures per million hours each
- 2 each hermetic sealed power switch transistors, $\lambda=0.033$ failures per million hours each
- 2 each plastic power transistors, $\lambda=0.026$ failures per million hours each
- 4 each plastic signal transistors, $\lambda=0.0052$ failures per million hours each
- 2 each hermetic sealed power diodes, $\lambda=0.064$ failures per million hours each
- 8 each plastic power diodes, $\lambda=0.019$ failures per million hours each
- 6 each hermetic sealed switch diodes, $\lambda=0.0024$ failures per million hours each
- 32 each composition resistors, $\lambda=0.0032$ failures per million hours each
- 3 each potentiometers, commercial, $\lambda=0.3$ failures per million hours each
- 8 each pulse type magnets, 130C rated, $\lambda=0.044$ failures per million hours each
- 12 each ceramic capacitors, commercial, $\lambda=0.042$ failures per million hours each
- 3 each film capacitors, commercial, $\lambda=0.2$ failures per million hours each
- 9 each Al electrolytics, commercial, $\lambda=0.48$ failures per million hours each

Example of Reliability Calculation - Power System


Example of Reliability Calculation - Power System


| Single System Availabilty |  |  |
| :---: | :---: | :---: |
| Component | MTBF | Availability |
| PS Controller | 110,000 | 0.9999818 |
| Power Supply | 60,000 | 0.9999667 |
| Transductor 1 | 381,500 | 0.9999948 |
| Transductor 2 | 381,500 | 0.9999948 |
| Cables | 14,000,000 | 0.9999999 |
| System | 32,184 | 0.9999379 |

## Reliability Software

## Relex by Relex Software

See Reference Appendix for web link to this manufacturers products

## RelCalc by T-Cubed

See Reference Appendix for web link to this manufacturers products

Reliability/Availability Improvement By Redundancy


- Two types - Standby and Active

1. Standby - the redundant parts are off and only operate when the first part fails. This requires more vigilance on the part of the control system and is not covered here.
2. Active - the redundant part(s) are on, albeit operating at a reduced power level until asked to assume increased or full load. This is easier to implement than Standby redundancy and is the more common method. We will examine this further

The general, exponential form of the Binomial Distribution for $m$ out of $n$ parts is

```
\(R(t)=\sum_{k=m}^{n}\left(\frac{n!}{(n-k)!k!}\right)\left(e^{-\lambda t}\right)^{k}\left(1-e^{-\lambda t}\right)^{n-k}\)
\(\lambda=\) constant \(=\) failure rate
\(k=\) index counter
\(m=\) minimum number of power modules needed for operation
\(n=\) total number of power modules in the system
Special cases occurs when \(m=n\) or when \(m=n=1\)
\(R(t)=e^{-n \lambda t} \quad R(t)=e^{-\lambda t}\)
```

Binomial Expansion 2 out of 3 example

$$
\begin{aligned}
& R_{2 / 3}(t)=\sum_{k=m=2}^{n=3}\left(\frac{n!}{(n-k)!k!}\right)\left(e^{-\lambda t}\right)^{k}\left(1-e^{-\lambda t}\right)^{n-k} \\
& k=2 \\
& \frac{3!}{1!2!} e^{-2 \lambda t}\left(1-e^{-\lambda t}\right)=3 e^{-2 \lambda t}\left(1-e^{-\lambda t}\right)
\end{aligned}
$$

3 cases, probability of success, probability of failure

$$
k=3
$$

$\frac{3!}{0!3!} e^{-3 \lambda t}\left(1-e^{-\lambda t}\right)^{0}=1 e^{-3 \lambda t}$
1 case, probability of success, no failure


$$
R_{2 / 3}(t)=3 e^{-2 \lambda t}-2 e^{-3 \lambda t}
$$

Derivation
When $\lambda(t)$ is a function of time
General form $R(t)=e^{-\lambda(t) t}$

$$
\begin{aligned}
& \frac{d R(t)}{d t}=-\frac{d \lambda(t)}{d t} e^{-\lambda(t) t}-\lambda(t) e^{-\lambda(t) t} \\
& \frac{d \lambda(t)}{d t} \text { is } \ll \lambda(t) \\
& \frac{d R(t)}{d t}=-\lambda(t) e^{-\lambda(t) t} \text { but } e^{-\lambda(t) t}=R(t) \\
& \\
& \lambda(t)=\frac{-\frac{d R(t)}{d t}}{R(t)} \text { If } \lambda \text { is a constant then the above reduces to } \lambda(t)=\lambda \\
& \operatorname{MTBF}(t)=\frac{R(t)}{-\frac{d R(t)}{d t}}
\end{aligned}
$$

## Availability Improvement By Oversizing and Redundancy

For the $m$ out of $n$ case, where $m \neq n$
$n$ quantity of $\frac{m}{n}$ rated power supplies. Each power supply operates at $\frac{m}{n}$ rated $P_{R}$
$P_{O}=\frac{m}{n} P_{R}$
$M T B F_{O}=\frac{P_{R}}{P_{O}} M T B F_{R}=\frac{n}{m} M T B F_{R} \quad \lambda_{O}=\frac{m}{n} \lambda_{R}$ linear relationship is conservative

$$
\begin{aligned}
& R_{O m / n}(t)=\sum_{k=m}^{n}\left(\frac{n!}{(n-k)!k!}\right)\left(e^{-\lambda_{o} t}\right)^{k}\left(1-e^{-\lambda_{O} t}\right)^{n-k}=n e^{-m \lambda_{O} t}-m e^{-n \lambda_{O} t} \\
& \operatorname{MTBF}_{O m / n}(t)=\frac{n e^{-m \lambda_{O} t}-m e^{-n \lambda_{O} t}}{m n \lambda_{O} e^{-m \lambda_{O} t}-m n \lambda_{O} e^{-n \lambda_{O} t}} \\
& A_{O m / n}(t)=\frac{M T B F_{O m / n}(t)}{M T B F_{O m / n}(t)+M T T R}
\end{aligned}
$$

## Active Redundancy - One Full Rated Power Supply

For the case of 1 power supply with a power rating equal to the required operational power
$P_{R}=P_{O}$
$M T B F_{R}=M T B F_{o}$
$\lambda_{R}=\lambda_{o}$
$R_{o}=e^{-\lambda o_{o} t}=e^{-\lambda_{R} t}$
$A_{O}=\frac{M T B F_{O}}{M T B F_{O}+M T T R}=\frac{M T B F_{R}}{M T B F_{R}+M T T R}$

## Active Redundancy - One Out of Two Case

For the $m=1$ out of $n=2$ case
2- full rated rated power supplies. Each power supply operates at $\frac{1}{2}$ rated $P_{R}$
$M T B F_{O}=\frac{P_{R}}{P_{O}} M T B F_{R}=2 M T B F_{R} \quad \lambda_{O}=\frac{1}{2} \lambda_{R}$
$R_{01 / 2}(t)=2 e^{-\lambda \theta_{O} t}-e^{-2 \lambda_{O} t}$
$\operatorname{MTBF}_{01 / 2}(t)=\frac{2 e^{-\lambda_{O} t}-e^{-2 \lambda_{O} t}}{2 \lambda_{O} e^{-\lambda_{O} t}-2 \lambda_{O} e^{-2 \lambda_{O} t}}$
$A_{O 1 / 2}(t)=\frac{M T B F_{O 1 / 2}(t)}{M T B F_{O 1 / 2}(t)+M T T R}$

## Active Redundancy - Two Out of Three Case

For the $m=2$ out of $n=3$ case
3-1/2 rated power supplies. Each power supply operates at 2/3 rated $P_{R}$

$$
\begin{aligned}
& {M T B F_{O}}=\frac{P_{R}}{P_{O}}{M T B F_{R}}=\frac{3}{2}{M T B F_{R}}^{R_{O 2 / 3}(t)=3 e^{-2 \lambda_{o} t}-2 e^{-3 \lambda_{O} t}} \quad \lambda_{O}=\frac{2}{3} \lambda_{R} \\
& \operatorname{MTBF}_{O 2 / 3}(t)=\frac{3 e^{-2 \lambda_{O} t}-2 e^{-3 \lambda_{O} t}}{6 \lambda_{O} e^{-2 \lambda_{O} t}-6 \lambda_{O} e^{-3 \lambda_{o} t}} \\
& A_{O 2 / 3}(t)=\frac{M T B F_{O 2 / 3}(t)}{M T B F_{O 2 / 3}(t)+M T T R}
\end{aligned}
$$

For the $m=3$ out of $n=4$ case
4-3/4 rated power supplies. Each power supply operates at 3/4 rated $P_{R}$

$$
\begin{aligned}
& \operatorname{MTBF}_{O}=\frac{P_{R}}{P_{O}}{M T B F_{R}=\frac{4}{3} M T B F_{R} \quad \lambda_{O}=\frac{3}{4} \lambda_{R}}_{R_{O 3 / 4}(t)=4 e^{-3 \lambda_{o} t}-3 e^{-4 \lambda_{O} t}}^{M_{O 3 / 4}(t)=\frac{4 e^{-3 \lambda_{O} t}-3 e^{-4 \lambda_{O} t}}{12 \lambda_{O} e^{-3 \lambda_{O} t}-12 \lambda_{O} e^{-4 \lambda_{o} t}}} \\
& A_{O 3 / 4}(t)=\frac{M T B F_{O 3 / 4}(t)}{M T B F_{O 3 / 4}(t)+M T T R}
\end{aligned}
$$

## Active Redundancy - Four Out of Five Case

For the $m=4$ out of $n=5$ case
5-4/5 rated power supplies. Each power supply operates at 4/5 rated $P_{R}$

$$
\begin{aligned}
& {M T B F_{O}}=\frac{P_{R}}{P_{O}}{M T B F_{R}=\frac{5}{4} M T B F_{R} \quad \lambda_{O}=\frac{4}{5} \lambda_{R}}_{R_{O 4 / 5}(t)=5 e^{-4 \lambda_{O} t}-4 e^{-5 \lambda_{O} t}}^{M_{O H B F}{ }_{O 4 / 5}(t)=\frac{5 e^{-4 \lambda_{O} t}-4 e^{-5 \lambda_{O} t}}{20 \lambda_{O} e^{-4 \lambda_{O} t}-20 \lambda_{O} e^{-5 \lambda_{O} t}}} \\
& A_{O 4 / 5}(t)=\frac{M T B F_{O 4 / 5}(t)}{M T B F_{O 4 / 5}(t)+M T T R}
\end{aligned}
$$

## Active Redundancy Power Supply Reliability Summary

|  | $P S$ | Redundant Power Supplies |  |  |
| :---: | :---: | :---: | :---: | :---: |
| $1 F R$ | $\lambda_{O}=\lambda_{R}$ | $R_{O}=e^{-\lambda} O^{t}$ | $M T B F_{o}=M T B F_{R}$ | $A_{o}=\frac{M T B F_{O}}{M T B F_{O}+M T T R}$ |
| 1/2 | $\lambda_{O}=\frac{1}{2} \lambda_{R}$ | $R_{O 1 / 2}=2 e^{-\lambda_{O} t}-e^{-2 \lambda_{O} t}$ | $\operatorname{MTBF}_{O 1 / 2}(t)=\frac{2 e^{-\lambda} O^{t}-e^{-2 \lambda_{O} t}}{2 \lambda_{O} e^{-\lambda_{O} t}-2 \lambda_{O} e^{-2 \lambda_{O} t}}$ | $A_{O 1 / 2}(t)=\frac{M T B F_{O 1 / 2}(t)}{M T B F_{O I / 2}(t)+M T T R}$ |
| 2/3 | $\lambda_{O}=\frac{2}{3} \lambda_{R}$ | $R_{02 / 3}=3 e^{-2 \lambda} O^{t}-2 e^{-3 \lambda_{O} t}$ | $\operatorname{MTBF}_{02 / 3}(t)=\frac{3 e^{-2 \lambda} O^{t}-2 e^{-3 \lambda} O^{t}}{6 \lambda_{o} e^{-2 \lambda_{O} t}-6 \lambda_{o} e^{-3 \lambda_{O} t}}$ | $A_{O 2 / 3}(t)=\frac{M T B F_{O 2 / 3}(t)}{M T B F_{O 2 / 3}(t)+M T T R}$ |
| $3 / 4$ | $\lambda_{O}=\frac{3}{4} \lambda_{R}$ | $R_{o 3 / 4}=4 e^{-3 \lambda_{O}^{t}}-3 e^{-4 \lambda_{O} t}$ | $\operatorname{MTBF}_{O 3 / 4}(t)=\frac{4 e^{-3 \lambda_{O} t}-3 e^{-4 \lambda_{O} t}}{12 \lambda_{O} e^{-3 \lambda_{O} t}-12 \lambda_{o} e^{-4 \lambda_{O} t}}$ | $A_{o 3 / 4}(t)=\frac{M T B F_{03 / 4}(t)}{M T B F_{O 3 / 4}(t)+M T T R}$ |
| 4/5 | $\lambda_{O}=\frac{4}{5} \lambda_{R}$ | $R_{04 / 5}=5 e^{-4 \lambda_{O} t}-4 e^{-5 \lambda_{O} t}$ | $\operatorname{MTBF}_{04 / 5}(t)=\frac{5 e^{-4 \lambda_{o} t}-4 e^{-5 \lambda_{o} t}}{20 \lambda_{O} e^{-4 \lambda_{o} t}-20 \lambda_{o} e^{-5 \lambda_{O} t}}$ | $A_{04 / 5}(t)=\frac{M T B F_{\text {O4/5 }}(t)}{M T B F_{04 / 5}(t)+M T T R}$ |

Active Redundancy MTBF Plot


Time in hours

Active Redundancy - Availability
Configuration Availability


Time in hours


Time in hours

Active Redundancy - Number of PS Failures
Number of Failures


Time in hours

## Homework Problem \# 18

Two inverter stages in an uninterruptible power supply are to be connected in parallel. each is capable of full-load capability. The calculated failure rate of each stage is $\lambda=200$ failures per million hours. A. What is the probability that each inverter will remain failure free for a mission time of 1000 hours and B) What is the probability that the system will operate failure free for 1000 hours?

Solution:

## Homework Problem \# 19

For a critical mission, 3 power supplies, each capable of supplying the total required output, are to be paralleled. The power supplies are also decoupled such that a failure of any power supply will not affect the output. The calculated failure rate of each power supply is 4 per million hours.
A) What is the probability that each power supply will operate failure free for 5 years? B) What is the probability that the system will operate failure free for 5 years? Solution below.

SLAC Next-Generation

## High Availability Power Supply

Dave MacNair SLAC National Accelerator Laboratory Power Conversion Department (PCD)



Redundancy is Essential

5000 Nonredundant Vs 5000 1/2 Redundant

## 



Time in hours

## Hot Swap is Essential



Non-redundant - PEP II, SPEAR 3, LCLS (1994-2006)

- Power supply quantity is hundreds, not thousands
- Power supply availability budget is modest 98\%
- Non-redundant supplies satisfied availability budget
- Redundant power systems not readily available from industry
- Redundant systems would not fit within cost and schedule constraints

Redundant - KEK ATF 2 (2006-2008)

- Mock-up of ILC Final Focus accelerator
- Magnet power supplies ILC-like




## ATF2 Current and Field Recovery Plots



- During power module loss measured 6A magnet current drop at 150A
- 100 Gauss drop at 3.1 kilogauss. 200mS recovery with no overshoot, no re-standardize needed


## Goals

- All components $N+1$ modular and redundant
- Power module hot-swappable
- Unipolar or bipolar output from a single unipolar bulk voltage source
- Imbedded controller with digital current regulation
- Capable of driving superconducting magnets
- High bandwidth for use in BBA or closed orbit correction systems
- High stability and precision output current
- High accuracy read-backs
- Scalable to higher output levels

Applications

- ILC and other future accelerators

Next Generation Power Supply Block Diagram


## Next Generation Power Module Schematic



Next Generation Positive Output Current (Q1-Q2-Q3-Q4)


Next Generation Negative Output Current (Q2-Q1-Q4-Q3)


Power Modules Connected for Unipolar Output


## Power Modules Connected for Bipolar Output




- Input: 48 V
- Output V: 0 to 40 V
- Output I: 0 to 33A
- Output P: 0 to 1,320W
-2"X4"X8"


Next Generation Controller


VOLTAGE LOOP ERROR VOLTAGE






| STAMFORD LIEAR ACCELERATOR CENTER <br>  <br>  |  |  |  CONTROLER CARD |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |  |
|  | pate | Srparifl |  |  |  |
| 5,Feren more | E见边 |  |  |  |  |
|  |  |  | SA-125-347-98 | VER 1.0 | A |



To date

- Five power modules with embedded controllers have been built
- The modules have been tested individually and run as pairs
- Demonstrated
- 4 modules, $40 \mathrm{~V}, 100 \mathrm{~A}, 4,000 \mathrm{~W}$ unipolar output then reconfigure
- 4 modules, 40V, 33A, 1,320W bipolar output


## Future

- Design the outer current control loop components
- Demonstrate operation of a completely redundant power supply
- MTBF previously discussed relates to the laws of large quantities and 50\% confidence limits
- Confidence intervals are bounded with upper and lower limits. The broader the limits, the higher the confidence
- Electronic equipment, a one-sided, lower limit is appropriate
$t=$ time in hours
$f=$ number of failures
$M T B F_{\text {Predicted }}=t / f$
$K_{L}$ from chi-square distribution
$M T B F_{L L}=M T B F_{\text {Predicted }} * K_{L}$


## $K_{L}$ Multipliers For MTBF Confidence Levels

| Failures | Lower Limit $K_{L}$ |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $f$ | $60 \%$ | $70 \%$ | $80 \%$ | $90 \%$ | $95 \%$ |
| 1 | 0.620 | 0.530 | 0.434 | 0.333 | 0.270 |
| 2 | 0.667 | 0.600 | 0.515 | 0.422 | 0.360 |
| 3 | 0.698 | 0.630 | 0.565 | 0.476 | 0.420 |
| 4 | 0.724 | 0.662 | 0.598 | 0.515 | 0.455 |
| 5 | 0.746 | 0.680 | 0.625 | 0.546 | 0.480 |
| 500 | 0.965 | 0.954 | 0.942 | 0.930 | 0.915 |

Excerpted and abridged from W. Grant Ireson, Reliability Handbook, McGraw-Hill, NY 1966

If a power supply is to operate for 3 years before the first failure, what is the MTBF prediction for an $80 \%$ confidence level? Repeat for a $90 \%$ confidence level.

Solution:
3 years $=26280$ hours $=$ MTBF $_{\text {Test }}$
From the confidence limit table $K_{L}=0.434$ for $80 \%$ and $f=1$
Therefore, $\quad M_{\text {M }} F_{80 \%}=$ MTBF $_{\text {Predicted }} * 0.434 \geq 11,406$ hours
For $\quad M^{2} B F_{90 \%}=M T B F_{\text {Predicted }} * 0.333 \geq 8,751$ hours

It is desired to claim with $90 \%$ confidence that the actual MTBF of a power supply is 2500 hours. What must be the predicted MTBF if the test is terminated at the first test failure?

Fault Modes And Effects Criticality Analysis (FMECA)

## FMECA is

- A systematic way to prioritize the addressing of "weak links".
- An inductive, bottoms-up method of analyzing a system design or manufacturing process in order to properly evaluate the potential for failures

It Involves

- Identifying all potential failure modes, determining the end effect of each potential failure mode, and determining the criticality of that failure effect.


## 3 Major Iterations

- Used in the Design, Fabrication and Operation Stages

Fault Modes And Effects Criticality Analysis (FMECA)

| Part Name/ \# | Part Function | Potential Failure Mode | Potential Effects of Failure | S E V | Potential Causes of Failure | O | Design Evaluation Technique | D | $\begin{array}{l\|\|} \mathbf{R} \\ \mathbf{P} \\ \mathbf{N} \\ \hline \end{array}$ | W | W $H$ Y |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Coils | Provide magnetic field | coil to coil or coil to magnet steel short | magnet goes off line | 5 | coils moved during installation of magnet or adjacent beamline component, or alignment of magnet | 5 | protype test | 1 | 25 | 3 | 2 |
| Coils | Provide magnetic field | klixon trip due to overheating | magnet goes off line | 5 | inadequate water pressure differential across magnet | 5 | prototype test, calculation | 1 | 25 | 1 | 2 |
| Coils | Provide magnetic field | klixon trip due to overheating | magnet goes off line | 5 | too many loads on water circuit | 5 | prototype test, calculation | 1 | 25 | 1 | 2 |
| Coils | Provide magnetic field | klixon trip due to overheating | magnet goes off line | 5 | conducter sclerosis | 3 | n/a | 1 | 15 | 4 | 9 |
| Coils | Provide magnetic field | klixon trip due to overheating | magnet goes off line | 5 | foreign object in water line or coil which blocks water flow | 2 | n/a | 1 | 10 | 4 | 8 |
| Coils | Provide magnetic field | klixon trip due to overheating | magnet goes off line | 5 | damaged (crimped) coil which restricts water flow | 2 | n/a | 1 | 10 | 3 | 8 |
| Coils | Provide magnetic field | water leak | magnet goes off line due to ground fault | 5 | water hose brakes because of radiation damage | 5 | n/a | 1 | 25 | 4 | 3 |
| Coils | Provide magnetic field | water leak | magnet goes off line due to ground fault | 5 | corrosion in aluminum/copper conductor | 2 | n/a | 1 | 10 | 4 | 9 |
| Coils | Provide magnetic field | water leak | magnet goes off line due to ground fault | 5 | erosion of coil from excess water velocity | 4 | n/a | 1 | 20 | 4 | 2 |
| Coils | Provide magnetic field | water leak | magnet goes off line due to ground fault | 5 | break in braze joint between copper block and coil | 3 | prototype test | 1 | 15 | 3 | 8 |
| Fittings | Make water connection | water leak | magnet goes off line due to ground fault | 5 | cracked fittings from incorrect installation procedure | 4 | n/a | 1 | 20 | 3 | 8 |
| Jumpers | Connection between coils | short at jumper | magnet goes off line due to ground fault | 5 | sloppy installation | 5 | n/a | 1 | 25 | 3 | 8 |
| Jumpers | Connection between coils | short at jumper | magnet goes off line due to ground fault | 5 | poor design | 5 | design review, prototype | 1 | 25 | 1 | 2 |
| Jumpers | Connection between coils | loose jumpers | excessively high temperatures leading to melting of materials | 5 | poor design or incorrect procedures used at installation | 5 | n/a | 1 | 25 | 3 | 8 |

Fault Modes And Effects Criticality Analysis (FMECA)




## Maintainability

Cold swap - input bus and power supply must be off when it is exchanged
Warm swap - input bus is on but power supply is off when exchanged Hot swap - input bus is on and power supply is on when exchanged. Typically used with redundant, full rated power supplies


## 11. Power Supply Specifications

| List of Specifications for to be given to the Power Supply Designer |  |
| :--- | :--- |
| Requirement | Example |
| 1. Site conditions | Elevation, ambient temperature range, <br> humidity, seismic requirements |
| 2. Intended use and system | Storage ring accelerator dipole magnet <br> power supply |
| 3. Function | DC or pulsed, voltage or current source |
| 4. Load parameters and description | Inductance, capacitance and resistance |
| 5. Output ratings | Maximum voltage, current, operating or <br> pulse time, pulse width and repetition rate |
| 6. Input voltage and phases | 208V, 1 $\phi \quad$ 208V, 3 $\phi$ 480V, $3 \phi$ |
| 7. Efficiency | Up to 94\% achievable at full load output |

## List of Specifications for to be given to the Power Supply Designer

| Requirement | Example |
| :--- | :--- |
| 8. Input power factor | Up to 0.95 achievable for 6 pulse <br> Up to 0.97 achievable for 12 pulse |
| 9. Input line THD | < $5 \%$ voltage <br> <24\% current |
| 10. Conducted EMI 10kHz to 30MHz | MIL-STD-461E <br> FCC Class A Industrial <br> FCC Class B Residential |
| 11. Line regulation | 0.05 \% of rated output voltage change for a <br> $5 \% ~ l i n e ~ v o l t a g e ~ c h a n g e . ~ R e c o v e r y ~ i n ~ 500 ~$ |
| 12. Short-term (1 to 24 hour) stability | Allowable voltage or current deviation - 10s <br> of ppm achievable |
| 13. Output voltage ripple (PARD) | DC to I MHz, peak-to-peak, 0.05 \% of <br> rated voltage output |

## Power Supply Specifications

| List of Specifications for to be given to the Power Supply Designer |  |
| :--- | :--- |
| Requirement | Example |
| 14. Output pulse amplitude stability | I nanosecond for solid-state converters. <br> 10s of nanoseconds for thyratron triggers |
| 15. Output pulse - to pulse deviation in <br> time (jitter) | 0.05 \% of rated output voltage change for <br> 10 \% line change. Recovery in 500 $\mu$ |
| 16. Load regulation | Analog, mixed analog-digital, all digital <br> Communication bus |
| 17. Type of control system | -Low input voltage - loss of input phase <br> - Output over voltage - over current <br> - Excessive ground current <br> -Insufficient cooling air flow - cabinet over <br> temperature |
| 18. Interlocks |  |
| -Insufficient cooling water flow - cooling |  |
| water over temperature |  |$|$


| List of Specifications for to be given to the Power Supply Designer |  |
| :--- | :--- |
| Requirement | Example |
| 19. Interlocks (continued) | $\begin{array}{l}\text { •MPS fault } \\ \\ \\ \hline\end{array}$ |
| •PPS violated |  |$\}$


| List of Specifications for to be given to the Power Supply Designer |  |
| :--- | :--- |
| Requirement | Example |
| 22. Front panel displays | •Output voltage <br> •Output current <br> •Ground current <br> - Voltage or current mode <br> - Current limited operation |
| 23. Component deratings | Voltage, current and power |
| 24. Mean time between failure (MTBF) | MTBF = l/ (sum of all parts failure rates) |
| 25. Mean time to repair (MTTR) | Establish from MTBF and operational <br> Availability requirement |
| 26. Availability |  |
| 27. Maintainability |  |

## Power Supply Specifications

| List of Specifications for to be given to the Power Supply Designer |  |
| :---: | :---: |
| Requirement | Example |
| 28. Physical size | Based on output power - typically 1 to $4 \mathrm{~W} / \mathrm{cu}$ in |
| 29. Rack or free-standing | < 17 kW rack-mounted <br> > 17 kW free-standing |
| 30. Compliance with UL or other nationally-recognized inspection/test laboratories | Underwriters Laboratories - UL <br> National Recognized Test Laboratory NRTL |
| 31. Seismic | Must satisfy site earthquake design criteria Damage criteria and response spectra curves - separate or combined accelerations |
| 32. Quality Assurance | Must satisfy project quality assurance/quality control criteria |

# Section 12-References 

| References | Used in |
| :--- | :---: |
| Elements Of Power System Analysis, Stevenson, McGraw-Hill | Textbook |
| IEEE 90 - IEEE Standard Computer Dictionary: A Compilation of <br> IEEE Standard Computer Glossaries. Institute of Electrical and <br> Electronics Engineers. New York, NY: 1990 | Textbook |
| "Power Electronic Converter Harmonics", Derek Paice, IEEE <br> Press, 1996 | Textbook |
| Rectifier Circuits Theory And Design, Johannes Schaefer, John <br> Wiley \& Sons, Inc NY | Textbook |
| Switchmode Power Supply Handbook, Keith Billings, McGraw-Hill, <br> February 1999, ISBN 0070067198 | Textbook |
| EMI and Emissions: Rules, Regulations and Options, Daryl Gerke <br> and Bill Kimmel, Electronic Design News, February 2001 | Section 3 |
| EMI Control Methodology and Procedures, Donald White and <br> Michel Mardiguian, Interference Control Technologies, 4th Edition | Section 3 |
| http://www.iijnet.or.jp/murata/index.html for feedtrhu filters | Section 3 |


| References | Used in |
| :--- | :---: |
| "Case Studies on Mitigating Harmonics in ASD Systems to Meet <br> IEEE519-1992 Standards", Mahesh Swamy, Steven Rossiter, <br> Michael Spencer, Michael Richardson - IEEE Industry Application <br> Society Proceedings October 1994 | Section 3 |
| IEEE 519 - 1992 "Standard Practices and Requirements for <br> Harmonic Control in Electrical Power Systems" | Section 3 |
| Circuit Techniques for Improving the Switching Loci of Transistor <br> Switches in Switching Regulators,E.T. Calkin and B.H. Hamilton, <br> IEEE Transactions On Industry Applications, July 1976 | Section 4 |
| How to Select a Heatsink <br> http://www.aavidthermalloy.com/technical/papers/pdfs/select.pdf | Section 4 |
| IGBT Theory: <br> http://www.elec.gla.ac.uk/groups/dev mod/papers/igbt/igbt.html | Section 4 |
| Magnetics Designer for Transformers, chokes and inductors, <br> Intusoft Corporation <br> http://www.i-t.com/engsw/intusoft/magdesgn.htm | Section 4 |

Useful Textbooks And References

| References | Used in |
| :--- | :---: |
| Power Electronics Modeling Software, Integrated Engineering <br> Software, http://www. integratedsoft.com | Section 4 |
| PSPICE simulator for switching regulators, Linear Technologies, <br> http://www.linear.com/insider | Section 4 |
| PSPICE circuit simulator, Micro-Cap, Spectrum Software, <br> http://www.spectrum-soft.com | Section 4 |
| Zero Voltage Switching Resonant Power Supplies <br> http://www-s.ti.com/sc/psheets/slua159/slua159.pdf | Section 4 |
| SCSI information http://www.Scsita.org/aboutscsi/index01.html | Section 5 |
| MIL-STD-1629 "Procedures for Performing a Failure Mode, <br> Effects, and Criticality Analysis". | Section 7 |
| RelCalc by T-Cubed | Section 7 |
| Relex by Relex Software |  |

## Useful Textbooks And References

| References | Used in |
| :--- | :---: |
| Table of Laplace Transforms <br> http://www.vibrationdata.com/Laplace.htm | Section 6 |
| Table of Fourier Transforms <br> http://mathworld.wolfram.com/FourierTransform.html | Section 6 |
|  |  |
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# Section 13-Homework Problems 

Referring to the one-line diagram below, determine the line currents in the:
A. Generator
B. Transmission Line
C. M1
D. M2


## Homework Problem \# 2

A waveform $v(t)$ was analyzed and found to consist of 6 components as shown here.

a. Write the Fourier series for $v(t)$ in terms of $\omega=(2 * \pi) / T$
b. Show the harmonic content graphically by plotting the frequency spectrum
c. Give the numerical result of

$$
\begin{array}{ll}
b_{3}=\frac{2}{T} \int_{0}^{T} v(t) \sin 3 \omega t d t & \text { Help }: \int \sin ^{2}(3 \omega t) d t=\frac{1}{3} * \frac{\frac{1}{2} \cos (3 \omega t) \sin (3 \omega t)+\frac{3}{2} \omega t}{\omega} \\
b_{4}=\frac{2}{T} \int_{n}^{T} v(t) \sin 4 \omega t d t & \text { Help }: \int \cos (4 \omega t) \sin (4 \omega t) d t=\frac{-1}{8} \frac{\cos ^{2}(4 \omega t)}{\omega}
\end{array}
$$

## Homework Problem \# 3

Each waveform below can be written as a Fourier series. The result depends upon the choice of origin. For each of the 6 cases, state the type of symmetry present, non-zero coefficients and the expected harmonics.



## Homework Problem \# 4

A $10 \mathrm{~kW}, 3 \phi$ power supply has an efficiency of $90 \%$ and operates with a leading power factor of 0.8. Determine the size of a added inductor needed to improve the power factor to 0.95 .

## Homework Problem \# 5

A uniform magnetic field $B$ is normal to the plane of a circular ring 10 cm in diameter made of \#10 AWG copper wire having a diameter of 0.10 inches. At what rate must $B$ change with time if an induced current of $10 A$ is to appear in the ring? The resistivity of copper is about $1.67 \mu \Omega-c m$.

## Homework Problem \# 6

A 1000kVA, 12.47 kV to $480 \mathrm{~V}, 60 \mathrm{~Hz}$ three phase transformer has an impedance of 5\%. Calculate:
a. The actual impedance and leakage inductance referred to the primary winding
b. The actual impedance and leakage inductance referred to the secondary winding
c. The magnetizing inductance referred to the primary winding

Calculate the output voltage in the circuit shown below.



Assume ideal components in the phase-controlled circuit above. For a purely resistive load:
A. Explain how the circuit operates
B. Draw the load voltage waveform and determine the boundary conditions of the delay angle $\alpha$
C. Calculate the average load voltage and average load current as a function of $\alpha$
D. Find the RMS value of the load current.


Assume ideal components in the phase-controlled circuit above. For a purely inductive load:
A. Explain how the circuit operates.
B. Draw the load voltage and load current waveforms and determine the boundary conditions of the delay angle $\alpha$
C. Calculate the average load voltage and average load current as a function of $\alpha$
D. Find the RMS value of the load current.

## Homework Problem \# 10

Given the circuit below:

$h(t)=\frac{v_{\text {out }}(t)}{v_{\text {in }}(t)} \quad H(j \omega)=\frac{V_{\text {out }}(j \omega)}{V_{\text {in }}(j \omega)}$

Sketch $|H(j \omega)|$ versus $\omega$

## Homework Problem \# 11

A 100 kW power supply is $80 \%$ efficient. Approximately $50 \%$ of the power supply heat loss is removed by cooling water.

- How much heat is dissipated to building air and how much heat is removed by the water system.
- Calculate the water flow rate needed to limit the water temperature rise to $8^{\circ} \mathrm{C}$ maximum.


## Homework Problem \#12

A. An artificial transmission line can be formed using lumped Ls and Cs. Calculate the delay of an artificial line composed of 8 sections of inductances $L=4 m H$ per section and capacitance $C=40 p F$ per section.
B. The frequency of a signal applied to a two-wire transmission cable is $3 G H z$.

What is the signal wavelength if the cable dielectric is air? Hintrelative permittivity of air is 1

What is the signal wavelength if the cable dielectric has a relative permittivity of 3.6?

## Homework Problem \#13

For the transmission line shown below, calculate the Reflection Coefficients $\Gamma$, the reflected voltages and the voltage and current along the line versus time.


## Homework Problem \# 14

A controlled impedance transmission line often drives a kicker. The kicker is usually well modeled as an inductor. A matching circuit can be built around the kicker and its inductance so that this circuit, including the kicker magnet, has constant, frequency independent, impedance which is matched to the transmission line.

Assuming that the transmission line impedance is $Z_{0}$ and the kicker inductance is $L_{\text {Kicker }}$ derive the values of R1, R2, and C necessary to make a frequency independent (constant) impedance $Z_{0}$.


## Homework Problem \#15

A. What is the significance of the value $\sqrt{\frac{\mu_{0}}{\varepsilon_{0}}}$ ?
$B$. What is the significance of the values $\frac{1}{\sqrt{\mu_{0} \varepsilon_{0}}}$ and $\sqrt{L^{*} C}$
C. Calculate the speed of light in mediums with dielectric constants of: $\varepsilon_{r}=1 \quad \varepsilon_{r}=2 \quad \varepsilon_{r}=4 \quad \varepsilon_{r}=8 \quad \varepsilon_{r}=16$

## Homework Problem \# 16

Homework Problem:
A. At least 2 of 4 parallel power supplies in an accelerator must continue to operate for the system to be successful. Let $R_{i}=0.9$. Find the probability of success.
B. Solve for three out four for success
C. Solve for four out of four for success

## Homework Problem \# 17

Calculate the MTBF of a "typically commercial" 5 kW , switchmode power supply with EMI filter and appropriate electromechanical safety features amounting to $10 \%$ of the total number of components. The power supply operates at 50C ambient temperature. The power supply consists of the following components with the listed failure rates.:

- 2 each ICs, plastic linear, $\lambda=3.64$
- 1 each opto-isolator, $\lambda=1.32$
- 2 each hermetic sealed power switch transistors, $\lambda=0.033$
- 2 each plastic power transistors, $\lambda=0.026$
- 4 each plastic signal transistors, $\lambda=0.0052$
- 2 each hermetic sealed power diodes, $\lambda=0.064$
- 8 each plastic power diodes, $\lambda=0.019$
- 6 each hermetic sealed switch diodes, $\lambda=0.0024$
- 32 each composition resistors, $\lambda=0.0032$
- 3 each potentiometers, commercial, $\lambda=0.3$
- 8 each pulse type magnets, 130 C rated, $\lambda=0.044$
- 12 each ceramic capacitors, commercial, $\lambda=0.042$
- 3 each film capacitors, commercial, $\lambda=0.2$
- 9 each Al electrolytics, commercial, $\lambda=0.48$


## Homework Problem \# 18

Two inverter stages in an uninterruptible power supply are to be connected in parallel. each is capable of full-load capability. The calculated failure rate of each stage is $\lambda=200$ failures per million hours. A. What is the probability that each inverter will remain failure free for a mission time of 1000 hours and B) What is the probability that the system will operate failure free for 1000 hours?

Solution:

## Homework Problem \# 19

For a critical mission, 3 power supplies, each capable of supplying the total required output, are to be paralleled. The power supplies are also decoupled such that a failure of any power supply will not affect the output. The calculated failure rate of each power supply is 4 per million hours.
A) What is the probability that each power supply will operate failure free for 5 years? B) What is the probability that the system will operate failure free for 5 years? Solution below.

## Homework Problem \# 20

It is desired to claim with $90 \%$ confidence that the actual MTBF of a power supply is 2500 hours. What must be the predicted MTBF?


[^0]:    Multiples of Circuit Breaker Continuous Current Rating

