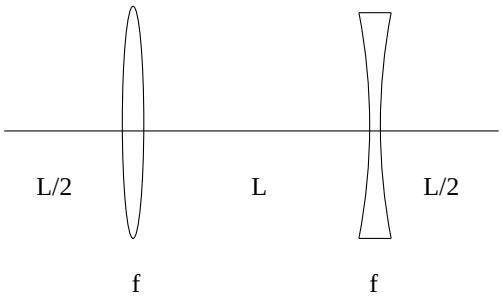
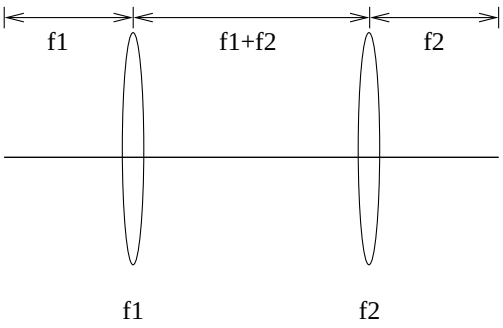
Homework 5

Exercise 1



Consider the quadrupole doublet shown above. Let $\frac{L}{f} = \sqrt{2}$. Using the thin lens approximation, find the transfer matrix for the three drifts and two quadrupoles shown in the figure above. What is the transfer matrix for the motion in the vertical plane? Now, write all matrix elements in terms of L, eliminating f. Does this combination focus in the horizontal plane, the vertical plane, or both?

Exercise 2 (From Brown & Servranchx, SLAC-PUB-3957)



The figure above shows the set-up for a telescope, magnifying only in one plane. Four quadrupoles can be used to make a two plane telescope. Calculate the transfer matrix to show that the position of a particle, x, exiting the telescope will be $x = -Mx_0$, where $M = \frac{f_2}{f_1}$ and x_0 is the position of the particle at the entrance of the telescope. The lengths of the drifts of the telescope in the figure are all given in terms of the focal lengths of the quadrupoles. If the position error of a particle is reduced to half of what it was coming into the telescope, what happens to the angle of the particle?

Exercise 3
A FODO cell has $\beta_{max} = 100$ m, $\beta_{min} = 25$ m, and the focal length of the quadrupoles is $f = 25$ m. What is the natural chromaticity of the FODO cell?

Exercise 4
In a small proton accelerator, a transverse beam size measurement is done when the machine is at the injection energy, 400 MeV. The momentum spread of the beam is $\frac{\Delta p}{p} = 1 \times 10^{-3}$. A horizontal beam sigma of $\sigma_x = 10$ mm is measured at a location where the amplitude function is a maximum, $\beta_{max} = 34$ m, and the dispersion, D, is 4 m.

a) What is the 39% beam emittance? (The 39% beam emittance encompasses 39% of the beam, which may be considered Gaussian.)

b) A measurement of σ_x is also done at a minimum beta location, where $\beta_{min} = 5$ m, and $D = 1$ m. What is the value of σ_x measured there?

c) What is the normalized horizontal emittance, ε_N, of the beam?