Unit 3 - Lecture 6
Non-resonant Accelerators

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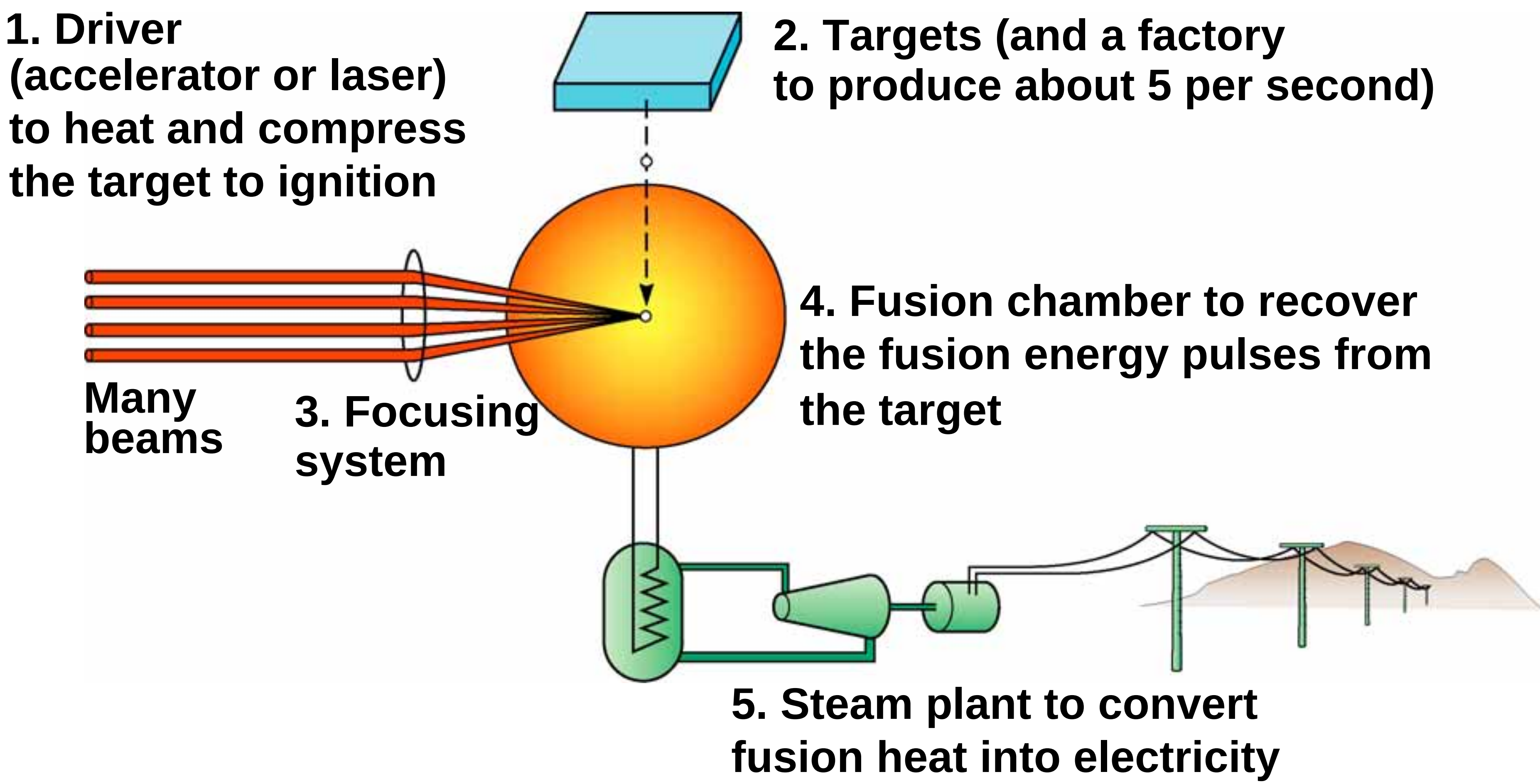
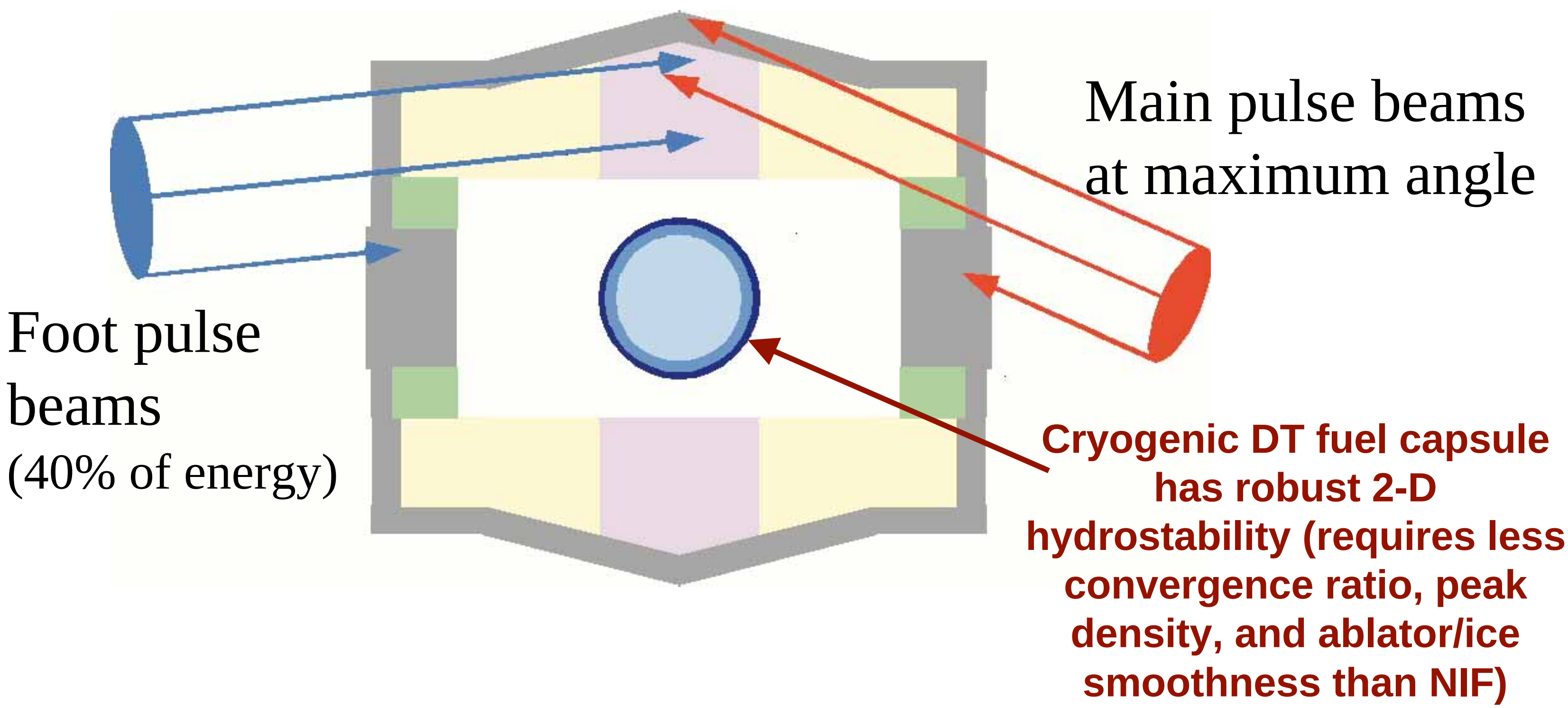
Present motivations

Components of an inertial fusion power plant



Target design is a variation of the distributed radiator target (DRT)

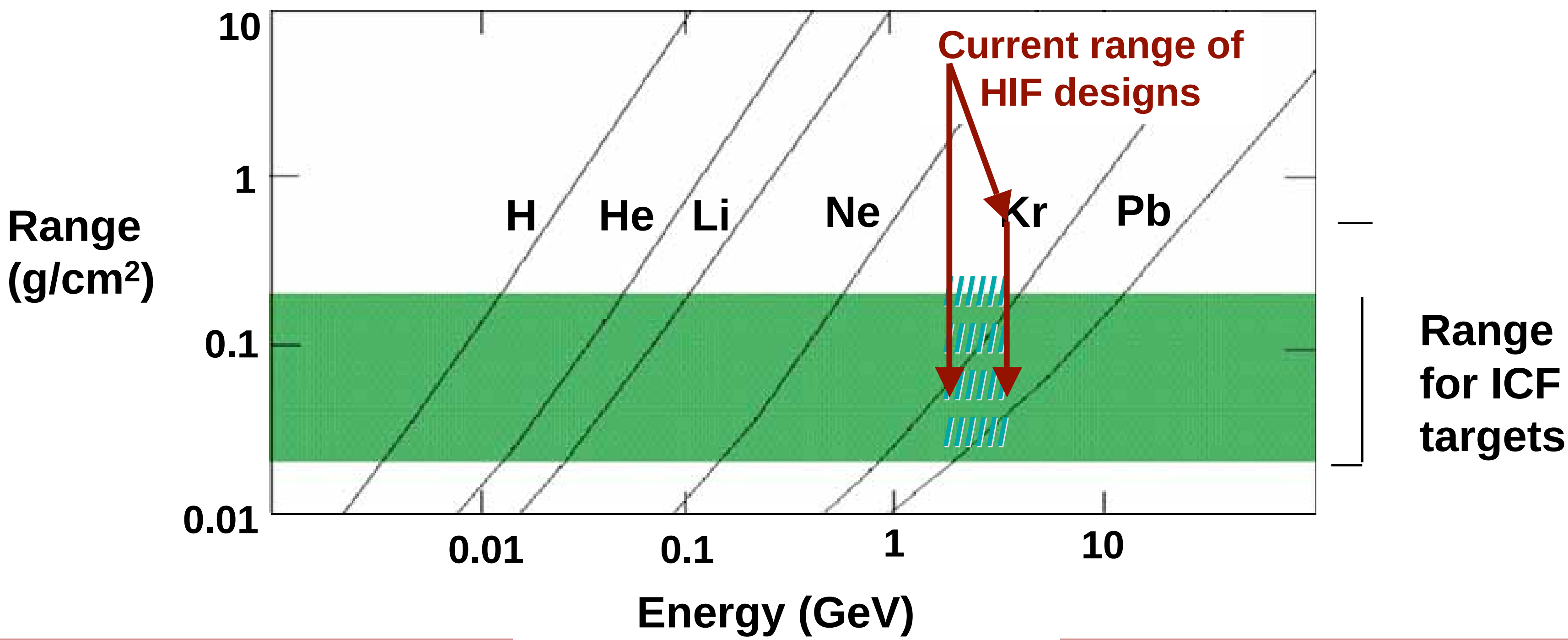


New design allows beams to come in from larger angle, ~ 24° off axis.

Yield = 400 MJ, Gain = 57 at E_{driver} = 7 MJ

Heavier ions ==> higher voltage ==> lower current beams

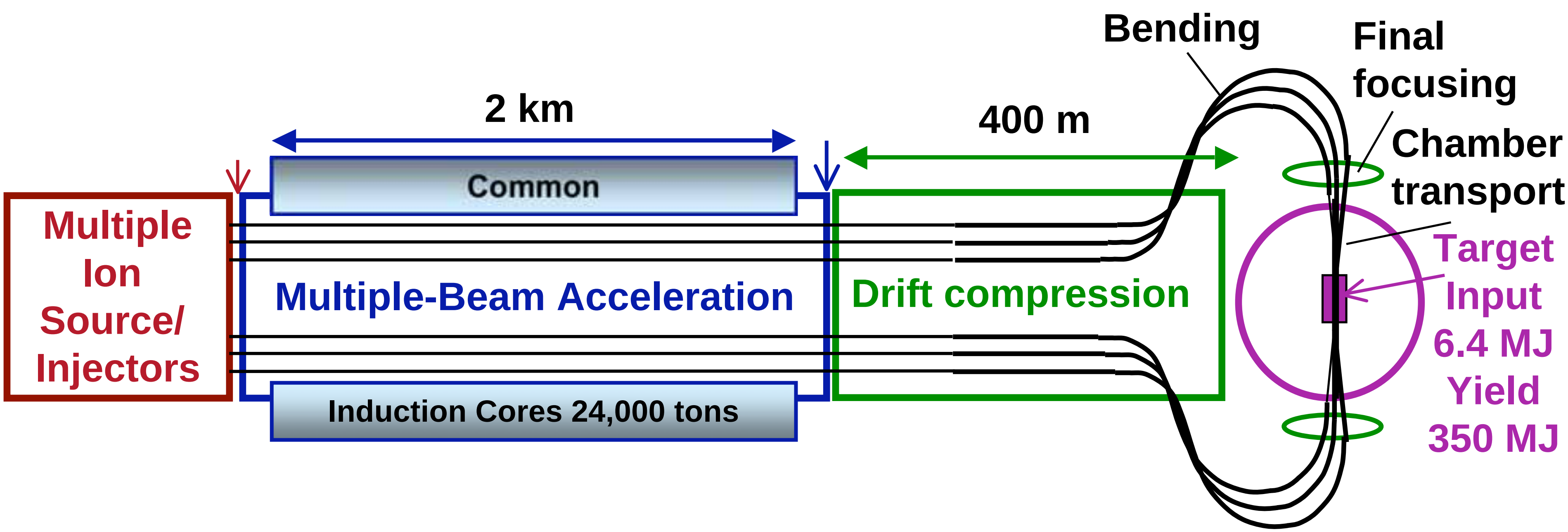
- Collective effects are reduced with heaviest ions
  - More energy/particle ==> fewer particles (~10^{15} total ions).
- Cost tradeoff: lower mass ions ==> lower voltage ==> lower cost
  - Compromise with 2.5 GeV Xenon.
- SC magnets can confine beam against its space charge during acceleration.



Beam requirements for HIF

- Representative set of parameters for indirect-drive targets
  - 500 Terawatts of beam power
  - beam pulse length ~ 10 ns
  - range 0.02 - 0.2 g/cm^2
  - focus such a large beam to a spot of ~1-5 mm radius
  - desired focal length ~6 m (maximum chamber size
- Basic requirements ==> certain design choices
  - parallel acceleration of multiple beams
  - acceleration of needed charge in a single beam is uneconomical
  - emittance required to focus single large beam extremely difficult

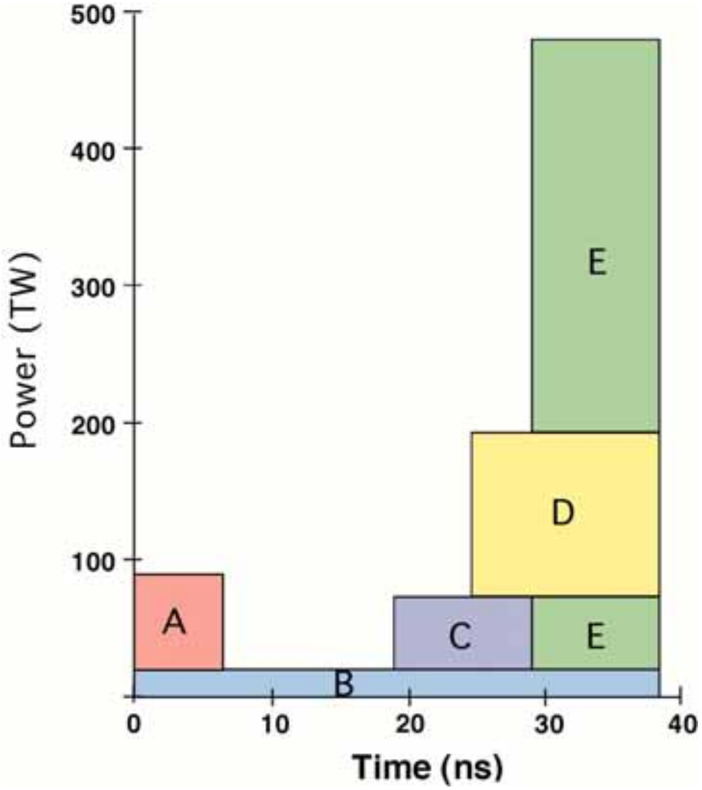
2.5 GeV 112-beam fusion driver: 6.4 MJ of Xe^{+1}



24,000 tonne induction cores

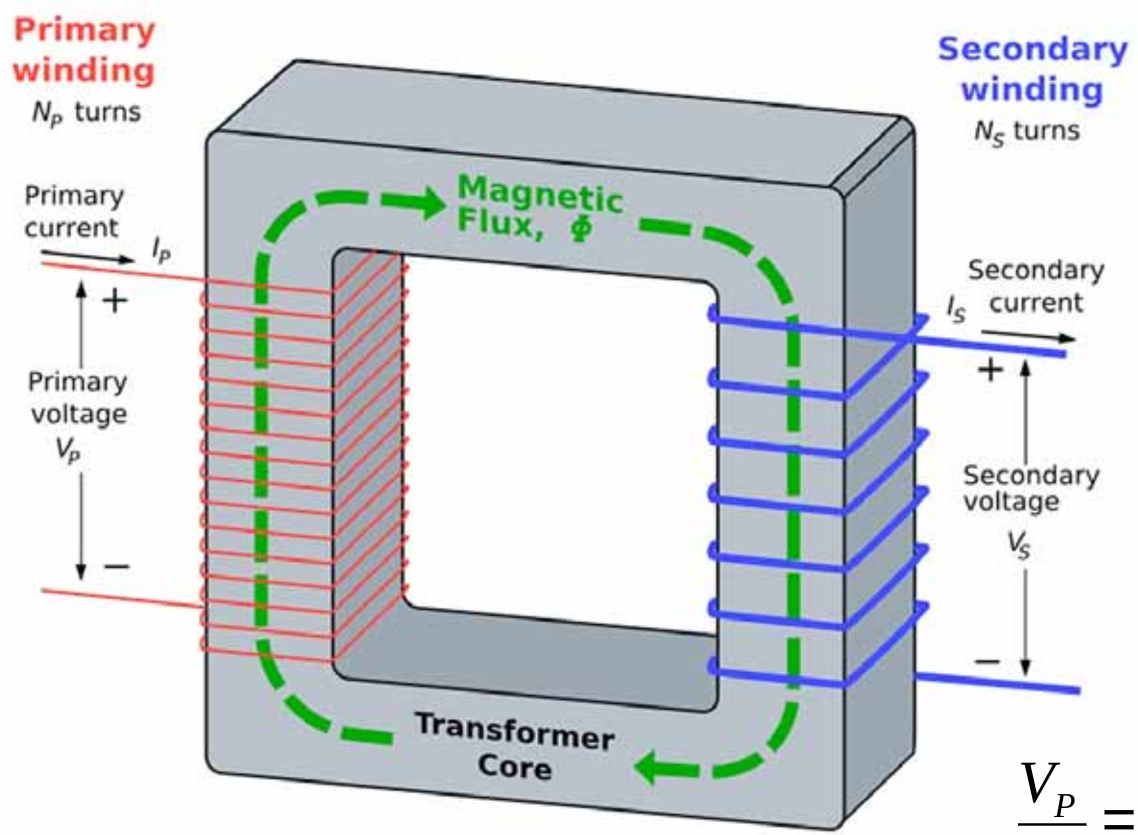
$720M hardware, $1 B direct, $2.1 B total capital cost

How could one produce such energetic monolithic pulses?



Transformers are highly efficient and can drive large currents

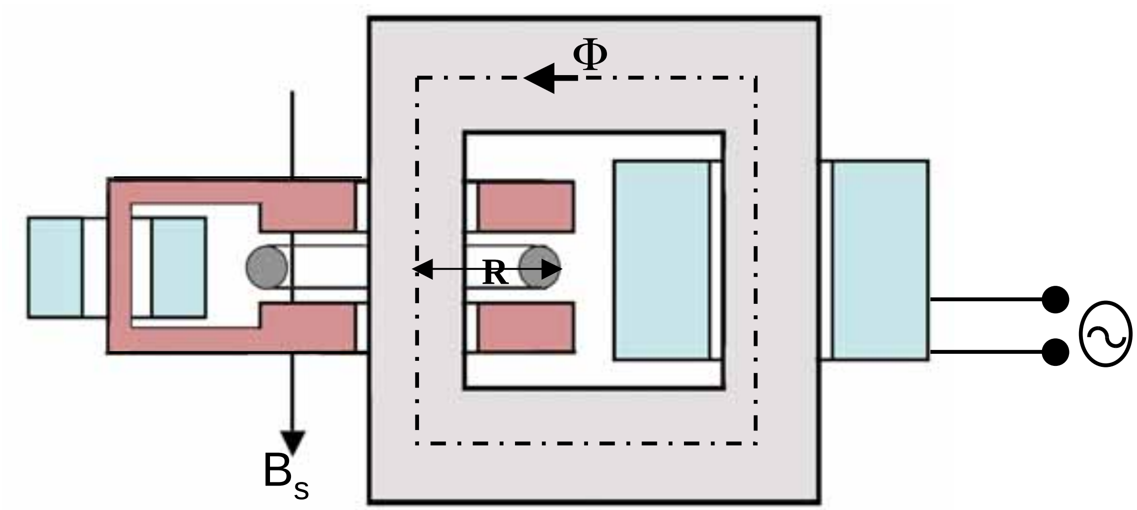
Large units can transfer > 99% of input power to the output



$$\frac{V_P}{V_S} = \frac{N_P}{N_S} = \frac{I_S}{I_P}$$

Image of step-down transformer from Wikipedia

Recall the ray transformer realized as the Betatron (D. Kerst, 1940)



The beam acts as a 1-turn secondary winding of the transformer

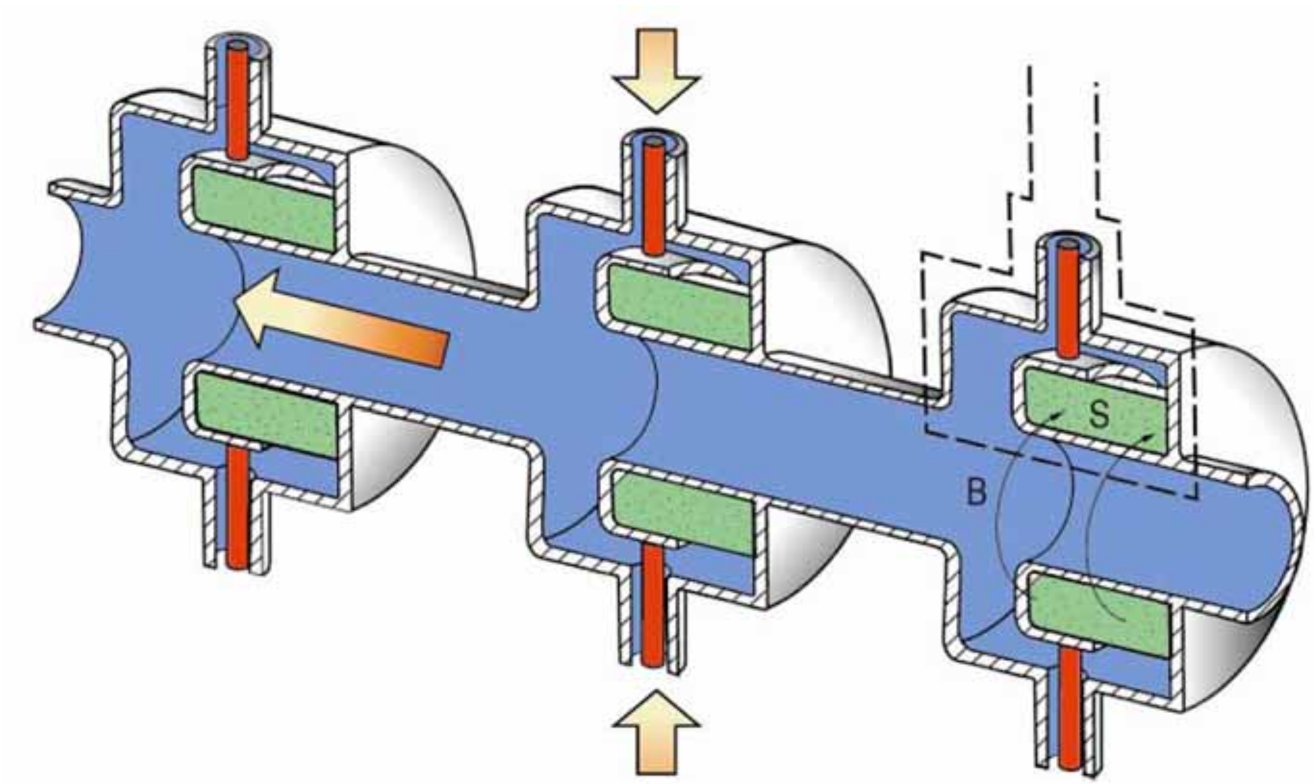
Magnetic field energy is transferred directly to the electrons

For the orbit size to remain invariant

$$\dot{\Phi} = 2\pi R^2 \dot{B}_s$$

This was good for up to 300 MeV electrons. What about electrons or ions?

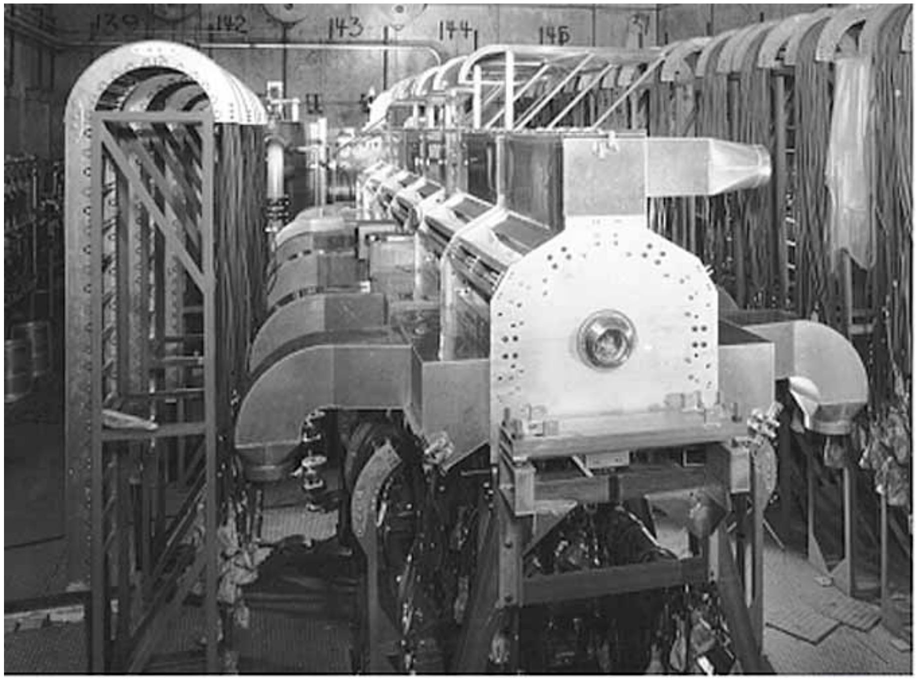
Linear Betatron: Linear Induction Accelerator could accelerate ions



N. Christofilos

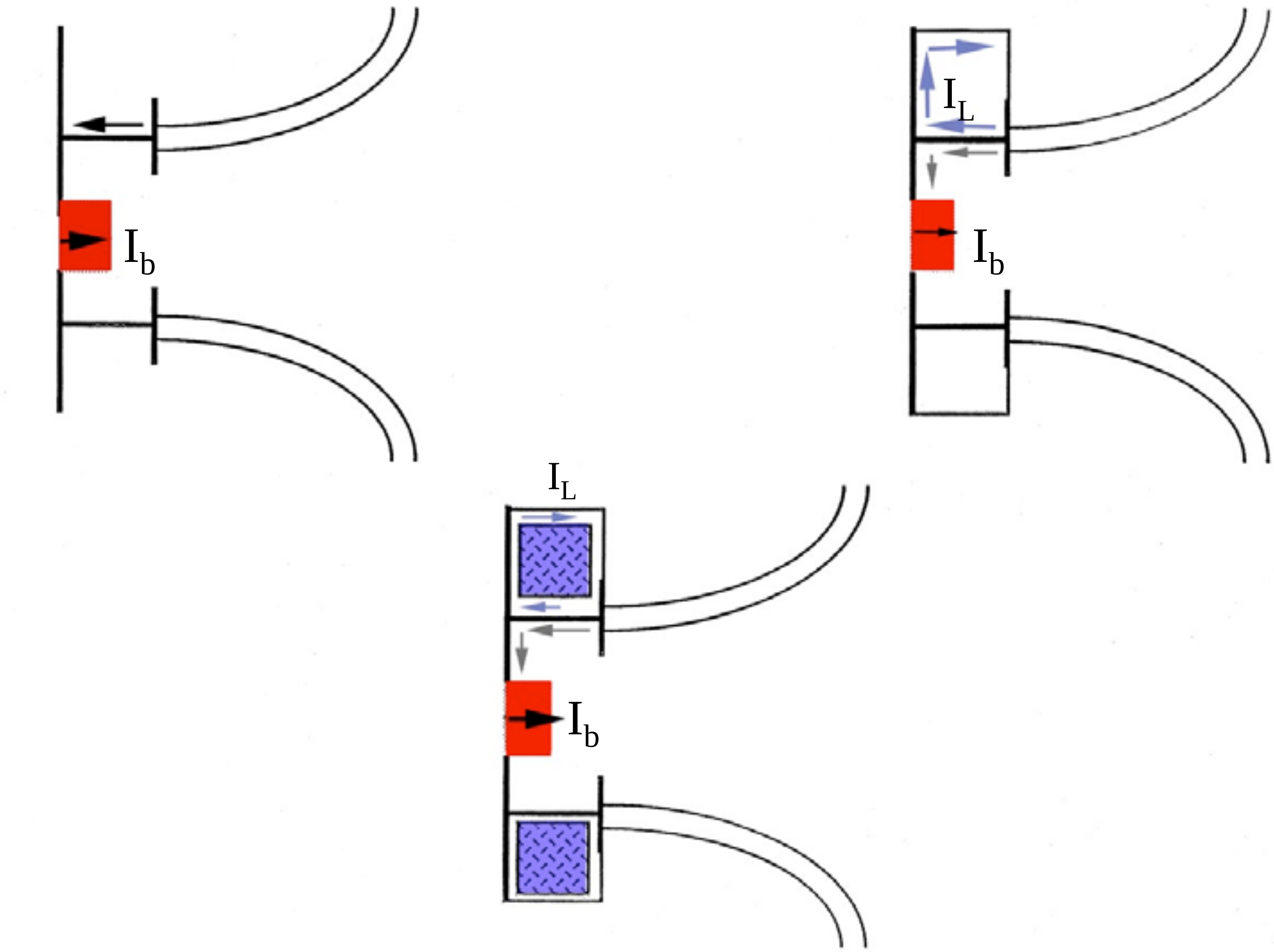
$$\oint_C \mathbf{E} \cdot d\mathbf{l} = -\frac{\partial}{\partial t} \int_S \mathbf{B} \cdot d\mathbf{s}$$

Linear induction accelerators & fusion



Astron-I Induction linac (1963)
& the Astron CTR experiment

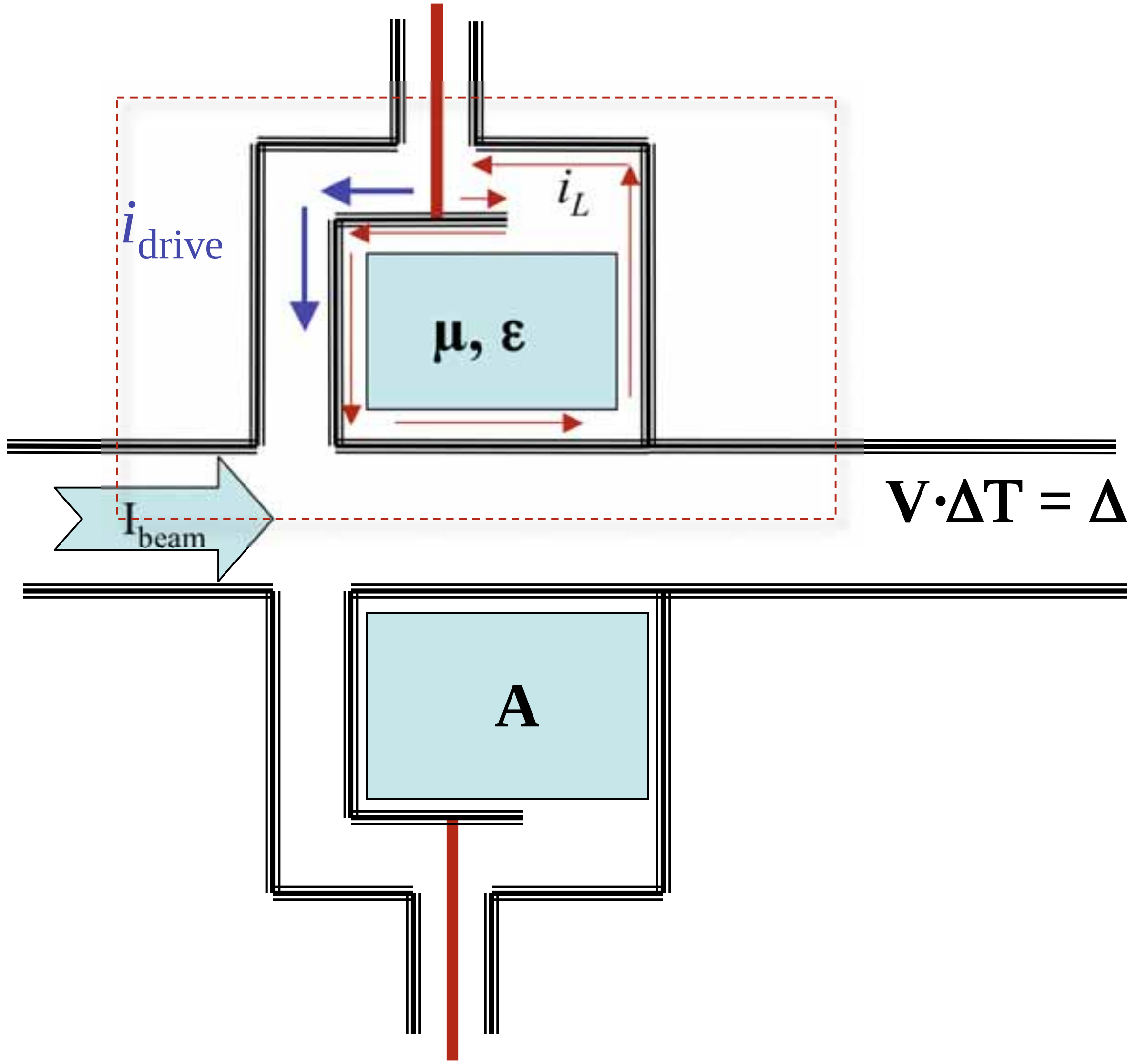
Principle of inductive isolation



Properties of inductive geometry

1. Leakage inductance: $L = (\mu/2\pi) \ln(R_o/R_i)$
   a) $i_L = (V_o/L)t$
2. Ferromagnetic core reduces the leakage current and slows the speed of the shorting wave until the core saturates
3. Load current does not encircle the core
   a) Pulse drive properties not core properties limit I_b
4. Field across the gap is quasi-electrostatic
5. Within the core electrostatic & inductive voltages cancel
   a) The structure is at ground potential

Current flow in the induction core



$$V \cdot \Delta T = \Delta B \cdot A$$

Realistic cross-section of a small induction cell

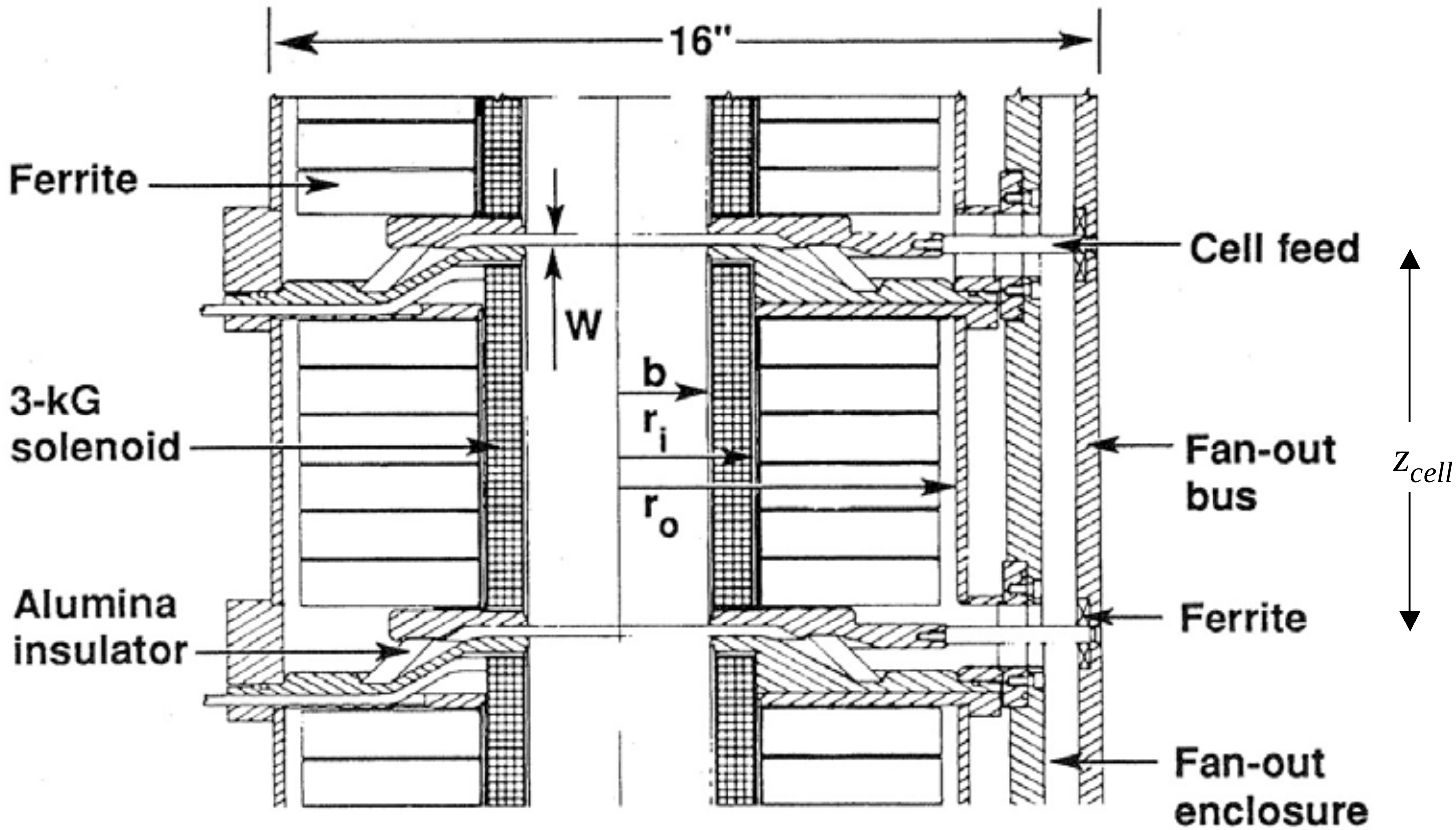
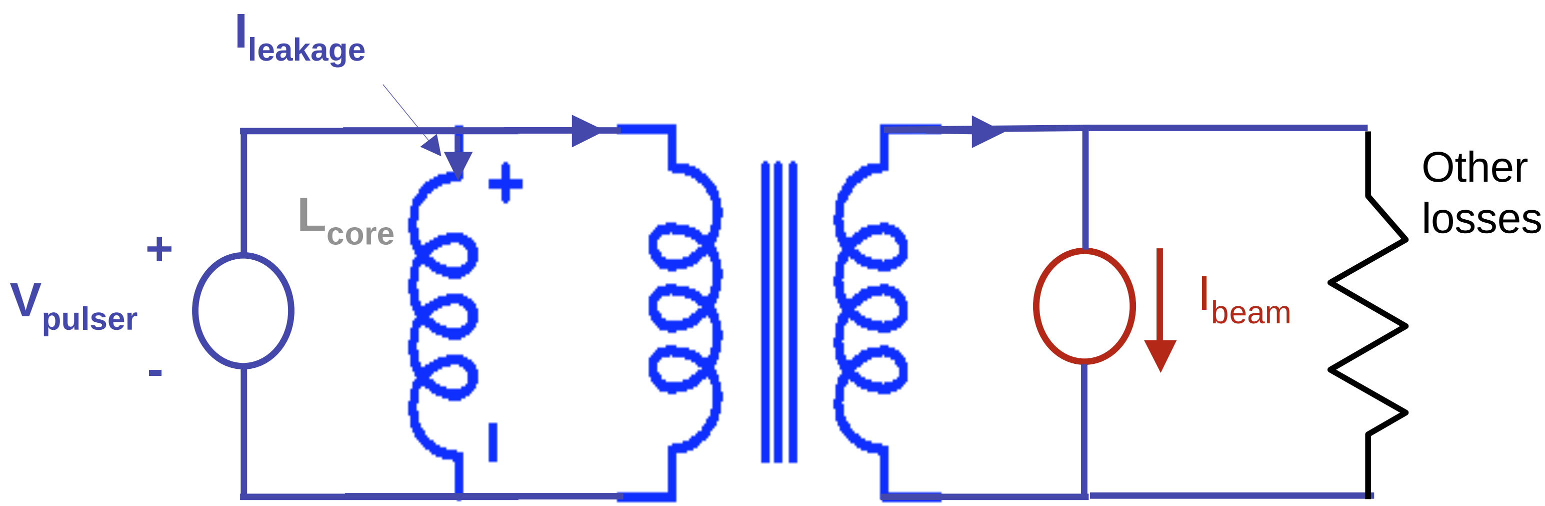


Fig. 5 Typical cross section of low-gamma ten-cell module.

What is the equivalent circuit



Characteristics of coaxial transmission lines

Wave velocity:

$$v_g = 1 \Big/ \sqrt{\mu\varepsilon} = c \Big/ \sqrt{\mu_r \varepsilon_r}$$

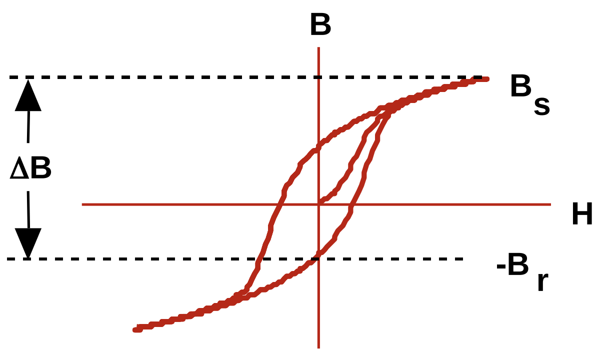
Core impedance:

$$Z_{core} = \sqrt{\mu / \varepsilon} = 120\pi \sqrt{\mu_r / \varepsilon_r} \text{ Ohms}$$

Characteristic impedance:

$$Z = \left(\frac{L}{C}\right)^{1/2} = \frac{Z_{core}}{2\pi} \ln\left(\frac{r_o}{r_i}\right) = \frac{1}{2\pi}\sqrt{\frac{\mu}{\varepsilon}} \ln\left(\frac{r_o}{r_i}\right) = 60\sqrt{\frac{\mu_r}{\varepsilon_r}} \ln\left(\frac{r_o}{r_i}\right)$$

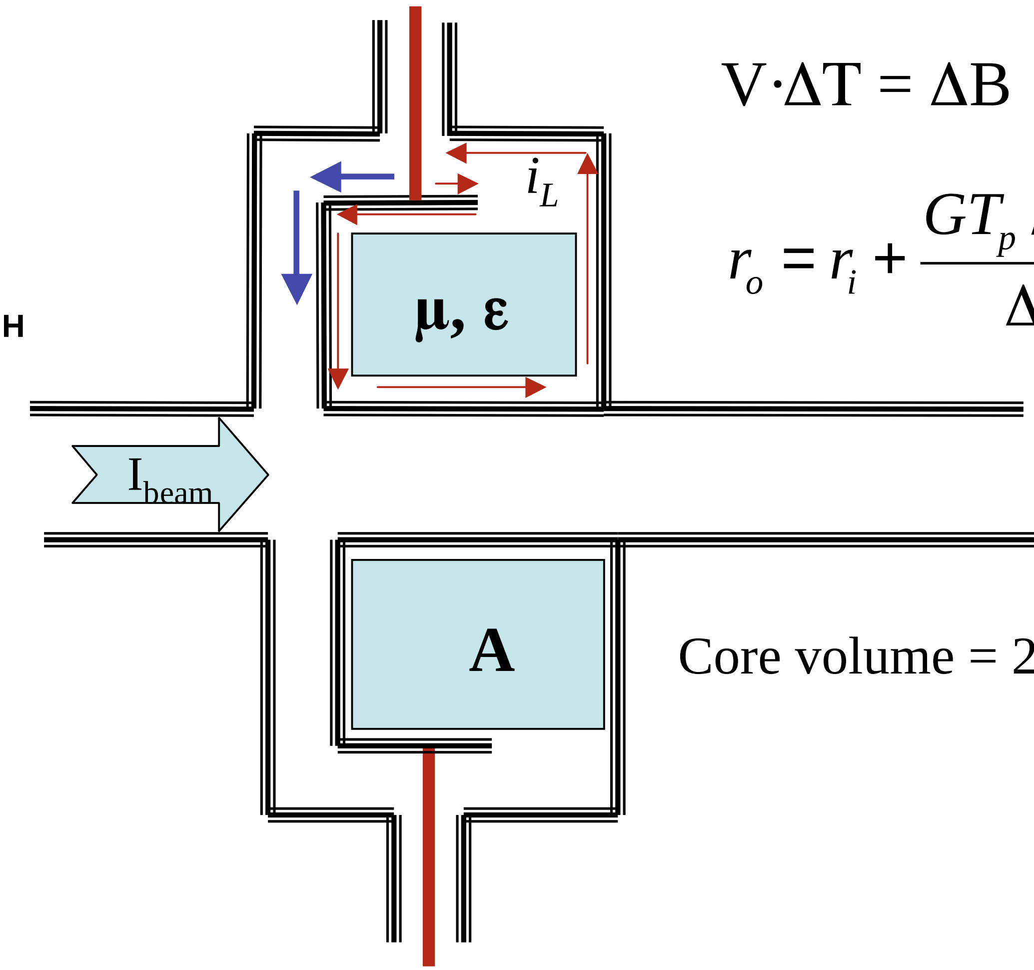
Volt-seconds, gradient (G) & inner radius set the induction core size



Core hysteresis loop

Leakage current magnetizes core

$$i_L = \frac{V}{L_c} t$$

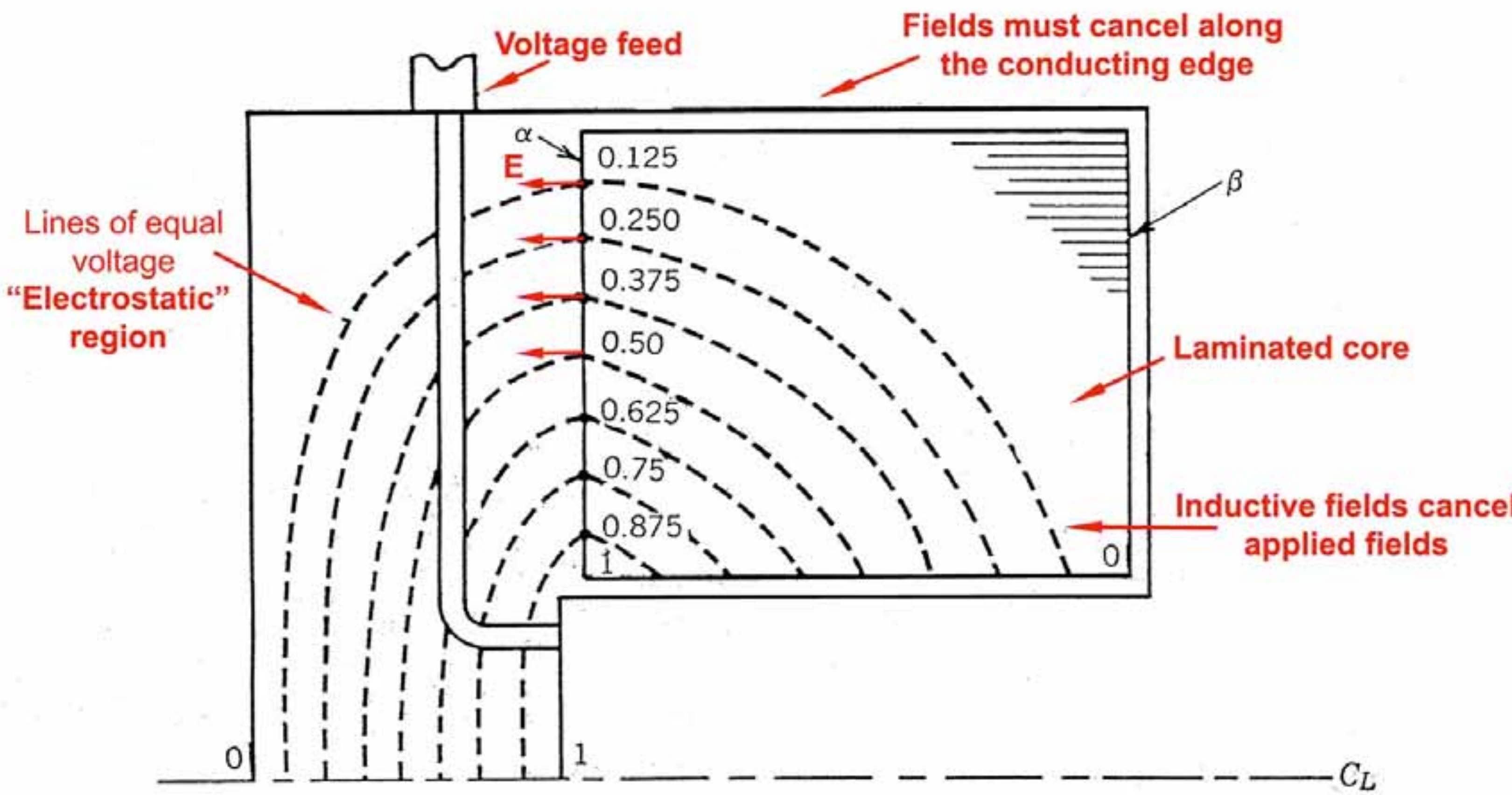


$$V \cdot \Delta T = \Delta B \cdot A \Longrightarrow$$

$$r_o = r_i + \frac{G T_p / f_{pack}}{\Delta B}$$

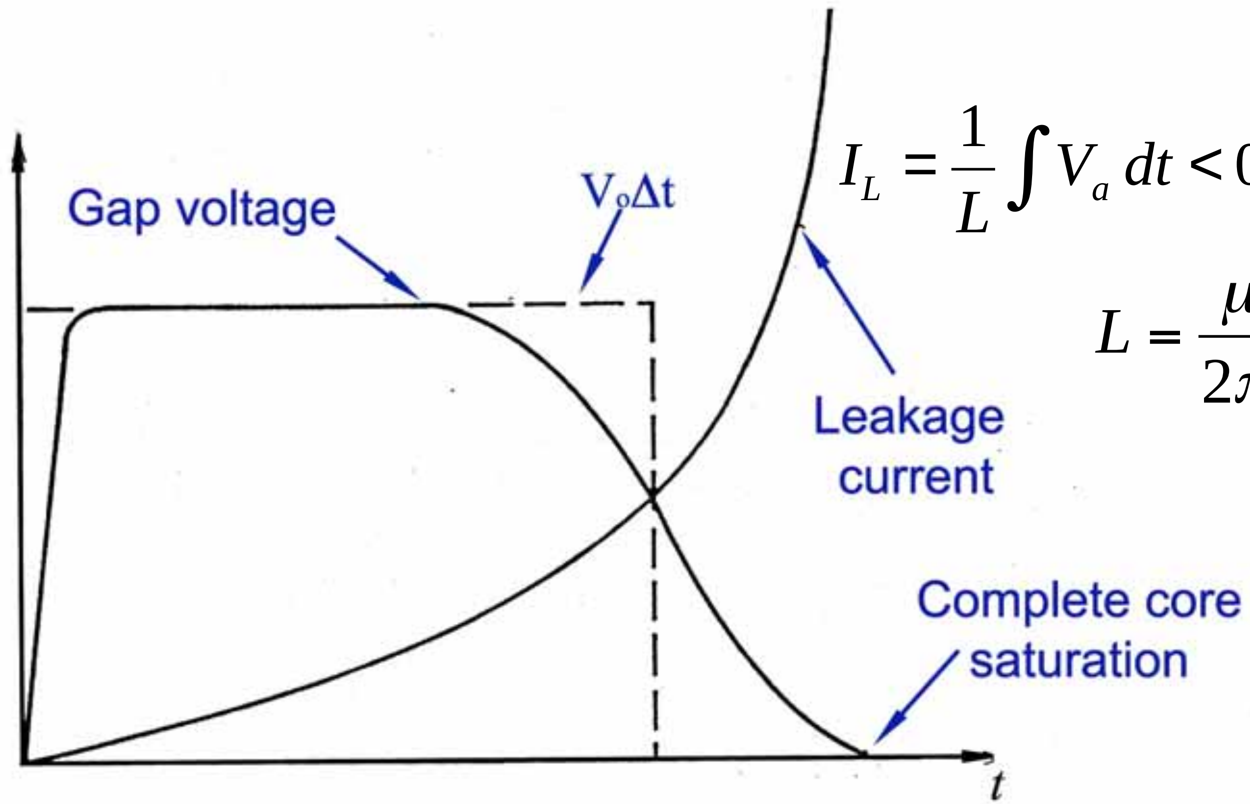
$$\text{Core volume} = 2\pi A (r_o + r_i)$$

Distribution of voltages in induction core
(no local saturation)



Laminating the core reduces eddy current losses & allows fields to penetrate through the core

Voltage & leakage current behavior at saturation

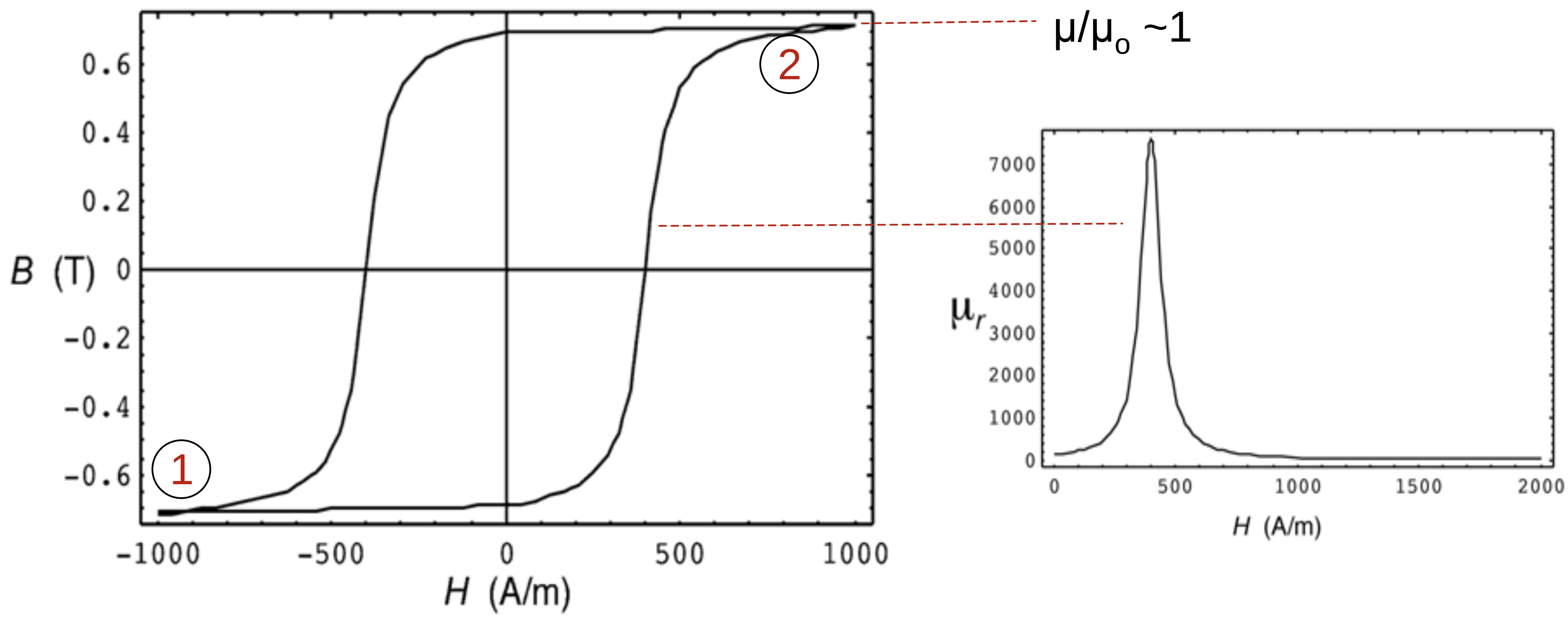


$$I_L = \frac{1}{L}\int V_a \, dt < 0.2 I_{beam}$$

$$L = \frac{\mu}{2\pi} z_{cell} \ln\frac{r_o}{r_i}$$

$$\Re_{core\ loss} = \frac{\pi}{\mu} G T_p^2 \left(\ln\frac{r_o}{r_i}\right)^{-1}$$

Hysteresis losses in induction cell

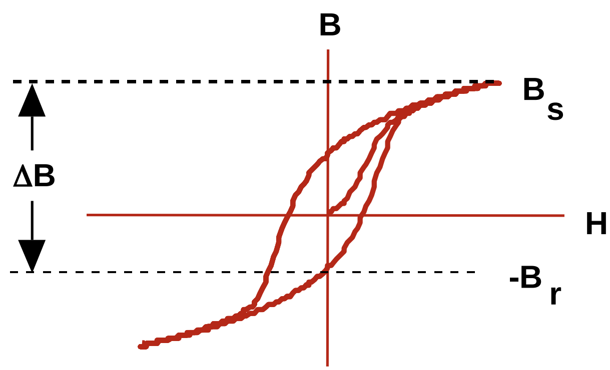


B-H hysteresis model curve at τ_{sat}=500 nsec for Co-amorphous

State 1 => State 2: Drive
State 2 => State 1: Reset
Area = Hysteresis loss

Resetting the cores

- Before the core can be pulsed again it must be reset to $-B_r$
- Properties of the reset circuit
  - Achieve $V\Delta t$ product > B_r+B_s
  - Supply unidirectional reverse current through the axis of the core
  - Have high voltage isolation so that the reset circuit does not absorb energy during the drive voltage pulse
    - Depends on the type of pulse forming line used in primary circuit

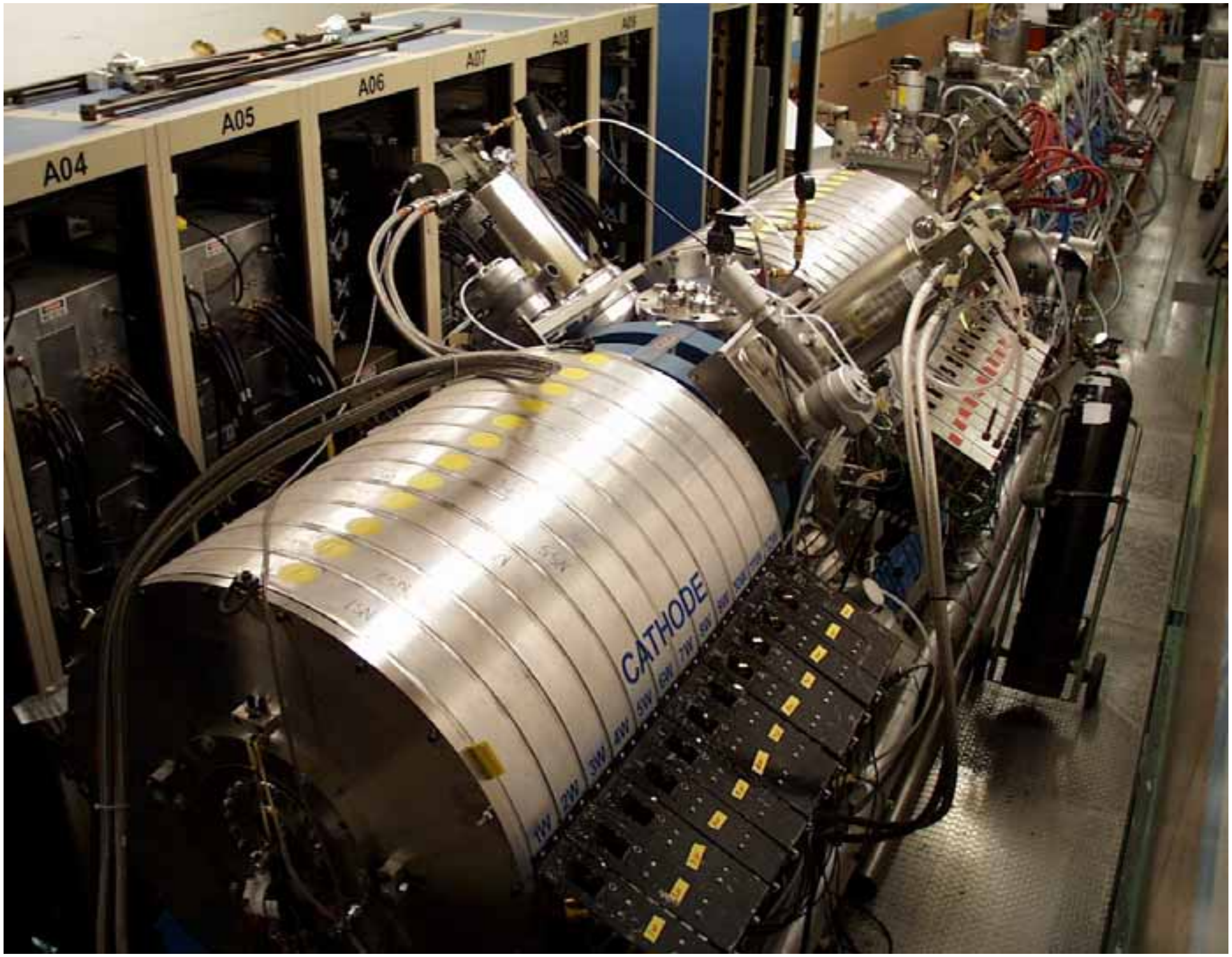


Core hysteresis loop

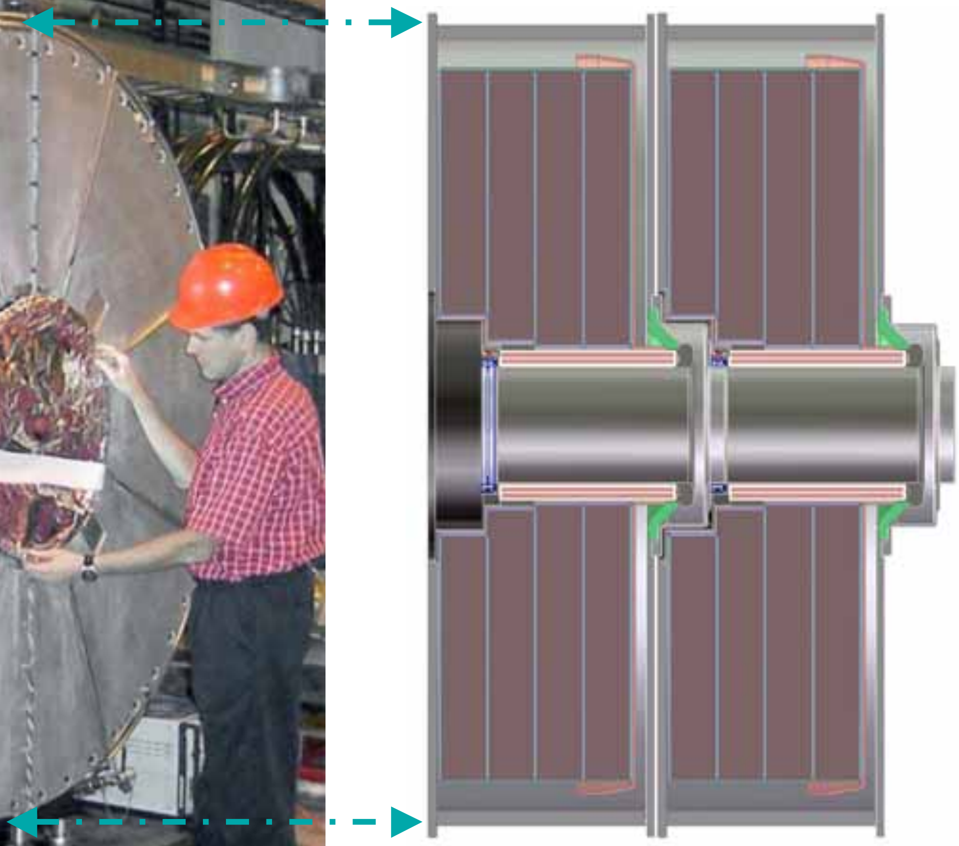
ETA-II Cell Modification

Metglass
replacement core

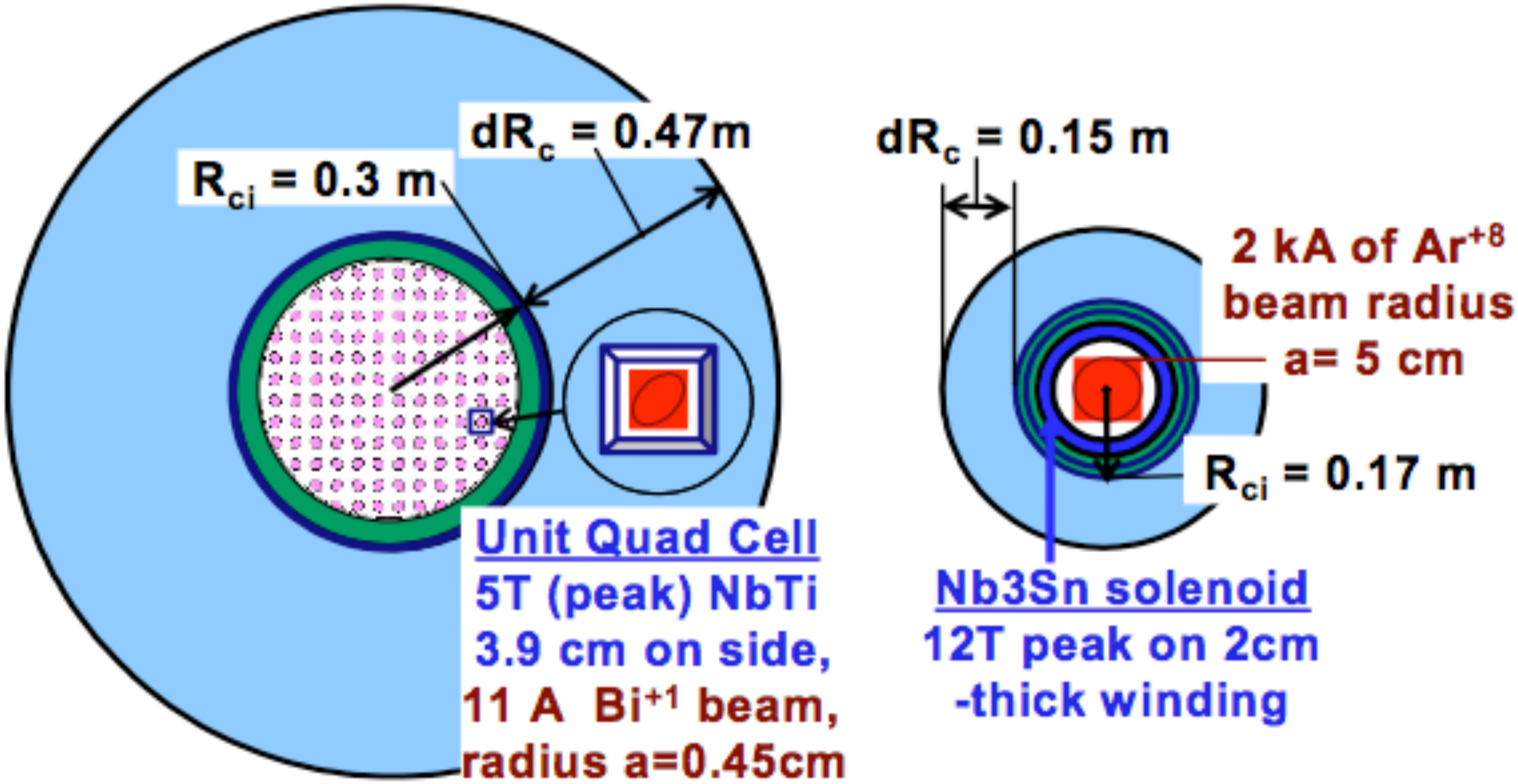
The RTA Injector (1 MeV, 1 KA, 375 ns)



Double 200 kV, 1.6 μs DARHT cell is of the scale needed for HIF



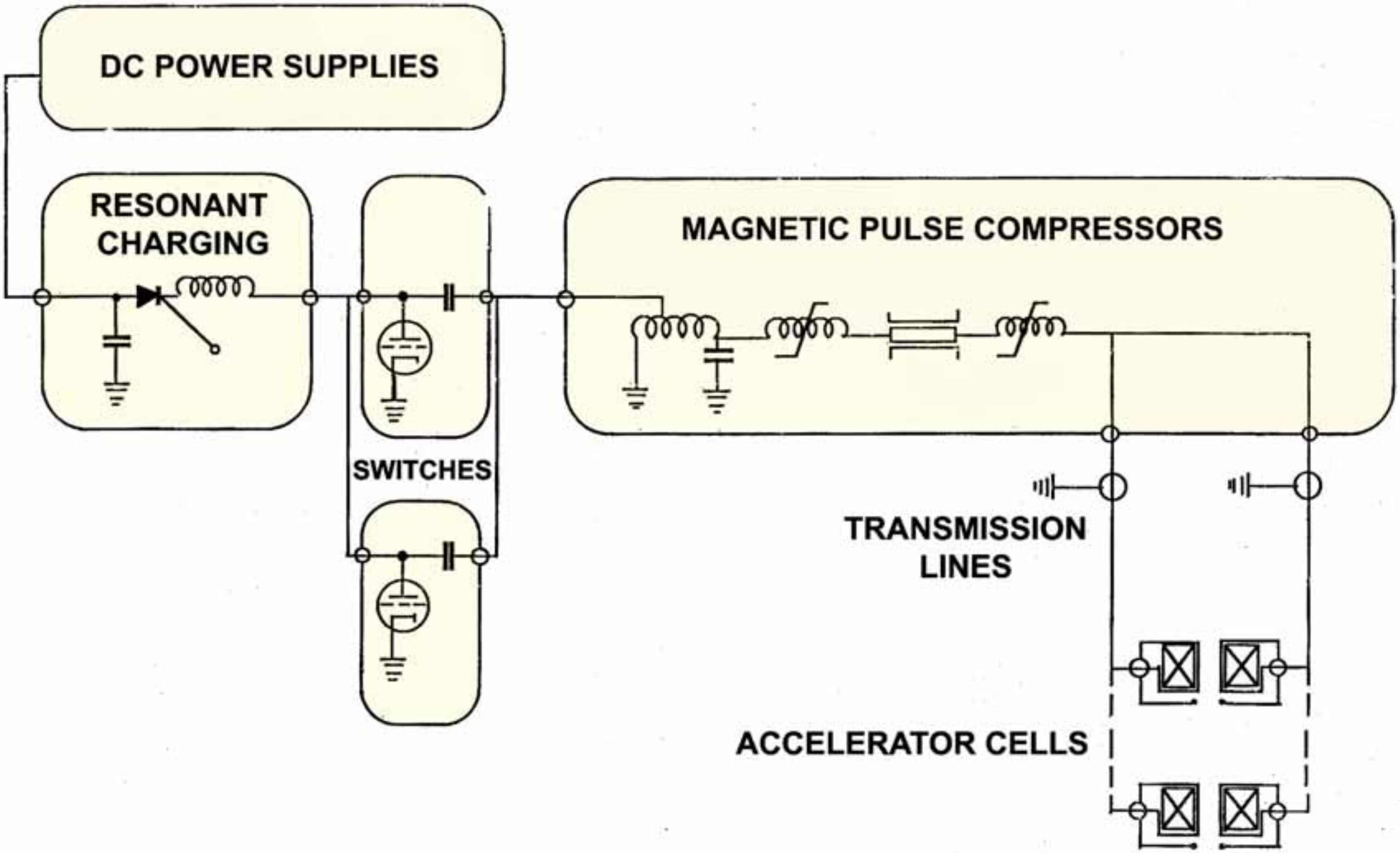
Comparing 1MJ HIF linac driver example cross-sections



Multi-beam Quad (MQ) driver, an RPD-like design scaled down to produce 1MJ of 4 GeV Bi+ ions in a single pulse.

Modular Solenoid (MS) driver system, one of 40 linacs, to produce 1MJ total of 500 MeV Ar+8 with five pulses per linac.

Schematic of induction linac power system



Why HIF Chose Induction

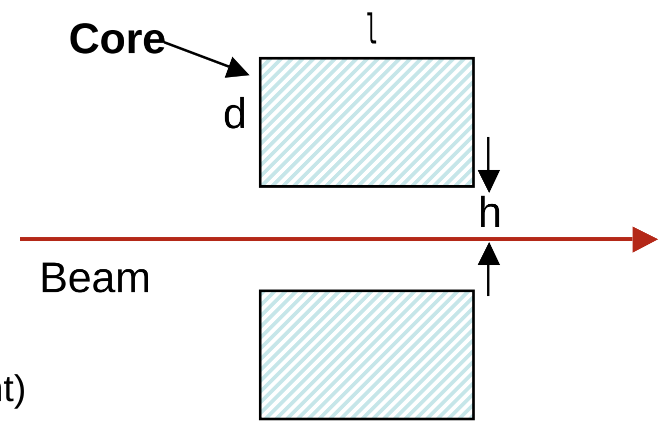
Induction linacs handle high currents naturally.

$$\text{Efficiency} = \frac{I\,l\,d\,\frac{\Delta B}{\tau}\,\tau}{I\,l\,d\,\frac{\Delta B}{\tau}\,\tau + \mathbf{w}\,\pi\, l\,d(2h + d)}$$

Voltage across gap

Loss function (frequency dependent)

Core volume



$$\eta_{LIA} = \frac{I\Delta B}{I\Delta B + w\pi(2h + d)}$$

Efficiency increases as current increases

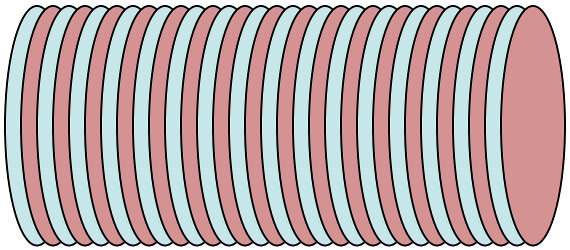
==> Multiple beams within single induction core

General Envelope Equation
for
Cylindrically Symmetric Beams

Can be generalized for sheet beams and beams with quadrupole focusing

Assumptions for the derivation

Divide beam into disks



- Rays are paraxial ($v_\perp/c \ll 1$)
- Axisymmetry
- No mass spread with a disk
- Small angle scattering
- Uniform B_z
- Disks do not overtake disks

Particle equations

$$\dot{\mathbf{p}} = q\left(\mathbf{E} + \mathbf{v} \times \mathbf{B}\right) + \delta\mathbf{F}_{scat}$$

$$\mathbf{p} = \gamma m \mathbf{v}$$

$$So, \quad \frac{\mathrm{d}}{\mathrm{dt}}(\gamma m \mathbf{v}) - q\left(\mathbf{E} + \mathbf{v} \times \mathbf{B}\right) = \delta\mathbf{F}_{scat} \qquad \text{(EoM)}$$

$$Define \quad \mathrm{w} = \gamma m c^2$$

- Paraxial implies

$$\mathrm{v}_\perp/\mathrm{c} << 1$$

and

$$I_{beam} << I_{Alfven} = \gamma\beta \frac{ec}{r_e} = 17{,}000\ \gamma\beta \text{ Amps}$$

Next write the particle equation of motion

- Define the cyclotron frequency & the betatron frequency

$$\omega_c = \frac{qB_z}{\gamma m} \quad \text{and} \quad \omega_\beta = \frac{\beta c B_\vartheta - E_r}{r}$$

- By Maxwell's equations

$$B_r = -\frac{r}{2}\frac{\partial B_z}{\partial z}$$

$$E_\vartheta = -\frac{r}{2}\frac{\partial B_z}{\partial t}$$

$$\frac{dB_z}{dt} \equiv \dot{B} = \frac{\partial B_z}{\partial t} + \beta c \frac{\partial B_z}{\partial z}$$

- The EoM for a beam particle is

$$\frac{\dot{\gamma}}{\gamma}\mathbf{v} + \dot{\mathbf{v}} + \omega_\beta^2 \mathbf{r} + \omega_{\mathbf{c}} \hat{\mathbf{z}} \times \mathbf{v} + \frac{1}{2\gamma}\frac{d}{dt}(\gamma\omega_c)\hat{\mathbf{z}} \times \mathbf{r} = \frac{1}{\gamma m}\delta\mathbf{F}_{scat}$$

Take moments of the EoM

- Three moment equations:
  1. **v** • EoM = Energy equation
  2. **r** • EoM = Virial equation
  3. **r** × EoM = Angular momentum equation

- Next take rms averages of the moment equations
  - ➙ Yields equations in R, V, L and their derivatives

- Ansatz: The radial motions of the beam are self similar
  - ➙ The functional shape of J(r) stays fixes as R changes

Last steps

- Angular momentum conservation implies

$$P_\vartheta = \gamma L + \gamma\omega_c \frac{R^2}{c} = \text{ constant}$$

- The energy & virial equations combine to yield

$$\ddot{R} + \frac{\dot{\gamma}}{\gamma}\dot{R} + \frac{U}{R} + \frac{\omega_c^2 R}{4} - \frac{E^2}{\gamma^2 R^3} = \frac{1}{\gamma^2 R^3}\int_{t_o}^{t} dt' \left(\frac{2\gamma R^2}{m}\varepsilon' \right)$$

where

$$U = \left\langle \omega_\beta^2 r^2 \right\rangle = I / I_{Alfven}$$

and

$$E^2 = \gamma^2 R^2 \left(V^2 - (\dot{R})^2 \right) + P_\vartheta^2$$