Last homework

Exercise 1

Calculate the space charge force for a particle at an arbitrary radius r, within a uniform, cylindrical beam of radius a and infinite extent in the longitudinal direction. Take the number of particles per length to be λ. The steps in the problem are as follows;

- Find the electric field at r using Gauss' law, $\oint E \cdot da = \frac{q_{enclosed}}{\varepsilon_0}$. Choose a Gaussian surface through a point of interest, and where there is a constant electric field at the surface.
- Find the magnetic field at r using Ampere's law, $\oint B \cdot dl = \mu_0 i_{enclosed}$. Choose a loop through a point of interest, where there is a constant magnetic field around the loop.
- Stuff the E and B fields obtained into the Lorentz force law, $\vec{F} = q[\vec{E} + \vec{v} \times \vec{B}]$

Exercise 2

Find the space charge tune shift of a single particle in this rigid beam using the result of the previous problem and Hill's equation, $x'' + \left(\frac{\nu_0}{R}\right)^2 x = \frac{F_{sc}}{\gamma m_0 v^2}$. The variation of the focusing force around the ring has been replaced with the average oscillation frequency per length, since the independent variable in the equation of motion is length. This is the machine tune divided by the ring radius R, since $\frac{2\pi\nu}{2\pi R}$ is the phase variation per length around the machine. Similarly, the space charge force is divided by $\gamma m_0 v^2$ to convert from units of force/time to units of meter^{-1}.

- Change r to x in the space charge force expression from the previous problem, and move the normalized $\frac{F_{sc}}{\gamma m_0 v^2}$ to the right-hand-side of Hill's equation. Collect the terms in x, and identify the new frequency of motion.
- The space charge tune shift $\Delta\nu$ is small compared to ν_0. Use the approximation $\nu_0^2 - \nu_{sc}^2 = (\nu_0 + \nu_{sc})(\nu_0 - \nu_{sc}) \approx 2\nu_0 \Delta\nu$ to find $\Delta\nu$, the space charge tune shift.

Exercise 3

What is a limitation of the analysis you did to find the space charge tune shift?

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Exercise 4

Calculate the ratio between the synchrotron radiation power radiated by a particle in the Large Hadron Collider (LHC), the proton collider at CERN, and the one radiated by a particle in the Advanced Light Source (ALS), the electron storage ring in Berkeley. The magnet bending radius is 2810 m and 5 m and the particle energy is 7000 GeV and 1.9 GeV for the LHC and the ALS respectively. (Remember that the electron mass is 9.1095×10^{-31} Kg while the proton one is 1.6726×10^{-27} Kg.)