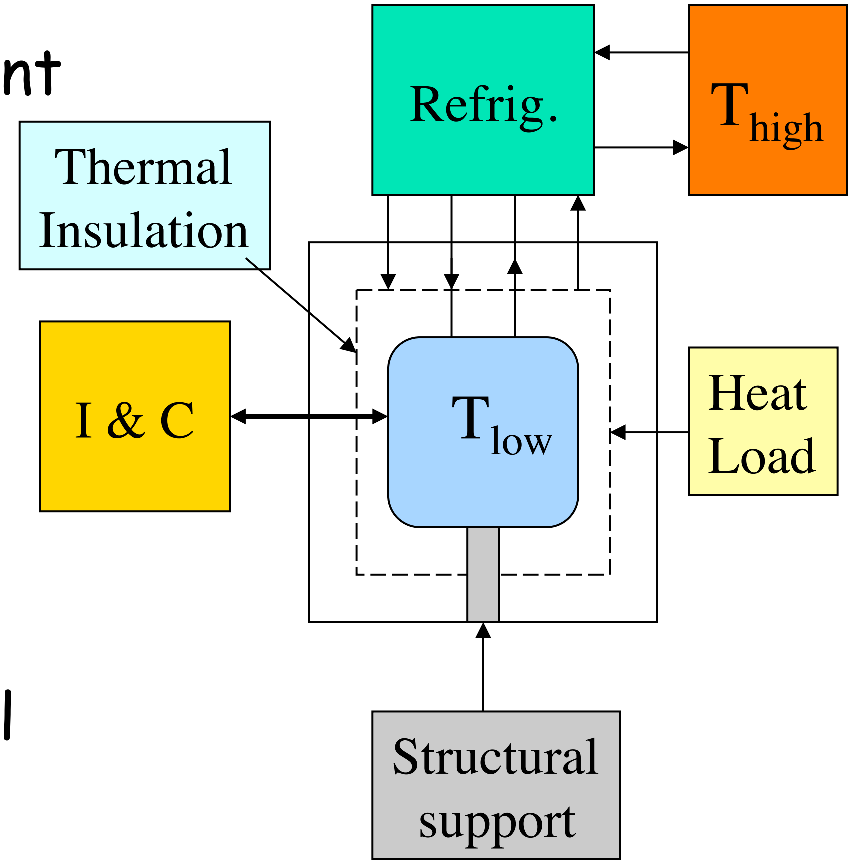
5.1 Cryogenic system design

- Low temperature environment
- Source of refrigeration
- Heat exchange medium
- Thermal insulation
- Structural support
- Instrumentation and control



Thermal Insulation Systems

- Solid foam insulation
- Powder insulation
- Vacuum
  - Radiation heat transfer
  - Gas conduction/convection
- Multi-layer insulation
  - Radiation shields (active and passive)
  - MLI

Solid Foam Insulations

- Solid foam insulations are not used very often in cryogenics because they have relatively poor performance
- Since these materials are typically gas filled, their thermal conductivity is > k_{air} ~ 25 mW/m K.

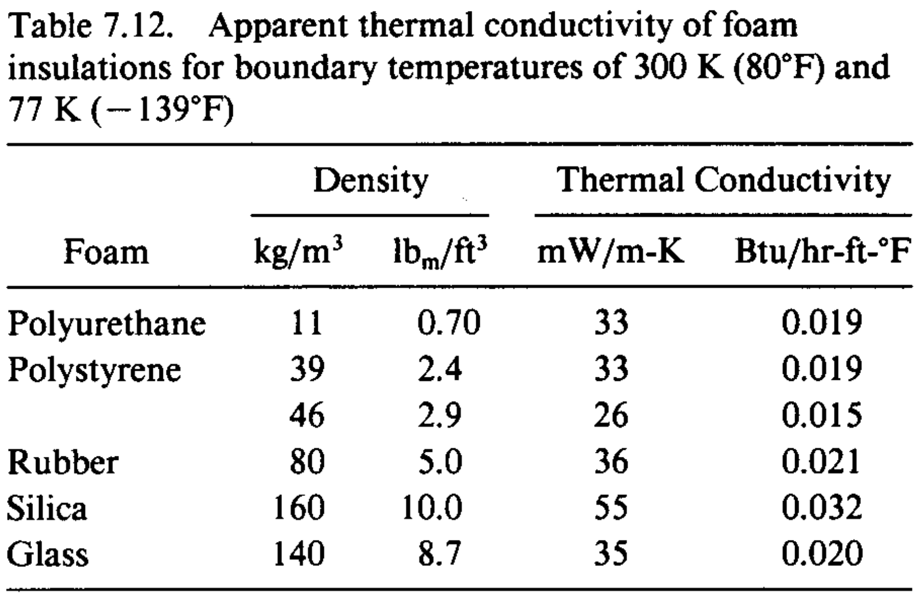


Table 7.12. Apparent thermal conductivity of foam insulations for boundary temperatures of 300 K (80°F) and 77 K (−139°F)

	Density		Thermal Conductivity	
Foam	kg/m^3	lb_m/ft^3	mW/m-K	Btu/hr-ft-°F
Polyurethane	11	0.70	33	0.019
Polystyrene	39	2.4	33	0.019
	46	2.9	26	0.015
Rubber	80	5.0	36	0.021
Silica	160	10.0	55	0.032
Glass	140	8.7	35	0.020

<u>Example:</u>

Consider a Polystyrene LN_2 vessel with 20 mm wall and 1 m^2 surface area.

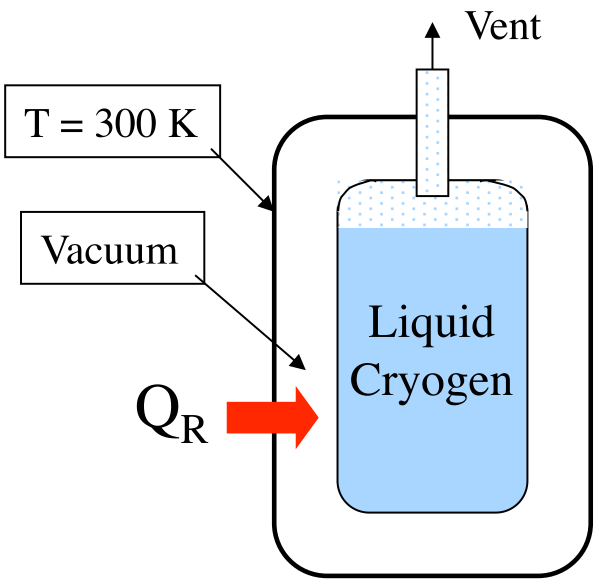
Heat leak: $Q = kA\Delta T/L$
= 33 mW/m K x 1 m^2 x (300 – 77) K / 0.02 m = 368 W

h_{fg} (LN_2) = 200 J/g; ρ ~ 800 g/L

dm/dt = 1.84 g/s (8.3 L/hr)

Vacuum Insulation

- High performance insulation systems all involve some level of vacuum.
  - How low vacuum is needed?
- Even for perfect vacuum, thermal radiation can still contribute significantly to total heat leak
  - $Q_R \sim T^4$ so process is dominated by high temperature surfaces (usually 300 K)



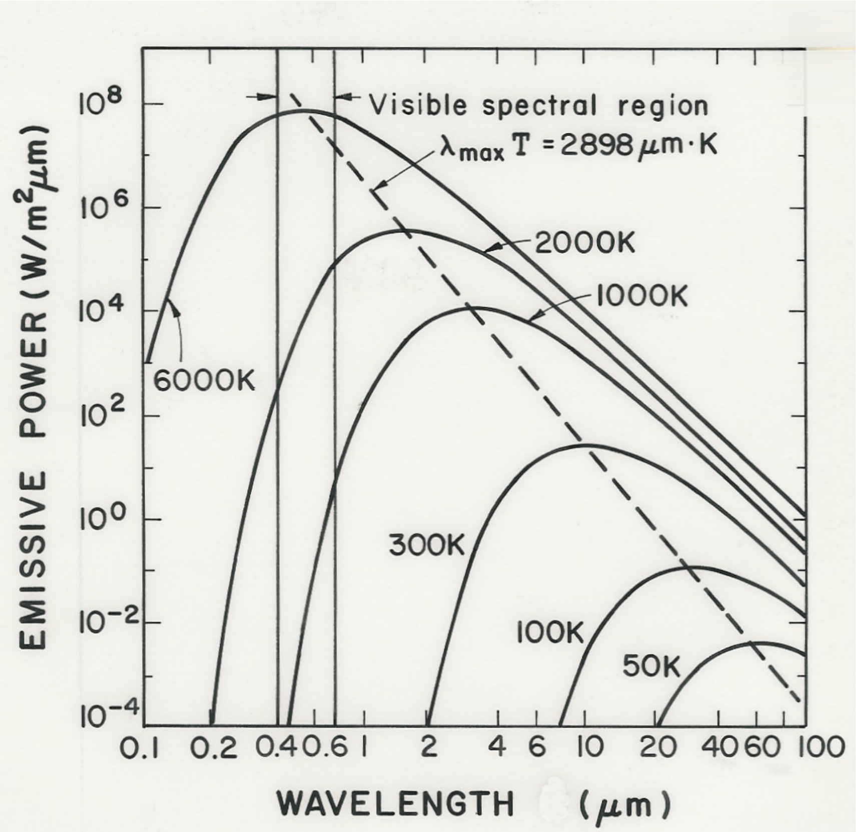
Thermal Radiation

- Radiation from room temperature is one of the main heat loads in cryogenic systems
- Black body spectrum is ideal emitted power versus wavelength of radiation
- Integral of spectrum is total emitted power

$$E_b = \int_0^\infty e_b(T,\lambda)\,d\lambda = \sigma T^4$$

where $\sigma = 5.67 \times 10^{-8}$ W/m^{2}K^4, the Stefan-Boltzman constant

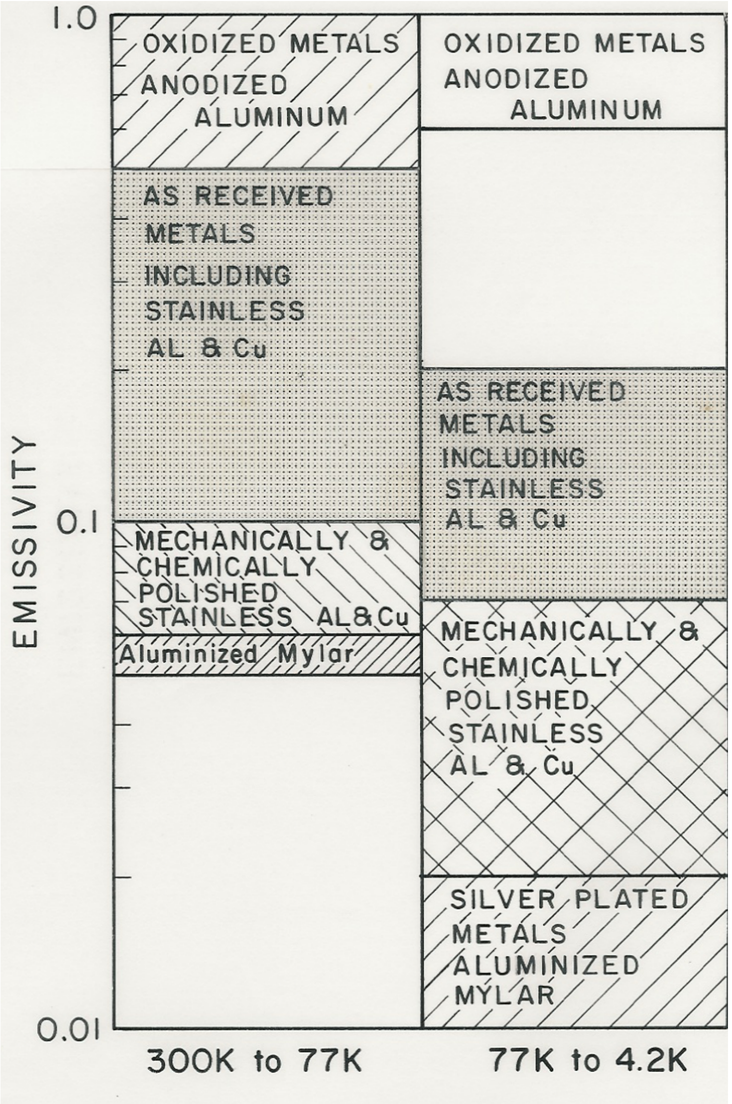
$$e_b(T,\lambda) = \frac{8\pi hc}{\lambda^5}\left(\frac{1}{e^{hc/\lambda k_B T} - 1}\right)$$



Radiant Emissivity (ε)

- Emissivity is the property of a surface material that determines the fraction of radiant flux that is absorbed or emitted.
- ε depends on material conductivity, temperature
- ε is also a function of wavelength, but engineering usually relies on average values measured for range of temperatures
- For a real surface,

$$q_r \cong \varepsilon \sigma T^4$$



Radiation heat transfer

- Net heat transfer for two facing black body surfaces

$$q_r = \sigma A[(T + \Delta T)^4 - T^4]$$

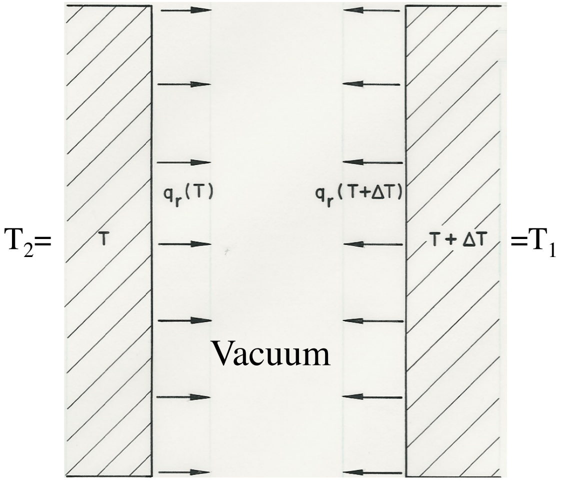
- For non-black bodies, the heat exchange between surfaces depends on the emissivity of each surface:

$$q_r = \left(\frac{\varepsilon_1 \varepsilon_2}{\varepsilon_1 + \varepsilon_2 - \varepsilon_1 \varepsilon_2}\right)\sigma\left(T_1^4 - T_2^4\right)$$

For $\varepsilon_1 \sim \varepsilon_2 = \varepsilon$ and $\varepsilon << 1$, ( ) $\sim \varepsilon/2$

- Example: Radiant heat transfer between 300 K and 77 K
  $\varepsilon \sim 0.05$,
  $q = 0.05/2 \times 5.67 \times 10^{-8} \times (300^4 - 77^4) = 11.4$ W/m^2
  h_{fg} (LN$_2$) = 200 kJ/kg and the density, ρ = 800 kg/m^3
  volume consumption = 11.4 W/m^2/200 J/g = 0.06 g/sm^2
  or about $\frac{1}{4}$ liter/hour of LN$_2$ (much better than foam)
  Note if the low T surface were at 4 K in Helium, the liquid consumption would be larger because h_{fg}(LHe) is about 21 J/g

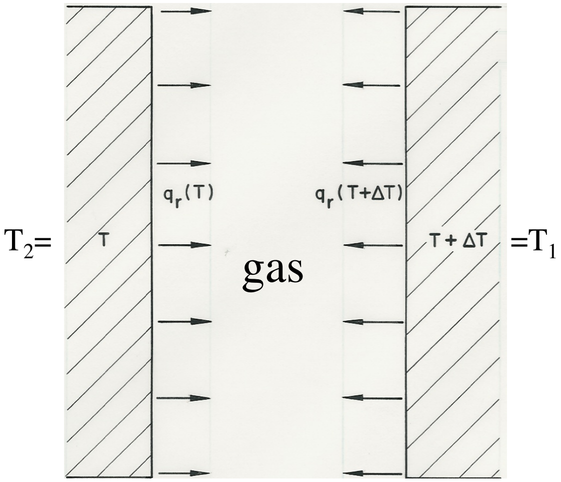
Two surfaces facing each other with vacuum between



Photon radiation exchange

Heat exchange with imperfect vacuum

- Residual heat leak due to gas conduction can contribute significantly to heat loading to a cryogenic system
  - At pressures near 1 Atm, the heat transfer is by natural convection
  - At lower pressure, convection is reduced, but gas conduction still can transfer considerable heat, k (T). This regime occurs for gas densities where the mean free path is less than the wall spacing.
- In addition to radiation heat transfer, gas conduction due to poor vacuum can seriously affect thermal performance



Gas conduction heat transfer

- At pressures below about 1 Pa, the mean free path of the molecule begins to exceed the distance between surfaces and heat is carried by Molecular-Kinetic processes

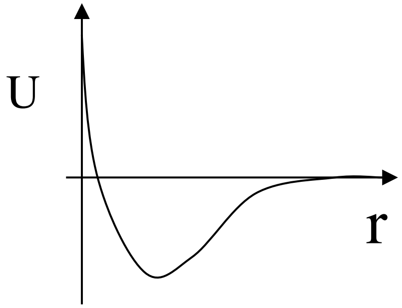
$$l \approx \frac{1}{n\sigma_{tot}} \approx \frac{k_B T}{\pi d^2 p}$$

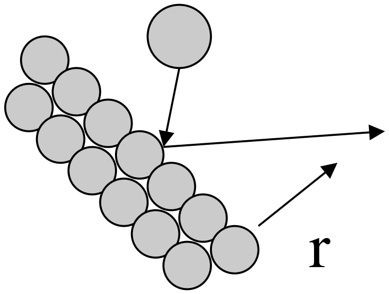
Where d is the molecule diameter and p is the pressure

  - For helium gas at 1 Atm (100 kPa) and 300 K, l ~ 60 nm
  - For helium at 1 Pa and 300 K, l ~ 6 mm, a distance comparable to spacing in containers
- In the molecular kinetic regime, the heat exchange depends on
  - Number of molecules striking the surface/unit time
  - The thermal equalization of the molecule with the surface
  - Probability that the molecule sticks to the surface

Adsorption & Accommodation Coef.

- Molecules are attracted to solid surfaces by Van der Waal's forces just as with intermolecular interactions





- α is the accommodation coefficient that measures the amount that a molecule comes in thermal equilibrium with the wall.

$$\alpha = \frac{T_i - T_e}{T_i - T_w} \leq 1$$

T_i is the temperature of the incident molecule
T_e is the temperature of the emitted molecule
T_w is the temperature of the wall

- For heat exchange between two surfaces, it is necessary to use an average accommodation coefficient,

$$\overline{\alpha} = \frac{\alpha_1 \alpha_2}{\alpha_1 + \alpha_2 - \alpha_1 \alpha_2}$$

Note: If the surfaces are not of equal area, geometric corrections are required for this formula

Gas conduction heat exchange

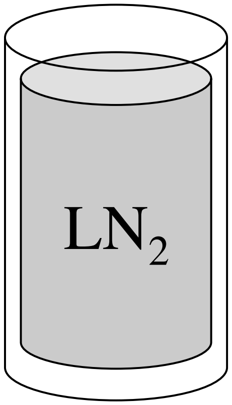
- In the molecular-kinetic regime, heat transfer between two parallel surfaces can be calculated using the expression,

$$q = \frac{\bar{\alpha}}{4}\frac{\gamma+1}{\gamma-1}\left(\frac{2R}{\pi M T}\right)^{1/2} p\left(T_1 - T_2\right) \qquad \text{Where } \gamma = C_p/C_v$$

- Values for accommodation coefficients:
  - α decreases with cleaner surfaces
  - α increases with decreasing temperature to ~1 at $T \sim T_{NBP}$
  - For rough calculations, α ~ 0.5 is practical

Surface condition	Transport gas	Temperature (K)	Accommodation coefficient
Very clean	helium	300	< 0.1
Engineering	helium	300	0.3
Engineering	helium	20	0.6
Engineering	nitrogen	250	0.7

Example of gas conduction heat transfer



- Consider a 100 liter ($A = 1\ m^2$) cryostat for storing liquid nitrogen.
  - Calculate the consumption of LN_2 if the vessel is only vacuum insulated. (h_{fg} = 198 kJ/kg). Radiant heat transfer between 300 K and 77 K (assume $\varepsilon \sim 0.05$)

    $q = 0.05/2 \times 5.67 \times 10^{-8} \times (300^4 - 77^4) = 11.4$ W (0.06 g/s)
  - Calculate the consumption if the vessel had a poor vacuum with helium at $p \sim 0.1$ Pa (10^{-6} Atm)

$$q = \frac{\bar{\alpha}}{4}\frac{\gamma+1}{\gamma-1}\left(\frac{2R}{\pi M T}\right)^{1/2} p\left(T_1 - T_2\right) \approx 7.6\frac{p\Delta T}{\bar{T}^{1/2}} \approx 12W$$

Adds to the radiation heat transfer doubling the heat load

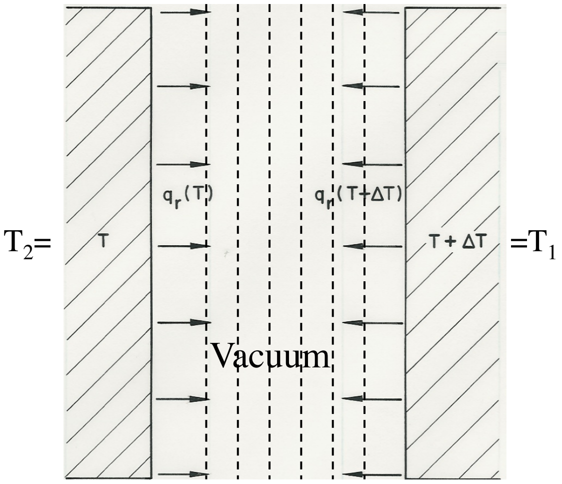
Conclusion: Good vacuum is highly desirable

Multi layer shielding

- Adding shielding between the radiant surfaces can significantly reduce the heat transfer. For n shields with emissivity e, the heat exchange is

$$q_r = \left(\frac{\varepsilon}{(n+1)(2-\varepsilon)} \right) \sigma \left(T_1^4 - T_2^4 \right)$$

which for $\varepsilon \ll 1$, reduces the q_r by a factor of 1/n +1

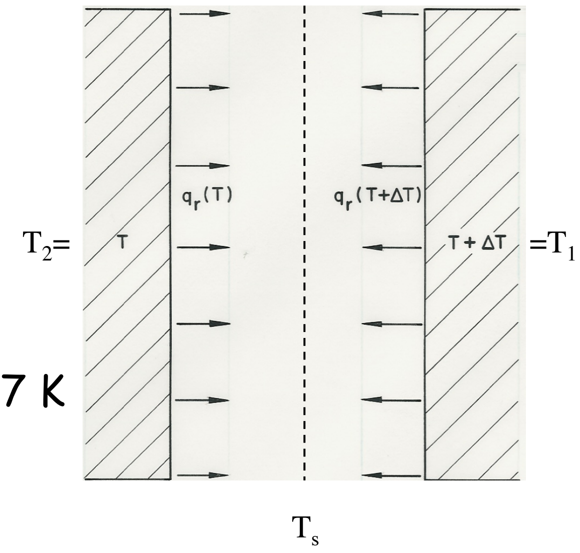


- Note that the shield temperatures are not equally distributed because the heat exchange is not linear. Consider one shield and all emissivities = ε in steady state;

$$q_r(1,s) = \frac{\varepsilon}{2} \sigma \left(T_1^4 - T_s^4 \right) = q_r(s,2) = \frac{\varepsilon}{2} \sigma \left(T_s^4 - T_2^4 \right)$$

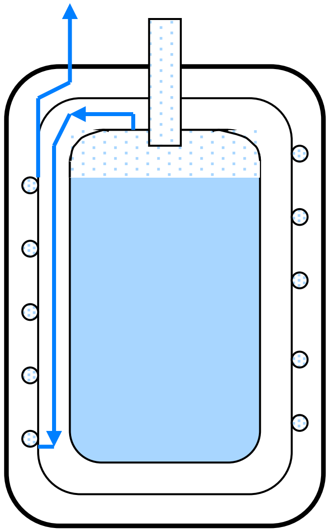
or

$$T_s = \left(\frac{T_1^4 + T_2^4}{2} \right)^{1/4}$$ ~ 252 K for T_1 = 300 K and T_2 = 77 K



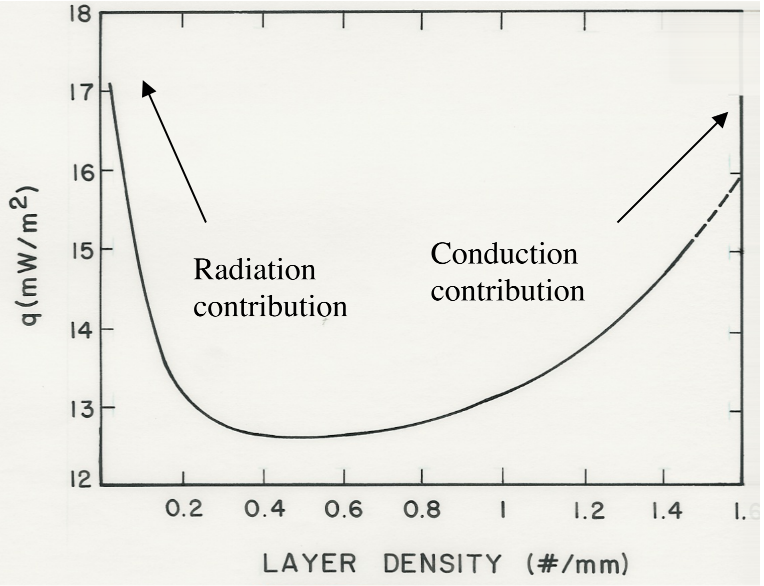
Refrigerated radiation shields

- There is significant thermodynamic advantage to actively cooling radiation shields in a cryogenic system. Examples:
  - LN_2 shield cooling in a cryostat
  - Vapor cooling in LHe storage vessels
  - Refrigerated shields
- Why would you want to do this?
  - Thermodynamic advantage of removing heat at higher temperature (COP)
  - Reduce boil-off of expensive fluid (LHe)
  - Can be done in conjunction with active cooling of other components (structural supports, current leads)



Multilayer Insulation (MLI)

- MLI is a material developed to approximate thermally insulated shields.
- MLI consists of aluminum (5 to 10 nm thick) on Mylar film usually with low density fibrous material between layers
- Insulation must operate in vacuum
- Heat transfer is by a combination of conduction and radiation
- MLI must be carefully installed covering all surfaces with parallel layers, not wrapped since conduction along layer will produce a thermal short
- Engineering applications must include factor of safety compared to ideal data

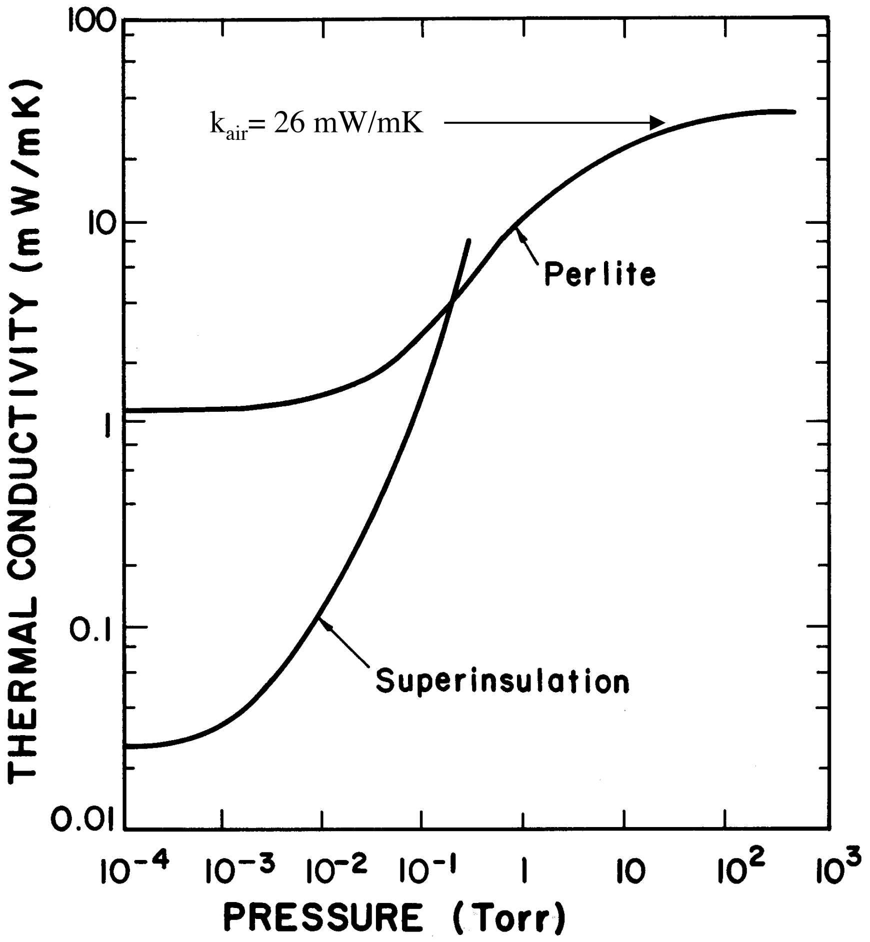


Radiation heat load for different densities between 4.2 K and 77 K

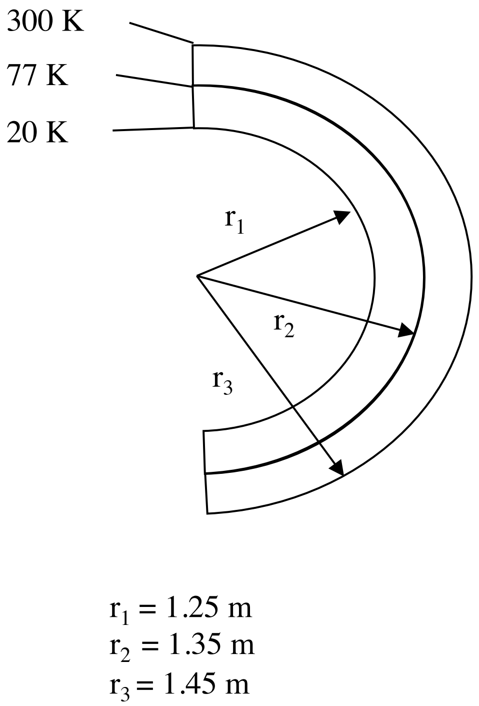
Recommended conservative values:
q_r (77 K, 4 K) ~ 50 to 100 mW/m^2
q_r (300 K, 77 K) ~ 1 to 1.5 W/m^2

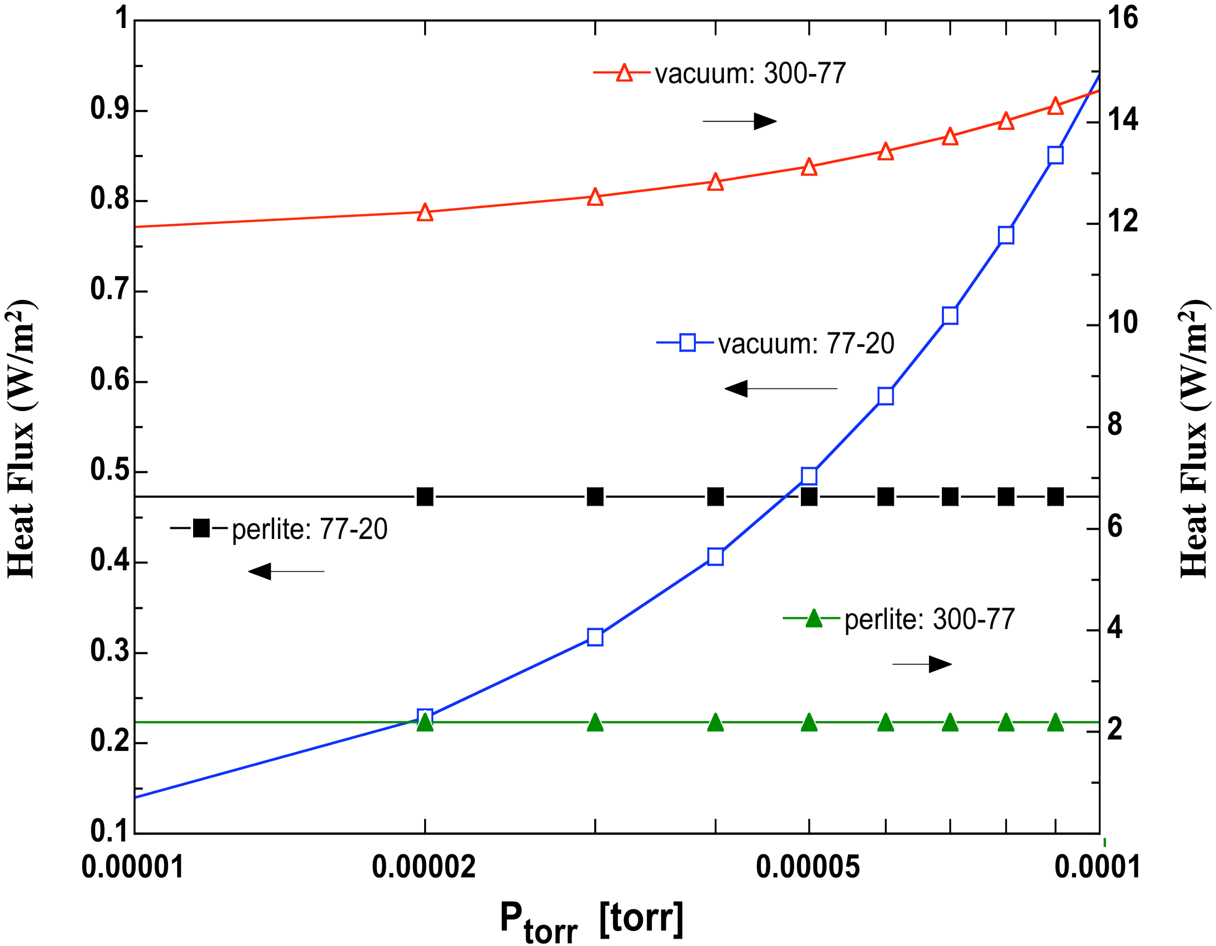
Powder insulations (perlite, glass bubbles)

- Powder insulations were developed for ease of installation in less stringent operating conditions.
- Perlite is a commercial powder of random size and shape (cheap)
- Hollow glass micro-spheres (3M) of 50 to 200 μm in diameter
- Vacuum requirements are less critical. Good performance at p ~ 0.1 Torr compared to 10^{-4} torr for MLI
- Perlite is mostly used for less stringent cryogenic vessels such as LNG containers or LN_2 and LO_2.
- NASA is planning to build new storage containers with glass bubbles



Perlite or Vacuum: which is the better insulation?

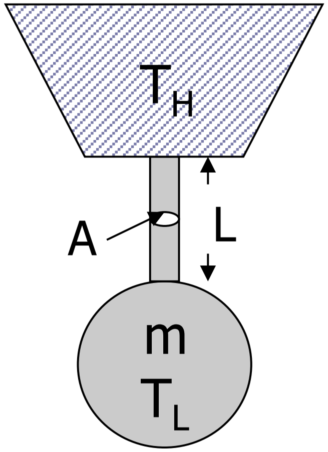




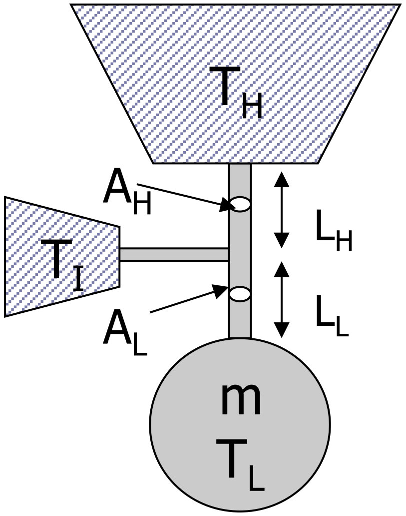
- Below T=77 K and P = 5×10^{-5} torr, pure vacuum provides superior insulation

Structural supports

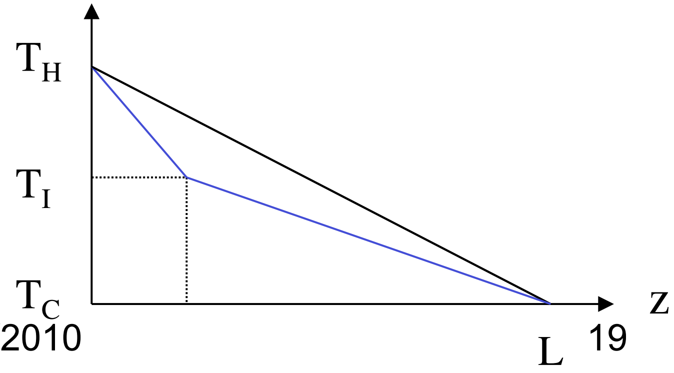
Simple support:



Actively cooled support:

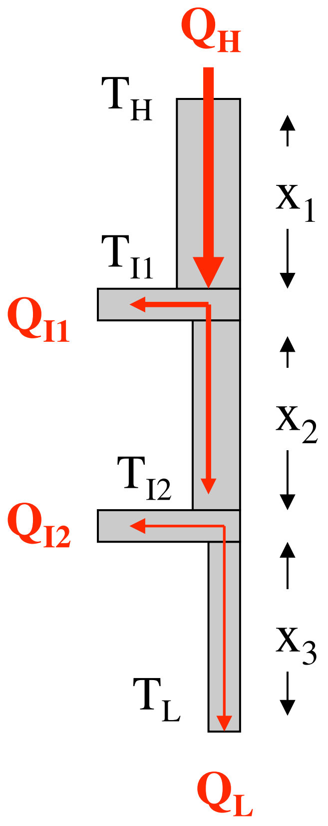


- Simple support is appropriate for small masses where the conduction heat leak is not large
- For large mass, an actively cooled support is preferred to reduce heat load at the lowest temperatures where the thermodynamic efficiency is low
    - Position for the intermediate cooling station
    - Thermodynamic optimization

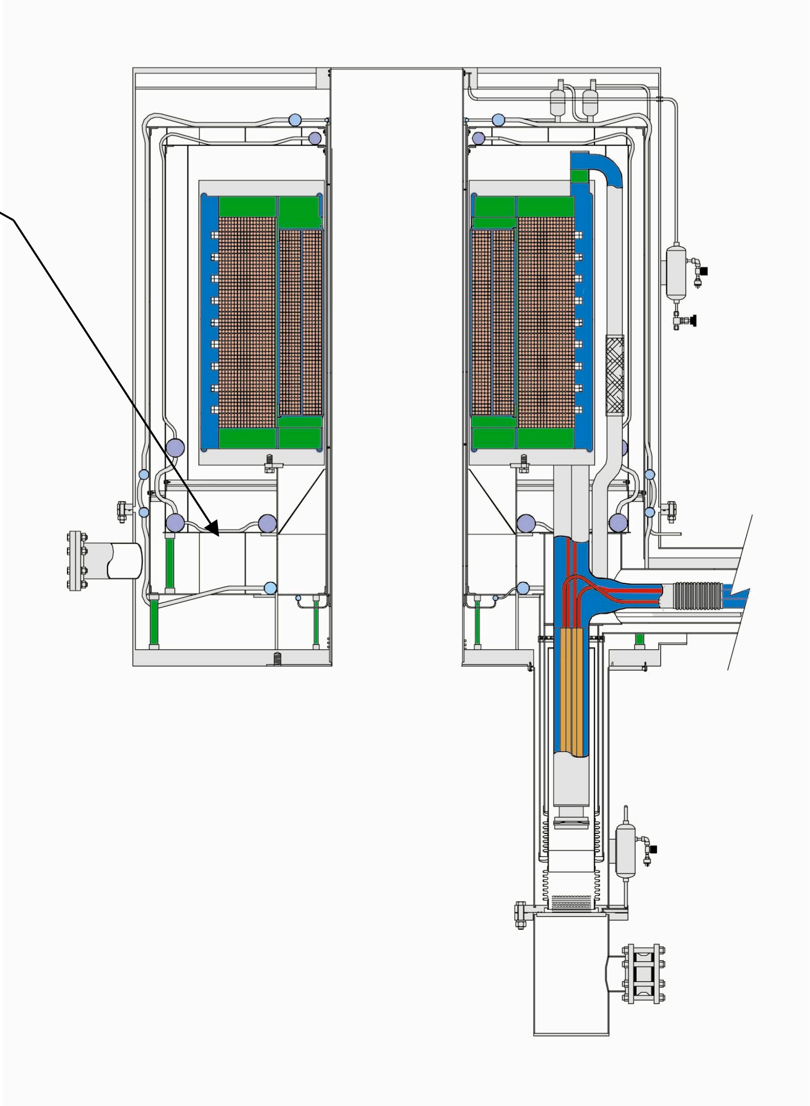


Optimization of mechanical supports

- Considerations:
  - Intermediate cooling stations (number, T_I, x)
  - Variation of thermal conductivity, k(T)
  - Temperature dependent mechanical properties, σ(T)
    - Only an advantage if loads occur when support is cold
- Procedure
  - Full optimization is based on assumptions about efficiency of refrigeration
  - Vary T_i and x_i to minimize total refrigeration
- Typical practical solution (easier)
  - Intermediate temperatures are known based on available refrigeration system (e.g. 80 K (LN_2), 20 K)
  - Vary position (x_i) to match available refrigeration



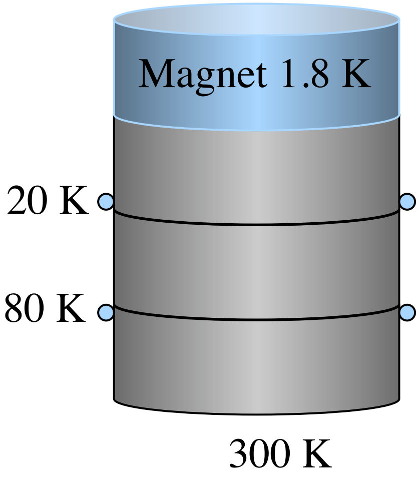
Example: 45 T Hybrid Magnet Cryostat



- Magnet loads are supported by refrigerated ss support column
    - 80 K by LN_2 natural circulation loop
    - 20 K by He gas forced circulation loop.
- Wall thickness decreases in low temperature sections
    - Increased strength of ss
    - Major portion of load only present when magnet is energized
- Location of refrigeration stations was optimized so that equivalent refrigeration is equal at each.

Support Tube for Hybrid

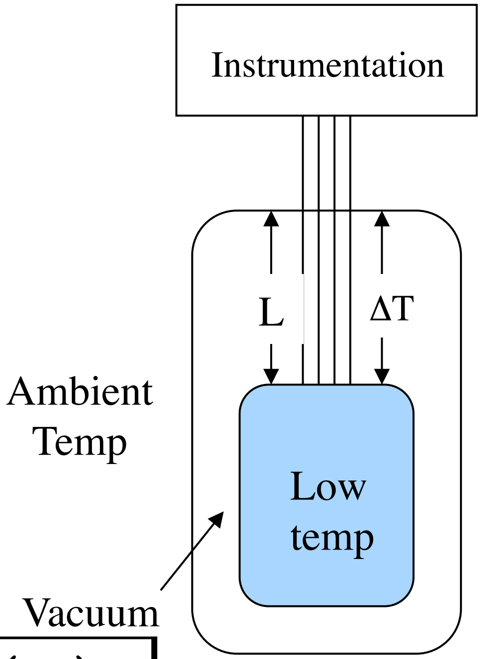
- Since load only occurs when magnet is energized, the structure takes advantage of increased material at low temperature
- Design load 6.3 MPa
- Cooling supplied by refrigerator at 20 K and LN_2 natural circulation loop (80 K)



Temperature	σ (MPa)	k_{ave}(W/m K)	Cross section (m^2)	Length (m)	Q (W)
1.8 K to 20 K	300	0.9	0.021	0.3	1.2
20 K to 80 K	240	5.6	0.026	0.3	30
80 K to 300 K	150	12.4	0.042	0.3	380

Instrumentation leads

- Most cryogenic systems have instrumentation
  - Monitor and control function (T, P, flow)
  - Measure performance of device (B, V, I)
- Instrumentation leads can significantly impact the thermal performance of a cryo system
  - Conduction heat leak
  - Joule heating in lead
- Proper lead design is important to ensure good measurement
  - Material selection
  - Thermal anchoring

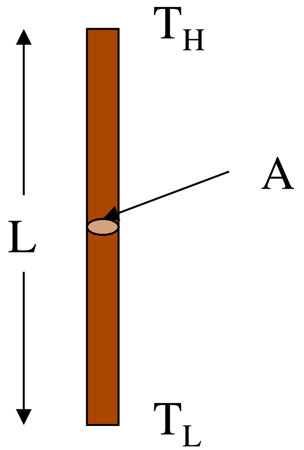


Gauge	D(mil)	D (mm)
24	20	0.51
28	12.6	0.32
32	8	0.2
36	5	0.127
40	3.1	0.08

Thermally optimized current lead

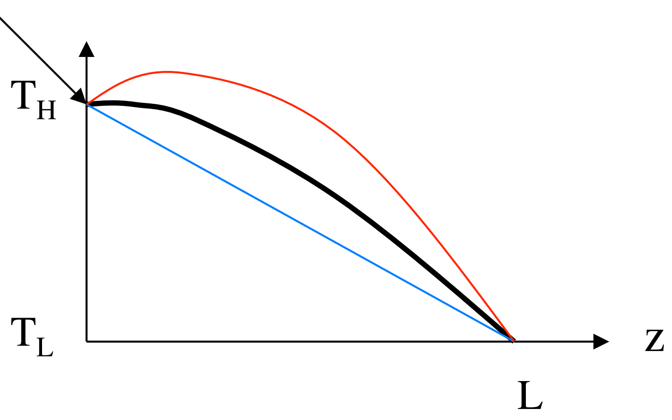
- Balance the heat generation with the conduction

$$\frac{d}{dx}\left(k(T)\frac{dT}{dx}\right)+\rho(T)\left(\frac{I}{A}\right)^2=0$$



- Minimize entropy generation occurs for dT/dx = 0 at T_H (x=0)

$$Q=I\left[2\int_{T_L}^{T_H}k(T)\rho(T)dT\right]^{1/2}$$

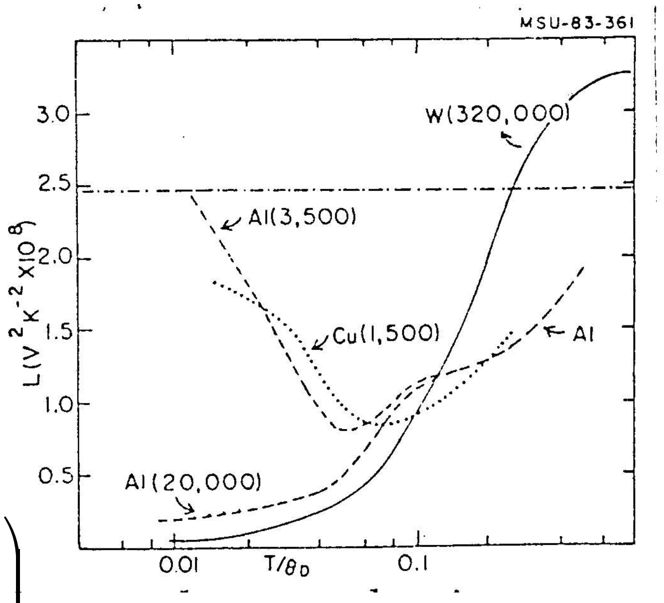


- For Wiedemann-Franz law materials

$$\frac{k(T)\rho(T)}{T}=L_0$$

Q ~ constant for all materials

Thermally optimized lead (continued)



- For W-F materials, exact solution:

$$\frac{Q}{I} = \sqrt{L_0\left(T_H^2 - T_L^2\right)}$$

$\sim$ 47 mW/A for $T_H = 300$ K, $T_L = 4$ K

- Optimum (L/A) for W-F materials

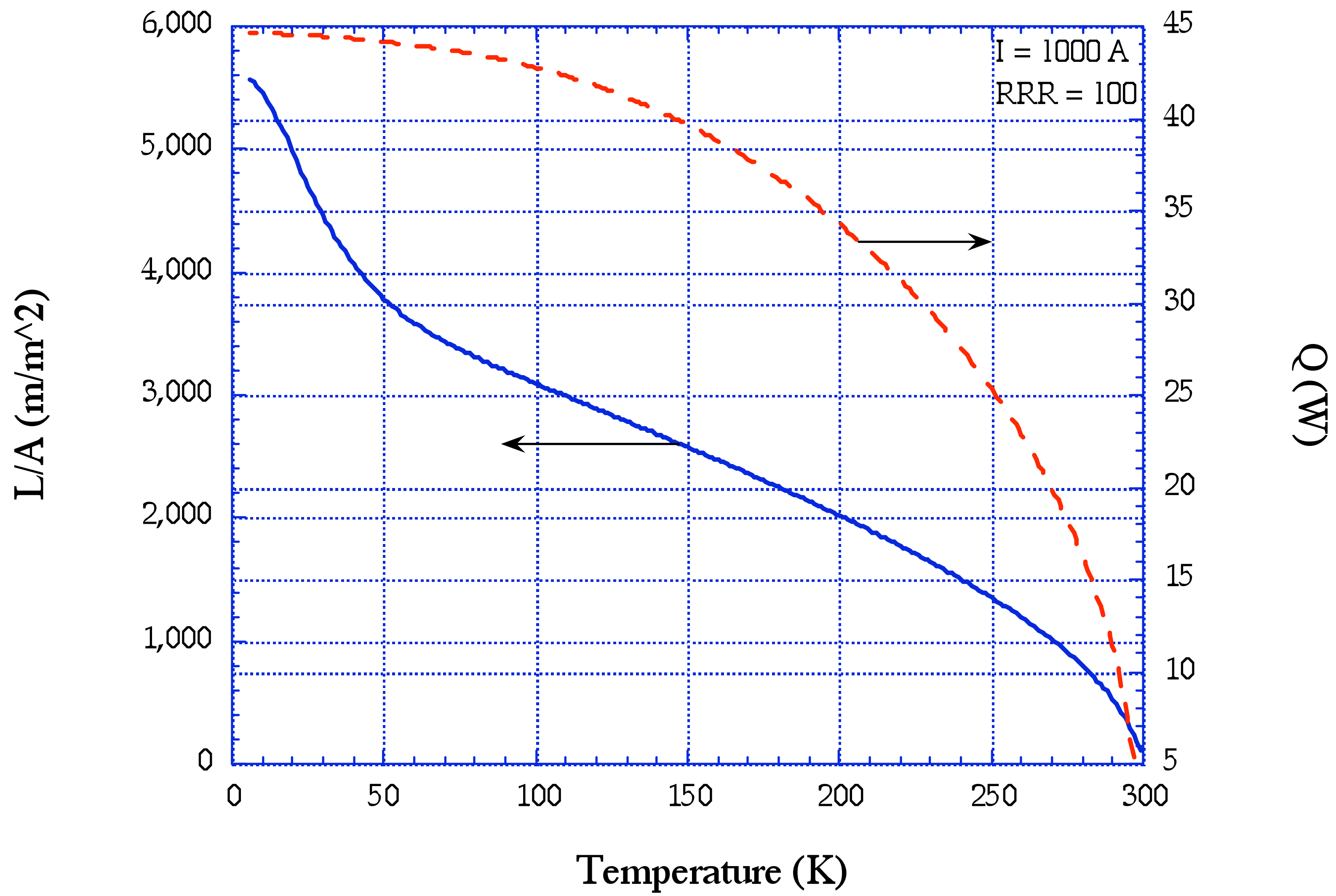
$$\frac{L}{A} = \frac{1}{I}\int_{T_L}^{T_H} \frac{k}{\sqrt{L_0\left(T_H^2 - T^2\right)}}dT = \frac{k}{\sqrt{L_0} I}\left(1.57 - \sin^{-1}\left(\frac{T_L}{T_H}\right)\right)$$

$$\sim 10^4\,(k/I),\ m^{-1}$$

Example: For a lead I = 1 A and L = 1 m, A = 0.25 mm^2 (d = 0.56 mm)
J = 4 A/mm^2

High current leads benefit from active cooling, which allows higher current density

Conduction Cooled Lead: Sample Results



Conduction Cooled Lead: Conclusions

- An 'optimized' lead is optimized for a single (maximum) current
- $Q_{c,\ min} \sim I$
- $Q_{c,\ min}$ is a function of T_h, T_c, I, and (weakly) on material choice
- JL = constant dependent only on T_h, T_c, and mtl. choice
- $L/A \sim 1 / I$

Vapor Cooled Lead – Scaling Rules

- Minimum heat leak:
  - As with conduction cooled leads, $Q_{min} \sim I$
  - Dependence of Q_{min} on coolant is dominated by (C_L / C_p)
- Optimized aspect ratio:
  - $L/A_{opt} \sim 1/I$ smaller current → larger aspect ratio
  - L/A_{opt} dependence on coolant: colder range → larger aspect ratio