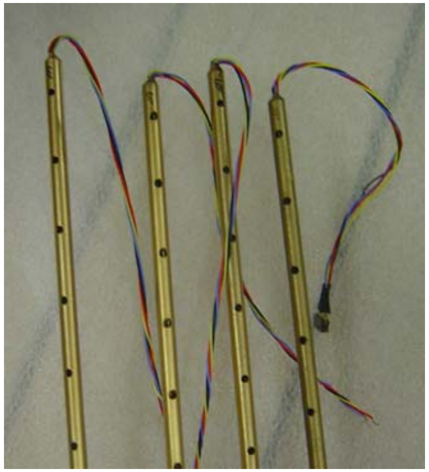
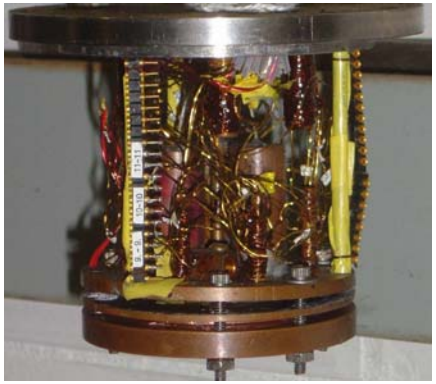


4.1 Instrumentation: Thermometry

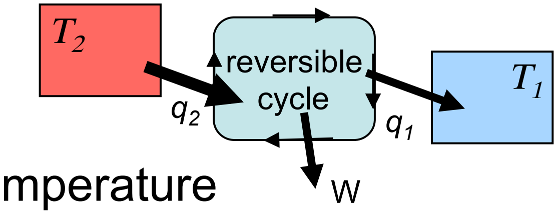


Outline

- Temperature measurement
  - ITS 90
  - Thermocouples
    - Fundamentals
    - Commercial configurations
    - Thermopiles
  - Resistance thermometers
    - Considerations & Options
    - Uncertainties
    - Do’s and don’ts
    - Transient calibration
  - Vapor pressure thermometers

International Temperature Scale 1990

- Temperature is defined in relation to physical (thermodynamic) quantities:
  - Heat flow in a cyclic process: $T_2/T_1 = q_2/q_1$
    - Provided concept of absolute (and zero) temperature
  - associated with 'ideal gas' behavior PV = nRT
    - For a closed constant volume, T P (gas bulb thermometers)



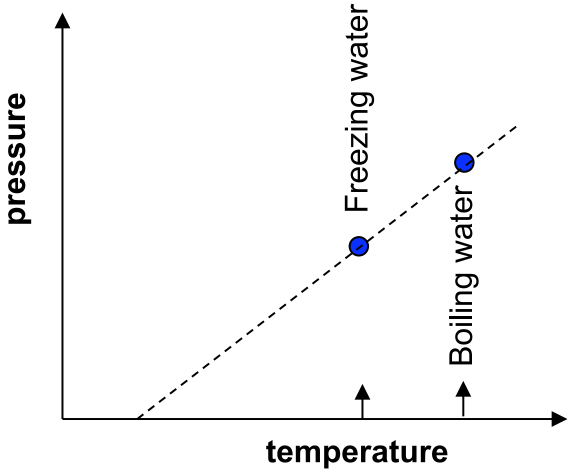
- Definition:
  - Unit of 1 Kelvin = $\frac{1}{273.16}$ of the thermodynamic temperature of the triple point of water
  - T(°C) = T(K) – 273.15 ⇐ (ice point)
- *So far it has proved advantageous to define an empirical scale which can be reproduced accurately and is internationally agreed upon. The best estimate of its difference from the thermodynamic scale is supplied*

Temperature Scales - History

- Guillaume Amontons – 1703: gas thermometer

For a fixed volume
near room
temperature
T = const x P

Suggestion of
coldest possible
temperature, T_o at
zero gas pressure



- Anders Celcius – 1744: defines T = 0° and T = 100°

- John Dalton, Guy-Lussac – 1802: *if we continue the fixed-volume 1°C step decrease in temperature below 0°C, 273.15 steps will bring us to zero pressure*

Temperature Scales - History

- William Thomson – 1848: absolute temperature scale, in the sense that it would be independent of any material properties

$$W = Q\mu\left(T_1 - T_2\right)$$

- Thomson considers Q as constant (water wheel)
- 1851 – James Joule convinces Thomson that

$$W = Q_{hot} - Q_{cold}$$

- 1854 – Joule suggests that $\mu = \dfrac{1}{T_{gas}} \quad \Rightarrow \quad \dfrac{Q_H}{Q_C} = \dfrac{T_H}{T_C}$

$$T_{gas} = \frac{Pv}{R} \quad (absolute\ temperature)$$

ITS 90

- Between 0.65 K and 5.0 K T_{90} is defined by vapor pressure relations of ^{3}He and ^{4}He

$$T_{90}(K) = A_0 + \sum_{i=1}^{9} A_i \left\{ \frac{\ln\left[P(Pa)\right] - B}{C} \right\}^i$$

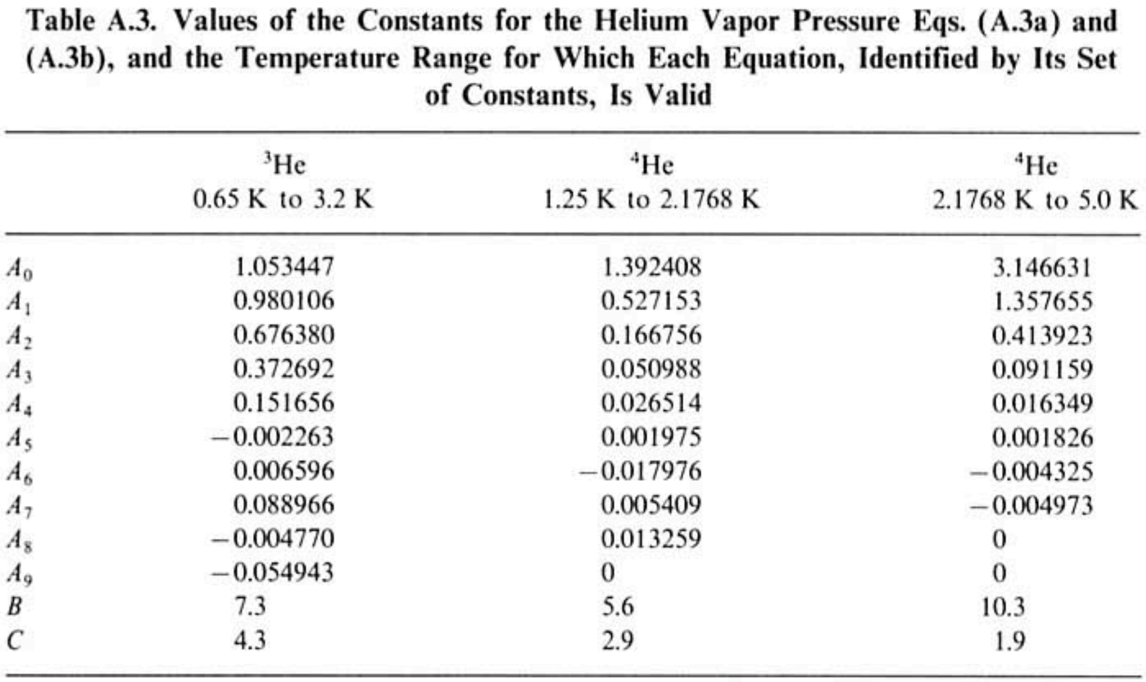
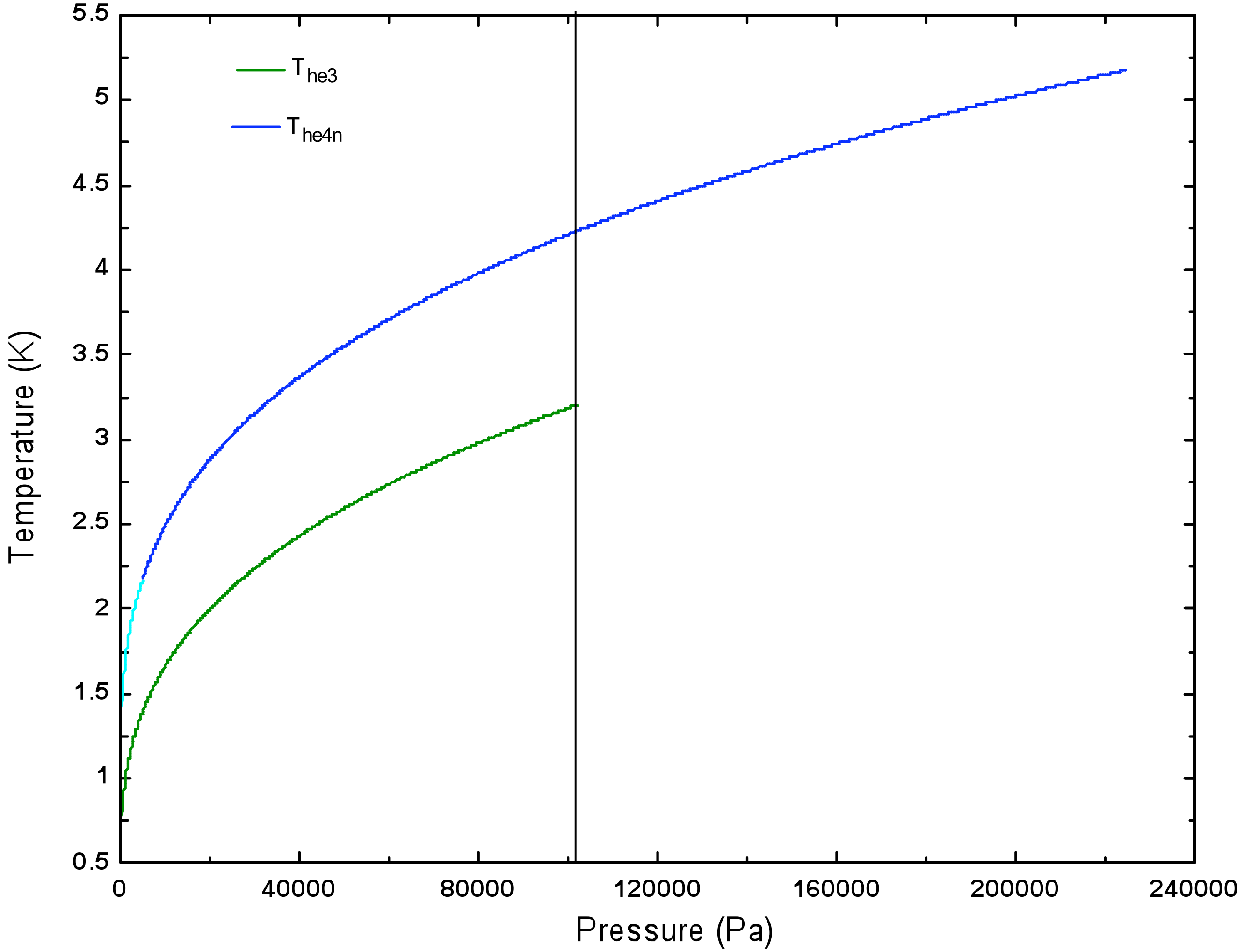


Table A.3. Values of the Constants for the Helium Vapor Pressure Eqs. (A.3a) and (A.3b), and the Temperature Range for Which Each Equation, Identified by Its Set of Constants, Is Valid

	^{3}He 0.65 K to 3.2 K	^{4}He 1.25 K to 2.1768 K	^{4}He 2.1768 K to 5.0 K
A_0	1.053447	1.392408	3.146631
A_1	0.980106	0.527153	1.357655
A_2	0.676380	0.166756	0.413923
A_3	0.372692	0.050988	0.091159
A_4	0.151656	0.026514	0.016349
A_5	−0.002263	0.001975	0.001826
A_6	0.006596	−0.017976	−0.004325
A_7	0.088966	0.005409	−0.004973
A_8	−0.004770	0.013259	0
A_9	−0.054943	0	0
B	7.3	5.6	10.3
C	4.3	2.9	1.9

From Modern Gas-Based Temperature and Pressure Measurements by F. Pavese & G. Molinar
Plenum Press, NY 1992

Helium Vapor Pressure Curves



ITS 90

- Between 3.0 K and the triple point of neon, T_{90} is defined by means of a helium gas thermometer calibrated at three temperatures:
  - Triple point of neon: 24.5561 K
  - Triple point of equilibrium hydrogen: 13.8033 K
  - A temperature between 3.0 K and 5.0 K determined via a ^{3}He or ^{4}He vapor pressure thermometer

$$T_{90} = \frac{a + bp + cp^2}{1 + B_x\left(T_{90}\right)n/V}$$

- Between the triple point of equilibrium hydrogen and the freezing point of silver (961.78 K) T_{90} is defined by means of platinum resistance thermometers calibrated at specified sets of defining fixed points and using specified interpolations procedures.
- Above the freezing point of silver, T_{90} is defined in terms of a defining pixed point and the Planck radiation law.

ITS 90 – fixed points

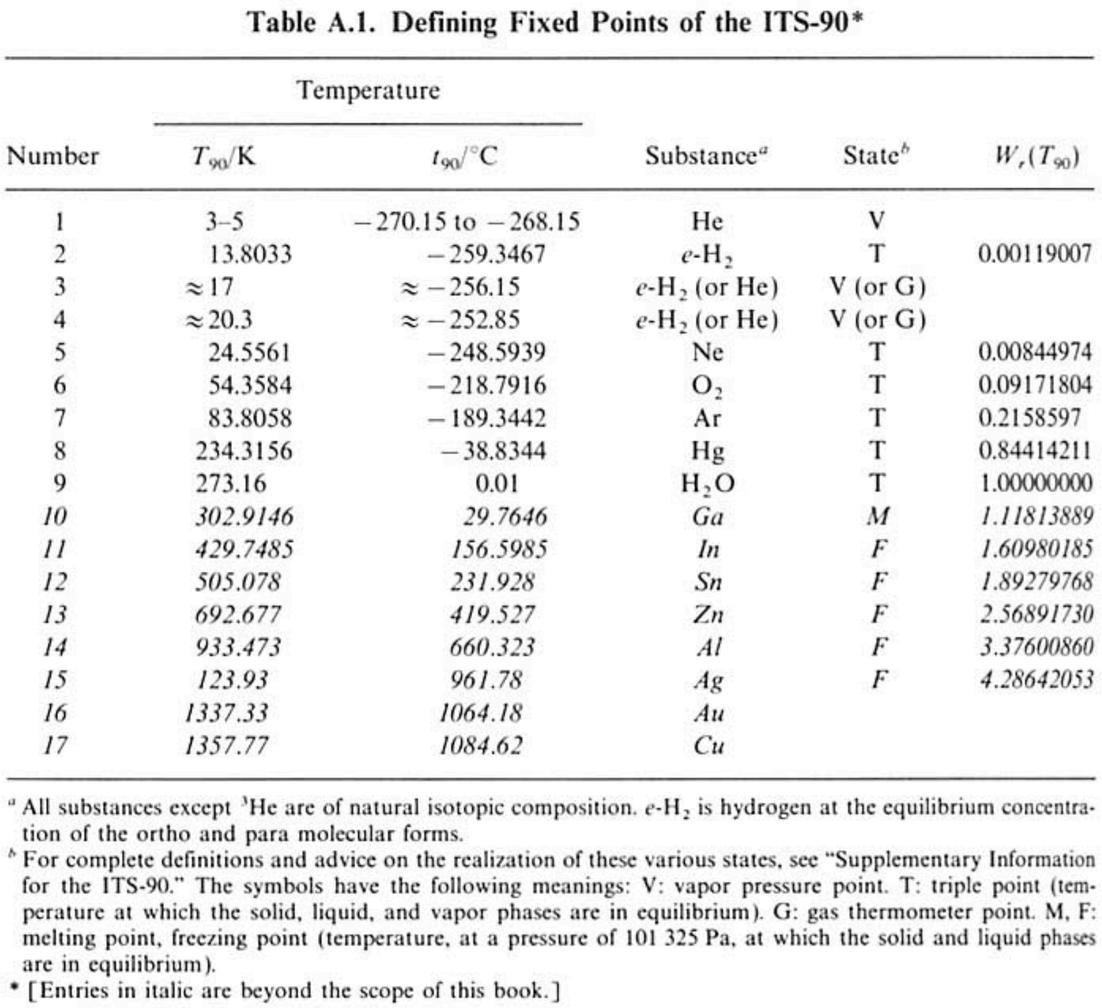


Table A.1. Defining Fixed Points of the ITS-90*

Number	Temperature		Substance[a]	State[b]	$W_r(T_{90})$
	T_{90}/K	t_{90}/°C			
1	3–5	−270.15 to −268.15	He	V	
2	13.8033	−259.3467	e-H_2	T	0.00119007
3	≈17	≈ −256.15	e-H_2 (or He)	V (or G)	
4	≈20.3	≈ −252.85	e-H_2 (or He)	V (or G)	
5	24.5561	−248.5939	Ne	T	0.00844974
6	54.3584	−218.7916	O_2	T	0.09171804
7	83.8058	−189.3442	Ar	T	0.2158597
8	234.3156	−38.8344	Hg	T	0.84414211
9	273.16	0.01	H_2O	T	1.00000000
10	*302.9146*	*29.7646*	*Ga*	*M*	*1.11813889*
11	*429.7485*	*156.5985*	*In*	*F*	*1.60980185*
12	*505.078*	*231.928*	*Sn*	*F*	*1.89279768*
13	*692.677*	*419.527*	*Zn*	*F*	*2.56891730*
14	*933.473*	*660.323*	*Al*	*F*	*3.37600860*
15	*123.93*	*961.78*	*Ag*	*F*	*4.28642053*
16	*1337.33*	*1064.18*	*Au*		
17	*1357.77*	*1084.62*	*Cu*		

[a] All substances except ^{3}He are of natural isotopic composition. e-H_2 is hydrogen at the equilibrium concentration of the ortho and para molecular forms.

[b] For complete definitions and advice on the realization of these various states, see "Supplementary Information for the ITS-90." The symbols have the following meanings: V: vapor pressure point. T: triple point (temperature at which the solid, liquid, and vapor phases are in equilibrium). G: gas thermometer point. M, F: melting point, freezing point (temperature, at a pressure of 101 325 Pa, at which the solid and liquid phases are in equilibrium).

* [Entries in italic are beyond the scope of this book.]

From <u>Modern Gas-Based Temperature and Pressure Measurements</u> by F. Pavese & G. Molinar
Plenum Press, NY 1992

Thermocouples - Fundamentals

- *Seebeck, 1821: Two wires of dissimilar metals joined at both ends display a continuous current if one end is heated. If the circuit is broken, a voltage is established which is a function of the junction temperature and the composition of the metals.*

- *Because temperature measurements via thermocouples are common and deceptively simple, many errors in their use and interpretation are also common. To avoid these, it is helpful to understand the physical principles behind the signal generated by a thermocouple …*

Thermocouples - Fundamentals

- The electric current, J, associated with the flow of electrons is given by:

$$J = \sigma\left(\nabla\tilde{\mu} / e - \alpha\nabla T\right)$$

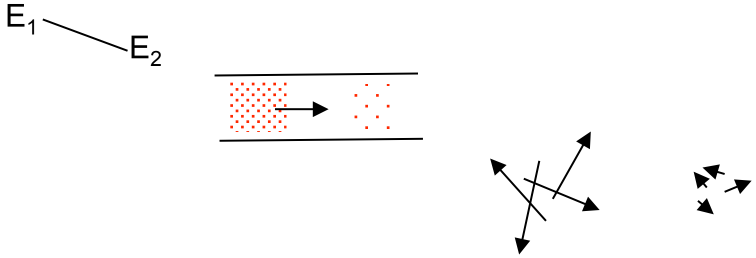
- The thermal current, q, associated with the same flow of electrons is:

$$q = T\alpha J - k\nabla T = \frac{T\sigma\alpha}{e}\nabla\tilde{\mu} - \left(T\sigma\alpha^2 + k\right)\nabla T$$

Here

- α, the Seebeck coefficient, is a measure of the tendency of electric currents to carry heat and for heat currents to induce electrical currents.
- $\tilde{\mu} = \mu - e\phi$, where e is the electric charge, ϕ the electric potential, and μ is the chemical potential (which is a function of composition and temperature).
- σ and k are the electric and thermal conductivity respectively

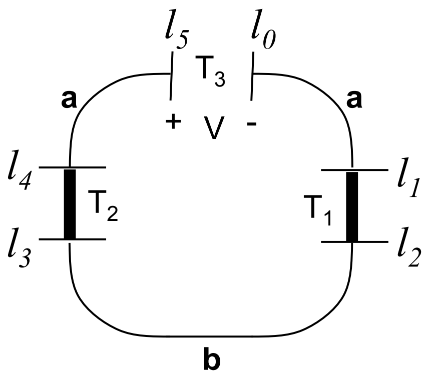
- The net motion of the electrons arises from three different gradients:
  - $\nabla\phi$ (voltage)
  - $\nabla\mu$ (concentration gradient)
  - ∇T (thermal energy gradient)



Thermocouples - Fundamentals

- Consider the circuit as shown connected to a potentiometer: the two ‘leads’ have the same composition and temperature, therefore they have the same chemical potential

$$\mu(l_5) = \mu(l_0) \quad or \quad \tilde{\mu}(l_5) - \tilde{\mu}(l_0) = -e\left[\phi(l_5) - \phi(l_0)\right]$$



- With zero current flow, J = 0, we have for any position along the path from l_0 to l_5:

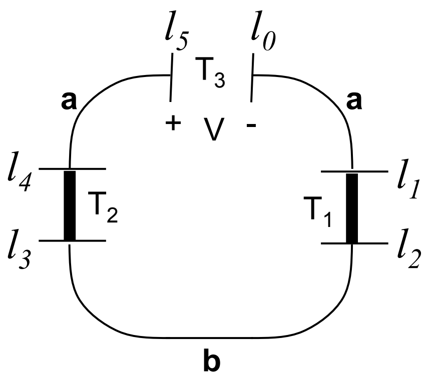
$$\tilde{\mu}(l) = \tilde{\mu}(l_0) + e\int_0^l \alpha(l,T)\frac{dT}{dl}dl$$

- Combining the above two equations, we have:

$$\phi(l_5) - \phi(l_0) = -\int_{l_0}^{l_5} \alpha(l,T)\frac{dT}{dl}dl$$

- Note that an open circuit voltage arises from regions where $\frac{dT}{dl} \neq 0$

Thermocouples



- Voltage difference between points 0 and 5:

$$\phi(l_5)-\phi(l_0) = -\int_{0}^{l_5} \alpha(l,T)\frac{dT}{dl}\,dl$$

- open circuit voltage arises from regions of dT/dl ≠ 0

$$\phi(l_5)-\phi(l_0) = -[\int_{0}^{l_1}\alpha_a(T)\frac{dT}{dl}\,dl + \int_{l_1}^{l_2}\alpha(l,T)\cancelto{0}{\frac{dT}{dl}}\,dl +$$

$$\int_{l_2}^{l_3}\alpha_b(T)\frac{dT}{dl}\,dl + \int_{l_3}^{l_4}\alpha(l,T)\cancelto{0}{\frac{dT}{dl}}\,dl + \int_{l_4}^{l_5}\alpha_a(T)\frac{dT}{dl}\,dl]$$

Note that

$$\int_{T_3}^{T_1}\alpha_a dT + \int_{T_2}^{T_3}\alpha_a dT$$

$$= \int_{T_1}^{T_2} -\alpha_a dT$$

- Note that although α_a and α_b are known, $\alpha(l,T)$ in the joint is unknown. Thus, the joints must be in regions where dT/dl = 0. Then:

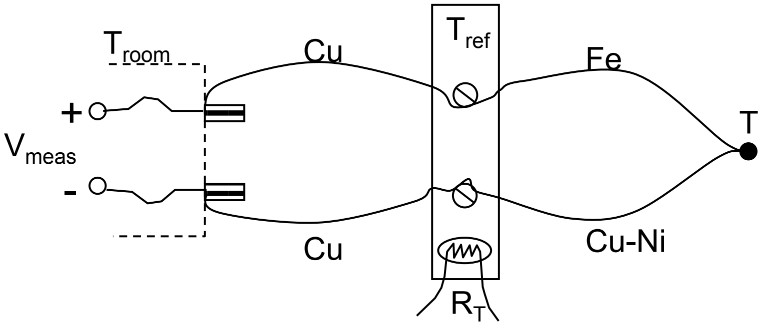
$$\phi(l_5)-\phi(l_0) = -\int_{T_1}^{T_2}[\alpha_b(T)-\alpha_a(T)]dT$$

$$= \int_{T_1}^{T_2}[\alpha_a(T)-\alpha_b(T)]dT = \int_{T_1}^{T_2}\alpha_{ab}(T)dT$$

- $\int_0^{T_2}\alpha_{ab}(T)dT$ is found in the tables! It represents the difference in the voltage generated by material a and material b, both spanning the temperatures 0°C and T_2.

Commercial Configurations

- Software compensation:



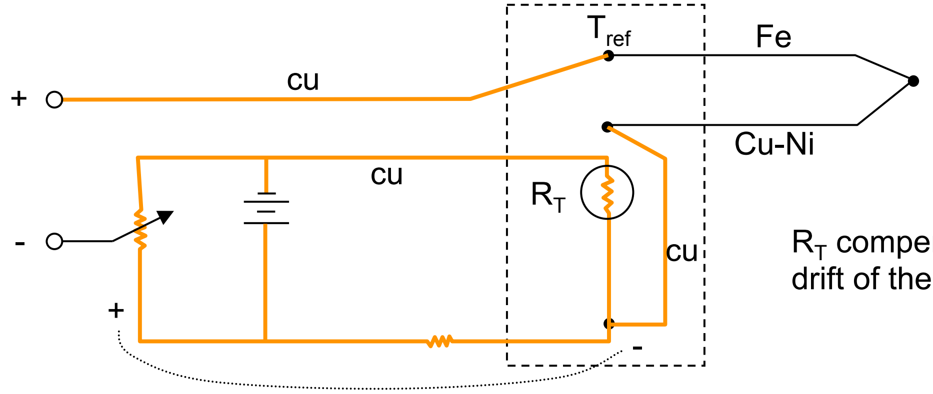
$$V_{meas} = \int_{T_{room}}^{T_{ref}} \alpha_{cu}\, dT + \int_{T_{ref}}^{T} \alpha_{Fe,C}\, dT + \int_{T_{ref}}^{T_{room}} \alpha_{cu}\, dT = \int_{T_{ref}}^{T} \alpha_{Fe,C}\, dT$$

$$\underbrace{\int_{T_{ref}}^{T} \alpha_{Fe,C}\, dT}_{V_{meas}} = \underbrace{\int_{T_o}^{T} \alpha_{Fe,C}\, dT}_{V(T)} - \underbrace{\int_{T_o}^{T_{ref}} \alpha_{Fe,C}\, dT}_{V_{ref}\ \text{(known)}}$$

- Measure R_T to obtain $T_{ref,}$ and then, V_{ref}
- Solve for V(T), use tables to determine T

Commercial Configurations

- Hardware compensation

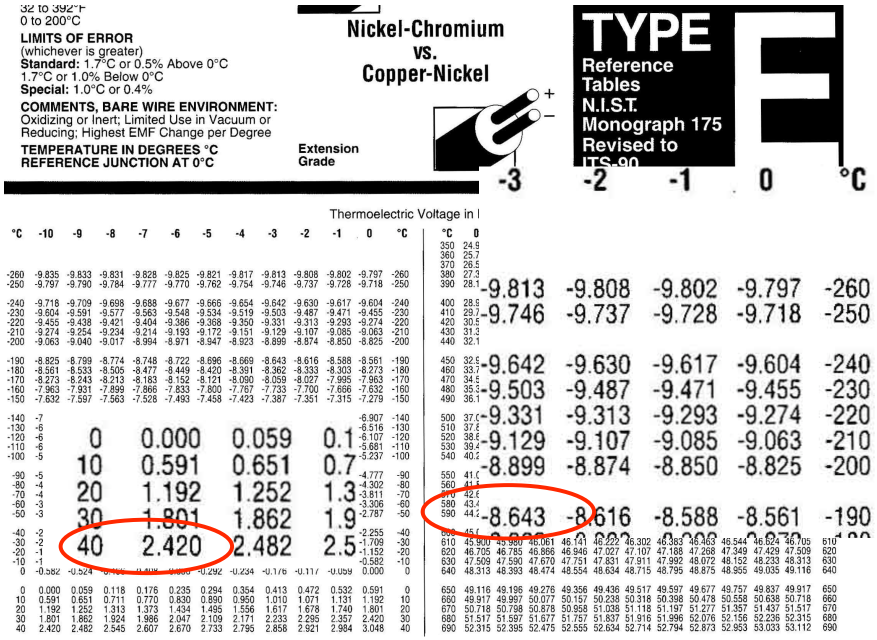


R_T compensates for temperature drift of the reference plate

- A specific voltage is created to buck, or cancel, V_{ref} allowing V(T) to be read directly
  - Advantage: faster than software compensation
  - Disadvantage: compensation voltage is specific to only one type of thermocouple wire at a time

Thermocouple - Example

- A type E thermocouple is used to measure T = 80 K with T_{ref} = 40 °C, (313 K)
  - What voltage is measured at the meter?
  - What is the reference voltage?
  - If the voltage resolution is 0.01 mV, what is the temperature resolution?



32 to 392°F
0 to 200°C

LIMITS OF ERROR
(whichever is greater)
Standard: 1.7°C or 0.5% Above 0°C
1.7°C or 1.0% Below 0°C
Special: 1.0°C or 0.4%

COMMENTS, BARE WIRE ENVIRONMENT:
Oxidizing or Inert; Limited Use in Vacuum or Reducing; Highest EMF Change per Degree

TEMPERATURE IN DEGREES °C
REFERENCE JUNCTION AT 0°C

Nickel-Chromium
vs.
Copper-Nickel

Extension Grade

TYPE E
Reference Tables
N.I.S.T. Monograph 175
Revised to ITS-90

Thermoelectric Voltage in

°C	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	0	°C
-260	-9.835	-9.833	-9.831	-9.828	-9.825	-9.821	-9.817	-9.813	-9.808	-9.802	-9.797	-260
-250	-9.797	-9.790	-9.784	-9.777	-9.770	-9.762	-9.754	-9.746	-9.737	-9.728	-9.718	-250
-240	-9.718	-9.709	-9.698	-9.688	-9.677	-9.666	-9.654	-9.642	-9.630	-9.617	-9.604	-240
-230	-9.604	-9.591	-9.577	-9.563	-9.548	-9.534	-9.519	-9.503	-9.487	-9.471	-9.455	-230
-220	-9.455	-9.438	-9.421	-9.404	-9.386	-9.368	-9.350	-9.331	-9.313	-9.293	-9.274	-220
-210	-9.274	-9.254	-9.234	-9.214	-9.193	-9.172	-9.151	-9.129	-9.107	-9.085	-9.063	-210
-200	-9.063	-9.040	-9.017	-8.994	-8.971	-8.947	-8.923	-8.899	-8.874	-8.850	-8.825	-200
-190	-8.825	-8.799	-8.774	-8.748	-8.722	-8.696	-8.669	-8.643	-8.616	-8.588	-8.561	-190
-180	-8.561	-8.533	-8.505	-8.477	-8.449	-8.420	-8.391	-8.362	-8.333	-8.303	-8.273	-180
-170	-8.273	-8.243	-8.213	-8.183	-8.152	-8.121	-8.090	-8.059	-8.027	-7.995	-7.963	-170
-160	-7.963	-7.931	-7.899	-7.866	-7.833	-7.800	-7.767	-7.733	-7.700	-7.666	-7.632	-160
-150	-7.632	-7.597	-7.563	-7.528	-7.493	-7.458	-7.423	-7.387	-7.351	-7.315	-7.279	-150
-140	-7										-6.907	-140
-130	-6										-6.516	-130
-120	-6										-6.107	-120
-110	-6										-5.681	-110
-100	-5										-5.237	-100
-90	-5										-4.777	-90
-80	-4										-4.302	-80
-70	-4										-3.811	-70
-60	-3										-3.306	-60
-50	-3										-2.787	-50
-40	-2										-2.255	-40
-30	-2										-1.709	-30
-20	-1										-1.152	-20
-10	-1										-0.582	-10
0	-0.582	-0.524				-0.292	-0.234	-0.176	-0.117	-0.059	0.000	0
0	0.000	0.059	0.118	0.176	0.235	0.294	0.354	0.413	0.472	0.532	0.591	0
10	0.591	0.651	0.711	0.770	0.830	0.890	0.950	1.010	1.071	1.131	1.192	10
20	1.192	1.252	1.313	1.373	1.434	1.495	1.556	1.617	1.678	1.740	1.801	20
30	1.801	1.862	1.924	1.986	2.047	2.109	2.171	2.233	2.295	2.357	2.420	30
40	2.420	2.482	2.545	2.607	2.670	2.733	2.795	2.858	2.921	2.984	3.048	40

°C	0											
350	24.9											
360	25.7											
370	26.5											
380	27.3											
390	28.1											
400	28.9											
410	29.7											
420	30.5											
430	31.3											
440	32.1											
450	32.9											
460	33.7											
470	34.5											
480	35.3											
490	36.1											
500	37.0											
510	37.8											
520	38.6											
530	39.4											
540	40.2											
550	41.0											
560	41.8											
570	42.6											
580	43.4											
590	44.2											
600	45.0											
610	45.900	45.980	46.061	46.141	46.222	46.302	46.383	46.463	46.544	46.624	46.705	610
620	46.705	46.785	46.866	46.946	47.027	47.107	47.188	47.268	47.349	47.429	47.509	620
630	47.509	47.590	47.670	47.751	47.831	47.911	47.992	48.072	48.152	48.233	48.313	630
640	48.313	48.393	48.474	48.554	48.634	48.715	48.795	48.875	48.955	49.035	49.116	640
650	49.116	49.196	49.276	49.356	49.436	49.517	49.597	49.677	49.757	49.837	49.917	650
660	49.917	49.997	50.077	50.157	50.238	50.318	50.398	50.478	50.558	50.638	50.718	660
670	50.718	50.798	50.878	50.958	51.038	51.118	51.197	51.277	51.357	51.437	51.517	670
680	51.517	51.597	51.677	51.757	51.837	51.916	51.996	52.076	52.156	52.236	52.315	680
690	52.315	52.395	52.475	52.555	52.634	52.714	52.794	52.873	52.953	53.033	53.112	690

0	0.000	0.059	0.1
10	0.591	0.651	0.7
20	1.192	1.252	1.3
30	1.801	1.862	1.9
40	2.420	2.482	2.5

-3	-2	-1	0	°C
-9.813	-9.808	-9.802	-9.797	-260
-9.746	-9.737	-9.728	-9.718	-250
-9.642	-9.630	-9.617	-9.604	-240
-9.503	-9.487	-9.471	-9.455	-230
-9.331	-9.313	-9.293	-9.274	-220
-9.129	-9.107	-9.085	-9.063	-210
-8.899	-8.874	-8.850	-8.825	-200
-8.643	-8.616	-8.588	-8.561	-190

T = 80 K = -193 °C

$V_{meas} = V(T) - V_{ref} = -8.643 - 2.420 = -11.063$ mV

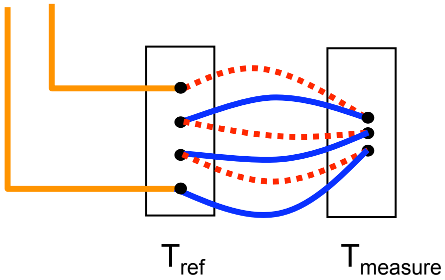
$V_{ref} = 2.420$ mV

@ 80 K, dV/dT = 26.8 μV/K

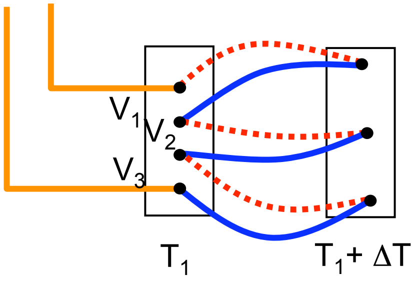
With dV = 10 μV, then dT = 0.37 K

Thermopiles

- To increase signal:



  - A series connection of ‘n’ pairs
    - produces n times the emf
    - Reduces temperature error by ~ $\frac{1}{\sqrt{n}}$
- To determine a spatially averaged temperature



$$V_1 = \alpha\left[\left(T_1 + \Delta T - \delta\right) - T_1\right]$$

$$V_2 = \alpha\left[\left(T_1 + \Delta T\right) - T_1\right]$$

$$V_3 = \alpha\left[\left(T_1 + \Delta T + \delta\right) - T_1\right]$$

$$V_{total} = 3\alpha\Delta T$$

Cryogenic Thermocouples

- Commonly used types for cryogenic temperatures:
  - Type E: Ni-Cr, Cu-Ni (constantan)
    - Highest α of types E, K, T (best down to 40 K)
    - Low thermal conductivity
  - Type K: Ni-Cr, Ni-Al
    - 8.5 μV/K @ 20 K (vs. 4.1 μV/K for type E)
  - Type T: Cu, Cu-Ni
  - Ag-Fe: high thermal power, but power decreases with time if stored at room temperature
- Important notes regarding use of thermocouples:
  - Voltage arises from region where $dT/dl \neq 0$
  - Joints must be made in regions where $dT/dl = 0$
  - If used in presence of magnetic fields, ensure that along the TC path, temperature is constant in regions of changing field, or field is constant in regions of changing temperature. ($\alpha = \alpha(H,T)$)
  - Minimize number of joints
  - Avoid dissimilar material joints at instrumentation feed-thru's
  - Heat sink TC wire before reaching the point of measurement

Thermometers - Considerations

- Temperature range
- Type of signal: voltage, capacitance
- Temperature sensitivity: change in signal per change in temperature
- Response time: size, thermal mass
- Mounting package
- Magnetic field sensitivity
- Strain sensitivity
- Repeatability (thermal cycling)
- Long term stability
- Radiation resistance
- Calibration
- Excitation requirement
- Cost

Thermometers - options

- Diodes (semiconductors): Si, Gas, GAlAs
  - Temperature dependent forward bias voltage
  - Small, fast response
  - Constant current source (10μA)
  - Very field dependent
  - Moderate sensitivity over large T-range
- PTC resistors (metal): Pt, Rh-Fe
  - Positive temperature coefficient
  - Very stable
  - Large size, slow response
  - Sensitive to magnetic fields
  - Fairly good sensitivity
  - Strain sensitive
- NTC resistors (semiconductors): CGR, GR, CR, RuO_2, Cernox™
  - Negative temperature coefficient
  - High sensitivity over limited temperature range (CGR, GR, CR)
  - Negligible field dependence (CGR, Cernox™, RuO_2), large field dependence (GR)
  - Strain sensitive, thus encapsulated, with thermal sensing through the leads
  - Moderate response
- Capacitors
  - Insensitive to magnetic field
  - Sensing circuit requires care and attention

Resistance Thermometers

- Which thermometers would you choose for the following situations?
  - Winding of Tevatron magnet:
    Cernox™, CGR, Rox™
  - Fluids experiment in helium II:
    CGR, GR, Cernox™
  - Characterize performance of LH_2 liquefier:
    GR, Rh-Fe, Cernox™
  - Cool-down study of an 80 K cryocooler:
    Pt, TC, Si-diode, Cernox™

Thermometers

- Factors contributing to uncertainty:
  - Sensor sensitivity:

$$S_T = \frac{\%\,change\ in\ signal}{\%\ change\ in\ T} \equiv dimensionless\ sensitivity$$

  - Voltmeter uncertainty

$$\frac{U_{T,V}}{T} = \frac{U_V/V}{S_T} = \%uncertainty\ in\ V \bullet \frac{\%change\ in\ T}{\%change\ in\ V}$$

  - Current source uncertainty

$$\frac{U_{T,I}}{T} = \frac{(U_I/I)}{S_T}$$

  - Calibration uncertainty – see mfc.
  - Thermal noise – usually negligible
  - Electromagnetic noise: $emf = \frac{dB}{dt} \bullet A$
    - •Twisted pairs
    - •Shielding – connect shield at one end only – preferably at signal source

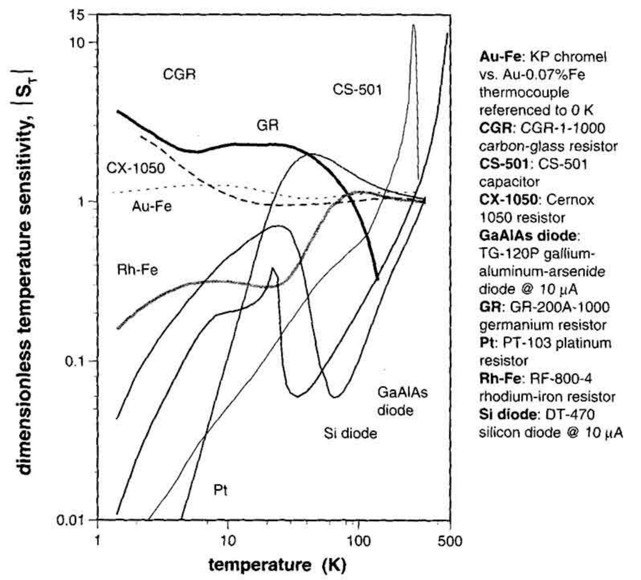
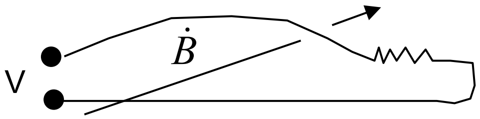
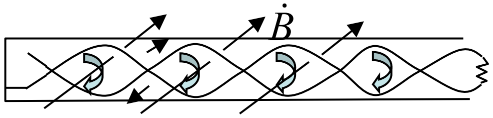


Figure 1. Absolute dimensionless temperature sensitivities of representative Lake Shore sensors.

Problem:



Solution:



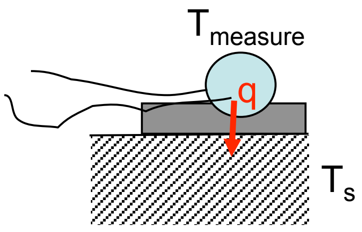
- Combined total uncertainty: $U_T = \left[(U_{T,V})^2 + (U_{T,I})^2 + (U_{T,Cal})^2 + (U_{T,therm\_noise})^2\right]^{1/2}$

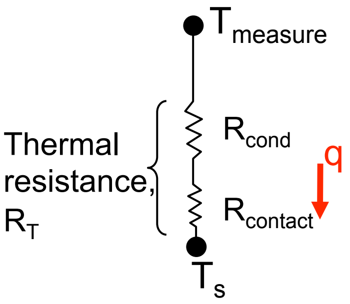
Thermometers

- Factors contributing to error (i.e. bias)

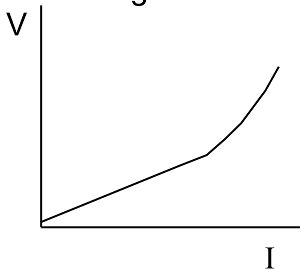
  "A thermometer always indicates a temperature intermediate between that of the region being investigated and any other environment with which the thermometer has thermal communication."

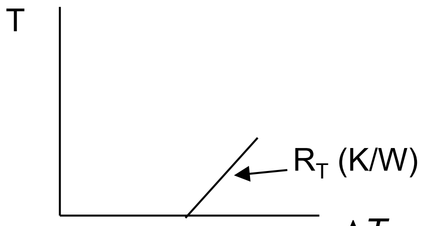
  - Self heating





As the current is increased, if T is no longer constant, then ρ is no longer constant

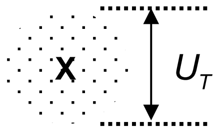




$$q(W) = I^2 R_e = \frac{\Delta T}{R_T}$$

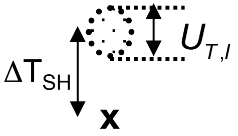
A compromise must be made between signal uncertainty and self-heating error

Low I:



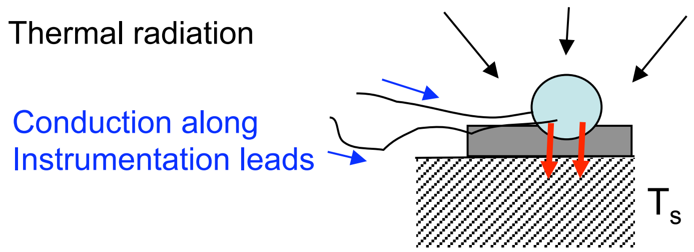
$$U_{T,I} = T\frac{U_I/I}{S_T}$$

High I:



Thermometers:
error factors (cont)

- Parasitic heat leak



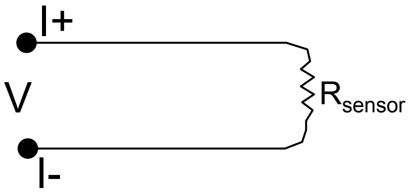
Example: 1 pair of 26 AWG (d=0.4 mm) copper wire, L = 1 m, ΔT = 220 K

$$q_{cond} = \frac{A}{L}\int k\left(T\right)dT = 2\frac{\pi\left(4x10^{-4}\right)^2}{4}92x10^3 = 23\,mW$$

$\Rightarrow$ ΔT = 230 mK, when $R_T \sim 10$ K/W

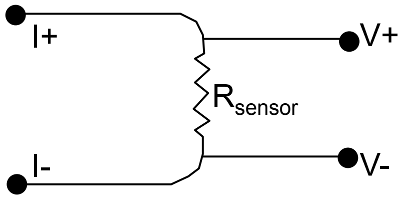
- Lead resistance

Problem:



$V = I(R_{sensor} + R_{leads})$

Solution: 4-wire connection



Thermometers:

error factors (cont)

- Thermal emf

Problem: the Seebeck coefficient of different materials, in regions of ΔT produces a thermal emf, even when no current is flowing

Solution: reversing polarity, or multiple current levels

$$V = \left[\left(V_1 + V_{emf}\right) - \left(-V_1 + V_{emf}\right)\right]/2 \qquad \left(reverse\ polarity\right)$$

$$V = \left[\left(I_1 R + V_{emf}\right) - \left(I_2 R + V_{emf}\right)\right] \qquad \left(known\ values\ of\ I\right)$$

$$R = \frac{(V)}{(I_1 - I_2)}$$

Thermometers: Do’s & Don’ts

- Thermally anchor leads as close to measurement temperature as possible (5 -10 cm length)
- Use twisted, shielded leads to minimize electromagnetic noise (connect shield at one end only)
- Minimize conduction heat load by using long lengths, small diameters, low thermal conductivity materials
- Follow recommended excitation levels to avoid self heating
- Isolate low-level signal leads from high-level signal leads
- Reverse polarity to cancel thermal emf components

Transient Calibration

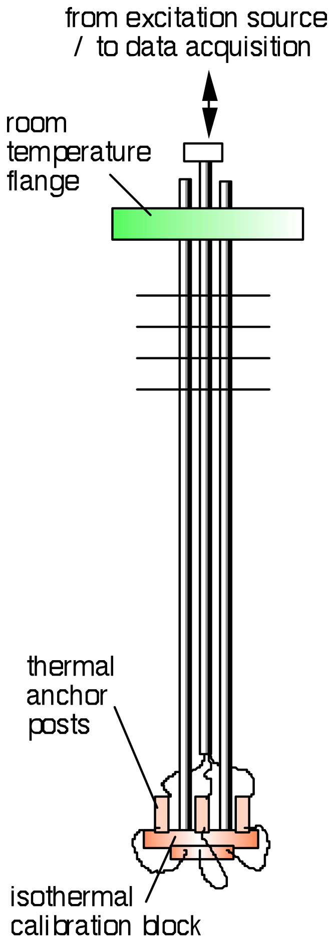
- Motivation
  - Calibration via multiple ‘steady state’ reference temperature points requires significant time (and cryogen resources)
  - For some applications extreme temperature resolution is not required
- Idea
  - Match resistance of un-calibrated sensors with temperatures determined from calibrated sensors through matched transient temperature sweep
- Theoretical foundation
  - Solids undergoing a transient temperature change may be modeled as an isothermal mass under the “lumped capacitance” approximation if Bi << 0.1.

    h = convective heat transfer coefficient

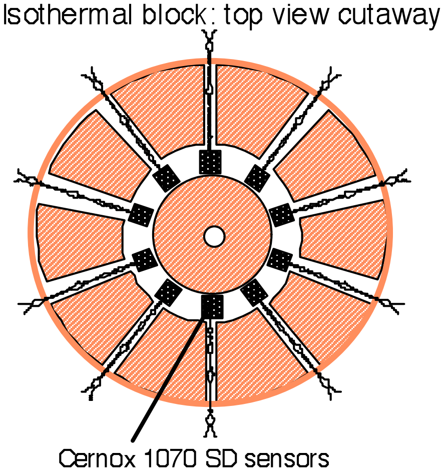
    k = thermal conductivity

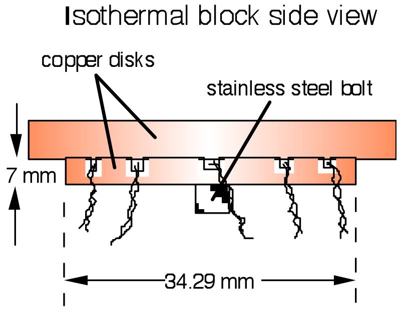
    L = length scale = volume/surface area
  - The time dependence of the temperature change is governed by $\tau = mC/hA$, but the spatial dependence of temperature at any instant in time is governed by the Biot number alone.

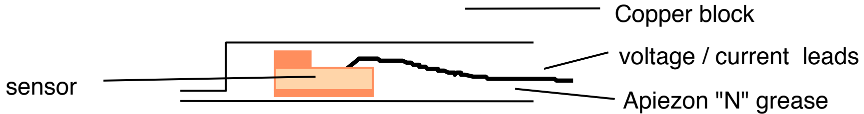
Transient Calibration: Procedure



- Mount calibrated and uncalibrated sensors in an “isothermal” copper block.
- Cool to cryogenic (liquid helium) temperature.
- Apply 0 - 10μA square wave excitation signal and begin data acquisition.
- Extract calibration rig from dewar smoothly over ~ 60 minute time span.





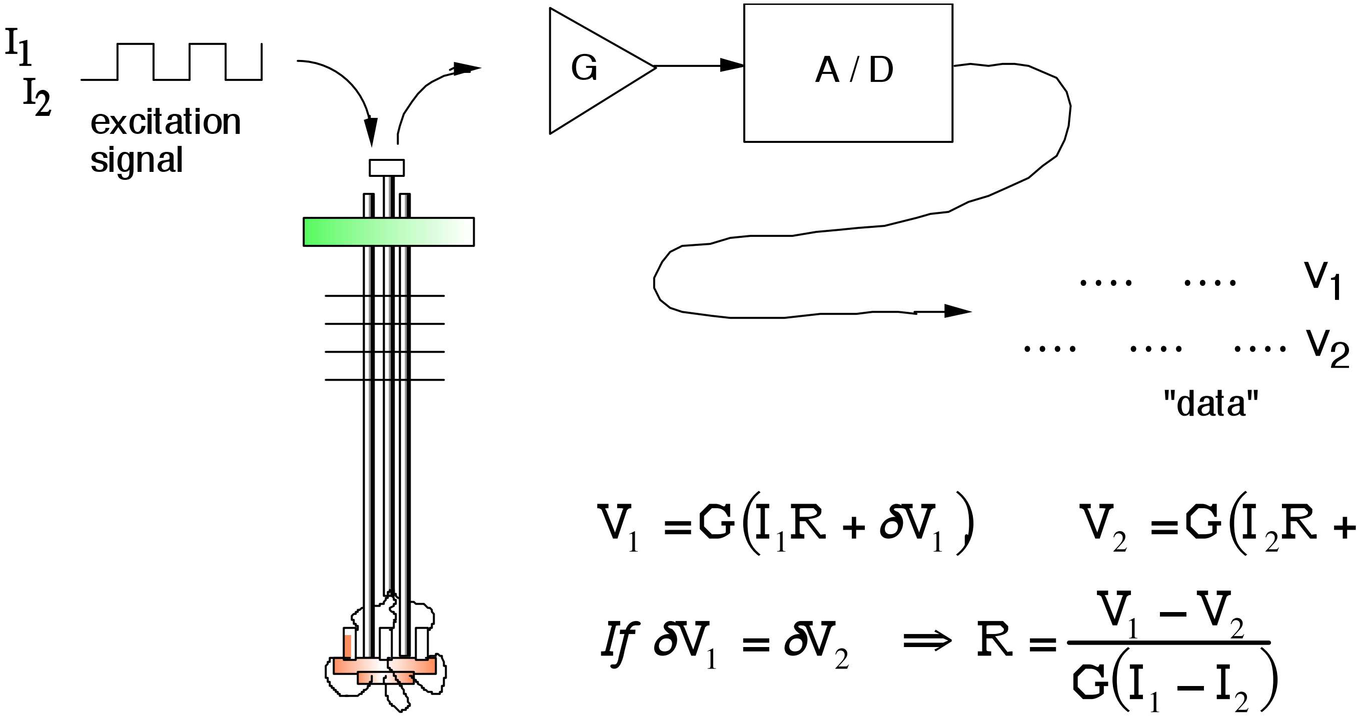


Mounting detail

In the case of the Cernox $^{\text{TM}}$ 1070 SD sensor, the primary heat transfer is through the base, rather than through the leads

Transient Calibration: Eliminating Parasitic Voltages

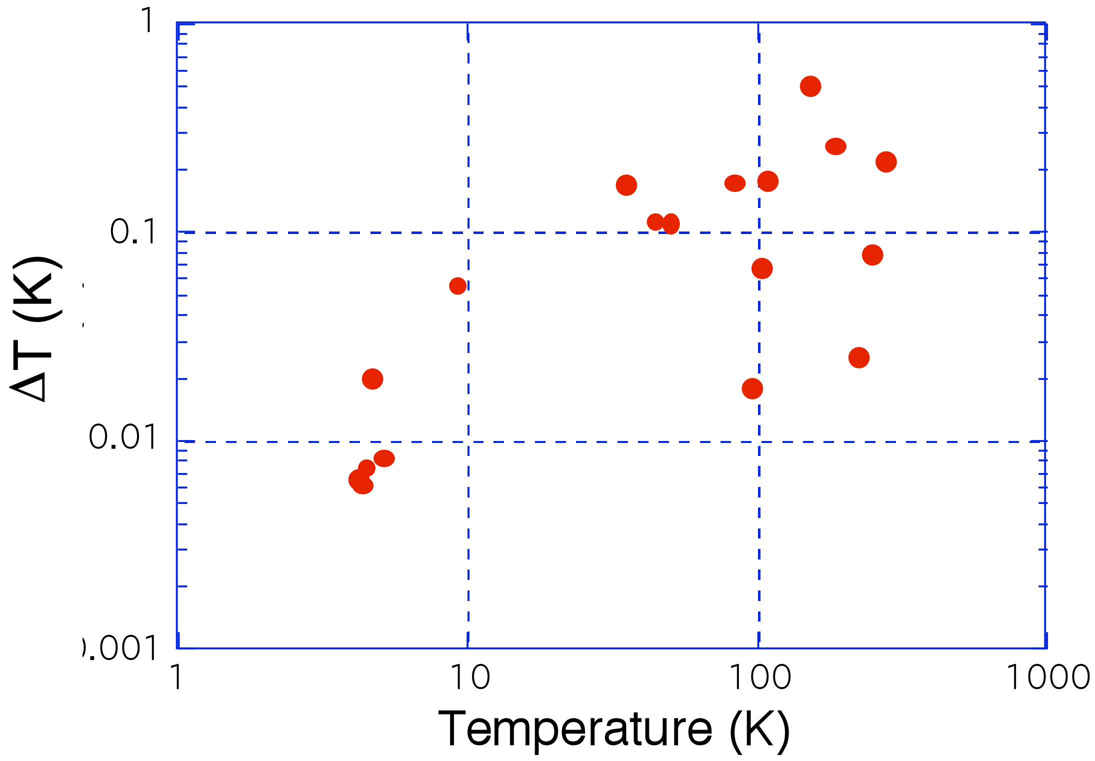
- Apply square wave excitation signal: 0 - 10 µA, 0.1 Hz
  - self heating test w/ 10 µA revealed temperature offset at 4.2K is < 1 mK
  - Frequency must be large enough to avoid changes in δV over one period



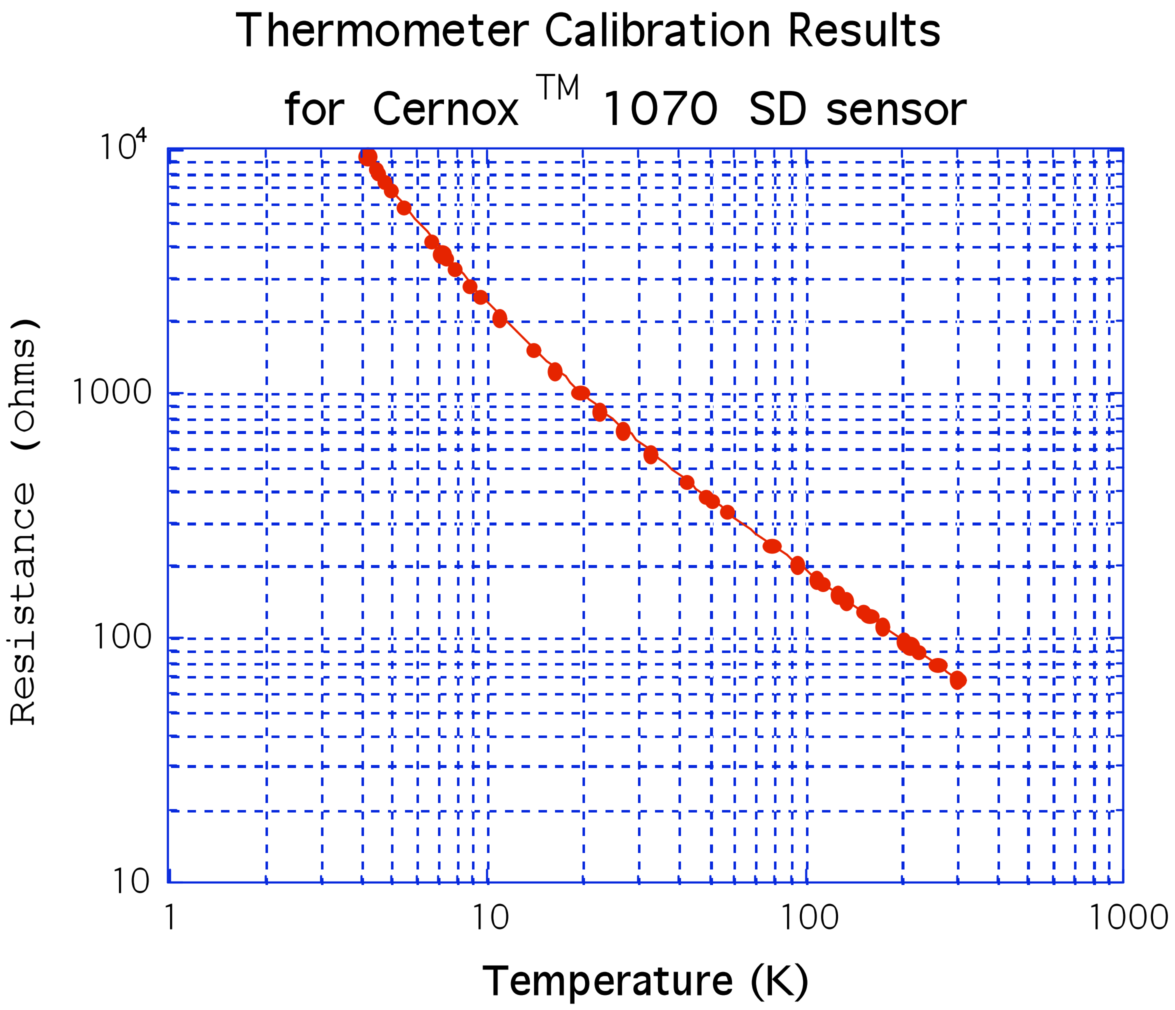
$$V_1 = G(I_1 R + \delta V_1) \qquad V_2 = G(I_2 R + \delta V_2)$$

$$\textit{If}\ \delta V_1 = \delta V_2 \quad \Rightarrow \quad R = \frac{V_1 - V_2}{G(I_1 - I_2)}$$

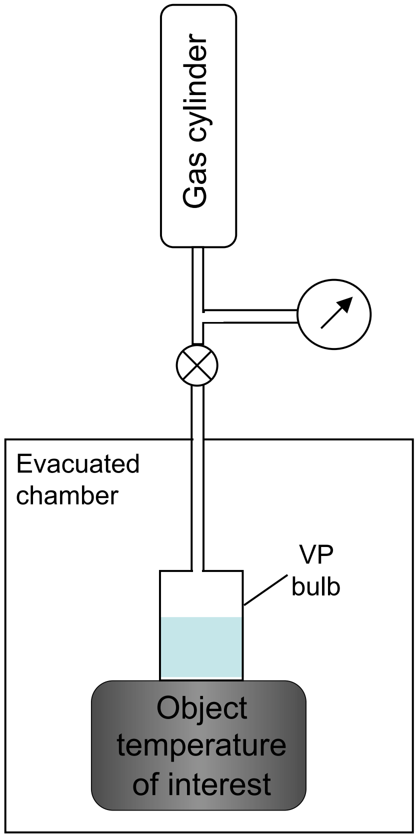
Transient Calibration: Measured ΔT between calibrated sensors



Transient Calibration: Sample Results



Vapor Pressure Thermometry



- Condense gas into bulb (maintain free surface)
- Measure pressure at room temperature (pressure is determined by coldest location)
- Use vapor pressure relationship to determine temperature
- Problems to avoid:
  - Loss of free surface
  - Cold-spots along capillary
  - Thermo-molecular pressure gradient in capillary

Vapor Pressure Thermometry

- Ensure free surface:
  - Maintain more than ~ 10% fill at T_{max} (P_{max}): *the problem to avoid is that at some T' < T_{max} the saturated pressure will be such that all moles of the liquid will evaporate to produce the saturated pressure. Above this temperature the bulb will behave as a 'gas bulb' rather than as a vapor pressure bulb.*
  - Let $N_t = N_a + N_b + N_c + N_L$ = total # of moles in the system (1)
    - N_a = # moles in ambient plumbing, storage vessel

      $$N_a = (P_{\max} V_a)/(RT_a)$$

    - N_b = # moles of gas in the vapor pressure bulb

      $$N_b = 0.9*(P_{\max} V_b)/\left[RT_{\max}\left(1 + B(T)P_{\max}\right)\right] \quad or \quad 0.9*(\rho_{gas} V_b)/M$$

      - B = 1st virial coefficient, M = molecular weight
    - N_c = # moles of gas in the capillary

      $$N_c = (P_{\max} V_c)/(RT_{c,ave})$$

    - N_L = # moles of liquid in the vapor pressure bulb

      $$N_L = 0.1*\left[\rho_{liquid}(T_{\max})V\right]/M$$

Vapor Pressure Thermometry

- Ensure free surface (cont.)
  - Must also ensure that at $T_{min}(P_{min})$, the bulb is not full of liquid

$$N_t - \frac{P_{\min}}{R}\left(\frac{V_a}{T_a} + \frac{V_c}{T_{c,ave}}\right) < \frac{\rho_L\left(T_{\min}\right)V_b}{M} \qquad (2)$$

  - Conditions (1) and (2) define allowable range for N_t and P_{charge}
  - Before cooldown, $N_t = (P_{charge}V_t)/(RT_a)$
- Size of capillary
  - It is usually desirable to make the capillary small (heat leak, uncertainty with $T_{c,ave}$)
  - But, the mean free path length $\lambda = \frac{kT_a}{\sqrt{2}\pi\delta^2 P_{\min}} \ll D_{capillary}$ to avoid the molecular flow regime.
  - Rarely a problem. At 1.5 K, $P_{sat,4He}$ ~ 500 Pa

$$\lambda = \frac{1.38x10^{-23}\,J}{\sqrt{2}\pi \quad K}\;\frac{300\,K}{\left(2.18x10^{-10}\,m\right)^2}\;\frac{m^3}{500\,J} = 4x10^{-5}\,m \approx 0.0015''$$

High Resolution Thermometry (JPL)

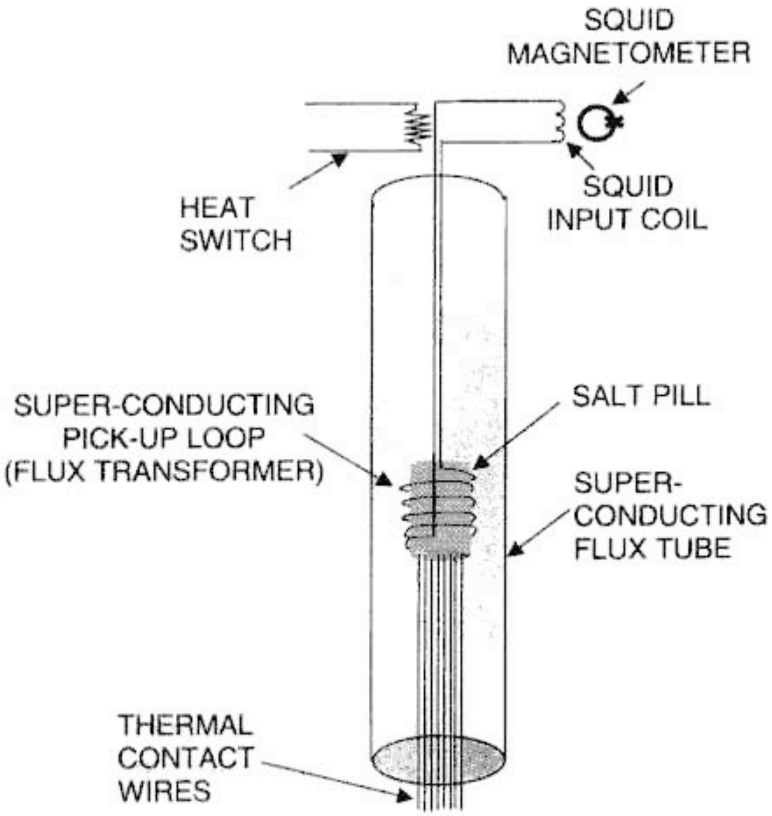


Fig. 1. Components of a high-resolution thermometer.