Unit 10 - Lectures 14

Cyclotron Basics

MIT 8.277/6.808 Intro to Particle Accelerators

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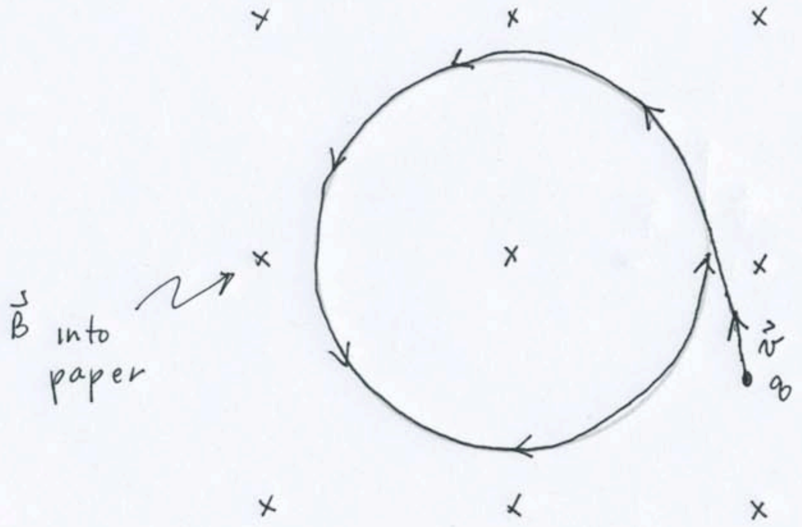
Outline

- Introduce an important class of circular particle accelerators: Cyclotrons and Synchrocyclotrons
- Identify the key characteristics and performance of each type of cyclotron and discuss their primary applications
- Discuss the current status of an advance in both the science and engineering of these accelerators, including operation at high magnetic field

Overall aim: reach a point where it will be possible for to work a practical exercise in which you will determine the properties of a prototype high field cyclotron design (next lecture)

Motion in a magnetic field

Shoot a charged particle into a perpendicular magnet field - what happens?

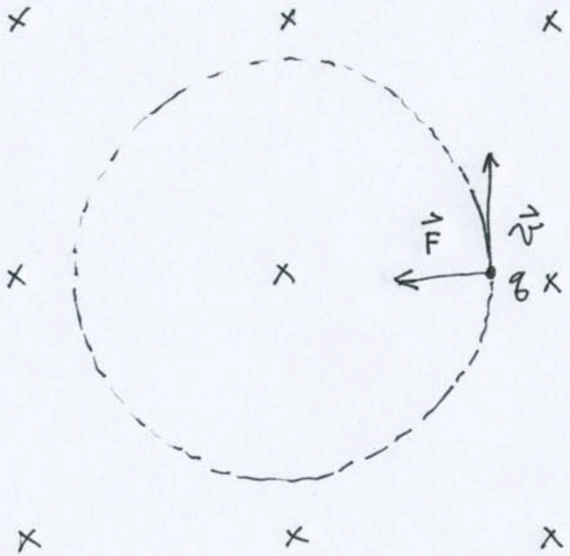


It moves in a closed circle - why?

Magnetic forces are perpendicular to the B field and the motion

It sees a sideways force - Lorentz Force

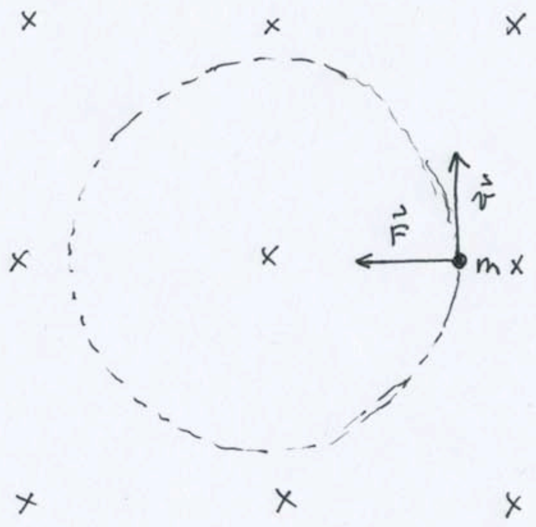
$\{\vec{F} = q\vec{v} \times \vec{B}\}$



$F = qvB$

Sideways force must also be *Centripedal*

IF you did not no (1) the particle had charge
(2) or that there was a magnetic field



$$F = m\frac{v^2}{r}$$

You could still infer a sideways (central force)

Governing Relation in Cyclotrons

- A charge q, in a uniform magnetic field B at radius r, and having tangential velocity v, sees a centripetal force at right angles to the direction of motion:

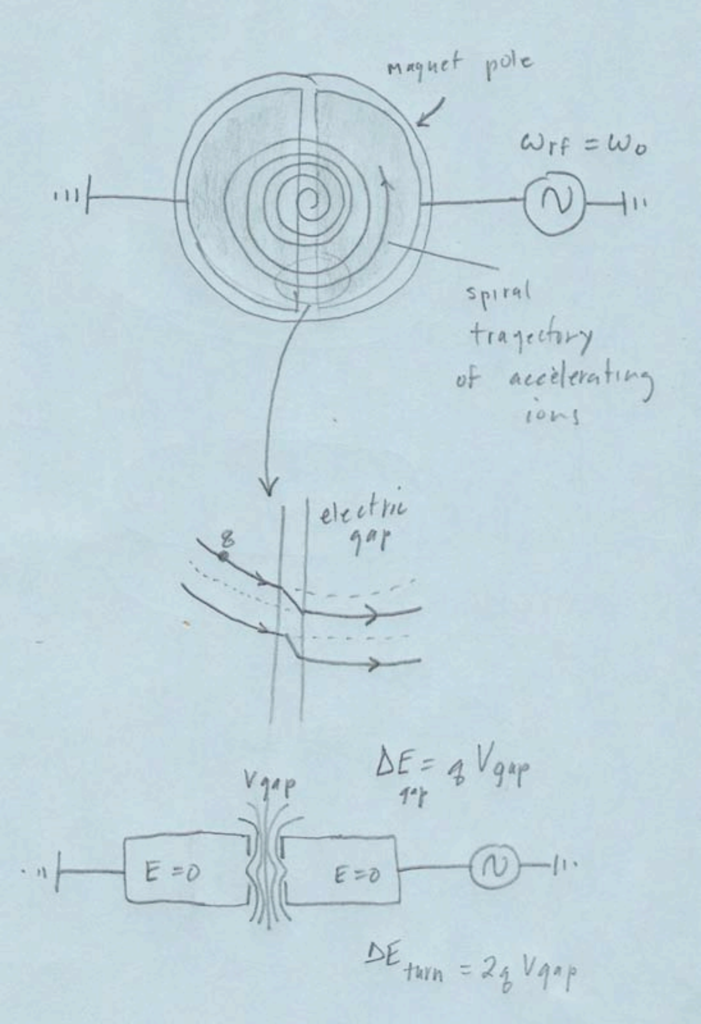
$$\frac{mv^2}{r}\hat{r} = q\vec{v} \times \vec{B}$$

- The angular frequency of rotation seems to be independent of velocity:

$$\omega = qB / m$$

Building an accelerator using cyclotron resonance condition

- A flat pole H-magnet electromagnet is sufficient to generate require magnetic field
- Synchronized electric fields can be used to raise the ion energies as ions rotate in the magnetic field
- Higher energy ions naturally move out in radius
- Highest possible closed ion orbit in the magnet sets the highest possible ion energy



There is a difficulty- we can't ignore relativity

- A charge q, in a uniform magnetic field B at radius r, and having tangential velocity v, sees a centripetal force at right angles to the direction of motion:

$$\frac{mv^2}{r}\hat{r} = q\vec{v} \times \vec{B}$$

- Picking an axial magnetic field B and azimuthal velocity v allows us to solve this relation:

$$mv^2/r = qvB \longrightarrow \omega = v/r = qB/m$$

- However:

$$m = \gamma m_0$$

$$\gamma = 1/\sqrt{1 - v^2/c^2}$$

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Relativistic Limit on Cyclotron Acceleration

- The mass in $\omega = qB/m$ is the relativistic mass $m = \gamma m_0$
- $\omega \approx$ **constant** only for very low energy cyclotrons

Proton Energy	% Frequency decrease
10 MeV	~1%
250 MeV	~21%
1.0 GeV	~52%

How to manage the relativistic change in mass?

There are 3 kinds of Cyclotrons:

- CLASSICAL: (original)
  - Operate at fixed frequency ($\omega = qB/m$) and ignore the mass increase
  - Works to about 25 MeV for protons ($\gamma \cong 1.03$)
  - Uses slowly decreasing magnetic field 'weak focusing'
- *SYNCHROCYCLOTRON:* let the RF frequency ω decreases as the energy increases
  - $\omega = \omega_0/\gamma$ to match the increase in mass ($m = \gamma m_0$)
  - Uses same decreasing field with radius as classical cyclotron
- *ISOCHRONOUS:* raise the magnetic field with radius such that the relativistic mass increase is just cancelled
  - Pick $B = \gamma B_0$ {this also means that B increases with radius}
  - Then $\omega = qB/m = qB_0/m_0$ is constant.
  - Field increases with radius- magnet structure must be different

Some Examples of Cyclotrons

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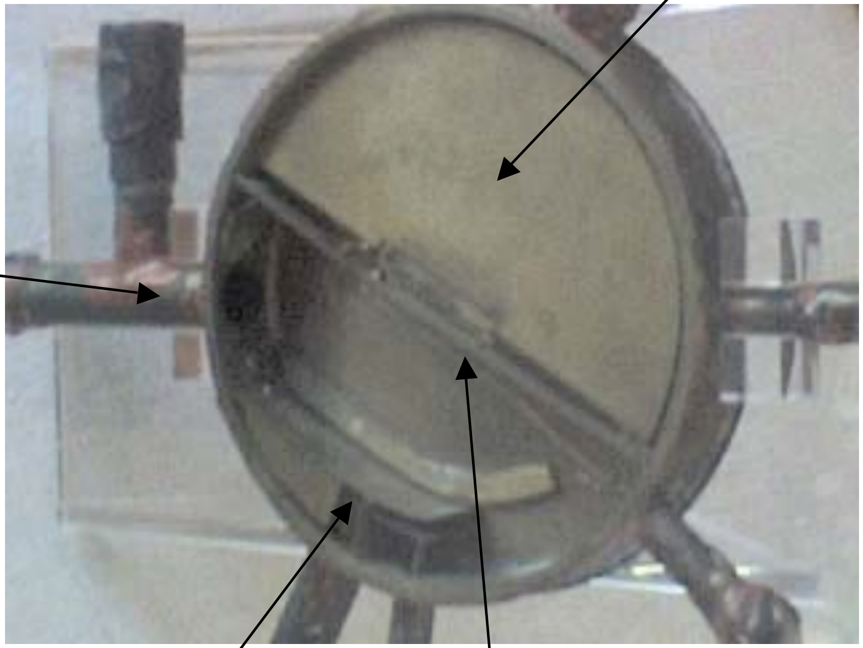
1932 Cyclotron

Evacuated Beam Chamber sits between magnet poles:



180° 'Dee'

Vacuum Port

Internal Energy Analyzer

Ion Source is a gas feed and a wire spark gap

The Largest…

- Gatchina Synchrocyclotron at Petersburg Nuclear Physics…

  1000 MeV protons and 10,000 tons

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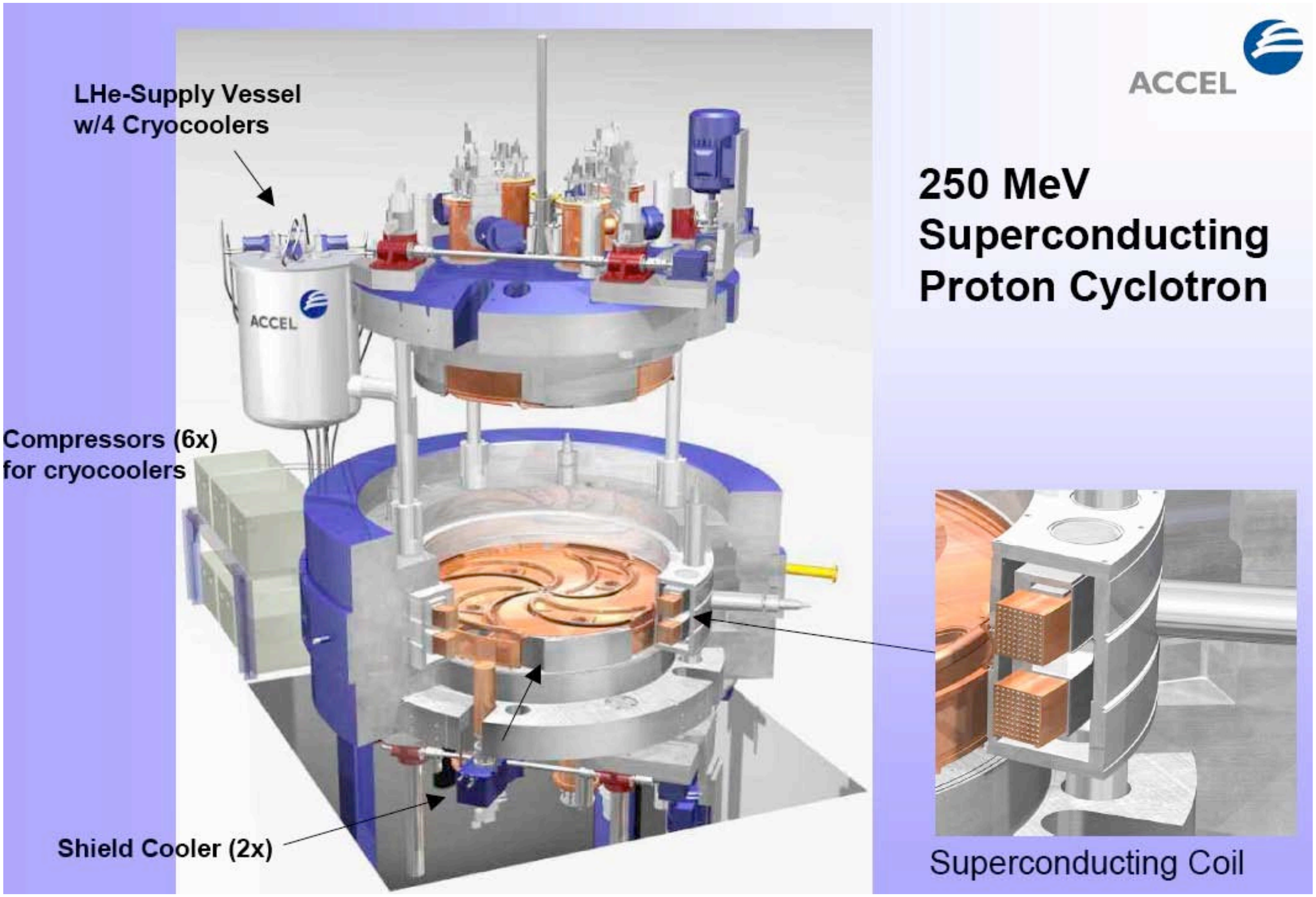
Superconducting Isochronous Cyclotron



250 MeV Superconducting Proton Cyclotron

Superconducting Coil

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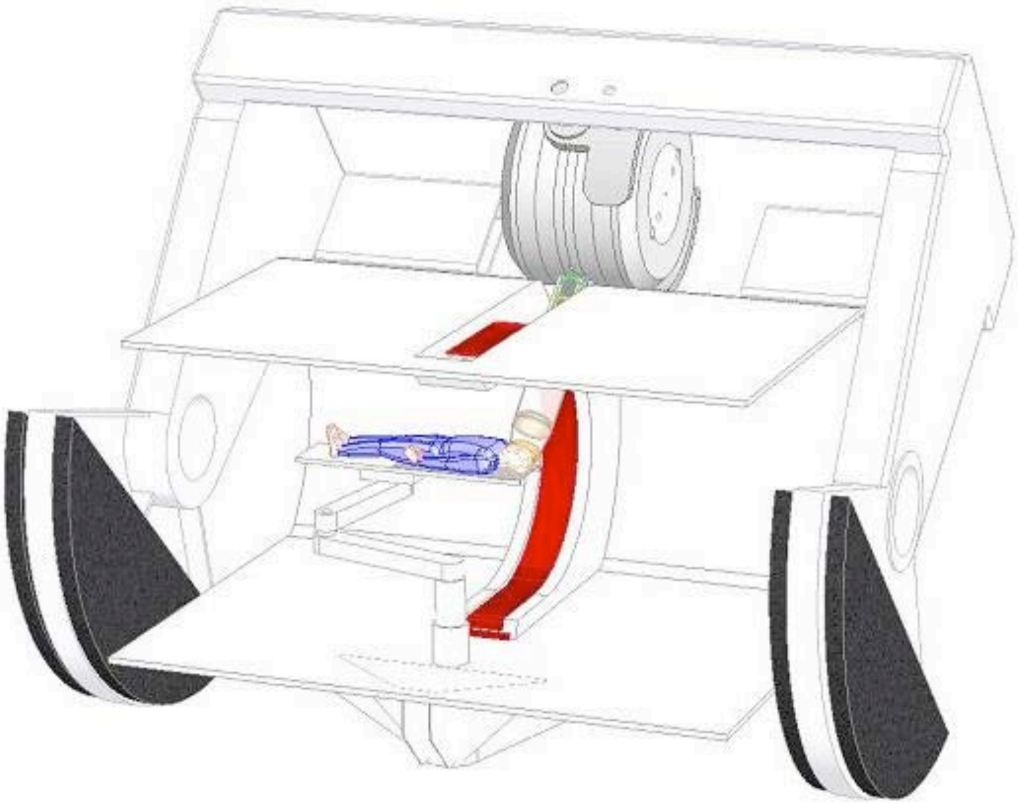
The Highest Magnetic Field…



- Still River Systems 9 Tesla, 250 MeV, synchrocyclotron for Clinical Proton Beam Radiotherapy

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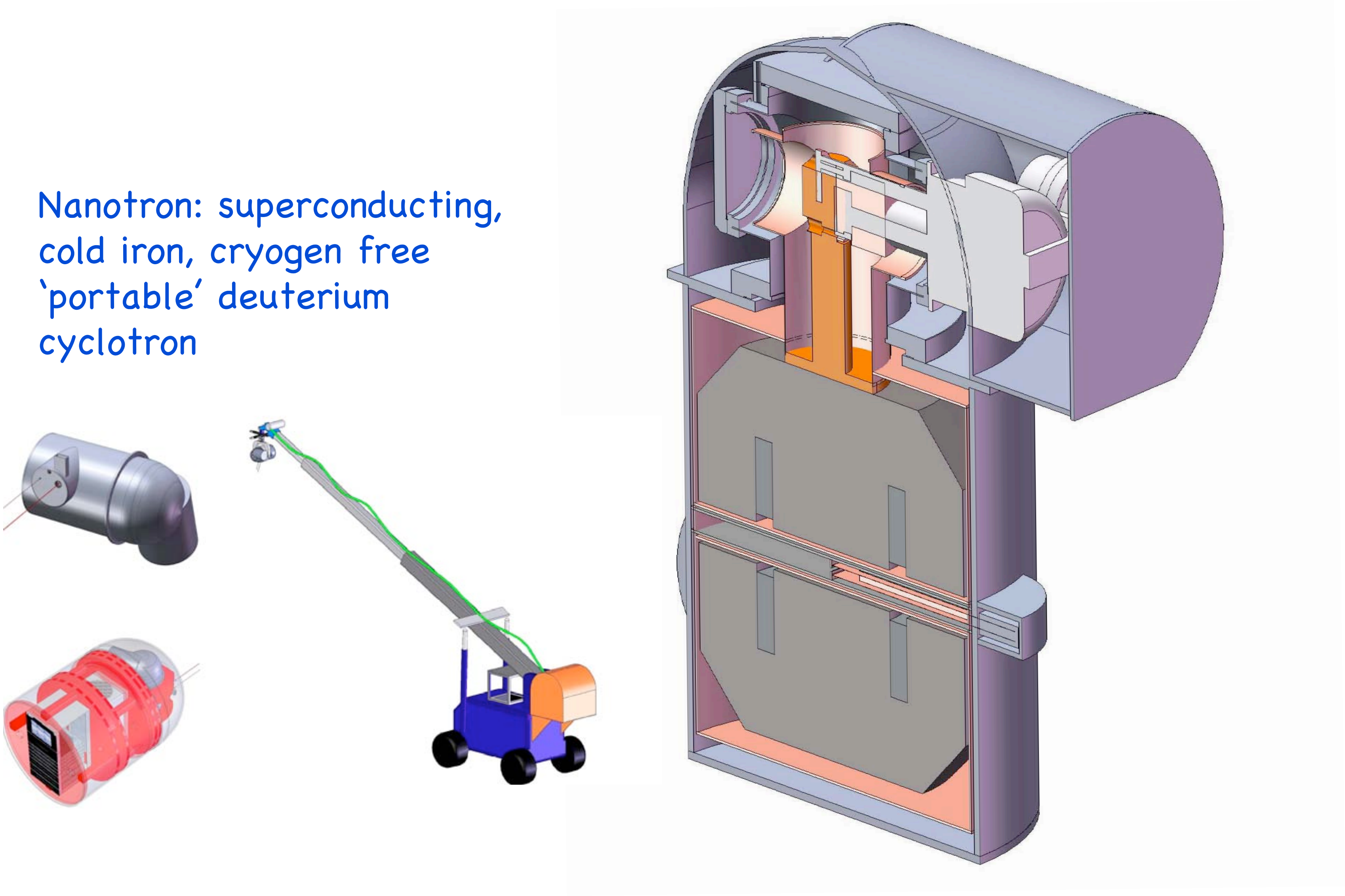
The Newest…

- Nanotron: superconducting, cold iron, cryogen free ‘portable’ deuterium cyclotron



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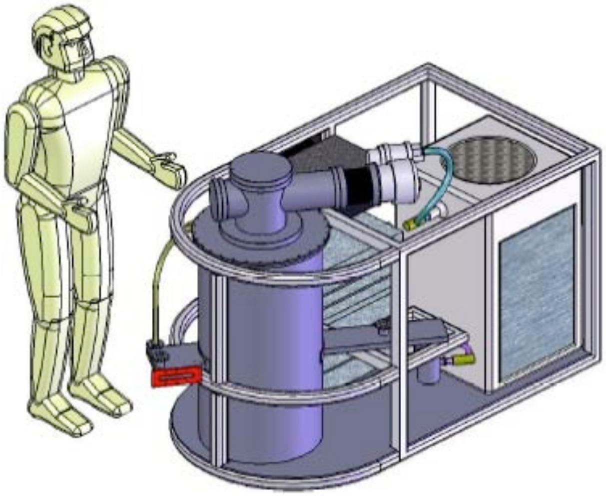
New Cyclotrons and Synchrocyclotrons are coming..

Isotron -for short lived PET isotope production:

- Protons or heavy ions
- 30-100 MeV
- Synchrocyclotron or isochronous cyclotron is possible



Also:

- Gigatron: 1 GeV, 10 mA protons for airborne active interrogation
- Megatron: 600 MeV muon cyclotron (requires a gigatron to produce muons and a reverse cyclotron muon cooler for capture for accel.)

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Key Characteristics of the Cyclotron 'Class'

Cyclotron utility is due to:

- Ion capture and Beam formation at low velocity, followed by acceleration to relativistic speeds in a single device
- Efficient use of low acceleration voltage makes them robust and uncritical; pulsed or CW operation allowed
- Beam characteristics are wrapped up in the design of the static magnetic guide field; ions have high orbital stability
- Ion species: H+ --> U; neg. ions (e.g. H^-), molecular ions (e.g. HeH^+)
- Intensities; picoamps (one ion per rf bucket) to milliamps
- γ: 0.01 --> 2.3

Have resulted in:

- 2nd largest application base historically and currently (electron linacs used in radiotherapy are 1st)
- Science (Nuclear, Atomic, Plasma, Archeology, Atmospheric, Space), Medicine, Industry, Security
- Highest energy CW accelerator in the world: K1200 heavy ion at MSU- 19.04 GeV ^{238}U

Key Characteristics- prob. most important:

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Classical Cyclotrons

- Weak focusing
- Phase stability
- Limited by Relativistic Mass Increase

How to manage the relativistic change in mass?

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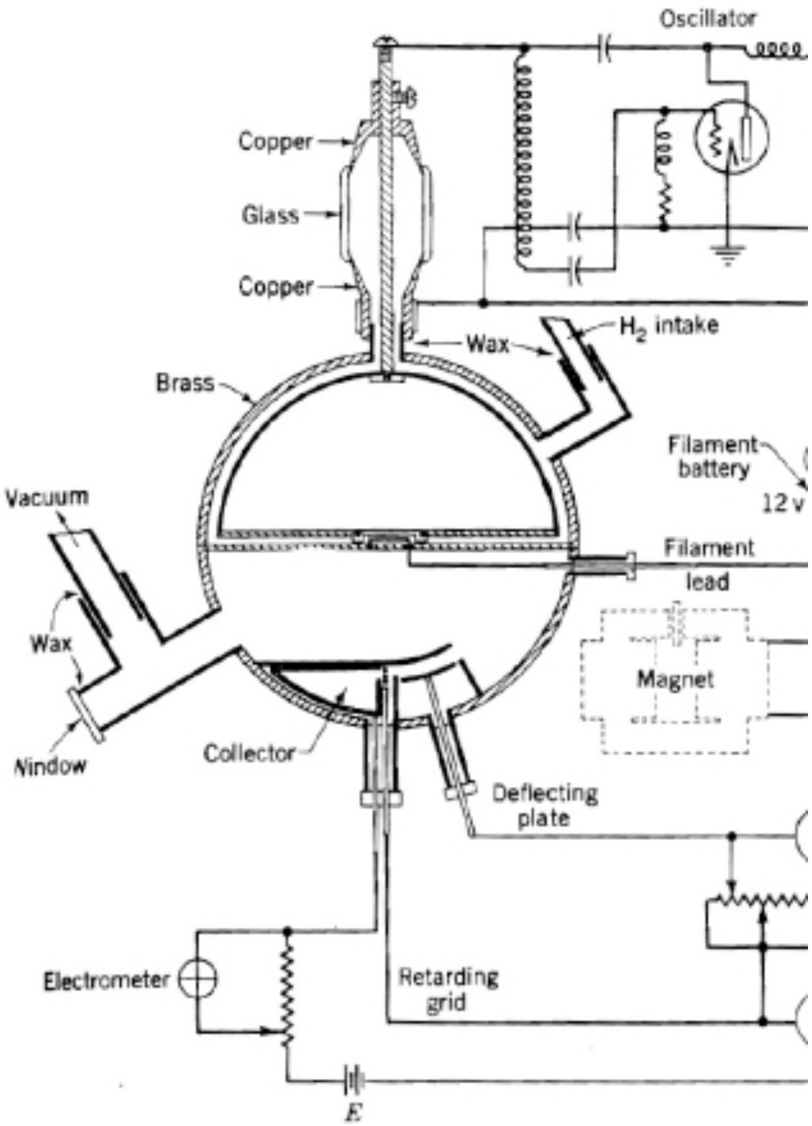
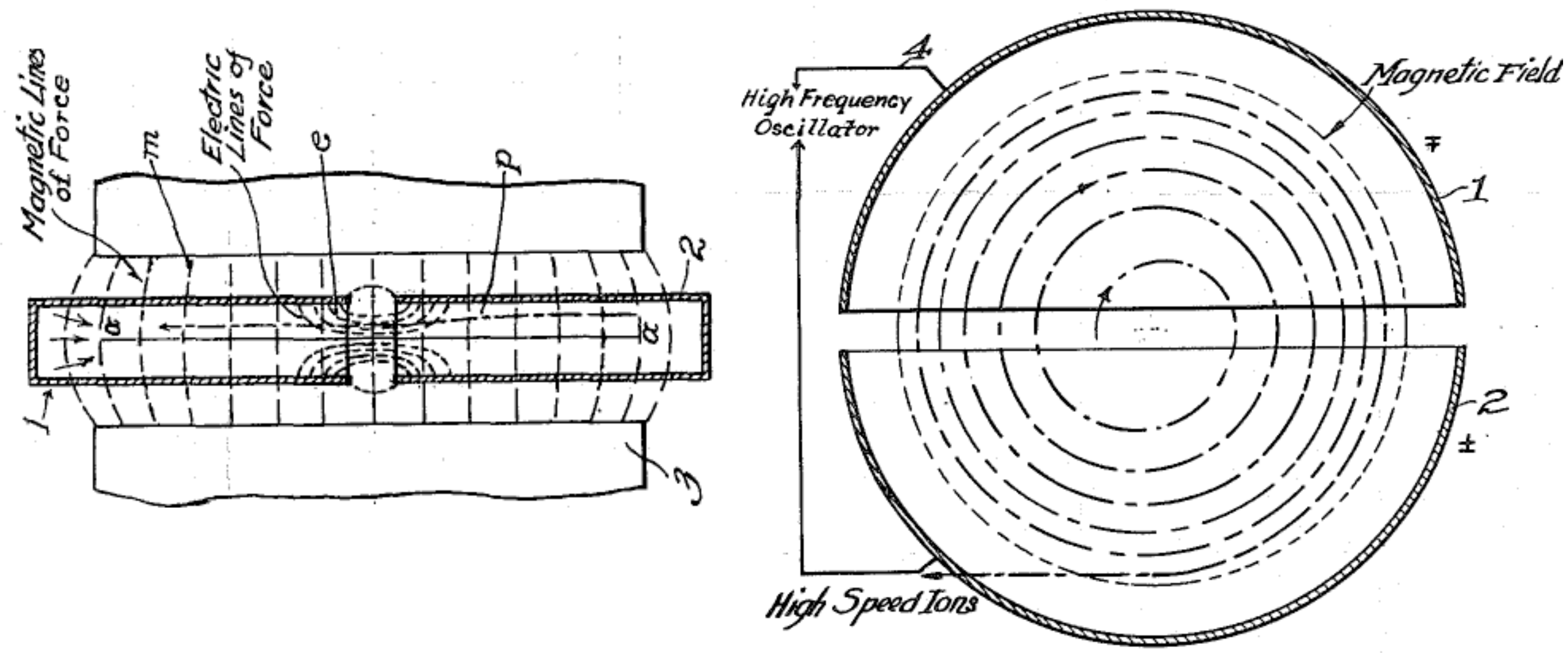
The 1931 Cyclotron…



Cyclotron Schematic Diagram (via Lawrence Patent)



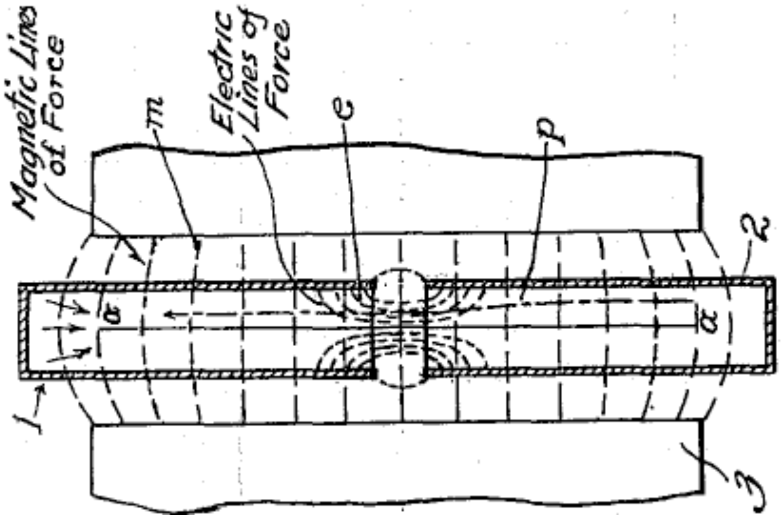
- A flat pole electromagnet (3) generates a vertical magnetic field (m)
- Ions (P) rotate in the mid-plane of an evacuated split hollow conductor (1-2)
- Time varying electric fields (4) applied to the outside of this conductor raise the ion energies as ions rotate in the magnetic field and cross the split line gap- the only place where electric fields (e) appear
- Higher energy ions naturally move out in radius
- Highest allowed closed ion orbit in magnet sets the highest possible ion energy

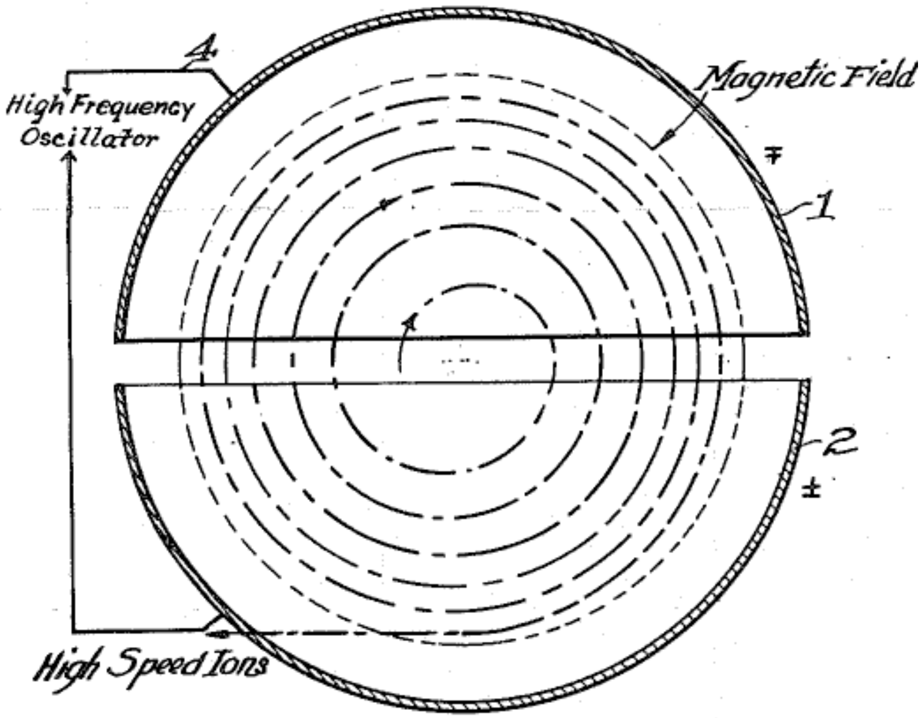
Let's break down the key phenomena that make cyclotrons work…

- We'll do this in a very 'raw' manner- using elementary properties of ions, conductors and electromagnetic fields

- Why choose this approach?
  - To demonstrate just how utterly simple cyclotrons are
  - To get to better appreciate the key challenges in making cyclotrons work
  - To understand how the advance machines just shown are possible

Magnetic Field Generation





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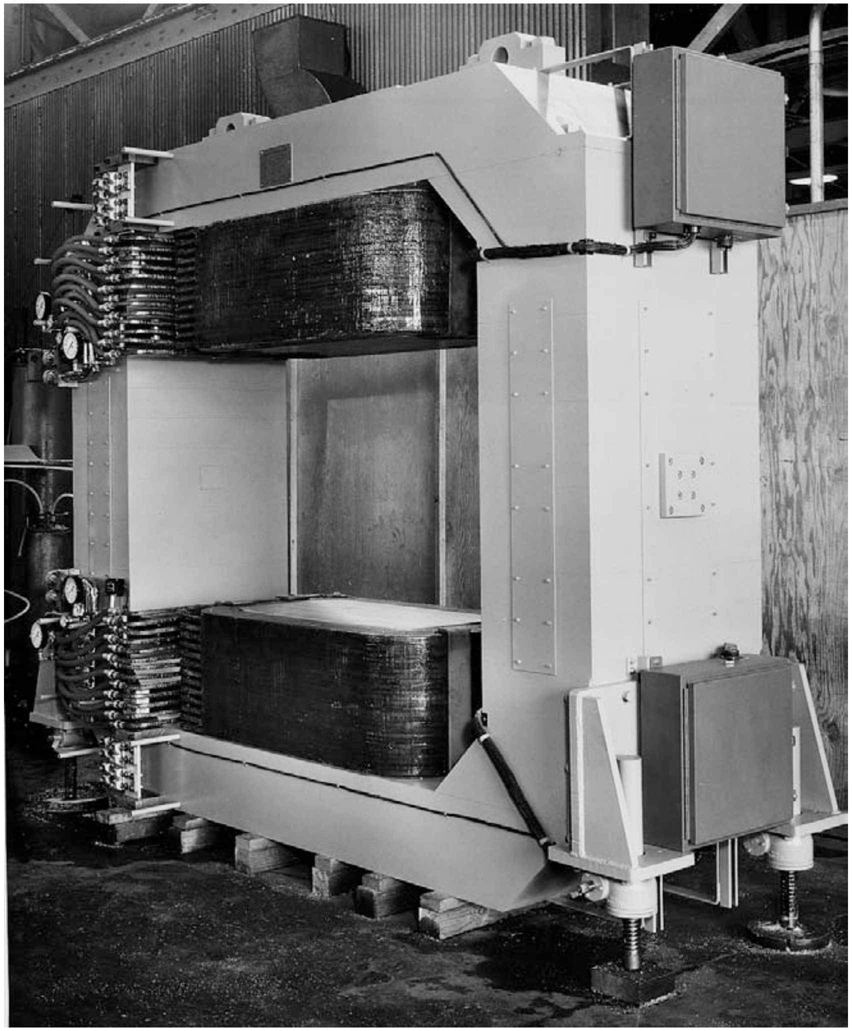
Typical large H Magnet



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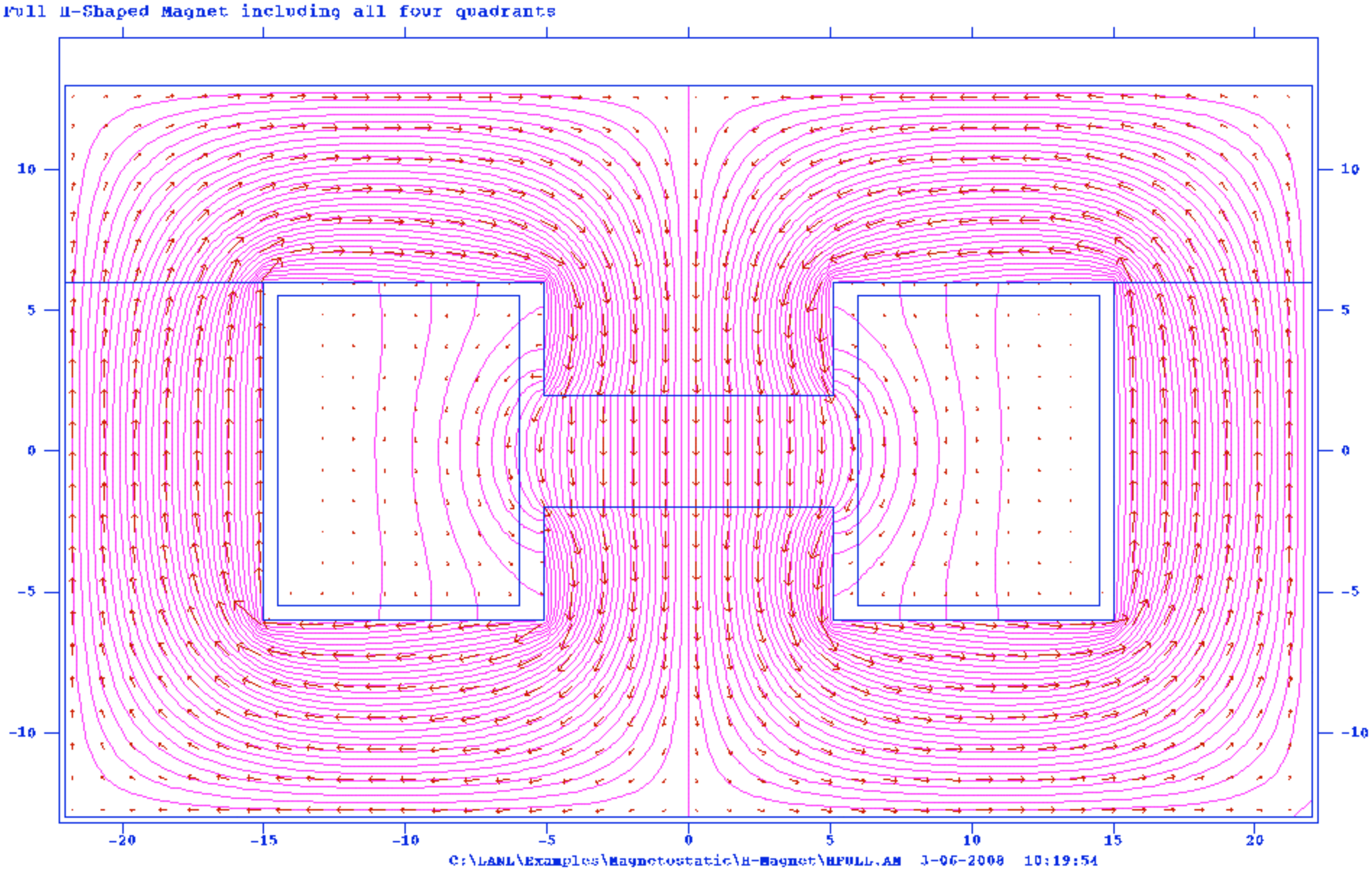
Magnetic field of a H Magnet



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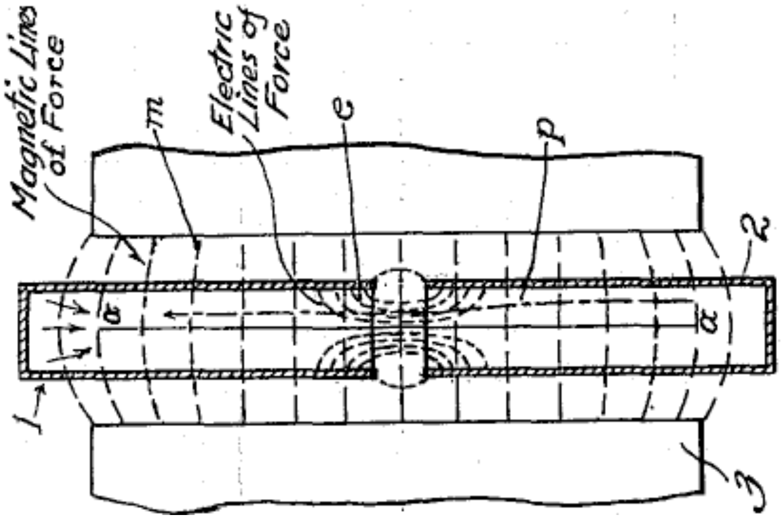
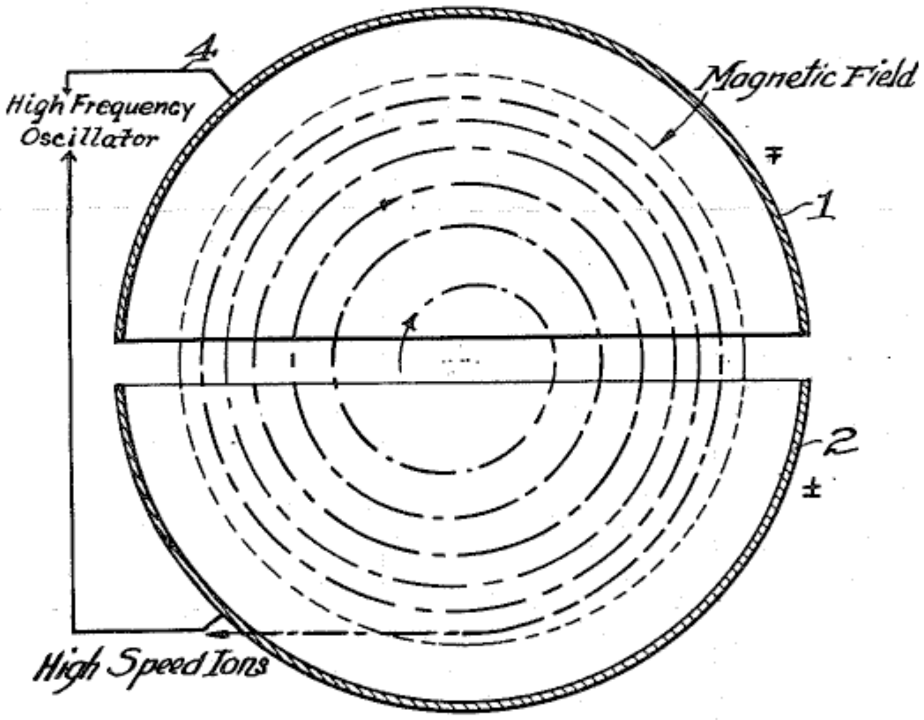
Ion Acceleration-- requires a bit more work…





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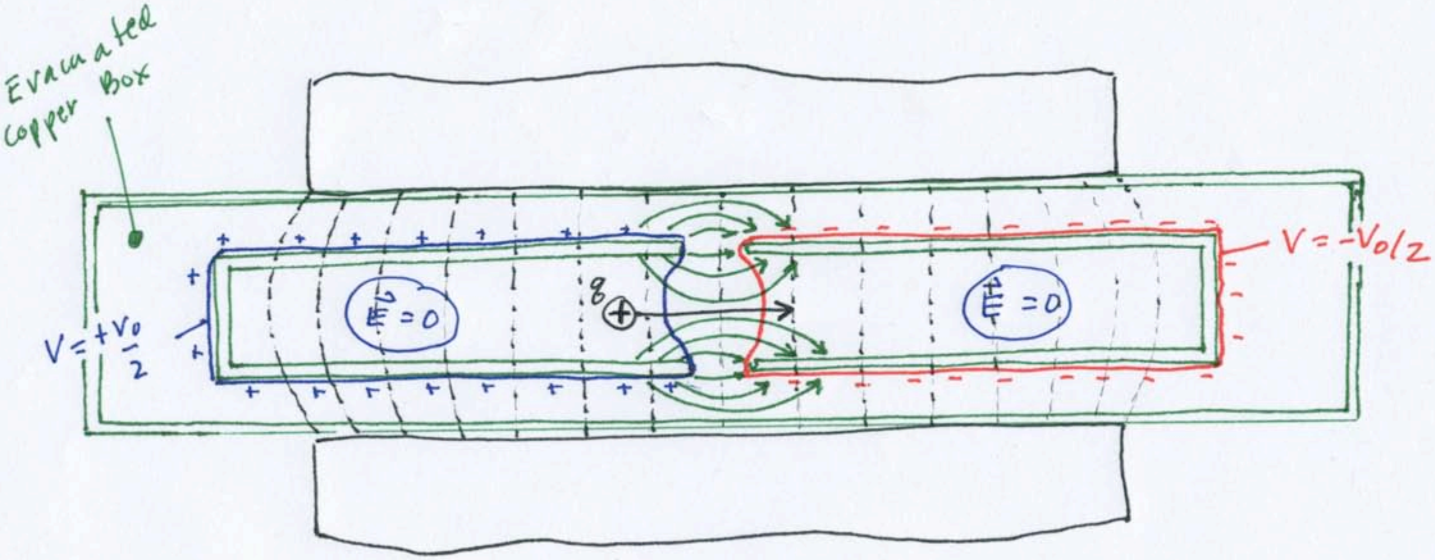
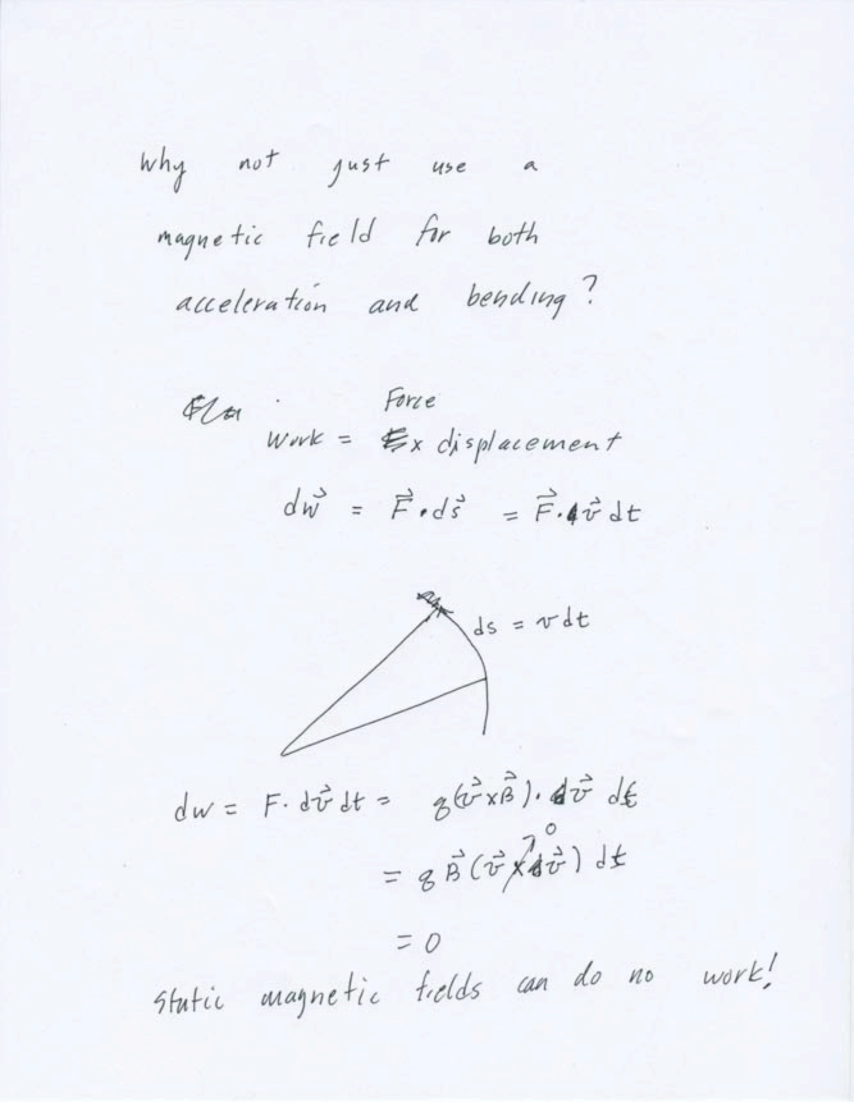
Acceleration really looks something like this…



$\vec{F} = q\vec{E}$ & $\Delta W = q\,\Delta V = q\,V_0$

Why not magnetic field only acceleration?



Why not just use a magnetic field for both acceleration and bending?

$\text{Work} = \text{Force} \times \text{displacement}$

$d\vec{w} = \vec{F}\cdot d\vec{s} = \vec{F}\cdot\vec{v}\,dt$

$ds = v\,dt$

$dw = F\cdot d\vec{v}\,dt = q(\vec{v}\times\vec{B})\cdot\vec{v}\,dt$

$= q\vec{B}\,\overset{0}{(\vec{v}\times\vec{v})}\,dt$

$= 0$

static magnetic fields can do no work!

Ion Orbital Rotation Frequency- numerically

- Consider an arbitrary positive ion of atomic species (A,Z) with Q orbital electrons removed. The ion cyclotron frequency would be:

$$f = \omega/2\pi = qB/2\pi m = \left(\frac{Q}{A}\right)\frac{e}{2\pi m_0}\frac{B}{\gamma}$$

- Where m_0 is the rest mass of a nucleon (~940 MeV). Evaluating the constants:

$$f = \left(\frac{Q}{A}\right)15.23MHz\frac{B}{\gamma}$$

- Some examples:
  - Low energy proton in 1 T field: 15.23 MHz
  - 250 MeV proton in 8.2T field: 98 MHz
  - 3.2GeV $^{40}Ar^{16+}$ ion in 5.5T field: 30.8 MHz

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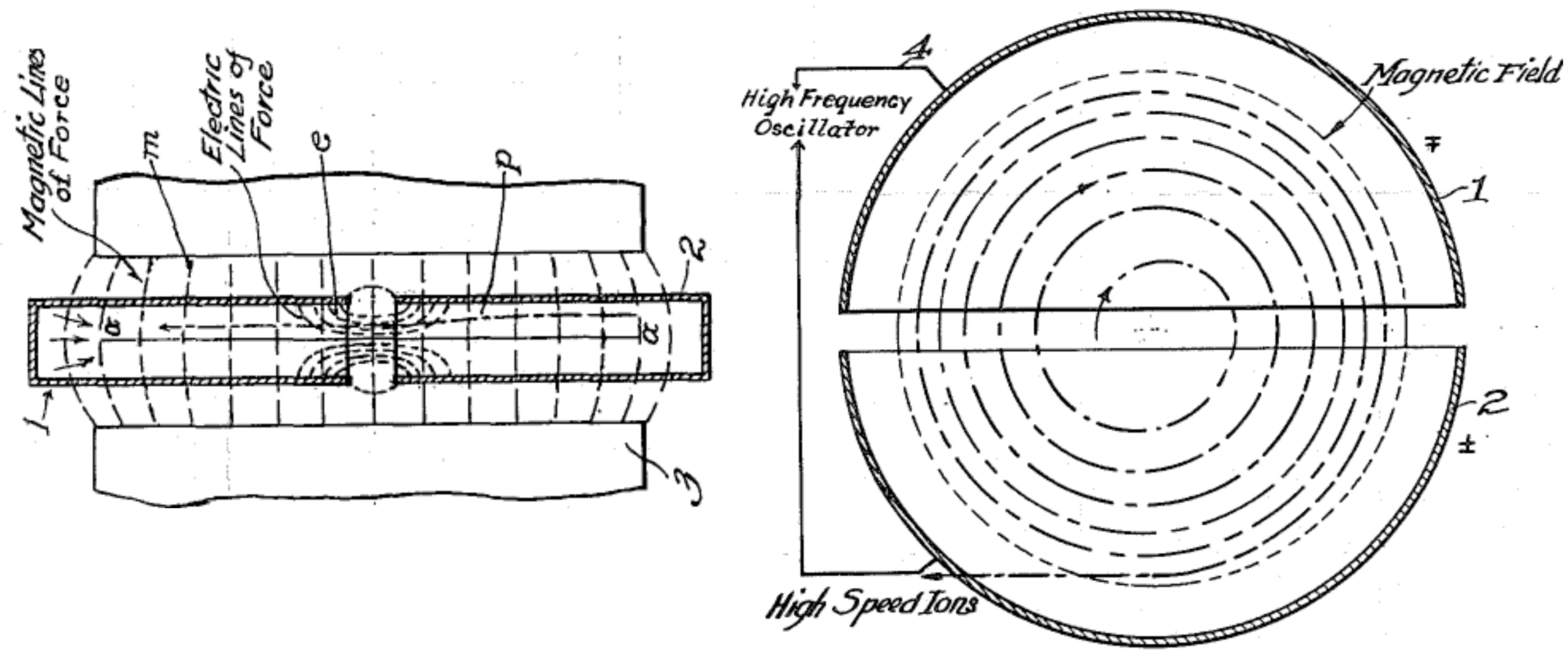
Ion Motion in a cyclotron



- A flat pole electromagnet (3) generates a vertical magnetic field (m)
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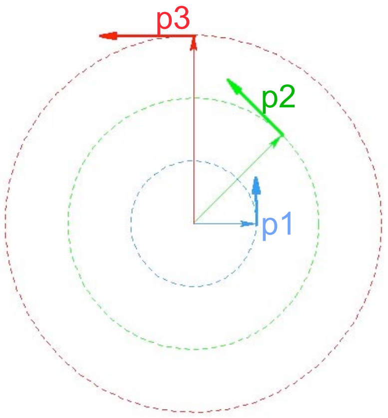
Alternative Expression in Momentum

- Again we equate the two expressions for the same force:

$$mv^2/r = qvB \longrightarrow p = mv = qBr$$

- The momentum at any radius is completely defined by the magnetic field there!



- Also, at the same field B,
    - If p3>p2>p1
    - Then r3>r2>r1

- Since $\omega = d\theta/dt = qB/m$, even though the three orbits are different in size, the ions will make 1 complete revolution at the same angular rate (unless $m = \gamma m_0$ is very different for the three momenta)

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Special Challenges in Cyclotrons

- Orbit Stability
- Initial beam Formation
- RF Acceleration
- Getting the beam out of the machine!
  - $p=erB$ --> $p/e=rB$
  - we call $\mathfrak{R}\equiv rB$ the magnetic rigidity or magnetic stiffness
  - We will see that $\mathfrak{R}$ shows up in the Cyclotron final energy formula- it's in $K_B=e^2r^2B^2/2m_0$-

In cyclotrons, the final energy is essentially set by the radius and B field at the point of beam extraction

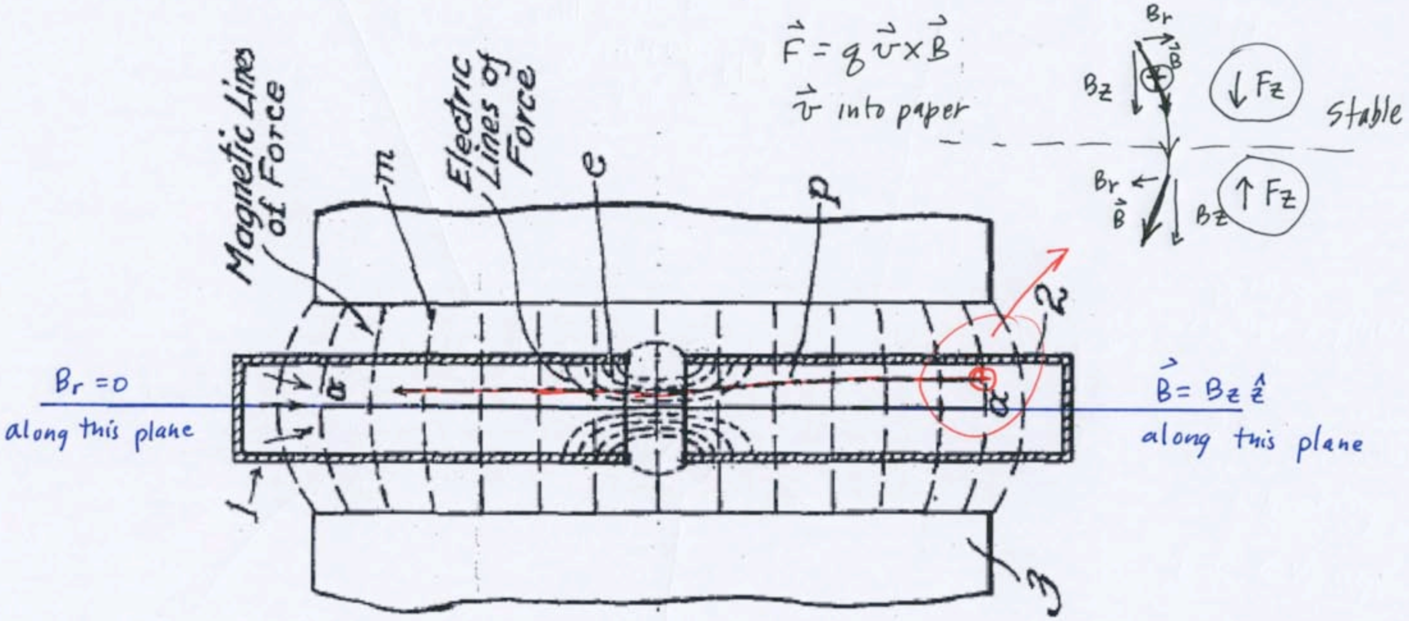
Built In Orbit Stability- Weak Focusing

What happens if an ion is above or below the middle of the magnet during acceleration?

$\boxed{F_z = -q v B_r}$

$\vec{F} = q\vec{v} \times \vec{B}$

$\vec{v}$ into paper



This is called weak focusing - it requires this field curvature

it means than B_z decreases with increasing r

The Field Index and Axial Stability

- An restoring force is required to keep ions axially centered in the gap
- We define the field index as:

$$n = -\frac{r}{B}\frac{dB}{dr}$$

  - One can show that an axial restoring for exists when n>0 (off median plane B_r has right sign)
  - Hence dB/dr<0 is required since B and r enter in ratios
  - This condition can be met with a flat pole H-Magnet

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Field Index n shows up in Equations of Motions

- Small oscillations of ions in r and z about equilibrium orbits:

$$\ddot{x} + (1-n)\omega^2 x = 0$$

$$\ddot{z} + n\omega^2 z = 0$$

- Have solutions :

$$x = x_m \sin(1-n)^{1/2}\omega t$$

$$z = z_m \sin n^{1/2}\omega t$$

- Where ω is the cyclotron frequency
- Betatron Frequencies (Tunes):

$$\nu_r = \omega_r / \omega = \sqrt{1-n}$$

$$\nu_z = \omega_z / \omega = \sqrt{n}$$

- Have real sinusoidal solutions for $0<n<1$; this condition is true in a classical cyclotron
- It's also referred to as a *weak focusing accelerator*

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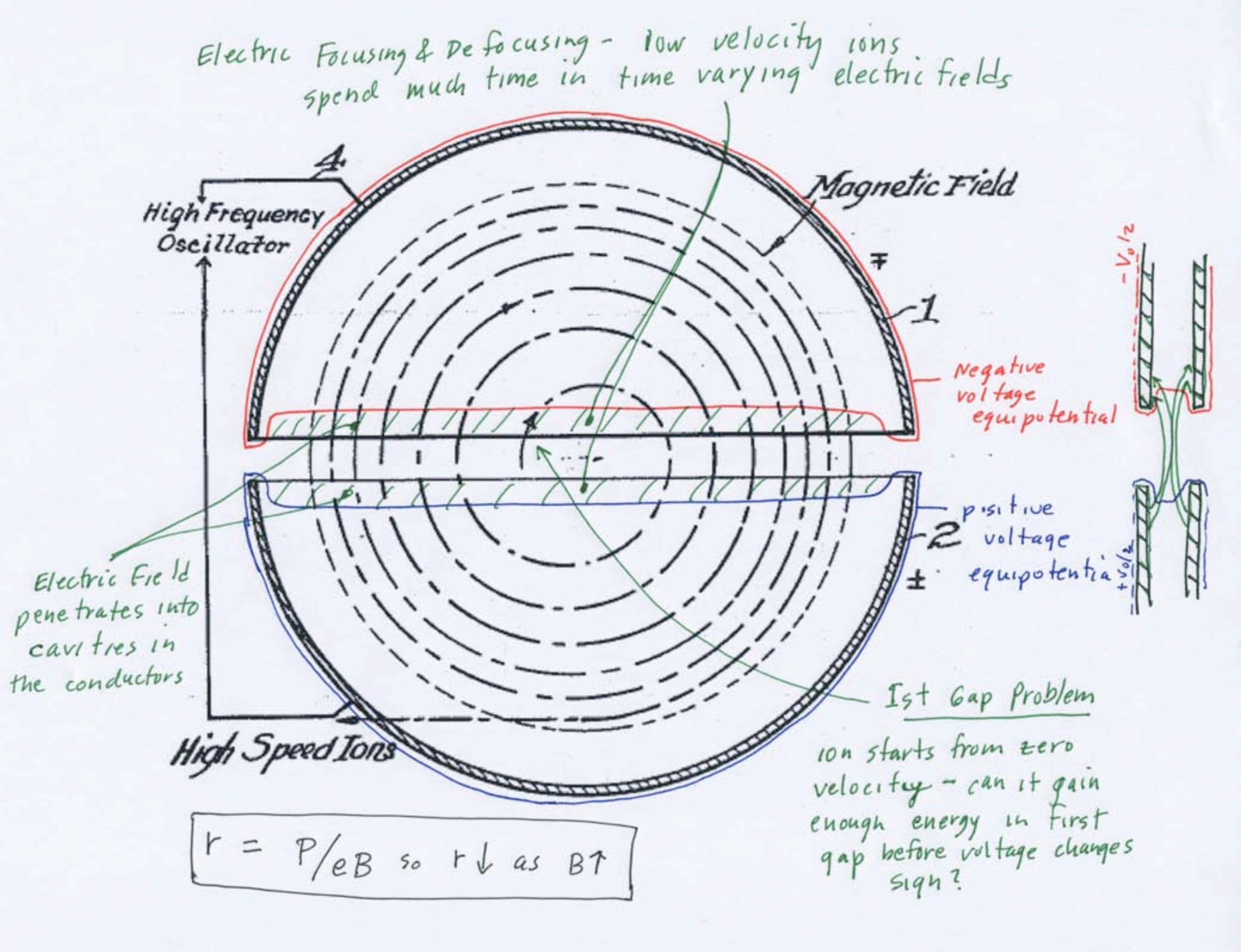
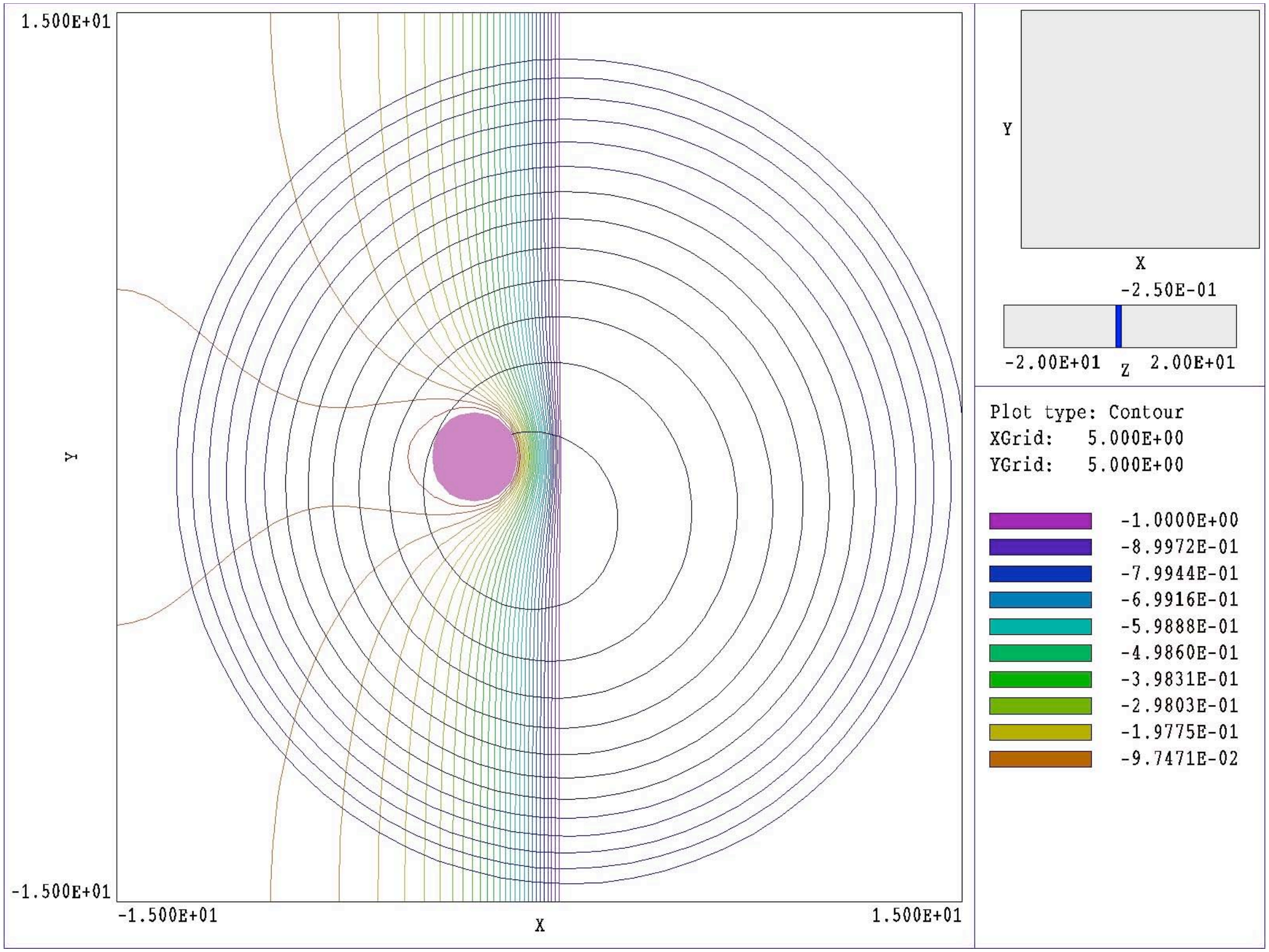
Initial Beam Challenge

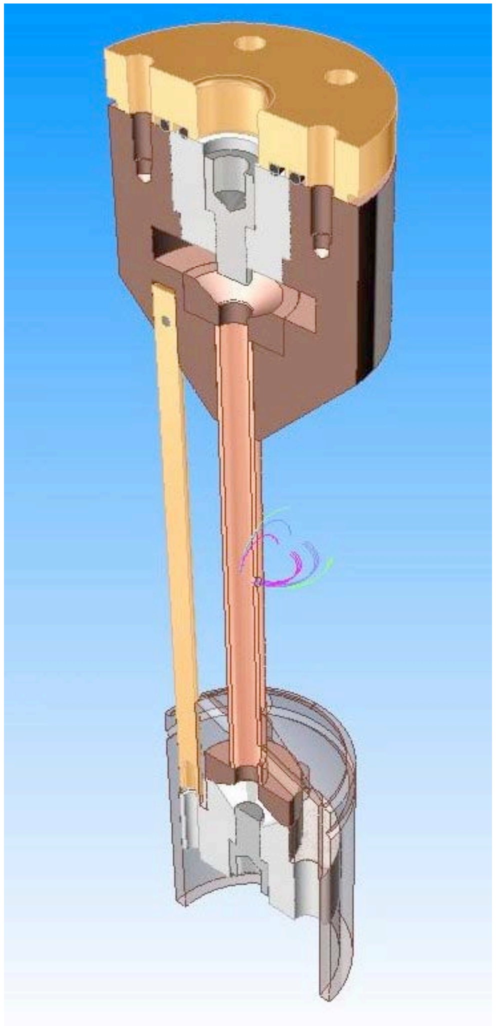


For Example: Initial Proton trajectories at 9T



Positive Ion Source must be compact

- Straight-forward field scaling of original 5.5 T ion source of K500 cyclotron
- Chimney diameter 3 mm
- Test ion source has extra support across median plane
- allows separated cathode geometry of Antaya thesis or Harper cyclotron
- Pulsed cathode lifetime expected to be months



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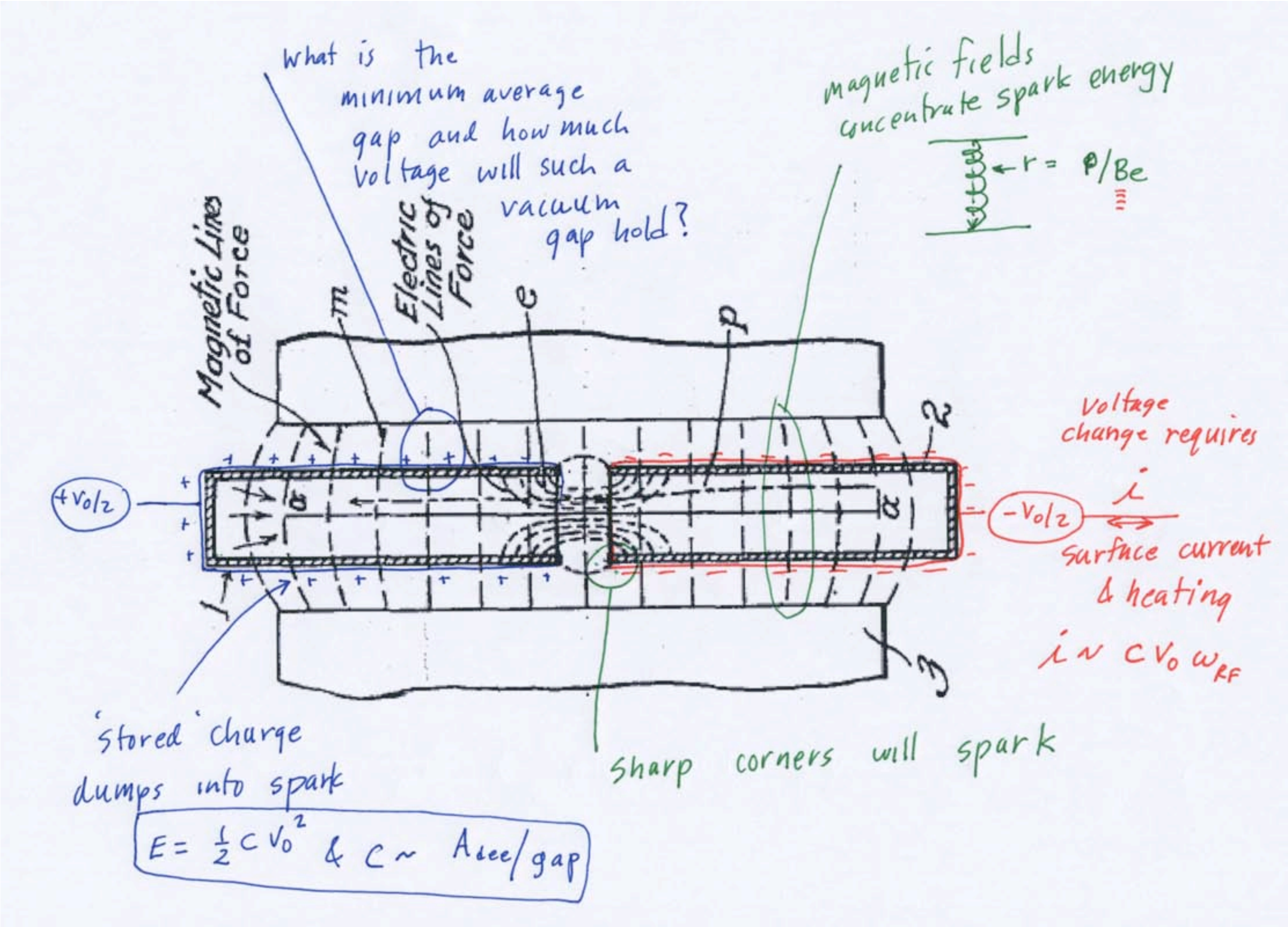
RF Acceleration Challenge



what is the minimum average gap and how much voltage will such a vacuum gap hold?

magnetic fields concentrate spark energy

$r = p/Be$

voltage change requires i surface current & heating

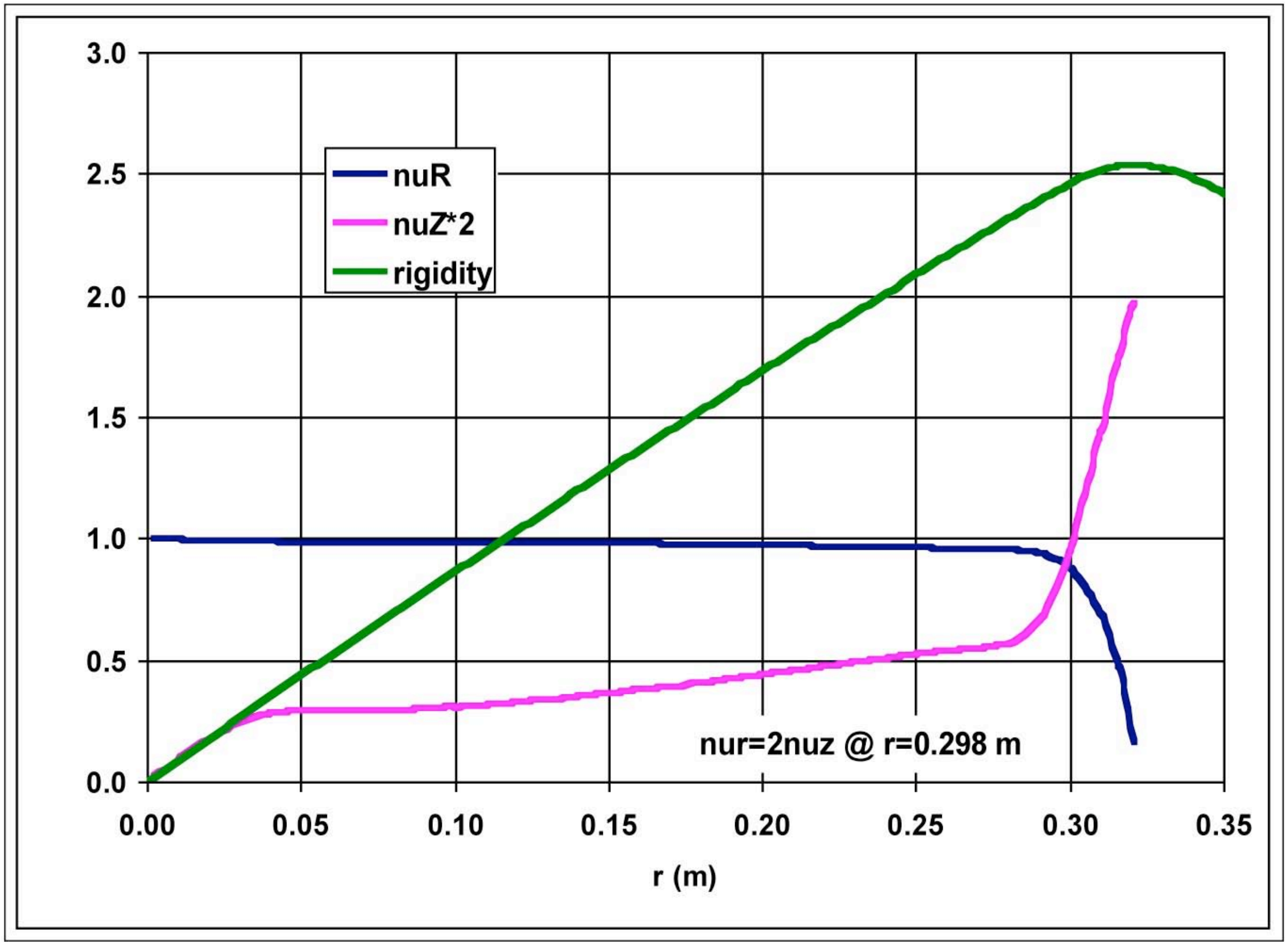
$i \sim C V_0 \omega_{RF}$

'stored' charge dumps into spark

$$\boxed{E = \frac{1}{2} C V_0^2 \ \& \ C \sim A_{dee}/gap}$$

Sharp corners will spark

Beam Extraction Challenge



Orbit Separation impacts Extraction

- Turn Number
  - Let E1 be the energy gain per revolution
  - Then the total number of revolutions required to reach a final kinetic energy T:
    - Let the average ion phase when crossing the acceleration gap phase be ϕ; V_0 is the peak voltage on the dee
    - Energy gain per gap crossing: $T_1=V_0\sin\phi$
    - Gaps per revolution: n
    - Turn number: $N=T/nT_1=T/(nV_0\sin\phi)$
    - 250 MeV protons; 17 KeV/turn: N~15,000
- Turn Spacing:
  - $dr/dN \sim r(T_1/T)$
  - 250 MeV protons r=0.3m: $dr/dN \cong 20$ microns!

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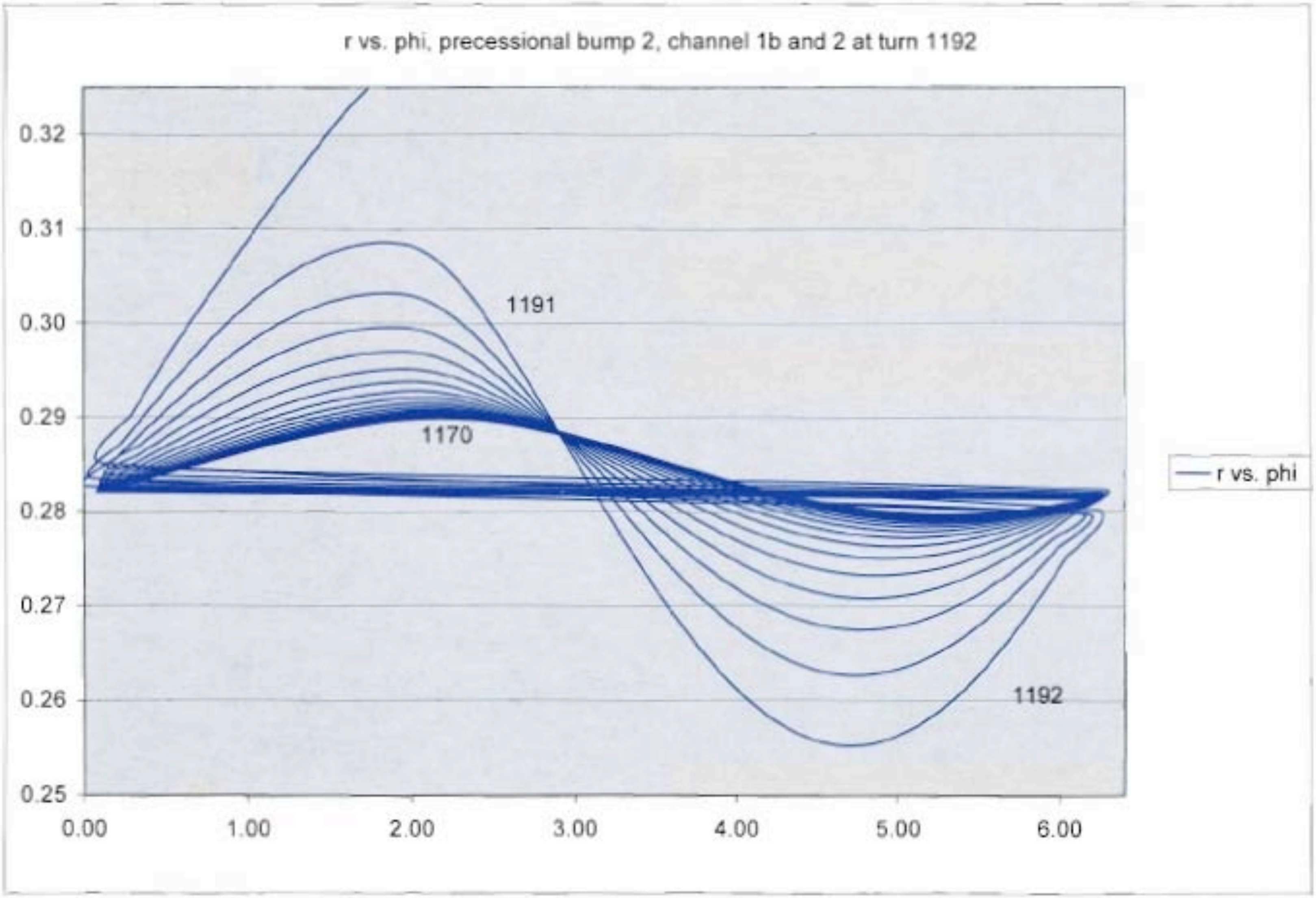
Beam Extraction: 5 micron orbit turn spacing to 1 cm in 20 orbit revolutions induced by field perturbation



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Phase Stable Acceleration aka *Phase Stability*

- 3 General Requirements:
  - required instantaneous acceleration voltage is less than the maximum *available* voltage
  - a change in ion momentum results in a change in ion orbit rotation period
  - rate of change of the frequency is less than a limiting critical value

- Second Condition is the most easily accessible:

$$\frac{d\tau}{\tau} = \left(\frac{1}{\alpha} - \frac{1}{\gamma^2}\right)\frac{dp}{p}$$

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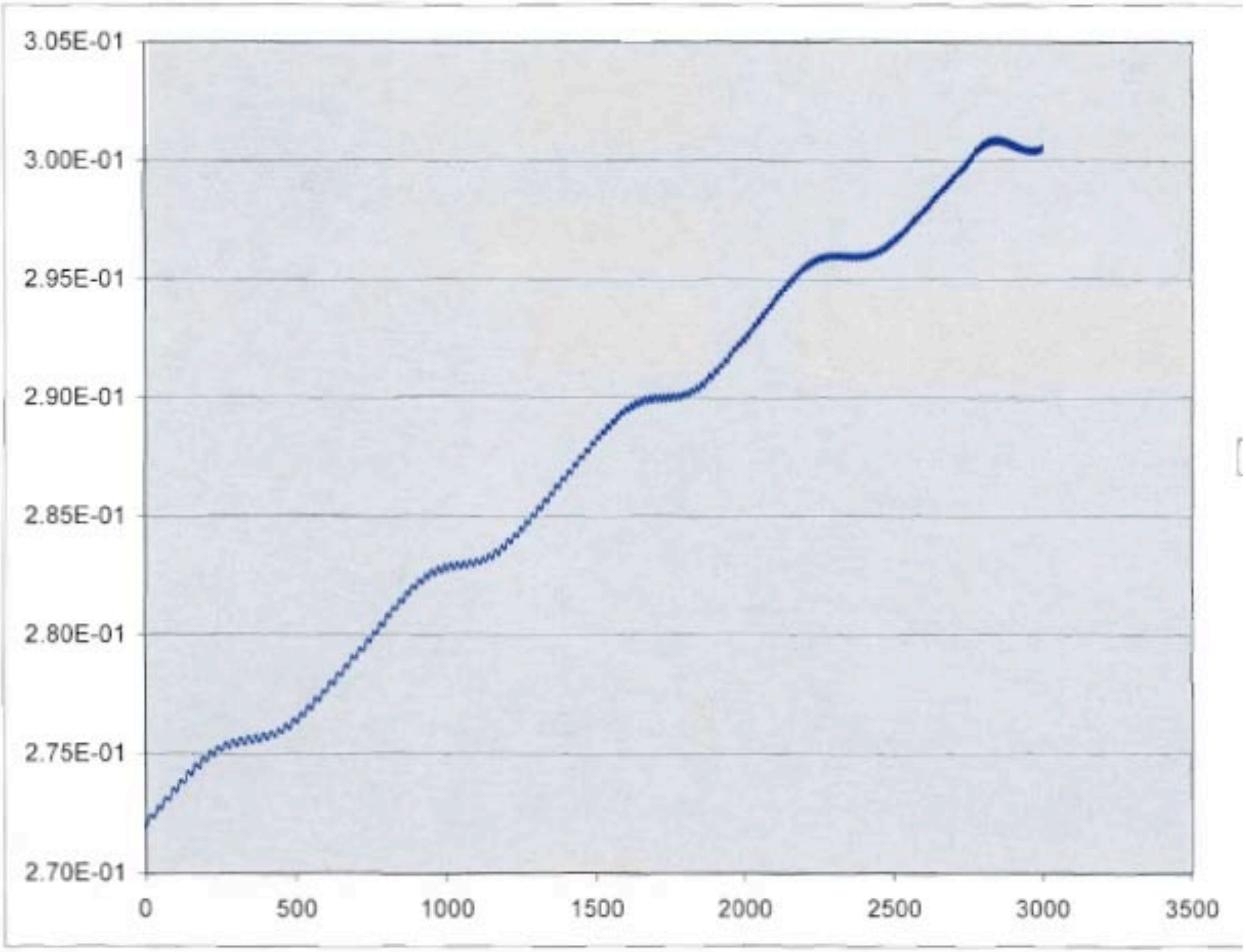
Acceleration in a 9T Guide Field



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Cyclotrons- Final Energy Scaling with Field and Radius

(The origin of Superconducting Cyclotrons
and Synchrocyclotrons)

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Cyclotron Energy Scales inversely with Field

- The final energy can be written as a power series expansion in the relativistic factor γ,
- The first term in this expansion is : $T_{final} \cong K_B Q^2/A$, for an ion of charge Qe and ion mass Am_0
- K_B represents the equivalent proton final energy for the machine, and is related to the ion momentum a.k.a. the particle rigidity $(B\rho)$:

$$K_B = (eB\rho)^2/2m_0$$

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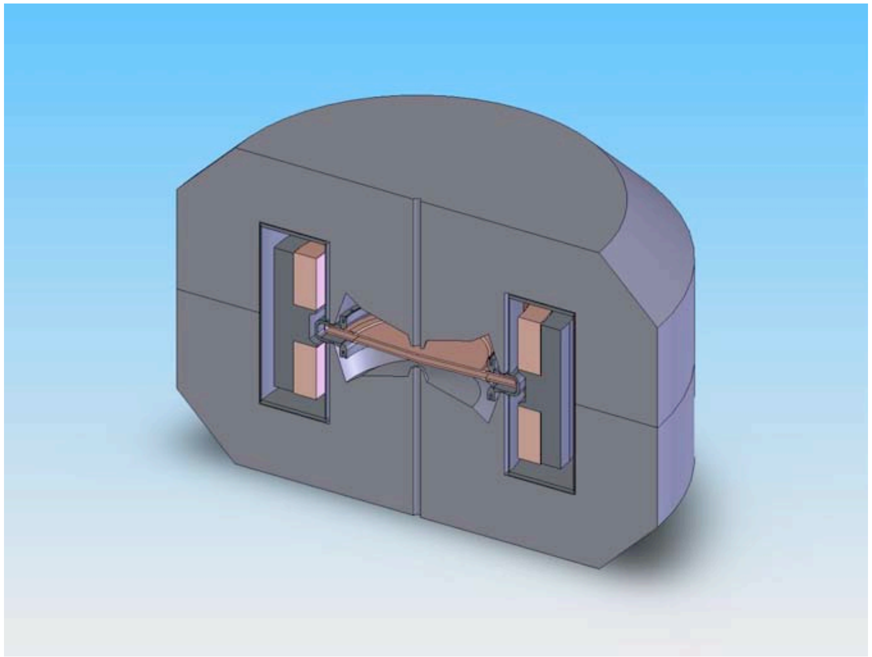
This inverse size scaling is approx. spherical



Almost (but not quite) spherical: Efficient cyclotron magnetic circuits include more iron laterally than axially

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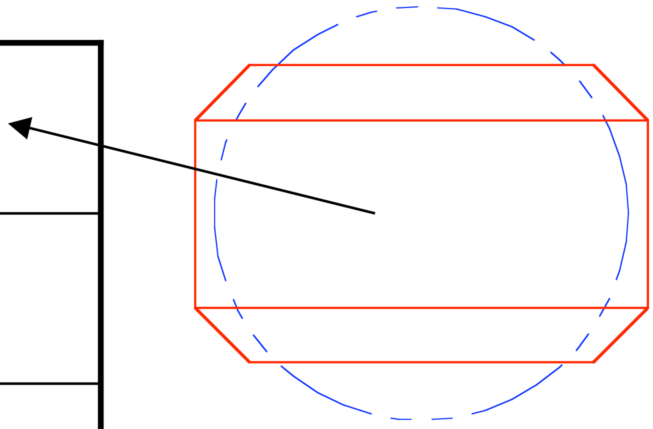
Radius and Field Scaling for Fixed Energy

B (T)	$r_{extraction}$ (m)	$(r_1/r)^3$
1	2.28	1
3	0.76	1/27
5	0.46	1/125
7	0.33	1/343
9	0.25	1/729



Classical Cyclotrons- Energy Limit

- Historically- E<25 MeV, and high acceleration voltages were required
- WHY?
  - Relativistic mass increase lowers the ion orbital frequency: $\omega = qB/\gamma m_0$
  - Ion frequency relative to the fixed RF frequency decreases (rotation time τ increases)
  - Ions arrive increasing late with respect to the RF voltage on the dee
  - Eventually crossing the gaps at wrong phase and decelerates
- 21 MeV proton : $\gamma_{final}=1.022$ seems small, but…
  - Angular rotation slip near full energy
    - $d\phi/dn = 360° \Delta\omega/\omega = 360° [mB_0/m_0B - 1] \approx 360[\gamma - 1] \dashrightarrow 8°$
  - An ion on peak phase is lost in 11 revolutions
  - Only solution- very high energy gain per turn - 360kV was required to reach 21 MeV in the LBL 60" Cyclotron!

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