Lecture 7

Microwave Network Analysis

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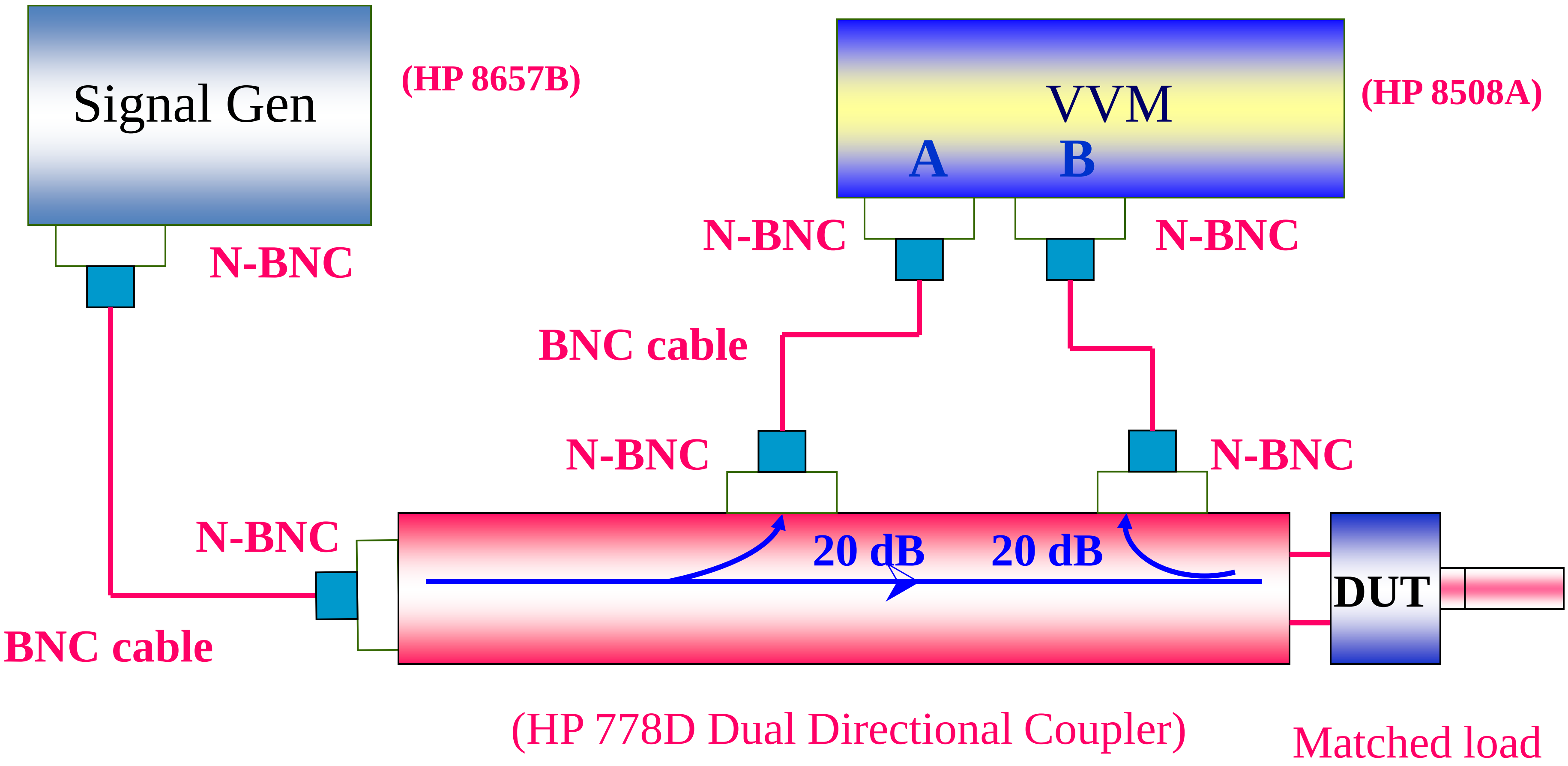
<u>VVM:</u> The vector voltmeter measures the magnitude of a reference and test voltage and the difference in phase between the voltages. Because it can measure phase, it allows us to directly measure the S-parameters of a circuit

Unfortunately, the use of the directional couplers and test cables connecting the measuring system to the vector voltmeter introduces unknown attenuation and phase shift into the measurements. These can be compensated for by making additional “calibration” measurements.


Signal Gen
(HP 8657B)
VVM
A
B
(HP 8508A)
N-BNC
N-BNC
N-BNC
BNC cable
N-BNC
N-BNC
N-BNC
20 dB
20 dB
DUT
BNC cable
(HP 778D Dual Directional Coupler)
Matched load

From the setup, it is seen that the voltage at channel A of the VVM (A^D) is proportional to the amplitude of the voltage wave entering the device under test (DUT) (a^D_1). Similarly, the voltage at channel B (B^D) is proportional to the amplitude of the voltage wave reflected from DUT (b^D_1). Thus we can write

$$A^D = K_A a_1^D$$

$$B^D = K_B b_1^D$$

Where K_A and K_B are constants that depend on the connecting cables. Since a^D_2 is zero because of the matched load at port 2, S11 is given by

$$S_{11} = \frac{b_1^D}{a_1^D} = \frac{B^D / K_B}{A^D / K_A}$$

To find K_A and K_B it is necessary to make a second measurement with a known DUT. This is called a “calibration” measurement. If the DUT is removed and replaced by a short circuit, the voltage at channel A (A^s) and channel B(B^s) are given by

$$A^S = K_A a_1^S$$

$$B^S = K_B b_1^S$$

Where a^s_1 is the amplitude of the voltage wave entering the short and b^s_1 is the amplitude of the voltage wave reflected from the short. However, for a short circuit the ratio of these amplitudes is –1 (reflection coefficient of a short). Thus

$$\frac{b_1^S}{a_1^S} = \frac{B^S / K_B}{A^S / K_A} = -1$$

$$\frac{K_B}{K_A} = -\frac{B^S}{A^S}$$

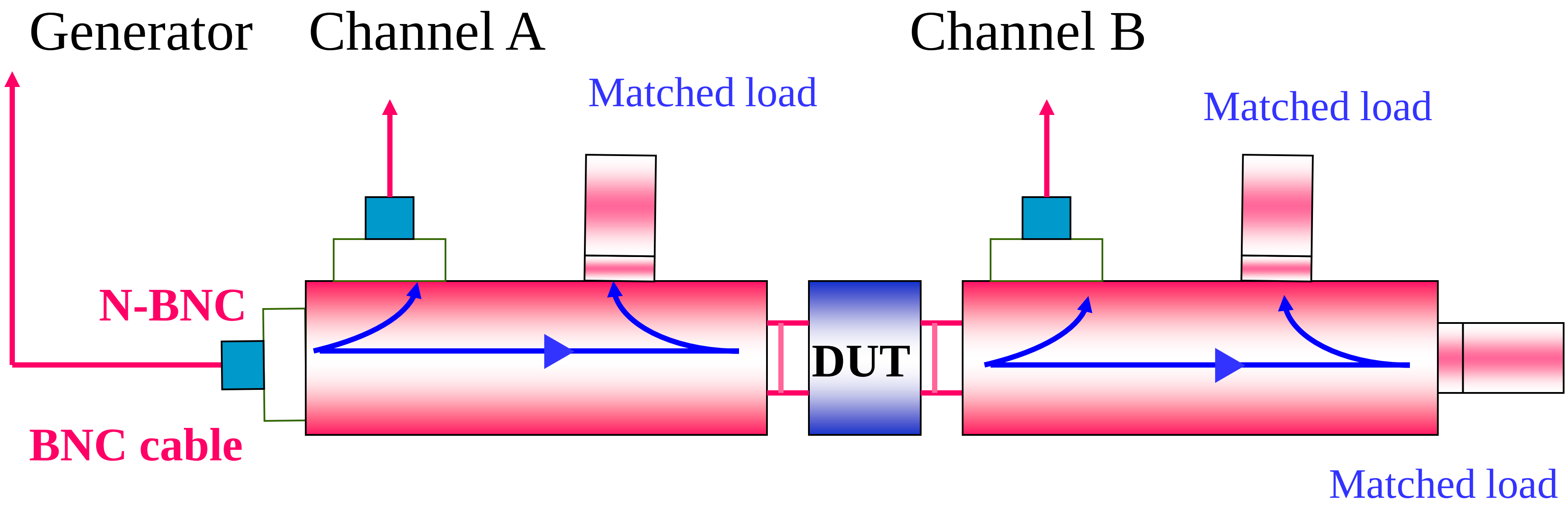
$$S_{11} = -\frac{\left(\dfrac{B^D}{A^D}\right)}{\left(\dfrac{B^S}{A^S}\right)}$$

Note: since VVM displays quantities in terms of magnitude and phase we can rewrite S_{11} as

$$\left(\frac{B^D}{A^D}\right) = \Gamma^D \angle \phi^D$$

$$S_{11} = \frac{\Gamma^D}{\Gamma^S} \angle\left(\phi^D - \phi^S - \pi\right)$$

$$\left(\frac{B^S}{A^S}\right) = \Gamma^S \angle \phi^S$$



The DUT is connected directly between two directional couplers. Voltage at A of the VVM is proportional to the voltage wave incident on the DUT while the voltage at B of the VVM is proportional to voltage wave transmitted through the DUT.

$$A^D = K_A a_1^D$$
$$B^D = K_B b_2^D$$
$$\Rightarrow \quad S_{21} = \frac{b_2^D}{a_1^D} = \frac{B^D / K_B}{A^D / K_A}$$

To find out the constants a calibration measurement must be made. Remove the DUT and connect both directional couplers directly together. The Known DUT in this case is just a zero-length guide with a transmission coefficient of unity. The measured voltages are:

$$A^E = K_A a_1^E$$
$$B^E = K_B b_2^E$$

where

$$\frac{b_2^E}{a_1^E} = \frac{B^E / K_B}{A^E / K_A} = 1$$

$$\therefore \frac{K_B}{K_A} = \frac{B^E}{A^E}$$

Transmission measurements: S_{12} or S_{21}

$$S_{21} = -\frac{\left(\dfrac{B^D}{A^D}\right)}{\left(\dfrac{B^E}{A^E}\right)}$$

$$S_{21} = \frac{T^D}{T^E}\angle\left(\theta^D - \theta E\right)$$

where

$$\left(\frac{B^D}{A^D}\right) = T^D \angle \theta^D$$

$$\left(\frac{B^E}{A^E}\right) = T^E \angle \theta^E$$

- Scattering Parameters (S-Parameters) plays a major role is network analysis

- This importance is derived from the fact that practical system characterizations can no longer be accomplished through simple open- or short-circuit measurements, as is customarily in low-frequency applications.

- In the case of a short circuit with a wire; the wire itself possesses an inductance that can be of substantial magnitude at high frequency.

- Also open circuit leads to capacitive loading at the terminal.

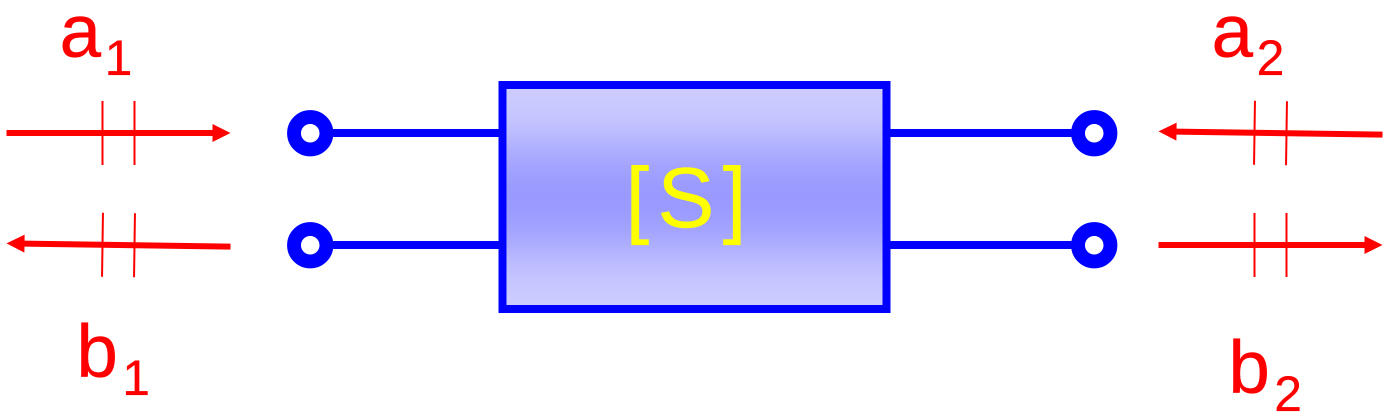
- In either case, the open/short-circuit conditions needed to determine Z-, Y-, h-, and ABCD-parameters can no longer be guaranteed.

- Moreover, when dealing with wave propagation phenomena, it is not desirable to introduce a reflection coefficient whose magnitude is unity.

- For instance, the terminal discontinuity will cause undesirable voltage and/or current wave reflections, leading to oscillation that can result in the destruction of the device.

- With S-parameters, one has proper tool to characterize the two-port network description of practically all RF devices without harm to DUT.

- S-parameters are power wave descriptors that permit us to define the input-output relations of a network in terms of incident and reflected power waves.



a_n – **normalized incident power waves**

b_n – **normalized reflected power waves**

$$a_n = \frac{1}{2\sqrt{Z_\circ}}\left(V_n + Z_\circ I_n\right) \qquad (1)$$

$$b_n = \frac{1}{2\sqrt{Z_\circ}}\left(V_n - Z_\circ I_n\right) \qquad (2)$$

- Index n refers either to port number 1 or 2. The impedance Z_0 is the characteristic impedance of the connecting lines on the input and output side of the network.

- Inverting (1) leads to the following voltage and current expressions:

$$V_n = \sqrt{Z_\circ}(a_n + b_n) \quad (3)$$

$$I_n = \frac{1}{\sqrt{Z_\circ}}(a_n - b_n) \quad (4)$$

- Recall the equations for power:

$$P_n = \frac{1}{2} Re\left\{V_n I_n^*\right\} = \frac{1}{2}\left(\left|a_n\right|^2 - \left|b_n\right|^2 \right) \quad (5)$$

- Isolating forward and backward traveling wave components in (3) and (4), we see

$$a_n = \frac{V_n^+}{\sqrt{Z_\circ}} = \sqrt{Z_\circ} I_n^+ \quad (6)$$

$$b_n = \frac{V_n^-}{\sqrt{Z_\circ}} = -\sqrt{Z_\circ} I_n^- \quad (7)$$

- We can now define S-parameters:

$$\begin{Bmatrix} b_1 \\ b_2 \end{Bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \begin{Bmatrix} a_1 \\ a_2 \end{Bmatrix} \quad (8)$$

$$S_{11} = \left.\frac{b_1}{a_1}\right|_{a_2=0} = \frac{\text{Refkected powe wave at port 1}}{\text{Incident power wave at port 2}} \qquad (9)$$

$$S_{21} = \left.\frac{b_2}{a_1}\right|_{a_2=0} = \frac{\text{Transmitted powe wave at port 2}}{\text{Incident power wave at port 1}} \qquad (10)$$

$$S_{22} = \left.\frac{b_2}{a_2}\right|_{a_1=0} = \frac{\text{Refkected powe wave at port 2}}{\text{Incident power wave at port 2}} \qquad (11)$$

$$S_{12} = \left.\frac{b_1}{a_2}\right|_{a_1=0} = \frac{\text{Transmitted powe wave at port 1}}{\text{Incident power wave at port 2}} \qquad (12)$$

- ❑ a_2=0, and a_1=0 ⇒ no power waves are returned to the network at either port 2 or port 1.

- ❑ However, these conditions can only be ensured when the connecting transmission line are terminated into their characteristic impedances.

- ❑ Since the S-parameters are closely related to power relations, we can express the normalized input and output waves in terms of time averaged power.

- ❑ The average power at port 1 is given by

$$P_1 = \frac{1}{2}\frac{\left|V_1^+\right|^2}{Z_\circ}\left(1-\left|\Gamma_{in}\right|^2\right) = \frac{1}{2}\frac{\left|V_1^+\right|^2}{Z_\circ}\left(1-\left|S_{11}\right|^2\right) \quad (13)$$

- The reflection coefficient at the input side is expressed in terms of S_{11} under matched output according:

$$\Gamma_{in} = \frac{V_1^-}{V_1^+} = \frac{b_1}{a_1}\bigg|_{a_2=0} = S_{11} \quad (14)$$

- This also allow us to redefine the VSWR at port 1 in terms of S_{11} as

$$VSWR = \frac{1+|S_{11}|}{1-|S_{11}|} \quad (15)$$

❑ We can identify the incident power in (13) and express it in terms of a_1:

$$\frac{1}{2}\frac{\left|V_1^+\right|^2}{Z_\circ} = P_{inc} = \frac{\left|a_1\right|^2}{2} \qquad (16)$$

Maximal available power from the generator

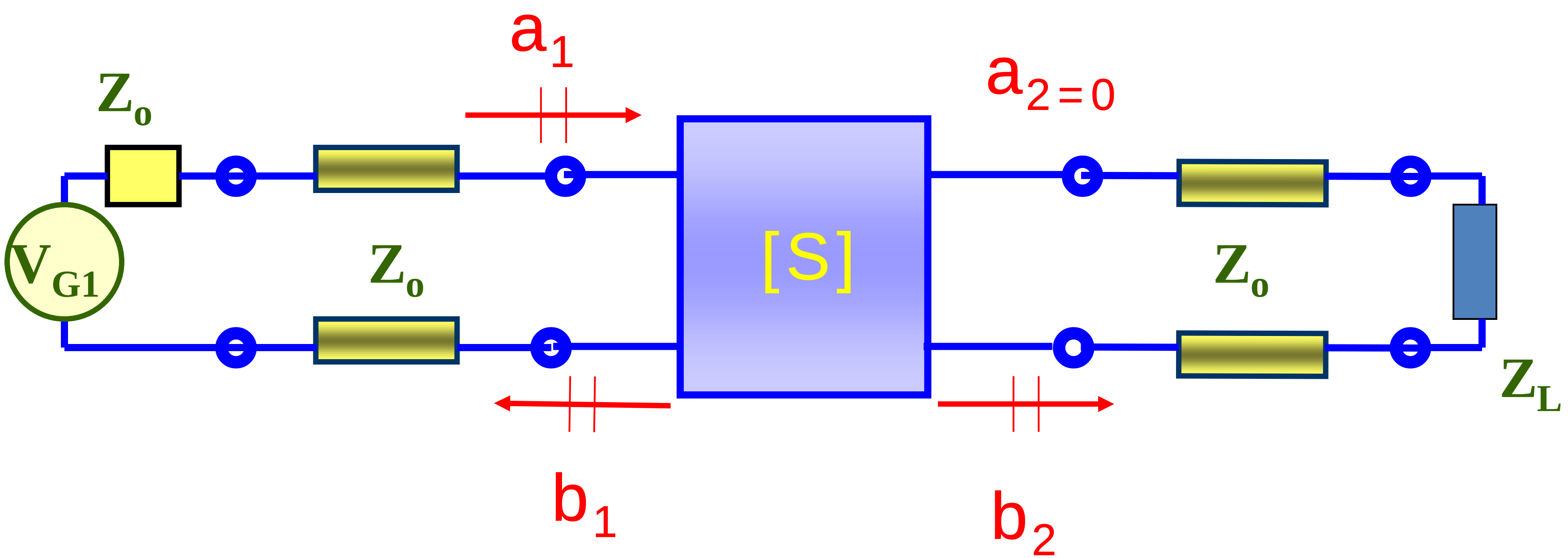
❑ The total power at port 1 (under matched output condition) expressed as a combination of incident and reflected powers:

$$P_1 = P_{inc} + P_{refl} = \frac{1}{2}\left(\left|a_1\right|^2 - \left|b_1\right|^2\right) = \frac{\left|a_1\right|^2}{2}\left(1 - \left|\Gamma_{in}\right|^2\right) \qquad (17)$$

- If the reflected coefficient, or S_{11}, is zero, all available power from the source is delivered to port 1 of the network. An identical analysis at port 2 gives

$$P_2 = \frac{1}{2}\left(|a_2|^2 - |b_2|^2\right) = \frac{|a_2|^2}{2}\left(1 - |\Gamma_{out}|^2\right) \qquad (18)$$

- S-parameters can only be determined under conditions of perfect matching on the input or the output side.



Measurement of S_{11} and S_{21} by matching the line impedance Z_o at port 2 through a corresponding load impedance $Z_L=Z_o$

- This configuration allows us to compute S_{11} by finding the input reflection coefficient:

$$S_{11} = \Gamma_{in} = \frac{Z_{in} - Z_{\circ}}{Z_{in} + Z_{\circ}} \qquad (19)$$

- Taking the logarithm of the magnitude of S_{11} gives us the return loss in dB

$$RL = -20\,log\left|S_{11}\right| \qquad (20)$$

- With port 2 properly terminated, we find

$$S_{21} = \left.\frac{b_2}{a_1}\right|_{a_2=0} = \left.\frac{V_2^- / \sqrt{Z_\circ}}{\left(V_1 + Z_\circ I_1\right) / \left(2\sqrt{Z_\circ}\right)}\right|_{I_2^+=V_2^+=0} \qquad (21)$$

- Since a_2=0, we can set to zero the positive traveling voltage and current waves at port 2.

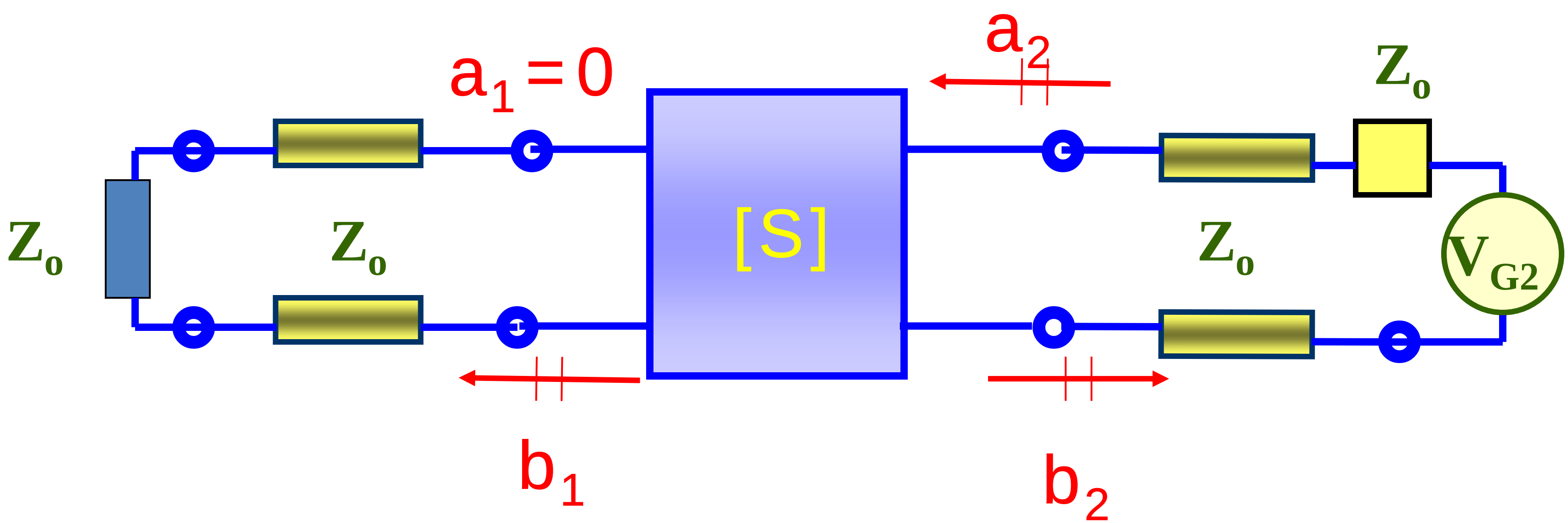
- Replacing V_1 by the generator voltage V_{G1} minus the voltage drop over the source impedance Z_o, V_{G1}-Z_oI_1 gives

$$S_{21} = \frac{2V_2^-}{V_{G1}} = \frac{2V_2}{V_{G1}} \qquad (22)$$

- The forward power gain is

$$G_{\circ} = |S_{21}|^2 = \left|\frac{V_2}{V_{G1}/2}\right|^2 \quad (23)$$

- If we reverse the measurement procedure and attach a generator voltage V_{G2} to port 2 and properly terminate port 1, we can determine the remaining two S-parameters, S_{22} and S_{12}.



- To compute S_{22} we need to find the output reflection coefficient Γ_{out} in a similar way for S_{11}:

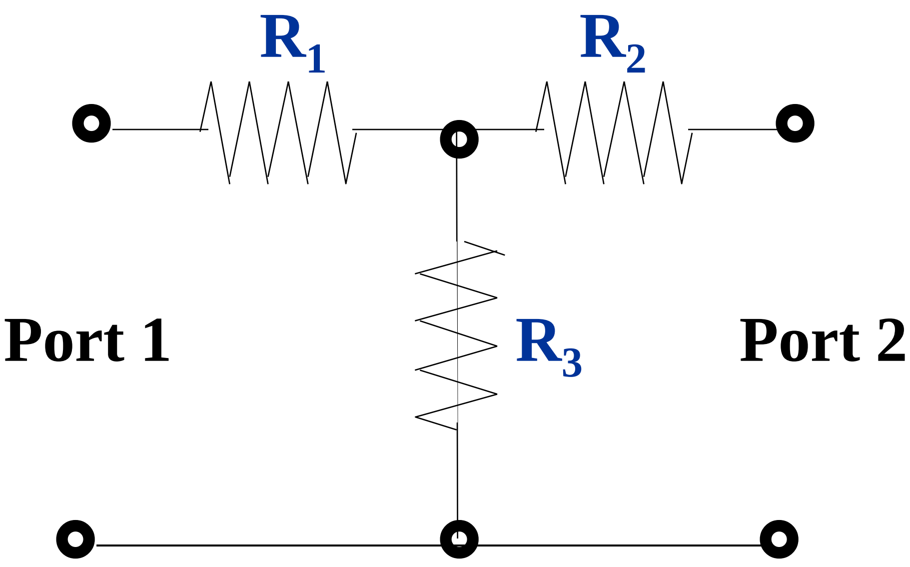
$$S_{22} = \Gamma_{out} = \frac{Z_{out} - Z_\circ}{Z_{out} + Z_\circ} \qquad (24)$$

$$S_{12} = \left.\frac{b_1}{a_2}\right|_{a_1=0} = \left.\frac{V_1^- / \sqrt{Z_\circ}}{\left(V_2 + Z_\circ I_2\right) / \left(2\sqrt{Z_\circ}\right)}\right|_{I_1^+ = V_1^+ = 0} \qquad (25)$$

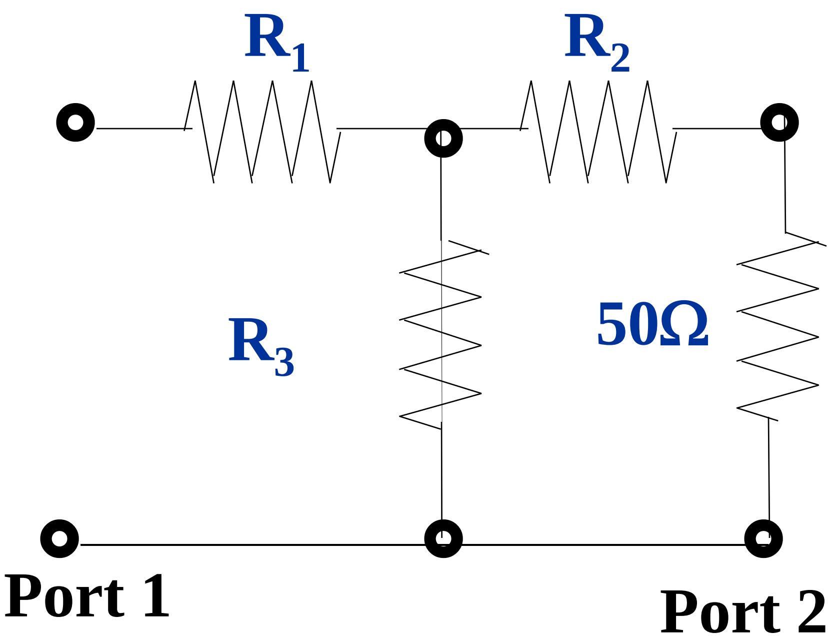
$$S_{12} = \frac{2V_1^-}{V_{G2}} = \frac{2V_1}{V_{G2}} \qquad (26) \qquad G_{or} = \left|S_{12}\right|^2 = \left|\frac{V_1}{V_{G2}/2}\right|^2 \qquad (27)$$

Reverse power gain

Find the S-parameters and resistive elements for the 3-dB attenuator network. Assume that the network is placed into a transmission line section with a characteristic line impedance of Z_o=50 Ω



An attenuator should be matched to the line impedance and must meet the requirement $S_{11} = S_{22} = 0$.

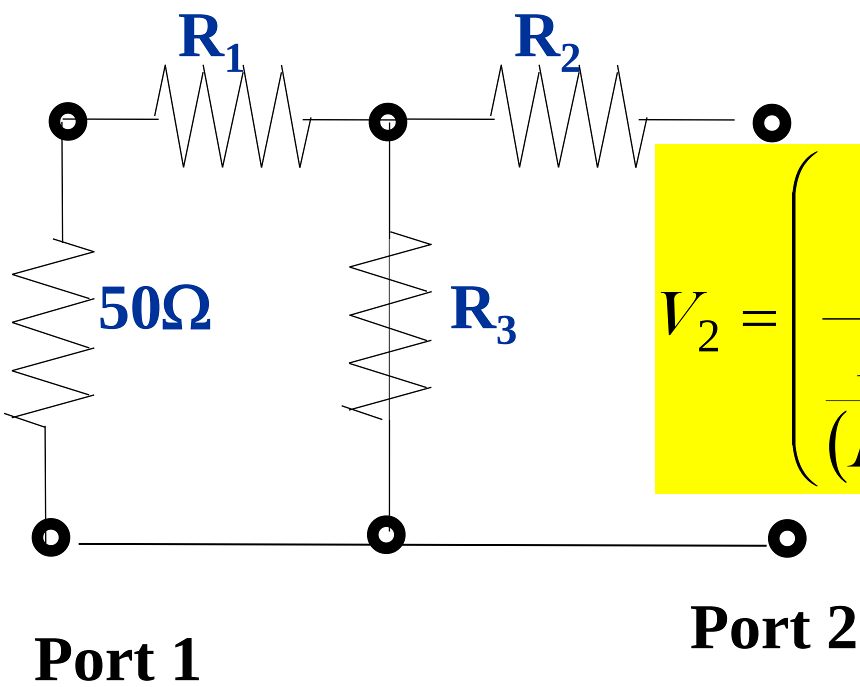


Circuit for S_{11} and S_{21}

$$Z_{in} = R_1 + \frac{R_3(R_2 + 50\Omega)}{(R_3 + R_2 + 50\Omega)} = 50\Omega$$

Because of symmetry, it is clear that $R_1 = R_2$.

We now investigate the voltage $V_2 = V^-_2$ at port 2 in terms of $V_1 = V^+_1$.



$$V_2 = \left(\frac{\dfrac{R_3\left(R_1 + 50\Omega\right)}{\left(R_3 + R_1 + 50\Omega\right)}}{\dfrac{R_3\left(R_1 + 50\Omega\right)}{\left(R_3 + R_1 + 50\Omega\right)} + R_1} \right) \left(\frac{50\Omega}{50\Omega + R_1} \right) V_1$$

For a 3 dB attenuation, we require

$$S_{21} = \frac{2V_2}{V_{G1}} = \frac{V_2}{V_1} = \frac{1}{\sqrt{2}} = 0.707 = S_{12}$$

Setting the ratio of V_2/V_1 to 0.707 and using the input impedance expression, we can determine R_1 and R_3

$$R_1 = R_2 = \frac{\sqrt{2}-1}{\sqrt{2}+1} Z_\circ = 8.58\Omega$$

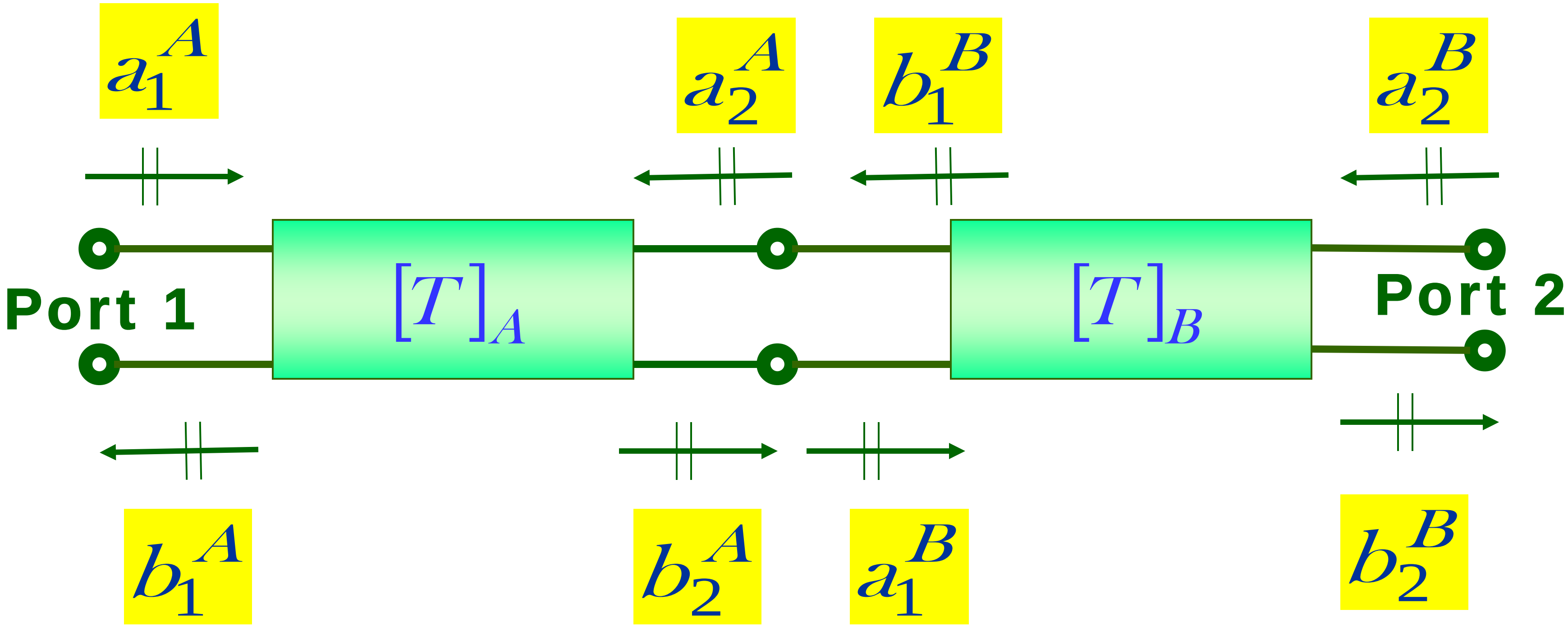
$$R_3 = 2\sqrt{2} Z_\circ = 141.4\Omega$$

Note: *the choice of the resistor network ensures that at the input and output ports an impedance of 50 Ω is maintained. This implies that this network can be inserted into a 50 Ω transmission line section without causing undesired reflections, resulting in an insertion loss.*

- To extend the concept of the S-parameter presentation to cascaded network, it is more efficient to rewrite the power wave expressions arranged in terms of input and output ports. This results in the chain scattering matrix notation. That is,

$$\begin{Bmatrix} a_1 \\ b_1 \end{Bmatrix} = \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix} \begin{Bmatrix} b_2 \\ a_2 \end{Bmatrix} \qquad (28)$$

- It is immediately seen that cascading of two dual-port networks becomes a simple multiplication.



Cascading of two networks A and B

If network A is described by

$$\begin{Bmatrix} a_1^A \\ b_1^A \end{Bmatrix} = \begin{bmatrix} T_{11}^A & T_{12}^A \\ T_{21}^A & T_{22}^A \end{bmatrix} \begin{Bmatrix} b_2^A \\ a_2^A \end{Bmatrix} \qquad (29)$$

And network B by

$$\begin{Bmatrix} a_1^B \\ b_1^B \end{Bmatrix} = \begin{bmatrix} T_{11}^B & T_{12}^B \\ T_{21}^B & T_{22}^B \end{bmatrix} \begin{Bmatrix} b_2^B \\ a_2^B \end{Bmatrix} \qquad (30)$$

$$\begin{Bmatrix} b_2^A \\ a_2^A \end{Bmatrix} = \begin{Bmatrix} a_1^B \\ b_1^B \end{Bmatrix} \quad (31)$$

Thus, for the combined system, we conclude

$$\begin{Bmatrix} a_1^A \\ b_1^A \end{Bmatrix} = \begin{bmatrix} T_{11}^A & T_{12}^A \\ T_{21}^A & T_{22}^A \end{bmatrix} \begin{bmatrix} T_{11}^B & T_{12}^B \\ T_{21}^B & T_{22}^B \end{bmatrix} \begin{Bmatrix} b_2^B \\ a_2^B \end{Bmatrix} \quad (31)$$

The conversion from S-matrix to the chain matrix notation is similar as described before.

$$T_{11} = \left.\frac{a_1}{b_2}\right|_{a_2=0} = \frac{a_1}{S_{21}a_1} = \frac{1}{S_{21}} \qquad (32)$$

$$T_{12} = -\frac{S_{22}}{S_{21}} \qquad (33)$$

$$T_{21} = \frac{S_{11}}{S_{21}} \qquad (34)$$

$$T_{22} = \frac{-\left(S_{11}S_{22} - S_{12}S_{21}\right)}{S_{21}} = \frac{-\Delta S}{S_{21}} \qquad (35)$$

Conversely, when the chain scattering parameters are given and we need to convert to S-parameters, we find the following relations:

$$S_{11} = \left.\frac{b_1}{a_2}\right|_{a_2=0} = \frac{T_{21}b_2}{T_{11}b_2} = \frac{T_{21}}{T_{11}} \qquad (36)$$

$$S_{12} = \frac{(T_{11}T_{22} - T_{12}T_{21})}{T_{11}} = \frac{\Delta T}{T_{11}} \qquad (37)$$

$$S_{21} = \frac{1}{T_{11}} \qquad (38)$$

$$S_{22} = -\frac{T_{12}}{T_{11}} \qquad (39)$$

To find the conversion between the S-parameters and the Z-parameters, let us begin with defining S-parameters relation in matrix notation

$$\{b\} = [S]\{a\} \quad (40)$$

Multiplying by $\sqrt{Z_\circ}$ gives

$$\sqrt{Z_\circ}\{b\} = \{V^-\} = \sqrt{Z_\circ}[S]\{a\} = [S]\{V^+\} \quad (41)$$

Adding $\{V^+\} = \sqrt{Z_\circ}\{a\}$ to both sides results in

$$\{V\} = [S]\{V^+\} + \{V^+\} = ([S] + [E])\{V^+\} \quad (42)$$

To compare this form with the impedance expression

$$\{V\} = [Z]\{I\}$$

We have to express $\{V^+\}$ in term of $\{I\}$. Subtract $[S]\{V^+\}$ from both sides of

$$\{V^+\} = \sqrt{Z_\circ}\{a\}$$

$$\{V^+\} - [S]\{V^+\} = \sqrt{Z_\circ}(\{a\} - \{b\}) = Z_\circ\{I\} \quad (43)$$

$$\{V^+\} = Z_\circ([E] - [S])^{-1}\{I\} \quad (44)$$

Substituting (44) into (42) yields

$$\{V\} = ([S]+[E])\{V^+\} = Z_\circ([S]+[E])([E]-[S])^{-1}\{I\} \quad (45)$$

or

$$[Z] = Z_\circ([S]+[E])([E]-[S])^{-1} \quad (46)$$

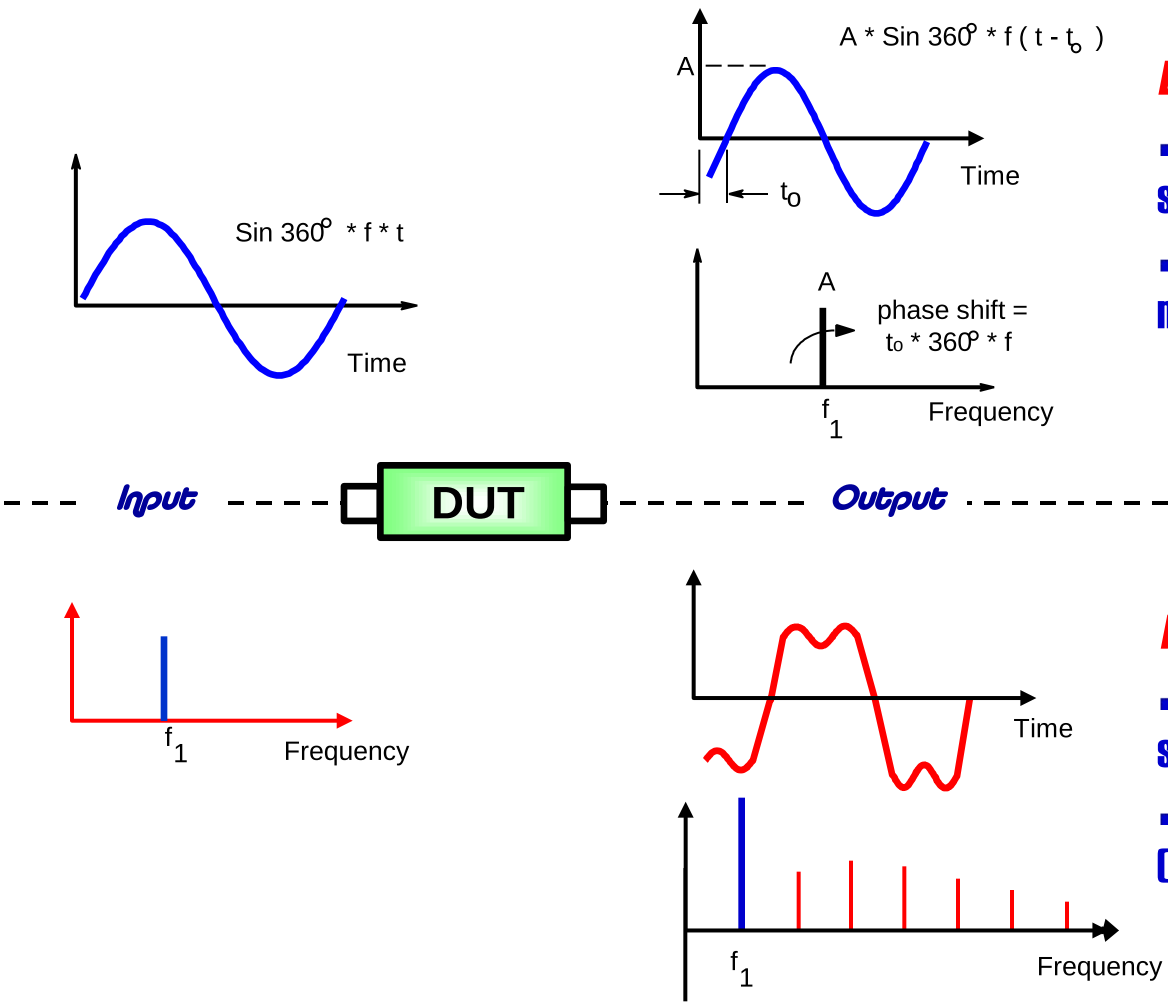
Explicitly

$$\begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} = Z_\circ \begin{bmatrix} 1+S_{11} & S_{12} \\ S_{21} & 1+S_{22} \end{bmatrix} \begin{bmatrix} 1-S_{11} & -S_{12} \\ -S_{21} & 1-S_{22} \end{bmatrix}^{-1}$$

$$= \frac{Z_\circ \begin{bmatrix} 1+S_{11} & S_{12} \\ S_{21} & 1+S_{22} \end{bmatrix}}{(1-S_{11})(1-S_{22})-S_{21}S_{12}} \begin{bmatrix} 1-S_{22} & S_{12} \\ S_{21} & 1-S_{11} \end{bmatrix} \quad (47)$$

Practical Network Analysis

Linear Versus Nonlinear Behavior



Linear behavior:

- input and output frequencies are the same (no additional frequencies created)
- output frequency only undergoes magnitude and phase change

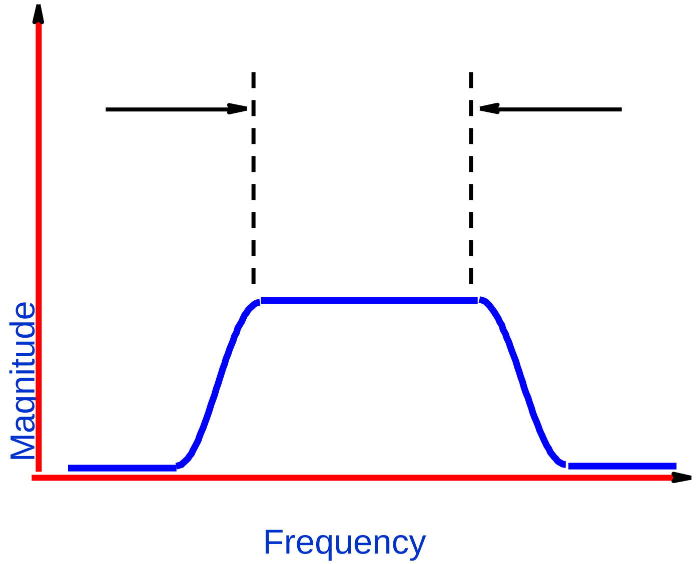
Nonlinear behavior:

- output frequency may undergo frequency shift (e.g. with mixers)
- additional frequencies created (harmonics, inter-modulation)

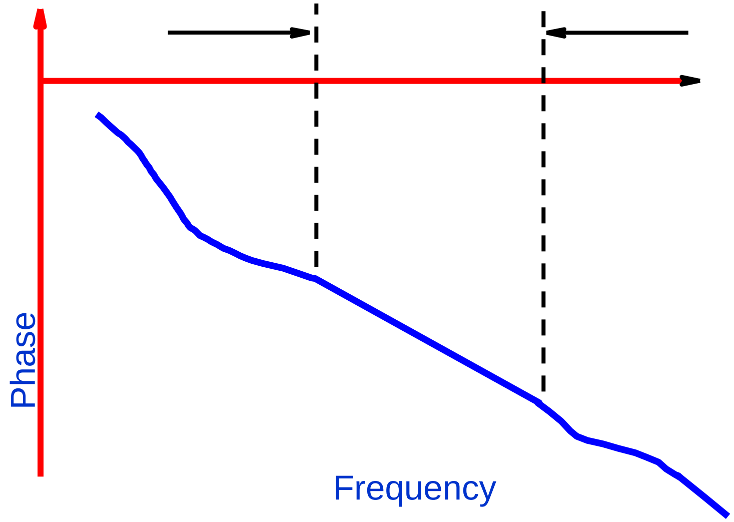
Criteria for Distortionless Transmission

Linear Networks

Constant amplitude over bandwidth of interest

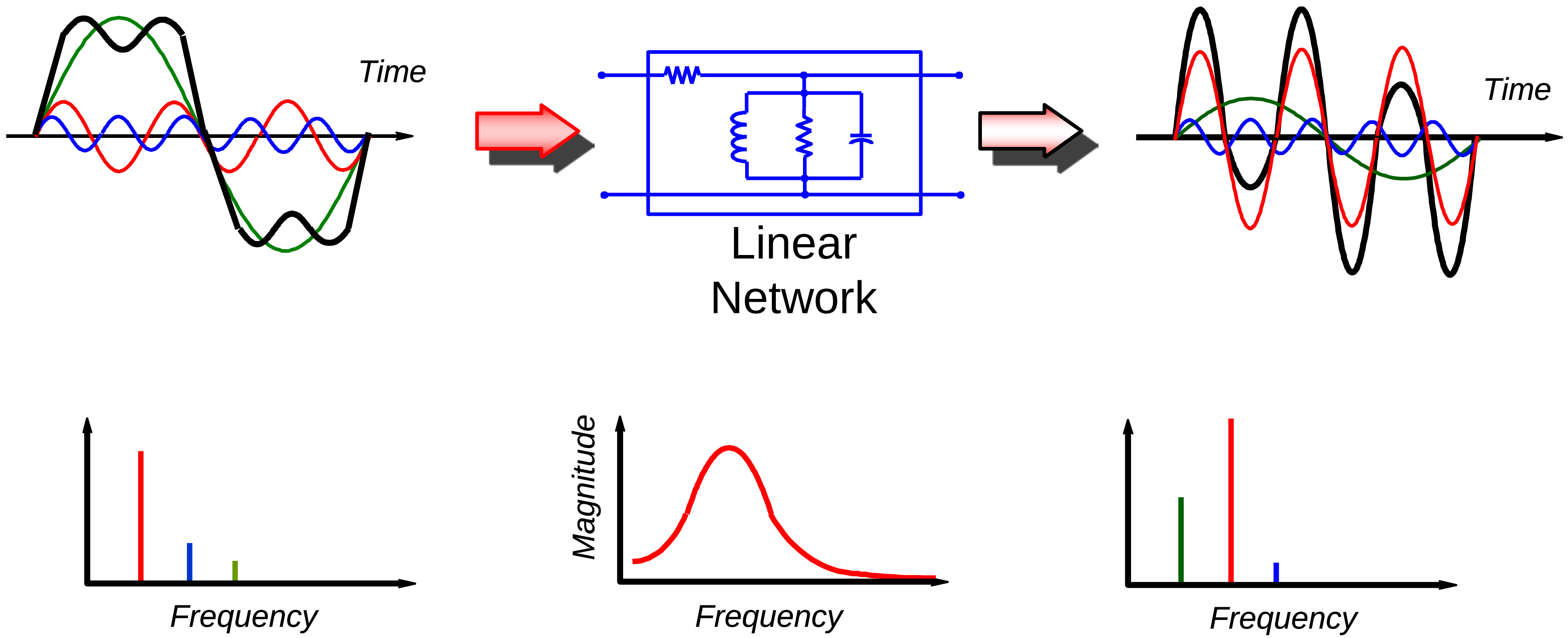


Linear phase over bandwidth of interest



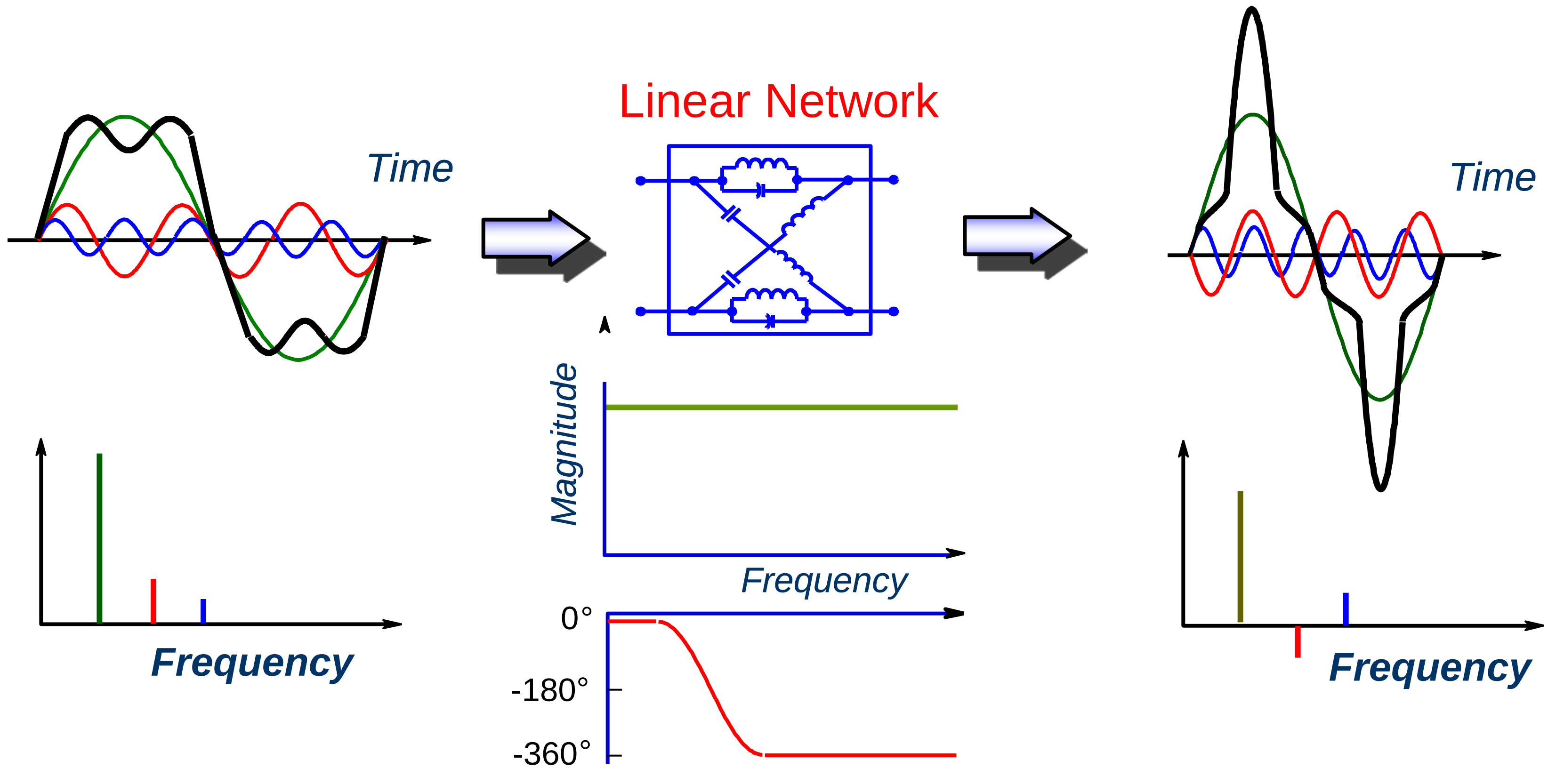
Magnitude Variation with Frequency

$$f(t) = \sin\omega t + \frac{1}{3}\sin 3\omega t + \frac{1}{5}\sin 5\omega t$$



Phase Variation with Frequency

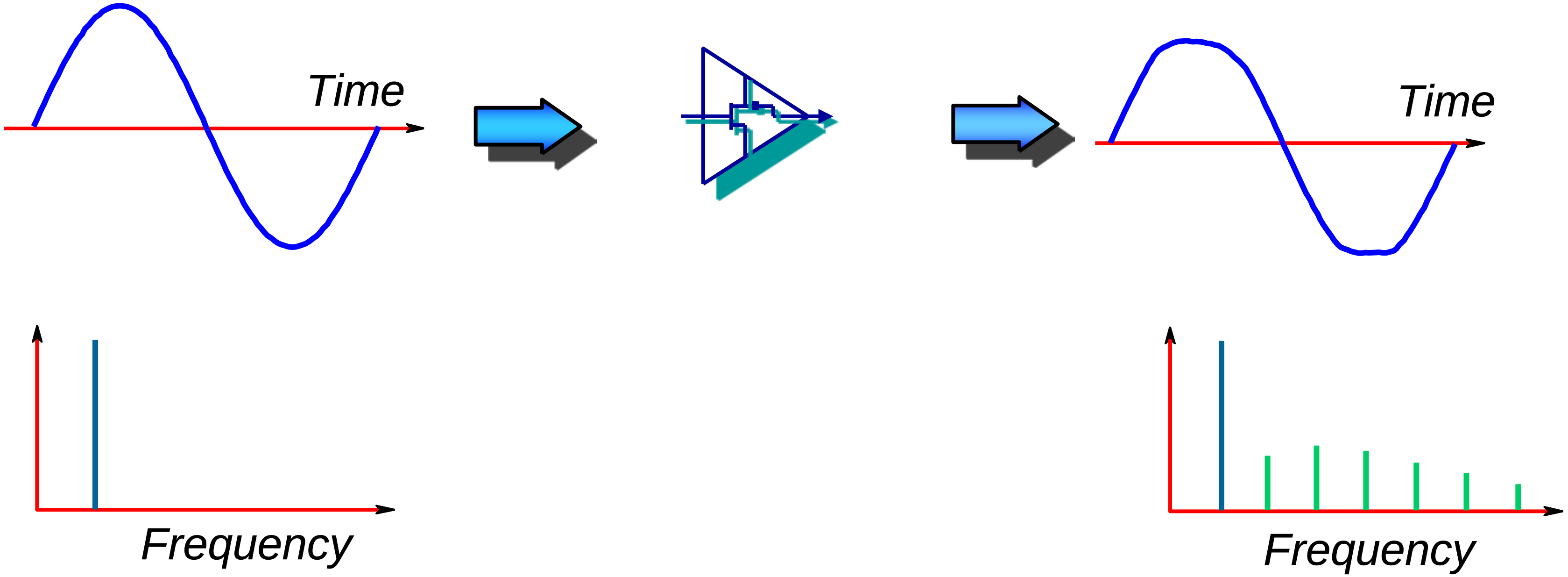
$$f(t) = \sin\omega t + \frac{1}{3}\sin 3\omega t + \frac{1}{5}\sin 5\omega t$$



Criteria for Distortionless Transmission

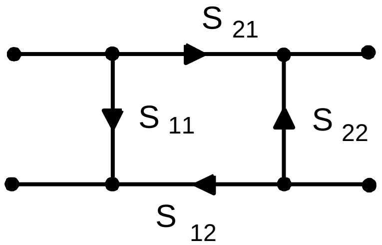
Nonlinear Networks

Saturation, crossover, inter-modulation, and other nonlinear effects can cause signal distortion



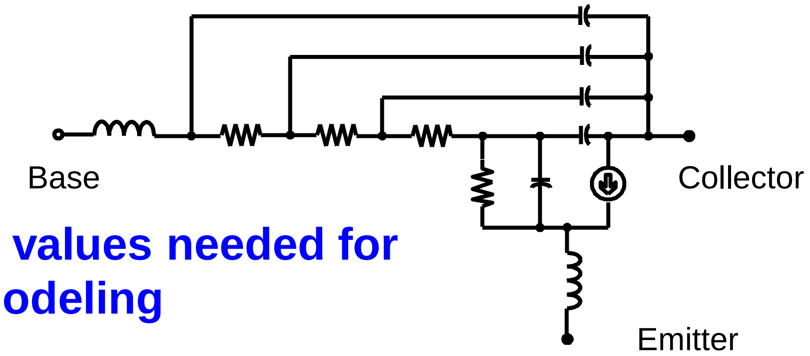
The Need for Both Magnitude and Phase

1. Complete characterization of linear networks

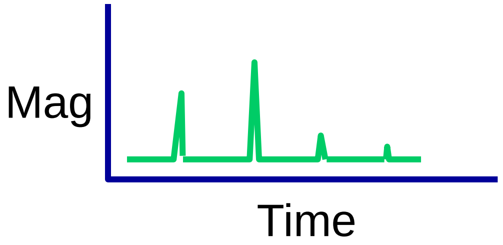


2. Complex impedance needed to design matching circuits

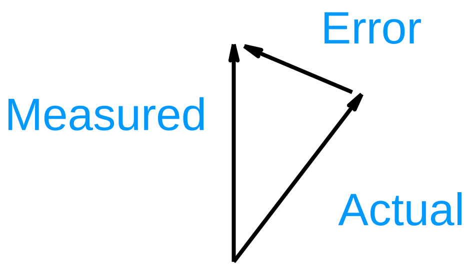
3. Complex values needed for device modeling



4. Time Domain Characterization

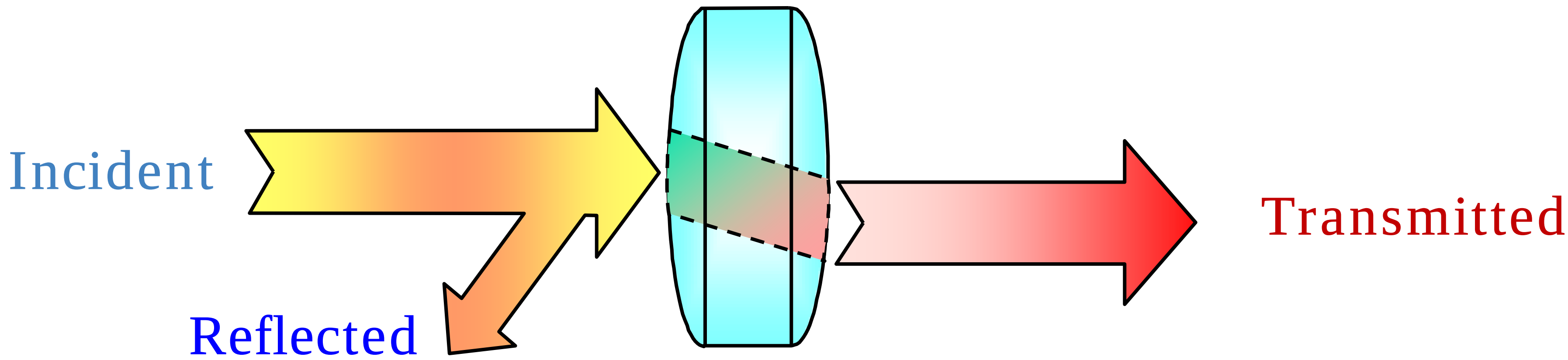


5. Vector Accuracy Enhancement

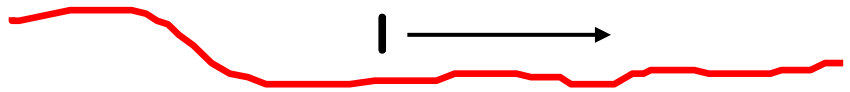


High-Frequency Device Characterization

Lightwave Analogy

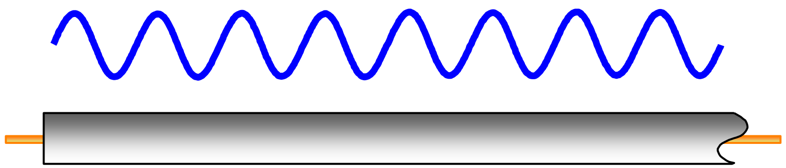


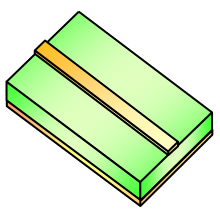
Transmission Line Review



Low Frequencies

- Wavelength >> wire length
- Current (I) travels down wires easily for efficient power transmission
- Voltage and current not dependent on position

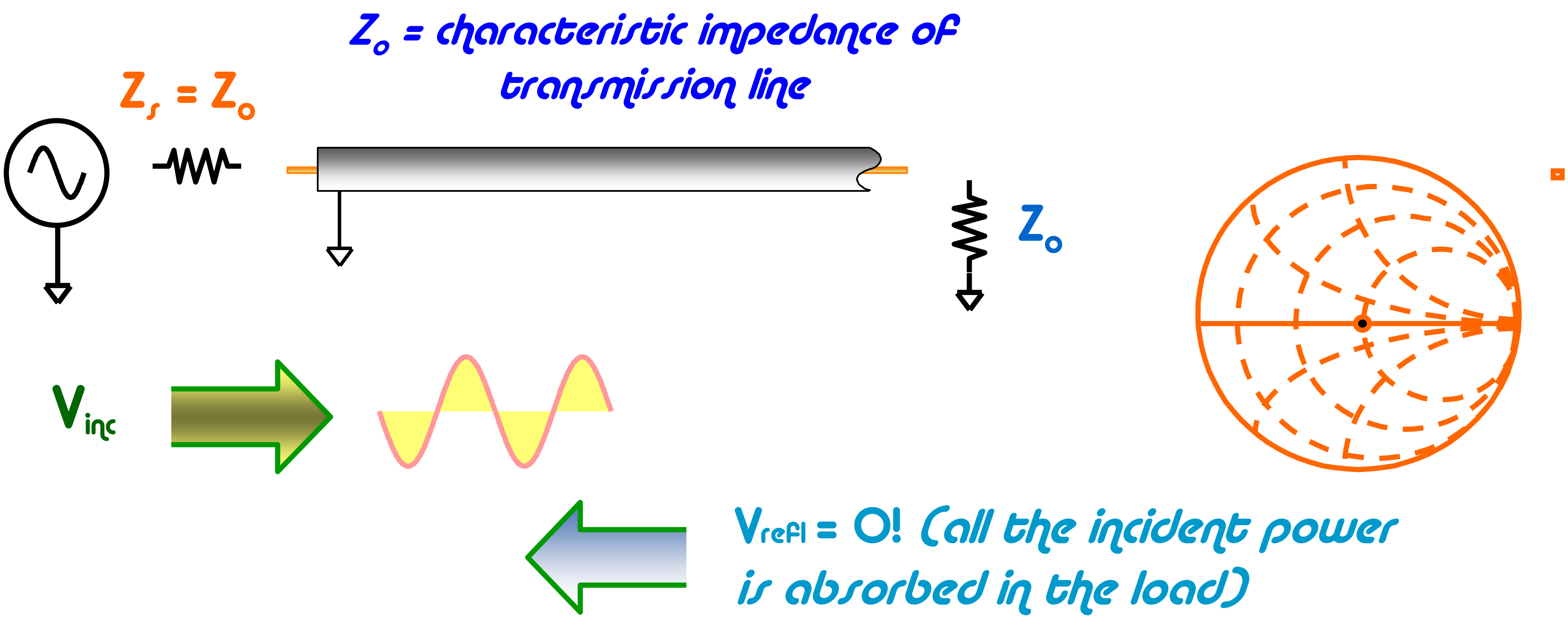




High Frequencies

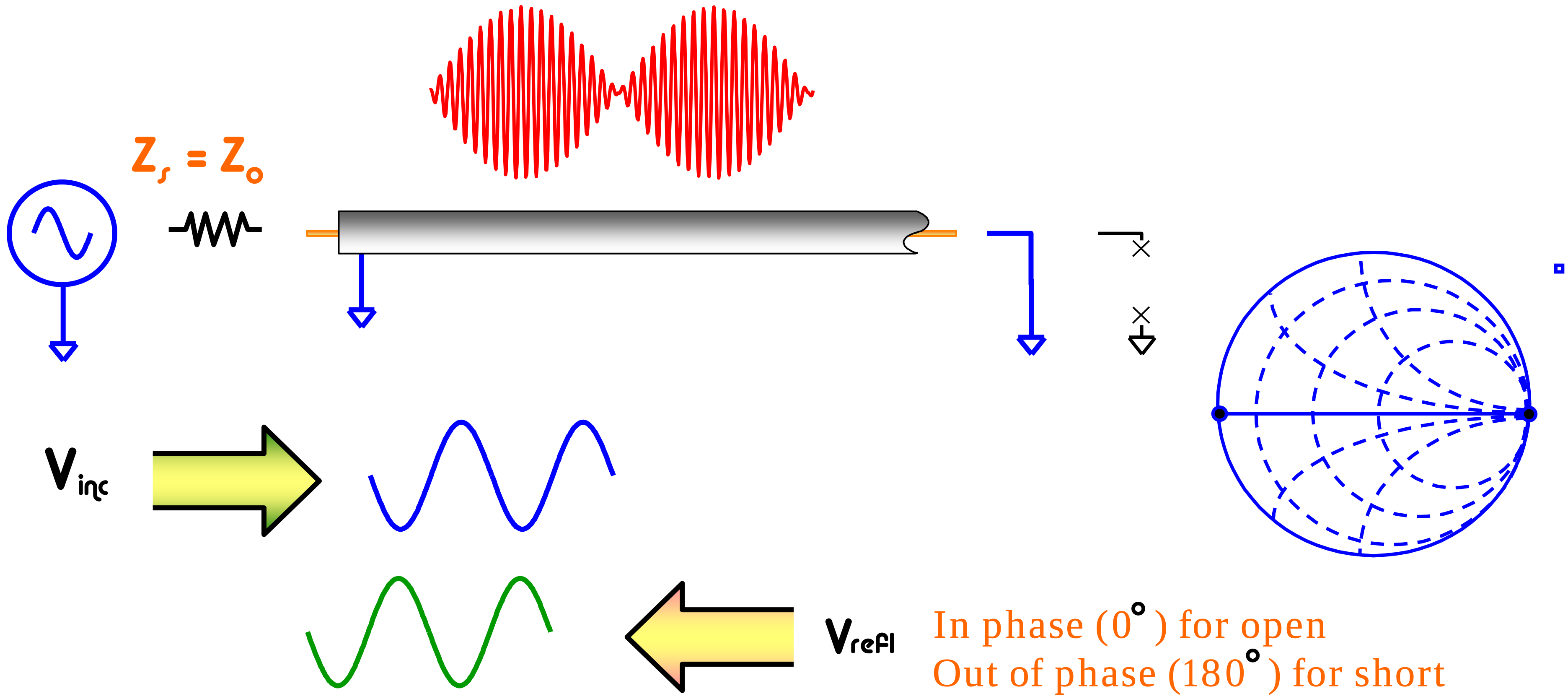
- Wavelength ≈ or << wire (transmission line) length
- Need transmission-line structures for efficient power transmission
- Matching to characteristic impedance (Z_0) is very important for low reflection
- Voltage dependent on position along line

Transmission Line Terminated with Z_o



For reflection, a transmission line terminated in Zo behaves like an infinitely long transmission line

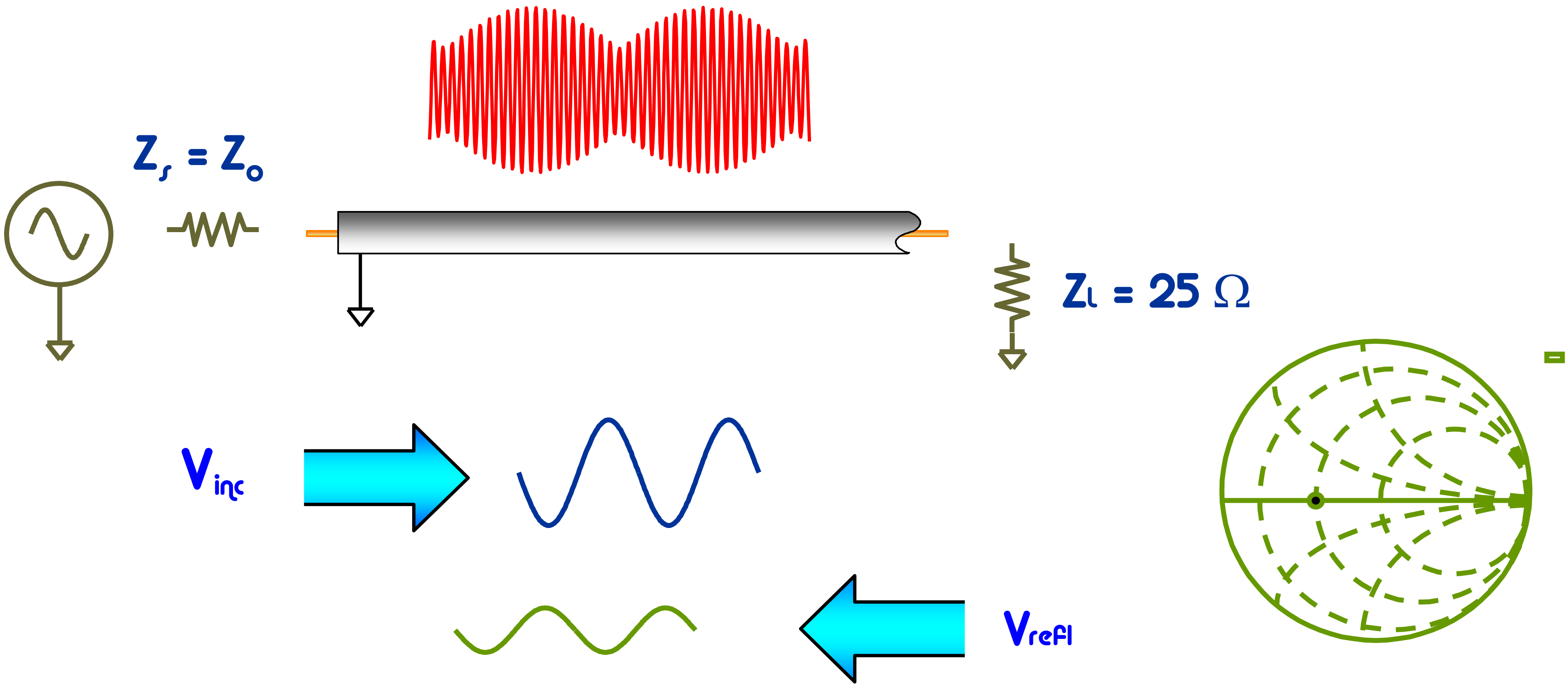
Transmission Line Terminated with Short, Open



In phase (0°) for open
Out of phase (180°) for short

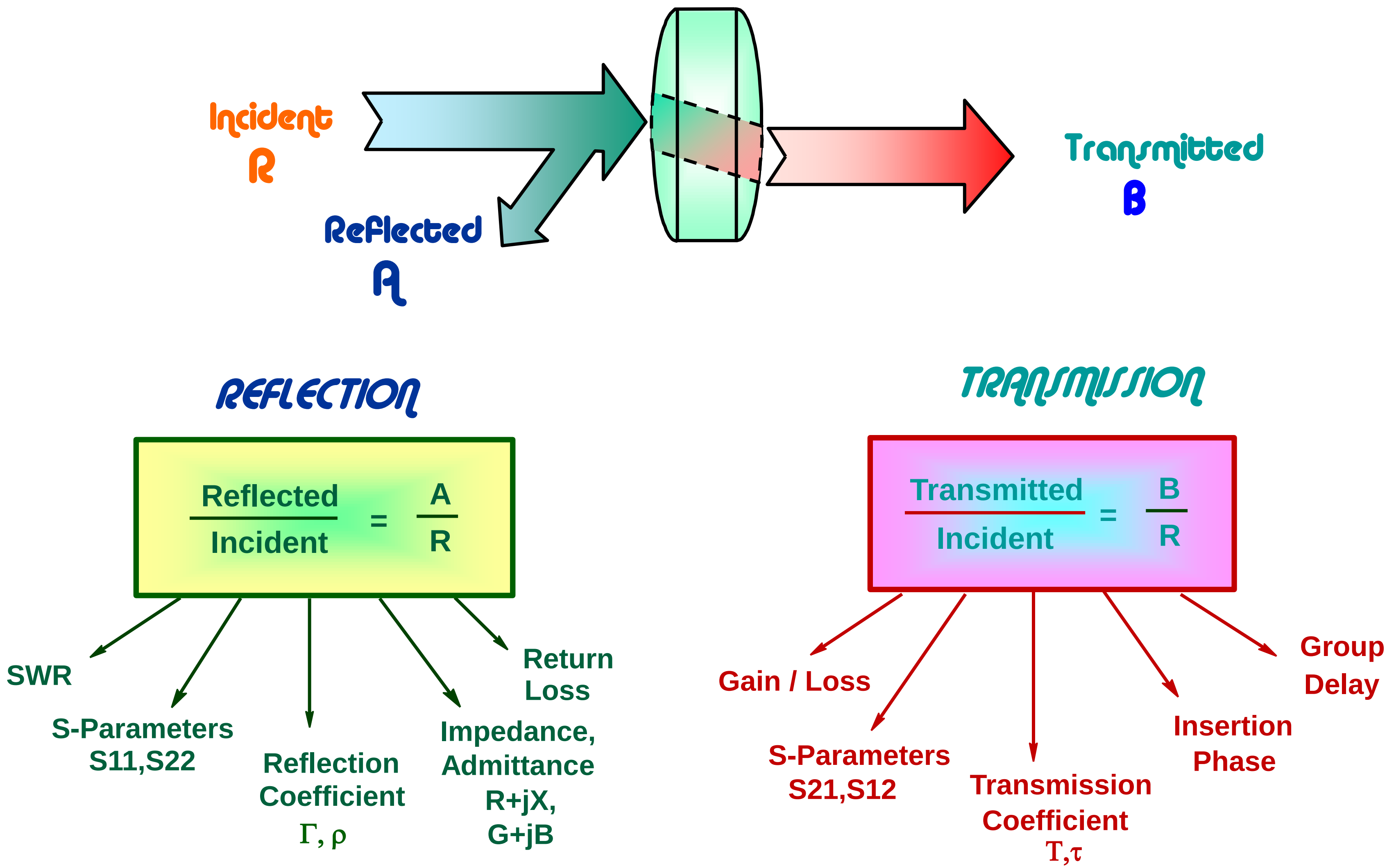
For reflection, a transmission line terminated in a short or open reflects all power back to source

Transmission Line Terminated with 25Ω



Standing wave pattern does not go to zero as with short or open

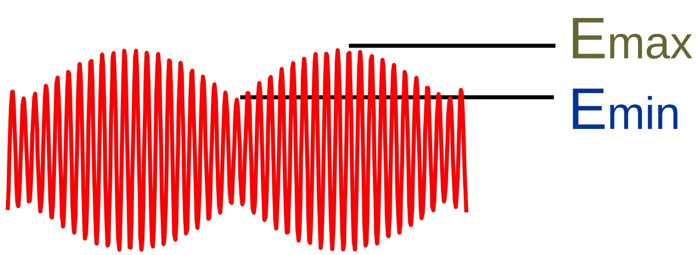
High-Frequency Device Characterization



Reflection Parameters

Reflection Coefficient $\Gamma = \dfrac{V_{reflected}}{V_{incident}} = \rho \angle \Phi = \dfrac{Z_L - Z_O}{Z_L + Z_O}$

Return loss = -20 log(ρ), $\rho = |\Gamma|$

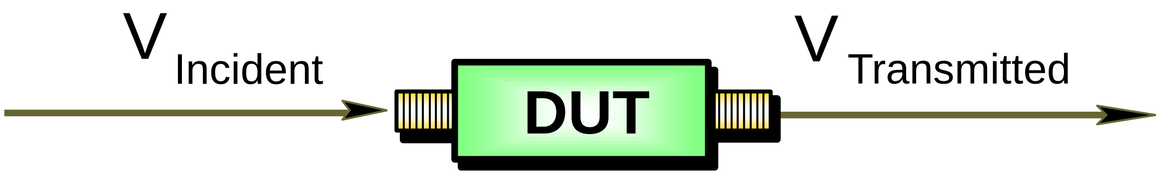


Voltage Standing Wave Ratio

$$\text{VSWR} = \frac{E_{max}}{E_{min}} = \frac{1+\rho}{1-\rho}$$

No reflection ($Z_L = Zo$)		Full reflection (Z_L = open, short)
0	ρ	1
∞ dB	RL	0 dB
1	VSWR	∞

Transmission Parameters



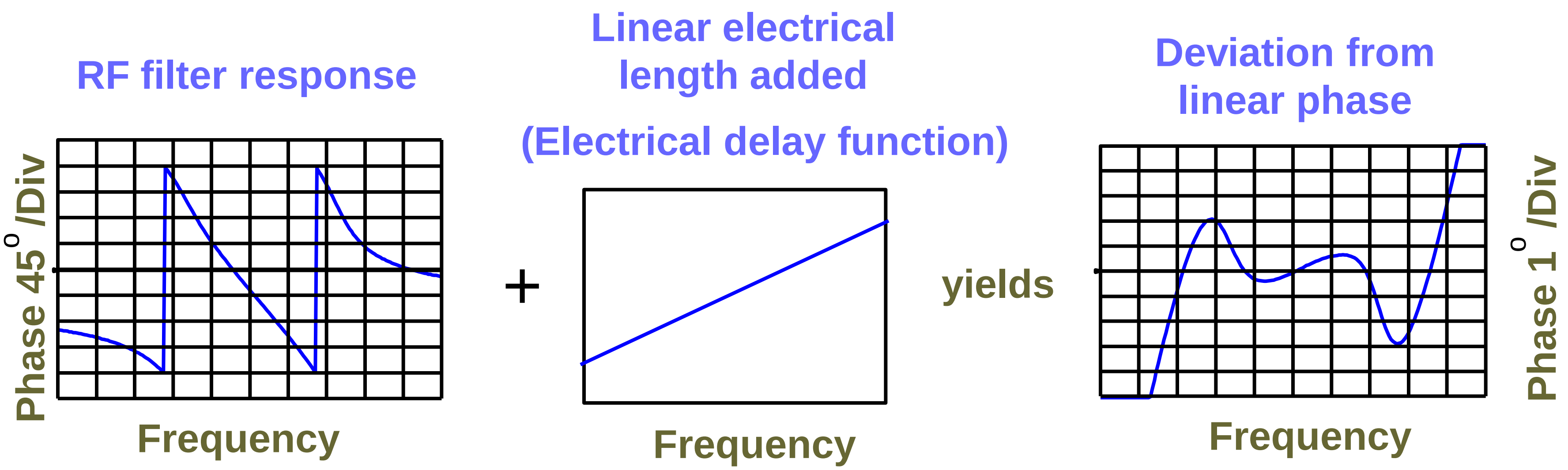
Transmission Coefficient $= T = \dfrac{V_{Transmitted}}{V_{Incident}} = \tau \angle \phi$

Insertion Loss (dB) $= -20 \, Log \left| \dfrac{V_{Trans}}{V_{Inc}} \right| = -20 \log \tau$

Gain (dB) $= 20 \, Log \left| \dfrac{V_{Trans}}{V_{Inc}} \right| = 20 \log \tau$

Deviation from Linear Phase

Use electrical delay to remove linear portion of phase response



Low resolution **High resolution**

Low-Frequency Network Characterization

H-parameters

$V_1 = h_{11}I_1 + h_{12}V_2$

$V_2 = h_{21}I_1 + h_{22}V_2$

Y-parameters

$I_1 = y_{11}V_1 + y_{12}V_2$

$I_2 = y_{21}V_1 + y_{22}V_2$

Z-parameters

$V_1 = z_{11}I_1 + z_{12}I_2$

$V_2 = z_{21}I_1 + z_{22}I_2$

$$h_{11} = \left.\frac{V_1}{I_1}\right|_{V_2=0}$$

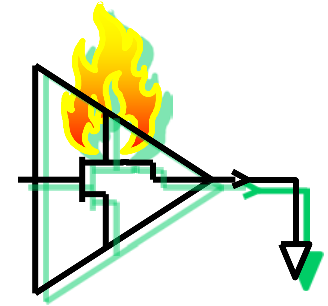
(requires short circuit)

$$h_{12} = \left.\frac{V_1}{V_2}\right|_{I_1=0}$$

(requires open circuit)

All of these parameters require measuring voltage and current (as a function of frequency)

Limitations of H, Y, Z Parameters (Why use S-parameters?)

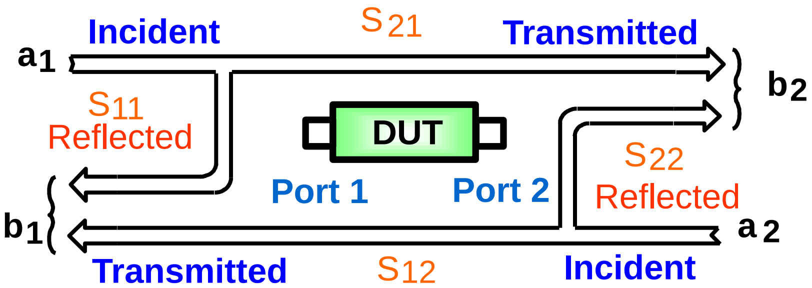


H,Y, Z parameters

- Hard to measure total voltage and current at device ports at high frequencies
- Active devices may oscillate or self-destruct with shorts opens

S-parameters

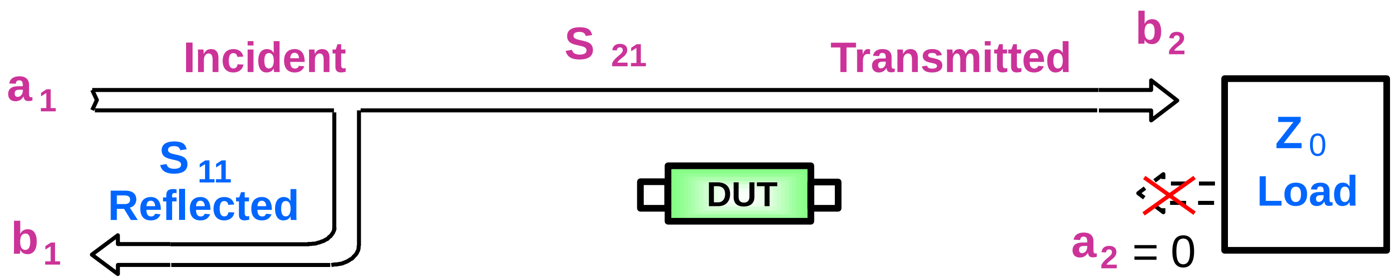
- Relate to familiar measurements
  (gain, loss, reflection coefficient ...)
- Relatively easy to measure
- Can cascade S-parameters of multiple
  devices to predict system performance
- Analytically convenient
  - CAD programs
  - Flow-graph analysis
- Can compute H, Y,or Z parameters from S-parameters if desired



$$b_1 = S_{11} a_1 + S_{12} a_2$$
$$b_2 = S_{21} a_1 + S_{22} a_2$$

Measuring S-Parameters

Forward

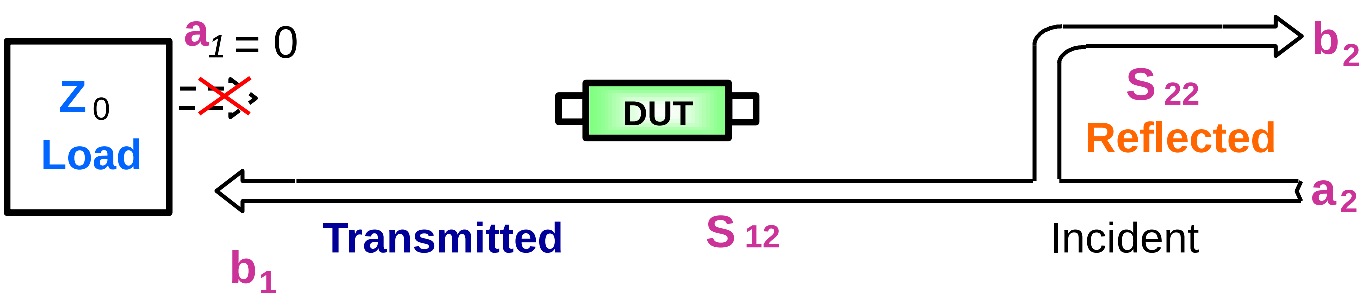


$$S_{11} = \frac{\text{Reflected}}{\text{Incident}} = \left.\frac{b_1}{a_1}\right|_{a_2 = 0}$$

$$S_{21} = \frac{\text{Transmitted}}{\text{Incident}} = \left.\frac{b_2}{a_1}\right|_{a_2 = 0}$$

$$S_{22} = \frac{\text{Reflected}}{\text{Incident}} = \left.\frac{b_2}{a_2}\right|_{a_1 = 0}$$

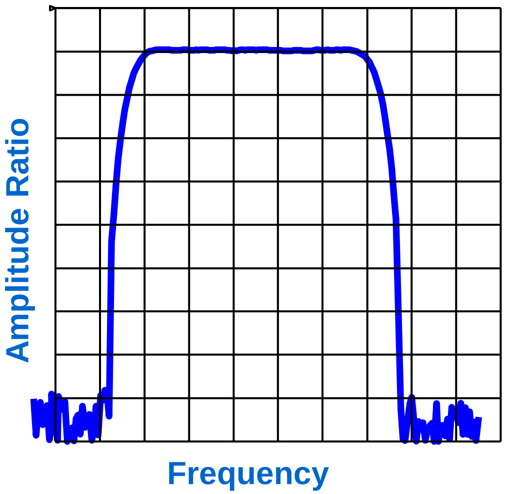
$$S_{12} = \frac{\text{Transmitted}}{\text{Incident}} = \left.\frac{b_1}{a_2}\right|_{a_1 = 0}$$

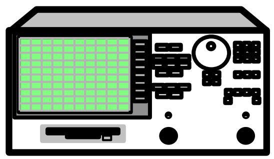


Reverse

What is the difference between *network* and *spectrum* analyzers?

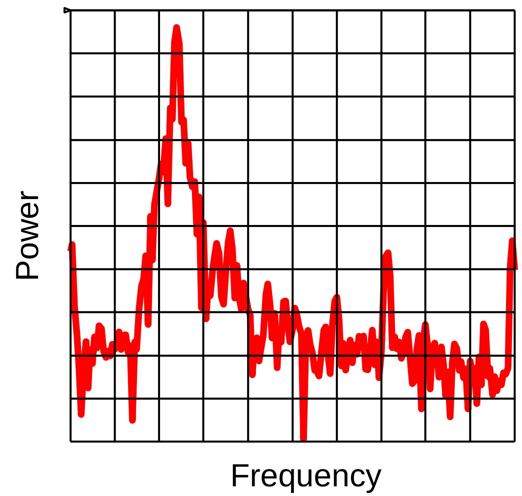
Hard: getting (accurate) trace
Easy: interpreting results

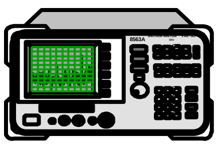




Measures known signal

Easy: getting trace
Hard: interpreting results





Measures unknown signals

Network analyzers:

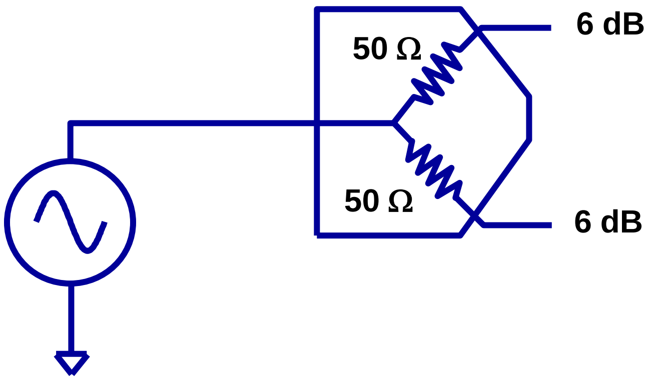
- measure components, devices, circuits, sub-assemblies
- contain source and receiver
- display ratioed amplitude and phase (frequency or power sweeps)

Spectrum analyzers:

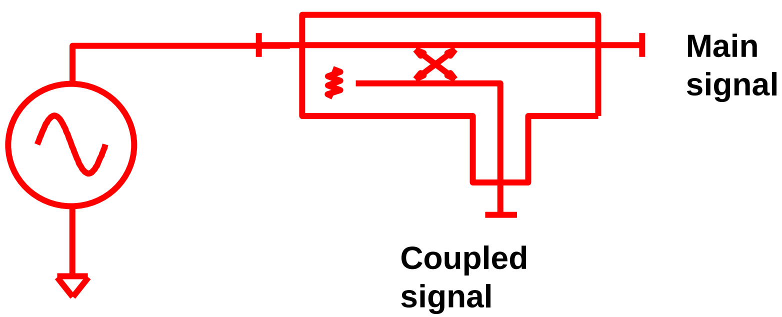
- measure signal amplitude characteristics (carrier level, sidebands, harmonics...)
- are receivers only (single channel)
- can be used for scalar component test (*no phase*) with tracking gen. or ext. source(s)

Signal Separation

Measuring incident signals for ratioing

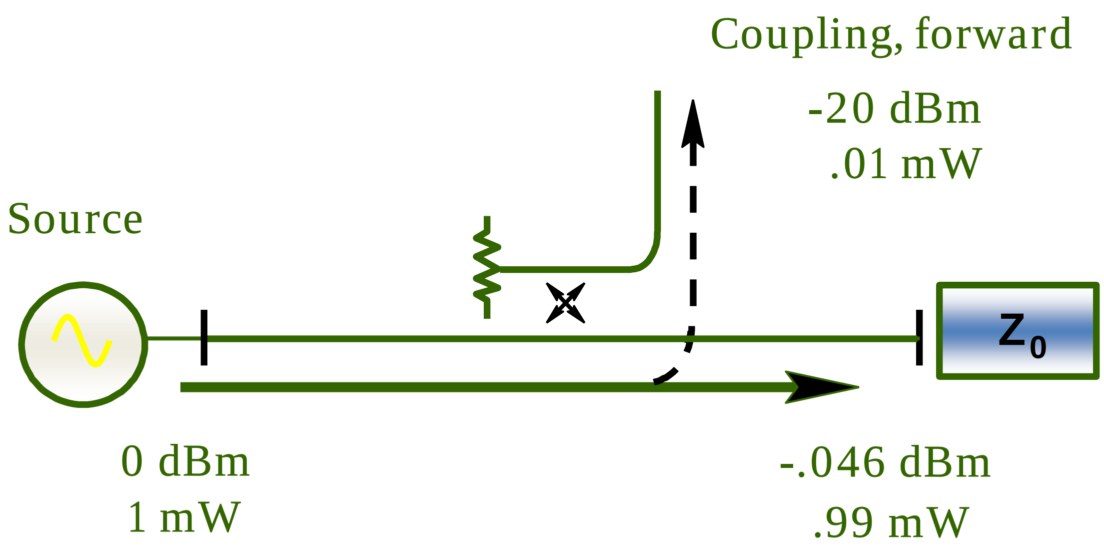


- *Splitter*
  - usually resistive
  - non-directional
  - broadband



- *Coupler*
  - directional
  - low loss
  - good isolation, directivity
  - hard to get low freq performance

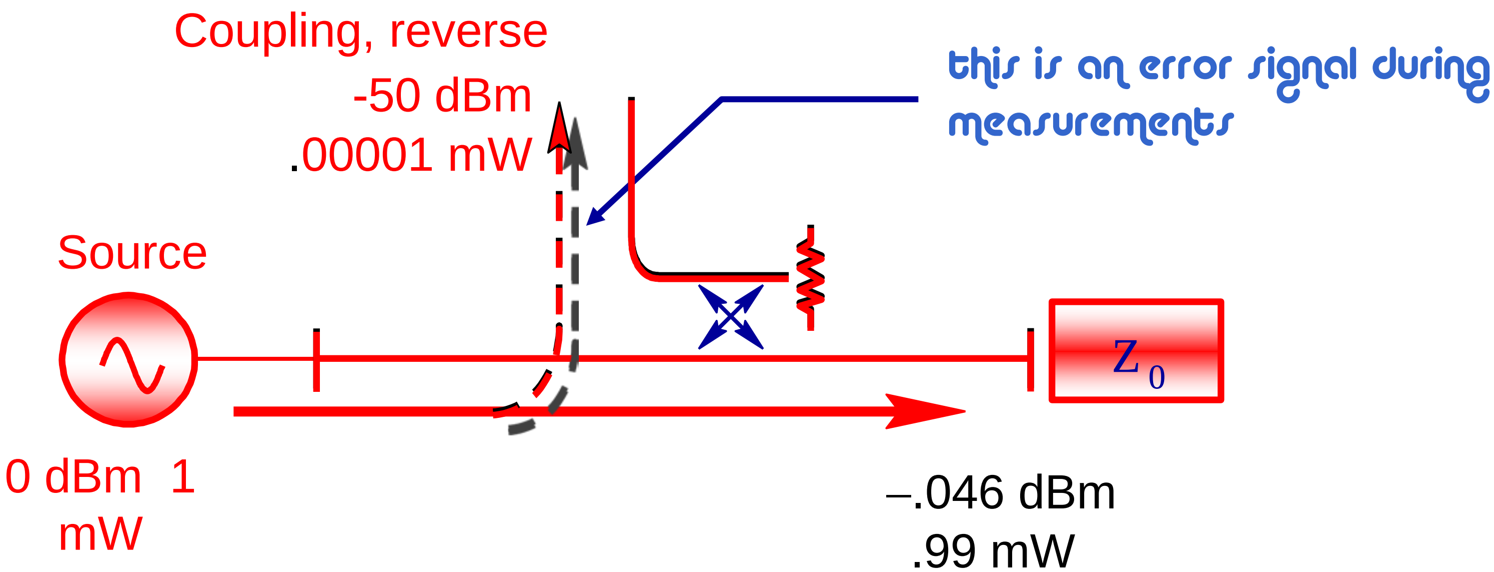
Forward Coupling Factor



Example of 20 dB Coupler

$$\text{Coupling Factor (dB)} = -10 \log \frac{P_{coupling\ forward}}{P_{incident}}$$

Directional Coupler Isolation (Reverse Coupling Factor)



Example of 20 dB Coupler "turned around"

$$\text{Isolation Factor (dB)} = -10 \log \frac{P_{\textit{coupled reverse}}}{P_{\textit{incident}}}$$

Directional Coupler Directivity

$$\text{Directivity (dB)} = 10 \log \frac{P_{coupled\ forward}}{P_{coupled\ reverse}}$$

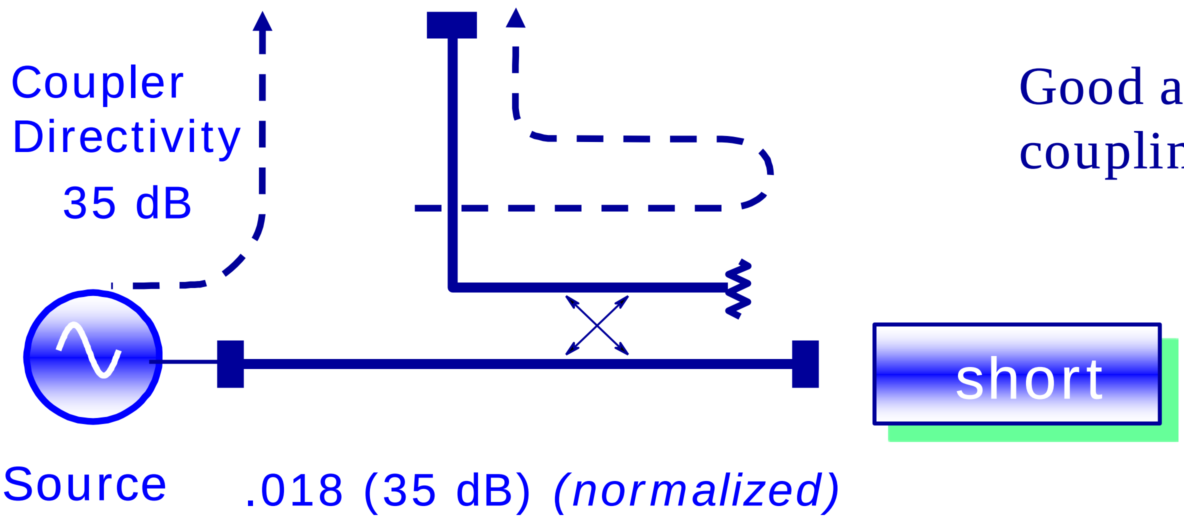
$$\text{Directivity} = \frac{\text{Coupling Factor}}{\text{Isolation}}$$

Directivity (dB) = Isolation (dB) - Coupling Factor (dB)

Example of 20 dB Coupler with 50 dB isolation:
Directivity = 50 dB - 20 dB = 30 dB

Measuring Coupler Directivity the Easy Way

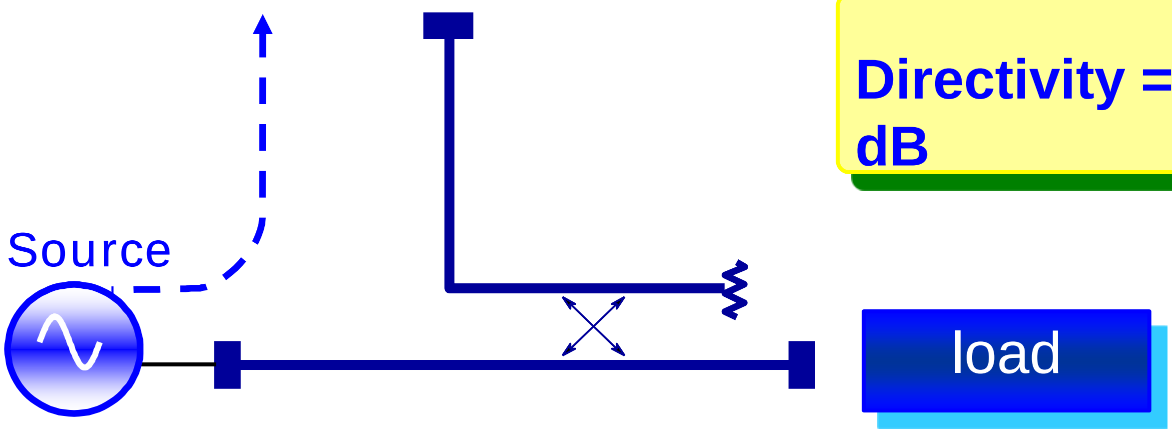
1.0 (0 dB) *(reference)*



Good approximation for coupling factors $\geq$ 10 dB

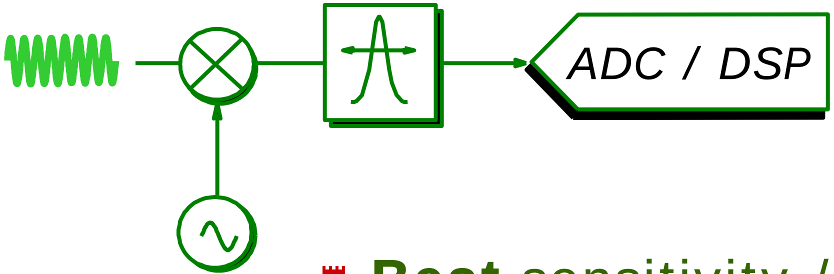
.018 (35 dB) *(normalized)*

Directivity = 35 dB - 0 dB = 35 dB

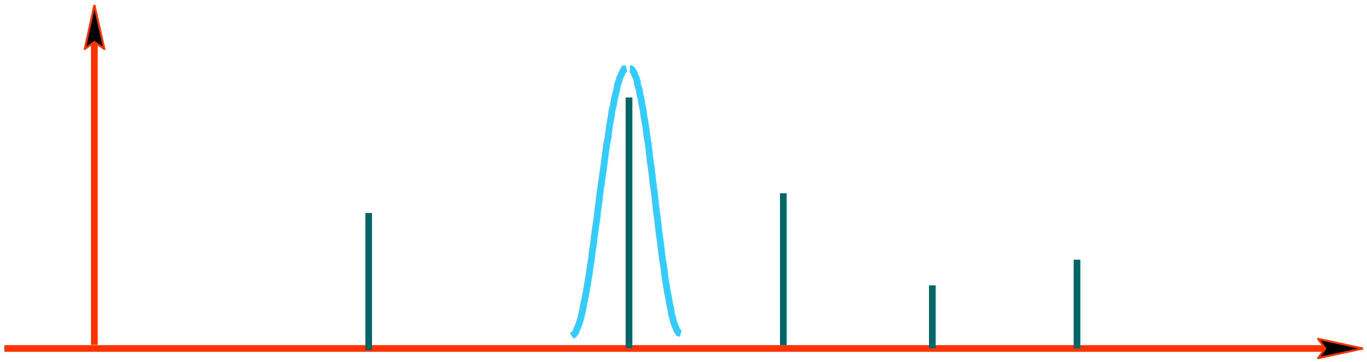


Assume perfect load

Narrowband Detection - Tuned Receiver

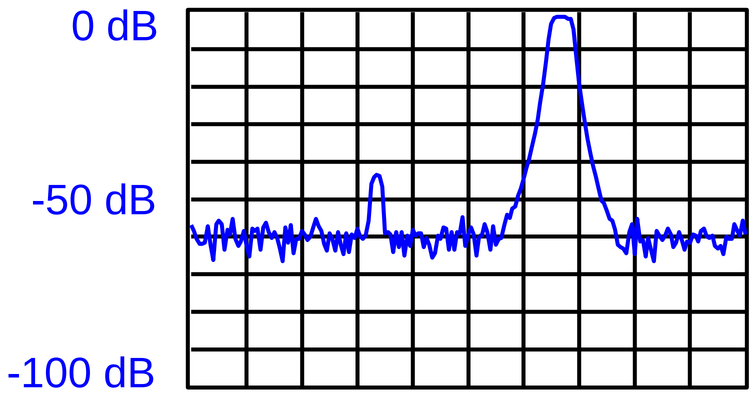


- **Best** sensitivity / dynamic range
- Provides harmonic / spurious signal **rejection**
- Improve dynamic range by increasing **power**, decreasing IF **bandwidth**, or **averaging**
- Trade off noise floor and measurement speed



Comparison of Receiver Techniques

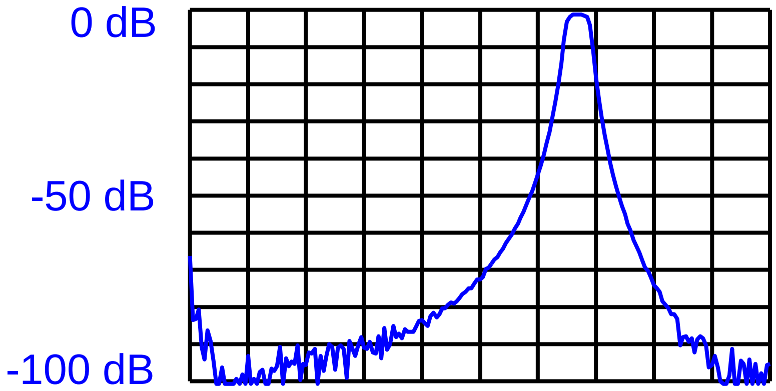
Broadband (diode) detection



-60 dBm Sensitivity

- higher noise floor
- false responses

Narrowband (tuned-receiver) detection



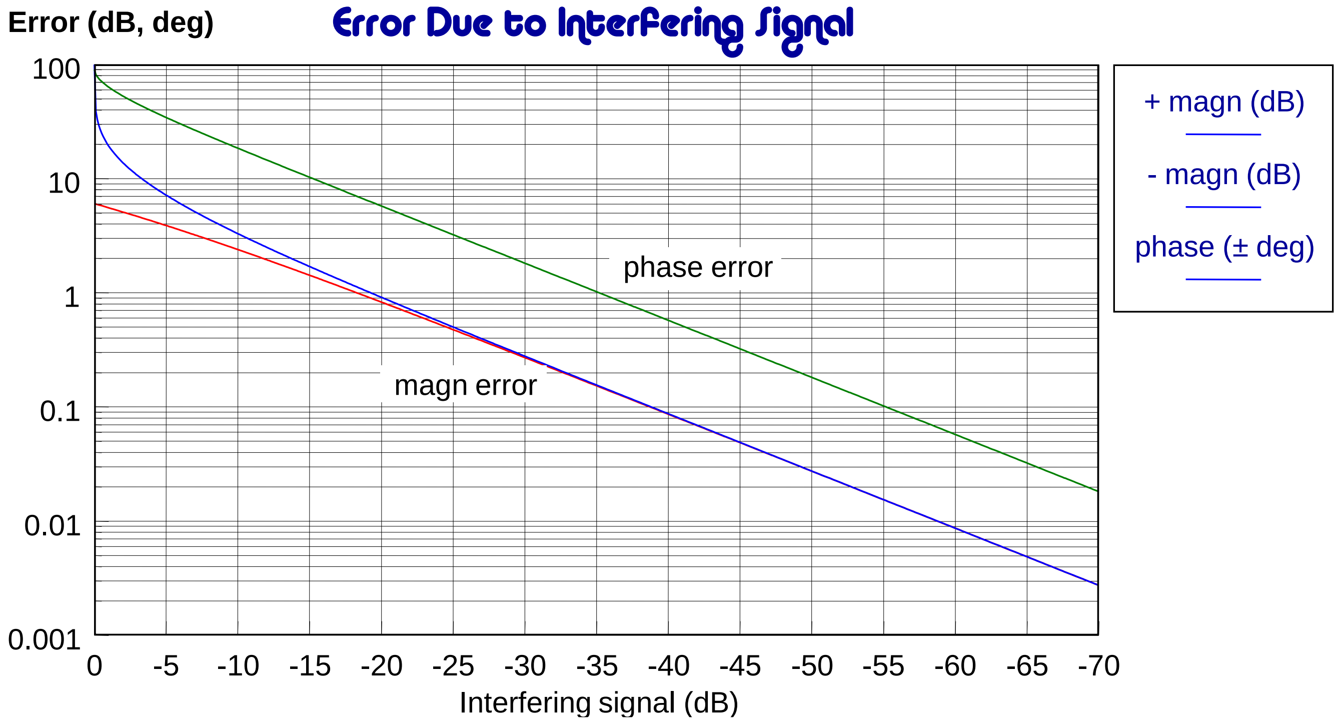
< -100 dBm Sensitivity

- high dynamic range
- harmonic immunity

Dynamic range = maximum receiver power - receiver noise floor

Dynamic Range and Accuracy

Dynamic range is very important for measurement accuracy !



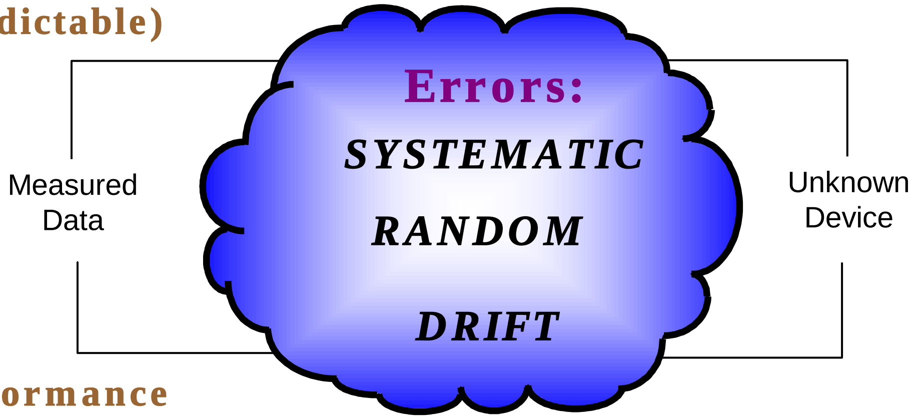
Measurement Error Modeling

Systematic errors

- due to imperfections in the analyzer and test setup
- are assumed to be time invariant (predictable)
- can be characterized (during calibration process) and mathematically removed during measurements

Random errors

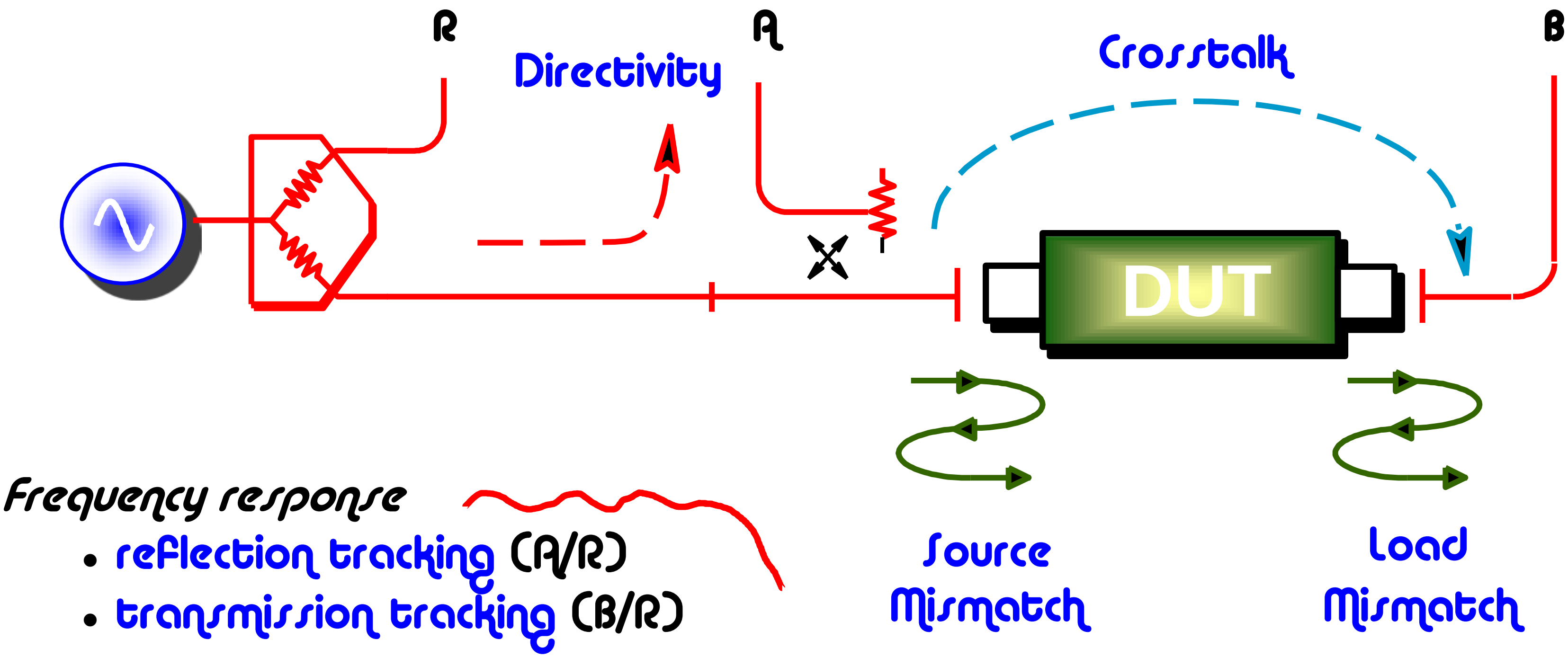
- vary with time in random fashion (unpredictable)
- cannot be removed by calibration
- main contributors:
  - instrument noise (source
  - phase noise, IF noise floor, etc.)
  - switch repeatability
  - connector repeatability



Drift errors

- are due to instrument or test-system performance changing *after* a calibration has been done
- are primarily caused by temperature variation
- can be removed by further calibration(s)

Systematic Measurement Errors



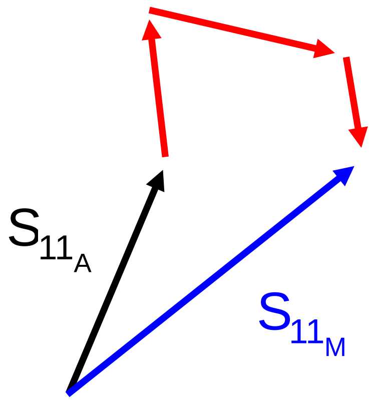
Six forward and six reverse error terms yields 12 error terms for two-port devices

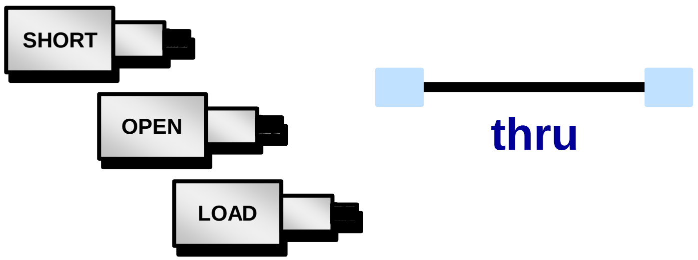
Types of Error Correction

Two main types of error correction:

- **response (normalization)**
  - simple to perform
  - only corrects for tracking errors
  - stores reference trace in memory, then does data divided by memory

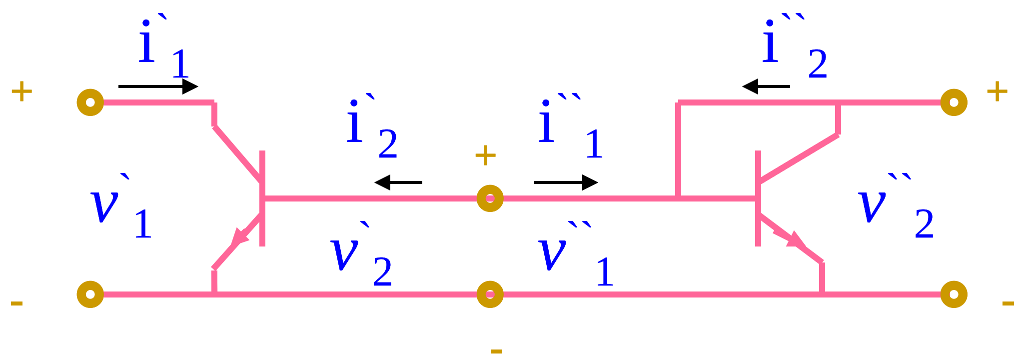
- **vector**
  - requires more standards
  - requires an analyzer that can measure phase
  - accounts for all major sources of systematic error

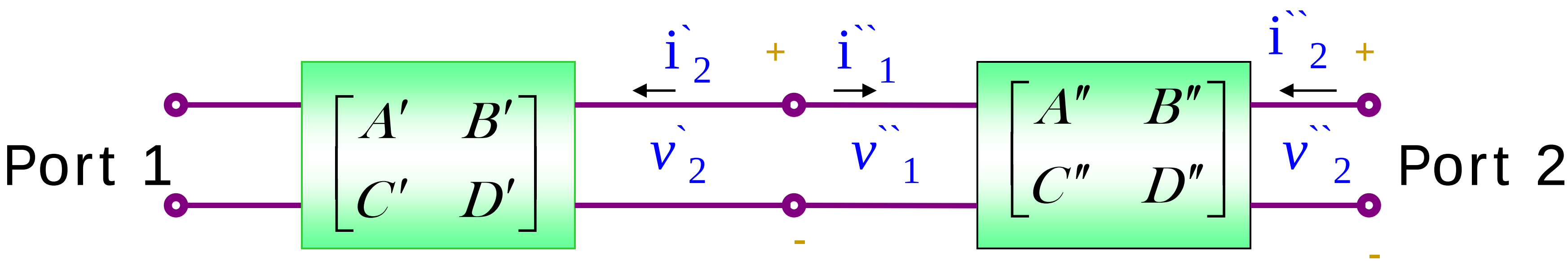




ABCD parameter representation

- Very useful when cascading networks

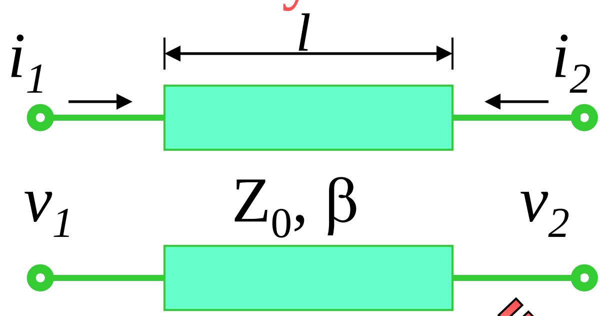




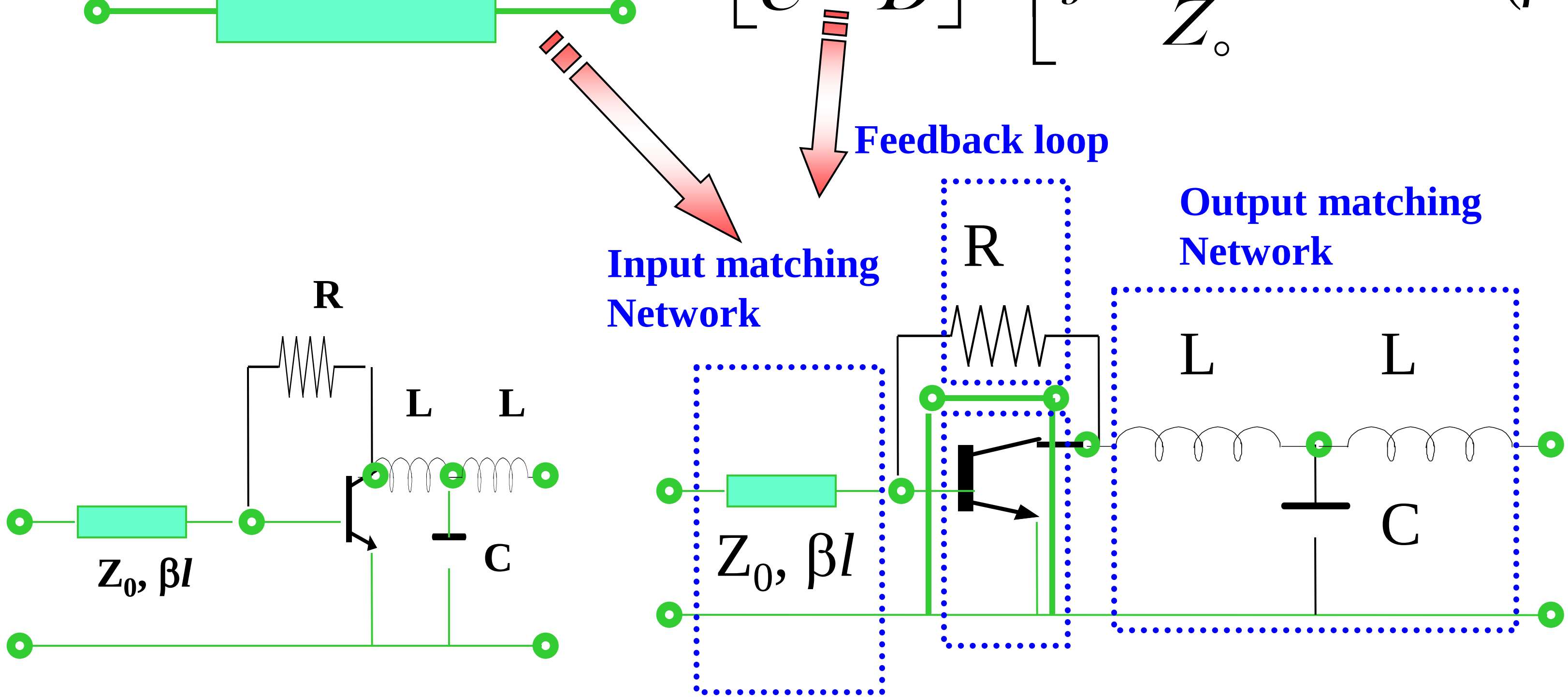
$$\begin{Bmatrix} v_1 \\ i_1 \end{Bmatrix} = \begin{bmatrix} A' & B' \\ C' & D' \end{bmatrix} \begin{bmatrix} A'' & B'' \\ C'' & D'' \end{bmatrix} \begin{Bmatrix} v_2'' \\ -i_2'' \end{Bmatrix}$$

ABCD parameter representation

- ABCD very useful for T.L.

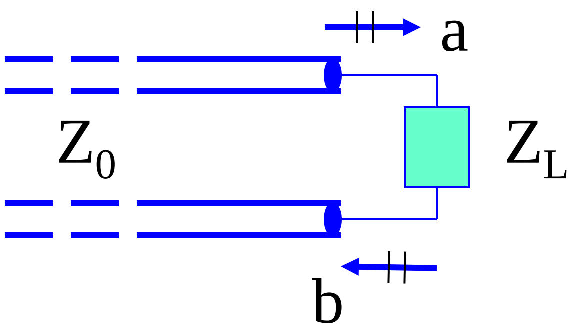


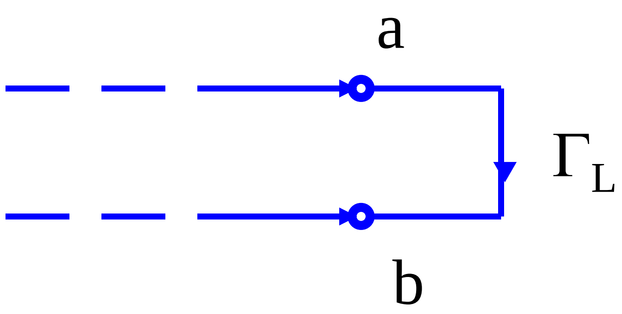
$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} \cos(\beta\ell) & jZ_\circ \sin(\beta\ell) \\ j\dfrac{\sin(\beta\ell)}{Z_\circ} & \cos(\beta\ell) \end{bmatrix}$$



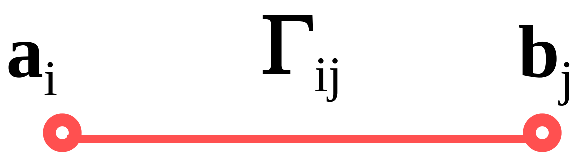
Signal Flow Computations

Complicated networks can be efficiently analyzed in a manner identical to signals and systems and control.





in general



Basic Rules:

We'll follow certain rules when we build up a network flow graph.

1. Each variable, a1, a2, b1, and b2 will be designated as a node.

2. Each of the S-parameters will be a branch.

3. Branches enter dependent variable nodes, and emanate from the independent variable nodes.

4. In our S-parameter equations, the reflected waves b1 and b2 are the dependent variables and the incident waves a1 and a2 are the independent variables.

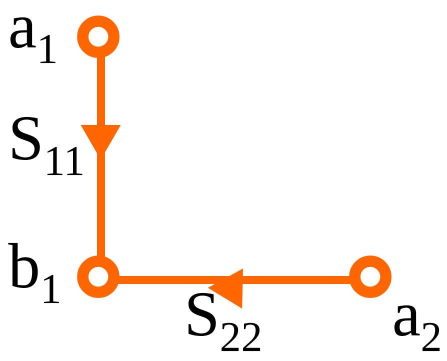
5. Each node is equal to the sum of the branches entering it.

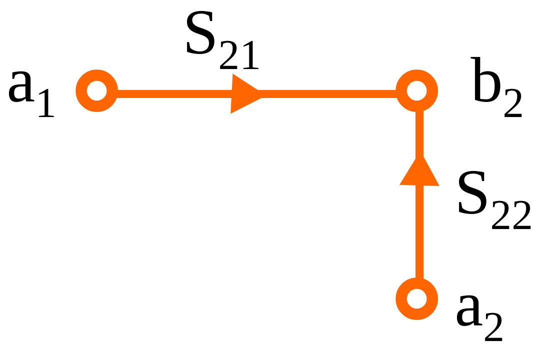
Let’s apply these rules to the two S-parameters equations

$$b_1 = S_{11}a_1 + S_{12}a_2$$

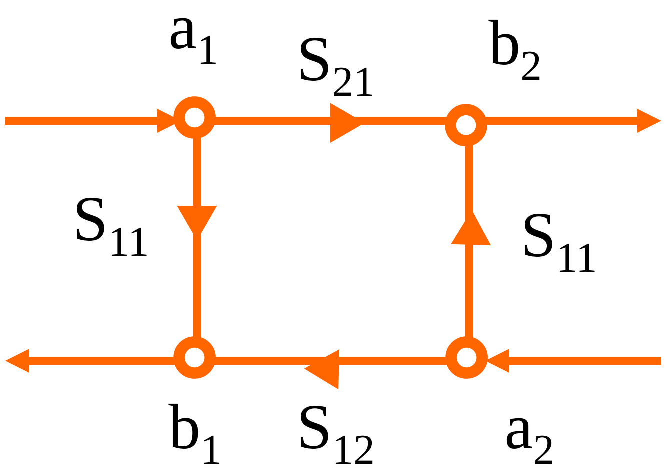
$$b_2 = S_{21}a_1 + S_{22}a_2$$

First equation has three nodes: b_1, a_1, and a_2. b_1 is a dependent node and is connected to a_1 through the branch S_{11} and to node a_2 through the branch S_{12}. The second equation is similar.



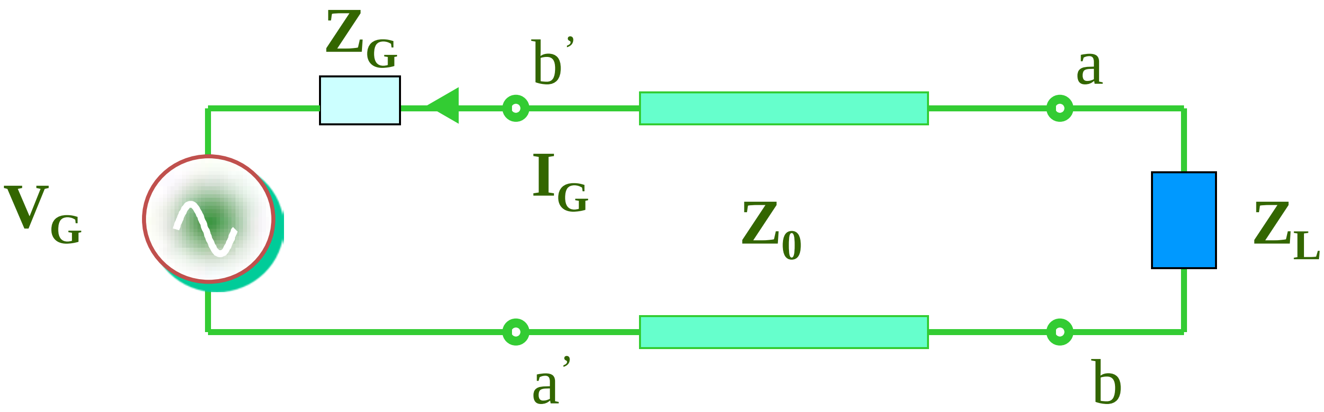


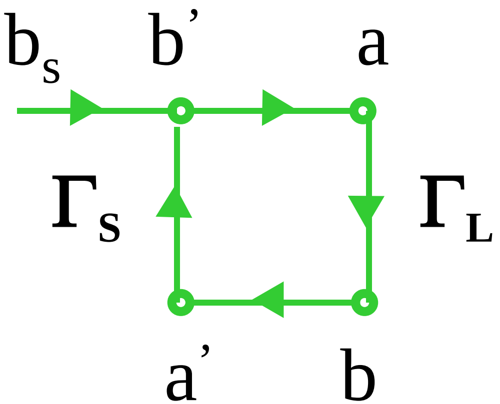
Complete Flow Graph for 2-Port



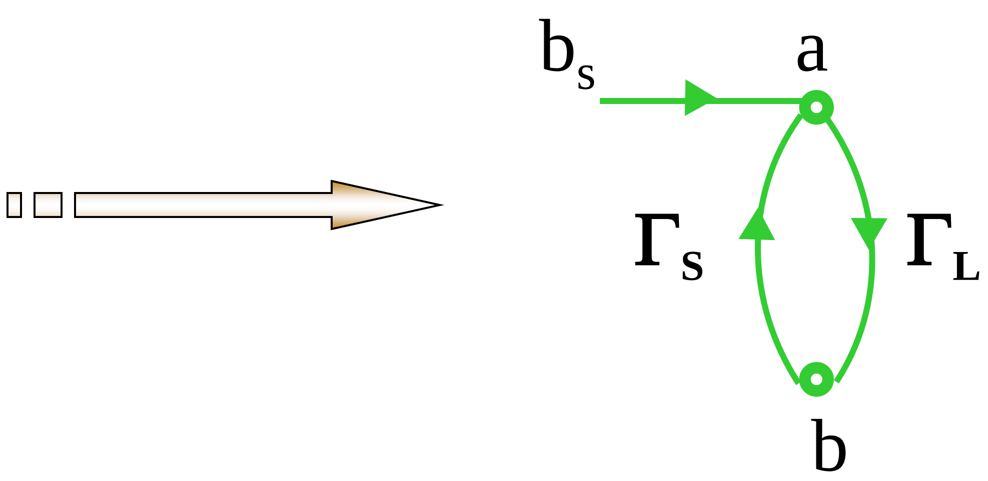
The relationship between the traveling waves is now easily seen. We have a_1 incident on the network. Part of it transmits through the network to become part of b_2. Part of it is reflected to become part of b_1. Meanwhile, the a_2 wave entering port two is transmitted through the network to become part of b_1 as well as being reflected from port two as part of b_2. By merely following the arrows, we can tell what's going on in the network. This technique will be all the more useful as we cascade networks or add feedback paths.

Arrangement for Signal Flow Analysis

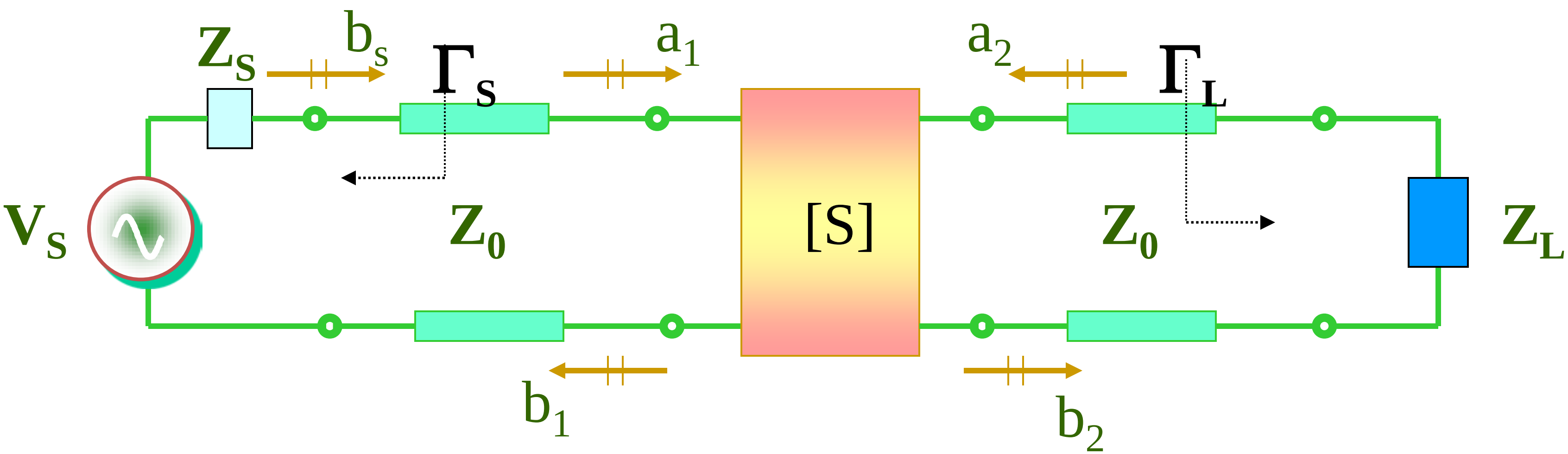


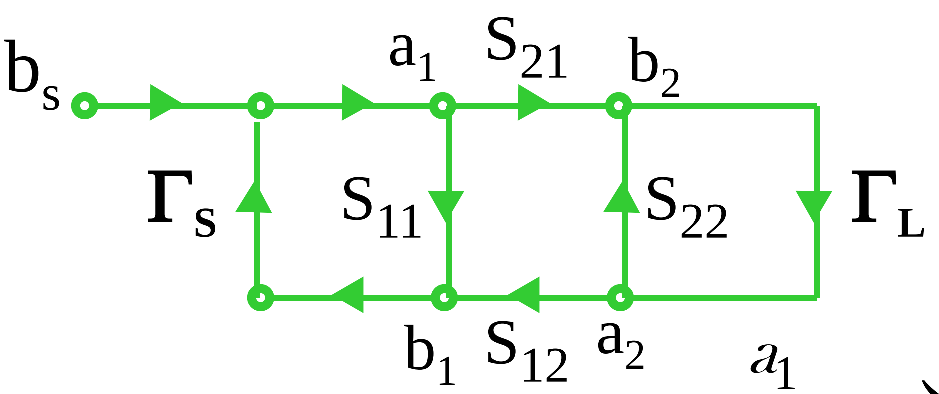


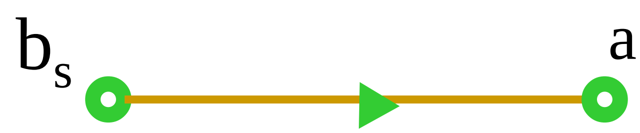
$$b_s = \frac{\sqrt{Z_\circ}}{Z_G + Z_\circ} V_G$$



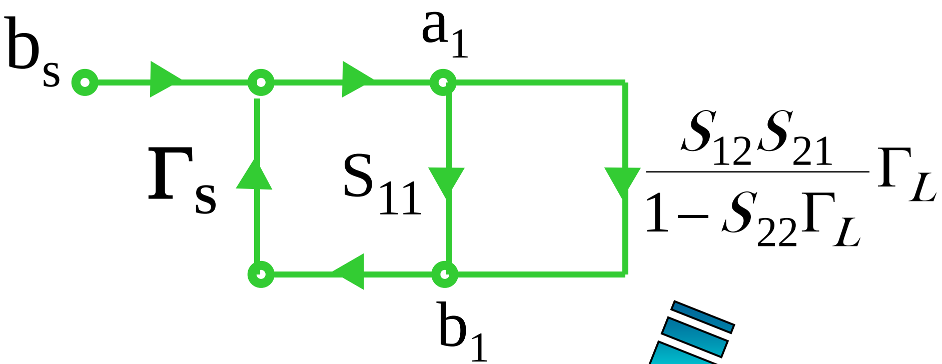
Analysis of Most Common Circuit

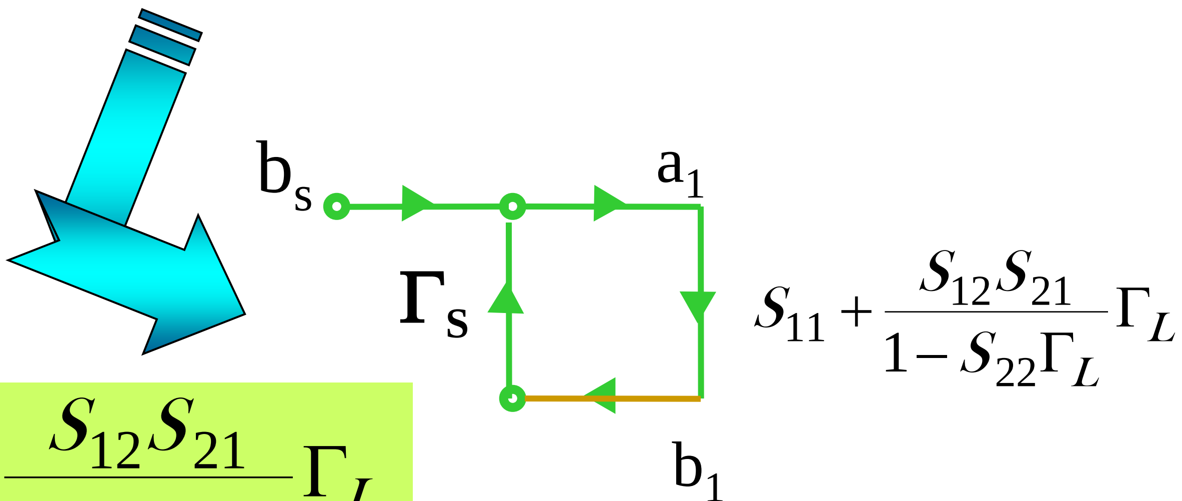






$$\frac{a_1}{b_s} \rightarrow \frac{1}{1-\left(S_{11}+\dfrac{S_{12}S_{21}}{1-S_{22}\Gamma_L}\Gamma_L\right)\Gamma_S}$$

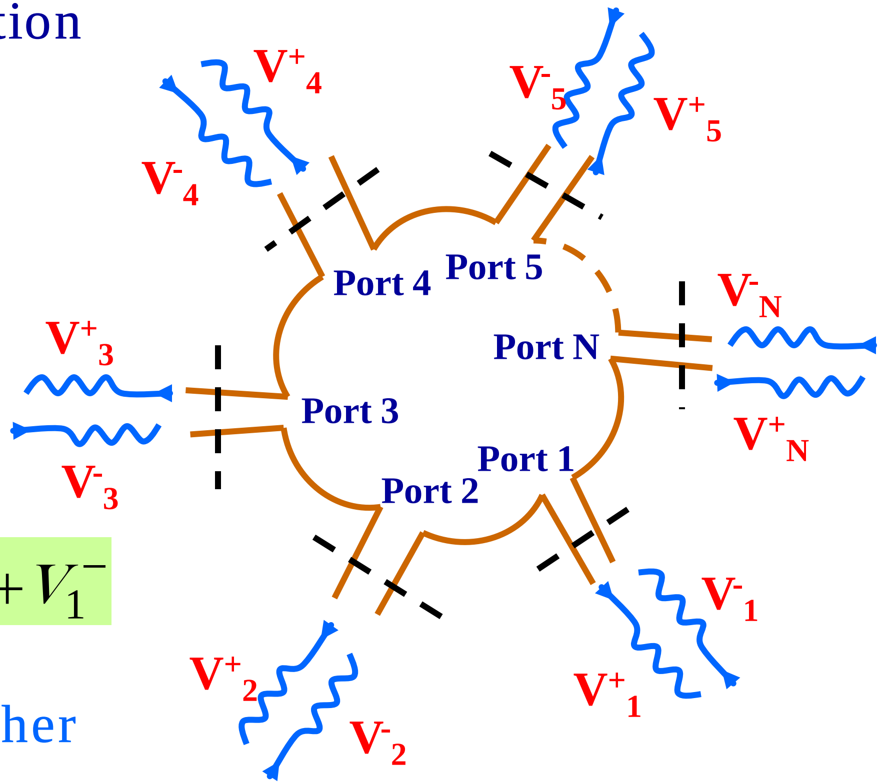




$$\Gamma_{in} = \frac{b_1}{a_1} = S_{11} + \frac{S_{12}S_{21}}{1-S_{22}\Gamma_L}\Gamma_L$$

Note: Only $\Gamma_L = 0$ ensures that S_{11} can be measured.

The scattered-wave amplitudes are linearly related to the incident wave amplitudes. Consider the N port junction



If the only incident wave is V^+_1 then

$$V_1^- = S_{11} V_1^+$$

S_{11} is the reflection coefficient

The total voltage is port 1 is $V_1 = V_1^+ + V_1^-$

Waves will also be scattered out of other ports. We will have

$$V_n^- = S_{n1} V_n^+ \quad n = 2,3,4,\ldots N$$

If all ports have incident wave then

$$\begin{bmatrix} V_1^- \\ V_2^- \\ \ldots \\ V_N^- \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} & S_{13} & \ldots & S_{1N} \\ S_{21} & S_{22} & S_{23} & \ldots & S_{2N} \\ \ldots & \ldots & \ldots & \ldots & \ldots \\ S_{N1} & S_{N2} & S_{N3} & \ldots & S_{NN} \end{bmatrix} \begin{bmatrix} V_1^+ \\ V_2^+ \\ \ldots \\ V_N^+ \end{bmatrix}$$

or

$$\left[V^-\right] = [S]\left[V^+\right]$$

$[S]$ is called the scattering matrix $\quad S_{ij} = \frac{V_i^-}{V_j^+} \;\; for \;\; V_k^+ = 0 \; (k \neq j)$

If we choose the equivalent Z_0 equal to 1 then the incident power is given by

$$\frac{1}{2}\left|V_n^+\right|^2$$

and the scattering will be symmetrical. With this choice

$$V = V^+ + V^-, I = I^+ + I^-$$

and

$$V^+ = \frac{1}{2}(V + I)$$

$$V^- = \frac{1}{2}(V - I)$$

V^+ and V^- are the variables in the scattering matrix formulation; but they are linear combination of V and I.

Other normalization are

$$\nu = \frac{V}{\sqrt{Z_\circ}} \qquad i = \frac{I}{\sqrt{Z_\circ}}$$

Just as in the impedance matrix there are several properties of the scattering matrix we want to consider.

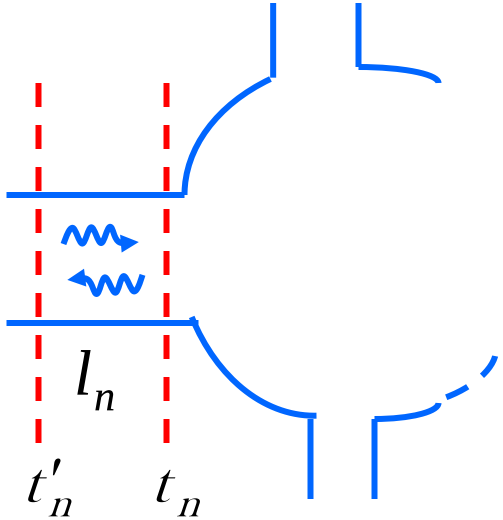
1. A shift of the reference planes
2. S matrix for reciprocal devices
3. S matrix for the lossless devices

Shift in the reference planes

Consider the following network, where t_n is the original location of the reference plane, and t'_n in the new location of the reference plane.

The electrical length between t_n and t'_n is

$\theta = \beta_n \ell_n$.



S_{mn} $m \neq n$ *must be multiplied by* $e^{-j\theta_n}$

S_{nn} *must be multiplied by* $e^{-j2\theta_n}$

Why is this a factor of 2?

$$V_n'^{+} = V_n^{+} e^{j\theta_n}$$

$$V_n'^{-} = V_n^{-} e^{-j\theta_n}$$

$$[S] = \begin{bmatrix} e^{-j\theta_1} & & & & \\ & e^{-j\theta_2} & & & \\ & & \cdot & & \\ & & & \cdot & \\ & & & & e^{-j\theta_N} \end{bmatrix} \begin{bmatrix} S_{11} & S_{12} & \cdots & S_{1N} \\ S_{21} & S_{22} & \cdots & S_{2N} \\ \cdots & \cdots & \cdots & \cdots \\ S_{N1} & S_{N2} & \cdots & S_{nn} \end{bmatrix} \begin{bmatrix} e^{-j\theta_1} & & & & \\ & e^{-j\theta_2} & & & \\ & & \cdot & & \\ & & & \cdot & \\ & & & & e^{-j\theta_N} \end{bmatrix}$$

Consider a 2×2 scattering matrix where two reference planes are shifted.

$$[S'] = \begin{bmatrix} e^{-j\theta_1} & 0 \\ 0 & e^{-j\theta_2} \end{bmatrix} \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \begin{bmatrix} e^{-j\theta_1} & 0 \\ 0 & e^{-j\theta_2} \end{bmatrix}$$

$$[S'] = \begin{bmatrix} S_{11}e^{-j\theta_1} & S_{12}e^{-j\theta_1}e^{-j\theta_2} \\ S_{21}e^{-j\theta_1}e^{-j\theta_2} & S_{22}e^{-j\theta_2} \end{bmatrix}$$

$$[S'] = \begin{bmatrix} S_{11}e^{-j2\theta_1} & S_{21}e^{-j\theta_1}e^{-j\theta_2} \\ S_{12}e^{-j\theta_1}e^{-j\theta_2} & S_{22}e^{-j2\theta_2} \end{bmatrix}$$

Proof of Symmetry of Scattering Matrix

For a reciprocal junction $S_{mn} = S_{nm}$ m≠n provided that $Z_0=1$

$P = \frac{1}{2}\left|V_n^+\right|^2$ **for all modes.**

$$V_n = V_n^+ + V_n^- \quad and \quad I_n = I_n^+ - I_n^-$$

$$\therefore I_n = V_n^+ - V_n^- \quad (Z_\circ = 1)$$

$$[V] = [V^+] + [V^-]$$

$$= [Z][I]$$

$$= [Z][V^+] - [Z][V^-]$$

Defining the unit matrix

$$[u] = \begin{bmatrix} 1 & & 0 \\ & 1 & \\ & & \cdot & \\ 0 & & & 1 \end{bmatrix}$$

$$[V^+] + [V^-] = [Z][V^+] - [Z][V^-]$$

Rearrange and factor

$$([Z]+[U])[V^-] = ([Z]-[U])[V^+]$$

or

$$[V^-] = ([Z]+[U])^{-1}([Z]-[U])[V^+]$$

but $[V^-] = [S][V^+]$ $\therefore [S] = ([Z]+[U])^{-1}([Z]-[U])$

Loss-less Junction

The total power leaving a junction must be equal to the total power entering the junction power.

$$\sum_{n=1}^{N} \left| V_n^- \right|^2 = \sum_{n=1}^{N} \left| V_n^+ \right|^2$$

but

$$V_n^- = \sum_{i=1}^{N} S_{ni} V_i^+$$

so that

$$\sum_{n=1}^{N} \left| \sum_{i=1}^{N} S_{ni} V_i^+ \right|^2 = \sum_{n=1}^{N} \left| V_n^+ \right|^2$$

Loss-less Junction

V^+_n are all independent incident voltages, so we choose V^+_n =0 except for n=i

$$\sum_{n=1}^{N}\left|\sum_{i=1}^{N} S_{ni}V_i^+\right|^2 = \sum_{n=1}^{N}\left|V_n^+\right|^2$$

$$\sum_{n=1}^{N}\left|S_{ni}V_i^+\right|^2 = \left|V_n^+\right|^2$$

$$\sum_{n=1}^{N}\left|S_{ni}\right|^2 = \sum_{n=1}^{N} S_{ni}S_{ni}^* = 1 \quad \forall i.$$

$$\begin{bmatrix} S_{1i} & S_{2i} & \dots & S_{Ni} \end{bmatrix}\begin{bmatrix} S_{1i}^* \\ S_{2i}^* \\ \dots \\ S_{Ni}^* \end{bmatrix} = 1$$

Loss-less 2 Port Junction

For this case [S] is unitary

$$\therefore \sum_{n=1}^{N} S_{nm} S_{np}^{*} = \delta_{mp}$$

or

$$\left.\begin{aligned} S_{11}S_{11}^{*} + S_{12}S_{12}^{*} &= 1 \\ S_{22}S_{22}^{*} + S_{12}S_{12}^{*} &= 1 \end{aligned}\right\} \Rightarrow |S_{11}| = |S_{22}|$$

$$S_{11}S_{12}^{*} + S_{12}S_{22}^{*} = 0$$

The magnitude of the input and output ports are equal in magnitude. Also

$$|S_{12}| = \sqrt{1 - |S_{11}|^{2}}$$

If we know $|S_{11}|$ then we can obtain $|S_{12}|$ and $|S_{22}|$.

Note: The fraction of power reflected at terminal *t*, is

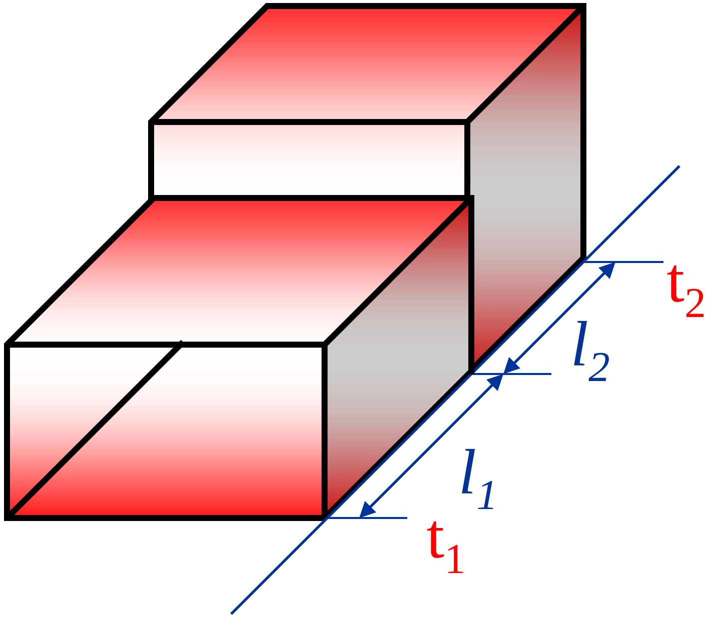
$$\frac{P_{refl}}{P_{inc}} = |S_{11}|^2$$

So that the insertion loss due to reflection is

$$IL = 10\,log\left(1 - |S_{11}|^2\right)$$

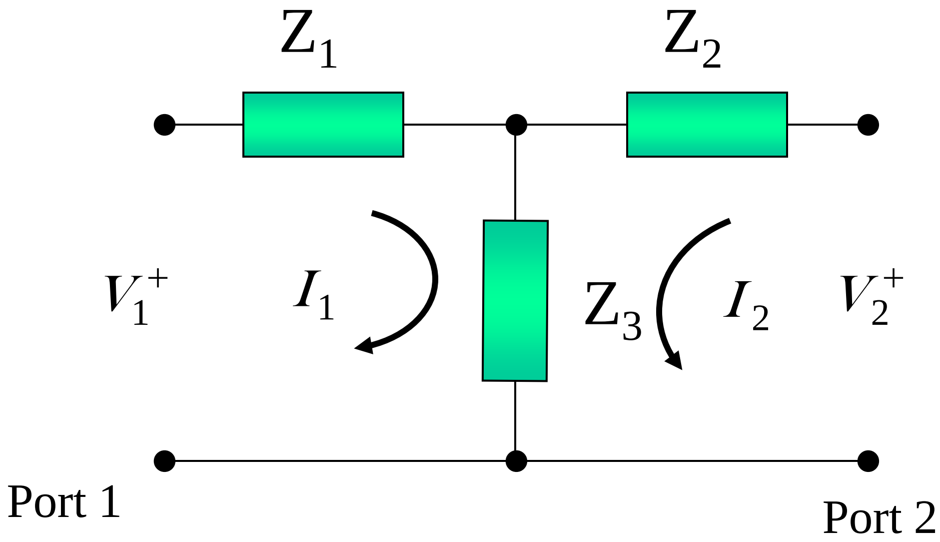
$$IL = 20\,log|S_{12}|$$

Example: two-port network



Assume TE_{10} modes at t_1 and t_2

Equivalent Circuit



Apply KVL:

$$V_1 = Z_1 I_1 + Z_3 I_1 + Z_3 I_2$$

$$V_2 = Z_2 I_2 + Z_3 I_2 + Z_3 I_1$$

If

$$Z_3 = Z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0}$$

$$Z_1 = Z_{11} - Z_{12}$$

$$Z_2 = Z_{22} - Z_{12}$$

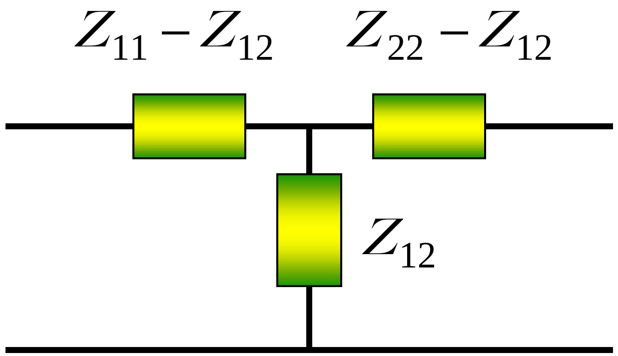
Then we have

$$V_1 = Z_{11} I_1 + Z_{12} I_2$$

$$V_2 = Z_{22} I_2 + Z_{12} I_2$$

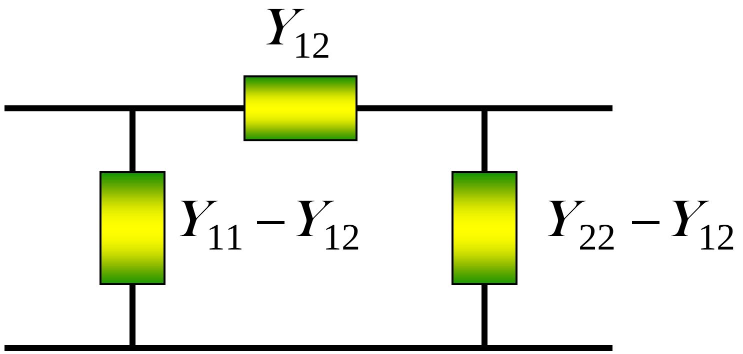
and

$$[V] = [Z][I]$$



This can be transformed into an admittance matrix

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{12} & Y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$



Traveling Wave:

$$V^{+} = Ae^{-\delta x}, V^{-} = Ae^{\delta x}$$

$$V(x) = V^{+}(x) + V^{-}(x)$$

Similarly for current:

$$I(x) = I^{+}(x) - I^{-}(x) = \frac{V^{+}(x)}{Z_{\circ}} - \frac{V^{-}(x)}{Z_{\circ}}$$

Reflection Coefficient:

$$\Gamma(x) = \frac{V^{-}(x)}{V^{+}(x)}$$

Introduce "normalized" variables:

$$v(x) = V(x)/\sqrt{Z_\circ}\ , i(x) = \sqrt{Z_\circ}\, I(x)$$

So that

$$v(x) = a(x) + b(x) \quad i(x) = a(x) - b(x) \quad \text{and} \quad b(x) = \Gamma(x)\, a(x)$$

This defines a single port network. What about 2-port?

2-port

$$b_1 = S_{11} a_1 + S_{12} a_2$$
$$b_2 = S_{21} a_1 + S_{22} a_2$$

Each reflected wave (b_1,b_2) has two contributions: one from the incident wave at the same port and another from the incident wave at the other port.

How to calculate S-parameters?

$$S_{11} = \left.\frac{b_1}{a_1}\right|_{a_2=0}$$

Input reflected coefficient with output matched.

$$S_{12} = \left.\frac{b_1}{a_2}\right|_{a1=0}$$

Reverse transmission coefficient with input matched.

$$S_{21} = \left.\frac{b_2}{a_1}\right|_{a_2=0}$$

Transmission coefficient with output matched.

$$S_{22} = \left.\frac{b_2}{a_2}\right|_{a_1=0}$$

Output reflected coefficient with input matched.

Generalized Scattering Matrix:

We define it via [b]=[S][a]. S-matrix depends on the choice of normalized impedance. Usually 50 Ω, but can be anything and can even be complex!

Calculating S_{ij}:

$$S_{ij} = \left.\frac{b_i}{a_i}\right|_{a_k=0, k\neq i, k=1,\ldots n} = \frac{V_i - Z^*_{\circ,i} I_i}{V_i + Z^*_{\circ,i} I_i} = \frac{Z_i - Z^*_{\circ,i}}{Z_i + Z_{\circ,i}}$$

Which is input reflected coefficient with all other ports matched.

$$S_{ki} = \left.\frac{bk}{a_i}\right|_{a_k=0, k\neq i, k=1,\ldots n}$$

is equal to transducer power gain from i to k with ports other than i matched.