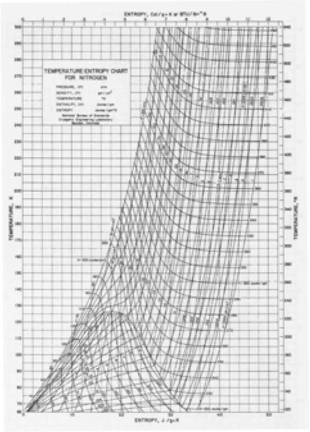
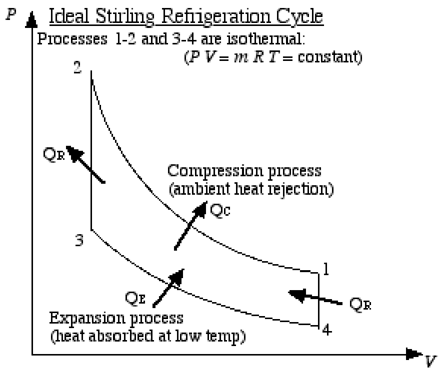
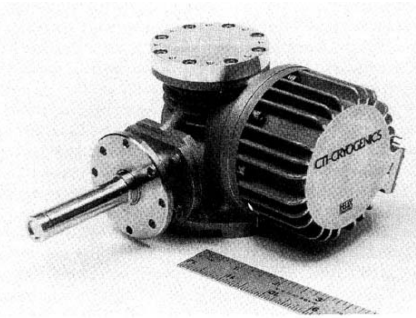
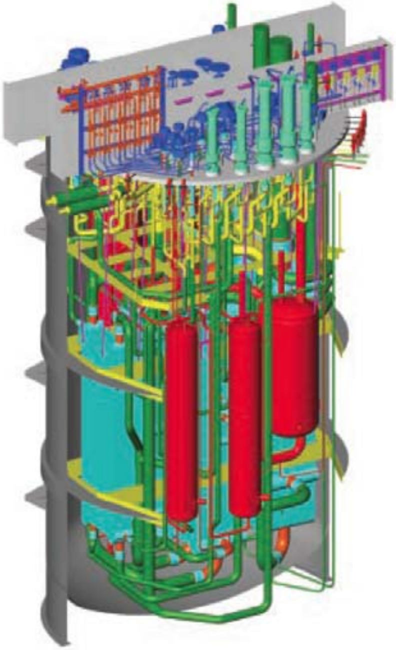






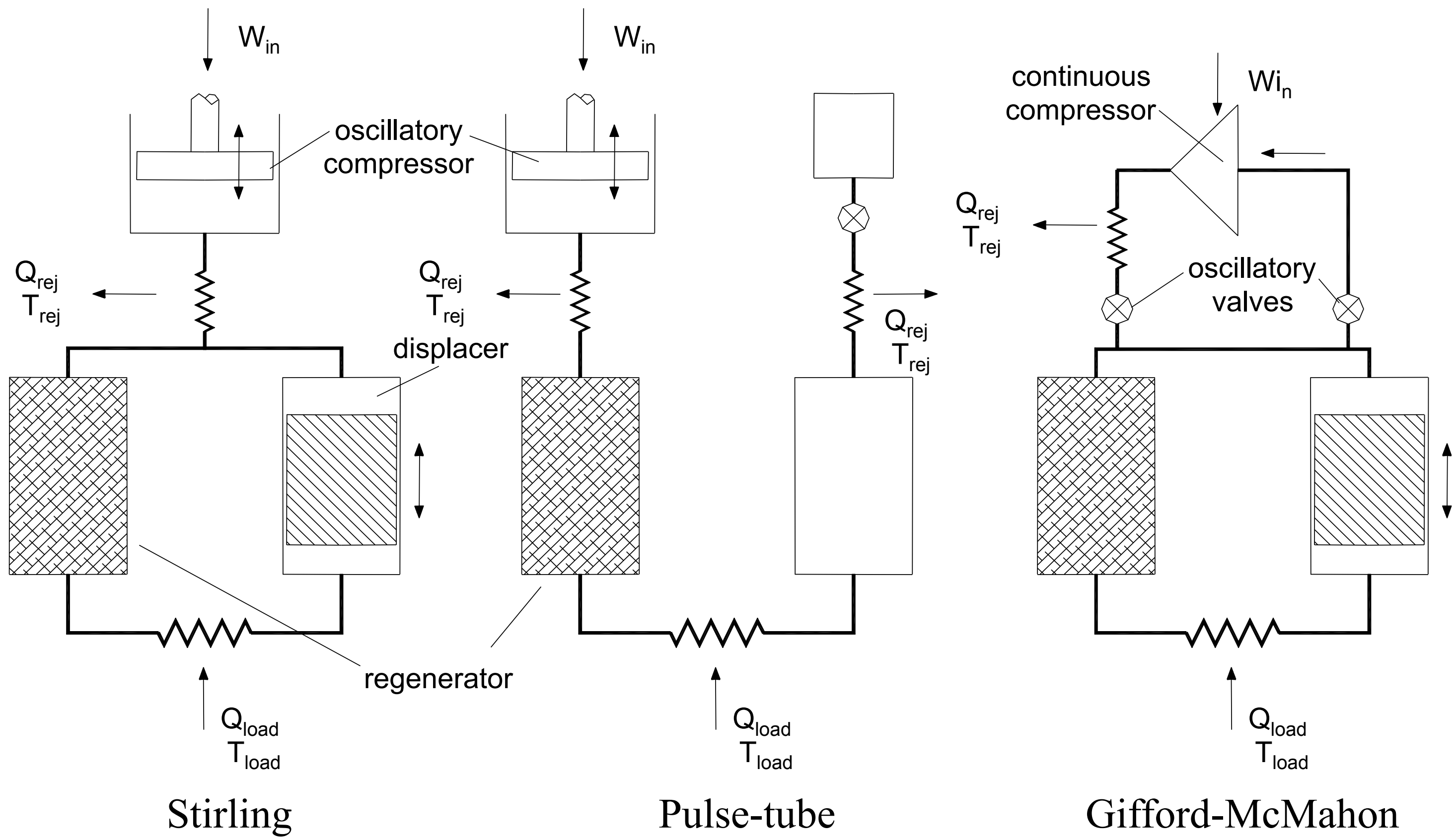


2.2 Cryocoolers

Outline

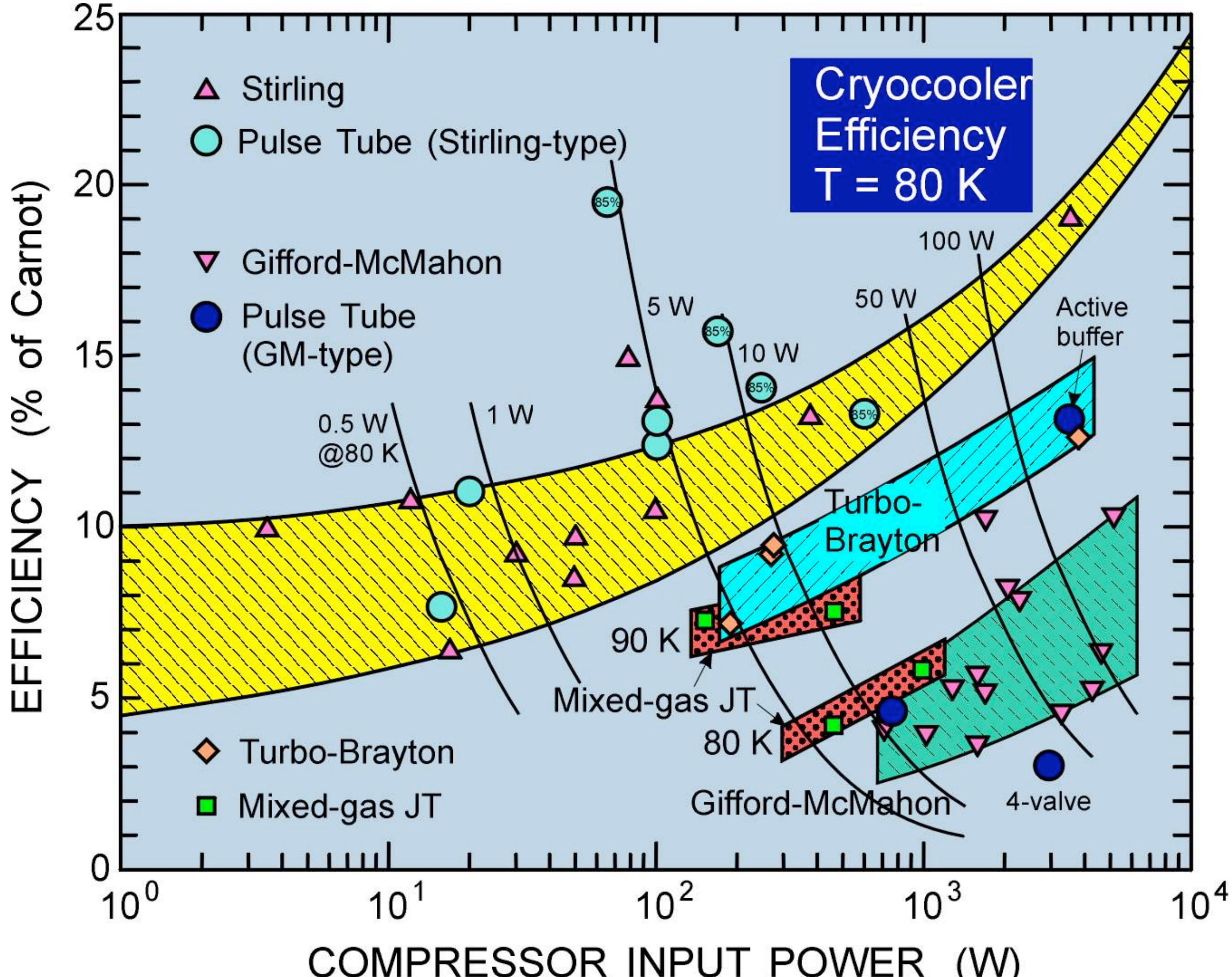
- Regenerative systems
  - Overview of regenerative coolers
  - Stirling Cryocoolers
  - Gifford-McMahon Cryocoolers
  - Pulse tube cryocoolers
- Introduction to EES

Regenerative Systems



- Oscillatory flow: frequencies 1 - 100 hz
- Regenerative heat exchangers: ideal – zero void volume, ΔP, high c_P
- Phase modulation (between pressure and flow waves) is crucial for performance

Cryocooler Actual Performance



From Ray Radebaugh (NIST) 1999

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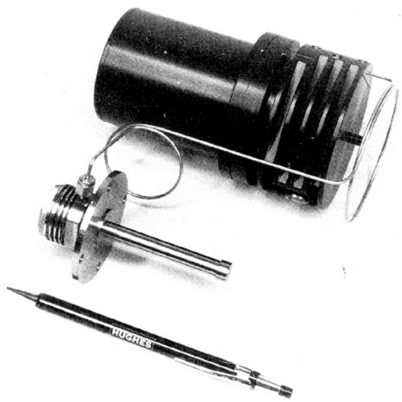
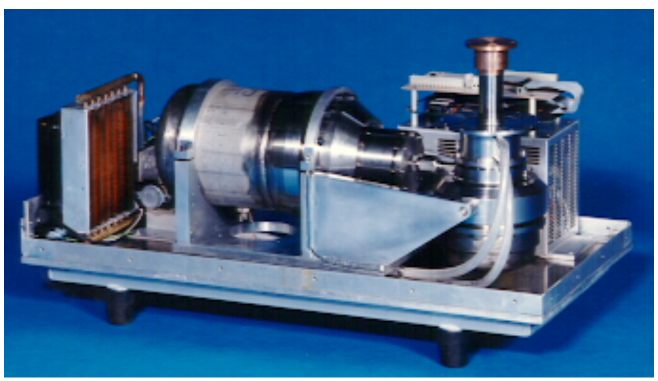
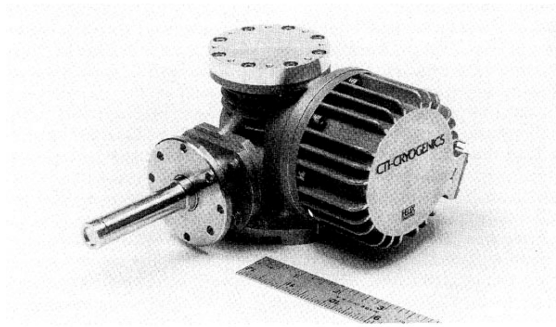
Stirling Cryocoolers

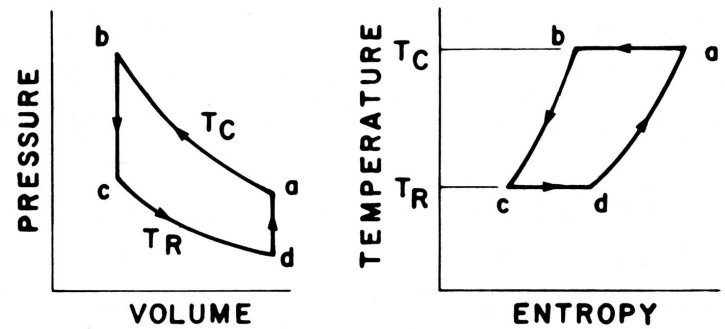


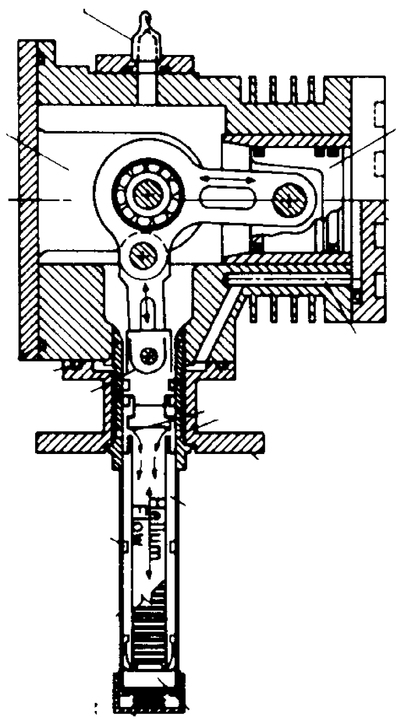


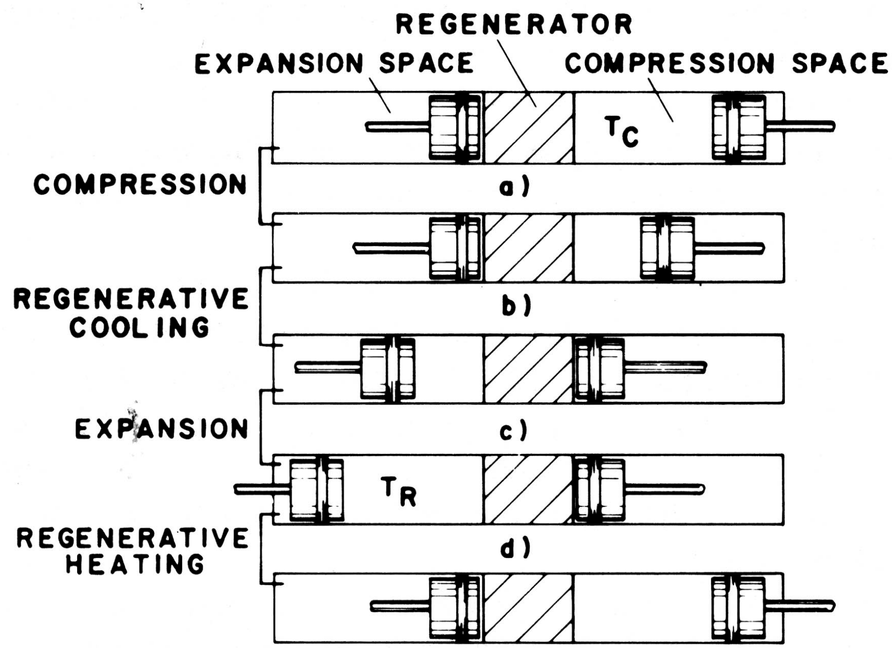
- Stirling cycle engine:
  - invented in 1815
  - 1950’s bid for auto industry
  - Today: 2.5 kW generators
- Stirling cryocoolers: 1946 -
- Ideal efficiency = Carnot
  - $COP = T_c / (T_h - T_c)$
- Primary uses:
  - tactical and security IR systems
  - medical and remote- location cryogen plants
- Potential cooling for large scale HTS applications
- Commercial sources:
  - Stirling (www.stirling.nl)
  - Sunpower(www.sunpower.com)
  - Stirling Technology Company (www.stirlingtech.com)

Stirling Cryocoolers

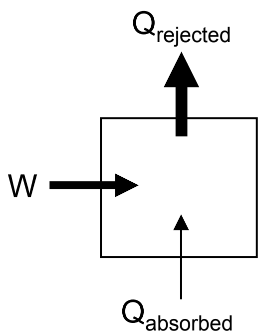




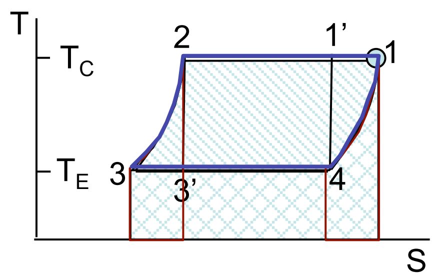




Stirling Cycle: Zero'th Order (ideal gas) Analysis



- Compare work and heat transfer for Stirling and Carnot cycles
- Use helium gas as quantitative example: T_C=300 K, T_E=100 K, P_1=1 atm., P_2=20 atm.
- Note that for an ideal gas in isothermal compression we have: $$s_2 - s_1 = -R\ln\left(\frac{P_2}{P_1}\right)$$



For an ideal cycle

$$W_{net} = Q_r - Q_a$$
$$= \oint \delta Q$$
$$= \oint TdS$$

	Carnot		Stirling	
process	ΔQ	helium (J/mol)	ΔQ	helium (J/mol)
1(1') – 2	$T_C(s_2\text{-}s_{1'})$	-627	$T_CR\ln(P_1/P_2)$	-4732
2 – 3(3')	0	0	$C_v(T_E\text{-}T_C)$	-2500
3(3') – 4	$T_E(s_4\text{-}s_{3'})$	209	$T_ER\ln(P_3/P_4)$	1577
4 – 1(1')	0	0	$C_v(T_C\text{-}T_E)$	2500
Net	$(T_C\text{-}T_E)(s_2\text{-}s_1)$	-418	$(T_C\text{-}T_E)$x $R\ln(P_1/P_2)$	-4981
COP	$T_E/(T_C\text{-}T_E)$	0.5	$T_E/(T_C\text{-}T_E)$	0.5

Stirling cycle processes more heat than Carnot cycle, but same efficiency

Stirling Cryocoolers

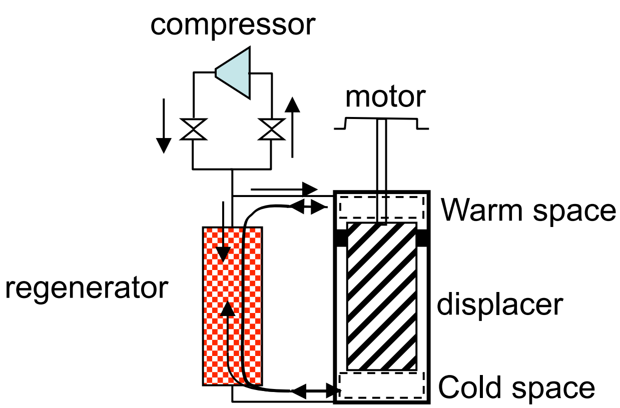
- How is real machine different from ideal?
  - Harmonic motion (vs. abrupt changes)
  - Regenerator void volume
  - Regenerator ineffectiveness
  - Pressure drop through regenerator
  - Non-isothermal compression and expansion
  - Non-zero ΔT between reservoir and heat exchangers
  - Constant temperature piston and cylinder walls
  - Isotropic pressure at all instants
- 1st order analysis (Schmidt, 1861)
  - includes
    - Harmonic motion
    - Regenerator void volume
  - Useful theoretical tool for parametric optimizations
- 2nd & 3rd order analyses – nodal simulations
  - SAGE (dgedeon@compuserve.com)

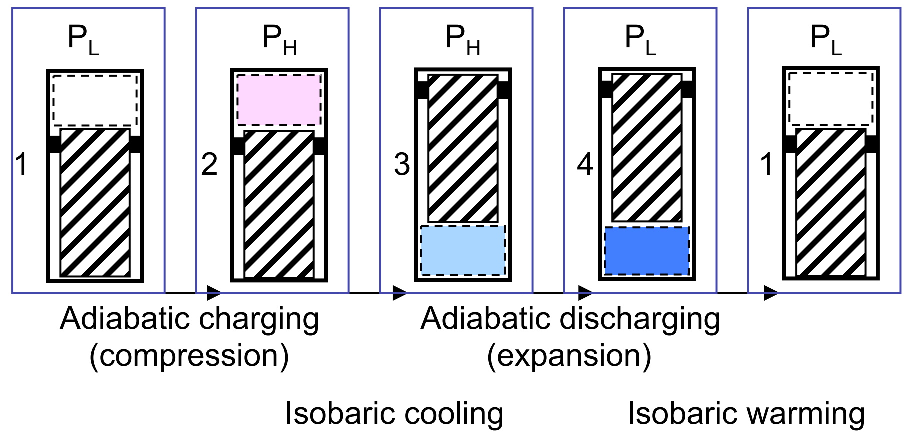
Gifford-McMahon Cryocooler

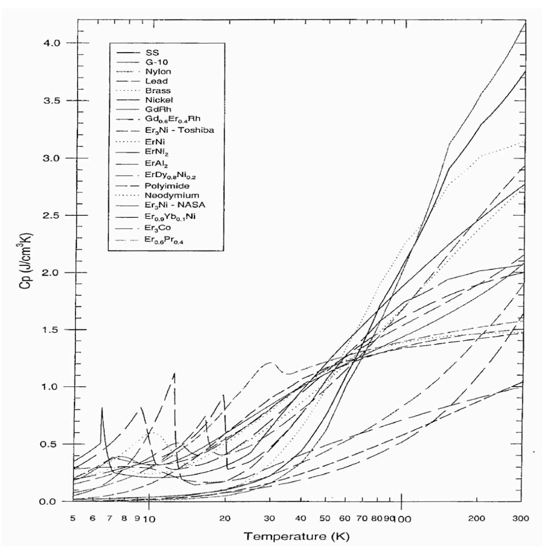
- Alternative to Stirling cryocoolers
  - Valving allows use of inexpensive compressors, and separation between cold head and compressor
  - Typical frequency 1 – 2 Hz
  - Somewhat reduced efficiency
  - Cooling power range:
    - ~ 1 watt @ 4.2 K : recondenser
    - 200 watts @ 80 K: cryo-pumps
- Primary uses:
  - Liquid nitrogen plants
  - Cryopumps
  - Conduction cooled s/c magnets - MRI, μSMES, HTS
  - Large scale HTS applications

Gifford-McMahon Cryocoolers

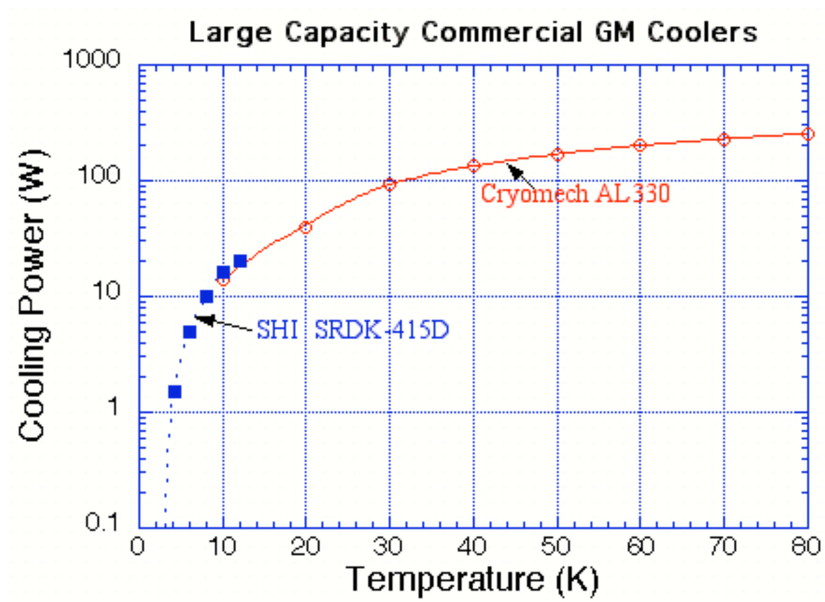
- Cycle description:





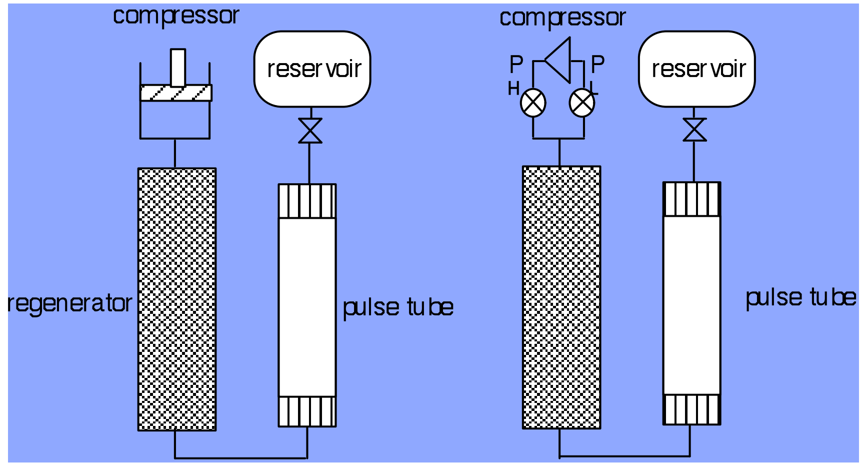


Materials research in the 80's and 90's has enabled 4 K GM machines with cooling capacity ~ 1 watt



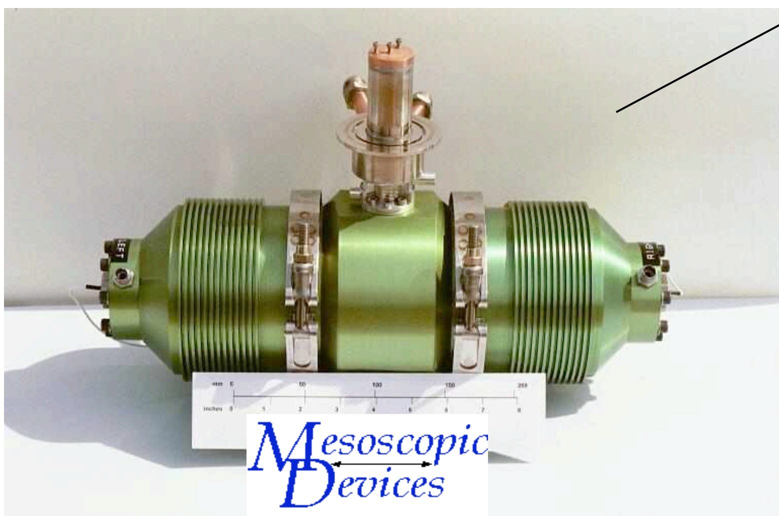
Pulse Tube Cryocoolers

- Two general types
  - Stirling type
    - High frequency ~ 60 Hz
    - High efficiency: 25% of Carnot
    - Operation down to 10 K
  - GM type
    - Low frequency ~ 1-2 Hz
    - Split design = very low vibration
    - Ideal for 4 K operation (≤ 1 watt)

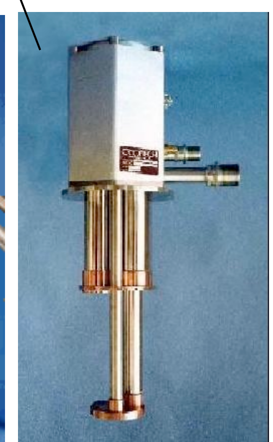


Stirling Type

GM Type

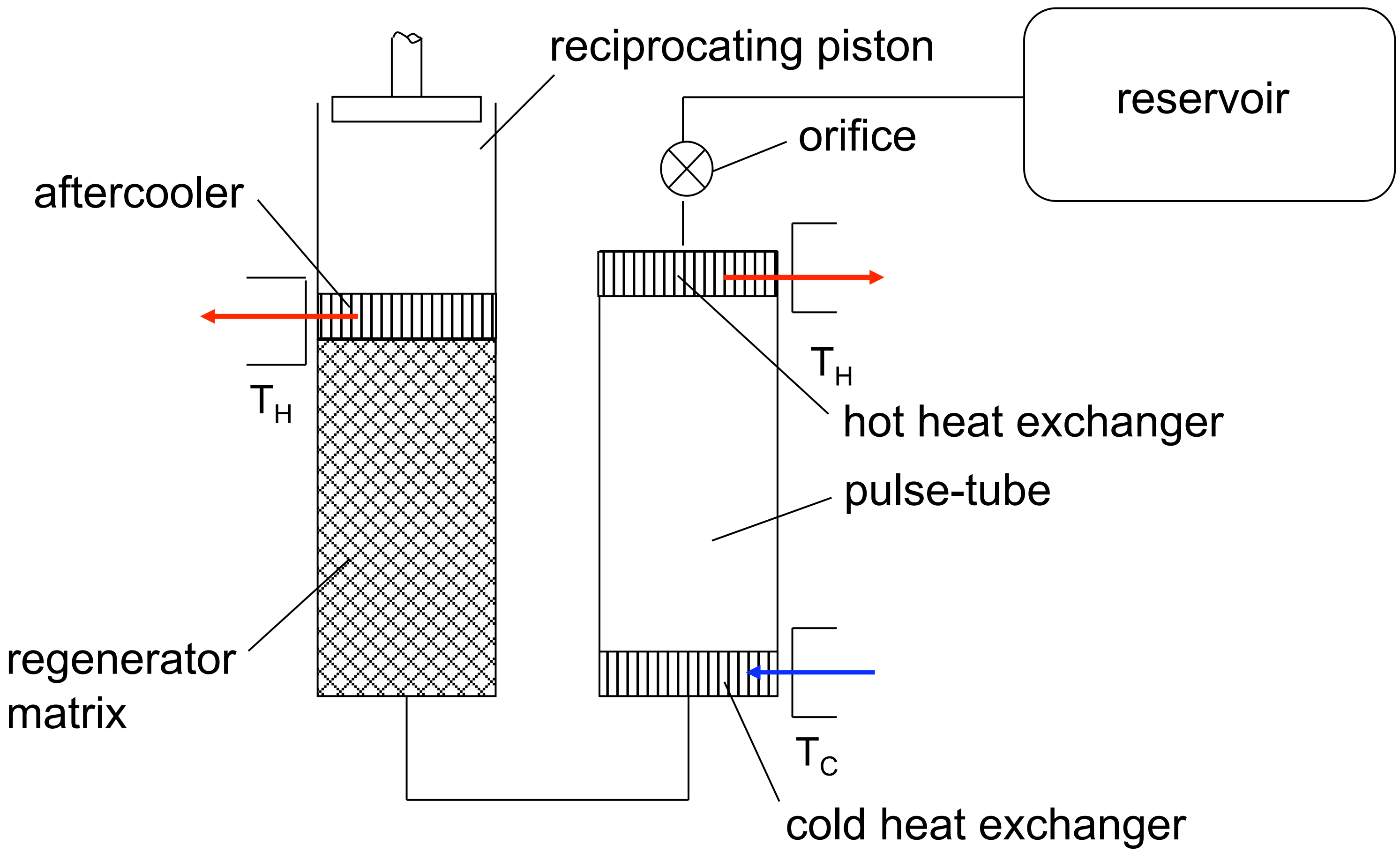


MD 200

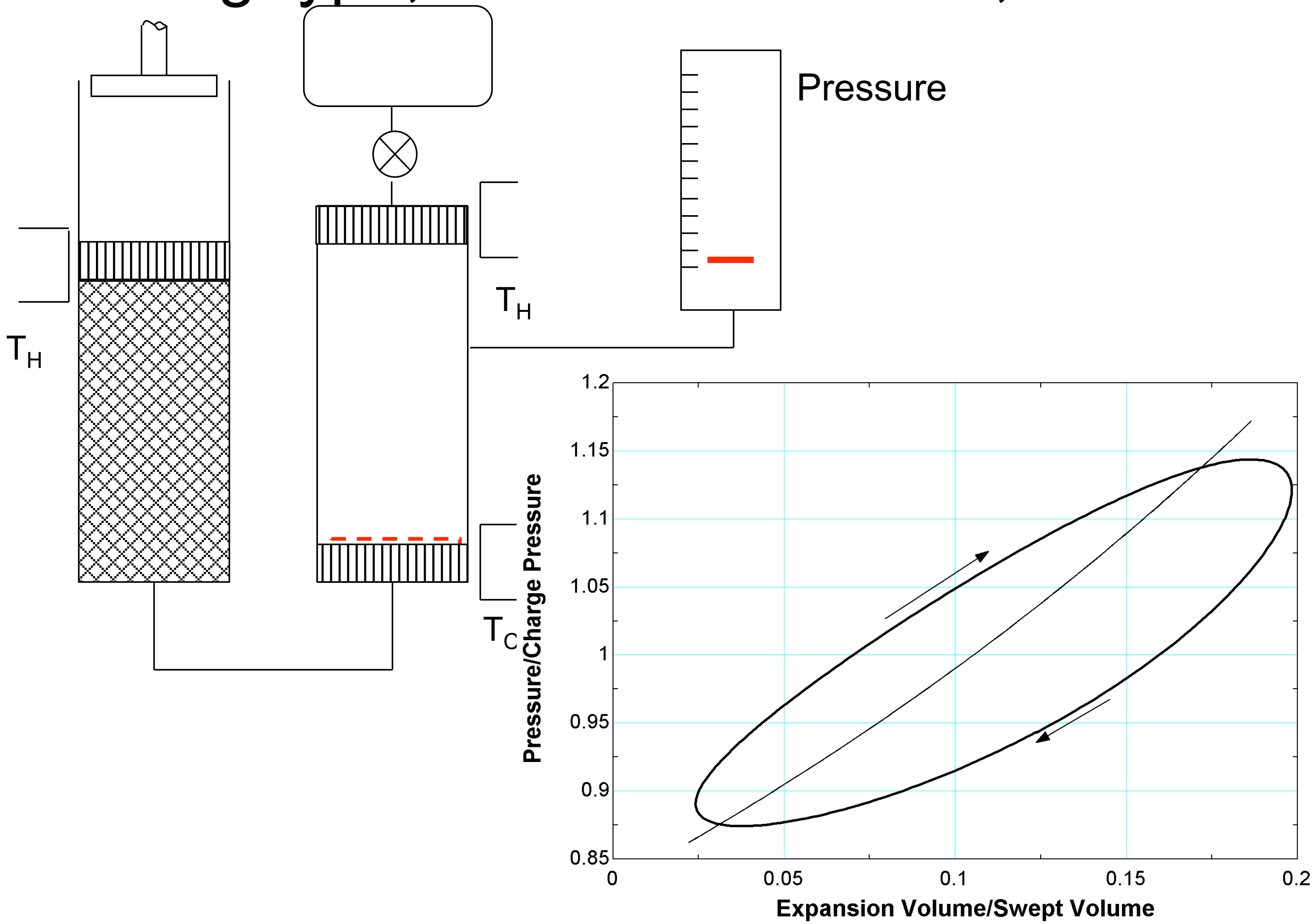


Cryomech PT405

Stirling-type, Orifice Pulse-Tube

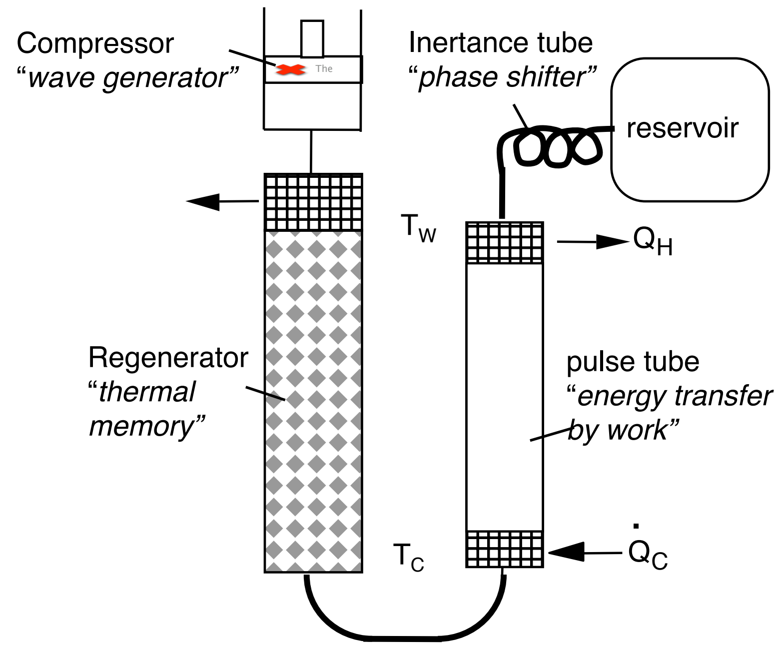


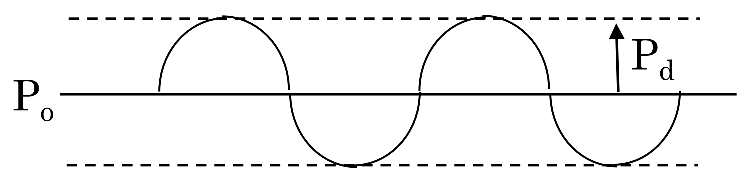
Stirling-type, Orifice Pulse-Tube, P-V work



Design Starting Point

- Desired performance:
  - $\dot{Q}_C$ @ T_C 25 watts @ 80 K
- Required definition:
  - T_w 300 K
  - P_o 2.5 MPa
  - P_r 1.3
  - f 60 Hz
- Design goals:
  - geometry (L, D) for each component
  - Acoustic power
  - θ : phase angle between mass flow & pressure oscillations





$$P_r = \frac{P_o + P_d}{P_o - P_d}$$

P_d = dynamic pressure amplitude

Acoustic Power

- Determined from amplitude and relative phase of pressure and mass flow oscillations:

$$\dot{W}_{ac,w} = \frac{1}{2} P_d \dot{V}_w \cos\theta = \frac{1}{2} \frac{P_d}{P_o} R T_w \dot{m}_w \cos\theta$$

- Coefficient of Performance (COP) relates desired cooling power to acoustic power provided by compressor:

$$COP = \frac{\dot{Q}_c}{\dot{W}_{ac,w}}$$

- Acoustic power flow in regenerator varies (nearly) linearly with temperature:

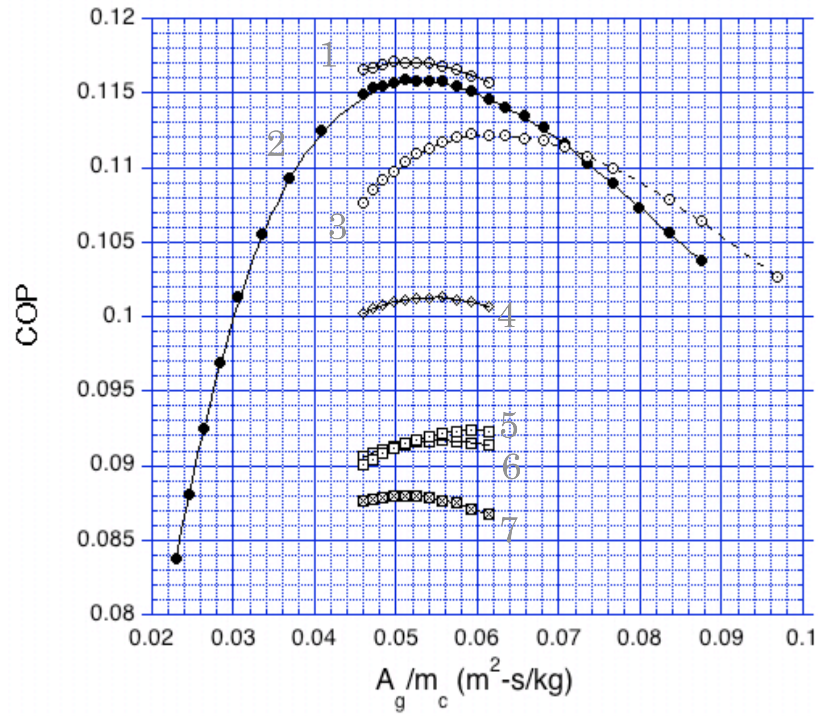
$$\dot{W}_{ac,T} = \frac{T}{T_w} \dot{W}_{ac,w}$$

Regenerator design

- Optimize heat exchanger performance by minimizing losses (ineffectiveness, pressure drop, conduction)
- Multiple parametric design studies using REGEN3.2 reveal:
  - COP depends on many variables, but in optimizing over these, one finds dominant dependence on T_c
  - With T_c=80K, T_w=300K, f=60Hz P_o=2.5MPa, P_r=1.3, we find:

$$COP_{opt} = 0.117$$
$$L_{opt} = 52\ mm$$
$$\left(\frac{A_g}{\dot{m}_c}\right)_{opt} = 0.052\frac{m^2 s}{kg}$$

T_c = 80 K, f = 60 Hz



#	L	P_o	P_r	θ_c
	(mm)	(MPa)	-	(°)
1	52	2.5	1.3	-30
2	45	2.5	1.3	-30
3	52	2.0	1.3	-30
4	52	2.5	1.3	-10
5	52	2.0	1.39	0
6	52	2.5	1.3	0
7	52	3.0	1.24	0

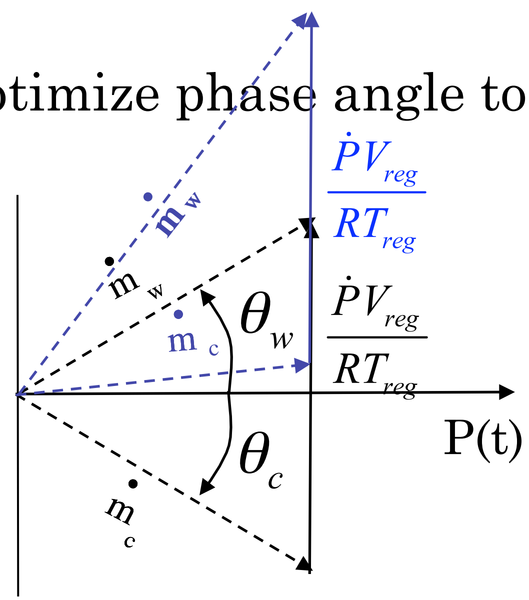
Regenerator (cont.)

- Optimized inverse mass flux $\left(\frac{A_g}{\dot{m}_c}\right)_{opt}$ enables scaling for desired $\dot{Q}_c$

$$A_g = \left(\frac{A_g}{\dot{m}_c}\right)_{opt} \dot{m}_c = \left(\frac{A_g}{\dot{m}_c}\right)_{opt} \underbrace{\frac{2}{T_w} \frac{\dot{Q}_c}{COP} \frac{P_o}{P_d} \frac{1}{R\cos\theta_c}}_{\text{from acoustic power relation}} = \frac{V_{reg}}{L_{reg}} \qquad (1)$$

not yet determined

- Optimize phase angle to minimize $\dot{m}_{ave}$ and associated losses



Dominant regenerator losses $\propto \dot{m}_{ave}$

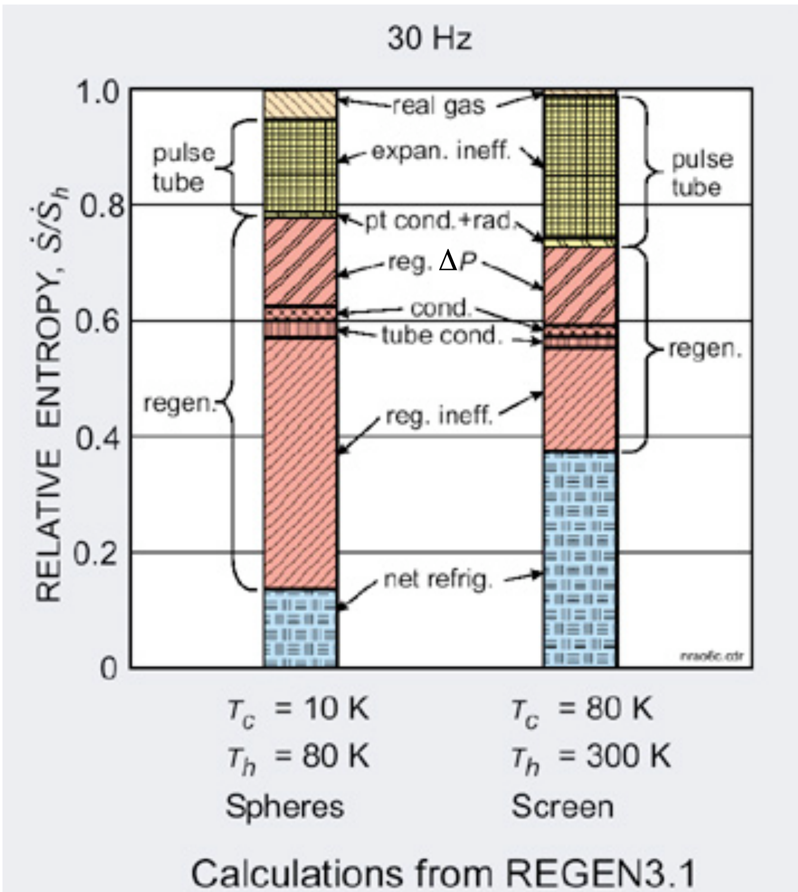
Note that when varying θ_c:

$\dot{m}\cos\theta$ remains constant (acoustic power)

$\dot{P}V_{reg} / RT_{reg}$ remains constant

$\dot{m}_{ave}$ minimized when $\theta_c = -\theta_w$

Losses in the Stirling-type Pulse tube

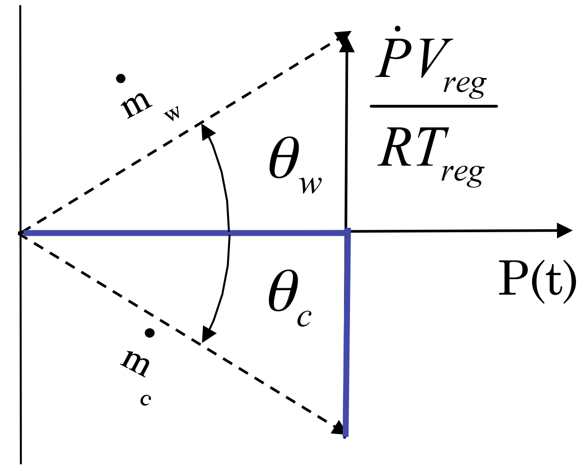


From Ray Radebaugh

- In the real system, entropy is generated in the regenerator and the pulse tube, reducing the amount of acoustic work that is available for enthalpy flow
- The major losses in the regenerator are proportional to the magnitude of mass flow through the regenerator.
- Optimimzed performance ⇒ minimize the magnitude of the mass flow through the regenerator.

Regenerator (cont.)

- $\theta_c = -\theta_w \quad \rightarrow \dot{m}_c \sin\theta_c = -\frac{1}{2}\frac{\omega P_d V_{reg}}{R T_{reg}}$



- From acoustic power relation

$$\dot{m}_c \cos\theta_c = \frac{2\dot{W}_{ac,c} P_o}{P_d R T_c}$$

- Combining and solving for θ_c

$$\theta_c = \arctan\left(\frac{-\omega P_d^2 V_{reg} T_c}{4\dot{W}_{ac,c} P_o T_{reg}}\right) \qquad (2)$$

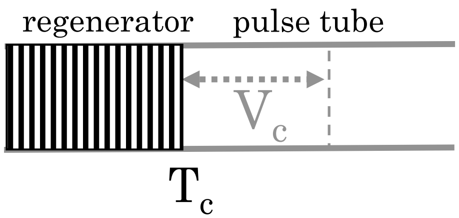
- Iterate (1) and (2) to solve for V_{reg} ($A_{reg} \rightarrow D_{reg}$) and θ_c

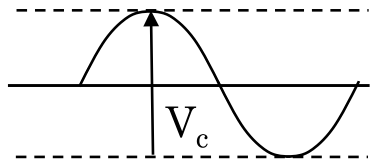
$$D_{reg} = 23.7 \text{ mm} \qquad \theta_c = -25° \qquad \dot{m}_c = 5.81 \text{ g/s}$$

- Notice that warm end parameters θ_w and $\dot{m}_w$ are now defined

Pulse Tube

- Gas flow from the regenerator extends a finite distance into the pulse tube, defining a 'cold end' volume, V_c





$$\vec{V}_c(t) \sim \frac{1}{2} V_c \sin(\omega t)$$

$$\dot{\vec{V}}_c(t) \sim \frac{1}{2} \omega V_c \cos(\omega t)$$

- V_c is related to cold end mass flow rate:

$$V_c = \frac{2\dot{V}_c}{\omega} = \frac{2\dot{m}_c R T_c}{\omega P_o}$$

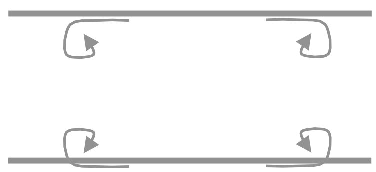
- "Rule of thumb": V_{pt} = 3 to 5 times V_c.

$T_c = 100$ K $\qquad$ $T_c = 10$ K

- For T_c = 80 K, $V_{pt} = 3.5 \mathrm{x} V_c$ = 7170 mm^3

Pulse tube (cont.)

- Two considerations for aspect ratio:
  - CFD models: $D_{pt} = D_{inertance} + f(D_{reg} - D_{inertance})$
  - Avoid turbulence at the walls



$$Re_{crit} = \frac{\rho u_{\mathrm{crit}} \delta}{\mu} = 280; \quad \delta = \sqrt{\frac{2v}{\omega}}$$

- Turbulence consideration defines minimum area (to avoid critical velocity) that should be checked at cold & warm end

$$A_{\min} = \frac{\dot{m}\delta}{Re_{crit}\,\mu}$$

- Larger of $A_{min,c}$ & $A_{min,w}$ defines area limit:

$$A_{min,c} = 1.34\text{x}10^{-4}\ \text{m}^2 \rightarrow D_{pt} = 13.1\ \text{mm}, \quad L_{pt} = 53.6\ \text{mm}$$

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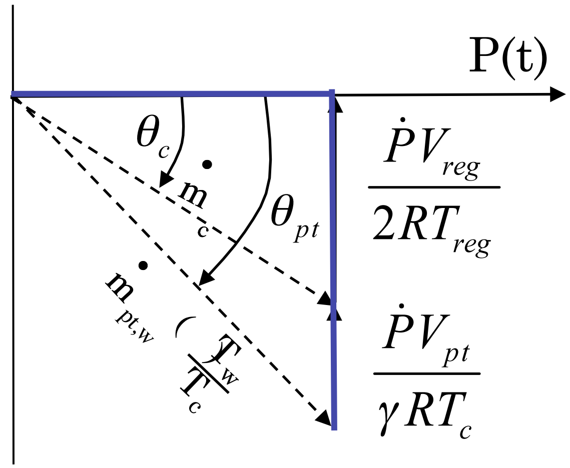
Pulse tube (cont.)

- Energy balance in pulse tube determines phasor diagram

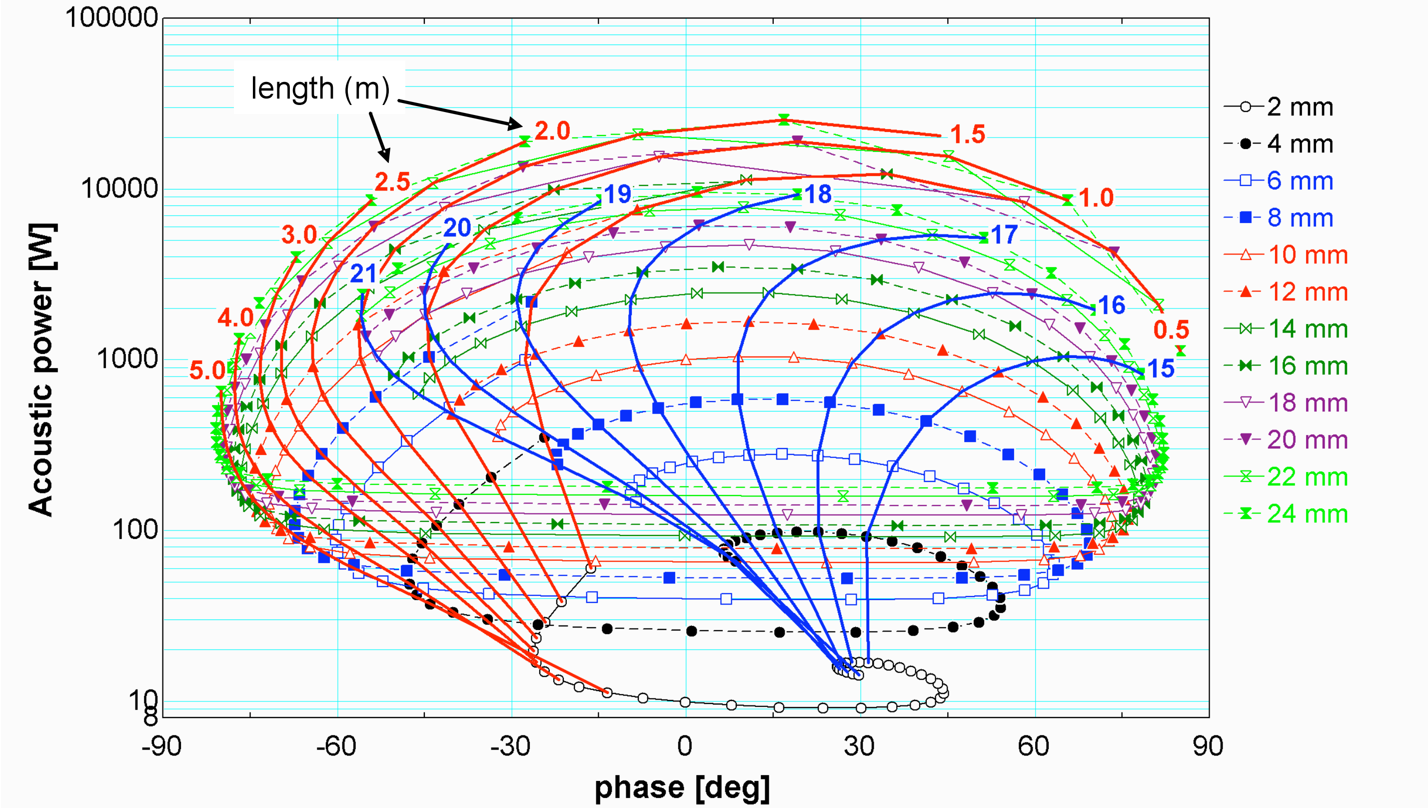


$$\theta_{pt} = \arctan\left[\frac{\omega P_d^2\left(V_{reg}\gamma T_c + 2V_{pt}T_{reg}\right)}{4\gamma T_{reg}\dot{W}_{ac,c}P_o}\right]$$

$$\theta_{pt} = -47°$$

Inertance tube

- The combination of P_o, P_r, f, θ_{pt} and $\dot{W}_{ac,c}$ uniquely defines necessary inertance tube geometry



For P_o=2.5MPa, P_r=1.3, f=60Hz, θ_{pt} = -47°, and $\dot{W}_{ac,c}$=57.1W: L_i=3.46m, D_i=5.61mm

Summary

- Approximate design for Pulse Tube Refrigerator:

T_c	80 K	COP	0.117	V_{pt}	7170mm^3
T_w	300 K	$\dot{W}_{ac,c}$	57.1 W	L_{pt}	53.6 mm
$\dot{Q}_c$	25 W	$\dot{W}_{ac,w}$	214 W	D_{pt}	13.1 mm
P_o	2.5 MPa	L_{reg}	52 mm	θ_{pt}	-47°
P_r	1.3	D_{reg}	23.7 mm	L_i	3.46 m
f	60 Hz	θ_c	-25°	D_i	5.61 mm

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Pulse Tubes: Future Directions & Commercial Sources

- R&D:
  - Phase shifting mechanisms - inertance tubes
  - Large capacity - modeling & losses
  - Performance improvements
- Sources:
  - GM-type
    - Cryomech, SHI (Sumitomo Heavy Industry), U of Giessen,
  - Stirling type
    - Atlas Scientific, STC, Sunpower, TRW, Martin-Marietta, Praxair, Sierra-Lobo