Special Topic:
Laser Acceleration

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Given at the 2009 Winter USPAS

Outline

- The Lawson-Woodward theorem
- One route to understanding the Lawson-Woodward theorem
    - Far-field antenna radiation pattern synthesis
    - Lawson’s argument for why open accelerator structures won’t work
- The reciprocity theorems
- Application to an Example
- Linear Accelerator Examples: a0<<1
- “Non-linear” accelerator examples: a0>1

Lawson-Woodward Theorem

***Colloquial version*:** A nonvanishing synchronous longitudinal field component can only occur in vacuum for TEM modes in the near field.

***Colloquial Version, Narrowly Applied to Accelerators*:** Boundaries must be present for net energy exchange to occur in first order for $v \rightarrow c$ beams traveling straight trajectories in vacuum.

Original version: J. D. Lawson, P. M. Woodward: EM fields in the neighborhood of a boundary (cf. aperture) may be represented by a combination of propagating and evanescent plane waves. Furthermore, the far field pattern is simply the Fourier transform of the aperture distribution. Propagating plane waves have $v_{ph} \geq c$ and strictly transverse polarization, hence no accelerating field. Evanescent plane waves have $E_{\parallel}/E_{\perp} \propto 1/\gamma \rightarrow 0$ as $\gamma \rightarrow \infty$, hence have decreasing accelerating field with increasing particle energy.

Schemes which do not violate Lawson-Woodward:

- IFEL (particles do not travel in straight lines)
- Ponderomotive Acceleration (Second-order process)
- Non-linear Compton Scattering (Multi-order process)
- LEAP (finite interaction length)

*Inverse Smith-Purcell (boundary within $\lambda\gamma$)

*Inverse Cerenkov (gas present to slow v_{ph})

One Route to L-W

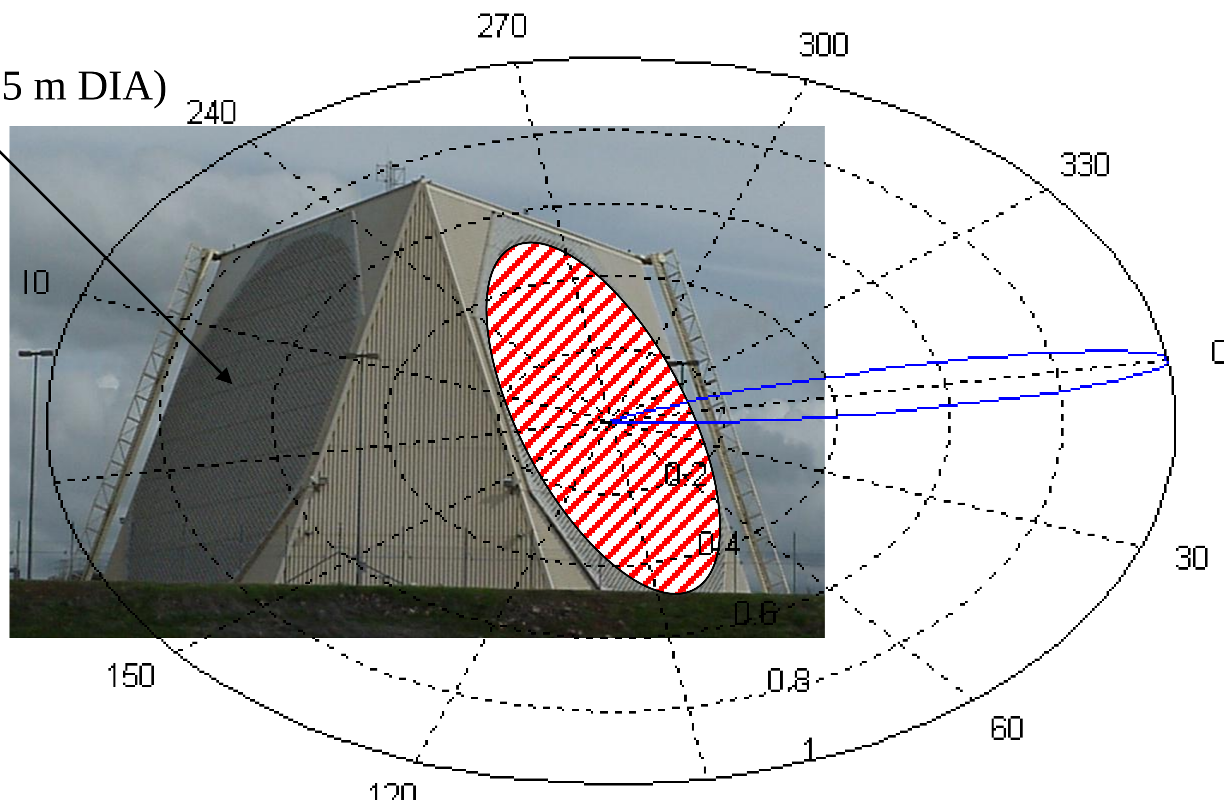
1. Derive an eigenfunction expansion suitable for expressing EM fields above an arbitrary boundary (first handled in the context of antenna design)
2. Identify general characteristics of the eigenfunctions
    1. Do they represent propagating or bound modes?
    2. Is there a non-vanishing field component *in the direction of propagation?*
    3. Is it synchronous?
3. Based on these general characteristics, deduce what situations cannot lead to particle acceleration for $v \rightarrow c$ particles.

One Route to L-W

The far-field antenna pattern synthesis problem:

Q: For a given far-field power distribution P(θ), what configuration of aperture currents is required to produce it?

Aperture (22.5 m DIA)



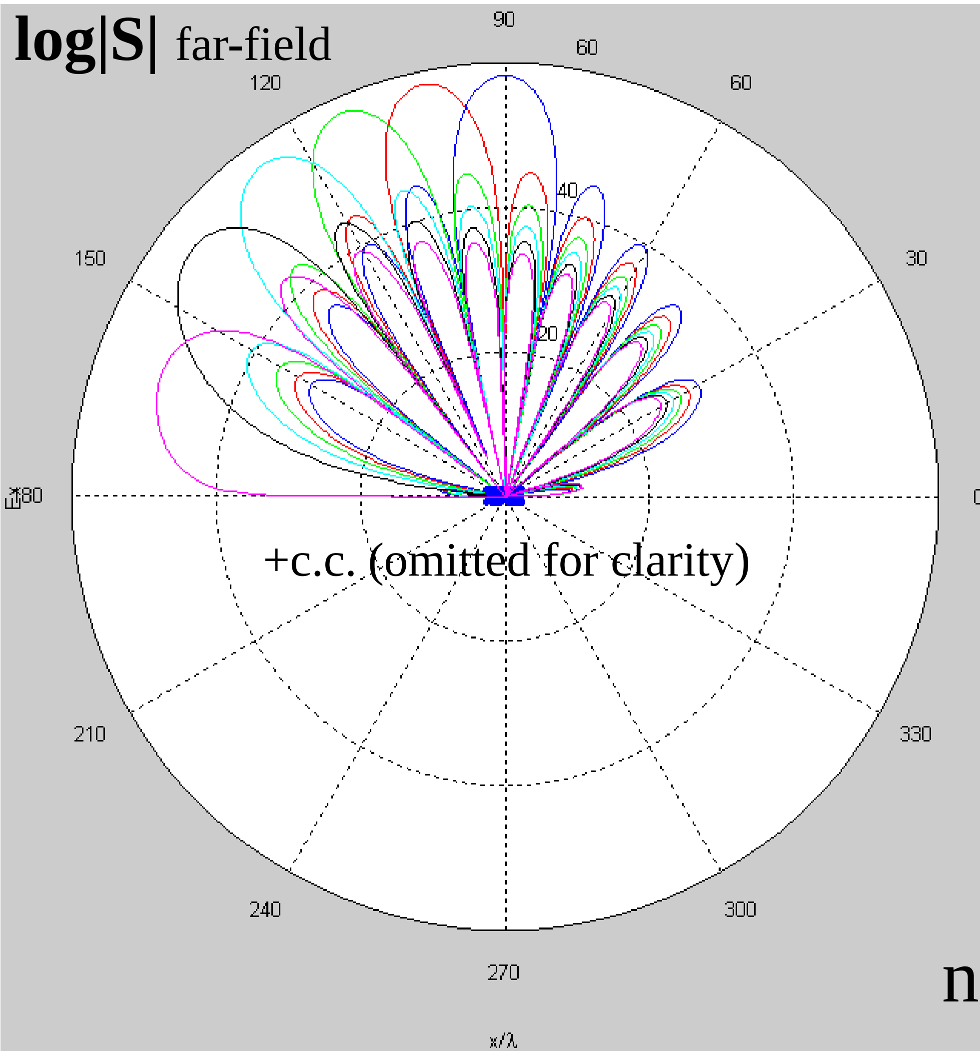
Pave Paws Phased-Array Radar Installation, Beale AFB (1792 dipoles, 340 W/dipole, 2.2° main lobe width)

➔50λ aperture

➔λ~45 cm (~650 MHz)

Woodward's Prescription

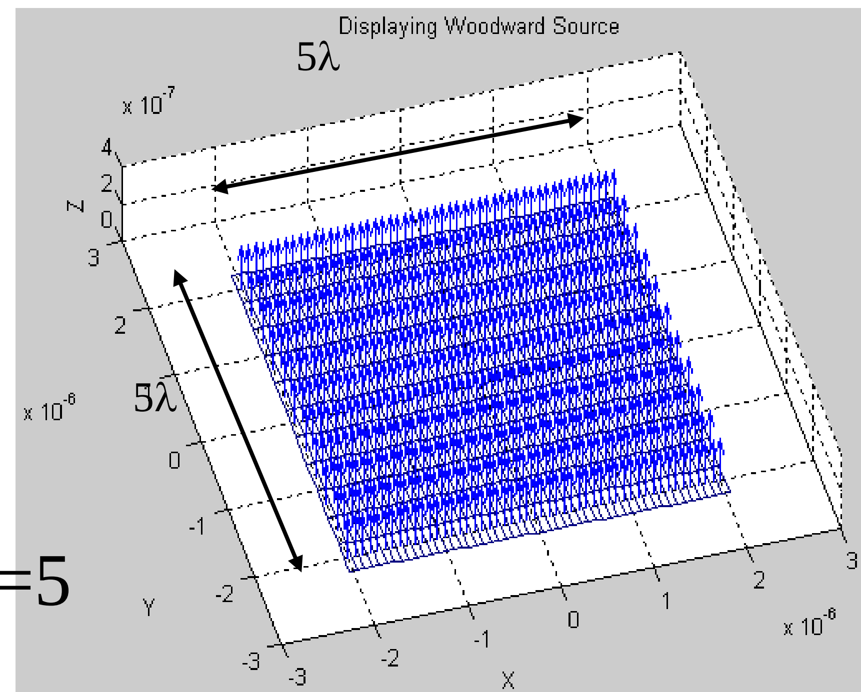
P. M. Woodward, "A Method of Calculating the Field over a Plane Aperture Required to Produce a Given Polar Diagram", *J.I.E.E.*, **93** (IIIA), 1554, (1947).



One can approximately synthesize any desired infinite far-field pattern by vectorially adding the patterns from the 2n+1 linearly independent (Fourier) harmonics of the radiation aperture.

Furthermore, one can synthesize any radiation pattern in the finite far-field to still greater precision by adding evanescent solutions.

n=5



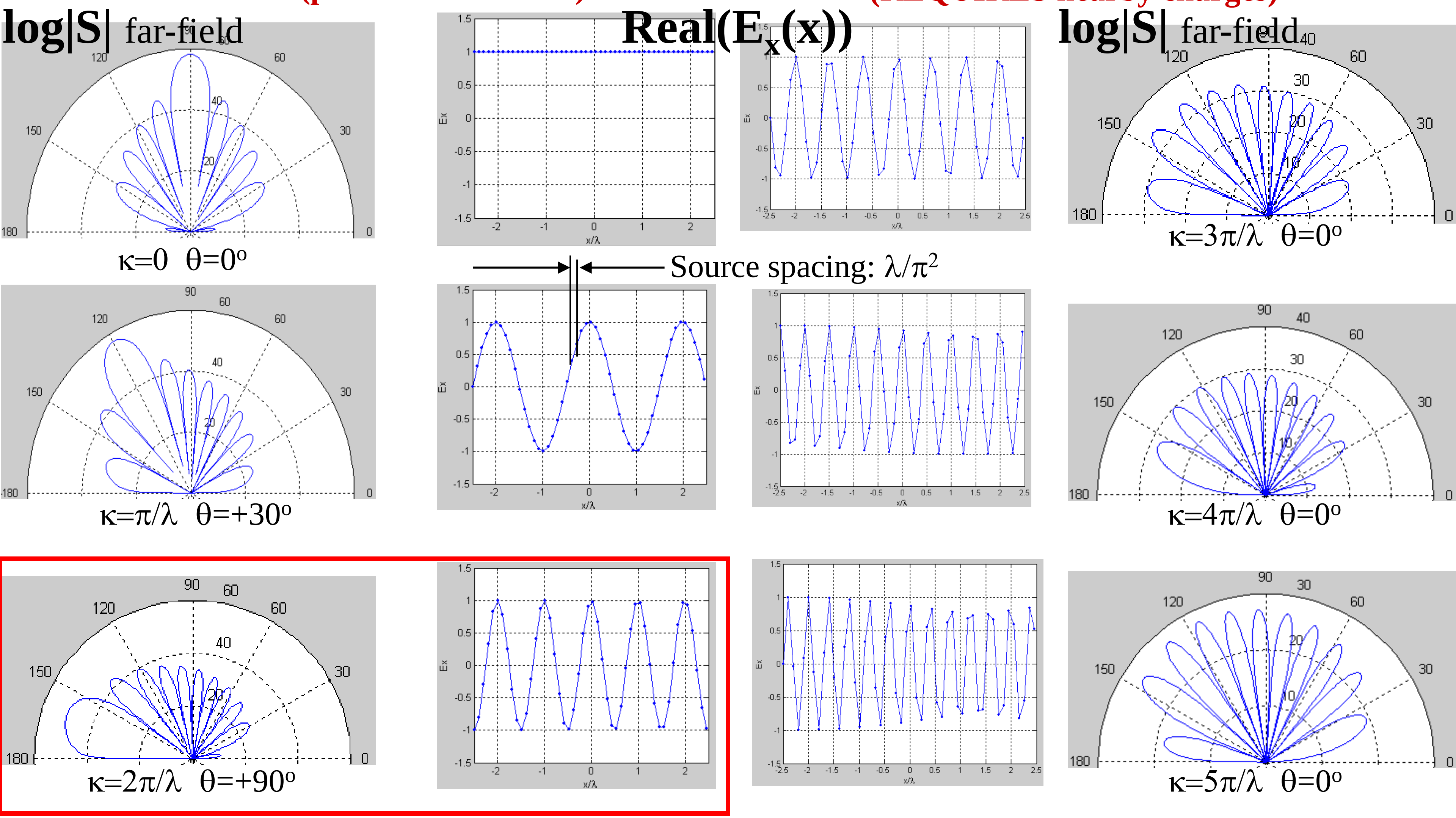
Propagating vs. Evanescent

Propagating

(pure vacuum mode)

Evanescent

(REQUIRES nearby charges)

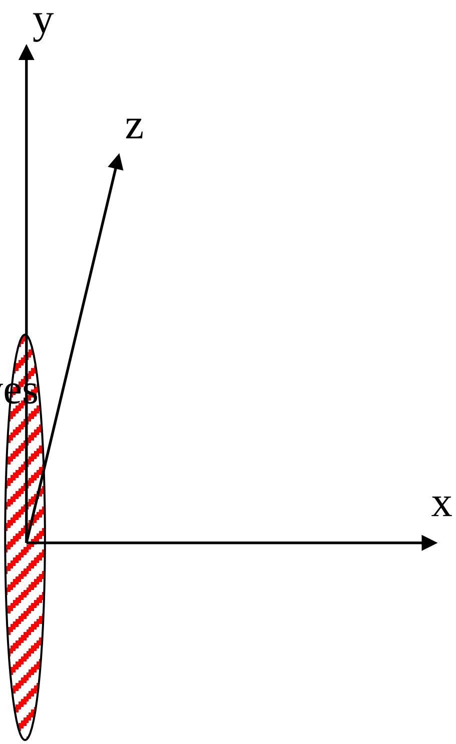


What's the Connection?

$$Z_0 H_y(0,\tilde{y}) = \int_{-\infty}^{\infty} p(\psi)\exp(-j2\pi\psi)\,d\psi$$

$$\tilde{y} \equiv \frac{y}{\lambda}$$

$$\psi = \sin\theta$$



Note: $|\sin\theta| \leq 1$ → propagating plane-waves

$|\sin\theta| > 1$ → evanescent waves

Rayleigh-Helmholtz Reciprocity Theorem (one of many)

Colloquial version: Mutual inductance is reciprocal when no nonlinear media are present. (paraphrase of J. D. Kraus, *Antennas*, 2nd Ed., McGraw-Hill, New York, p.410-11, (1950), with nonlinear media clause coming from J. R. Carson, "Reciprocal Theorems in Radio Communication", *Proc. IRE*, **17**(6), p.952ff, (1929).)

Colloquial Version, Narrowly Applied to Accelerators: If a structure accelerates beam, it will make the beam radiate, and the narrowband coupling impedance for each process will be the same.

Rigorous, general version: Given imposed quasistationary drive fields E_o' and E_o'' on two objects and a bounding surface S containing both objects within volume V, and ε,μ, and σ are all scalar and constant: (from S. Ballantine, *Proc. IRE*, **17**(6), p.929ff, (1929).)

$$\iiint [E_o' C'' - E_o'' C'] dv = \frac{c}{4\pi} \iint [E' \times H'' - E'' \times H']_n dS$$

$$C \equiv \tfrac{c}{4\pi} \nabla \times H$$

Original version: *"Let there be two circuits of insulated wire A and B and in their neighborhood any combination of wire circuits or solid conductors in communication with condensers. A periodic electromotive force in the circuit A will give rise to the same current in B as would be excited in A if the electromotive force operated in B."* Lord J. W. S. Rayleigh, *Theory of Sound*, v. II, Dover: New York, p. 145, (1894).

Many Variants…

$$E_o = \tfrac{c}{4\pi\lambda}\nabla\times H - E$$
$$\nabla\times E = -i\omega\mu H / c$$
$$\nabla\cdot\mu H = 0$$
$$\nabla\cdot\varepsilon E = 0$$
$$\lambda = \sigma + i\omega\varepsilon / 4\pi$$

$$\iiint [E_o' \cdot C'' - E_o'' \cdot C']dv = \frac{c}{4\pi}\iint [E' \times H'' - E'' \times H']_n dS = 0$$

No radiation from beyond surface S

$$C \equiv \tfrac{c}{4\pi}\nabla\times H = (\sigma + i\omega\varepsilon / 4\pi)(E + E_o)$$

$$E_o' \cdot C'' - E_o'' \cdot C' = \frac{c}{4\pi}\left[\nabla\cdot(E' \times H'') - \nabla\cdot(E'' \times H')\right]$$

Most general local form

$$E'' \cdot C' - E' \cdot C'' = E_o' \cdot C'' - E_o'' \cdot C' = 0$$

$$\int_\infty (E_o' \cdot J'' - E_o'' \cdot J')dv = 0$$

Impressed fields, no impressed charges, Ballantine form

$$\int_\infty (E'' \cdot J_o' - E' \cdot J_o'')dv = 0$$

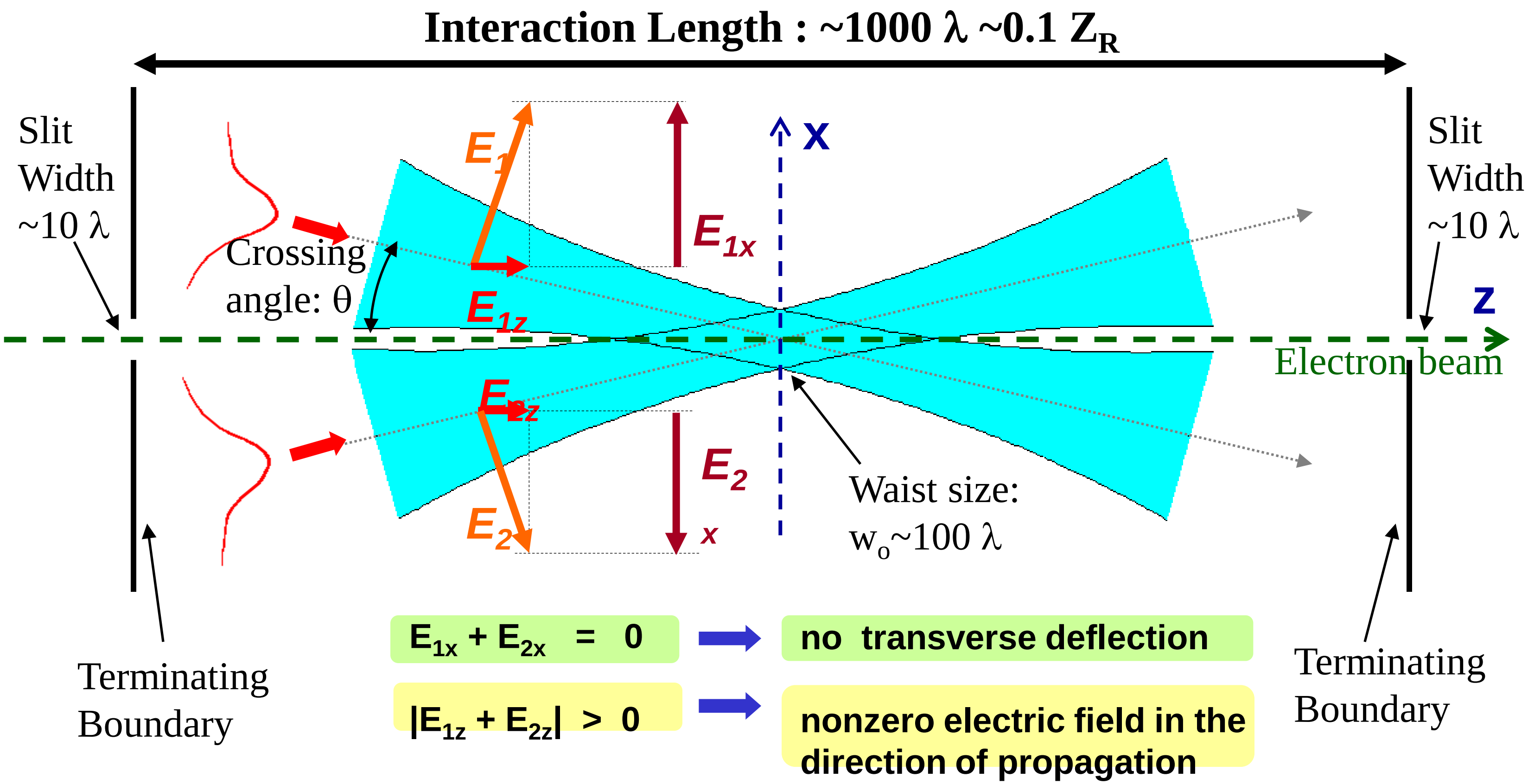
Impressed currents, no impressed fields

$$\int_\infty (\sigma + i\omega\varepsilon / 4\pi)(E_o' \cdot E'' - E_o'' \cdot E')dv = 0$$

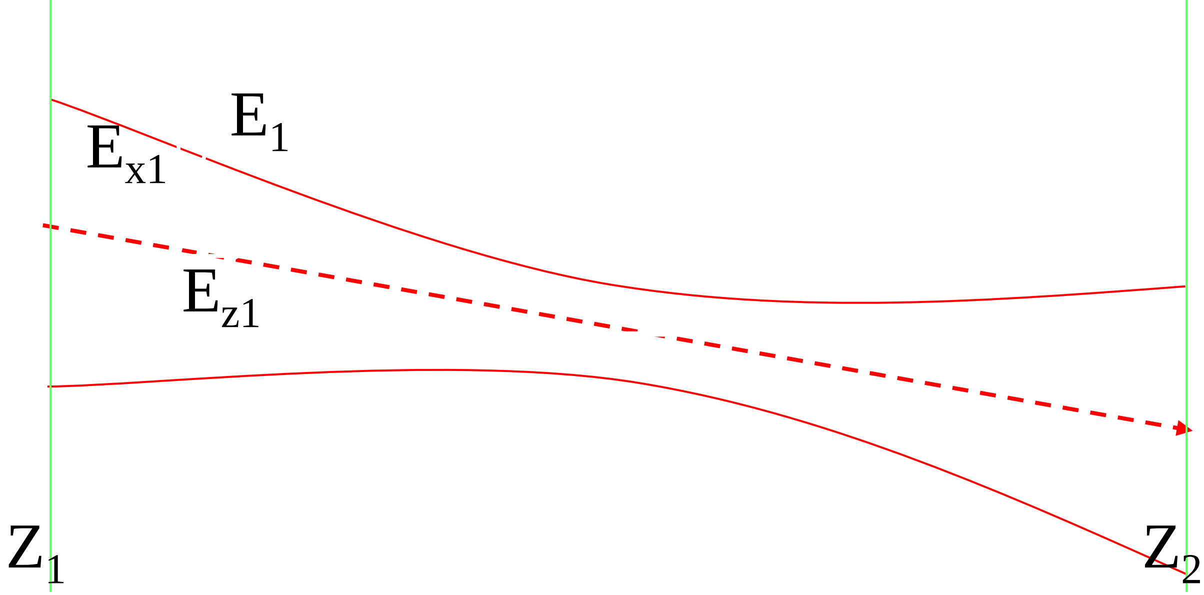
No other radiation beyond surface S

Specific Example:
Crossed-Gaussian Accelerator

Interferometric Acceleration



The laser beams are polarized in the XZ plane, and are out of phase by π



$$E_{x1} = \frac{E_{01} w_o}{w_1} \exp\left[-\frac{r_1^{\,2}}{w_1^2}\right] \cos(\psi_t)$$

$$E_{z1} = \frac{2E_{01} x_1}{k w_1^{\,2}} \exp\left[-\frac{r_1^{\,2}}{w_1^2}\right] \left(\sin(\psi_t) - \left[\frac{z_1}{Z_R}\right] \cos(\psi_t)\right)$$

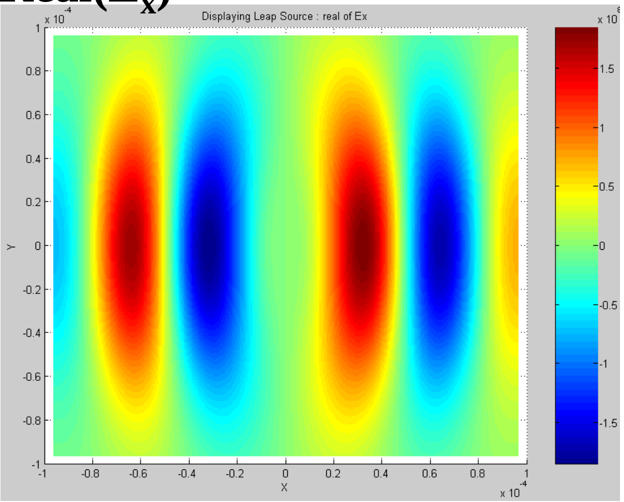
(paraxial approximation to first order in $1/w_o k \sim 10^{-3}$)

$$\psi_t = k z_1 - \omega t + z_1 r_1^{\,2} / (Z_R w_1^{\,2}) - \tan^{-1}(z_1 / Z_R) + \phi_o$$

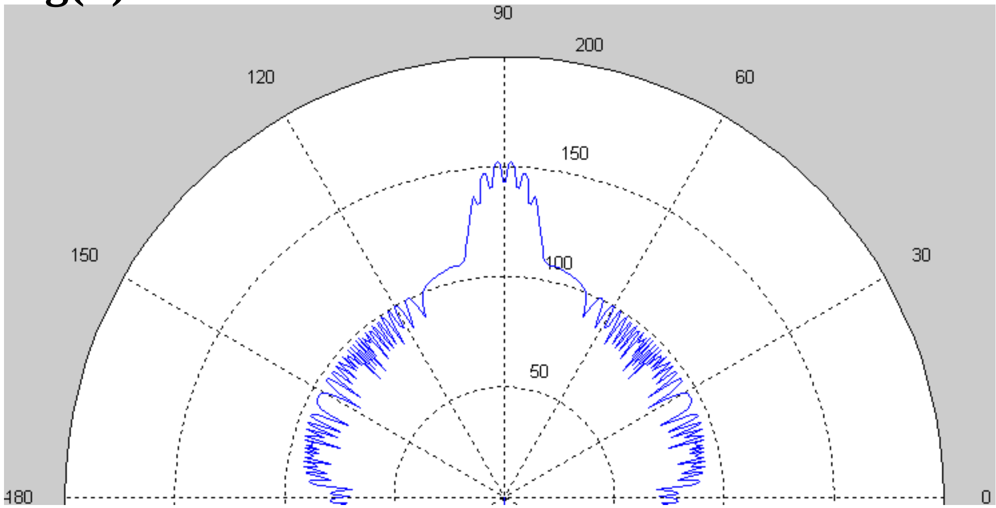
P. Sprangle, E. Esarey, J. Krall, A. Ting, *Opt. Comm.*, **124**, p.69ff, (1996).

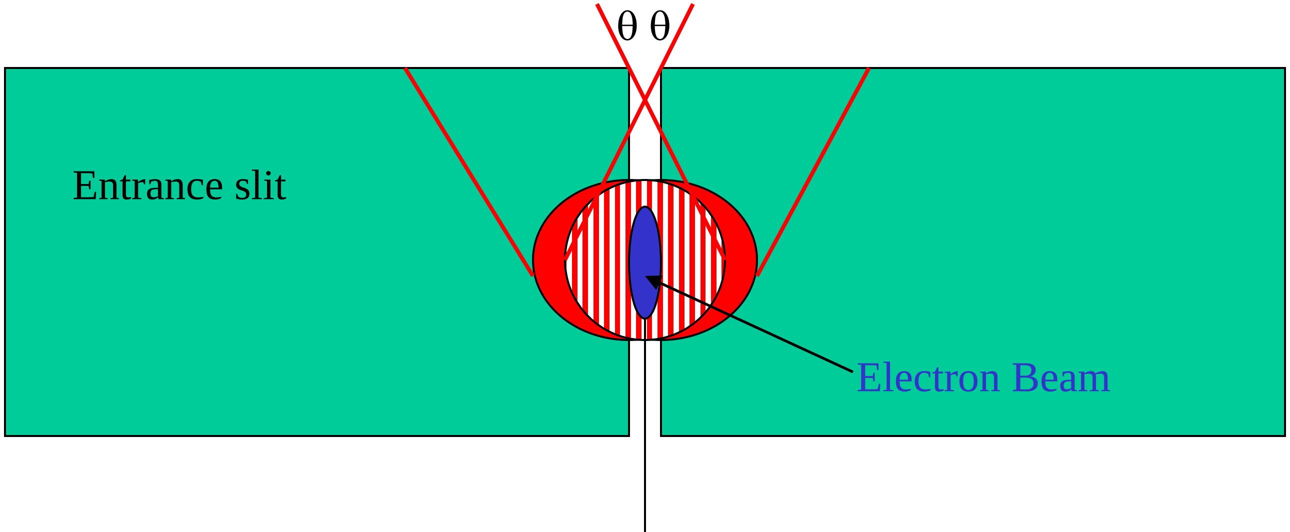
Crossed Gaussian Illumination

Real(E_x)

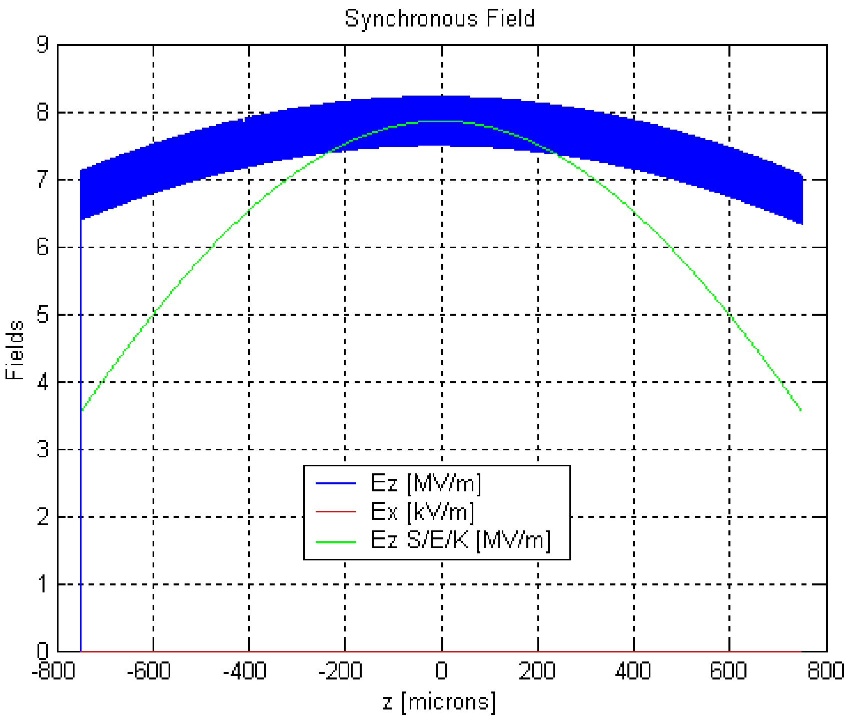


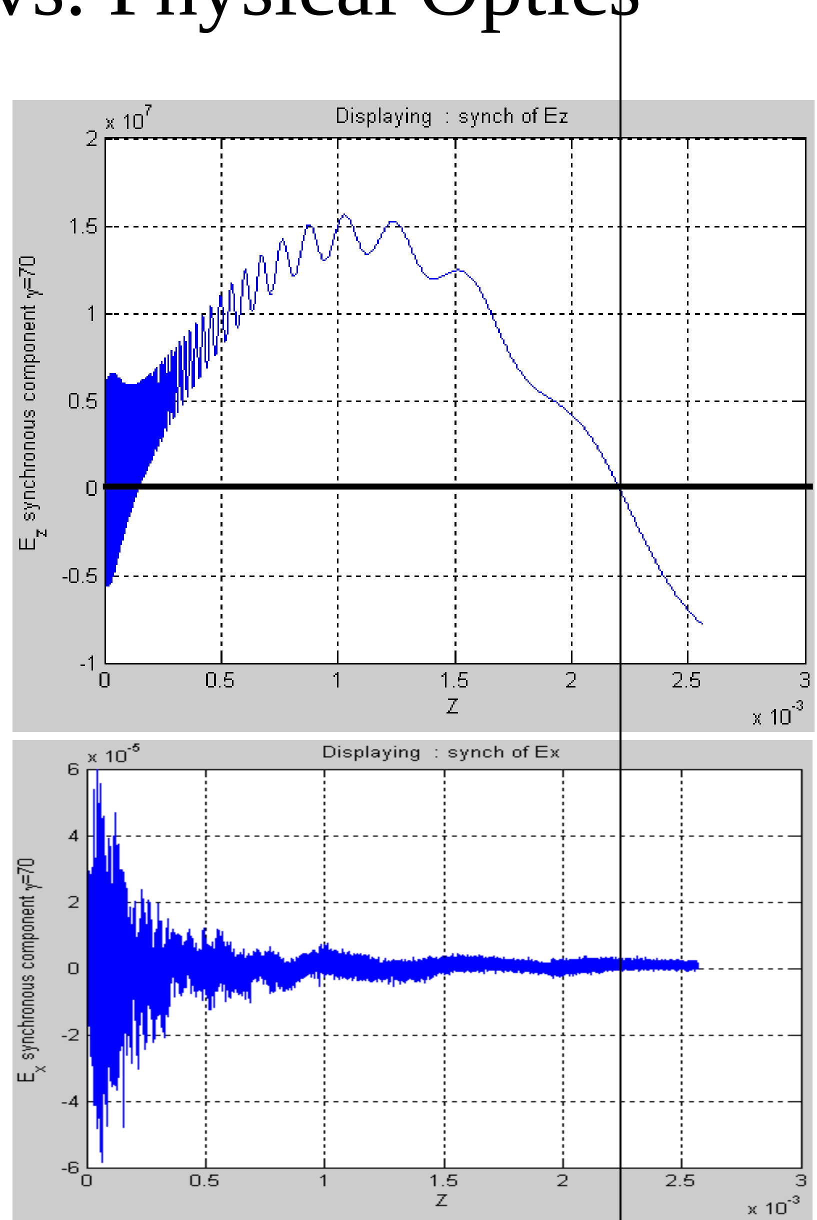
log(S) far-field





Analytic Field Calculations: Paraxial Approximation vs. Physical Optics

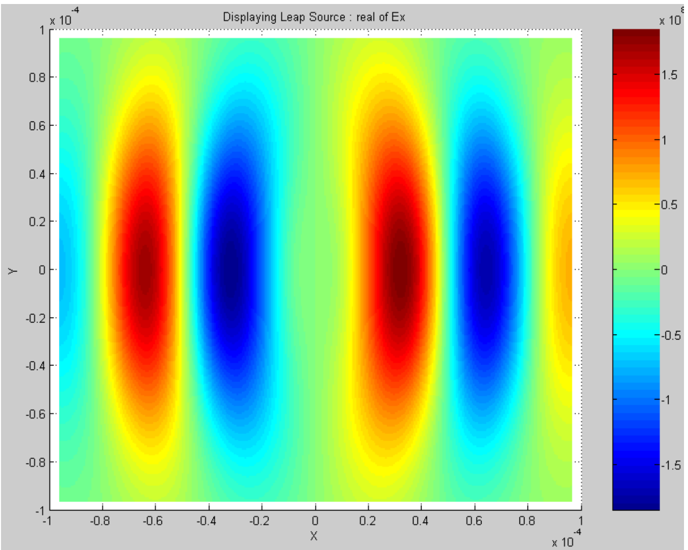




Analytic theory (Sprangle/Esarey/Krall 1996) , numerical integration of crossed TEM_{00} modes, and a brute-force vector diffraction calculation of fields

Accelerator vs. Radiator

Real(E_x)

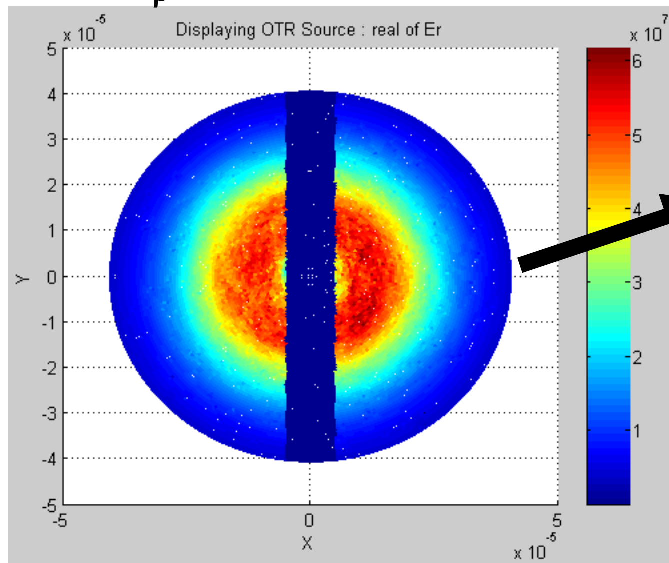


Crossed-Beam Accelerator

Crossed Gaussian beams, w_0=64μ,

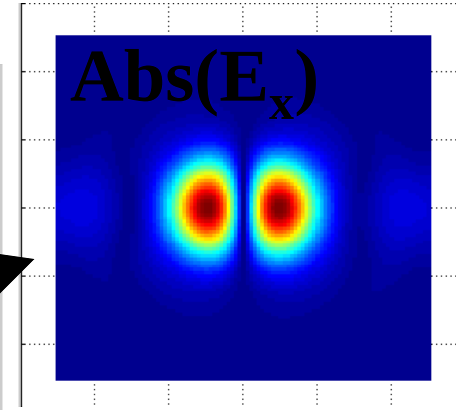
θ=11.5 mr

Real(E_ρ)



CTR Radiator, viewed narrowband

10μ x 10μ x λ/30 bunch, 10^4 particles



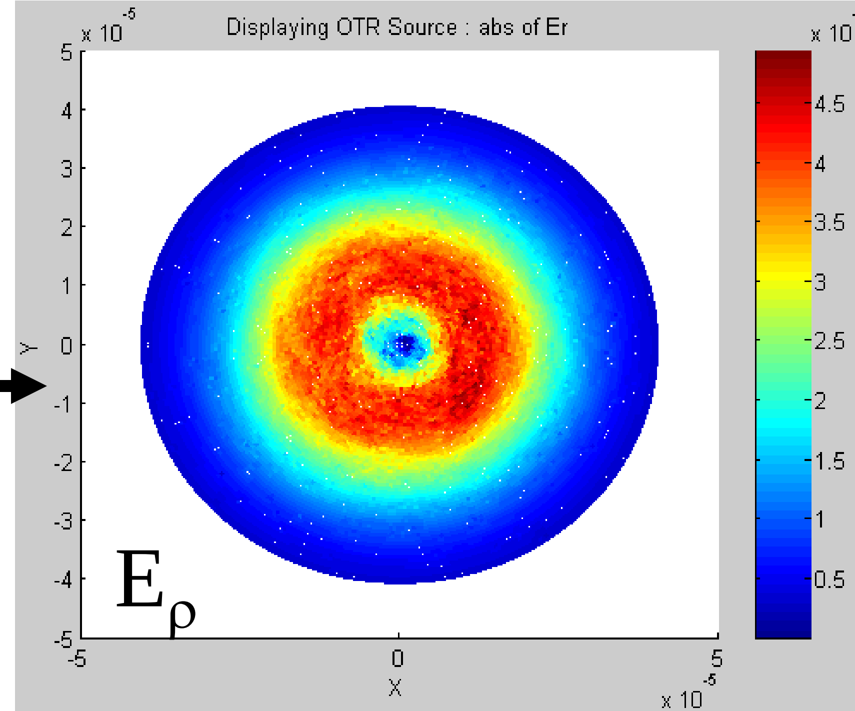
Far-field CTR
From Slit

$$E_{\{x,y\}} = \sum_{i=1}^{N=10^4} \frac{e\omega}{4\pi\varepsilon_o \gamma\beta^2 c^2} \frac{\{x_i, y_i\}}{R_i} \sqrt{\frac{2}{\pi}} \exp\left(\frac{j\omega z_i}{\beta c}\right) K_1\left(\frac{R_i \omega}{\gamma\beta c}\right)$$

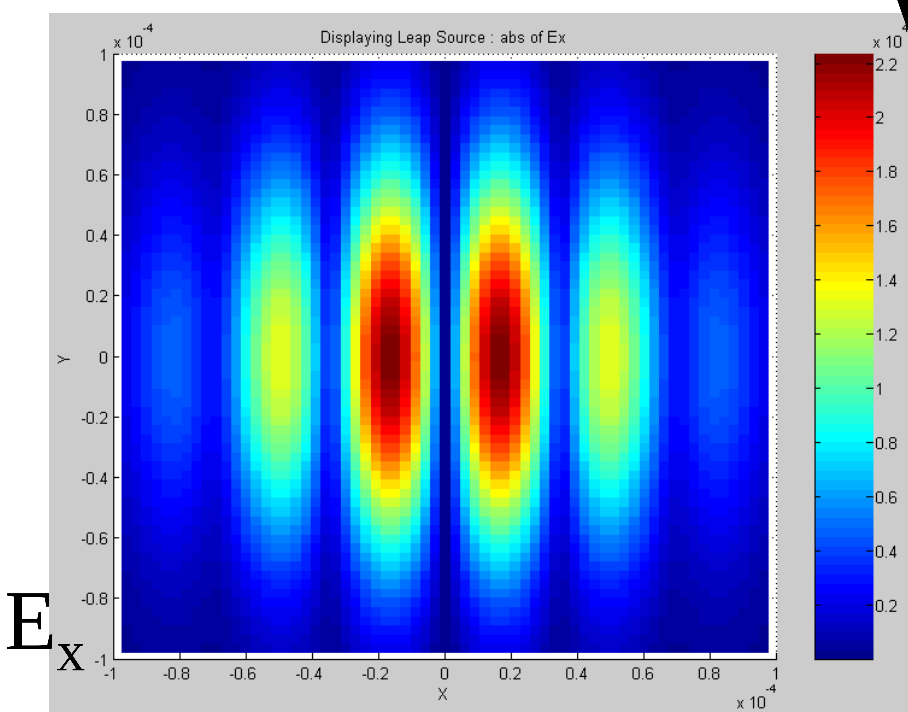
$$E_z = -j \sum_{i=1}^{N=10^4} \frac{e\omega}{4\pi\varepsilon_o \gamma^2 \beta^2 c^2} \sqrt{\frac{2}{\pi}} \exp\left(\frac{j\omega z_i}{\beta c}\right) K_0\left(\frac{R_i \omega}{\gamma\beta c}\right)$$

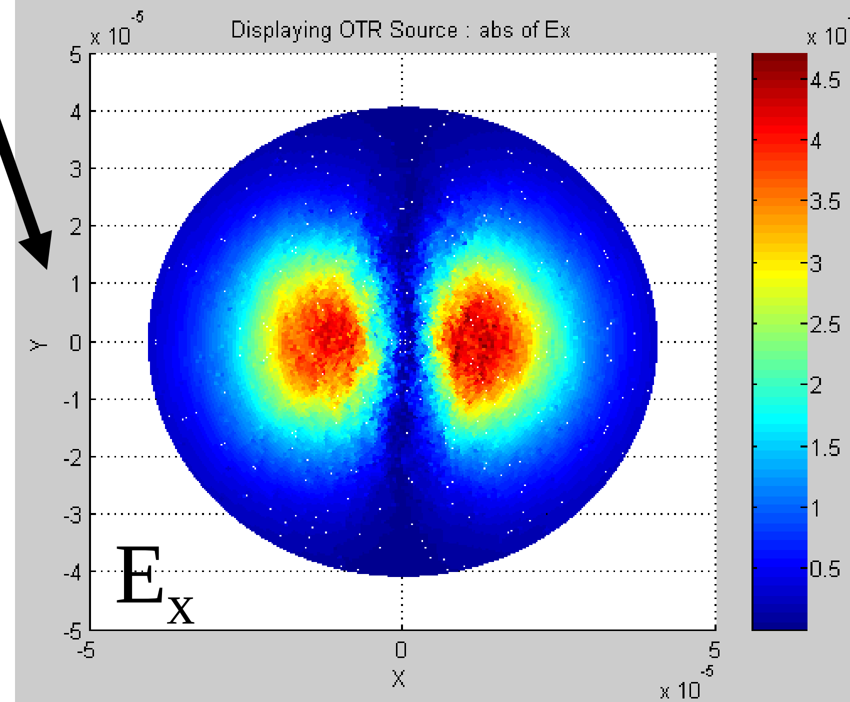
$$\vec{H} \approx Y\hat{k} \times \vec{E}$$

Sprangle-Esarey-Krall vs. Inverse Transition Radiation: Near-Field CTR at 800nm

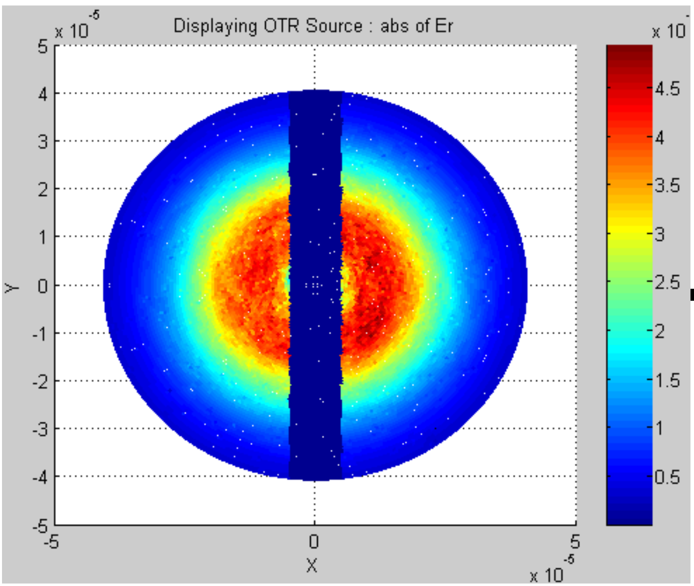


LEAP





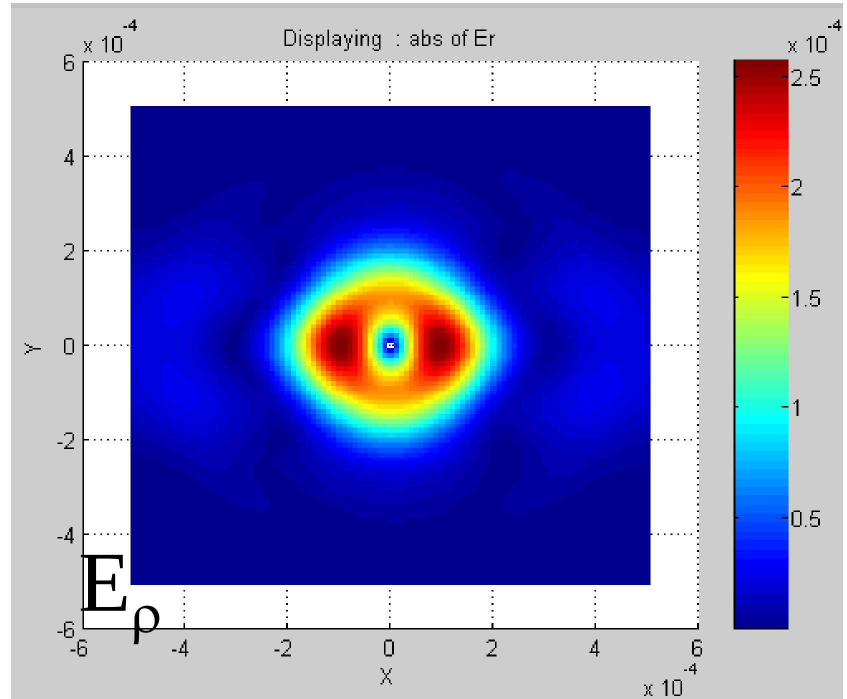
Inverse Transition Radiation: Far-Field

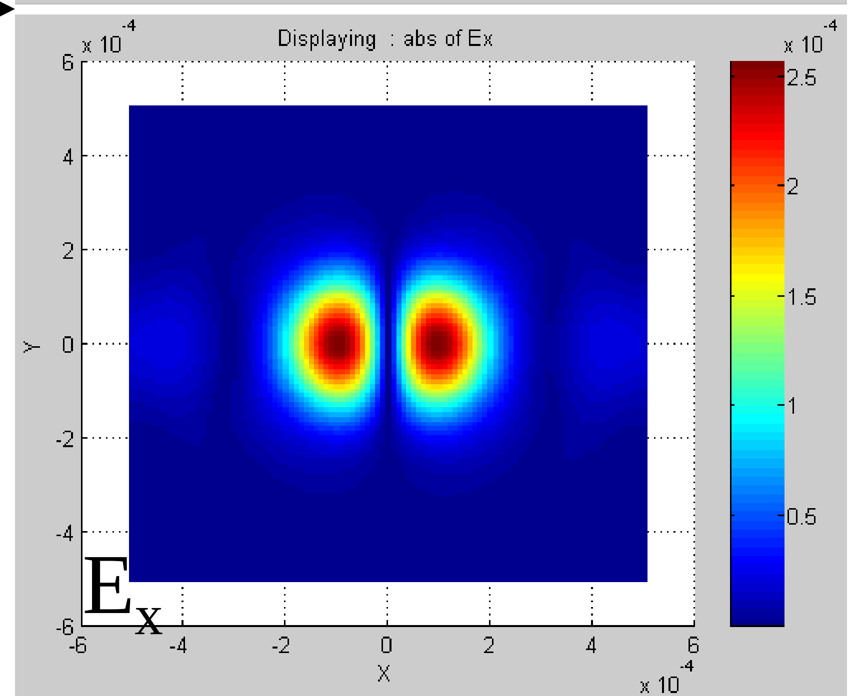


Propagate

12,500 λ

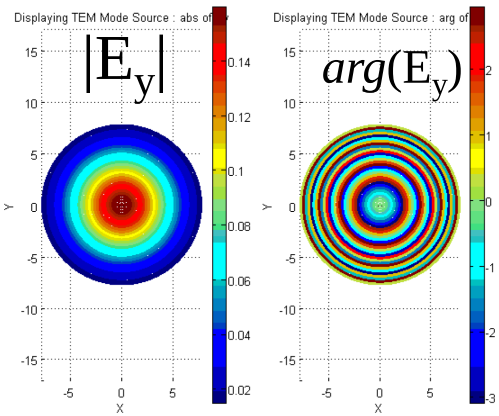
[1 cm=6.4 z_R]



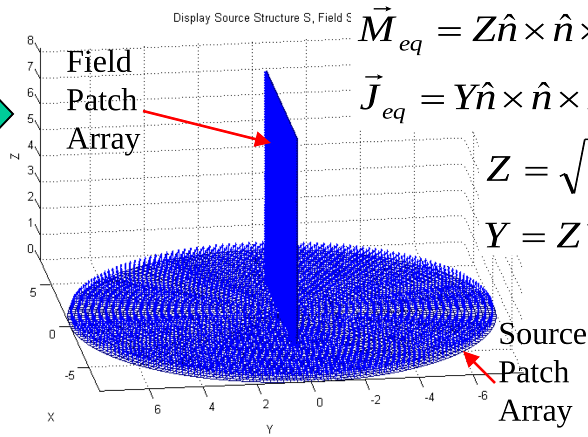


Vector Diffraction Code

A Matlab implementation of Huygen's Principle in l.i.h. media



1. Construct or load incident fields



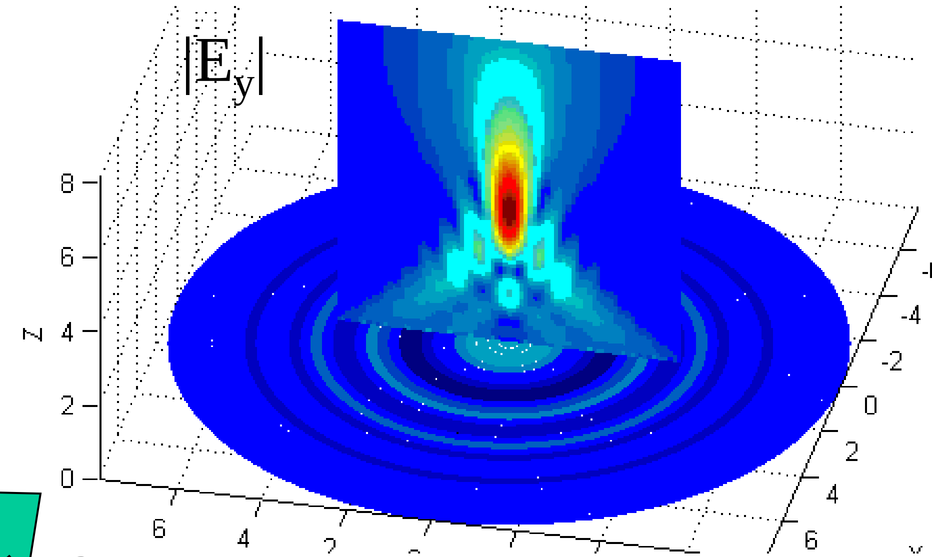
$$\vec{M}_{eq} = Z\hat{n}\times\hat{n}\times\vec{H}$$

$$\vec{J}_{eq} = Y\hat{n}\times\hat{n}\times\vec{E}$$

$$Z = \sqrt{\frac{\mu}{\varepsilon}}$$

$$Y = Z^{-1}$$

2. From incident fields, calculate surface currents on initial, discrete surface



3. Propagate to field points

$$\vec{E}(r) = -\int \vec{\vec{G}}_{EJ}(\vec{R})\cdot\vec{J}(\vec{r}\,')dS' - \int \vec{\vec{G}}_{EM}(\vec{R})\cdot\vec{M}(\vec{r}\,')dS'$$

$$\vec{H}(r) = -\int \vec{\vec{G}}_{HM}(\vec{R})\cdot\vec{M}(\vec{r}\,')dS' - \int \vec{\vec{G}}_{HJ}(\vec{R})\cdot\vec{J}(\vec{r}\,')dS'$$

$$\vec{\vec{G}}_{EJ}(\vec{R}) = jkZ\frac{e^{-jkR}}{4\pi R}\left[\left(\vec{\vec{1}} - \hat{R}\hat{R}\right)\left(1 - \frac{1}{k^2R^2} - j\frac{1}{kR}\right) + \hat{R}\hat{R}\left(\frac{2}{k^2R^2} + j\frac{2}{kR}\right)\right]$$

$$\vec{\vec{G}}_{EM}(\vec{R}) = \frac{e^{-jkR}}{4\pi R}\left(\frac{1}{R} + jk\right)\hat{R}\times\vec{\vec{1}}$$

$$\vec{\vec{G}}_{HM}(\vec{R}) = jk_oY\frac{e^{-jkR}}{4\pi R}\left[\left(\vec{\vec{1}} - \hat{R}\hat{R}\right)\left(1 - \frac{1}{k^2R^2} - j\frac{1}{kR}\right) + \hat{R}\hat{R}\left(\frac{2}{k^2R^2} + j\frac{2}{kR}\right)\right]$$

$$\vec{\vec{G}}_{EM}(\vec{R}) = -\frac{e^{-jkR}}{4\pi R}\left(\frac{1}{R} + jk\right)\hat{R}\times\vec{\vec{1}}$$

$$\hat{R}\hat{R} = \frac{1}{R}\begin{bmatrix}(x-x')^2 & (x-x')(y-y') & (x-x')(z-z') \\ (y-y')(x-x') & (y-y')^2 & (y-y')(z-z') \\ (z-z')(x-x') & (z-z')(y-y') & (z-z')^2\end{bmatrix}$$

L. Diaz, T. Milligan, *Antenna Engineering Using Physical Optics*, Artech House, London, (1996).

Fundamental Physics Considerations I

- Lawson-Woodward Theorem requires that one or more of:
  - Boundaries*
  - Gases
  - Periodic transverse motion of accelerated particles

  be present for linear acceleration ($\propto E$) to take place

- *Furthermore, since free-space modes are strictly TEM, efficient acceleration requires a structure that either strongly diffracts the TEM mode, or guides a TM-like mode ➔ boundaries must be very close to the beam ($r/\gamma\lambda<1$)

- Accelerating fields must not degrade transverse emittances → fields must be rotationally symmetric

Fundamental Physics Considerations II

- Good coupling impedance → strong fundamental-mode wakefield
- Stability against regenerative beam breakup → minimal higher order mode wakefields
- Higher stored energy (Q) in structure → Tighter dimensional tolerances

- Larger acceptance → larger aperture

Basic Technical Considerations I

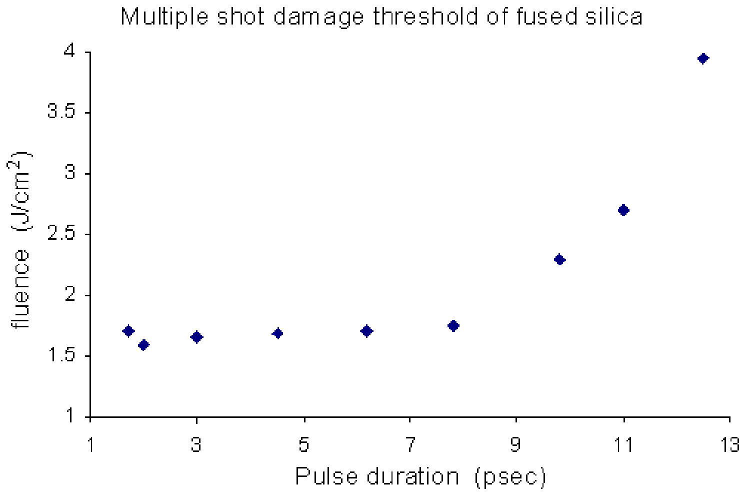
- For efficiency, accelerators should be designed at wavelengths to use the most efficient lasers
  - Yb:KGd(WO_4)$_2$, Yb:KY(WO_4)$_2$ → $\lambda \sim 1.0\ \mu m$
  - Erbium Fiber → $\lambda \sim 1.5\ \mu m$
  - Cr^{++}:ZnSe → $\lambda \sim 2.2\text{-}2.8\ \mu m$

- For economy of fabrication, accelerators should be designed at wavelengths were materials are low loss and amenable to lithographic or fiber drawing processes:
  - a-SiO_2 → $\lambda \sim 0.2\text{-}2.5\ \mu m$
  - c-Si → $\lambda > 1.5\ \mu m$

Basic Technical Considerations II

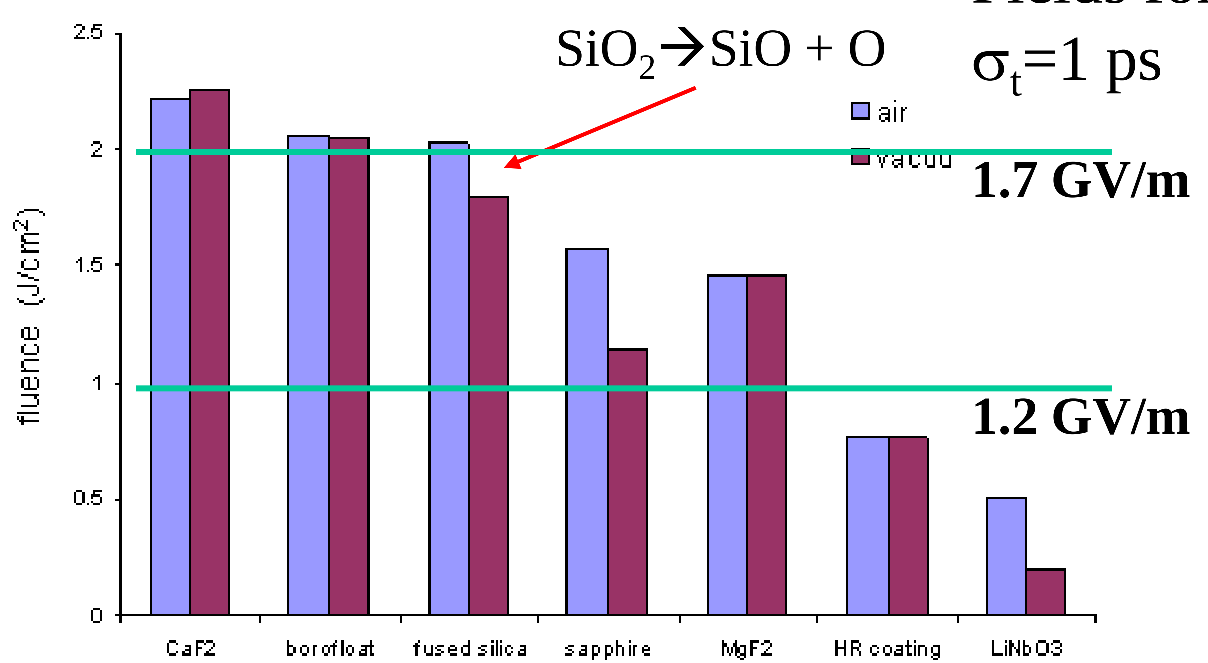
- Structure materials should have
  - High damage threshold → resistance to breakdown
  - High radiation resistance → resistance to high-radiation accelerator environment
  - Excellent optical linearity, even under large applied electric fields → minimal intensity-dependent dephasing
  - Good thermal conductivity, low thermal expansion → thermally stable under changing operating conditions
  - Amenability to fabrication → Lithography or fiber drawing

Short-Pulse Laser Damage of Dielectrics



- $t^{1/2}$ dependence for t>10 ps
- No t dependence for t<5 ps
- "Laser Conditioning" raises threshold ~10%
- Some materials perform worse under vacuum

material	damage threshold	
	air	vacuum
CaF_2	2.212	2.260
borofloat	2.064	2.054
fused silica	2.027	1.795
sapphire	1.574	1.152
MgF_2	1.455	1.455
800 nm HR	0.769	0.768
$LiNbO_3$	0.504	0.194

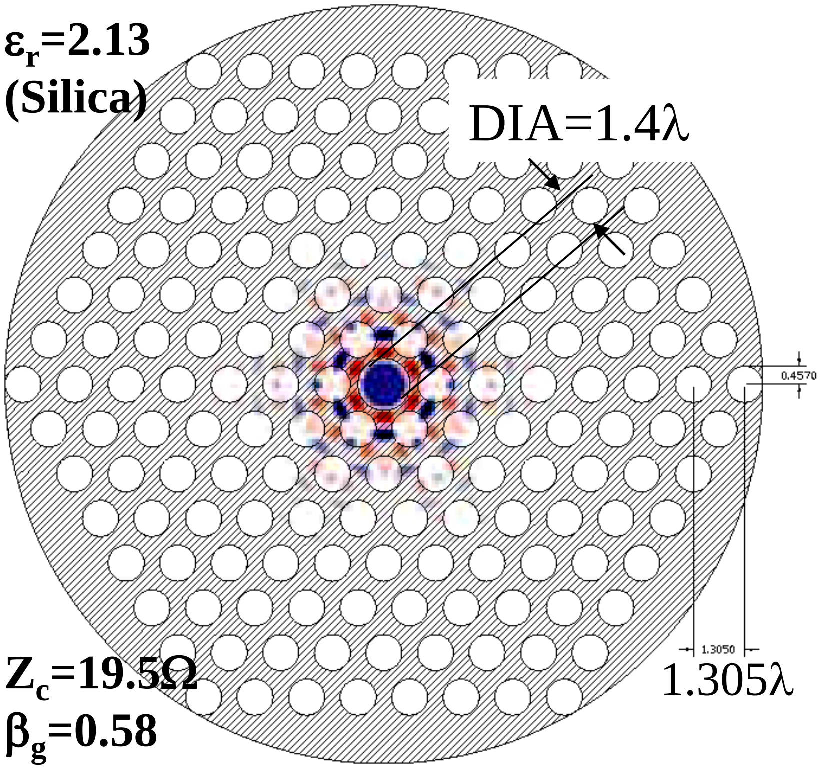


T. Plettner, "Proof-of-Principle Experiment for Crossed Laser beam Electron Acceleration in a Dielectric Loaded Vacuum Structure", Ph.D. Thesis, Stanford Univ., 2002.

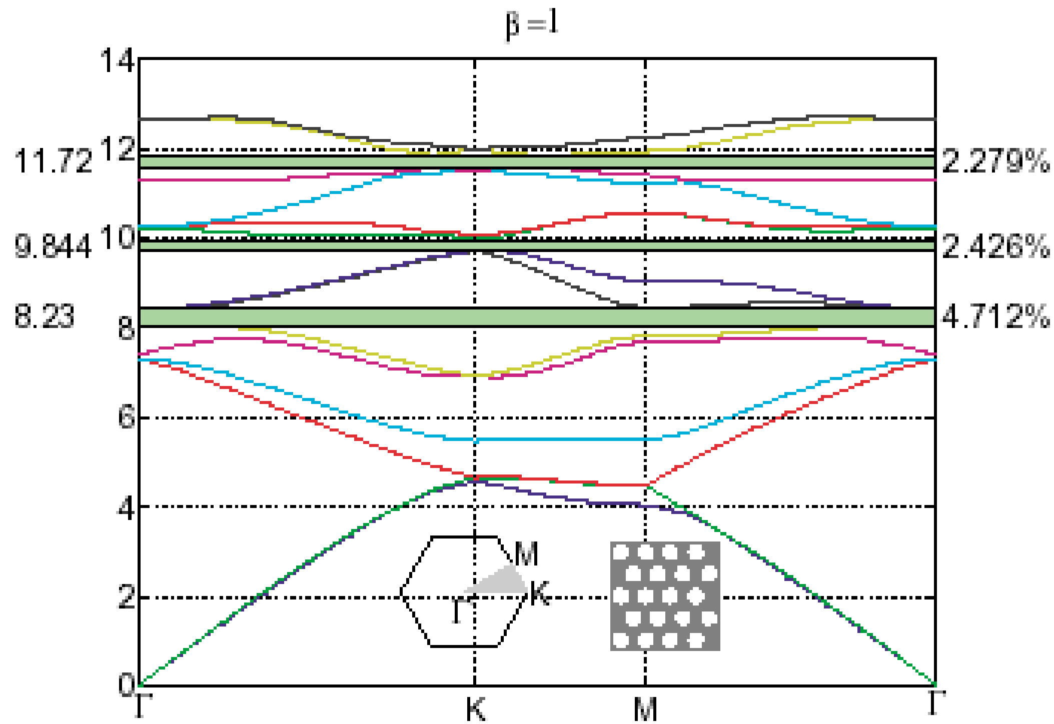
Examples of
Laser Accelerators

Second Example: Photonic Band Gap Structures

ε_r=2.13 (Silica)



Z_c=19.5Ω
β_g=0.58

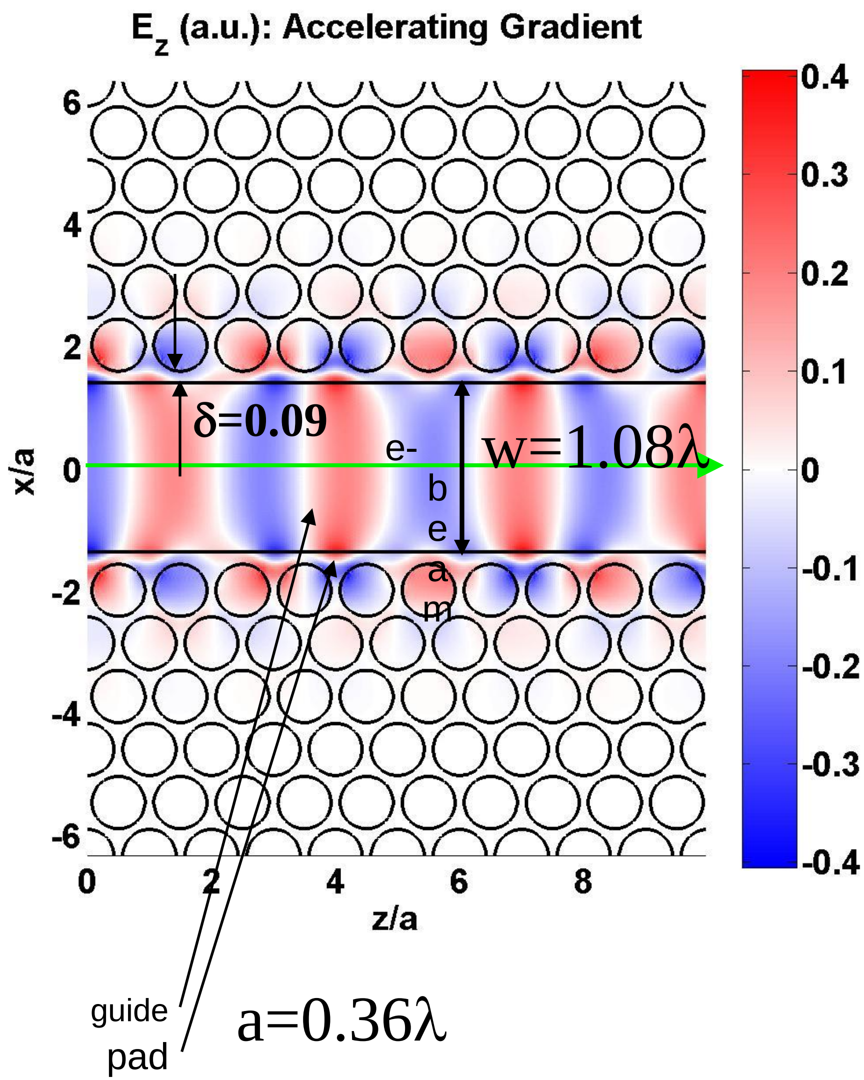


X. (Eddie) Lin, "Photonic Band Gap Fiber Accelerator", *Phys. Rev. ST-AB*, **4**, 051301, (2001).

- Can be designed to support a single, confined, synchronous mode
- All other modes at all other frequencies radiate strongly

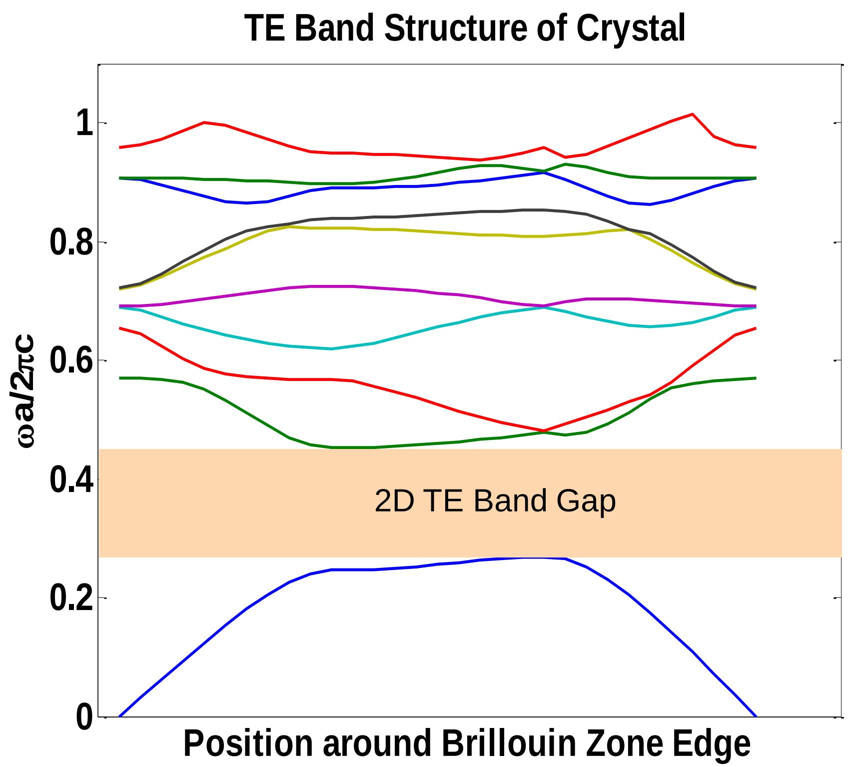
$$Z_c = \frac{|E_{\mathrm{acc}}|^2 \lambda^2}{2P}$$

2D Photonic Band Gap Structures



a=0.36λ

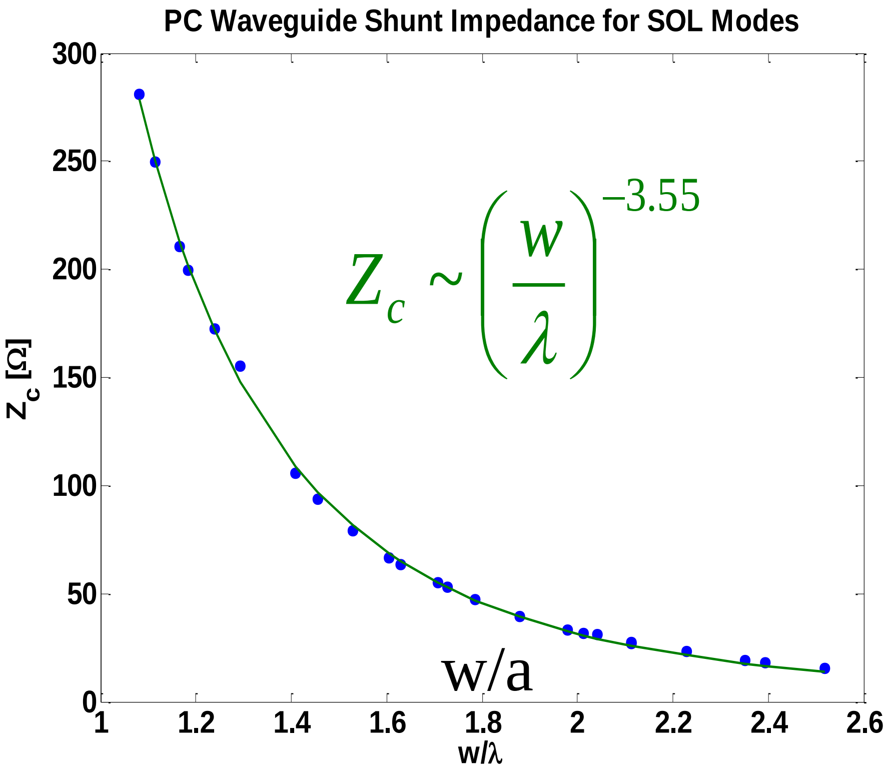
Ben Cowan, ARDB, SLAC

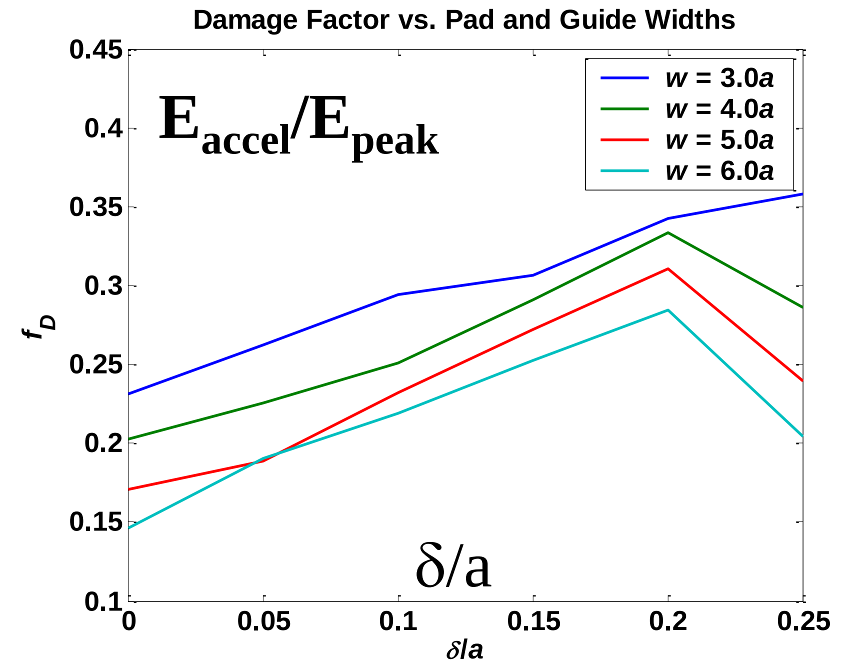


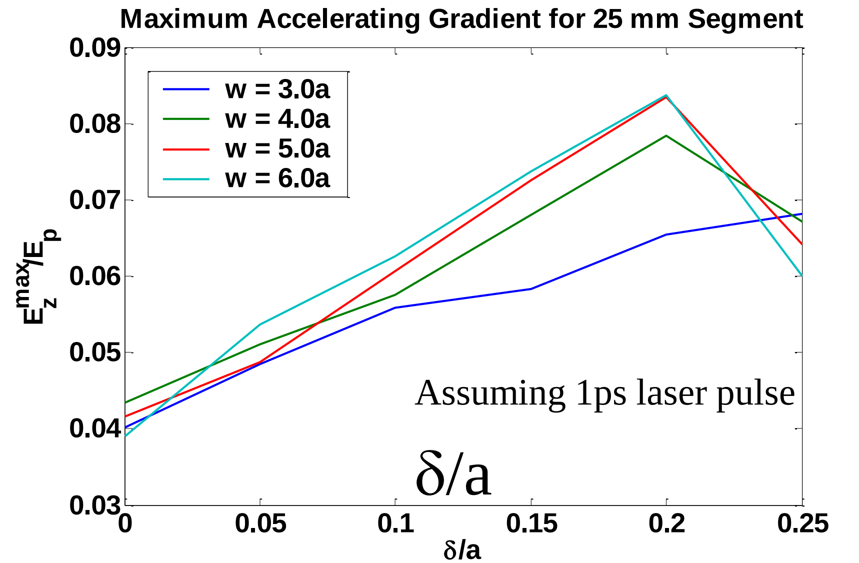
ε_r=12.1 (Silicon)

This geometry is designed for the lithographic process.

Impedance and Gradient Optimization

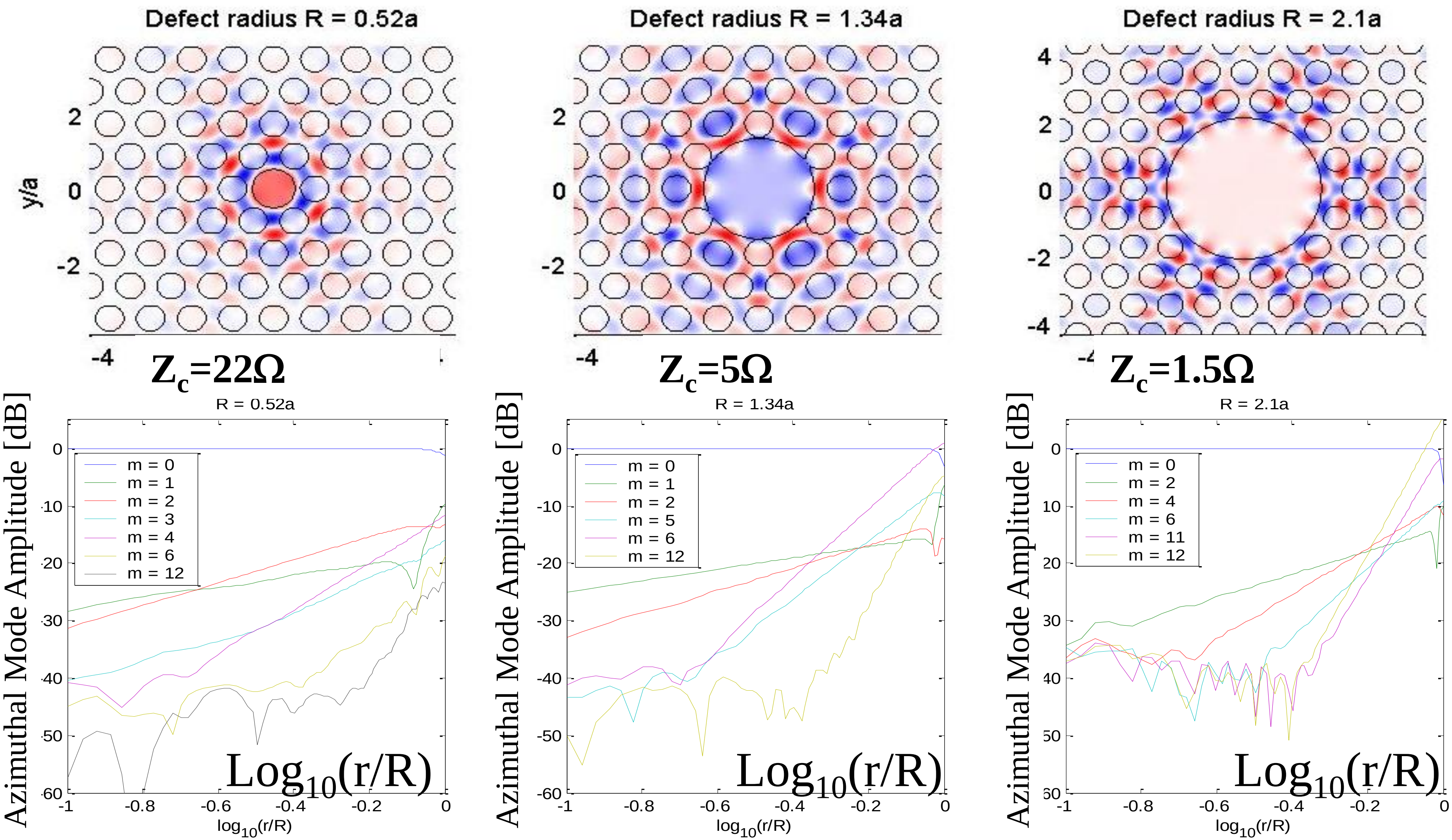






Ben Cowan, ARDB, SLAC

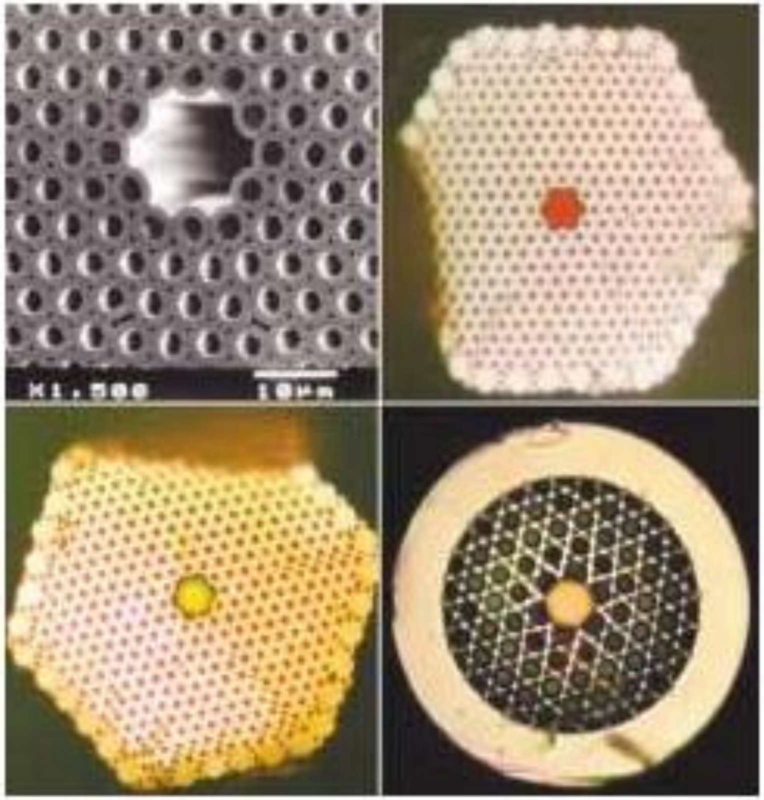
2D Fiber Structures



Mehdi Javanmard, ARDB, SLAC

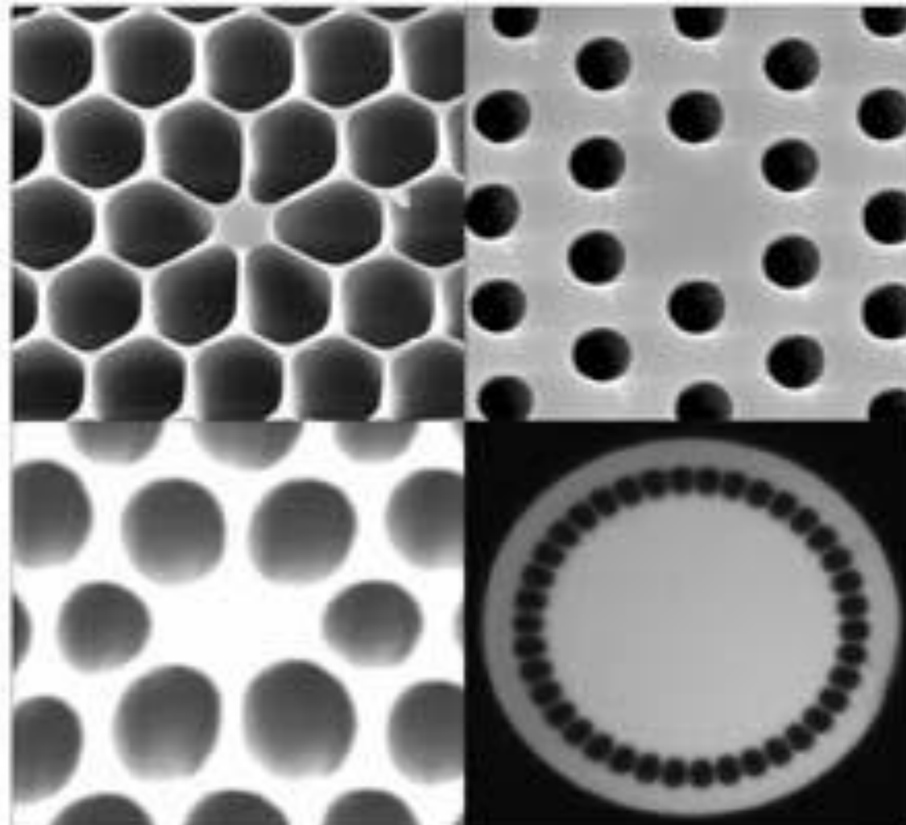
This geometry is designed for the fiber drawing process.

Fabricated Examples



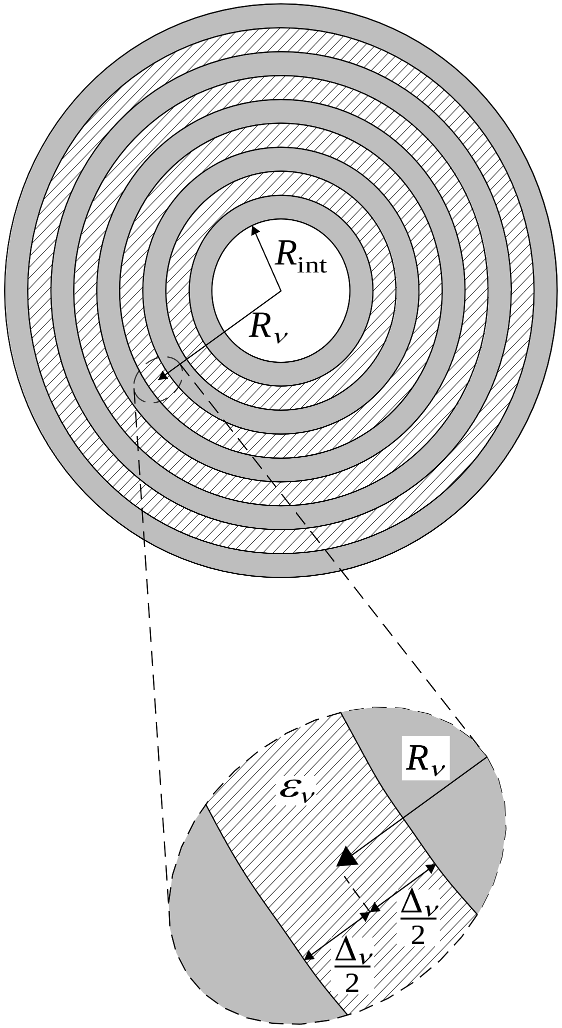
P. Russell, “Holey fiber concept spawns optical-fiber rennaissance”, *Laser Focus World,* Sept. 2002, p. 77-82.

a b c d



PCF structures vary according to application: (a) highly nonlinear fiber; (b) endlessly single-mode fiber; (c) polarization maintaining fiber; (d) high NA fiber. From René Engel Kristiansen (Crystal Fibre A/S), “Guiding Light with Holey Fibers,” *OE Magazine* June 2002, p. 25.

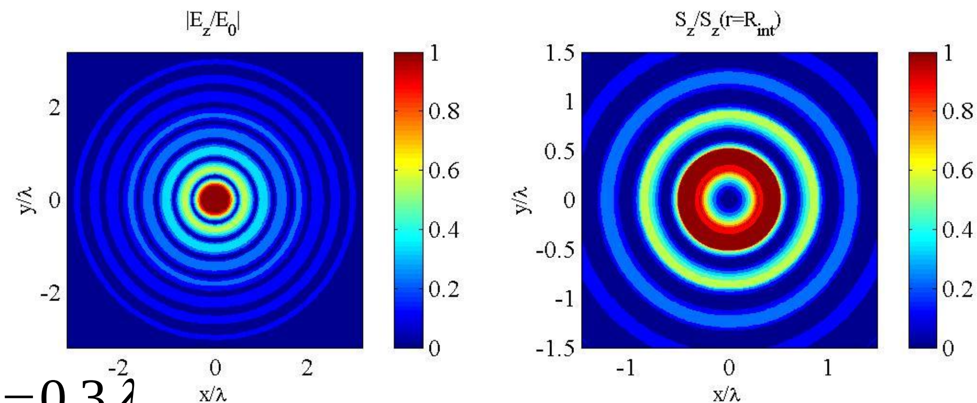
Hollow Fiber Bragg Accelerator



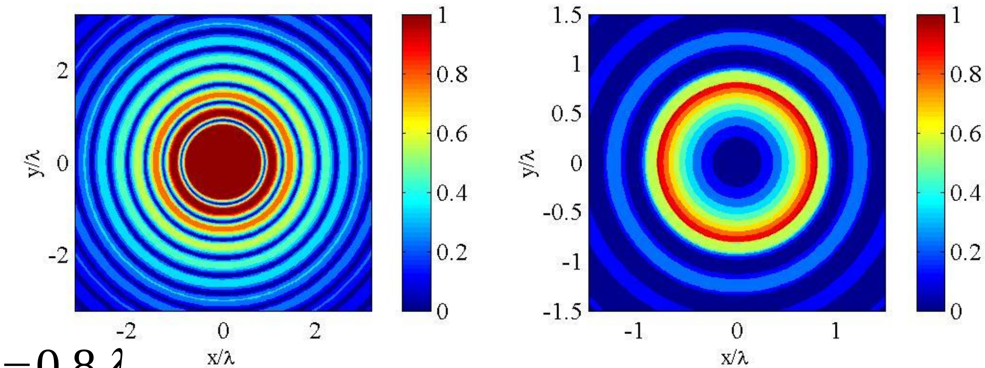
Concentric layers (ε_1,ε_2)

Each layer □ $\lambda / 4\sqrt{\varepsilon - 1}$

$v_{ph} = c$



$R_{\text{int}} = 0.3\lambda_0$



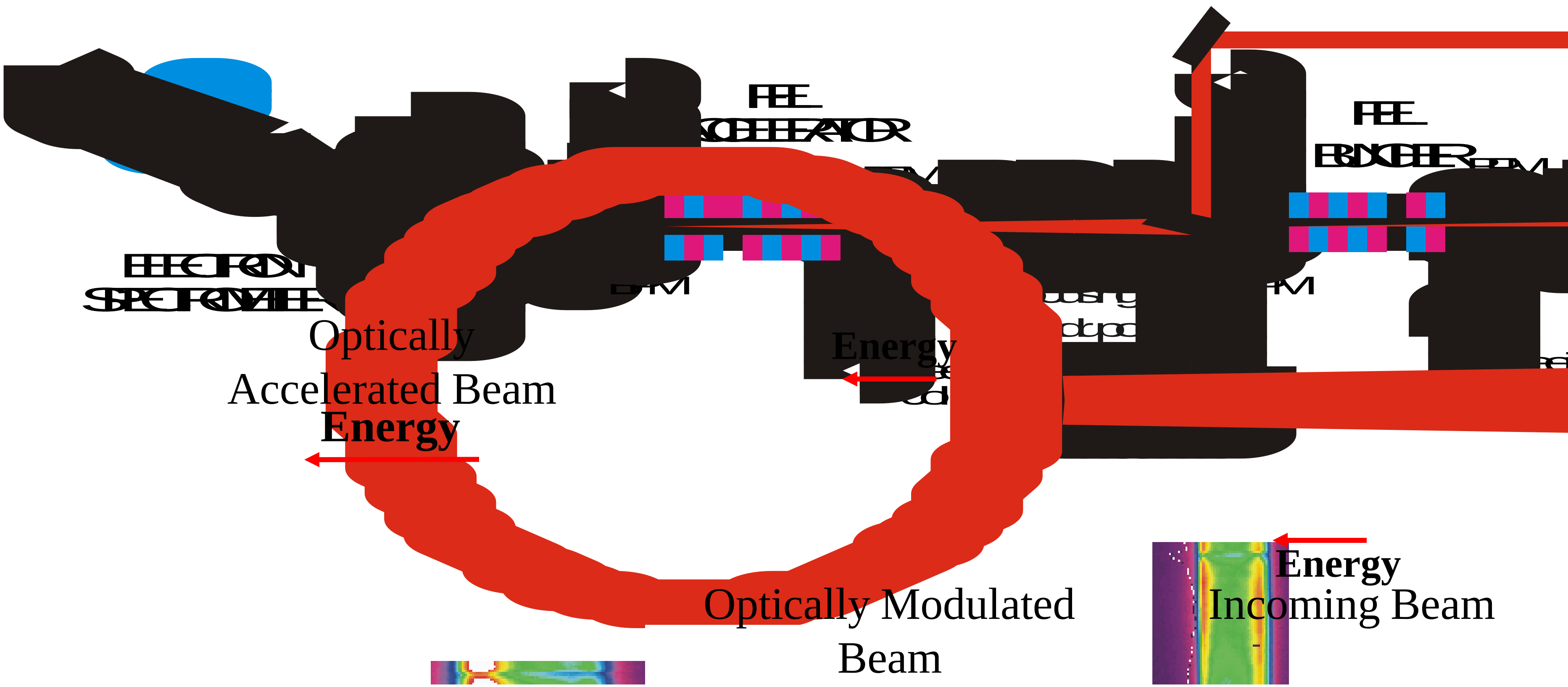
$R_{\text{int}} = 0.8\lambda_0$

Levi Schächter, The Technion

The Inverse Free Electron Laser

STELLA (Staged Electron Laser Acceleration) experiment at the BNL ATF

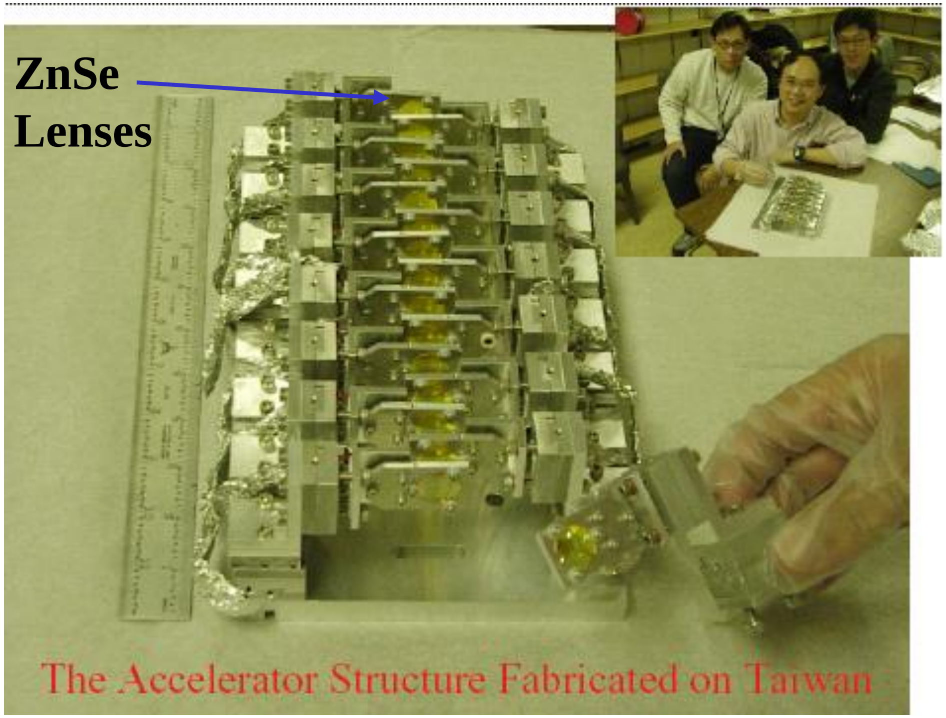
(STI Optronics/Brookhaven/Stanford/U. Washington)



W. Kimura, I. Ben-Zvi, in proc. of Adv. Accel. Conc. Conf., Santa Fe, NM, 2000.

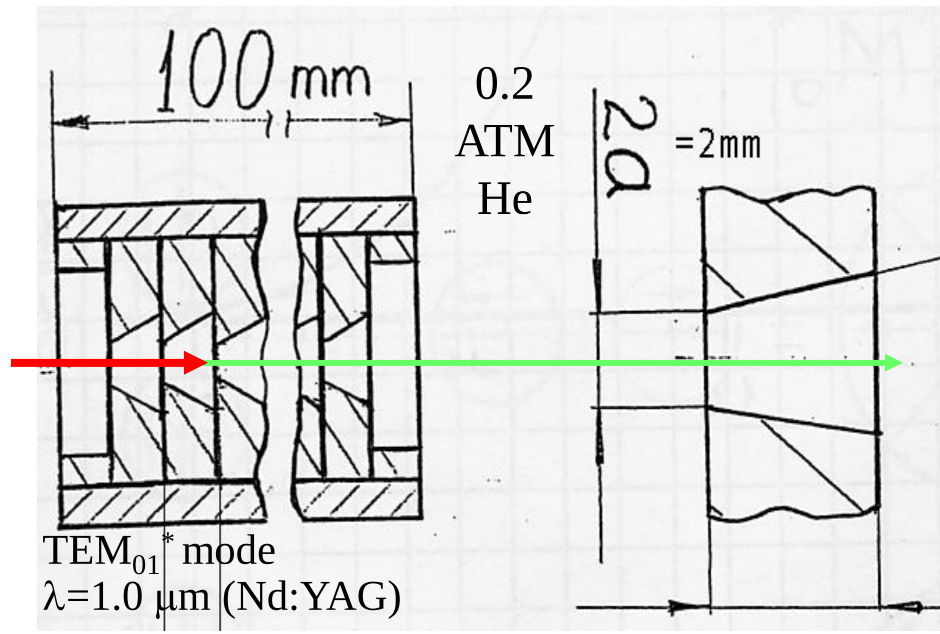
Multicell Linear Acceleration Experiments

Multiple ITR Accelerators
Y.-C. Huang, NTHU, Taiwan
(at Brookhaven)



Expected gain: 250 keV over 24cm.
Y-C. Huang, *et al*, Nat'l Tsinghua University.
Images from ATF User's Meeting, January 31, 2002.

Inverse Cerenkov Acceleration in Waveguide
(Unfolded Fabry-Perot Interferometer)
A. Melissinos, R. Tikhoplav (U. Rochester, Fermilab)

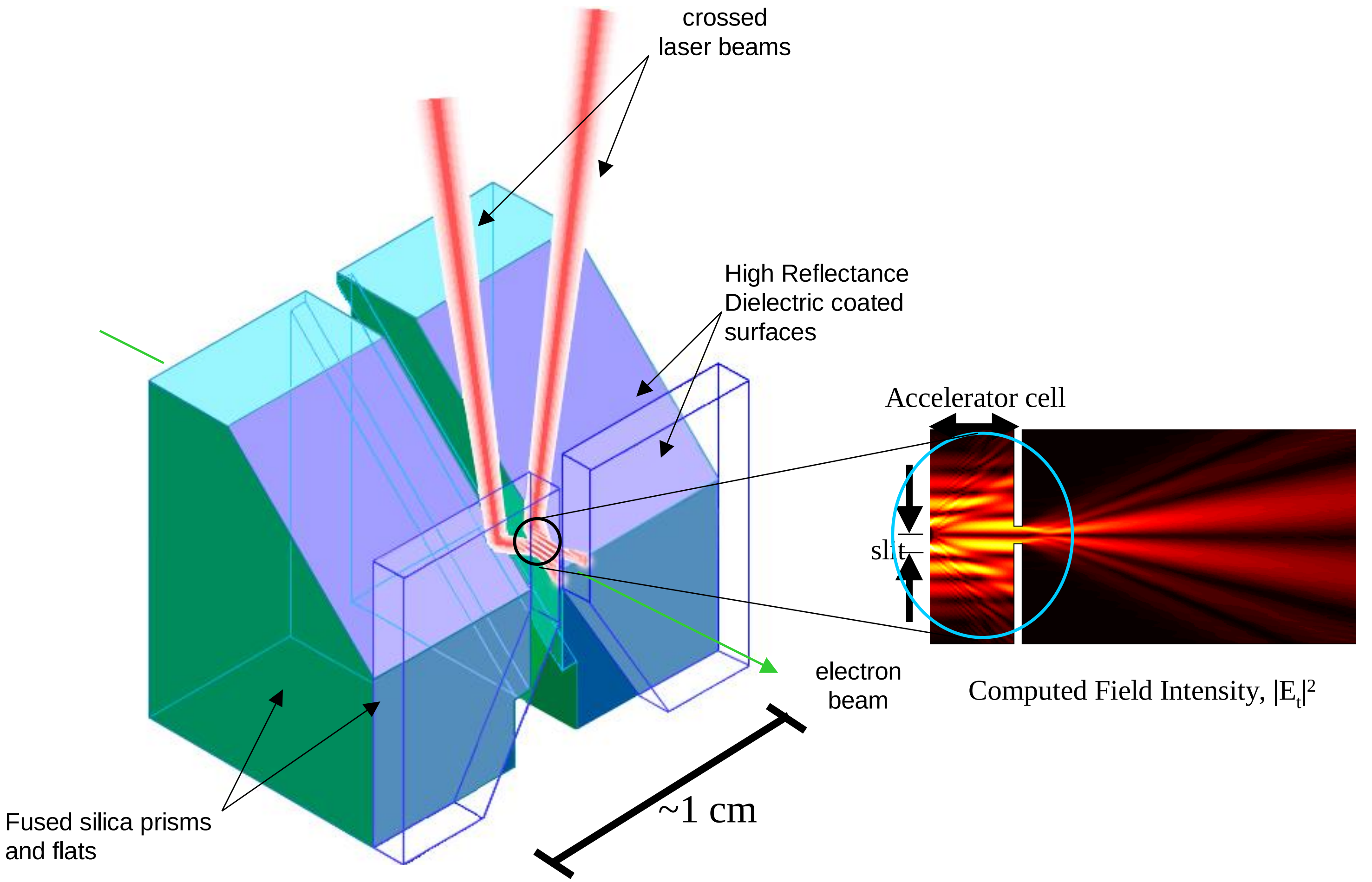


Status: Structure has been fabricated with 80% power transmission measured. Nd:YAG drive laser is under construction at Fermilab now.

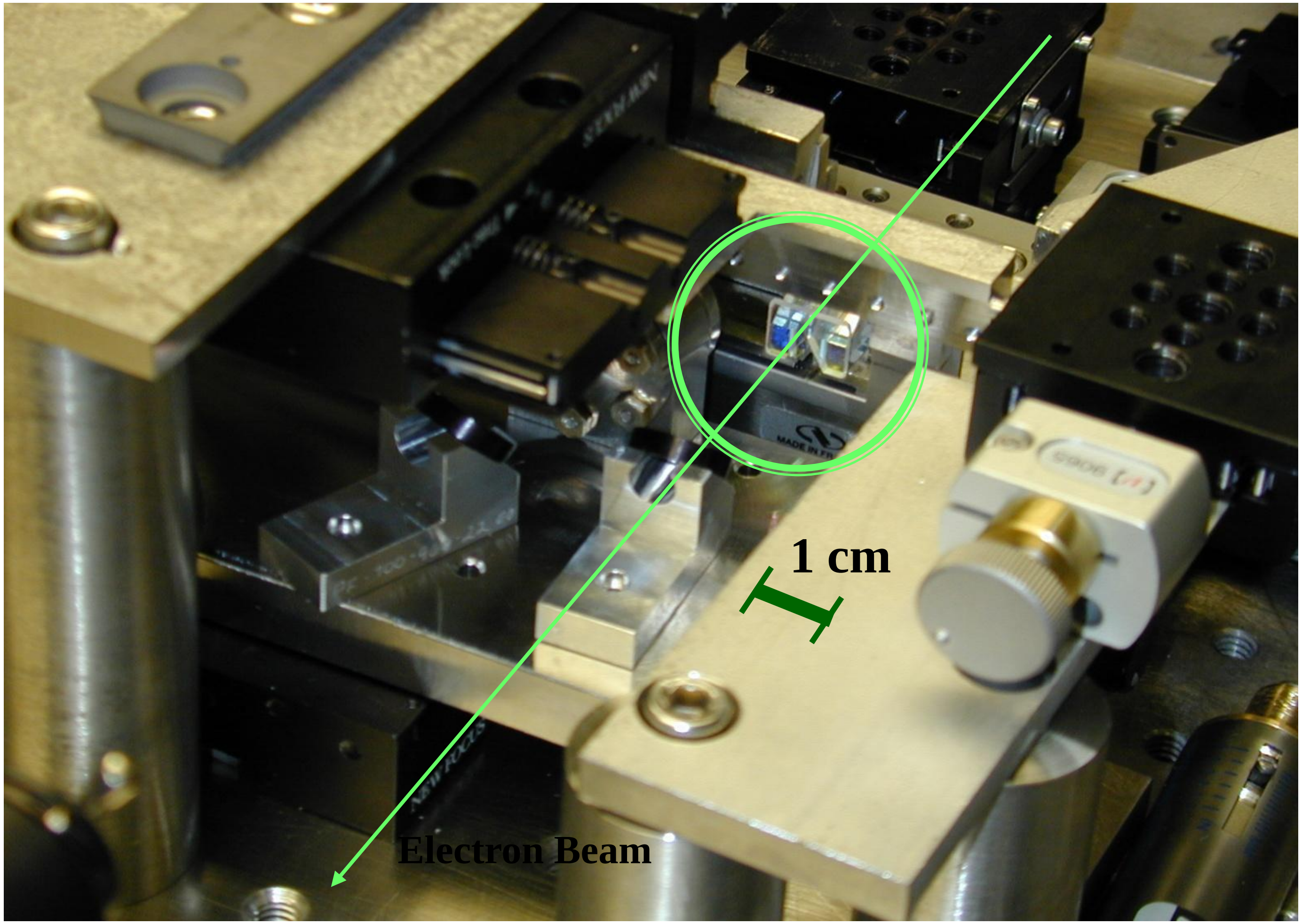
The LEAP Accelerator Cell

"Laser Electron Acceleration Project"

Stanford University (Appl Phys. & HEPL) / SLAC



The LEAP Accelerator Cell



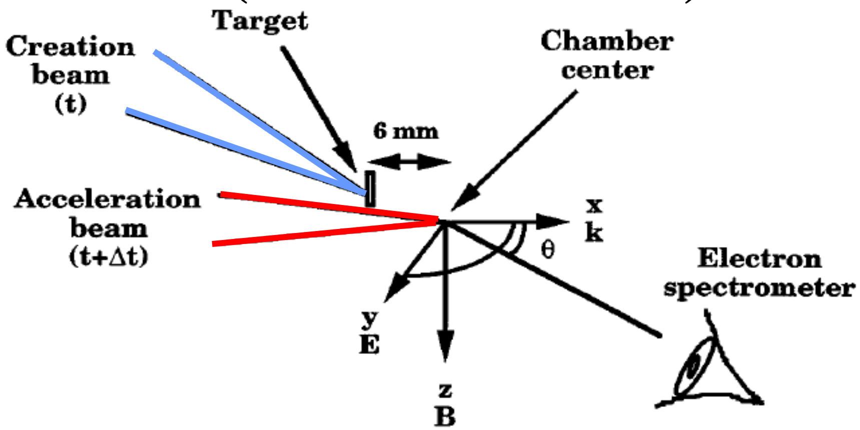
High Field Coupling Mechanisms

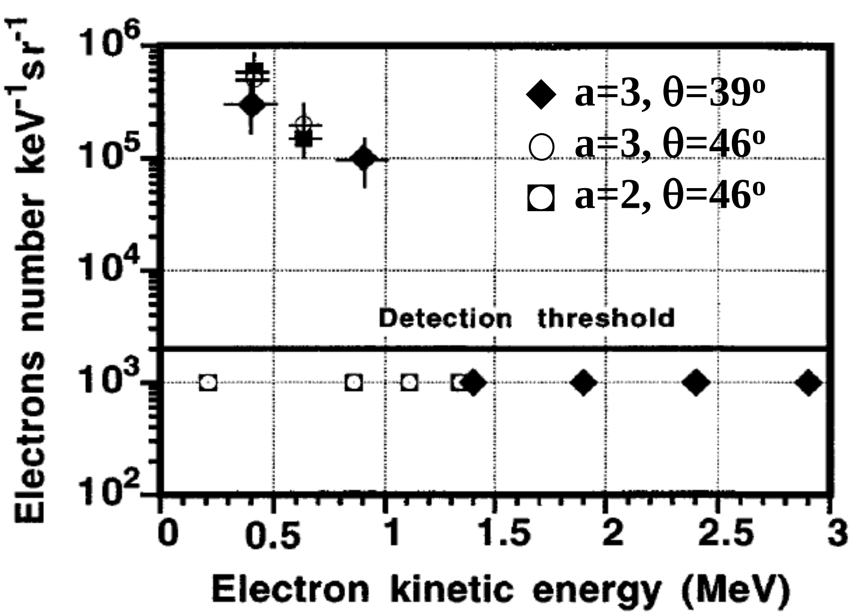
$a_o > 1$

No near-field structure required, but beam deflection often plays a vital role

High Field Acceleration Mechanisms

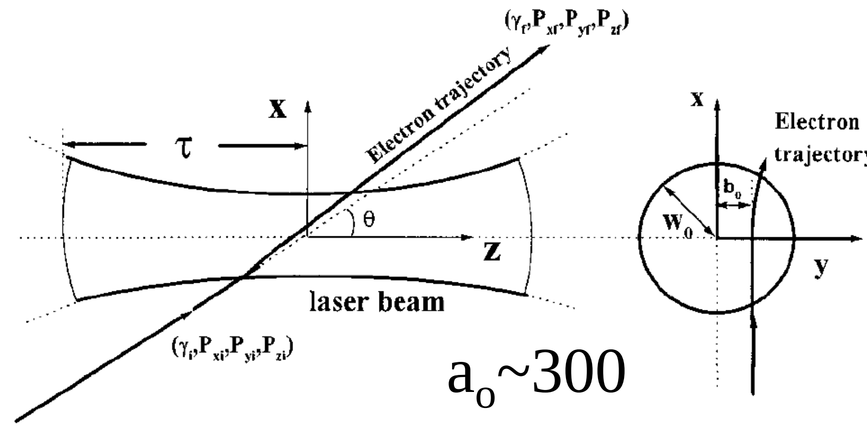
Ponderomotive Acceleration
(CEA-Lemeil-Valenton)

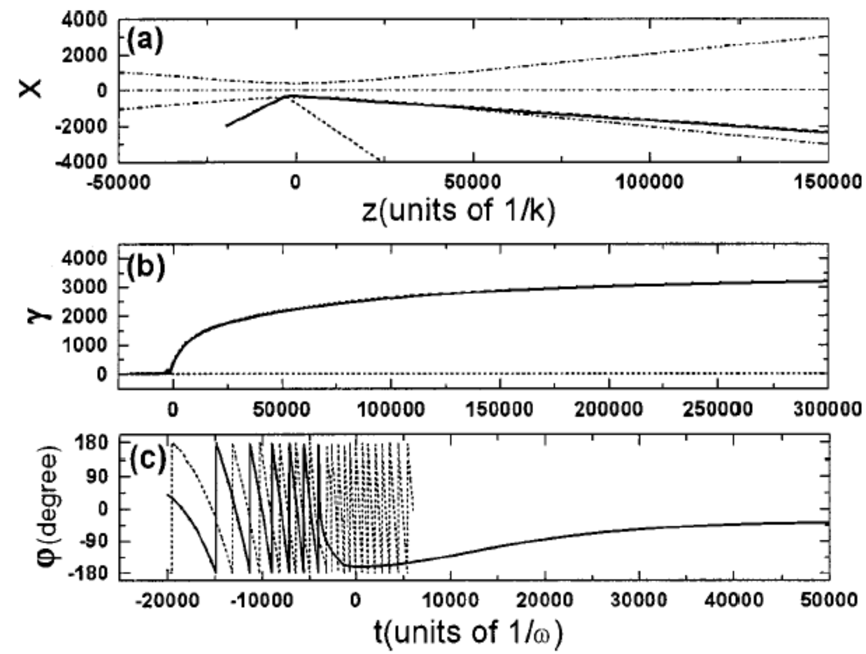




G. Malka, E. Lefebvre, J. Miquel, Phys. Rev. Lett. **78,** 3314 (1997).

Combined Ponderomotive and Nonlinear Compton Scattering

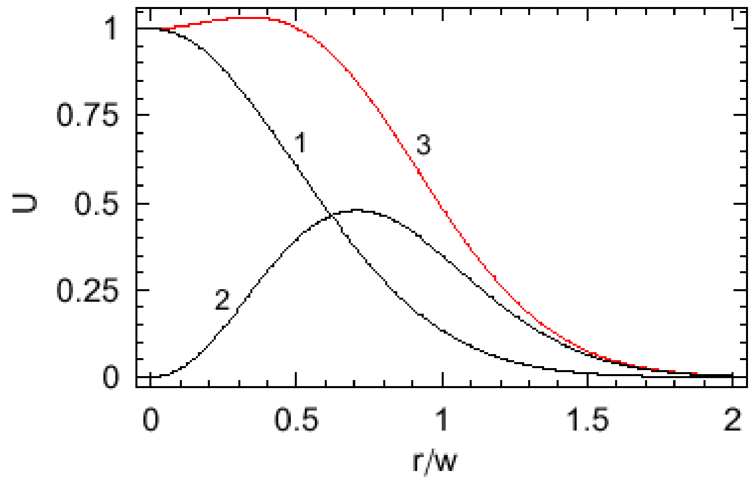




P. X. Wang, Y. K. Ho, *et al*, J. Appl. Phys. **91** (2) p. 856, (2002).

High Field Acceleration Mechanisms

Ponderomotive Acceleration and Focussing



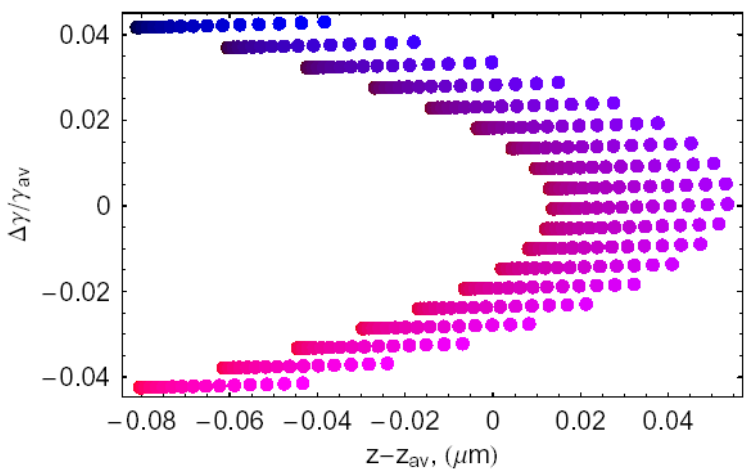
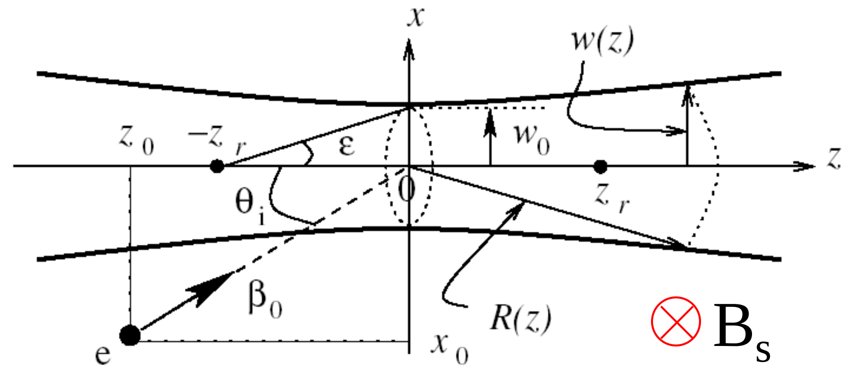
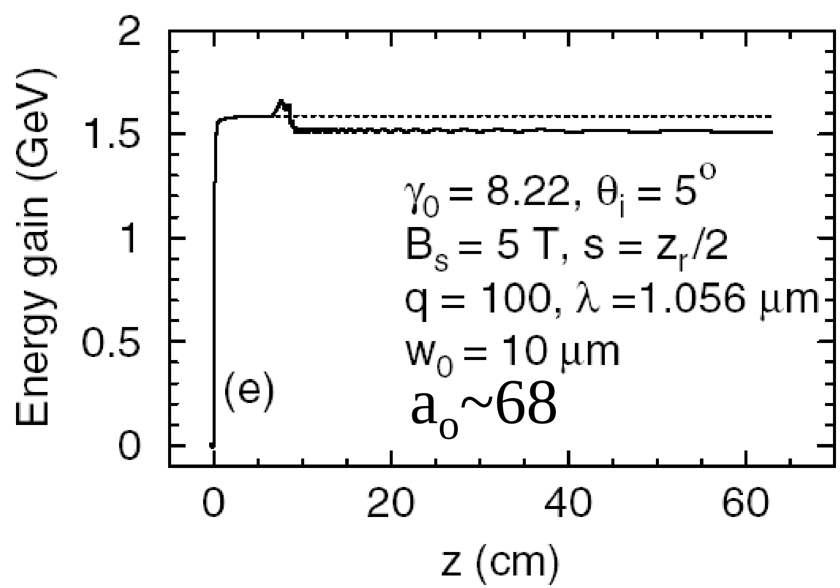


FIG. 4. Longitudinal phase space $a_0 \sim 5$

G. Stupakov, M. Zolotorev, Phys Rev Lett **86** 5274 (2001).

Ponderomotive Scattering with Deflection field to aid beam extraction

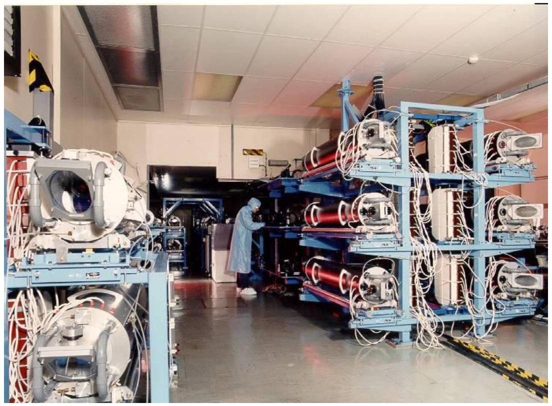




Y. Salamin, C. Keitel, Phys Rev Lett 88 095005 (2002).

Lasers at 1 PW and beyond

The GEKKO XII 1 PW (400 J) Nd:glass system at ILE Osaka. (a_o~30)



The Vulcan 100 TW (2600 J) Nd:glass system at RAL. Upgrading to 1 PW system.

(a_o~10→30)

The High Peak Power T^3 Laser 100 TW (1.9J) at JAERI Kansai. Upgrading to 1 PW system. (a_o~10→30)

Under Construction

National Ignition Facility (LLNL)
Nd:glass 600 TW (1.8 MJ)
(a_o~24→1300 [0.1 ps])

Laser MegaJoule (CEA-Limeil)
Nd:glass 600 TW (1.8 MJ)
(a_o~24→1300 [0.1 ps])

1 shot/8 hours!

Key Issues for Low Field Acceleration

- **What improvement in material damage threshold is possible?**
    - Are there better materials?
    - Do improved surface smoothness or material purity help?
- **What coupling impedance is possible?**
    - What structure geometries can be fabricated?
    - What geometric accuracy is possible?

- What electrical efficiency is possible?
- Can alignment and vibration tolerances be met?

Key Issues for High Field Acceleration

- **Can better laser media be manufactured?**
  - Broad(er) bandwidth
  - Higher damage and saturation fluences
  - Better thermal conductivity
  - Amenable to diode-pumping
- **How small a focal spot is possible?**
  - What surface accuracy is possible for very large optics?

- What beam quality is possible for what charge?

"One of the authors (W.W.H.), in his study of cavity resonators, was motivated by a desire to find a cheap method of obtaining high energy electrons. This cavity acceleration work was put aside, largely because of the change in standard of success caused by the advent of Kerst's betatron. . .

. . .By the end of the war many people were interested [in linear acceleration], possible reasons being: (a) wide-spread knowledge of cavity properties and technique, (b) the enormous pulsed powers made available by radar developments."

- E. L. Ginzton, W. W. Hansen, W. R. Kennedy, "A Linear Electron Accelerator", *Rev. Sci. Inst.*, **19**(2), p. 89, February 1948.

Final Thought