Computer Classes

Code

Computer classes are performed with particle-in-cell code BEAMPATH developed for beam dynamics simulation in linear accelerators and beamlines with space charge forces. Description of mathematical model is given in Nuclear Instriment and Methods in Physical Research, Vol 539, p.p. 455-489 (2005).

Execution of the code

Put in the same directory the executable file *beampath.exe* and file *beampath* containing initial data of the problem. In terminal window, type *./beampath.exe* and press *Enter* to start the program. Execution of the program can be observed at the terminal window via constantly changing lines with beam parameters. To start a new problem, all files generated by the code during previous run, have to be removed from the directory. Start the new run with *beampath.exe* and *beampath* files only in the directory.

Code output

During execution, code generates new files with beam parameters, particle coordinates, momentum, and field data. To plot results, open files using any available graphics program. Recommended graphics software: Pro Fit from http://www.quansoft.com/.

This software was exported from the United States in accordance with the Export Administration Regulations. Diversion contrary to U.S. law prohibited.

1. Beam expansion in free space

Simulation of uniformly charged proton beam expansion in free space with the following parameters:

Beam current	$I = 0.01$ A
Beam energy	$E = 150$ keV
Initial beam radius	$r_o = 0.08$ cm

1. Run the problem.
2. Open file `traject` and plot x(cm) versus z(cm)
3. Open file `run_beam` and plot x_envelope(cm) versus z(cm), y_envelope(cm) versus z(cm).
4. Open files `fort.1001`, `fort.1002`. Plot x(cm) versus y(cm) from both files.
5. Calculate analytically beam expansion at z = 5 cm, 10 cm, 15 cm using formulas

$$\bar{R} \approx 1 + 0.25Z^2 - 0.017Z^3 \qquad \bar{R} = \frac{r}{r_o} \qquad Z = 2\frac{z}{r_o}\sqrt{\frac{I}{I_c(\beta\gamma)^3}}$$

and compare with numerical data.

2. Beam focusing in quadrupole channel

Simulation of proton beam focusing in a magnetic FODO quadrupole channel with the following parameters:

Length of the lens	D =10 cm
Field gradient	$G_m = \pm\ 0.016$ Tesla/cm
Period of the structure	L = 80 cm
Beam energy	E = 150 keV
Beam current	I = 8.945 mA
Normalized emittance	ε = 0.1788 π cm mrad

1. Determine maximum and minimum beam envelopes in the channel with negligible current $I = 0$.

2. Run the program with $I = 0$.

3. Open file `traject` and plot x(cm) versus z(cm) and y(cm) versus z (cm). Open file `run_beam` and plot beam envelopes x-envelope, y-envelope versus z.

4. Open files `fort.1001`, `fort.1002`. Plot at the same plot x-y from each file. Plot at another plot x-px from each file.

5. Determine maximum and minimum beam envelopes in the channel with current I = 8.945 mA.

6. Repeat steps 2-3-4 for program run with current I = 8.945 mA.

3. Beam focusing in longitudinal magnetic field

Simulation of focusing of proton beam in longitudinal magnetic field with the following parameters:

Beam energy, E	150 keV
Normalized beam emittance, ε	0.56 π cm mrad
Beam current, I	87 mA
Equilibrium beam radius, R_e	1 cm

Beam equilibrium in magnetic field is defined by the equation:

$$B = \frac{2mc\beta\gamma}{qR_e}\sqrt{\left(\frac{\varepsilon}{\beta\gamma R_e}\right)^2 + \frac{2I}{I_c(\beta\gamma)^3}}$$

Beam oscillations around equalibrium radius $R = R_e + \xi$ are described by

$$\xi = \xi_o \cos\left(\sqrt{2\,\frac{(2+b)}{1+b}}\,\frac{\omega_L}{\beta c}\,z + \Psi_o\right)$$

where

$$b = \frac{2}{(\beta\gamma)}\,\frac{I}{I_c}\,\frac{R^2}{\varepsilon^2}$$

1. Define required magnetic field to provide beam equilibrium with current I = 0 and normalized emittance $\varepsilon = 0.56$ π cm mrad
2. Put the value of beam radius slightly larger than equilibrium value (R= 1.1 cm), and run the program.
3. Open file `run_beam` and plot x-envelope versus z. Compare beam envelope oscillations with analytical predictions.
4. Define required magnetic field to provide beam equilibrium with current I = 87 mA and normalized emittance $\varepsilon = 0.56$ π cm mrad
5. Repeat steps 2, 3 for beam run with current I = 87 mA and normalized emittance $\varepsilon = 0.56$ π cm mrad.

4. Beam transport in periodic structure of axial-symmetric lenses

Simulation of beam transport in a periodic sequence of thin magnetic lenses with the following parameters:

Distance between lenses	$S = 4$ m
Length of lens	$D = 13.42$ cm
Radius of aperture of the channel	$a = 1$ cm
Ions	Ar_{40}^{+7}
Beam voltage	24.42 kV

1. Determine acceptance of the channel.
2. Determine maximum normalized emittance of the beam which can be transported through the channel with negligible current. Determine required magnetic field in each lens B_{max} to transport such a beam. Determine initial radius of the matched beam assuming the beam enters the system at the distance of 2 m from the first lens.
3. Run the program with $I = 0$, and with beam emittance, and initial beam radius determined at step 2
4. Open file `traject` and plot x versus z. Open files `fort.1001, fort.1002` and plot x versus y from each file. Plot x versus px from each file.
5. Determine maximum current of the beam which can be transported through the system with negligible emittance. Determine required magnetic field in each lens B_{max} to transport such a beam. Determine initial radius of the matched beam assuming the beam enters the system at the distance of 2 m from the first lens.
6. Run the program with beam current, and beam radius determined at step 5. Put the value of beam emittance 1e-10.
7. Open file `traject` and plot x versus z. Open files `fort.1001, fort.1002` and plot x versus y from each file.

5. Beam emittance growth in focusing channel

Simulation of emittance growth of rms-matched beams with Water Bag, Parabolic, and Gaussian distributions in a uniform focusing channel with the following parameters:

Beam energy	$E = 80$ keV,
Beam current	$I = 18$ mA
Beam radius	R_o=1.5 cm
Normalized beam emittance	ε =0.07 π cm mrad

1. Estimate beam emittance growth of rms-matched beams with KV, Water Bag, Parabolic, and Gaussian distributions in uniform focusing channel:

$$\frac{\varepsilon_f}{\varepsilon_i} = \sqrt{1 + b\left(\frac{W_i - W_f}{W_o}\right)}$$

Distribution	Free Energy parameter $\frac{W_i - W_f}{W_o}$
KV	0
WB	0.01126
PB	0.02366
GS	0.077

2. Put parameter ITYPE='GS' and run the program.
3. Open file `run_beam` and plot x-emittance and y-emittance versus z. Compare results with analytical predictions.
4. Open files `fort.1001`, `fort.1002`, and plot beam distributions x versus px.
5. Put parameter ITYPE='WB', run the program, and repeat steps 3-4.
6. Put parameter ITYPE='PB', run the program, and repeat steps 3-4.
7. Put parameter ITYPE='KV', run the program, and repeat steps 3-4.

6. Gaussian beam uniforming in free space

Simulation of uniforming of Gaussian beam in free space under self space charge forces with the following parameters:

Beam current, I	0.018 A
Beam energy, E	80 keV
Initial beam radius , R_o	1.5 cm

1. Define an optimal distance of beam uniforming (η= 4), where

$$\eta = \frac{4I}{I_c(\beta\gamma)^3}\frac{z^2}{R_o^2}$$

2. Run the program.
3. Put the file `fort.1001` into directory PhaseSpaceDistributions. Rename the file to 2. Run the program ./a.exe.
4. Open generated file `dNdxdy` and plot column dN/dxdy versus x(cm), y(cm) through `Plot -> Data z(x,y)`.
5. Repeat steps 2, 3, 4 with files `fort.1002`, `fort.1003`.

7. Beam halo formation (particle-core interaction)

Single particle dynamics around beam core

1. Put initial data for particle `npart = 1, p = 0, x = 0.8`, and for beam envelope `b = 3.0317, delta = -0.2, tmax=200`. Run the program. Open files `particle_res, envelope_res` and plot at the same graph x versus t and r versus t.
2. Repeat step 1 for single particle with initial position `x = 1.071, x = 1.0828, x = 1.728`.

Stroboscopic image

3. Put `npart = 4, x = 0.8, 1.071, 1.0828, 1.728, nprint= 790, tmax=5000`. Run the program, open file `particle_res` and plot x versus px.

Dependence of maximum deviation of particle as a function of envelope amplitude

4. Put `delta = -0.1, b = 3.025, npart  = 1, x = 1.31, p = 0, nprint = 1, tmax=200`. Run the program. Open files particle_res, envelope_res and plot at the same graph x versus t and r versus t. Determine maximum particle deviation from the axis.

5. Repeat step 4 for the following prameters:

```
delta = -0.3, b = 3.055, npart  = 1, x = 0.9, p = 0, nprint = 1, tmax=200
delta = -0.4, b = 3.085, npart  = 1, x = 0.75, p = 0, nprint = 1, tmax=200
delta = -0.5, b = 3.095, npart  = 1, x = 0.601, p = 0, nprint = 1, tmax=200
```

6. Compare dependence of x_{max} versus delta with analytical predictions.

8. Beam bunching in RF field

Simulation of proton beam bunching with the following parameters:

Beam energy, E	750 keV
Normalized beam emittance, ε	0.632 π cm mrad
Beam current, I	1 A
Equilibrium beam radius, R_e	0.5 cm
RF Frequency	201 MHz
Modulation voltage U_1	50 kV
Gap width d	2.4 cm
Radius of aperture of drift channel a	1 cm

1. Determine required magnetic field for focusing of the beam with negligible current $I = 0$.
2. Run the program with I=0.
3. Open file `induced_current` and plot the value of I_1/I verus z.
4. Open file `fort.2001`. Plot pz versus T. Adjust limits for T (min=0.8, max=2) and for pz (min=0.038, max=0.042). Plot x versus T. Adjust limits for T (min=0.8, max=2) and for x (min= -1, max=+1).
5. Open file `fort.2003` and plot pz versus T. Adjust limits for T (min=12, max=13.5) and for pz (min=0.038, max=0.042). Plot x versus T. Adjust limits for T (min=12, max=13.5) and for x (min= -1, max=+1).

6. Determine required magnetic field for focusing of the beam with negligible current $I = 1$ A.
7. Run the program with I = 1A.
8. Open file `induced_current` and plot the value of I_1/I verus z.
9. Open file `fort.2001`. Plot pz versus T. Plot x versus T.
10. Open file `fort.2002` and plot pz versus T. Adjust limits for T (Tmin=8.9, Tmax=9.9) and for pz (pz_min=0.038, pzmax=0.042).
11. Compare obtained results with analytical treatment of beam bunching.

Analytical treatment of beam bunching

First harmonic of induced current $\frac{I_1}{I} = 2J_1(X)$

Bunching parameter $X = \frac{U_1 M_1}{2U_o}\left(\frac{\omega z}{v_o}\right)\frac{\sin(\omega_q \frac{z}{v_o})}{\omega_q \frac{z}{v_o}}$

Coupling coefficient of the beam with modulation gap $M_1 = \frac{\sin\frac{\theta_1}{2}}{\frac{\theta_1}{2}}$

Transit time angle through the gap $\theta_1 = \frac{\omega d}{v_o}$

Reduced plasma frequency of the beam $\omega_q = \sqrt{F_p}\,\omega_p$

Plasma frequency for an unbounded beam $\omega_p = \sqrt{\frac{q\rho}{m\varepsilon_o}} = \frac{2c}{R}\sqrt{\frac{I}{I_c\beta}}$

Form factor of reduction of plasma frequency $F_p = 2.56\frac{J_1^2(2.4\frac{R}{a})}{1 + \frac{5.76}{(\frac{\omega a}{v_o})^2}}$

Problems

Section 1

1.1. Calculate the value of critical beam current for (a) electrons, (b) protons, (c) Ar_{40}^{+7}.

1.2. Calculate expansion of proton beam in free space with current I = 10 mA, energy E = 150 keV, initial radius R_o = 0.08 cm at the distance of z = 15 cm.

1.3. Estimate number of particles in Debye sphere for proton beam with I = 1A, R =1 cm, energy 47 keV, T = 300° K.

1.4. Calculate normalized emittance of electron beam with radius R = 1 mm, emerged from the cathode at room temperature T = 300 K.

1.5. Calculate normalized beam emittance of Ar^{+5} ion beam with temperature kT_i = 3 eV immersed in longitudinal magnetic field B_z = 0.637 Tesla extracted with radius of R = 5 mm.

1.6. Calculate divergence of the Ar_{40}^{+8} beam with current I = 0.5 mA, extracted from the gap with radius of r_1 = 0.5 cm, gap width d = 3 cm, with extraction voltage U = 8.5 kV.

1.7 .Derive the following expressions:

$$<x^2> = \pi\ \beta_x \int_o^\infty r_x^3\ \rho_x(r_x^2)\, dr_x$$

$$<x'^2> = \pi \gamma_x \int_o^\infty r_x^3 \rho_x(r_x^2)\, dr_x$$

$$<x\ x'> = -\ \pi\ \alpha_x \int_o^\infty r_x^3\ \rho_x(r_x^2)\, dr_x$$

1.8. Calculate space charge field for the beams with the following space charge density

$$\rho(r) = \frac{4\, I}{3\pi\beta c R^2}(1 - \frac{2\, r^2}{3\, R^2})$$

$$\rho_b = \frac{3\, I}{2\pi c\, \beta R^2}\ (1 - \frac{r^2}{2\, R^2})^2$$

$$\rho\,(r_o) = \frac{2I}{\pi\, R_o^2\ \beta c}\ \ exp(-\,2\,\frac{r_o^2}{R_o^2}\,)$$

Section 2

2.1. Consider proton beam with energy of $E = 150$ keV, current $I = 8.945$ mA, and normalized emittance of $\varepsilon = 0.1788\ \pi$ cm mrad, propagating in a quadrupole FODO channel with aperture of $a = 1.4$ cm, magnetic lenses of length of $D = 10$ cm with field gradient in each lens of $G_m = 1.6$ Tesla/m, and with the period of the structure of $L = 80$ cm.

2.1.1. Determine averaged phase advance per period of the structure without space charge forces.

2.1.2. Determine acceptance and normalized acceptance of the channel.

2.1.3. Determine equilibrium beam radius in the channel, maximum and minimum beam envelopes, without space charge forces.

2.1.4. Determine the value of space charge parameter, depressed betatron tune shift, and average beam radius in the channel including space charge forces. Determine maximum and minimum beam envelopes in the channel with space charge forces.

2.1.5. Determine maximum beam current in the channel which can be transported with the given beam emittance.

2.2.1. Determine required longitudinal magnetic field to provide equilibrium of proton beam with the following parameters: beam energy $E = 150$ keV, beam current $I = 0$, normalized beam emittance $\varepsilon = 0.56\pi\,\text{cm}\,\text{mrad}$, equilibrium beam radius $R_e = 1$ cm.

2.2.2. Determine period of beam envelope oscillations around beam equilibrium radius for the beam with the above parameters.

2.3.1. Determine required longitudinal magnetic field to provide equilibrium of proton beam with the following parameters: beam energy $E = 150$ keV, beam current $I = 87$ mA, normalized beam emittance $\varepsilon = 0.56\pi \,\mathrm{cm\,mrad}$, equilibrium beam radius $R_e = 1$ cm.

2.3.2. Determine period of beam envelope oscillations around beam equilibrium radius for the beam with the above parameters.

2.4. Consider focusing structure of periodic sequence of thin magnetic lenses with distance between lens centers of $S = 4$ m, with the length of each lens of $D = 13.42$ cm and radius of aperture of the channel of $R_{max} = 1$ cm for transport of Ar_{40}^{+7} beam extracted with the voltage of 24.42 kV.

2.4.1. Determine acceptance of the channel

2.4.2. Determine maximum normalized emittance of the beam which can be transported through the channel with negligible current. Determine required magnetic field in each lens B_{max} to transport such a beam. Determine initial radius of the beam assuming the beam enters the system at the distance of 2 m from the first lens.

2.4.3. Determine maximum current of the beam which can be transported through the system with negligible emittance. Determine required magnetic field in each lens B_{max} to transport such a beam. Determine initial radius of the beam assuming the beam enters the system at the distance of 2 m from the first lens.

2.5. Derive expression for maximum beam current in quadrupole channel, in uniform magnetic field, and in periodic structure of axial-symmetric lenses.

2.6. Derive expression for Hamiltonian for particle motion in longitudinal magnetic field in Larmor system of coordinates.

Section 3.

3.1 Define optimal distance for uniforming of proton Gaussian beam in free space under self space charge forces with the following parameters: beam energy $E = 80$ keV, beam current $I = 18$ mA, initial beam radius R_o=1.5 cm.

3.2. Determine emittance growth of rms-matched beams with KV, Water Bag, Parabolic, and Gaussian distrbutions in a uniform focusing channel with the following parameters: beam energy $E = 80$ keV, beam current $I = 18$ mA, beam radius R_o=1.5 cm, normalized beam emittance ε =0.07 π cm mrad.

3.3 Derive expression for Gaussian beam uniforming in free space under space charge forces.

3.4 Derive expression for Hamiltonian of particle-core interaction. Perform averaging of Hamiltonian of particle-core interaction.

Section 4.

4.1. Consider proton beam bunching in drift space with the following parameters:

Beam energy, E	750 keV
Beam current, I	1 A
Beam radius, R_e	0.5 cm
RF Frequency	201 MHz
Modulation voltage U_1	50 kV
Gap width d	2.4 cm
Radius of aperture of drift channel a	1 cm

4.1.1. Determine transit time factor in RF gap.

4.1.2. Determine optimal beam drift length for beam bunching wth negligible current I=0

4.1.3. Determine the value of plasma frequency of the beam for beam current I = 1 A.

4.1.4 Determine reduce plasma frequency.

4.1.5. Determine optimal drift space for beam bunching with current $I = 1$ A. Determine the ratio of the first harmonic of induced beam current to beam current at the point of optimal bunching.

4.2. Determine maximum current of accelerated proton beam with energy of W = 150 keV in RF structure with wavelength of $\lambda = 85.7$ cm, accelerating field of E =20 kV/cm, synchronous phase of $\varphi_s = -60^o$, effective focusing gradient of $G_t = 280$ kV/cm^2. Compare results with that given by simplified model of uniform ellipsoid.

4.3. Derive Hamiltonian in RF field.

Recommended readings and references

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