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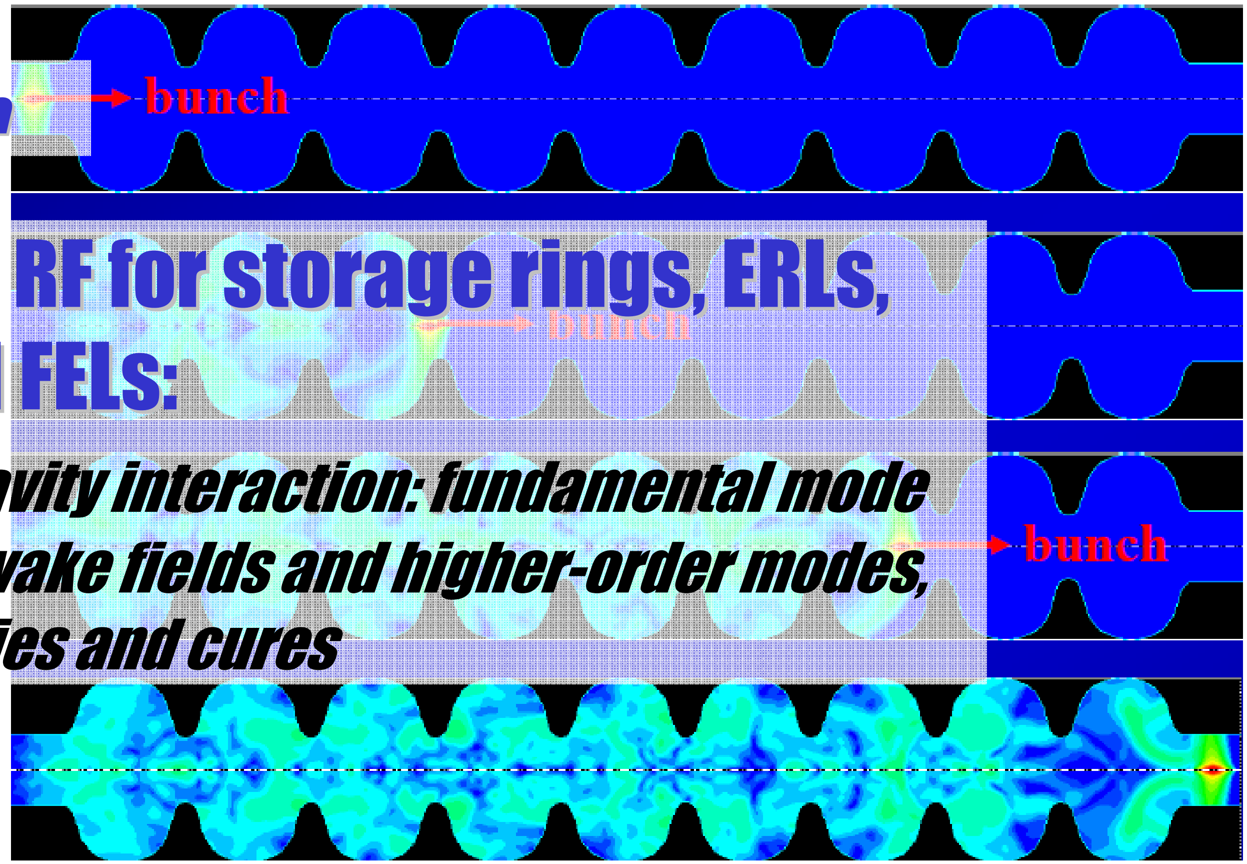
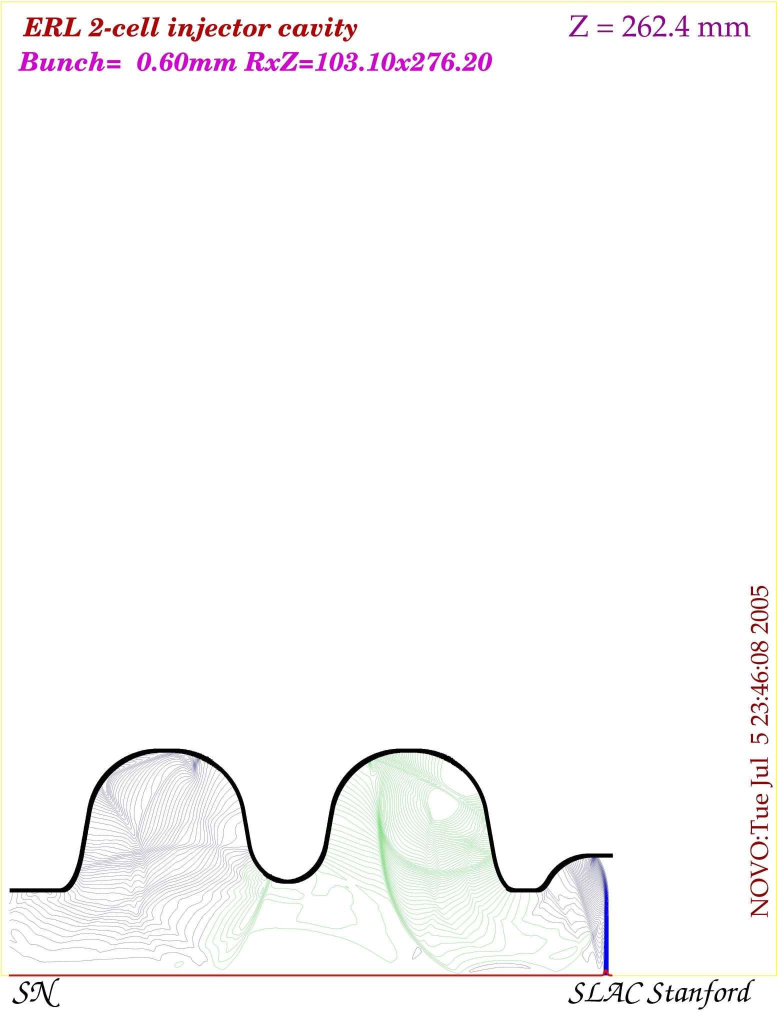
Superconducting RF for storage rings, ERLs, and linac-based FELs:

- **Lecture 6** ***Beam-cavity interaction: fundamental mode beam loading, wake fields and higher-order modes, beam instabilities and cures***



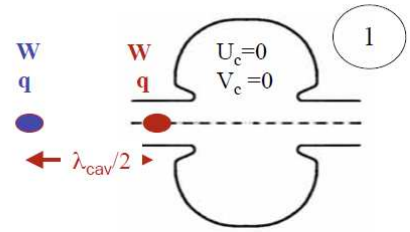
Beam-cavity interaction

- As bunch traverses a cavity, it deposits electromagnetic energy, which is described in terms of wakefields.
- The wakefields, in turn, can be presented as a sum of cavity eigenmodes.
- If a charge passes a cavity exactly on axis, it excites only monopole HOMs. For a point charge the HOM excitation depends only on the amount of charge and the cavity shape.
- Subsequent bunches may be affected by these fields and at high beam currents one must consider beam instabilities and additional heating of accelerator components.



Fundamental theorem of beam loading

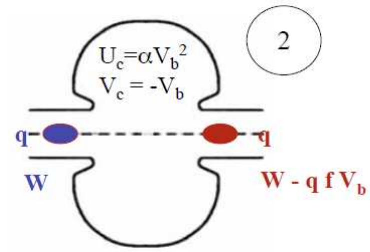
- This theorem relates the energy loss by a charge passing through a structure to the electromagnetic properties of modes of that structure.
- A point charge crosses a cavity initially empty of energy.
- After the charge leaves the cavity, a beam-induced voltage $V_{b,n}$ remains in each mode.
- By energy conservation the particle must have lost energy equal to the work done by the induced voltage on the charge.
- What fraction (f) of $V_{b,n}$ does the charge itself see?



For simplicity:

Assume that the change in energy of the particles does not appreciably change their velocity

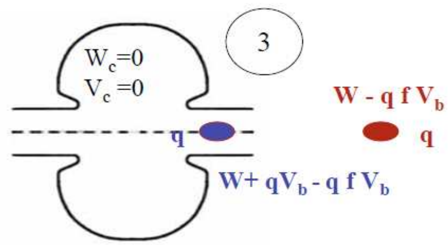
Half an rf period later, the voltage has changed in phase by π



Notice:

$\alpha V_b^2 = q\ f\ V_b \implies V_b = q\ f / \alpha$

V_b is proportional to q



By energy conservation:

$$W + qV_b - q\ fV_b + W - q\ fV_b = W + W$$

$$\implies f = 1/2$$

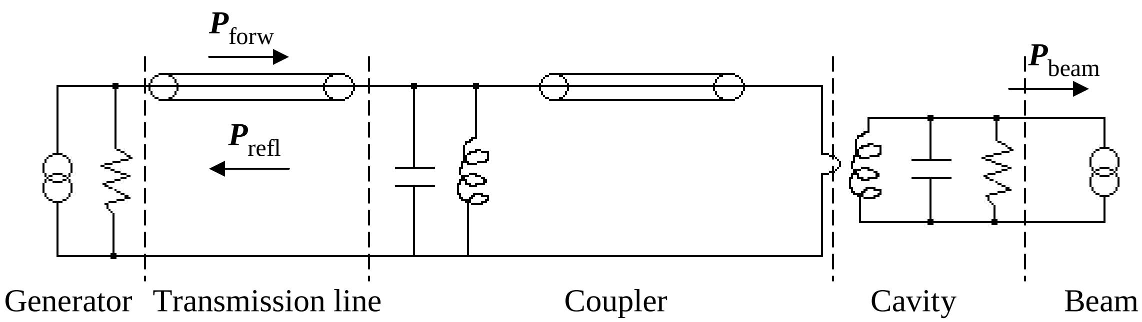
Note that **the second charge** has gained energy

$$\Delta W = 1/2\ qV_b$$

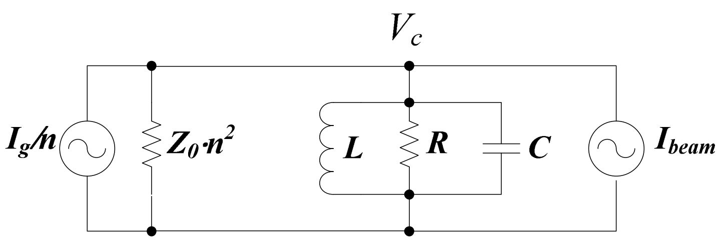
from longitudinal wake field of **the first charge**

Circuit model of the fundamental mode with beam

- At the fundamental mode frequency there are high fields induced by an RF power source therefore interaction with the fundamental mode is considered separately from HOMs.
- When considering beam interaction with the fundamental mode, it is convenient to use an equivalent circuit model:



which can be simplified to

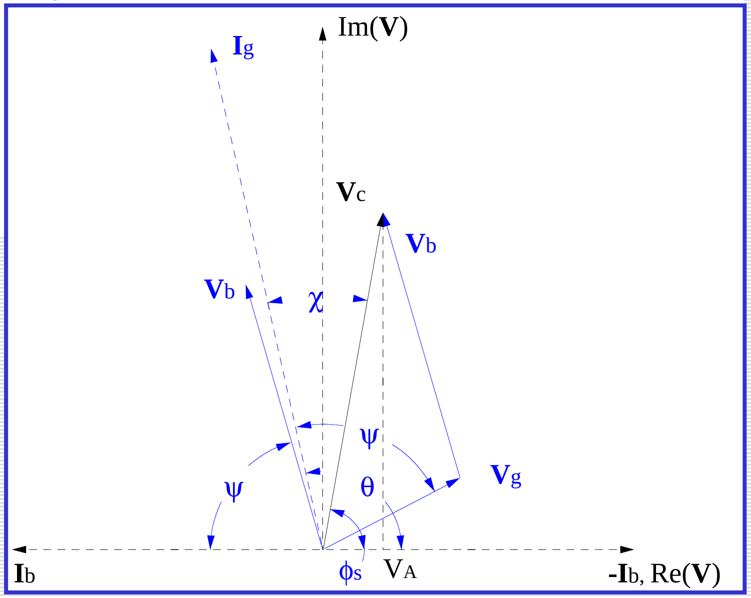


Phasor diagrams

- To obtain the total cavity voltage we need to add the generator-induced voltage and beam-induced voltage (this follows from the principle of linear superposition consequence of the linearity of Maxwell equations.)
- For the case of sinusoidally varying voltages (and currents), one must add them taking into account the relative phases. It is convenient to describe the voltages as vectors in the complex plane as

$$\mathbf{V} = Ve^{i(\omega t+\varphi)}$$

- This vector rotates counterclockwise in the complex plane and is called *phasor*.
- It is convenient to choose a frame that is rotating with a frequency ω, so that the phasors remain fixed in time.
- Then the component of any voltage that contributes to acceleration of the bunch is the projection of the voltage onto the real axis.



Beam loading of the fundamental mode

- From the equivalent circuit diagram and introducing cavity tuning angle ψ

$$\tan\psi = 2Q_\mathrm{L}\frac{\Delta\omega}{\omega}$$

and beam phase relative to RF wave crest φ_0, one can derive for the forward power (see RF_power_with_beam_loading.pdf)

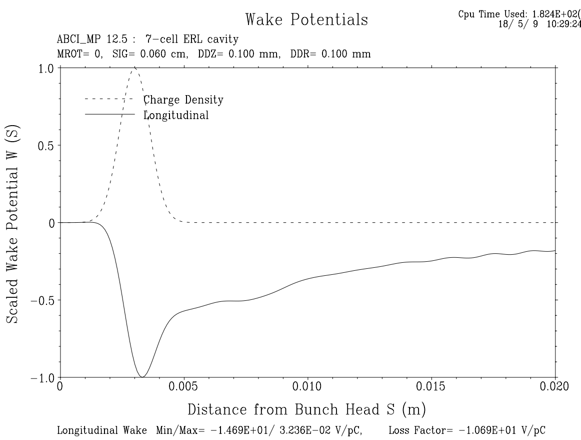
$$P_\mathrm{forw} = \frac{V_\mathrm{c}^2}{4\,R/Q\cdot Q_\mathrm{ext}}\cdot\frac{(\beta+1)^2}{\beta^2}\cdot\left\{\left[1+\frac{I_\mathrm{b}\,R/Q\cdot Q_\mathrm{L}}{V_\mathrm{c}}\cos\varphi_0\right]^2+\left[\tan\psi+\frac{I_\mathrm{b}\,R/Q\cdot Q_\mathrm{L}}{V_\mathrm{c}}\sin\varphi_0\right]^2\right\}$$

- The two terms correspond to active and reactive parts of the beam loading.
- In storage rings, where the beam is passing cavity off crest, the reactive beam loading is compensated by appropriate cavity detuning. The coupling β is chosen to achieve matching conditions at a maximum beam current.
- In ERLs, with two beams passing the cavity 180° apart, the beam loading is zero for perfect energy recovery and the cavity is tuned to resonance. Then RF power is determined by residual beam current phase and amplitude errors and by the cavity resonant frequency fluctuations due to environmental noise (microphonics).

Beam-cavity interaction: time domain

The details of the wakefields themselves are usually of a lesser interest that the integrated effect of a driving charge on a traveling behind it test particle as both particles pass through a structure (the cavity, for example). The integrated field seen by a test particle traveling on the same path at a constant distance s behind a point charge q is the longitudinal wake (Green) function $w(s)$. Then the wake potential is a convolution of the linear bunch charge density distribution $\lambda(s)$ and the wake function:

$$W(s) = \int_{-\infty}^{s} w(s - s')\lambda(s')ds'$$



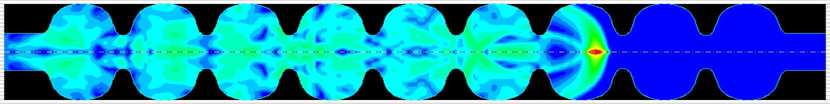
Once the longitudinal wake potential is known, the total energy loss is given by

$$\Delta U = \int_{-\infty}^{\infty} W(s)\lambda(s)ds$$

Now we can define a figure of merit, the loss factor, which tells us how much electromagnetic energy a bunch leaves behind in a structure:

$$k = \frac{\Delta U}{q^2}$$

The more energy it looses, the more is the likelihood of adverse effects on the subsequent bunches.



Loss factor of cavity modes

In the frequency domain, the loss factor can be represented as a sum of individual loss factors of cavity modes

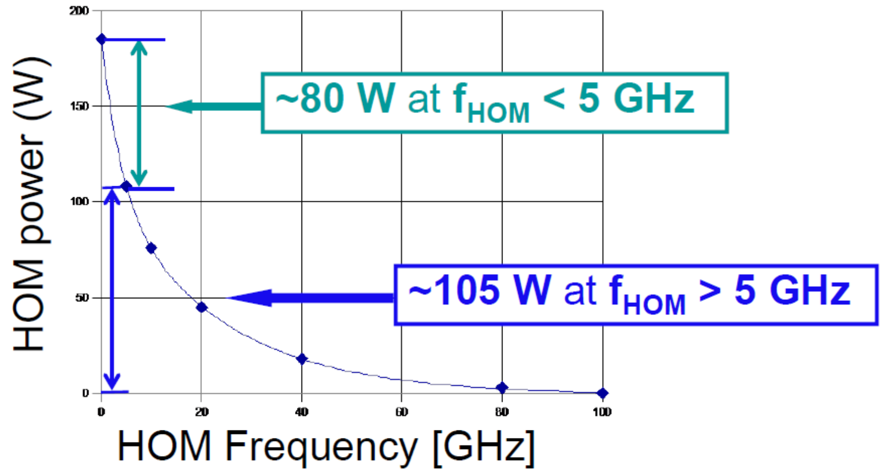
$$k = \sum_n k_n = \sum_n \frac{\omega_n}{2}\left(\frac{R}{Q}\right)_n$$

here R/Q is in circuit definition.

The loss factor can be used to calculate beam losses due to HOMs over the whole bunch spectrum. This approximation works usually quite well.

$$P_{HOM} = k_{HOM} \cdot q \cdot I_{av}$$

here q is the bunch charge, I_{av} is the average beam current.

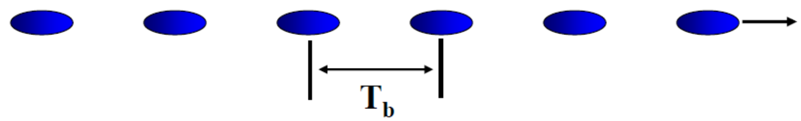


Example:

- 100 mA ERL beam
- 0.7 mm (rms) long 77 pC bunches
- 9-cell cavities with loss factor of 12 V/pC

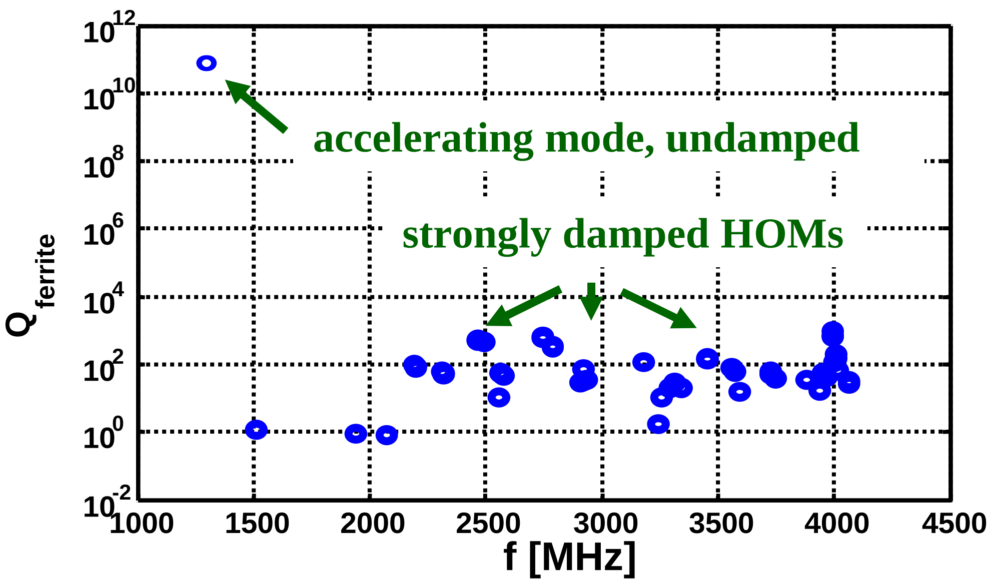
The HOM power loss is 185 W over a frequency range up to 100 GHz.

Beam-cavity interaction: frequency domain



- If the wakefields (HOMs) do not decay sufficiently between the bunches, then fields from subsequent bunches can interfere constructively (resonant effect, if $f_{HOM} \approx N/T_b$) and cause excessive HOM power loss and various instabilities.
- That is why practically all SRF cavities have special devices to damp HOMs (absorb their energy). For analysis of instabilities, it is more convenient to use frequency domain rather than time domain approach.

$$P_{HOM}^{res} = (R/Q)_{HOM} \, Q_{L,HOM} \, I_{beam}^2$$



Beam instabilities

Detrimental effects caused by beam-cavity interaction include:

- multi-bunch instabilities (longitudinal and transverse) in storage rings
- beam loading of the fundamental mode
- multipass beam break-up (BBU) instabilities (transverse and longiyudinal) in re-circulating linacs
- single-pass BBU in linacs
- resonant excitation of longitudinal HOMs
- increased beam energy spread
- additional cryogenic losses

In the following we will consider the two important examples of beam instabilities.

Multi-bunch instability in a storage ring

Let us consider a single-bunch beam interacting with a narrow-band resonance. The revolution time of a particle bunch depends on the average energy of particles within a bunch and the Fourier spectrum of the bunch current being made up of harmonics of the revolution frequency is therefore energy dependent. On the other hand, by virtue of the frequency dependence of the cavity impedance, the energy loss of a bunch in the cavity depends on the revolution frequency. We have therefore an energy dependent loss mechanism which can led to damping or growth of coherent longitudinal oscillations. This effect is generally referred to as Robinson instability. In case of M bunches one can generalize this to get M coupled-bunch modes with the phase shift between adjacent bunches for the mode number n

$$\Delta\phi_n = \frac{2\pi}{M}n, \quad n = 0, 1, \ldots, M-1$$

The exact location of the HOM resonant frequency ω_r relative to the nearest harmonic of revolution frequency $p\omega_0$ is of critical importance for the stability of the beam as one can see from the equation for the growth rate and the figure on the next slide

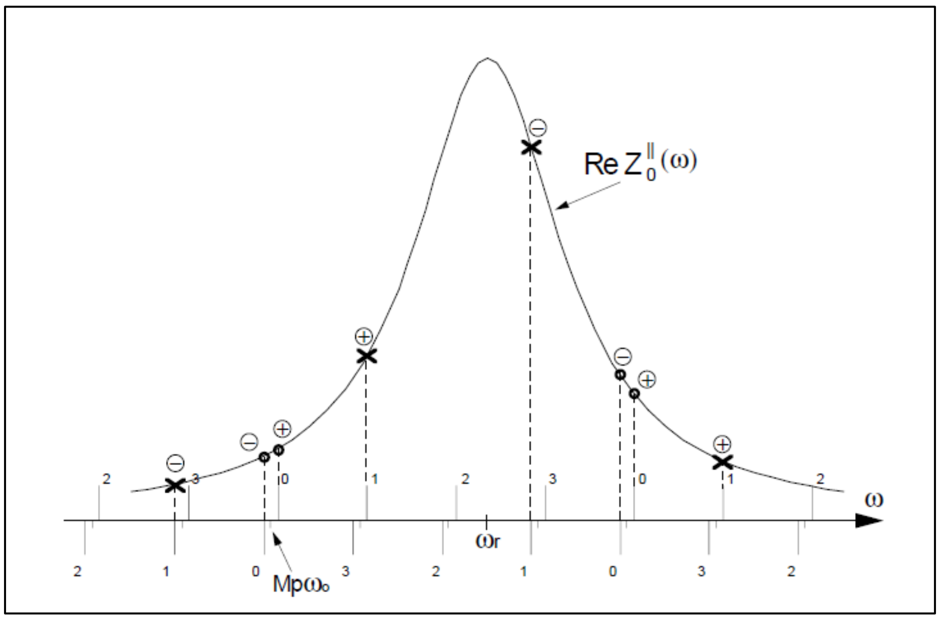
$$\begin{aligned}
\tau_n^{-1} &= \omega_s \frac{I_0}{2hV_c\cos(\phi_s)\omega_0} \sum_{p=-\infty}^{\infty} (pM\omega_0 + n\omega_0 + \omega_s) ReZ_0^{\|}(pM\omega_0 + n\omega_0 + \omega_s) \\
&= \omega_s \frac{I_0}{2hV_c\cos(\phi_s)\omega_0} \times \\
&\quad \times \sum_{p=0}^{\infty} \Big[(pM\omega_0 + n\omega_0 + \omega_s) ReZ_0^{\|}(pM\omega_0 + n\omega_0 + \omega_s) - (pM\omega_0 - n\omega_0 - \omega_s) ReZ_0^{\|}(pM\omega_0 - n\omega_0 - \omega_s) \Big] \\
&\approx \omega_s \frac{I_0}{2hV_c\cos(\phi_s)} \sum_{p=0}^{\infty} \Big[(pM + n) ReZ_0^{\|}(pM\omega_0 + n\omega_0 + \omega_s) - (pM - n) ReZ_0^{\|}(pM\omega_0 - n\omega_0 - \omega_s) \Big]
\end{aligned}$$

Multi-bunch instability in a storage ring (2)

here ω_s is the synchrotron oscillation frequency,

$$\omega_s = \omega_0 \sqrt{\frac{\eta \cdot h \cdot V_c \cos(\varphi_s)}{2\pi \cdot E_0}}$$

h is the RF harmonic number, η is the slippage factor, I_0 is the total beam current, V_c is the total cavity voltage (sum over all cavities), φ_s is the synchronous phase, Z_0 is the cavity impedance (sum over all cavities), E_0 is the beam energy.



Multi-bunch instability in a storage ring (3)

Assuming the worst case, when the HOM resonant frequency coincides with the "bad" sideband, so that the growth rate is determined just by one term in the equation, one can derive the following formula for the instability threshold current (τ_d is the "natural" damping time of oscillations, N_{cav} is the number of cavities)

High E_{acc} reduces number of cells

$$I_{th} = \frac{1}{\tau_d} \frac{2V_c \cos(\varphi_s) \cdot \omega_{rf}}{\omega_s \cdot \omega_r \cdot (R/Q)_{HOM} \cdot Q_{L,HOM} \cdot N_{cav}} \propto \frac{E_{acc}}{(R/Q)_{HOM} \cdot Q_{L,HOM} \cdot \omega \cdot \omega^{1/2}}$$

Choose a geometry that has low *R/Q* for HOMs

Design couplers that extract HOMs efficiently, use single-cell cavities

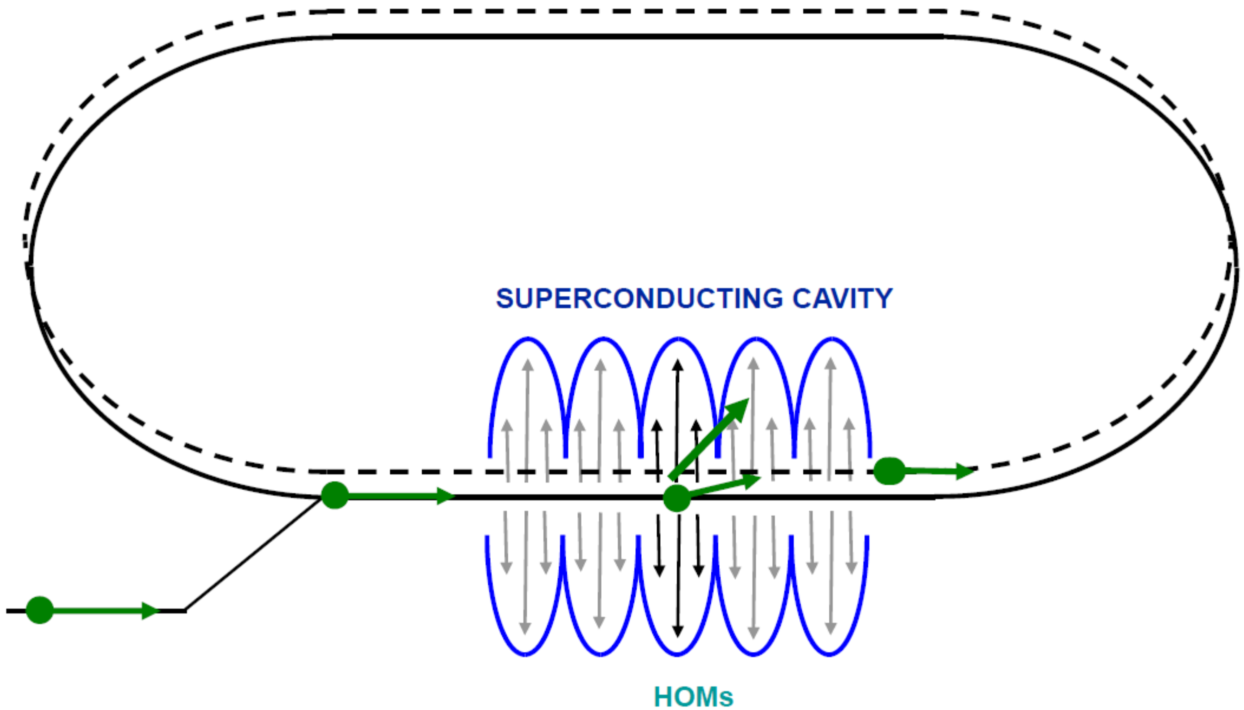
Low frequency reduces synchrotron frequency

Low frequency reduces number of cells

As we see from this formula, the beam instability threshold current is inversely proportional to the impedance of HOMs and frequency.

BBU in re-circulating linacs

If a particle enters a cavity on the axis when a dipole HOM has been excited, then the particle will leave with a deflection in the horizontal or vertical direction. The optics of the recirculation line will cause the transverse momentum imparted to the particle by the HOM to result in the particle entering the cavity with a transverse displacement when it returns back. The transverse offset can cause the particle to further excite the HOM and this process can continue until the particle collides with the cavity wall.



BBU in re-circulating linacs (2)

The threshold current at which a multipass BBU occurs is predicted by the approximate expression

$$I_{th}^{l} = \frac{-2pc}{e\cdot(R/Q)_m Q_{L,m} k_m M_{ij} \sin(\omega_m t_r + l\pi/2) e^{\omega_m t_r / 2Q_m}} \propto \frac{-2pc}{e(R/Q)_m Q_{L,m} k_m M_{12}}$$

for transverse BBU

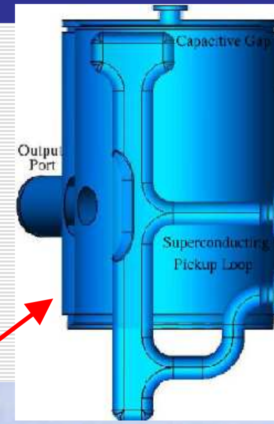
where for $i,j = 1,2$ or $3,4$ and if the mode m is the transverse HOM, this formula is for the transverse BBU;
for $i,j = 5,6$ and if the mode m is the monopole HOM, this formula is for the longitudinal BBU;
if the mode m is fundamental mode, it is for the beam-loading instability;
$l = 1$ for longitudinal HOMs and 0 otherwise;

p is the momentum of the particle, c is the speed of light, e is the charge of the electron, R/Q is the shunt impedance of the mode m, Q is the quality factor of the mode, $k = \omega/c$ is the wave number of the mode, and M_{12} is the transfer matrix element relating the transverse momentum at the cavity exit to the transverse displacement of the particle at the entrance of the same cavity during the next pass. The HOM of concern is the one which corresponds to the lowest threshold current.

One can see that similarly to the storage sing case, the threshold current is inversely proportional to the impedance of HOMs and frequency.

The HOM impedance must be controlled to achieve high beam currents!

HOM extraction/damping

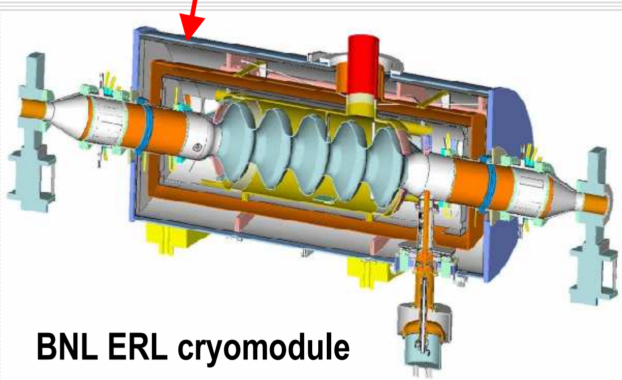


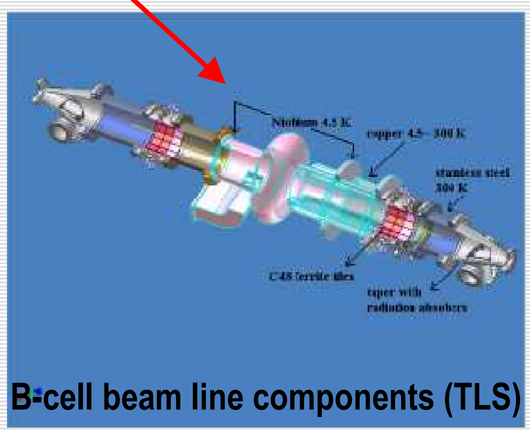
<u>Several approaches are used</u>:

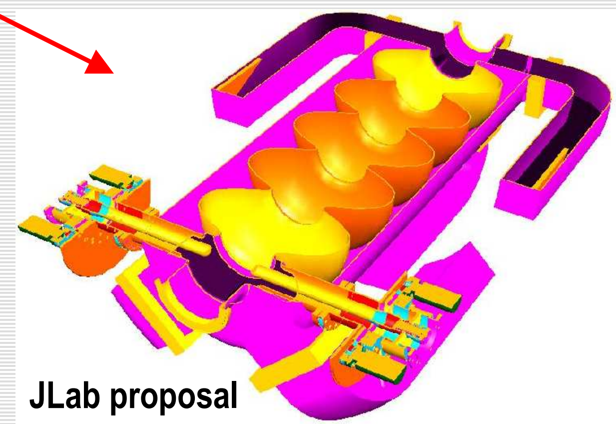
- Loop couplers (several per cavity for different modes/orientations)
- Waveguide dampers
- Beam pipe absorbers (ferrite or ceramic)

Will consider HOM damping designs later.


TESLA cavity loop coupler


BNL ERL cryomodule


B-cell beam line components (TLS)


JLab proposal

What have we learned?

- **Bunched beams interact strongly with accelerating structures.**
- **Beam loading of the fundamental mode must be taken into account when designing RF controls.**
- **High impedance of HOMs can lead to beam instabilities and must be damped to achive high beam currents.**

✪ **In the next lecture we will consider SRF system design and optimization.**