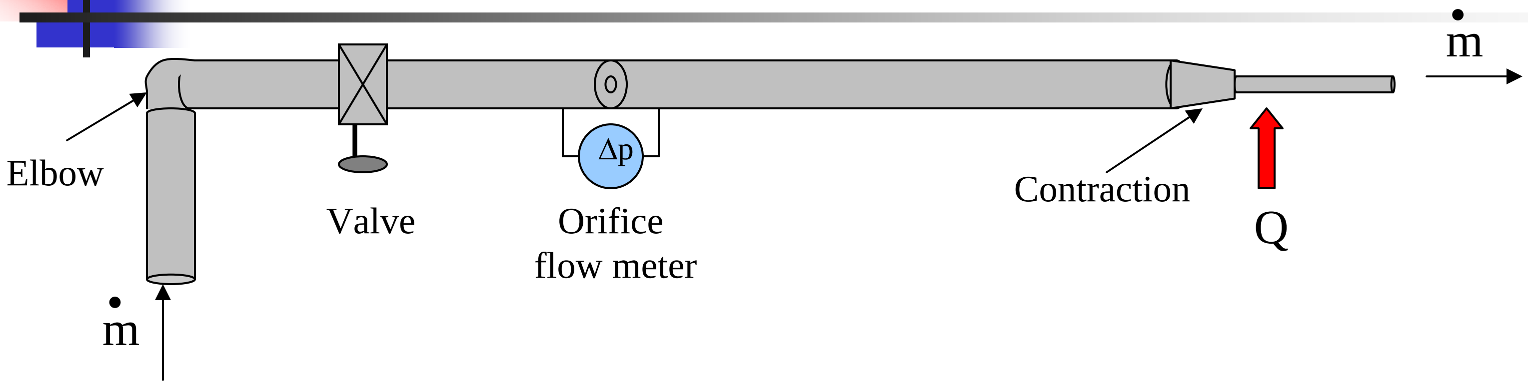
3.1 Cryogenic Fluid Mechanics

- Fluid flow commonly occurs in most cryogenic systems
  - Refrigeration systems: gas cycles including flow through heat exchangers
  - Fluid distribution systems: transfer lines, actively cooled thermal shields
  - Natural and forced circulation systems for large devices (magnets)
  - Forced flow superconductors for large magnets
- Cryogenic systems frequently experience a variety of fluid flow conditions:
  - Single phase, subcooled liquid (incompressible)
  - Compressible fluid flow (gases)
  - Two phase flow (liquid + vapor co-existing)
- Most fluid dynamics issues are "classical" in nature although compressible and two phase flow are common

Typical 1 D flow problem



- Input variables: $\dot{m}$ (mass flow rate), Q (heat load), inlet T & P
- Physical dimensions: length (L), tube diameters, other components (flow meters, valves, tube bends, etc.)
- Output Variables: outlet pressure & temperature, phases
- <u>Fluid dynamics problem</u>: determine the pressure drop (Δp) and adiabatic temperature change under given conditions
- <u>Heat transfer problem</u>: determine the temperature of the fluid and tube wall for given heat transfer rate, Q. The heat transfer coefficient (h) vs. flow conditions
- In most cases, these two problems are coupled (i.e. must be solved simultaneously)

One phase, incompressible flow

Hydraulic diameter

- Pressure drop: $\Delta p = 4 f_d \frac{L}{D_h}\left(\frac{1}{2}\rho u^2\right)$ where $D_h = \frac{4A_{flow}}{P}$
- f_d is the "Fanning Friction Factor", which depends on $\underbrace{\text{Reynolds number}}_{}$:

$$\mathrm{Re}_D \equiv \frac{\rho u D_h}{\mu}$$

<u>Smooth tube correlations:</u>

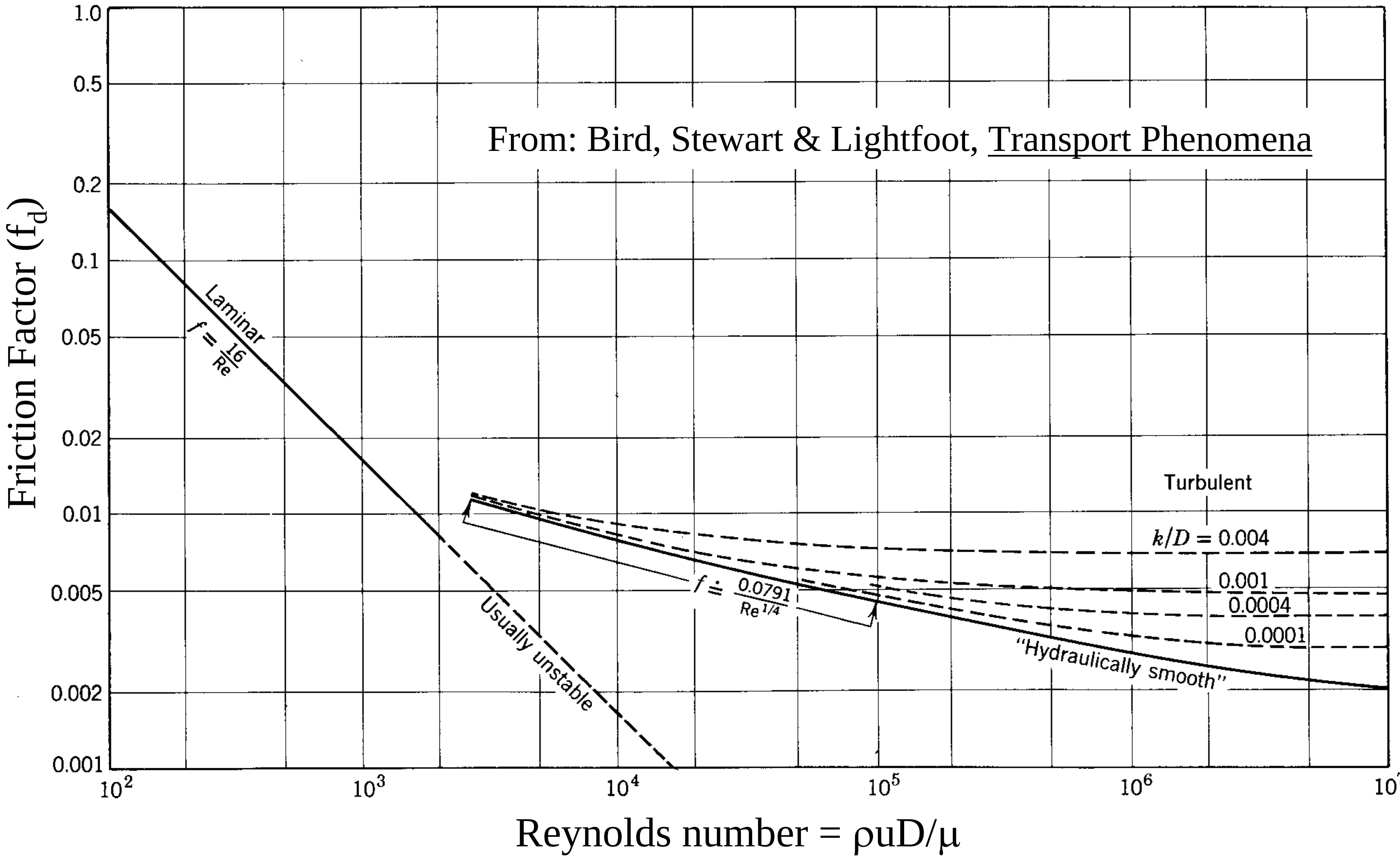
- $f_d = \frac{16}{\mathrm{Re}_D}$ for laminar flow $\mathrm{Re}_D < 2000$
- $f_d = \frac{0.0791}{\mathrm{Re}_D^{1/4}}$ for turbulent flow $2000 < \mathrm{Re}_D < 10{,}000$ (Blausius)
- $\frac{1}{f_d^{1/2}} = 1.737 \ln\left(\mathrm{Re}_D\, f_d^{1/2}\right) - 0.396$ for turbulent flow $10{,}000 < \mathrm{Re}_D$
  Von Karman-Nikuradse

<u>Rough tubes</u>: value depends on ratio (k/D = amplitude of roughness/diameter)

- $\frac{1}{\sqrt{f_C}} = -4\log_{10}\left(\frac{k}{3.7D} + \frac{1.25}{\mathrm{Re}_D\sqrt{f_C}}\right)$ Colebrook correlation for Re > 10000 and rough tubes

Fanning Friction Factor

Note: Darcy friction factor (ME) = 4 x Fanning friction factor (ChE)



Loss coefficients for components

$$\Delta p = K\left(\frac{1}{2}\rho u^2\right)$$

Similar to the tube pressure drop expression, with "K" incorporating all hydraulic losses

- Components
  - Valves
  - Elbows
  - Tees
  - Flow meters
- Comparison to flow in tube: K ~ $4f(L/D_h)$
- Equivalent length of 2" 90 elbow:

  L_{eq} ~ KD/4f

  = 0.95x2/4x0.005

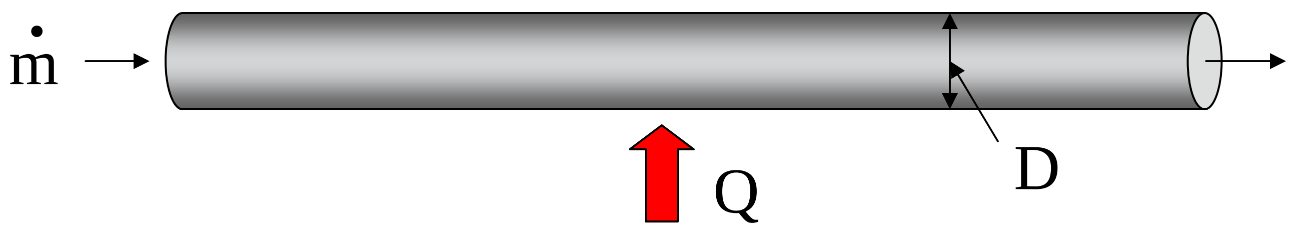
  ~ 100" (2.5 m)←

RESISTANCE COEFFICIENTS $K = \dfrac{h_m}{V^2/2g}$ FOR OPEN VALVES, ELBOWS, AND TEES

	Screwed				Flanged				
Nominal diameter, in	½	1	2	4	1	2	4	8	20
Valves (fully open):									
Globe	14	8.2	6.9	5.7	13	8.5	6.0	5.8	5.5
Gate	0.30	0.24	0.16	0.11	0.80	0.35	0.16	0.07	0.03
Swing check	5.1	2.9	2.1	2.0	2.0	2.0	2.0	2.0	2.0
Angle	9.0	4.7	2.0	1.0	4.5	2.4	2.0	2.0	2.0
Elbows:									
45° regular	0.39	0.32	0.30	0.29					
45° long radius					0.21	0.20	0.19	0.16	0.14
90° regular	2.0	1.5	$\boxed{0.95}$	0.64	0.50	0.39	0.30	0.26	0.21
90° long radius	1.0	0.72	0.41	0.23	0.40	0.30	0.19	0.15	0.10
180° regular	2.0	1.5	0.95	0.64	0.41	0.35	0.30	0.25	0.20
180° long radius					0.40	0.30	0.21	0.15	0.10
Tees:									
Line flow	0.90	0.90	0.90	0.90	0.24	0.19	0.14	0.10	0.07
Branch flow	2.4	1.8	1.4	1.1	1.0	0.80	0.64	0.58	0.41

Compressible fluid dynamics

- Liquids are incompressible and in many cases one can approximate gas flows with incompressible expressions assuming average properties. When is this OK? ($v/c \ll 1$ and $\Delta p/p \ll 1$, $\Delta T/T \ll 1$)
- For some problems, it is necessary to include compressibility effects, such that the the density, ρ = Function (p,T)



- For example, as the heat is applied to the flow, the temperature will increase resulting in an acceleration of the fluid.
- Also, isenthalpic flow ($Q = 0$, $W = 0$) of real gases can increase (or decrease) the temperature of the fluid (Joule-Thomson Effect)

Compressible fluid dynamics (cont.)

- Pressure drop: $$\frac{dp}{dx} = \underbrace{4\frac{f_d}{D}\left(\frac{1}{2}\rho u^2\right)}_{\text{friction}} + \underbrace{u^2\frac{d\rho}{dx}}_{\text{acceleration}} \qquad (1)$$
- Density depends on T, p through the equation of state for the fluid. These are related as: $$\frac{d\rho}{dx} = \rho\kappa\frac{dp}{dx} + \rho\beta\frac{dT}{dx}$$
- Since the pressure drop depends on u and dT/dx, we need a second equation to solve for 1-D flow. That is the "Stagnation Enthalpy":

$$q = -\frac{\rho u D}{4}\left(C_p\mu_j + u^2\kappa\right)\frac{dp}{dx} + \frac{\rho u D}{4}\left(C_p - u^2\beta\right)\frac{dT}{dx} \qquad (2)$$

Where, $\mu_j = -\frac{1}{C_p}\left.\frac{\partial T}{\partial p}\right)_h$ is the Joule-Thomson coefficient

Equs. (1) and (2) are simultaneous equations with unknowns T & p

V. Arp, Adv. in Cryo Engn. Vol 17, 342 (1972)

Simultaneous solution of fluid equations

- Separation of the two simultaneous equations yields,

$$\frac{dp}{dx}=\frac{-2f_dG^2/\rho D+4qG\beta/\rho D\left(C_p-u^2\beta\right)}{1-\left(G^2/\rho\right)\left(\kappa+\beta\phi\right)} \qquad (1')$$

$$\frac{dT}{dx}=\frac{4q/GD\left(C_p-u^2\beta\right)-2f_dG^2\phi/\rho D}{1-\left(G^2/\rho\right)\left(\kappa+\beta\phi\right)} \qquad (2')$$

Where we have defined the parameter,

$$\phi=\frac{\mu_jC_p+u^2\kappa}{C_p-u^2\beta} \quad \text{and} \quad G\equiv\frac{\dot{m}}{A}$$

Note that the dominant term in Eq. (1') is the friction ($2fG^2/\rho D$) and in Eq. (2') is the enthalpy flux ($4q/GDC_p$)

Approximate solution for sub-sonic flow

- For nearly ideal gases, $\kappa \sim p^{-1}$ and $\beta \sim -T^{-1}$ and for relatively slow velocity, $u \ll c$ (sound speed) the above two equations simplify,

$$\frac{dp}{dx} \approx -\frac{2G^2 f_d}{\rho D} + \frac{4qG\beta}{\rho D C_p} \qquad (1'')$$

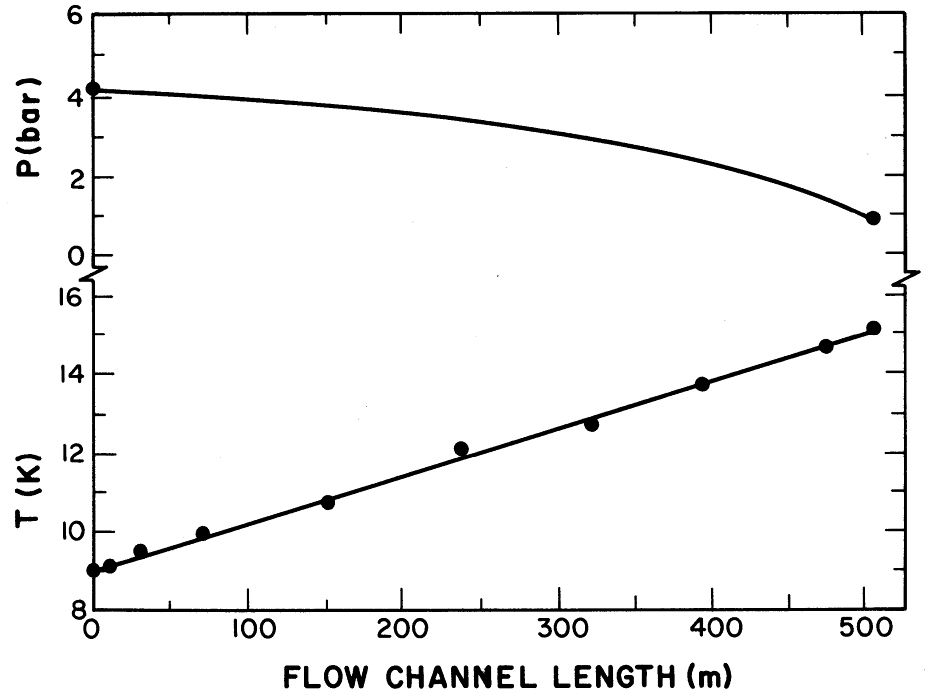
$$\frac{dT}{dx} \approx \frac{4q}{GDC_p} - \frac{2 f_d G^2}{\rho D}\mu_j \qquad (2'')$$

- It is straightforward to prepare a program to calculate dp/dx and dT/dx in a 1-D channel containing a compressible fluid of known properties
- Note that the 2nd term in the dp/dx equation (acceleration) has the effect of increasing the pressure gradient relative to the incompressible form (note that β is negative)
- The 2nd term in the dT/dx equation (isenthalpic expansion) can either increase or decrease the gradient relative to the incompressible solution depending on the magnitude and sign of μ_j (κ is positive)

Example: Compressible flow in a tube (1)

Supercritical helium

- Tube length = 500 m
- D = 4.8 mm
- m_dot = 0.98 gm/s
- Q = 0.074 W/m
- Solid lines are computed using compressible fluid mechanics
  - f = 0.007



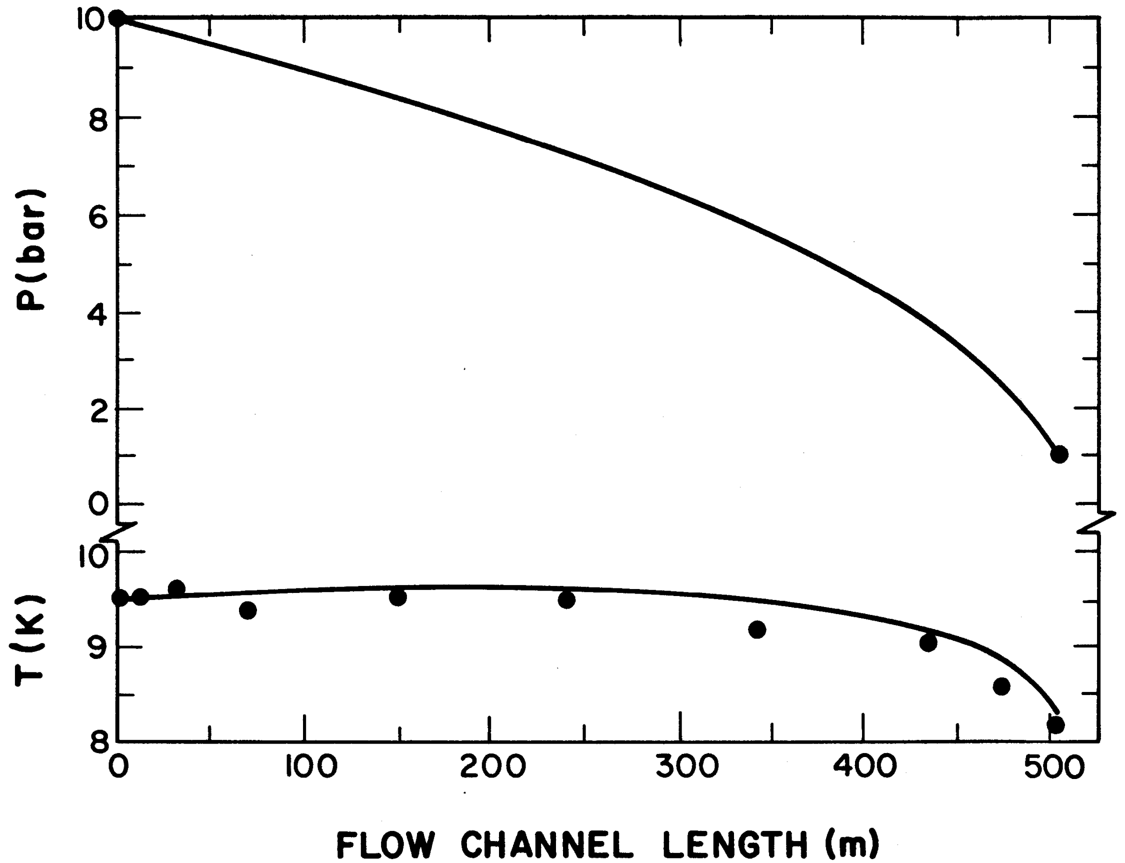
Note: this value of f was used to best fit the data and is slightly different from value predicted from correlation.

J.W. Dean, et al, Adv. Cryog. Engn. 23, 250 (1978)

To compare to incompressible solution, choose average values for T & p.

Example: Compressible flow in a tube (2)

- Length = 500 m
- D = 4.8 mm
- m_dot = 3 gm/s
- Q = 0.062 W/m
- Solid lines are computed using compressible fluid mechanics
  - f = 0.005

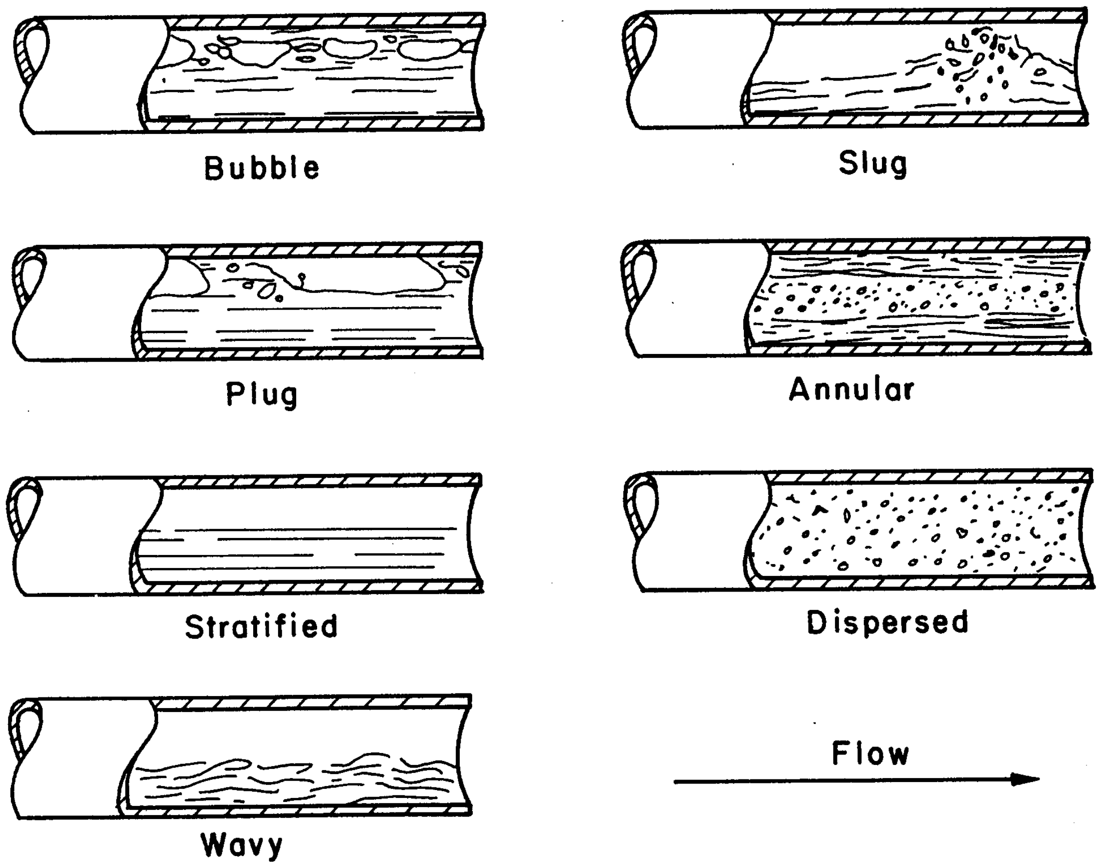


Example

Two phase flow

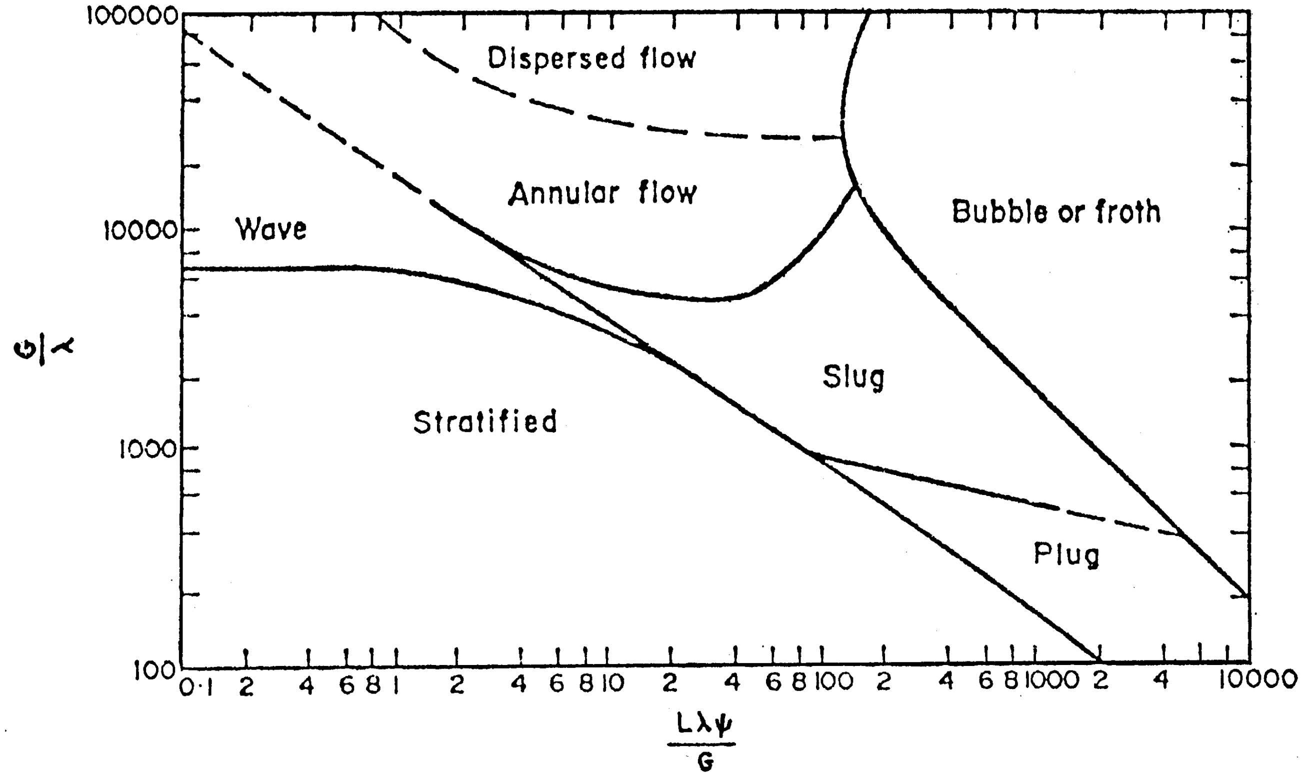
- Two-phase flow is an important topic in cryogenics since the liquids are volatile and are commonly distributed near saturation conditions
- Two-phase flow is a difficult problem as there are many variables that affect it:
  - Mass flow
  - Pressure & temperature relative to saturation
  - Gravity, system configuration
  - Heat transfer, rate of phase change
- This is mostly an empirical subject due to its complexity
- Numerous two-phase flow models exist, although much of this work has been carried out on conventional fluids (water/steam, water/air)

Flow Regimes (Baker)



All these flow regimes may occur in a horizontal pipe flow. The actual flow regime will depend on fluid velocities and properties.

Baker plot for air-water system



$\lambda = [(\rho_G/\rho_a)(\rho_L/\rho_w)]^{1/2}$

$\psi = (\sigma_w/\sigma_L)[(\mu_L/\mu_w)(\rho_w/\rho_L)^2]^{1/3}$

L = liquid mass flow rate per unit area, $lb_m/hr\text{-}ft^2$

G = vapor mass flow rate per unit area, $lb_m/hr\text{-}ft^2$

ρ_G = vapor density

ρ_L = liquid density

ρ_a = density of air = 1.20 kg/m^3 = 0.075 lb_m/ft^3

ρ_w = density of water = 998 kg/m^3 = 62.3 lb_m/ft^3

σ_L = surface tension

σ_w = water surface tension = 0.073 N/m

μ_L = liquid viscosity

μ_w = water viscosity = 0.001 Pa-s = 1 centipoise

Two-phase flow definitions

1. Void Fraction (α): Ratio of the local vapor volume to the total flow volume

$$\alpha = \frac{A_v}{A_l + A_v}$$

2. Flow Quality (χ): Ratio of the vapor mass flow rate to the total mass flow rate

$$\chi = \frac{\dot{m}_v}{\dot{m}_l + \dot{m}_v}$$

3. Slip Ratio (S): ratio of the vapor to liquid velocity

$$S = \frac{u_v}{u_l} = \left(\frac{\chi}{1-\chi}\right)\left(\frac{1-\alpha}{\alpha}\right)\left(\frac{\rho_l}{\rho_v}\right)$$

Pressure drop in two phase flow

$$\Delta p_T = \Delta p_{gr} + \Delta p_a + \Delta p_f$$

- Due to change in elevation (h): $\Delta p_{gr} = -g\int_0^h \langle\rho\rangle dx$
- Acceleration of the fluid stream:

$$\Delta p_a = G^2\left[\frac{\chi^2}{\alpha\rho_v} + \frac{\left(1-\chi^2\right)}{\left(1-\alpha\right)\rho_l}\right]_1 - G^2\left[\frac{\chi^2}{\alpha\rho_v} + \frac{\left(1-\chi^2\right)}{\left(1-\alpha\right)\rho_l}\right]_2$$

- Friction pressure drop depends on model for flow regime:

Define the two phase friction multiplier: $\phi_l^2 \equiv \dfrac{\left(dp/dx\right)_{2\phi}}{\left(dp/dx\right)_l}$

Homogeneous two phase flow model

- Assumptions (mixed flow model):
  - S = 1; i.e. the liquid and vapor velocities are equal
  - Thermodynamic equilibrium exists between the two phases
  - The friction contribution to the pressure drop reduces to the standard expression for each phase
  - Model works most effectively for helium at high Reynolds number (ρ_v/ρ_l is typ. ~ 0.1)

$$\Delta p = 4 f_d \frac{L}{D}\left(\frac{1}{2}\rho u^2\right)$$

where f_d is given by an appropriate correlation

The two phase flow friction multiplier:

For example: $f_d = \frac{0.0791}{\mathrm{Re}_D^{1/4}}$

$$\phi_l^2 = \frac{\Delta p_{2\phi}}{\Delta p_l} = \frac{\langle\rho\rangle u_{2\phi}^2}{\rho_l u_l^2}\frac{f_{2\phi}}{f_l}$$

Using Blausius correlation and $u_l = u_v$

$$\phi_l^2 = \left[1+\chi\left(\frac{\rho_l}{\rho_v}-1\right)\right]\left[1+\chi\left(\frac{\mu_l}{\mu_v}-1\right)\right]^{-1/4}$$

Lockhart Martinelli Correlation

- Assumptions (separated flow model)
  - Static pressure drop of two phases are equal
  - Fluid volume is a linear combination of two phases
  - Only applies to friction contribution to pressure drop
  - Works well for nitrogen & hydrogen at moderate flow rates
- Correlating parameter

$$X^2 = \frac{(dp/dx)_l}{(dp/dx)_v} = \left(\frac{\rho_v f_l}{\rho_l f_v}\right)\left(\frac{1-\chi}{\chi}\right)^2 \quad \text{where, for example: } f_l = \frac{0.0791}{\mathrm{Re}_D^{1/4}} = \frac{0.0791}{(\rho_l u_l D/\mu_l)^{1/4}}$$

X^2 is the ratio of the pressure drop for the pure liquid/vapor phases.

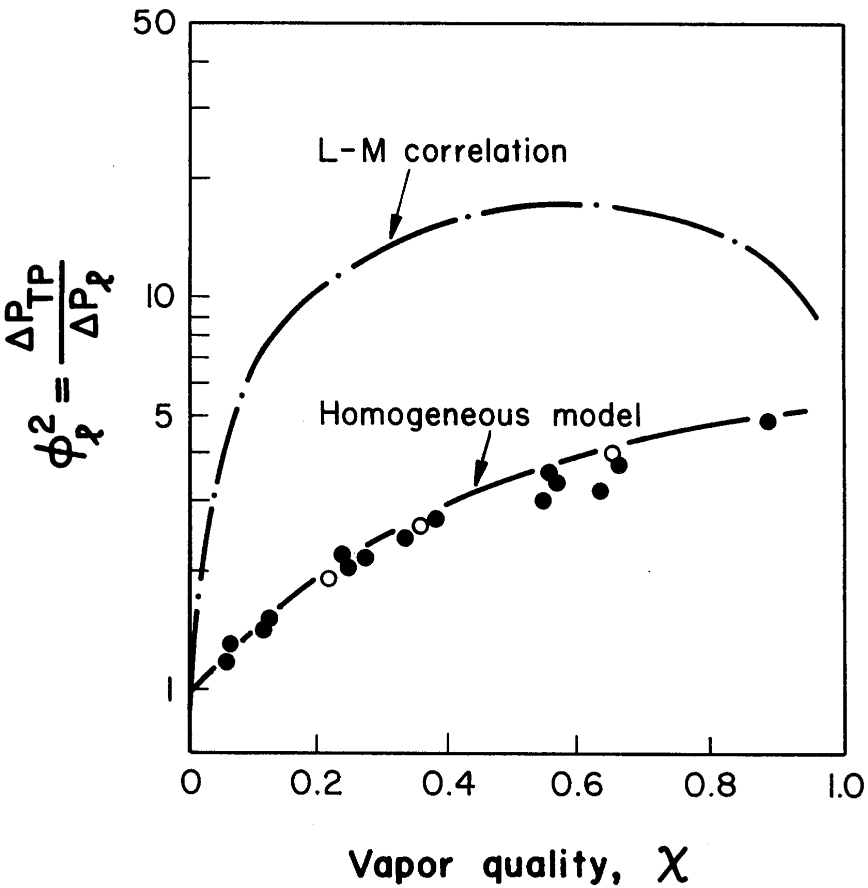
The two phase flow friction multiplier is then given as a correlation in terms of the factor X^2

$$\phi_l^2 = \frac{dp/dx)_{2\phi}}{dp/dx)_l} = \frac{X^2 + CX + 1}{X^2}$$

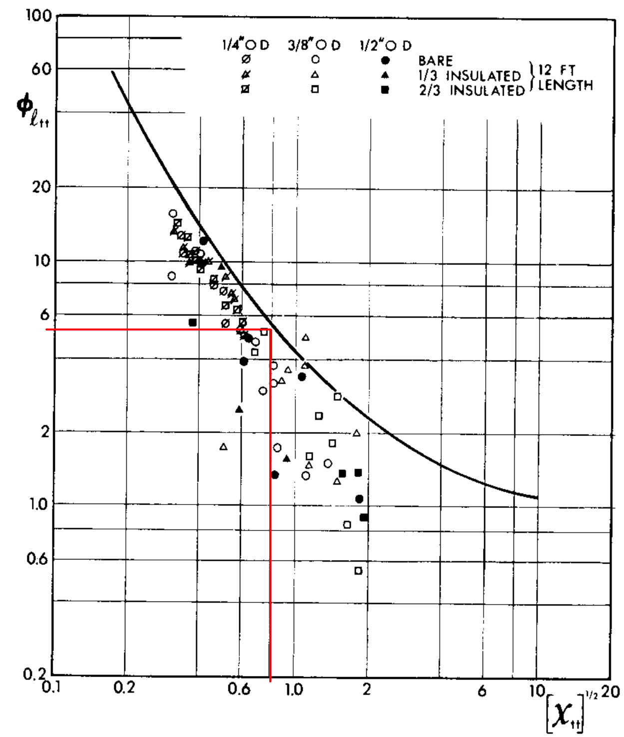
Where C = 20 if both phases are turbulent

Homogeneous Model vs LM Correlation

helium



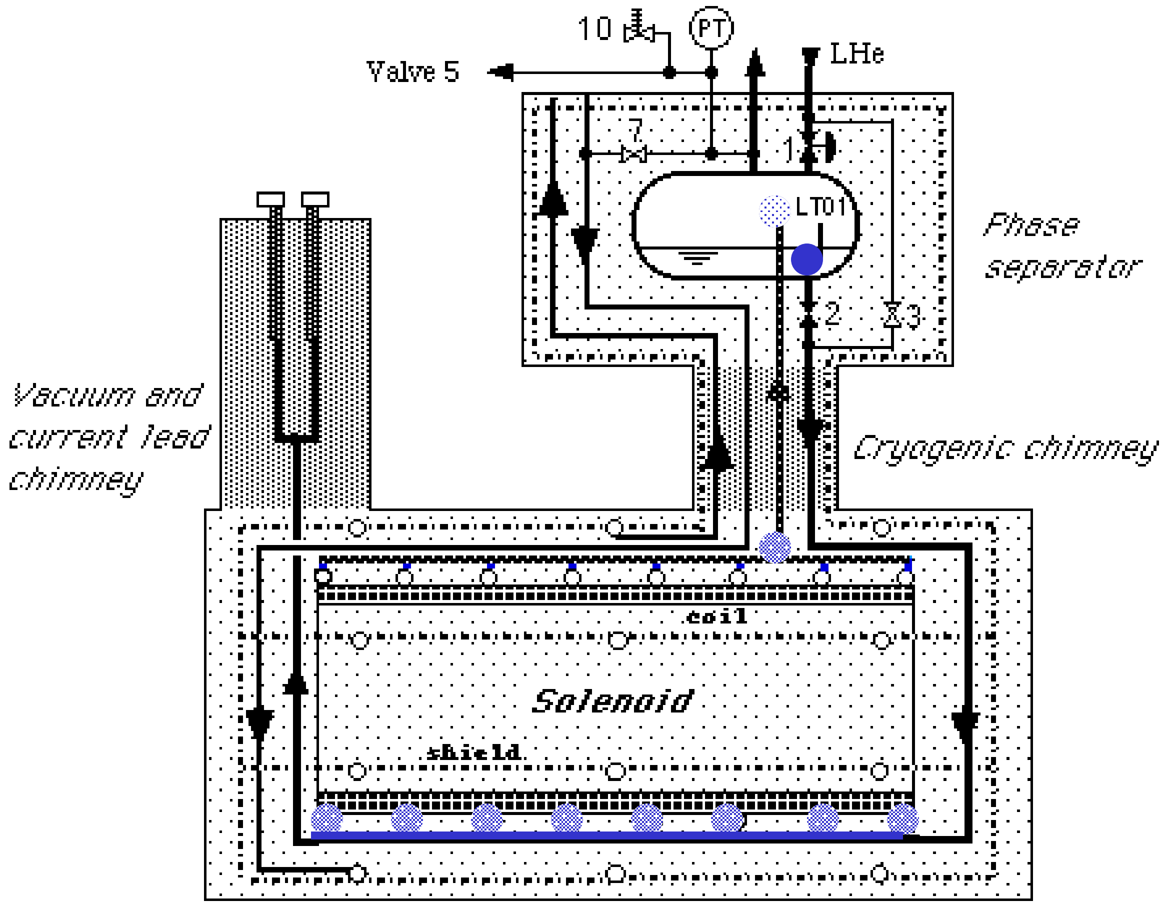
nitrogen



Note: $X_{tt} = \left(\frac{\rho_v}{\rho_L}\right)^{0.571} \left(\frac{\mu_L}{\mu_v}\right)^{0.145} \left(\frac{1-\chi}{\chi}\right)$

P. S. Shen & Y. Jao, Advances in Cryo. Engn. 15 , 378 (1970)

Natural circulation loops



Coil cooling system for CMS detector magnet at CERN-LHC

Natural Circulation Cooling Loop

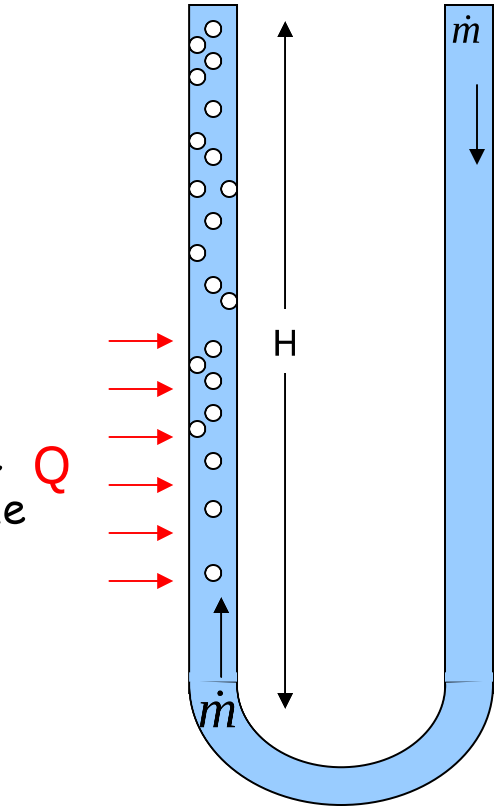
- Natural circulation loops are very useful in many cryogenic applications since they do not require a pump or other device to force the fluid flow
- Common applications include:
  - Cooling shields for large cryostats (NHMFL magnets)
  - Indirect cooling of large superconducting magnets such as the detector magnets for particle accelerators
- These cooling loops work based on the balance between the hydrostatic head difference as the driving pressure and the friction and acceleration pressure drop

$$\Delta p_d = \Delta p_f + \Delta p_a$$

- For a simple loop (right), the driving pressure is given by the difference in hydrostatic head on the two legs of the loop

$$\Delta p_d = \rho_l g H - g\int_0^H \langle \rho(z) \rangle dz$$

- The density varies with z because Q generates vapor and Δp_d increases with Q



Pressure drop through the loop

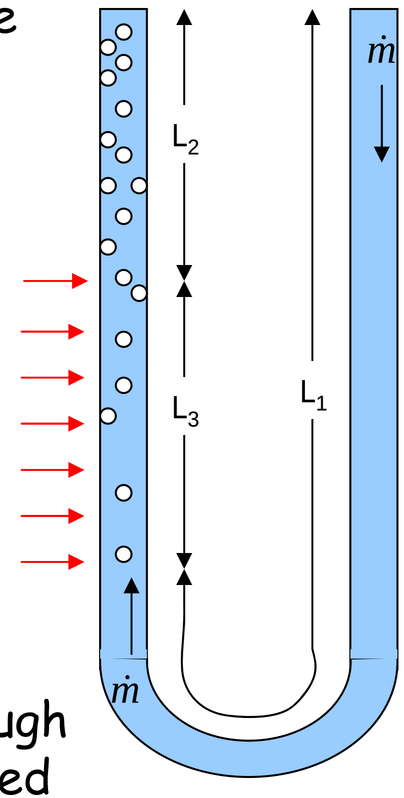
There are several contributions to the pressure drop in the natural circulation loop:

1. Single phase on the liquid side: $\Delta p_1 = f \dfrac{\dot{m}^2 L_1}{2\rho_l A^2 D_h}$
2. Adiabatic two phase flow: $\Delta p_{2p}^A = f \dfrac{\dot{m}^2 L_2}{2\rho_l A^2 D_h} \Phi_{2p}(\chi_{ex})$
3. Diabatic two phase flow: $\Delta p_{2p}^D = f \dfrac{\dot{m}^2 L_3}{2\rho_l A^2 D_h} \dfrac{1}{\chi_{ex}} \int_0^{\chi_{ex}} \Phi_{2p}(\chi) d\chi$ Q
4. Acceleration pressure drop: $\Delta p_a = \dfrac{\dot{m}^2}{A^2}(v_g - v_l)\chi_{ex}$

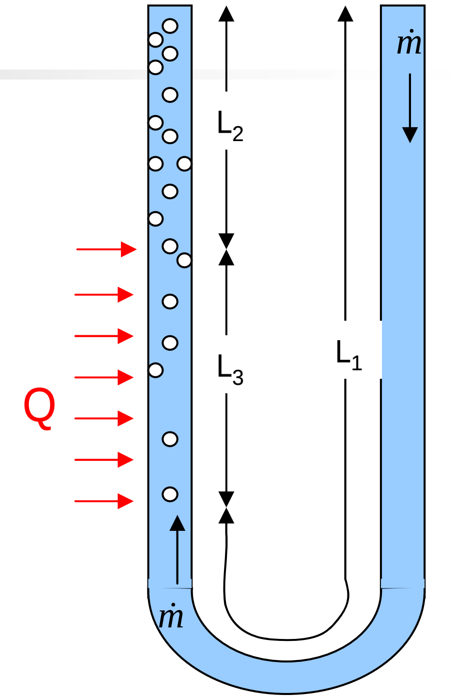
   v_g and v_l are the specific volumes of the gas and liquid

   The above equations relate to the total mass flow through the loop. However, the vapor mass flow, $\dot{m}_v$ is determined by the heat rate, Q. Where,

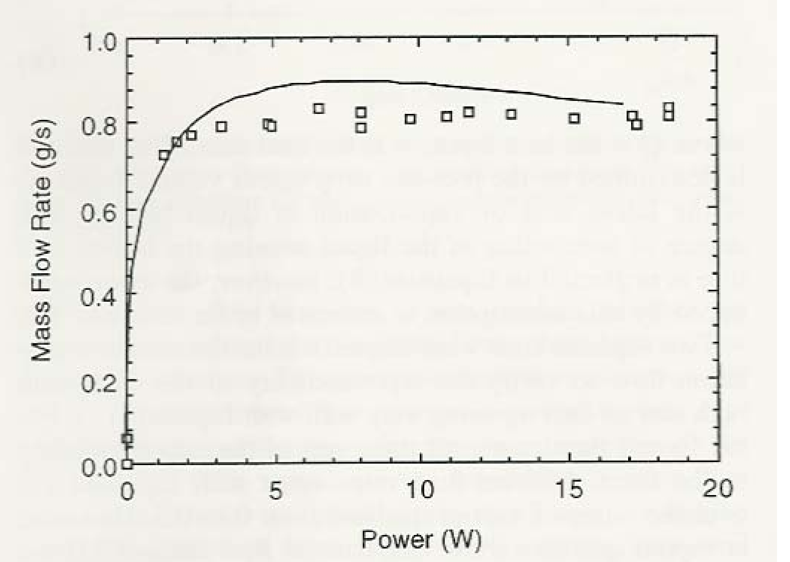
$$\dot{m}_v = Q \big/ h_{fg} \quad \text{and} \quad \chi_{ex} = \dot{m}_v \big/ \dot{m}$$



Natural circulation loop test (helium)

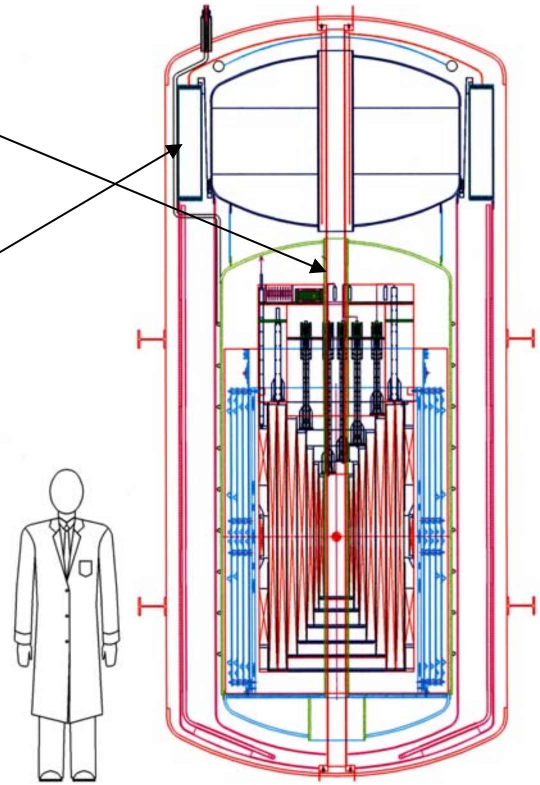


- Flow loop is ~ 5 mm ID and heated over part of its length.
- Simultaneous analysis of fluid dynamic equations
- Observation: mass flow almost independent of Q from 2 to 20 W
  - Increasing Q increases χ and Δp_d
  - Δp_f increases due to friction
- Very stable operation
- Model predicts behavior



Design of LN_2 cooled shield for 900 MHz NMR magnet

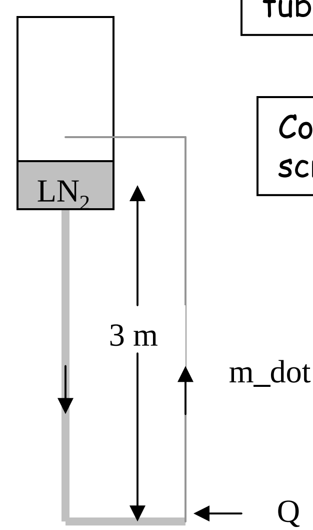
- 900 MHz NMR magnet is a large superconducting system under development at the NHMFL
- The cryostat is to have high thermal efficiency.
- A critical area is the inner warm bore, where the space between 300 K and 2 K (operating temperature) is small
- To reduce the heat load at 2 K, there is a **thermal shield** between 300 K and 2 K that is operating at LN_2 temperature
- The heat load on the LN_2 shield is sufficiently high that it must be actively cooled with liquid along its length
- LN_2 is supplied to the shield from the **LN_2 reservoir**
- Anticipated heat load on the inner bore is about 20 W

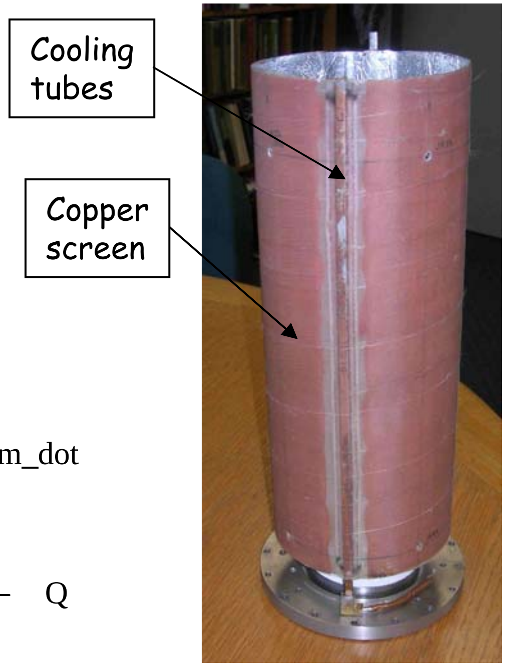


LN_2 shield design

- Shield is cooled by two tubes running axially along the length, ~ 3 m
- The supply line to the cooling tubes is large diameter, so one can neglect the pressure drop in the inlet side
- Circumferential conduction heat transfer maintains uniform temperature around shield. Used copper screen.
- Physical dimensions:
  - Shield diameter 100 mm
  - Length = 3 m
  - Thickness < 5 mm

A schematic of the simplified system is given:





- Assumed hydrodynamic conditions:
  - Turbulent flow throughout
  - Two phase flow on return leg only
  - Homogeneous flow conditions

Model Analysis

Flow rate through the cooling loop is dependent on the balance between hydrostatic head difference and friction pressure drop. The following equations apply:

1. Mass flow of the vapor: $\dot{m}_v = \frac{Q}{h_{fg}}$ ~ 20/200 J/g ~ 0.1 g/s and $\dot{m} = \frac{Q}{\chi h_{fg}}$

2. Driving pressure head: $\Delta p_d = \rho_l g H - g\int_0^H \langle\rho\rangle dz$

   where $\frac{1}{\langle\rho\rangle} = \frac{1-\chi}{\rho_l} + \frac{\chi}{\rho_v}$ and $\chi = m_v/m$ is the return vapor quality

   substituting for the average density, $\Delta p_d = \rho_l g H\left[1 - \left(\frac{\rho_l}{\rho_v}\chi + 1\right)^{-1}\right]$

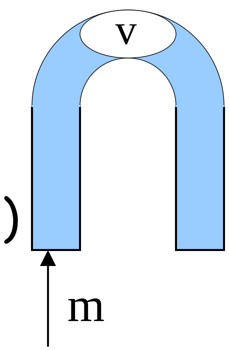
3. Friction loss due to adiabatic two phase flow

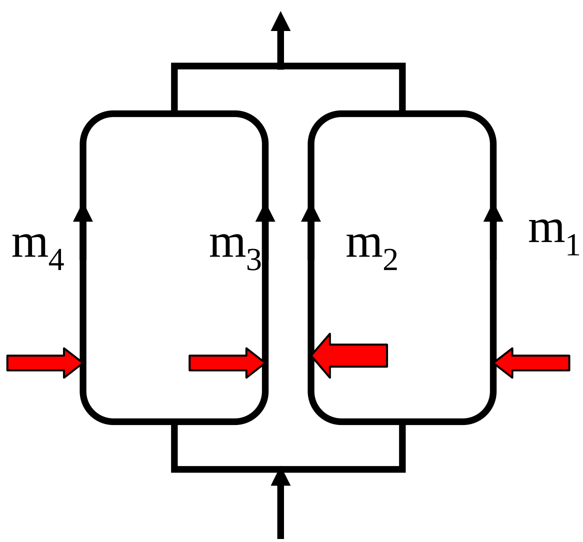
   $\Delta p_f = \frac{2 f H \dot{m}^2}{\rho_l A^2 d_h}\Phi_{2p}(\chi)$ where f is the friction factor, A = flow area and d_h

And for the homogeneous model; $\Phi_{2p} = \frac{\Delta p_{2p}}{\Delta p_l} = \left[1 + \chi\left(\frac{\rho_l}{\rho_v} - 1\right)\right]\left[1 + \chi\left(\frac{\mu_l}{\mu_v} - 1\right)\right]^{-1/4}$

Two phase flow summary

- Clearly, two phase flow is a complicated process given the number of variables involved:
  - Mass flow rate, pressure drop, heat transfer rate, temperature
  - Void fraction, flow quality, flow regime, orientation
- "Words of Wisdom"
  - Avoid creating "traps" that can collect vapor
  - Return two phase liquid above supply surface (phase separation)
  - Avoid parallel tubes with different hydraulic characteristics





Δp is the same for all channels, but vapor mass flow will increase for the channels with higher heat load. This will decrease the mass flow and could lead to "dry out' or vapor lock condition.

Cryogenic fluid distribution

- Major components
  - Vacuum jacket lines & bayonet connections
  - Low heat leak valves
  - Heat exchangers & vaporizers
  - Circulation pumps
  - Compressors and expansion engines
- Performance of each component affects overall thermodynamic efficiency of system
- Inefficiency (entropy generation) mostly due to:
  - Fluid friction
  - Heat exchange over finite ΔT