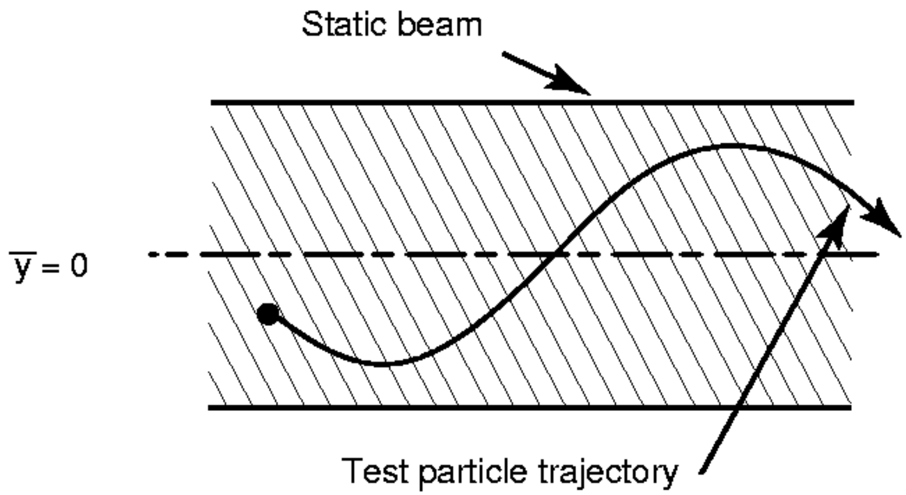
# Unit 8 - Lecture 19

# Limiting phenomena in rings: coherent & incoherent effects

William A. Barletta
Director, United States Particle Accelerator School
Dept. of Physics, MIT

# Types of tune shifts: Incoherent motion




- Center of mass does not move
- Beam environment does not “see” any motion
- Each particle is characterized by an individual amplitude & phase



From: E. Wilson Adams lectures

# Incoherent collective effects

- Beam-gas scattering
  - ➞ Elastic scattering on nuclei => leave physical aperture
  - ➞ Bremsstrahlung
  - ➞ Elastic scattering on electrons
  - ➞ Inelastic scattering on electrons

  leave rf-aperture

  =======> reduce beam lifetime
- Ion trapping (also electron cloud) - scenario
  - ➞ Beam losses + synchrotron radiation => gas in vacuum chamber
  - ➞ Beam ionizes gas
  - ➞ Beam fields trap ions
  - ➞ Pressure increases linearly with time
  - ➞ Beam -gas scattering increases
- Intra-beam scattering

# Intensity dependent effects

- Types of effects
  - Space charge forces in individual beams
  - Wakefield effects
  - Beam-beam effects

- General approach: solve

$$\boxed{x'' + K(s)x = \frac{1}{\gamma m \beta^2 c^2} F_{non-linear}}$$

- For example, a Gaussian beam has

$$F_{SC} = \frac{e^2 N}{2\pi\varepsilon_o \gamma^2 r}\left(1 - e^{-r^2/2\sigma^2}\right) \quad \textit{where } N = \textit{charge/unit length}$$

- For r < σ

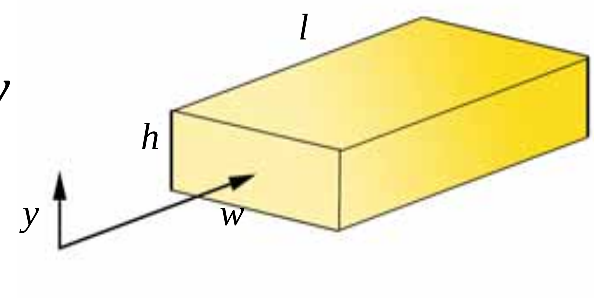
$$F_{SC} \approx \frac{e^2 N}{4\pi\varepsilon_o \gamma^2} r$$

# Beam-beam tune shift

$$\Delta p_y(y) \sim \left(F_{elec} + F_{mag}\right)\frac{l}{2c}$$ *(once the particle passes l/2 the other bunch has passed)*

- For $\gamma >> 1$, $F_{elec} \approx F_{mag}$

$$\mathrm{E} = \frac{N_{beam} e}{lwh} y$$




$$\therefore \Delta p_y(y) \approx 2\frac{e\mathrm{E}}{2c} = \frac{\mathrm{e}^2 N_B}{\varepsilon_o cwh} y$$

$$\Rightarrow \frac{\Delta p_y}{p_o} = \Delta y' \sim y$$ similar to gradient error $k_y \Delta s$ with $k_y \Delta s = \frac{\Delta y'}{y}$

- Therefore the tune shift is

$$\Delta Q = -\frac{\beta^*}{4\pi} k_y \Delta s \approx \frac{r_e \beta^* N}{\gamma wh}$$ where $r_e = \frac{e^2}{4\pi\varepsilon_o mc^2}$

- For a Gaussian beam

$$\boxed{\Delta Q \approx \frac{r_e}{2}\frac{\beta^* N}{\gamma A_{\text{int}}}}$$

# Effect of tune shift on luminosity

- The luminosity is

$$L = \frac{f_{coll} N_1 N_2}{4 A_{\text{int}}}$$

- Write the area in terms of emittance & $\beta$ at the IR

$$A_{\text{int}} = \sigma_x \sigma_y = \sqrt{\beta_x^* \varepsilon_x} \; 0 \sqrt{\beta_y^* \varepsilon_y}$$

- For simplicity assume that

$$\frac{\beta_x^*}{\beta_y^*} = \frac{\varepsilon_x}{\varepsilon_y} \Rightarrow \beta_x^* = \frac{\varepsilon_x}{\varepsilon_y} \beta_y^* \Rightarrow \beta_x^* \varepsilon_x = \frac{\varepsilon_x^2}{\varepsilon_y} \beta_y^*$$

- In that case

$$A_{\text{int}} = \varepsilon_x \beta_y^*$$

- And

$$L = \frac{f_{coll} N_1 N_2}{4 \varepsilon_x \beta_y^*} \sim \frac{I_{beam}^2}{\varepsilon_x \beta_y^*}$$

# Increase N to the tune shift limit

- We saw that

$$\boxed{\Delta Q_y \approx \frac{r_e}{2}\frac{\beta^* N}{\gamma A_{\text{int}}}}$$

or

$$N = \Delta Q_y \frac{2\gamma A_{\text{int}}}{r_e \beta^*} = \Delta Q_y \frac{2\gamma \varepsilon_x \beta^*}{r_e \beta^*} = \frac{2}{r_e}\gamma \varepsilon_x \Delta Q_y$$

Therefore the tune shift limited luminosity is

$$\boxed{L = \frac{2}{r_e}\Delta Q_y \frac{f_{coll} N_1 \gamma \varepsilon_x}{4\varepsilon_x \beta_y^*} \sim \Delta Q_y \left(\frac{IE}{\beta_y^*}\right)}$$

# Incoherent tune shift for in a synchrotron

Assume: 1) an unbunched beam (no acceleration), & 2) uniform density in a circular x-y cross section (not very realistic)

$$x'' + \left(K(s) + \underline{K_{SC}(s)}\right)x = 0 \quad \rightarrow Q_{x0} \ \text{(external)} + \Delta Q_x \ \text{(space charge)}$$

For small "gradient errors" $k_x$

$$\underline{\Delta Q_x} = \frac{1}{4\pi}\int_0^{2R\pi} k_x(s)\beta_x(s)ds = \frac{1}{4\pi}\int_0^{2R\pi} \underline{K_{SC}}(s)\beta_x(s)ds$$

where

$$K_{SC} = -\frac{2r_0 I}{ea^2\beta^3\gamma^3 c}$$

$$\Delta Q_x = -\frac{1}{4\pi}\int_0^{2\pi R}\frac{2r_0 I}{e\beta^3\gamma^3 c}\frac{\beta_x(s)}{a^2}ds = -\frac{r_0 RI}{e\beta^3\gamma^3 c}\left\langle\frac{\beta_x(s)}{a^2(s)}\right\rangle = -\frac{r_0 RI}{e\beta^3\gamma^3 c\varepsilon_x}$$



From: E. Wilson Adams lectures

# Incoherent tune shift limits current at injection

$$\Delta Q_{x,y} = -\frac{r_0}{2\pi\beta^2\gamma^3}\frac{N}{\varepsilon_{x,y}}$$

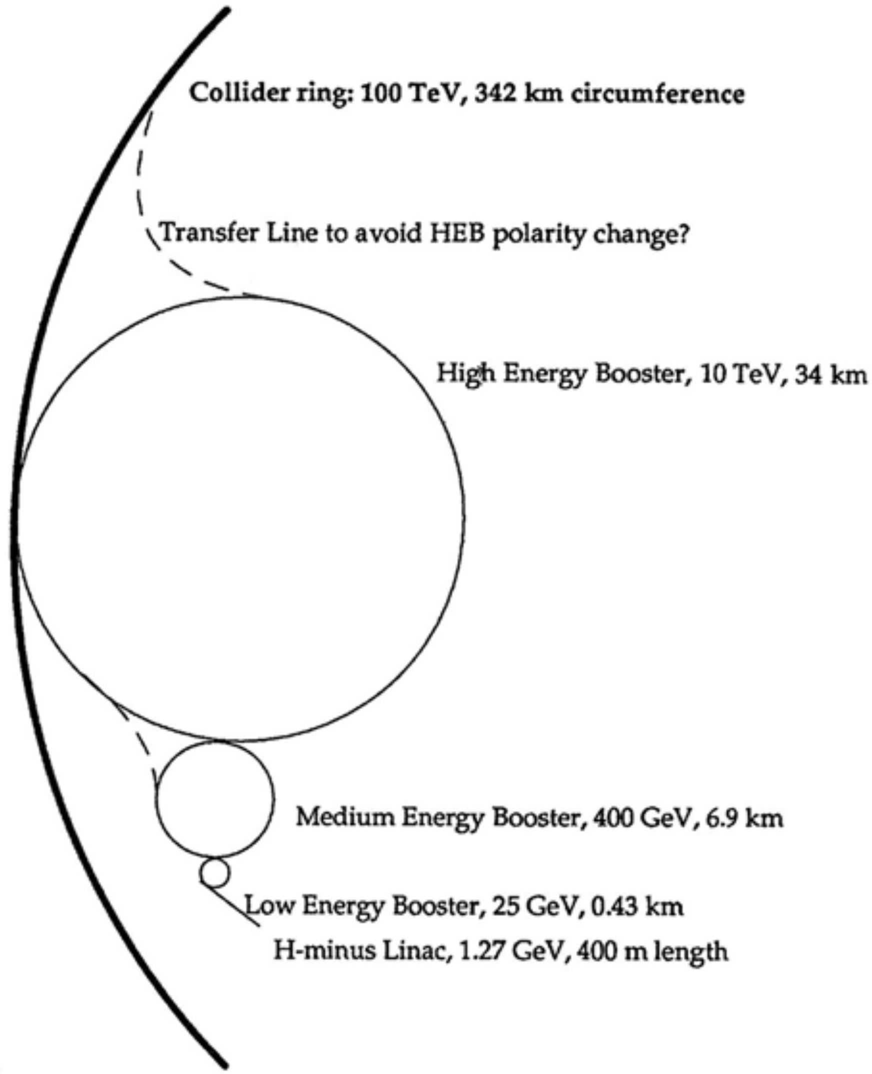
using I = (Neβc)/(2πR) with
N…number of particles in ring
$\varepsilon_{x,y}$….emittance containing 100% of particles

- ❖ "Direct" space charge, unbunched beam in a synchrotron
- ❖ Vanishes for $\gamma \gg 1$
- ❖ Important for low-energy hadron machines
- ❖ *Independent of machine size* 2πR for a *given N*
- ❖ *Overcome by higher energy injection ==> cost*



From: E. Wilson Adams lectures

# Injection chain for a 200 TeV Collider



# Beam lifetime

Based on F. Sannibale USPAS Lecture

# Finite aperture of accelerator ==> loss of beam particles

- Many processes can excite particles on orbits larger than the nominal.
  - ➙If new orbit displacement exceed the aperture, the particle is lost

- The limiting aperture in accelerators can be either *physical* or *dynamic*.
  - ➙Vacuum chamber defines the physical aperture
  - ➙Momentum acceptance defines the dynamical aperture



From: Sannibale USPAS Lecture

# Important processes in particle loss

- Gas scattering, scattering with the other particles in the beam, quantum lifetime, tune resonances, &collisions

- Radiation damping plays a major role for electron/positron rings
  - ➞For ions, lifetime is usually much longer
    - Perturbations progressively build-up & generate losses

- Most applications require storing the beam as long as possible

*==> limiting the effects of the residual gas scattering*

*==> ultra high vacuum technology*

From: Sannibale USPAS Lecture

# What do we mean by lifetime?

- Number of particles lost at time $t$ is proportional to the number of particles present in the beam at time $t$

$$dN = -\alpha N(t)\,dt \qquad with \quad \alpha \equiv constant$$

- Define the lifetime $\tau = 1/\alpha$; then $\boxed{N = N_0 e^{-t/\tau}}$

- Lifetime is the time to reduce the number of beam particles to $1/e$ of the initial value

- Calculate the lifetime due to the individual effects (gas, Touschek, …)

$$\frac{1}{\tau_{total}} = \frac{1}{\tau_1} + \frac{1}{\tau_2} + \frac{1}{\tau_3} + ....$$



From: Sannibale USPAS Lecture

# Is the lifetime really constant?

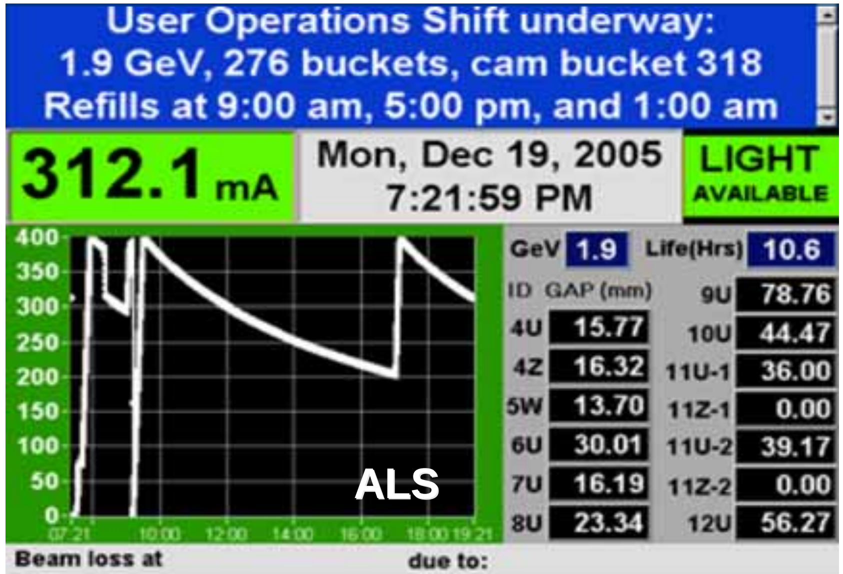
- In typical electron storage rings, lifetime depends on beam current
- Example: the *Touschek effect* losses depend on current.
  - When the stored current decreases, the losses due to Touschek decrease ==> lifetime increases
- Example: Synchrotron radiation radiated by the beam desorbs gas molecules trapped in the vacuum chamber
  - The higher the stored current, the higher the synchrotron radiation intensity and the higher the desorption from the wall.
  - Pressure in the vacuum chamber increases with current

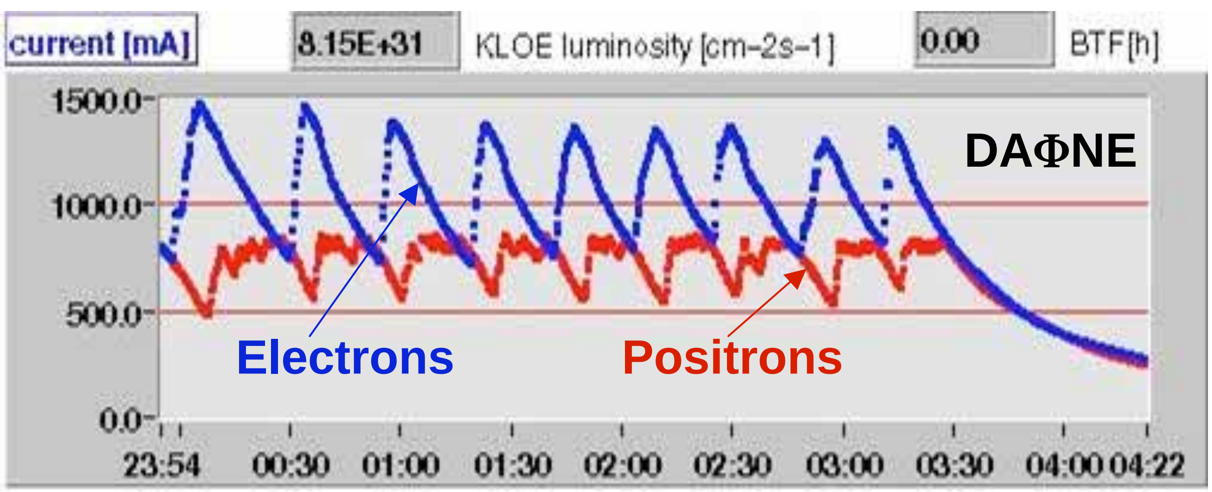
  ==> increased scattering between the beam and the residual gas

  ==> reduction of the beam lifetime

From: Sannibale USPAS Lecture

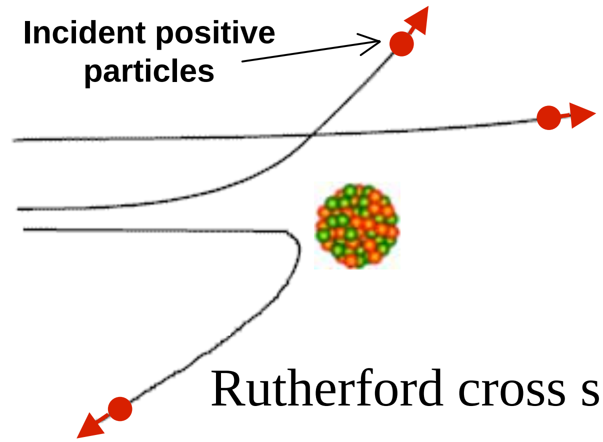
# Examples of beam lifetime measurements



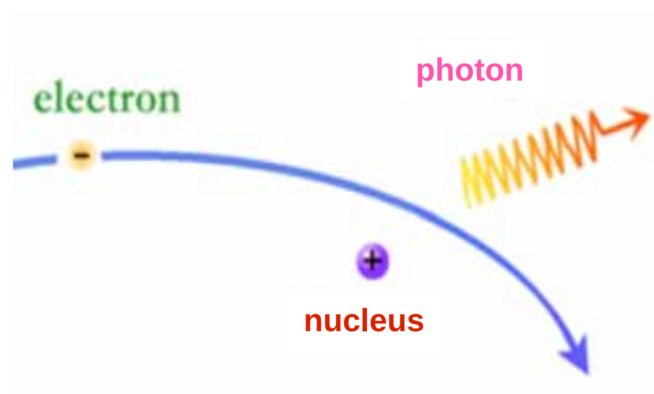
# Beam loss by scattering

- Elastic (Coulomb scattering) from residual background gas
  - Scattered beam particle undergoes transverse (betatron) oscillations.
  - If the oscillation amplitude exceeds ring acceptance the particle is lost




Rutherford cross section

- Inelastic scattering causes particles to *lose energy*
  - Bremsstrahlung or atomic excitation
  - If energy loss exceeds the momentum acceptance the particle is lost




Bremsstrahlung cross section

# Elastic scattering loss process

- Loss rate is $\left.\dfrac{dN}{dt}\right|_{Gas} = -\phi_{beam\ particles} N_{molecules} \sigma^*_R$

$$\phi_{beam\ particles} = \frac{N}{A_{beam} T_{rev}} = \frac{N}{A_{beam}} \frac{\beta c}{L_{ring}}$$

$$N_{molecules} = n A_{beam} L_{ring}$$

$$\sigma^*_R = \int_{Lost} \frac{d\sigma_{Rutherford}}{d\Omega} d\Omega = \int_0^{2\pi} d\varphi \int_{\theta_{MAX}}^{\pi} \frac{d\sigma_{Rutherford}}{d\Omega} \sin\theta \, d\theta$$

$$\frac{d\sigma_R}{d\Omega} = \frac{1}{(4\pi\varepsilon_0)^2} \left( \frac{Z_{beam} Z_{gas} e^2}{2\beta c\, p} \right)^2 \frac{1}{\sin^4(\theta/2)} \quad [MKS]$$

# Gas scattering lifetime

- Integrating yields

$$\left.\frac{dN}{dt}\right|_{Gas} = -\frac{\pi\, n\, N \beta\, c}{(4\pi\,\varepsilon_0)^2}\left(\frac{Z_{Inc} Z e^2}{\beta\, c\, p}\right)^2 \frac{1}{\tan^2(\theta_{MAX}/2)}$$

*Loss rate for gas elastic scattering* [*MKS*]

- For M-atomic molecules of gas $n = M\, n_0 \dfrac{P_{[Torr]}}{760}$

- For a ring with acceptance $\varepsilon_A$ & for small $\theta$ $\langle \theta_{MAX} \rangle = \sqrt{\dfrac{\varepsilon_A}{\langle \beta_n \rangle}}$

==>

$$\tau_{Gas} \cong \frac{760}{P_{[Torr]}} \frac{4\pi\varepsilon_0^2}{\beta\, c\, M\, n_0}\left(\frac{\beta\, c\, p}{Z_{Inc} Z e^2}\right)^2 \frac{\varepsilon_A}{\langle \beta_T \rangle} \quad [MKS]$$

# Inelastic scattering lifetimes

- Beam-gas bremsstrahlung: if $E_A$ is the energy acceptance

$$\boxed{\tau_{Brem[hours]} \cong -\frac{153.14}{\ln\left(\Delta E_A / E_0\right)} \frac{1}{P_{[nTorr]}}}$$

- Inelastic excitation: For an average $ß_n$

$$\boxed{\tau_{Gas[hours]} \cong 10.25 \frac{E^2_{0[GeV]}}{P_{[nTorr]}} \frac{\varepsilon_{A[\mu m]}}{\langle \beta_n \rangle_{[m]}}}$$

# Touschek effect: Intra-beam Coulomb scattering

- Coulomb scattering between beam particles can transfer transverse momentum to the longitudinal plane
  - If the $p_{||}+\Delta p_{||}$ of the scattered particles is outside the momentum acceptance, the particles are lost
  - First observation by Touschek at ADA $e^+e^-$ ring
- Computation is best done in the beam frame where the relative motion of the particles is non-relativistic
  - Then boost the result to the lab frame

$$\boxed{\frac{1}{\tau_{Tousch.}} \propto \frac{1}{\gamma^3}\frac{N_{beam}}{\sigma_x\sigma_y\sigma_S}\frac{1}{\left(\Delta p_A/p_0\right)^2} \propto \frac{1}{\gamma^3}\frac{N_{beam}}{A_{beam}\sigma_S}\frac{1}{\hat{V}_{RF}}}$$

# Transverse quantum lifetime

- At a fixed s, transverse particle motion is purely sinusoidal

$$x_T = a\sqrt{\beta_n}\,\sin\left(\omega_{\beta_n} t + \varphi\right) \qquad T = x, y$$

- Tunes are chosen in order to avoid resonances.
  - ➙ At a fixed azimuthal position, a particle turn after turn sweeps all possible positions between the envelope
- Photon emission randomly changes the “invariant” *a* & consequently changes the trajectory envelope as well.
- Cumulative photon emission can bring the particle envelope beyond acceptance in some azimuthal point
  - ➙ The particle is lost

# Quantum lifetime was first estimated by Bruck and Sands

$$\tau_{Q_T} \cong \tau_{D_T} \frac{\sigma_T^2}{A_T^2} \exp\left(A_T^2 / 2\sigma_T^2\right) \quad T = x, y$$

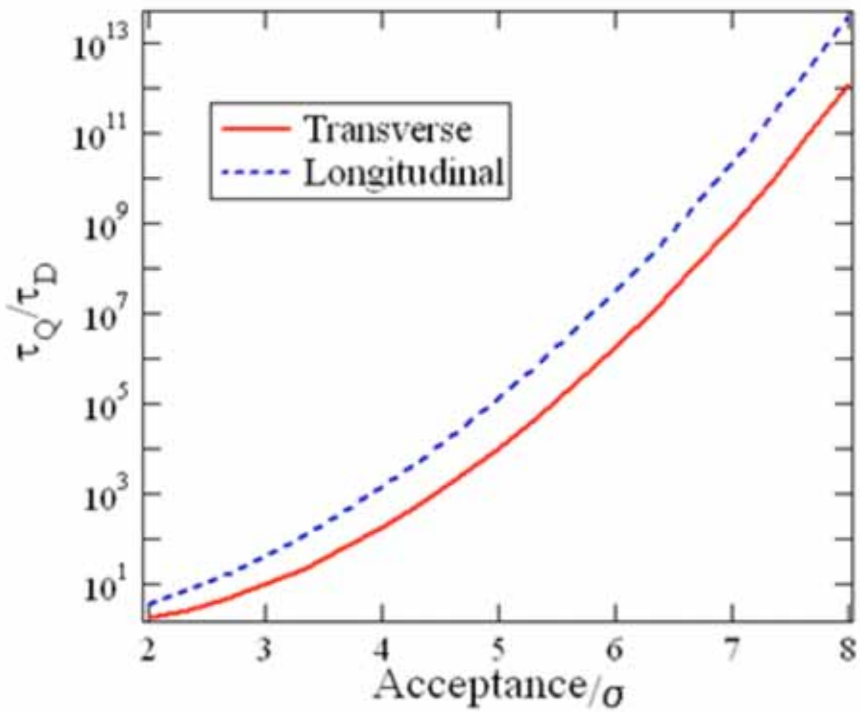
*Transverse quantum lifetime*

$$where \quad \sigma_T^2 = \beta_T \varepsilon_T + \left(D_T \frac{\sigma_E}{E_0}\right)^2 \quad T = x, y$$

$\tau_{D_T} \equiv$ *transverse damping time*

$$\tau_{Q_L} \cong \tau_{D_L} \exp\left(\Delta E_A^2 / 2\sigma_E^2\right)$$

*Longitudinal quantum lifetime*




- Quantum lifetime varies very strongly with the ratio between acceptance & rms size.

*Values for this ratio ≥6 are usually required*

# Lifetime summary

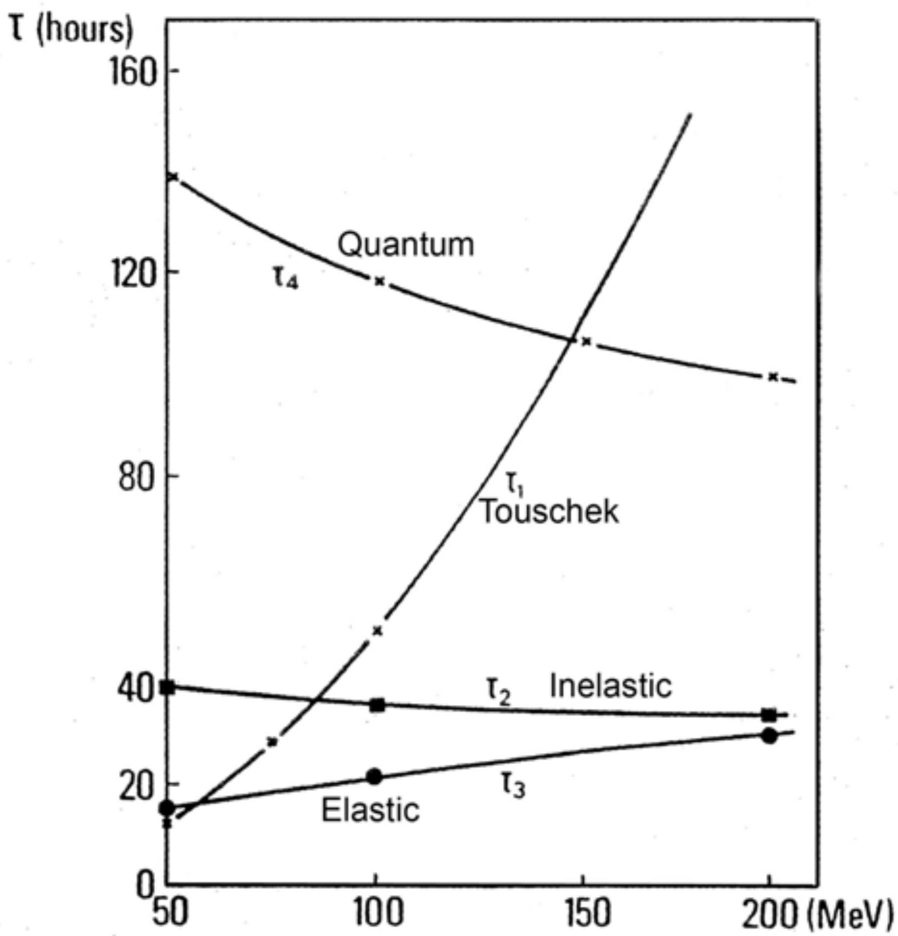


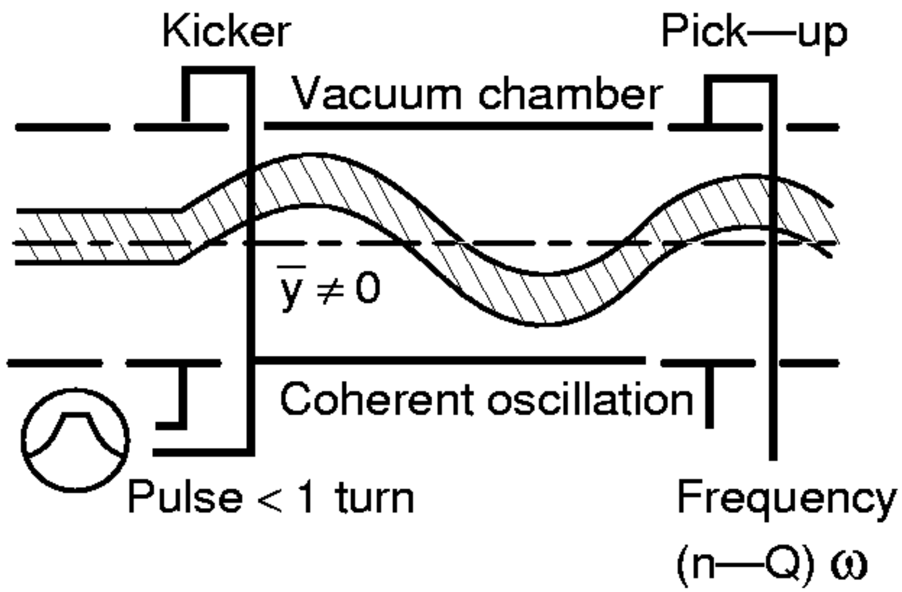

Fig. 1. Lifetime resulting from the different types of beam–gas interaction

# Coherent limitations on beams in storage rings

# Coherent effects are characterized by impedances

- Collective effects on the bunch as a whole driven by the collective forces generated by the beam
- Low Q diseases limit impedance for the ring
  - Transverse - "fast" head-tail effect
  - Longitudinal - Bunch lengthening "microwave" effect
- High Q diseases:
  - Multi-bunch instabilities
- Cures:
  - Vacuum chamber design
  - Lower current if possible
  - Landau damping

# Types of tune shifts: Coherent motion




Center of mass moves with a betatron oscillation

Beam environment (e.g. a position monitor) “sees” a “collective motion”

Within the coherent motion, each particles has its individual motion



From: E. Wilson Adams lectures

# Reminder about impedances

- Describe interaction of beam with the vacuum chamber by impedances

- Example: Longitudinal motion is described by a current I.
  - Modification of the motion is described by $\Delta I$

- Radiated field, $\Delta E \propto$ displacement => driving voltage

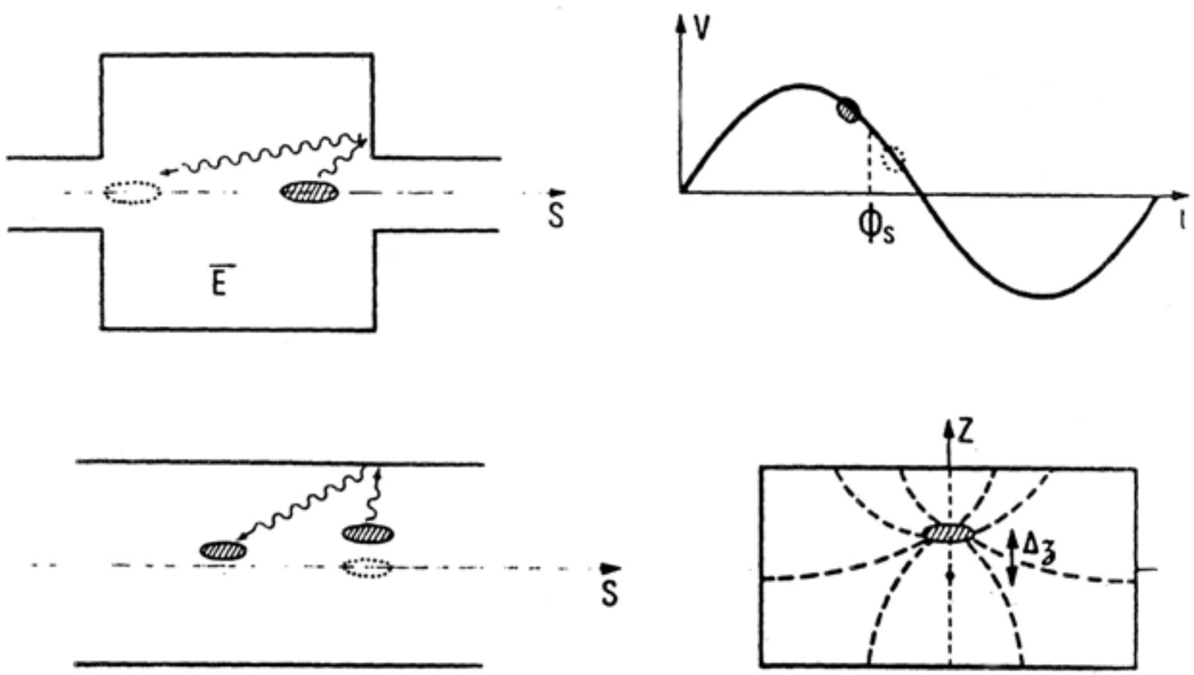
$$\Delta V = Z \, \Delta I$$

- Z is the Fourier transform of the wake-field
  - Longitudinal impedances are given in Ohms
  - Transverse impedances are given in Ohms / meter

# Impedance relations

- For a cylindrical vacuum chamber of radius b

$$Z_{\perp} = \frac{2c}{b^2}\frac{Z_{ll}}{\omega}$$



# Example of coherent bunch lengthening

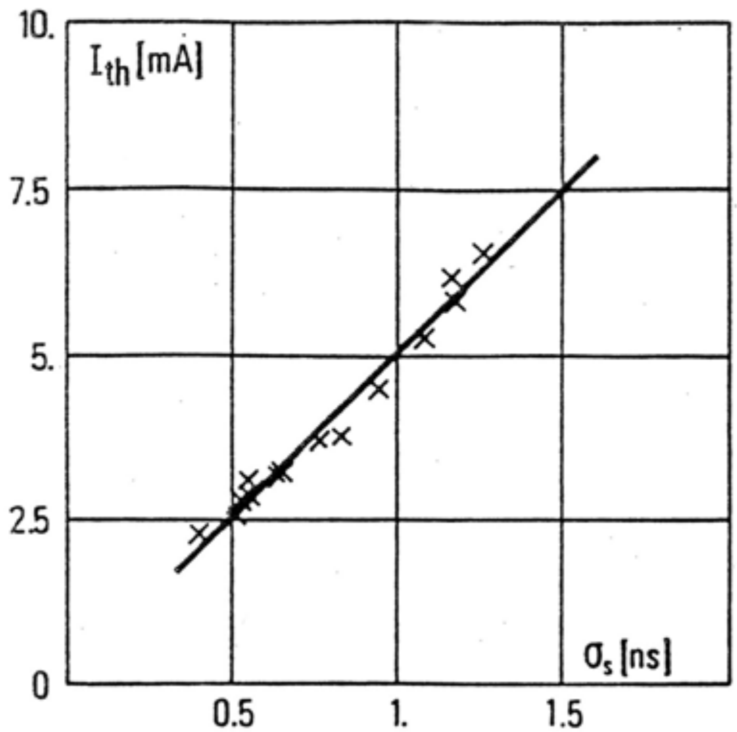



Fig. 9. Transverse instability threshold vs bunch length measured on ACO.

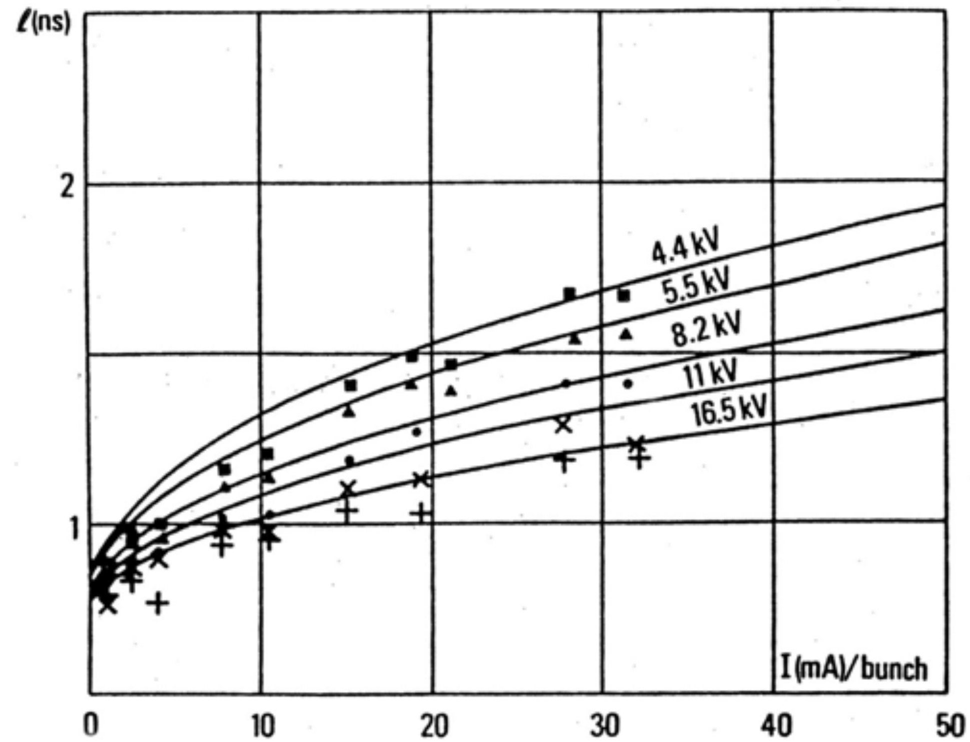



Fig. 13. Bunch length measurements on ACO.

# Phase mix or Landau damping

(Model from Barletta & Briggs)

# Very high order multipole: 1-D model

Motion in coordinate space

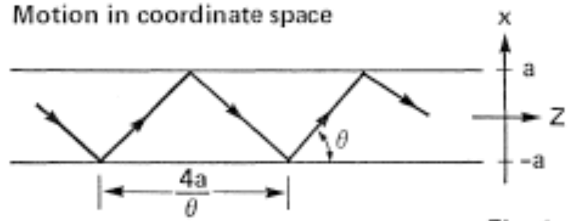



Fig. 1a

Motion in phase space:

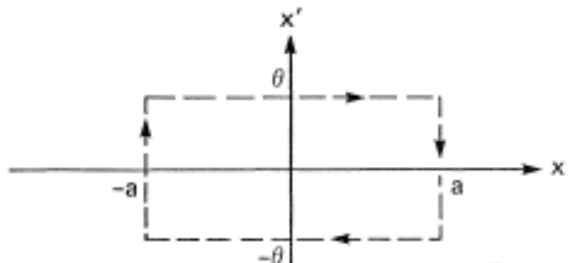



Fig. 1b

Motion in phase space for "real" multipole channel

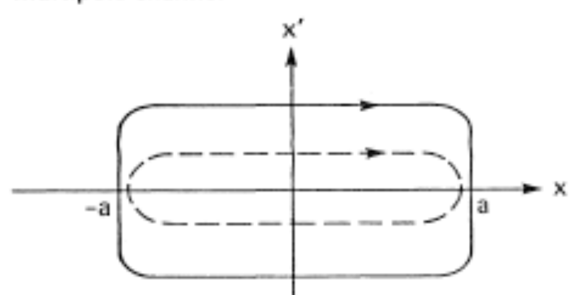
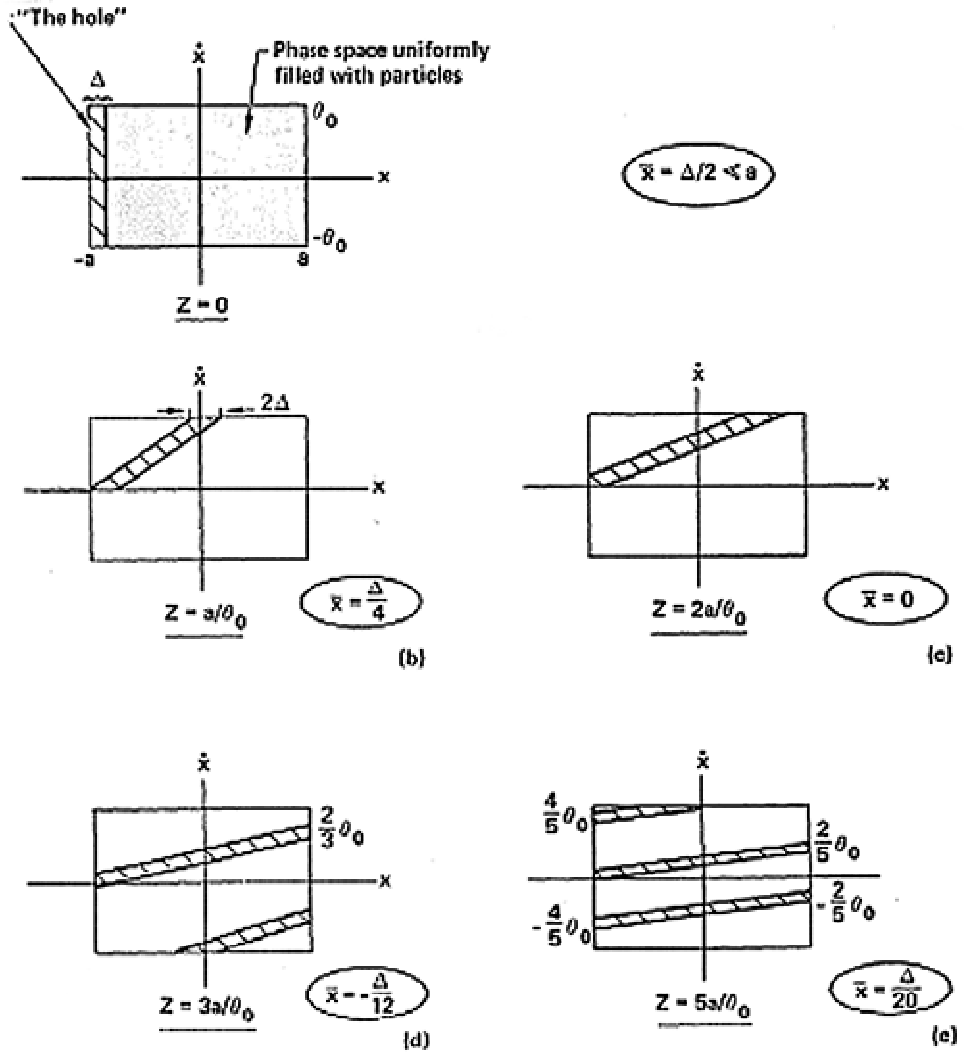
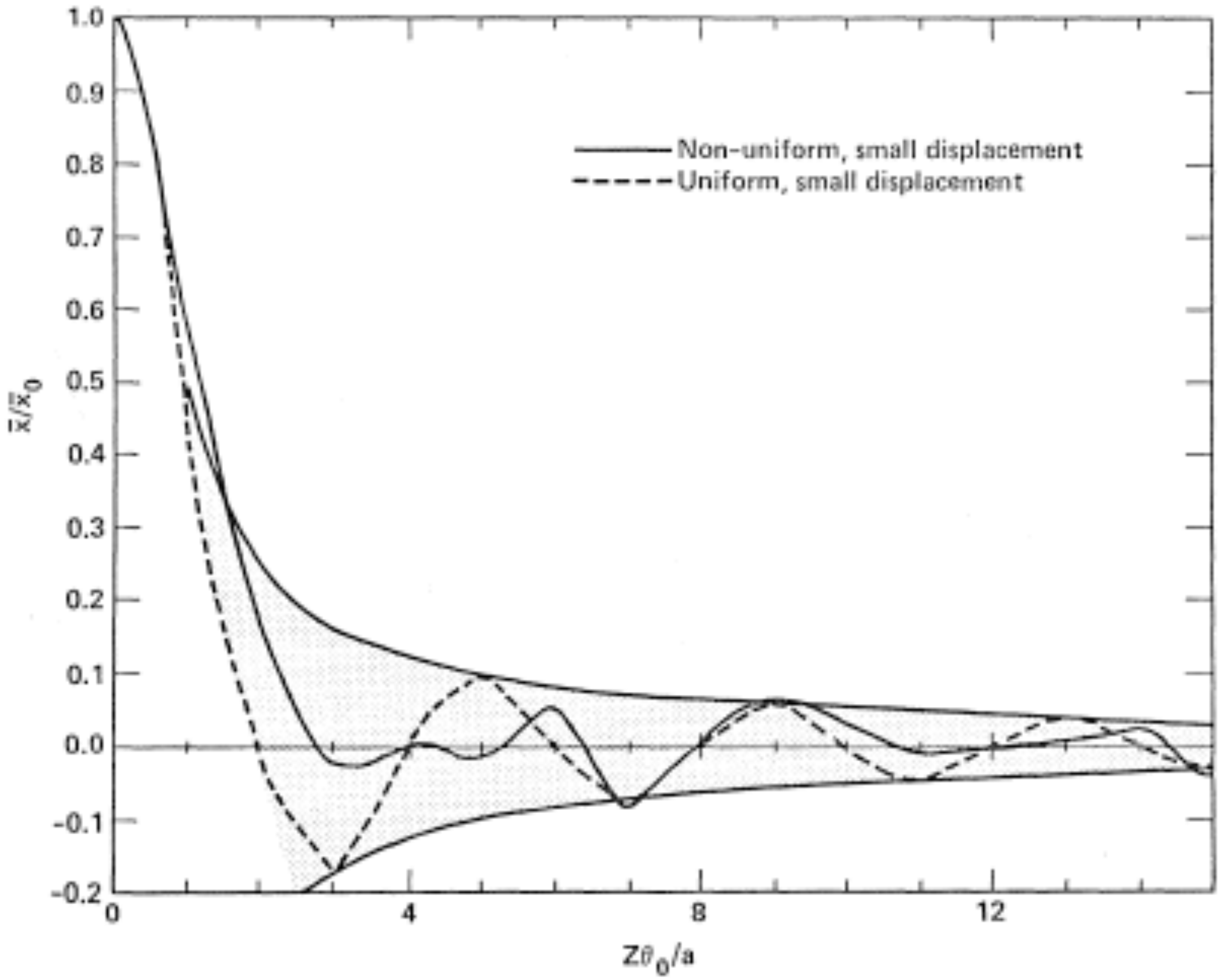



Fig. 1c

# Phase mix damping of small uniform displacement

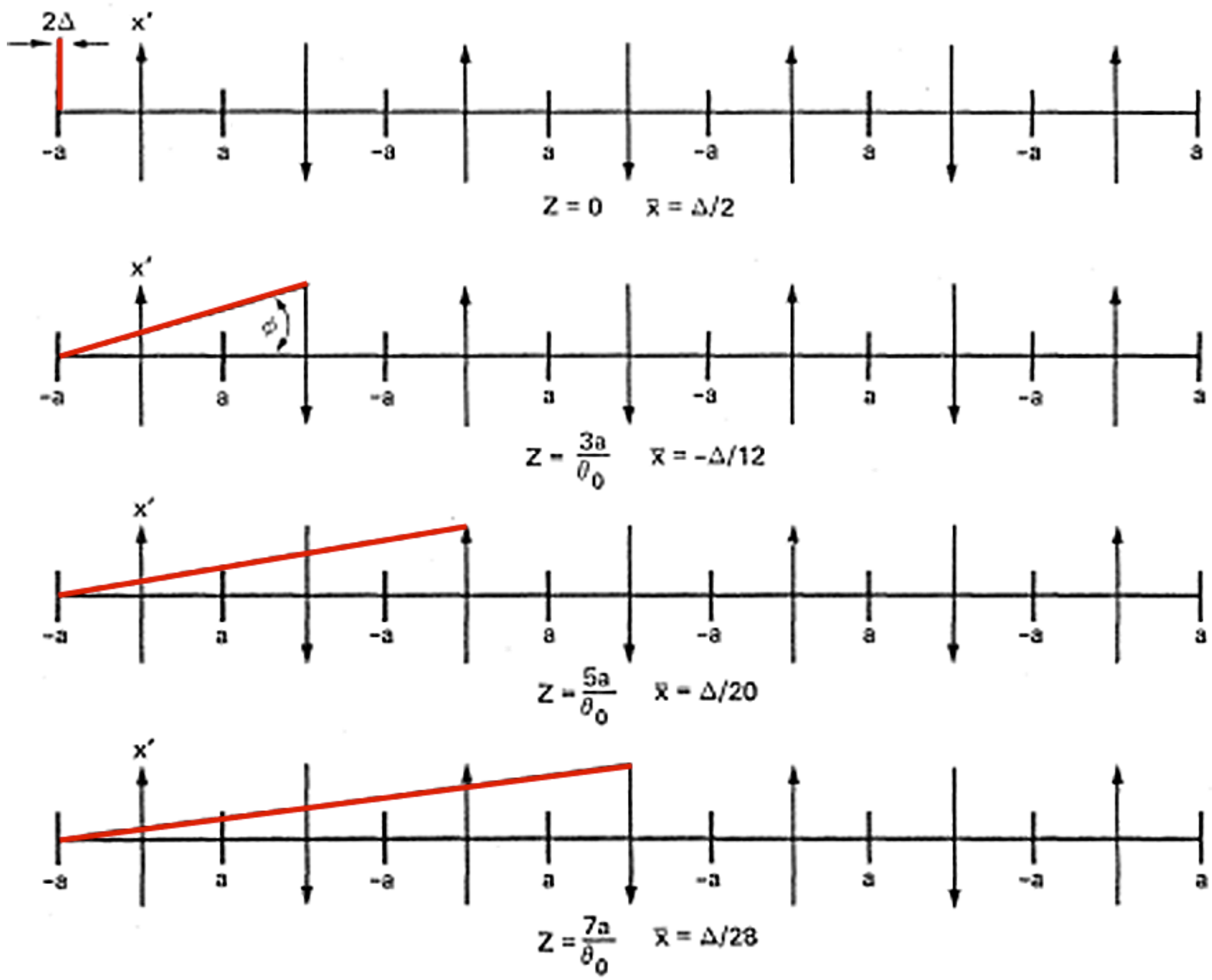


# Small uniform displacements damp as 1/z

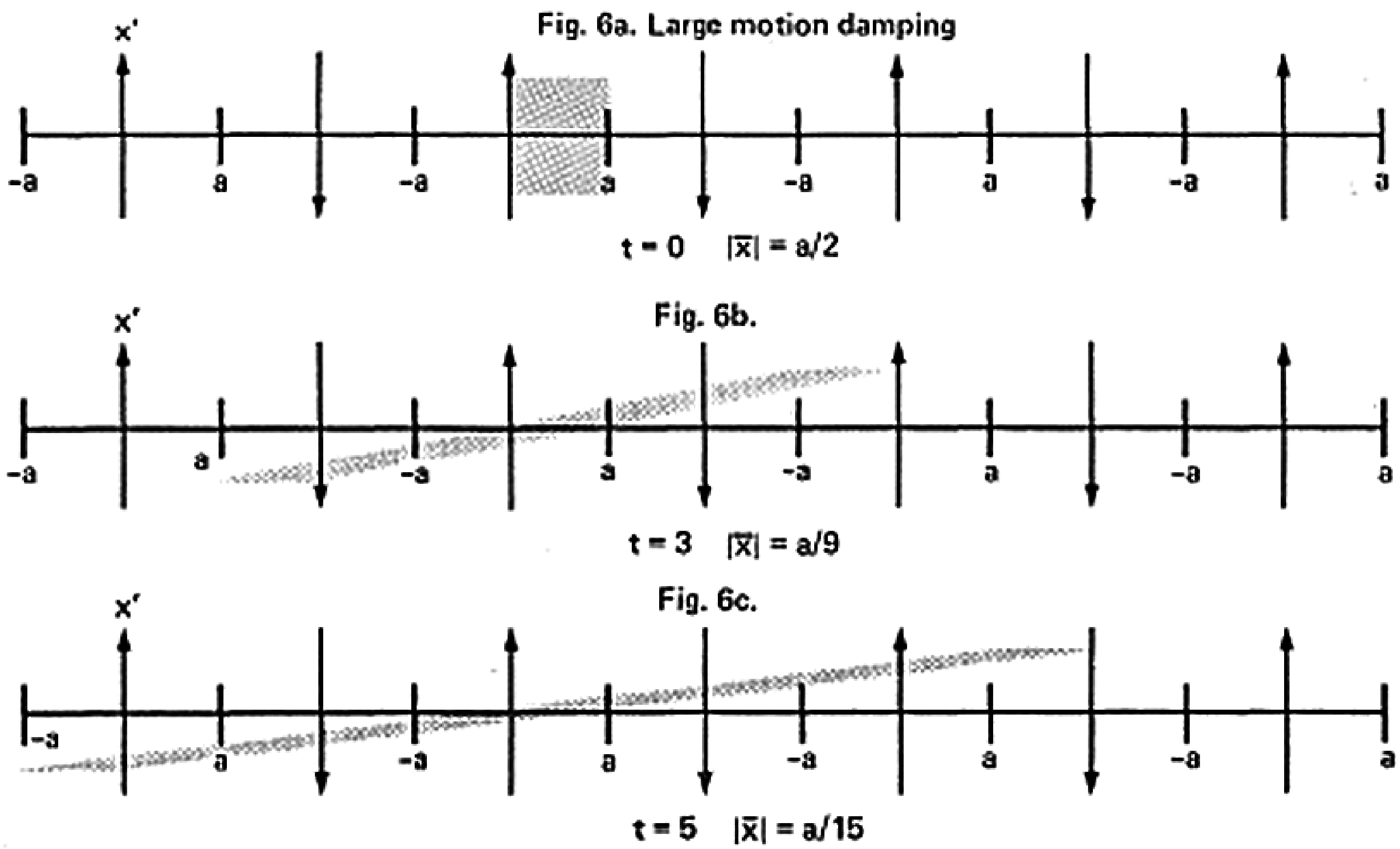


# Phase mix damping of small uniform displacement

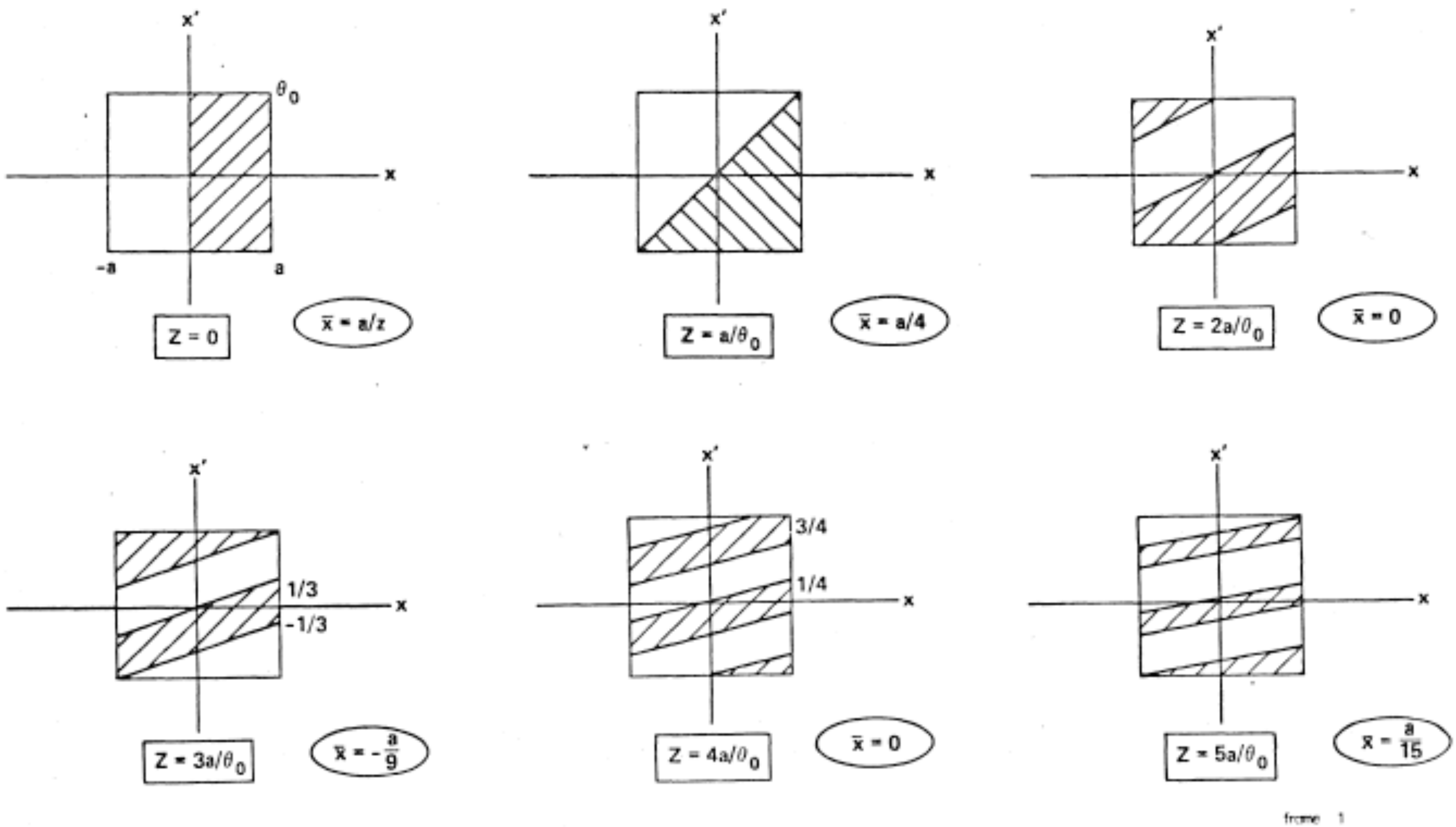
Fig. 5 Motion of phase space sliver



# Large displacements damp slower than *1/z*



# Phase space rapidly assumes asymptotic form



# Scattering reduces the damping rate

Fig. 10a

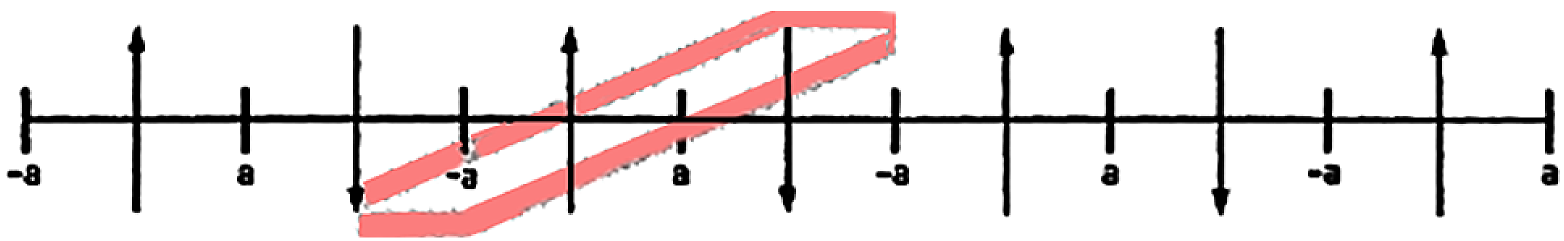



Fig. 10b

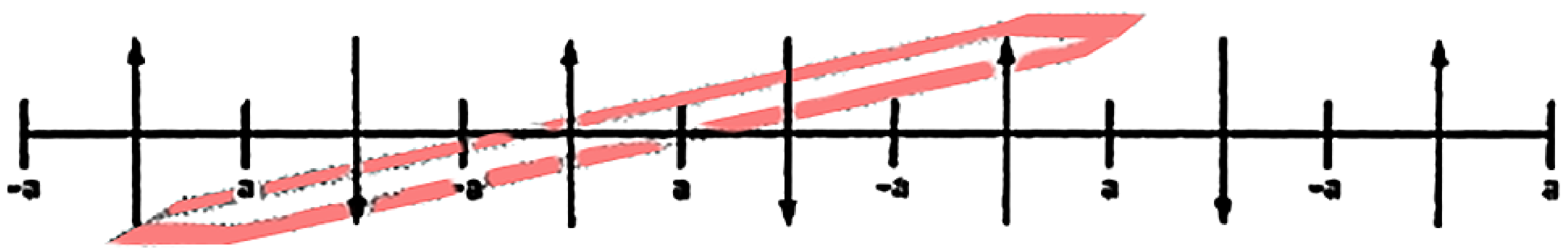
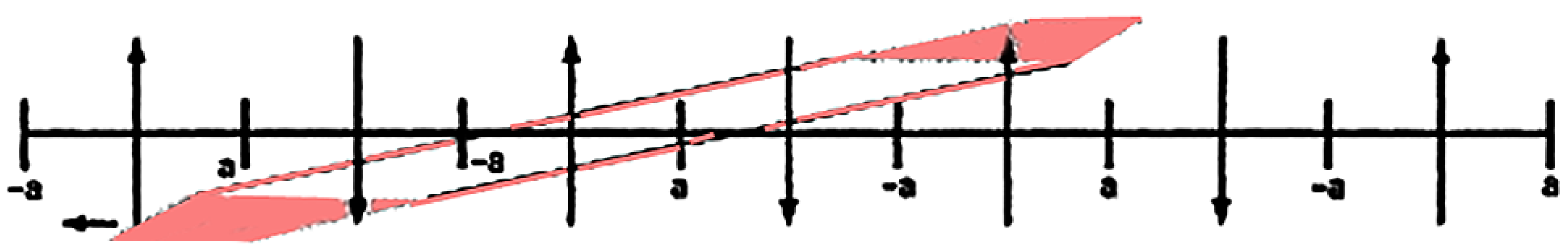
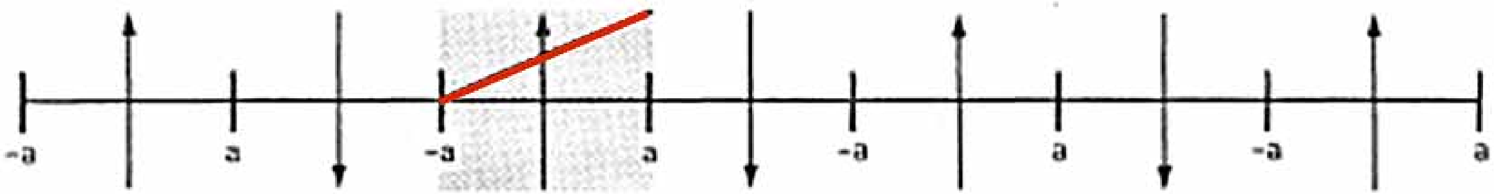


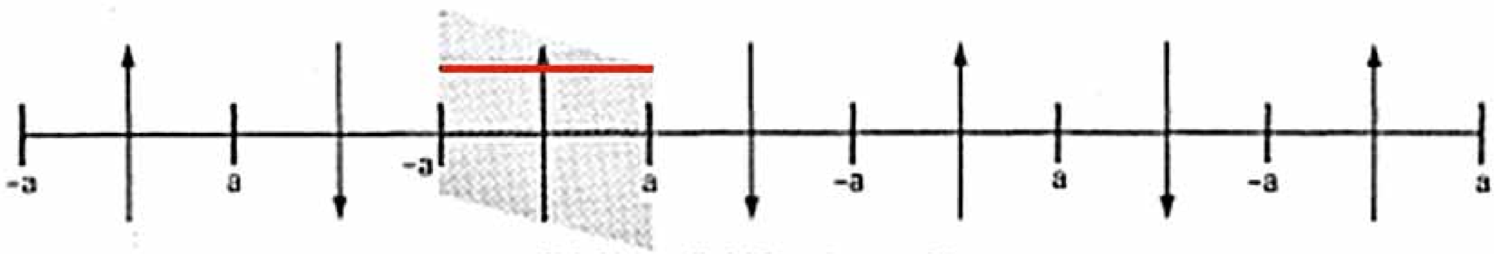

Fig. 10c



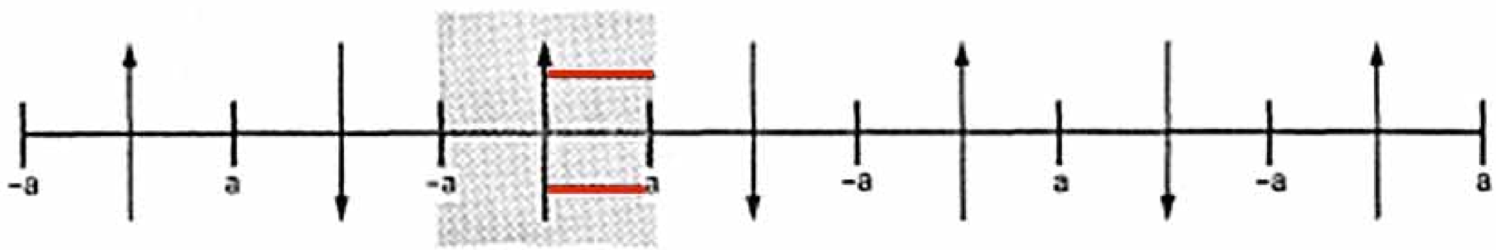
# Passage of beam through a convergent lens slows damping




(a) Phase fluid just before lens

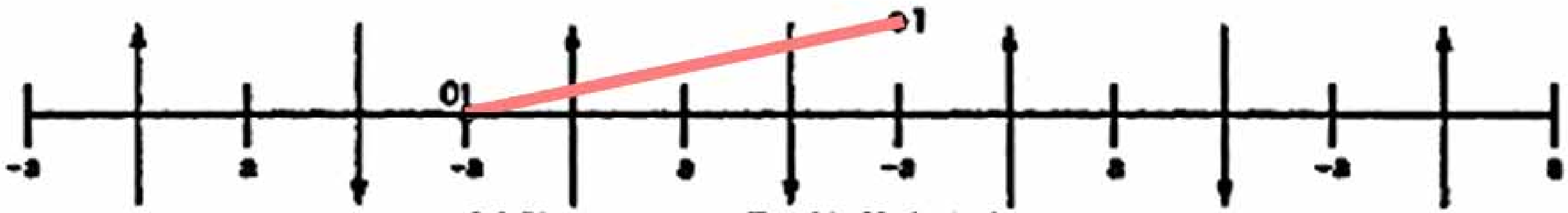



(b) Phase fluid just beyond lens

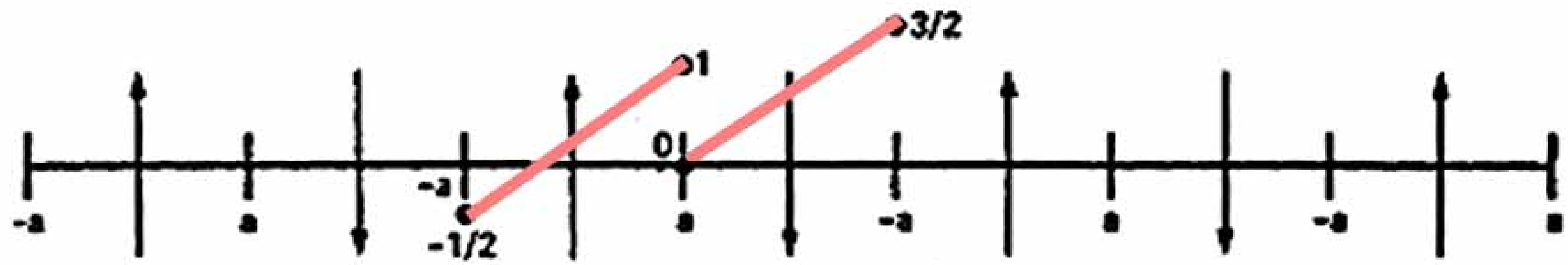



(c) Phase fluid at late time

# Evaluation of effect of divergent lens



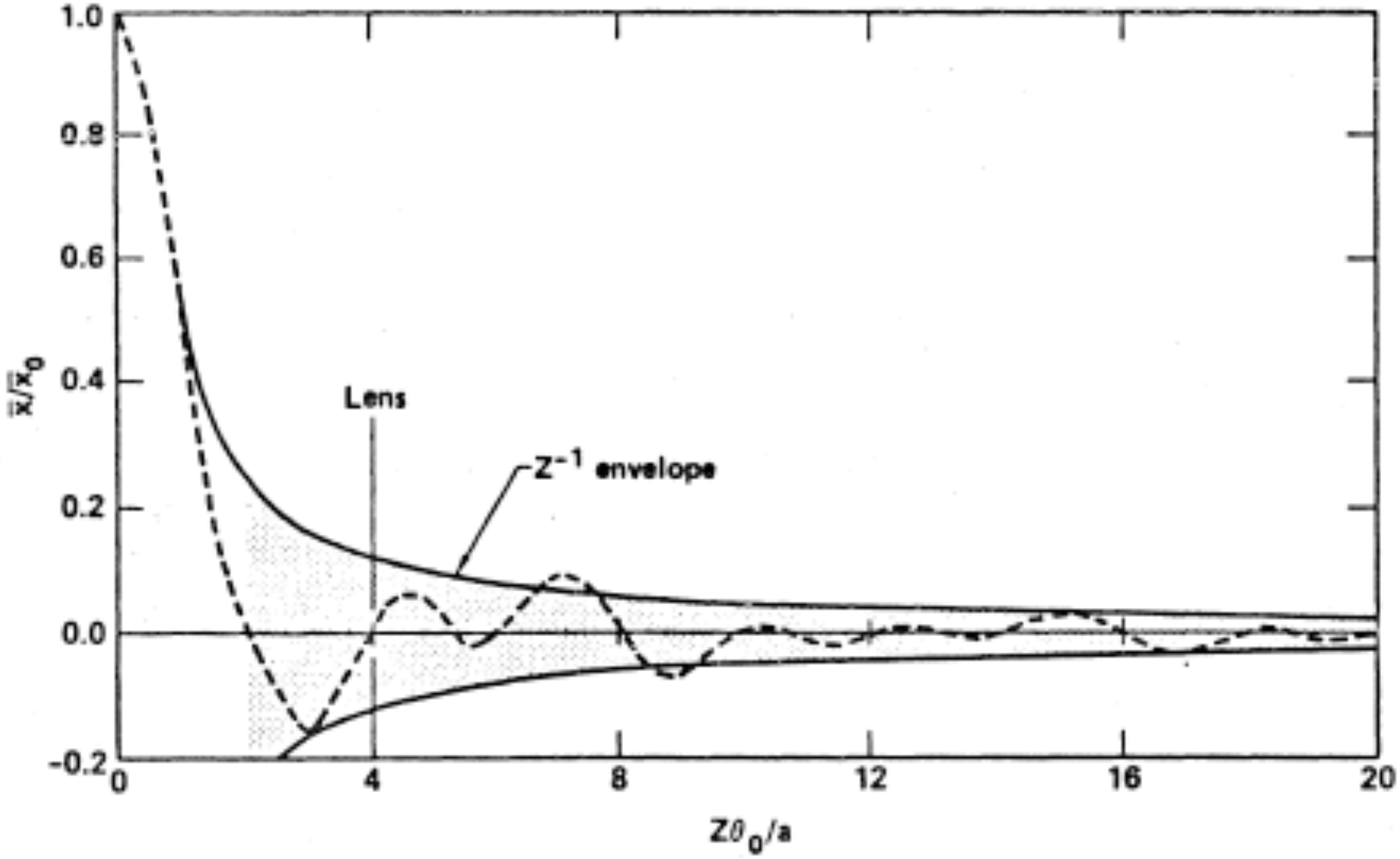

(a) Phase space at $Z = (4a/\theta_0)^-$ before lens




(b) Phase space at $Z = (4a/\theta_0)^+$ after lens

$$\frac{\bar{x}}{x_0} = -\left\{ -(-1)^{I(t/4)}\left(2-\frac{t_3}{2}\right)\left(\frac{t_3}{2} \bmod 2\right) + (-1)^{I(t/2)}(2-t_2)(t_2 \bmod 2) - (-1)^{I(3t/4)}\left(2-\frac{3t_1}{2}\right)\left(\frac{3t_1}{2} \bmod 2\right)\right\} \Big/ (8+6t) \qquad (4)$$

# Divergent lens speeds damping slightly



# Simple damping model - conclusions

- Mean displacement can damp rapidly upon entering an anharmonic transport channel
- Once phase fluid assumes asymptotic form damping proceeds very slowly
- If initial state matches asymptotic form, damping is slow from the start
- Damping rate depends on microscopic form of phase fluid
  - Beams with the same mean displacement can damp at quite different rates
  - Scattering is likely to slow damping
- The price of changing the damping rate is increasing the emittance of the beam