TM-CLASS CAVITY DESIGN

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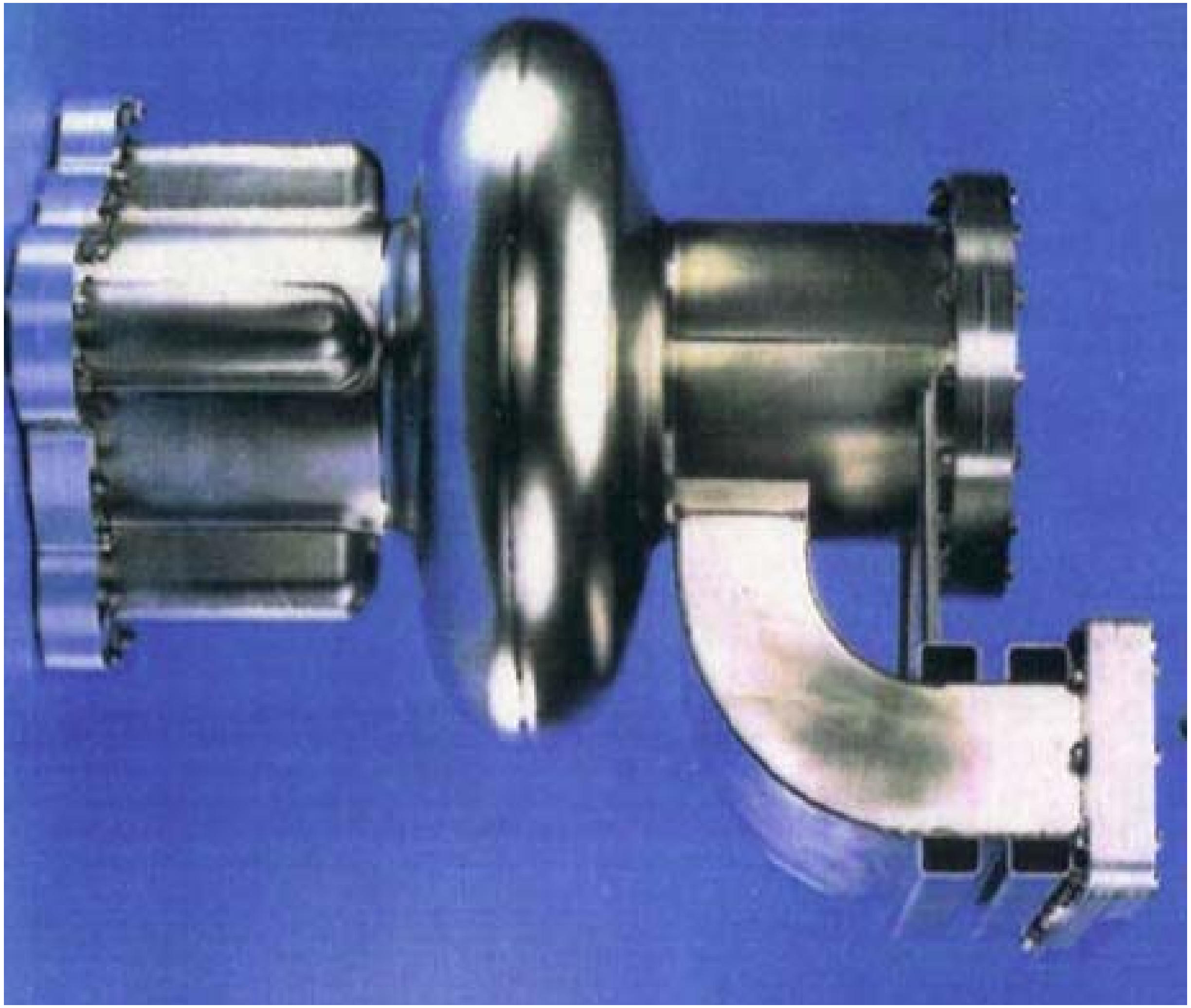
500 MHz, Single-cell



350 MHz, 4-cell, Nb on Cu

1500 MHz, 5-cell

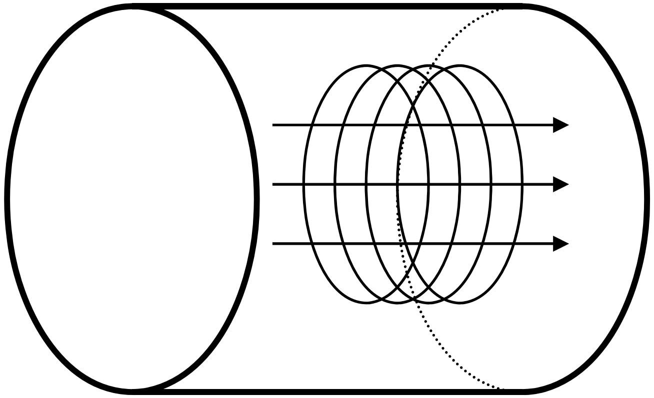
1300 MHz 9-cell

Pill Box Cavity

Hollow right cylindrical enclosure

Operated in the TM_{010} mode $\quad H_z = 0$

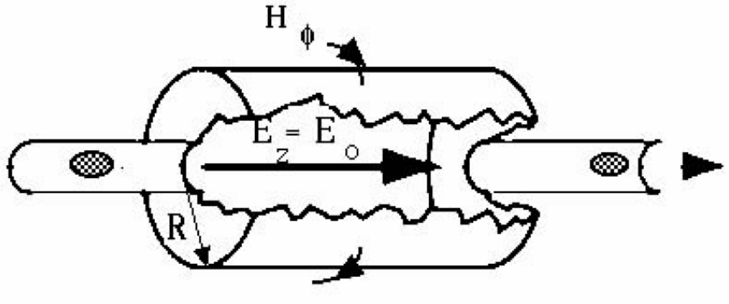
TM$_{010}$ mode



$$\frac{\partial^2 E_z}{\partial^2 r} + \frac{1}{r}\frac{\partial E_z}{\partial r} = \frac{1}{c^2}\frac{\partial^2 E_z}{\partial^2 t} \qquad \omega_0 = \frac{2.405c}{R}$$

$$E_z(r,z,t) = E_0 J_0\left(2.405\frac{r}{R}\right)e^{-i\omega_0 t}$$

$$H_\varphi(r,z,t) = -i\frac{E_0}{\mu_0 c}J_1\left(2.405\frac{r}{R}\right)e^{-i\omega_0 t}$$



Modes in Pill Box Cavity

- **TM_{010}**
  - **Electric field is purely longitudinal**
  - **Electric and magnetic fields have no angular dependence**
  - **Frequency depends only on radius, independent on length**
- **TM_{0mn}**
  - **Monopoles modes that can couple to the beam and exchange energy**
- **TM_{1mn}**
  - **Dipole modes that can deflect the beam**
- **TE modes**
  - **No longitudinal E field**
  - **Cannot couple to the beam**

TM Modes in a Pill Box Cavity

$$\frac{E_r}{E_0} = -\frac{n\pi}{x_{lm}}\frac{R}{L} J_l'\left(x_{lm}\frac{r}{R}\right)\sin\left(n\pi\frac{z}{L}\right)\cos l\varphi$$

$$\frac{E_\varphi}{E_0} = \frac{ln\pi}{x_{lm}^2}\frac{R^2}{rL} J_l\left(x_{lm}\frac{r}{R}\right)\sin\left(n\pi\frac{z}{L}\right)\sin l\varphi$$

$$\frac{E_z}{E_0} = J_l\left(x_{lm}\frac{r}{R}\right)\sin\left(n\pi\frac{z}{L}\right)\cos l\varphi$$

$$\omega_{lmn} = c\sqrt{\left(\frac{x_{lm}}{R}\right)^2 + \left(\frac{\pi n}{L}\right)^2}$$

$$\frac{H_r}{E_0} = -i\omega\varepsilon\frac{l}{x_{lm}^2}\frac{R^2}{r} J_l\left(x_{lm}\frac{r}{R}\right)\cos\left(n\pi\frac{z}{L}\right)\sin l\varphi$$

x_{lm} is the mth root of $J_l(x)$

$$\frac{H_\varphi}{E_0} = -i\omega\varepsilon\frac{R}{x_{lm}} J_l'\left(x_{lm}\frac{r}{R}\right)\cos\left(n\pi\frac{z}{L}\right)\cos l\varphi$$

$$\frac{H_z}{E_0} = 0$$

TM$_{010}$ Mode in a Pill Box Cavity

$$E_r = E_\varphi = 0 \qquad E_z = E_0 J_0\left(x_{01}\frac{r}{R}\right)$$

$$H_r = H_z = 0 \qquad H_\varphi = -i\omega\varepsilon E_0 \frac{R}{x_{01}} J_1\left(x_{01}\frac{r}{R}\right)$$

$$\omega = x_{01}\frac{c}{R} \qquad x_{01} = 2.405$$

$$R = \frac{x_{01}}{2\pi}\lambda = 0.383\lambda$$

TM$_{010}$ Mode in a Pill Box Cavity

Energy content

$$U = \varepsilon_0 E_0^2 \frac{\pi}{2} J_1^2(x_{01}) L R^2$$

Power dissipation

$$P = E_0^2 \frac{R_s}{\eta^2} \pi J_1^2(x_{01})(R+L)R$$

$$x_{01} = 2.40483$$

$$J_1(x_{01}) = 0.51915$$

Geometrical factor

$$G = \eta \frac{x_{01}}{2} \frac{L}{(R+L)}$$

TM010 Mode in a Pill Box Cavity

Energy Gain

$$\Delta W = E_0 \frac{\lambda}{\pi} \sin \frac{\pi L}{\lambda}$$

Gradient

$$E_{acc} = \frac{\Delta W}{\lambda / 2} = E_0 \frac{2}{\pi} \sin \frac{\pi L}{\lambda}$$

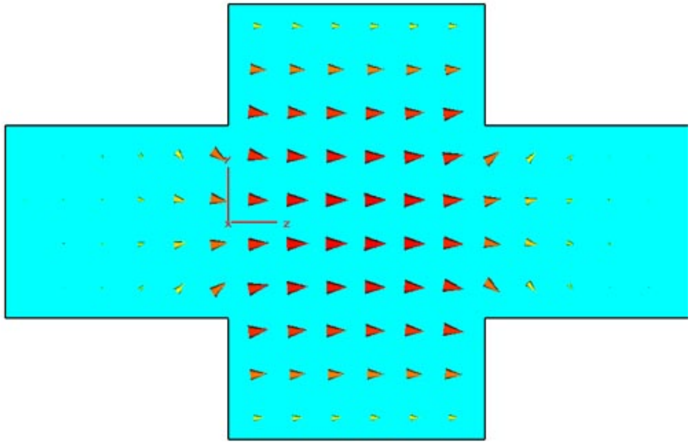
Shunt impedance

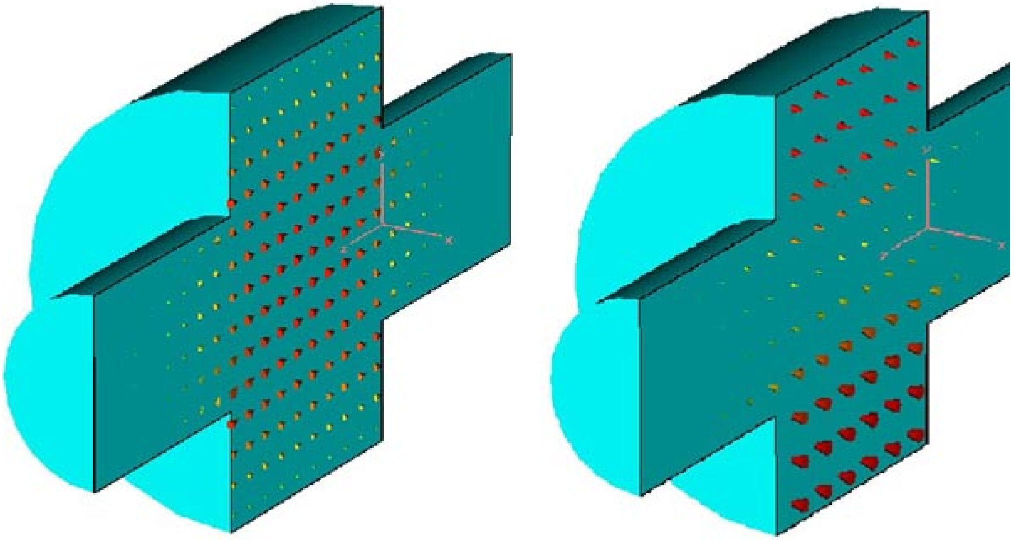
$$R_{sh} = \frac{\eta^2}{R_s} \frac{1}{\pi^3 J_1^2(x_{01})} \frac{\lambda^2}{R(R+L)} \sin^2\left(\frac{\pi L}{\lambda}\right)$$

Real Cavities

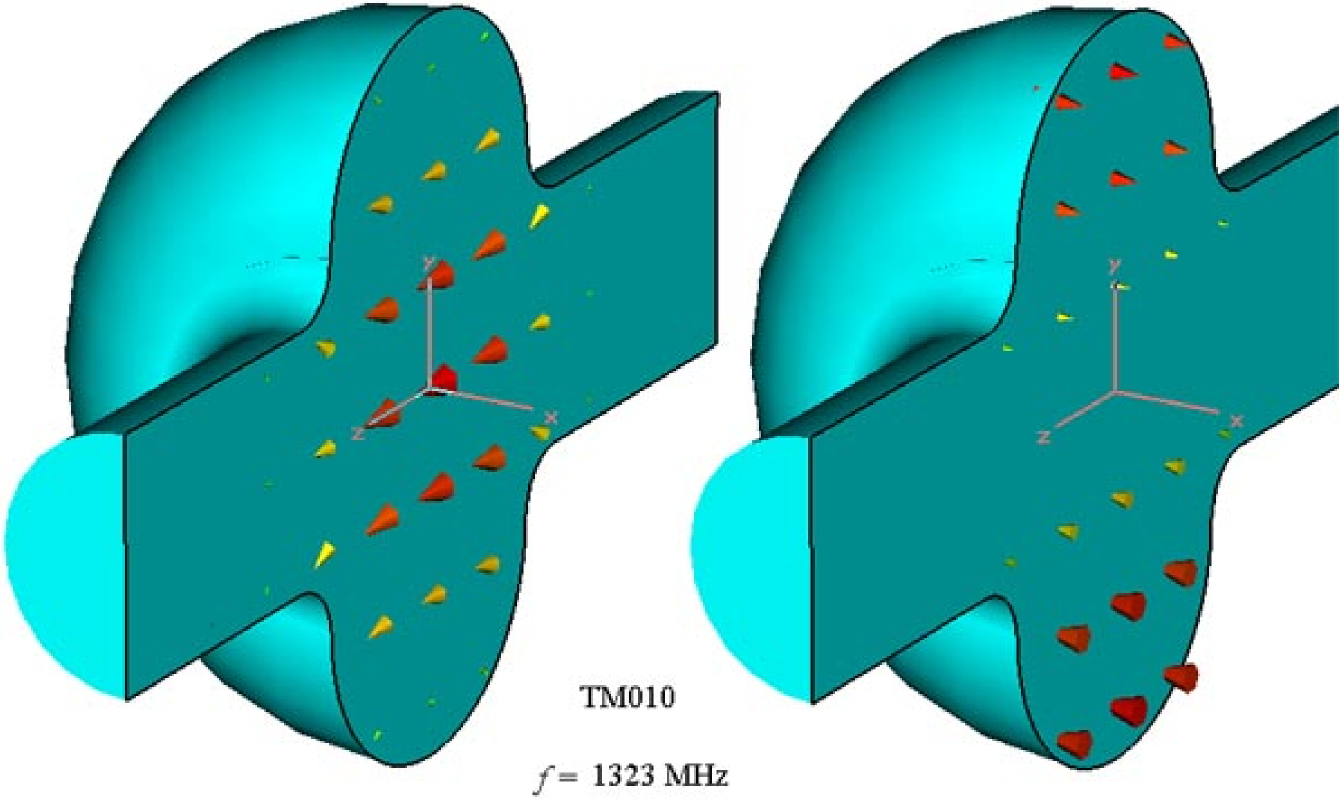
Beam tubes reduce the electric field on axis

- Gradient decreases
- Peak fields increase
- R/Q decreases



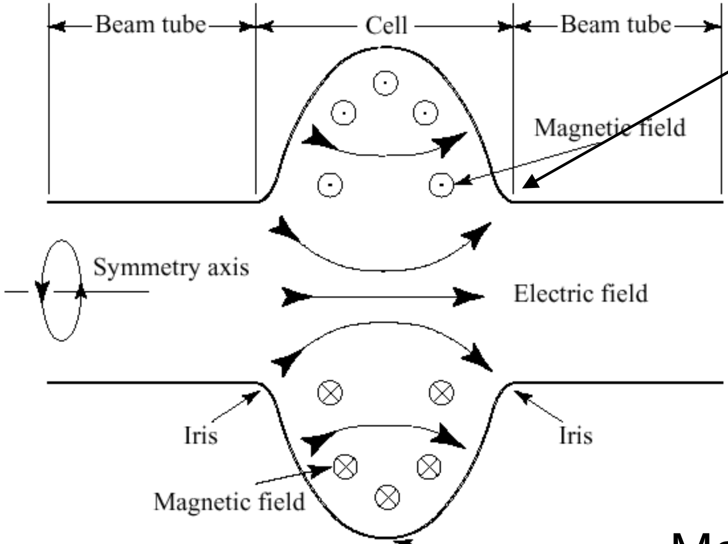


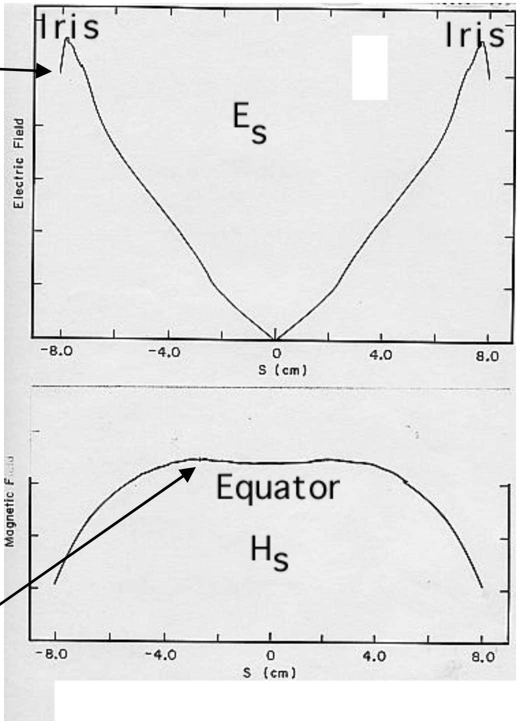
Real Cavities



Single Cell Cavities

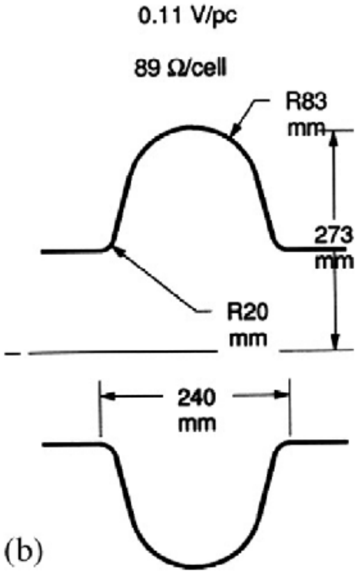
Electric field high at iris





Magnetic field high at equator

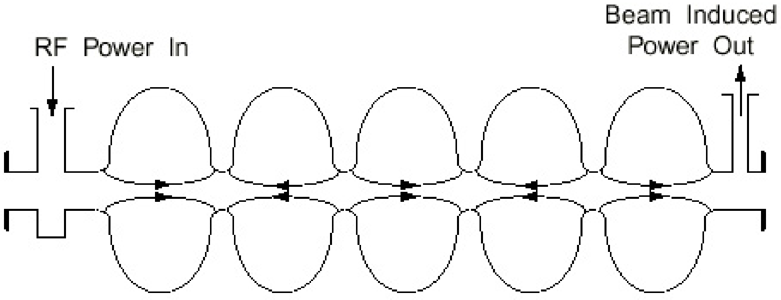
Single Cell Cavities



Superconducting
B-Factory Cell Shape

Quantity	Cornell SC 500 MHz	Pillbox
G	270 ohmΩ	$257\ \Omega$
R_{a}/Q_0	88 ohm/cell	$196\ \Omega/\mathrm{cell}$
$E_{\mathrm{pk}}/E_{\mathrm{acc}}$	2.5	1.6
$H_{\mathrm{pk}}/E_{\mathrm{acc}}$	52 Oe/MV/m	$30.5\ \mathrm{Oe}/(\mathrm{MV}/\mathrm{m})$

Multi-Cell Cavities



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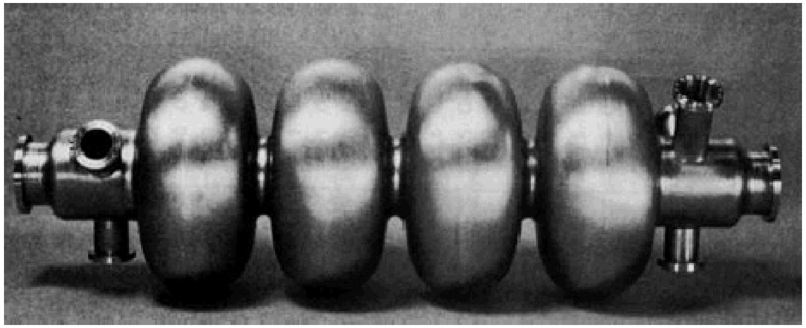
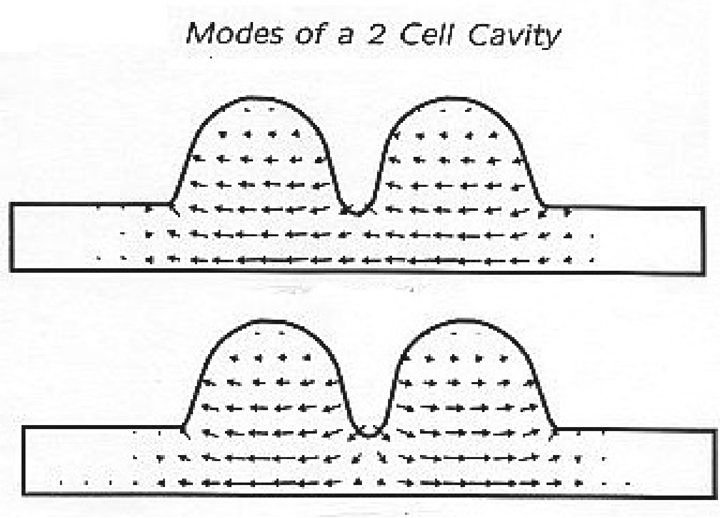
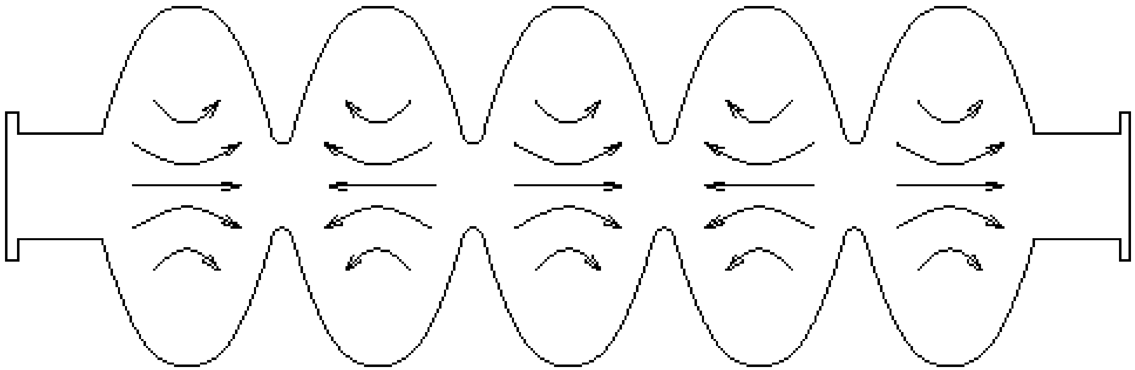


Multi-Cell Cavities





: Sketch of the electric field lines of the π-mode of a 5-cell

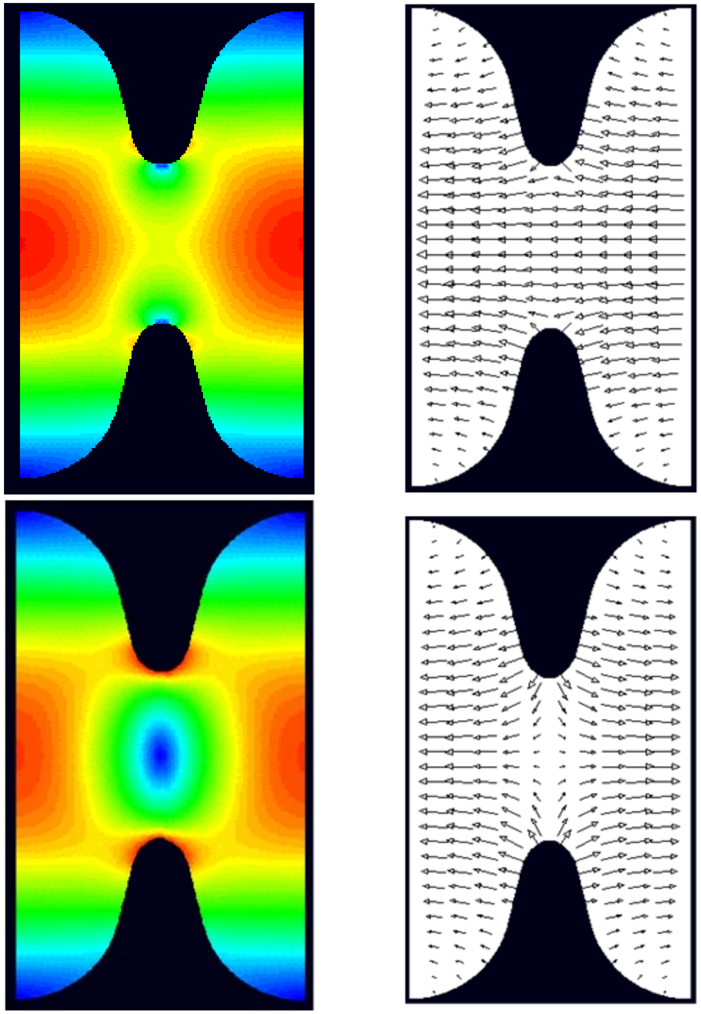
Cell-to-cell Coupling

0 mode

ω_0

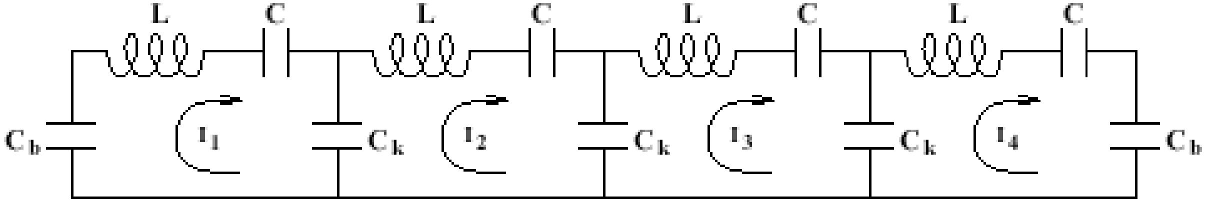
π mode

ω_π



$$k = \frac{2\left(\omega_\pi - \omega_0\right)}{\omega_\pi + \omega_0}$$

Multi-Cell Cavities



$$k = \frac{C}{C_k} \qquad C_b = C_k / 2$$

Mode frequencies:

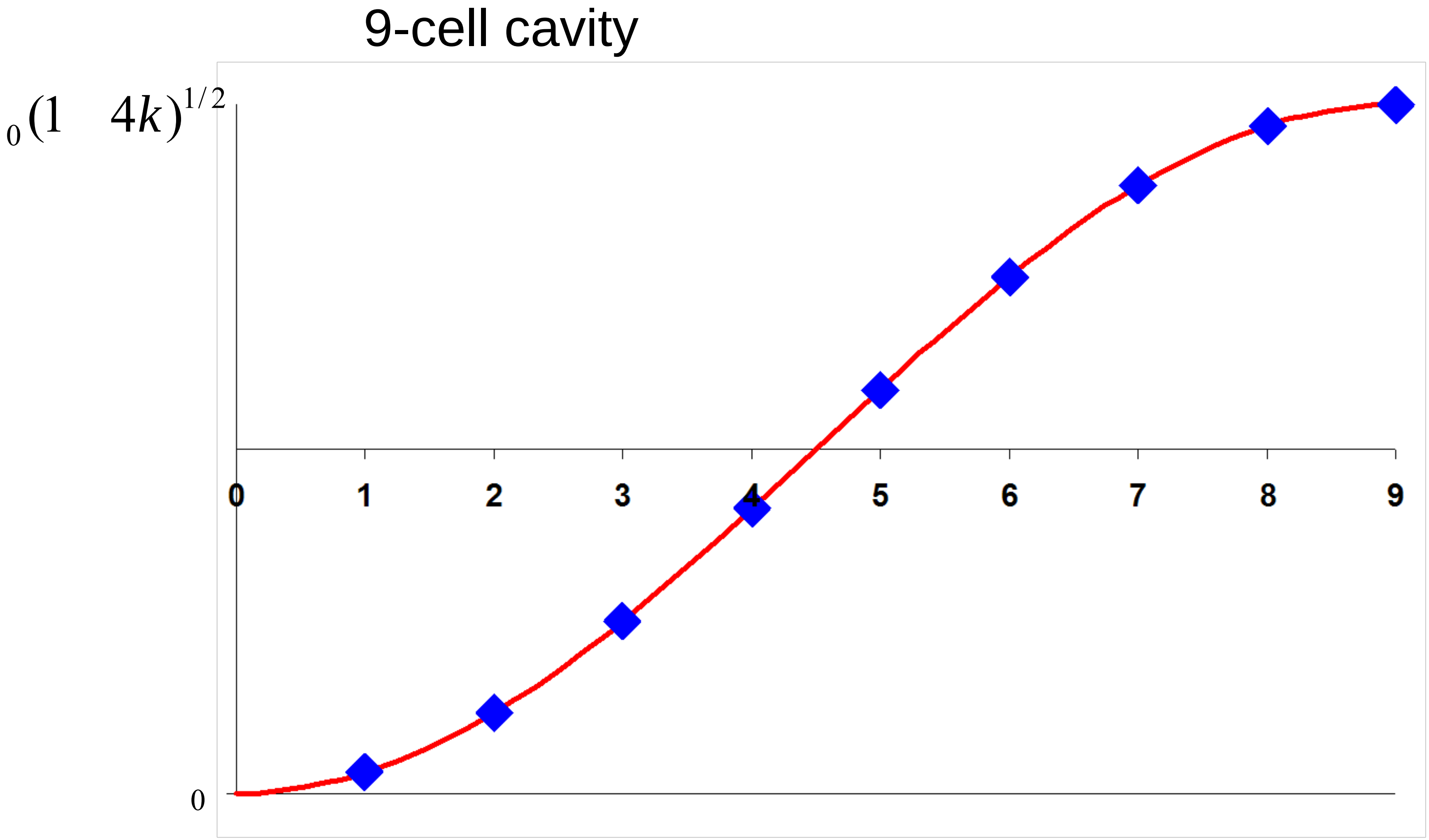
$$\frac{\omega_m^2}{\omega_0^2} = 1 + 2k\left(1 - \cos\frac{\pi m}{n}\right)$$

$$\frac{\omega_n - \omega_{n-1}}{\omega_0} \simeq k\left(1 - \cos\frac{\pi}{n}\right) \simeq \frac{k}{2}\left(\frac{\pi}{n}\right)^2$$

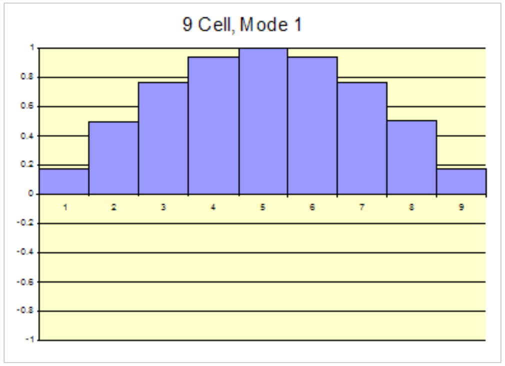
Voltages in cells:

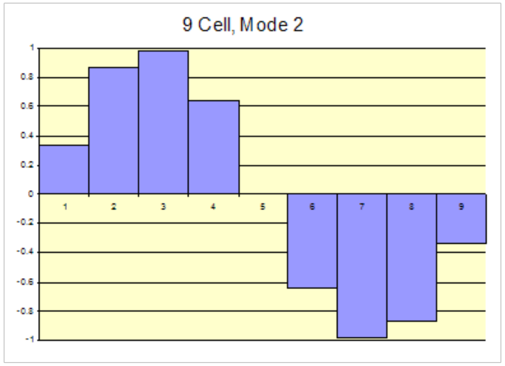
$$V_j^m = \sin\left(\pi m \frac{2j-1}{2n}\right)$$

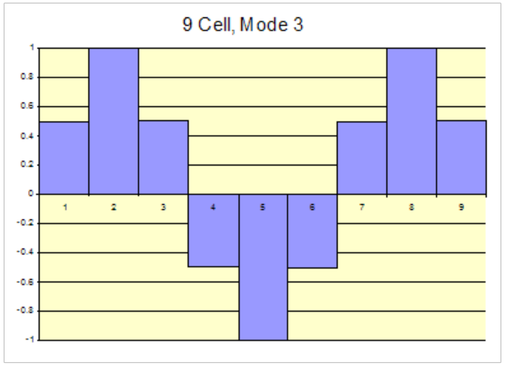
Pass-Band Modes Frequencies

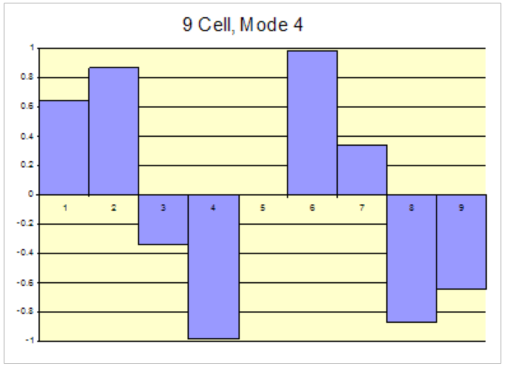


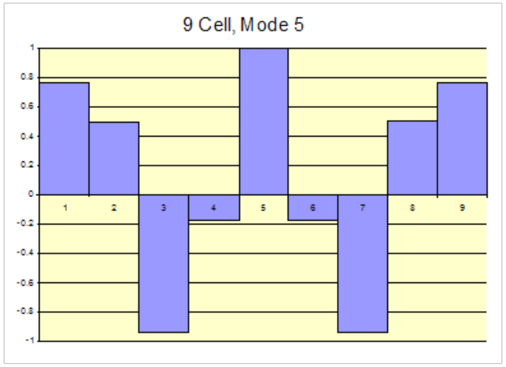
Cell Excitations in Pass-Band Modes

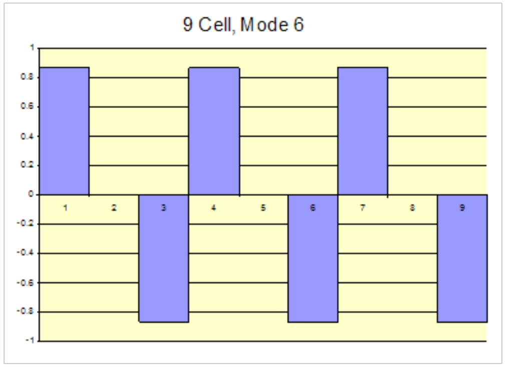

9 Cell, Mode 1

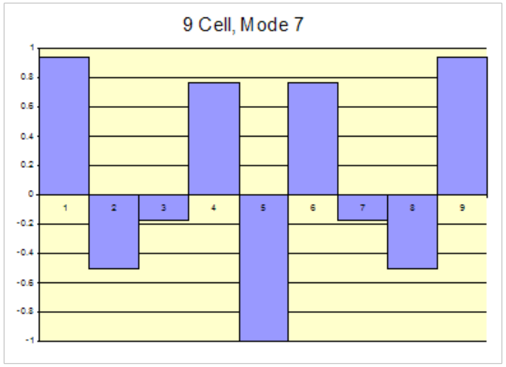

9 Cell, Mode 2

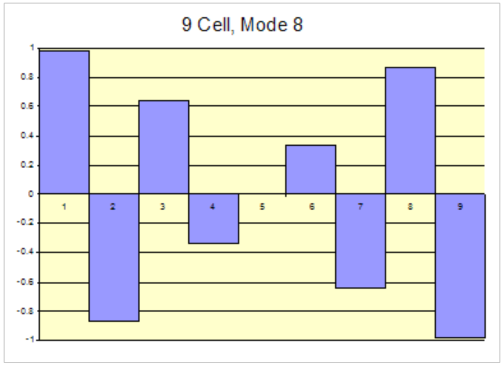

9 Cell, Mode 3

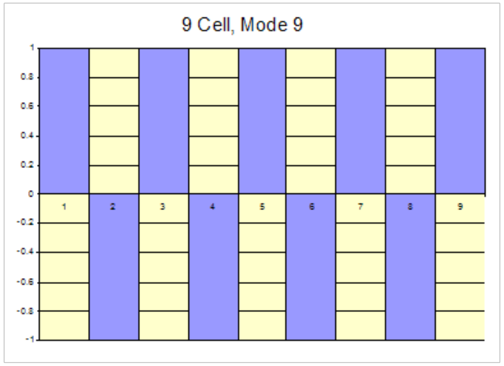

9 Cell, Mode 4


9 Cell, Mode 5


9 Cell, Mode 6


9 Cell, Mode 7


9 Cell, Mode 8


9 Cell, Mode 9

Field Flatness

Geometrical differences between cells causes a mixing of the eigenmodes

Sensitivity to mechanical deformation depends on mode spacing

$$\frac{\omega_n - \omega_{n-1}}{\omega_0} \simeq k\left(1 - \cos\frac{\pi}{n}\right) \simeq \frac{k}{2}\left(\frac{\pi}{n}\right)^2$$

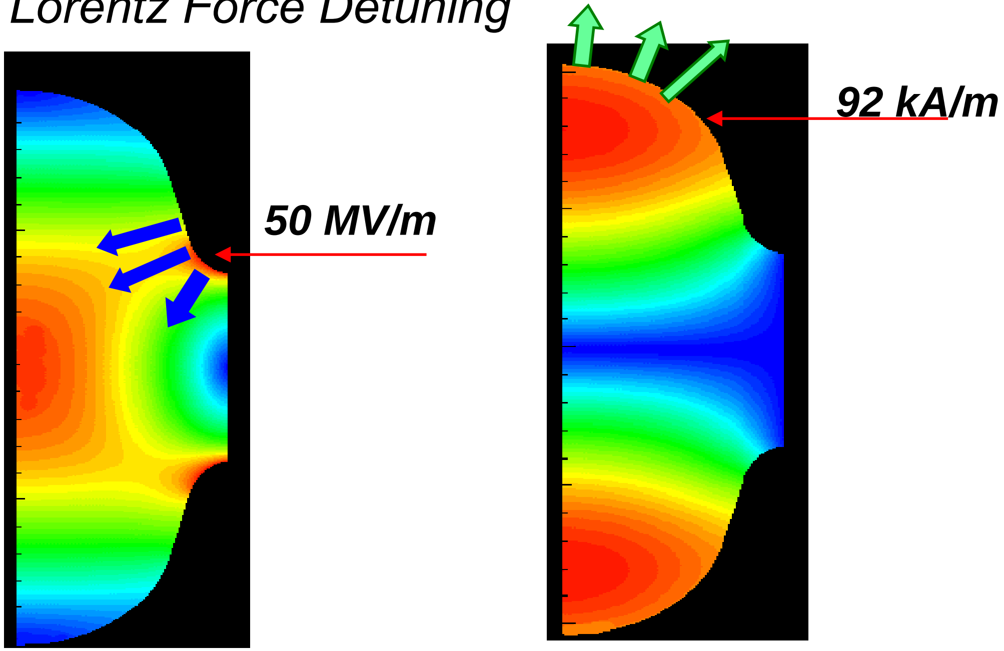
Mechanical Design

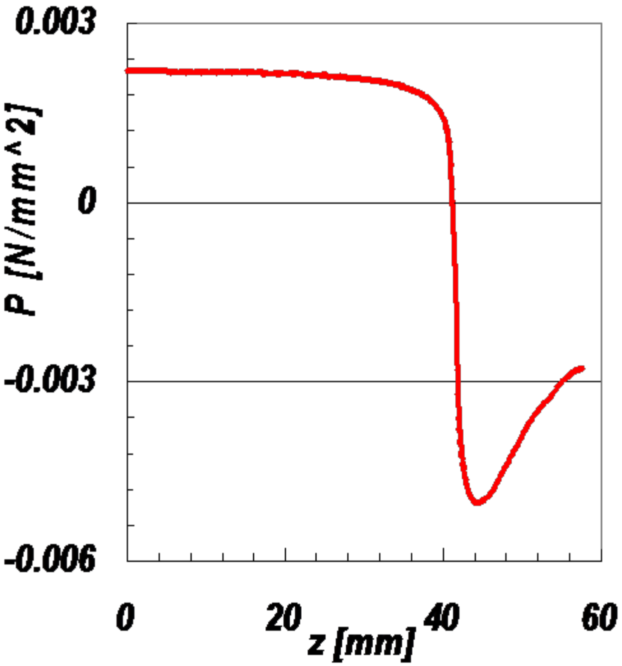
The mechanical design of a cavity follows its RF design:

- **Lorentz Force Detuning**
- **Mechanical Resonances**

Lorentz Force Detuning

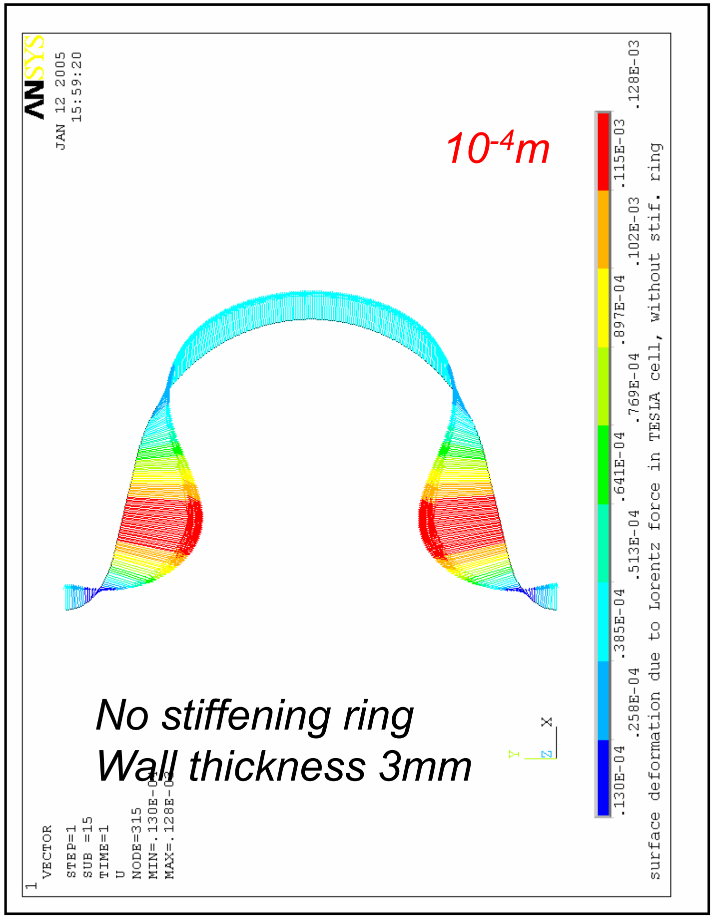
$$P = \frac{\mu_0 H_s^2 - \varepsilon_0 E_s^2}{4}$$





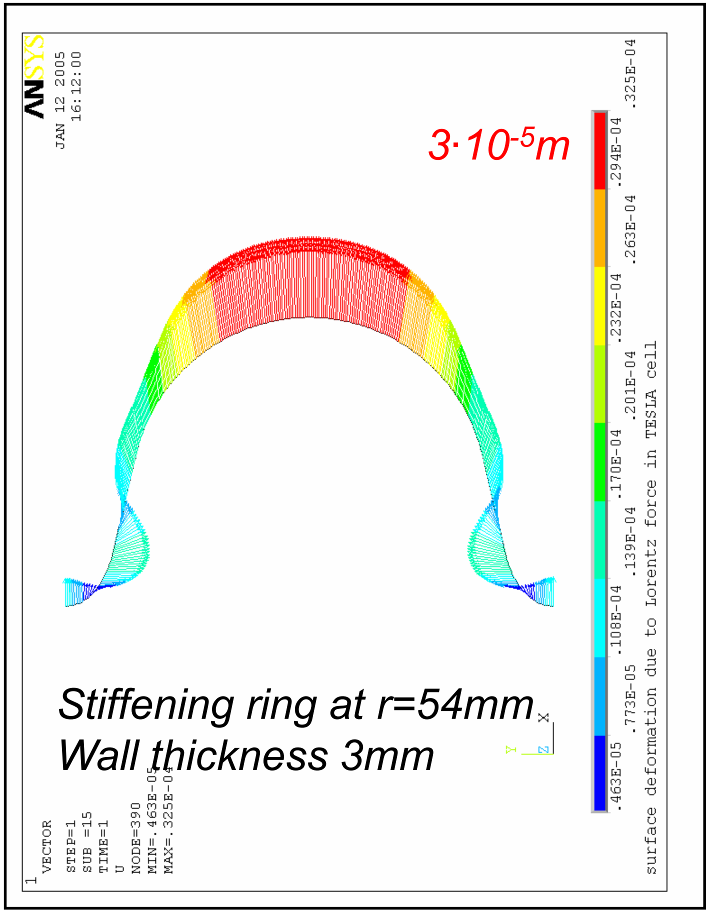
E and H at E_{acc} = 25 MV/m in TESLA inner-cup

Mechanical Design



10^{-4}m

No stiffening ring
Wall thickness 3mm



$3 \cdot 10^{-5}$m

Stiffening ring at r=54mm
Wall thickness 3mm

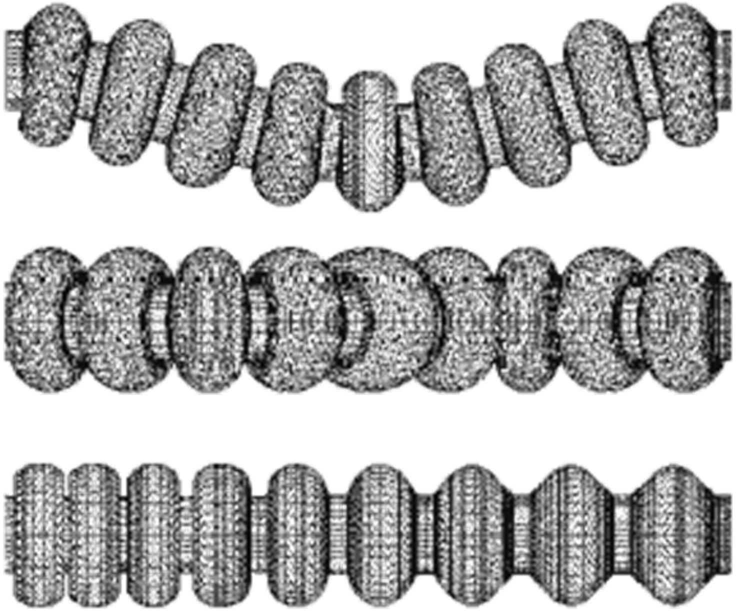
$k_L = -1$ Hz/(MV/m)^2

Essential for the operation of a pulsed accelerator

$$\Delta f = k_L (E_{acc})^2$$

Mechanical Design

Mechanical Resonances of a multi-cell cavity



60 Hz

152 Hz

Transverse modes

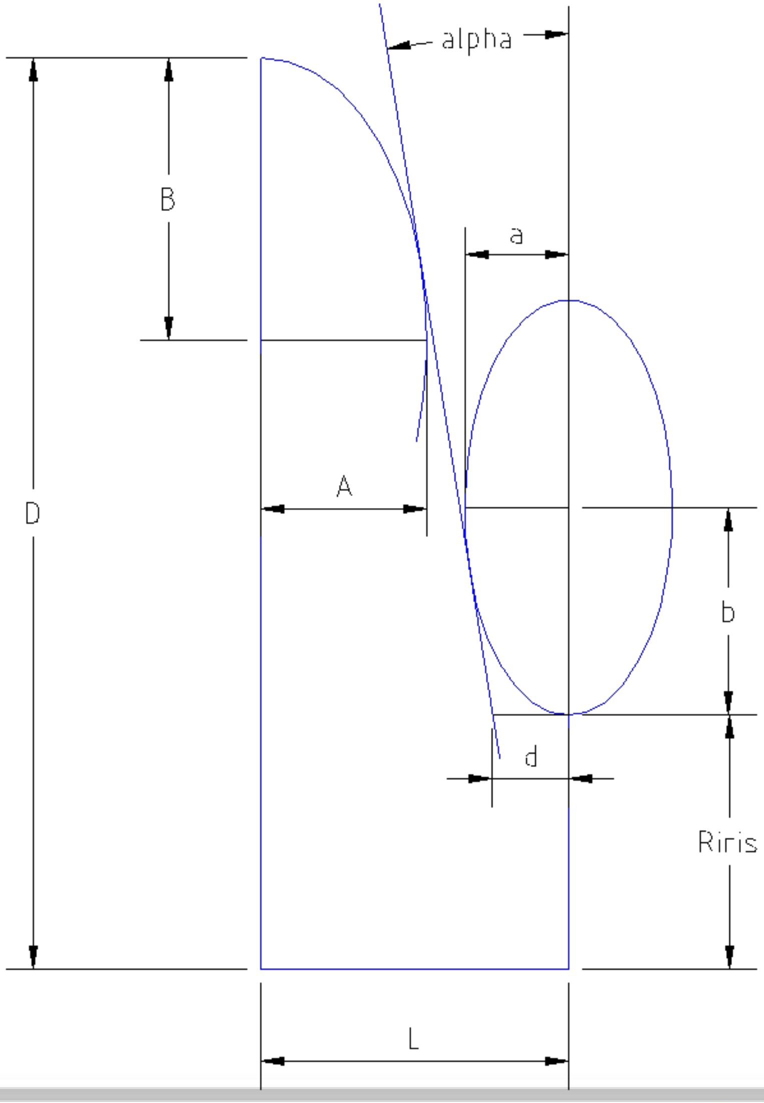
250 Hz *Longitudinal mode*

TESLA structure

The mechanical resonances modulate frequency of the accelerating mode.
Sources of their excitation: vacuum pumps, ground vibrations…

Cell Shape Parametrization

- Full parametric model of the cavity in terms of 7 meaningful geometrical parameters:
  - ✓ Ellipse ratio at the equator (R=B/A)
    ruled by mechanics
  - ✓ Ellipse ratio at the iris (r=b/a)
    Epeak
  - ✓ Side wall inclination (α)
    and position (d)
    Epeak vs. Bpeak tradeoff and coupling k
  - ✓ Cavity iris radius Riris
    coupling k
  - ✓ Cavity Length L
    β
  - ✓ Cavity radius D
    used for frequency tuning
- Behavior of all e.m. and mechanical properties has been found as a function of the above parameters

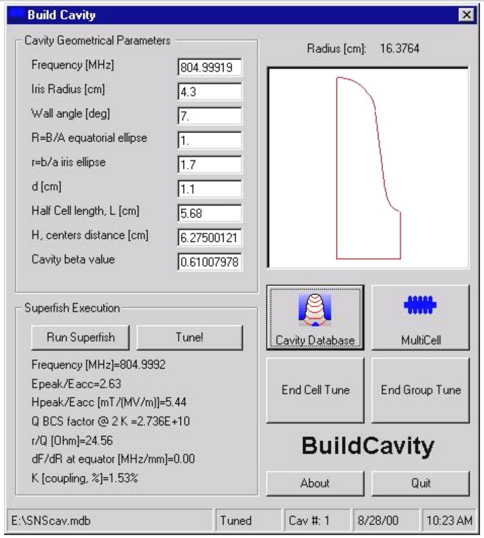


Tools used for the parametrization

Using of the parametric tool developed at *INFN Milano* for the analysis of the cavity shape on the electromagnetic parameters:

- All RF computations are handled by SUPERFISH
- Inner cell tuning is performed through the cell diameter, all the characteristic cell parameters stay constant: R, r, , d, L, Riris
- End cell tuning is performed through the wall angle inclination, , or distance, d.

  R, L and Riris are independently settable.
- Multicell cavity is then built to minimize the field unflatness, compute the effective and the final cavity performances.
- A proper file to transfer the cavity geometry to ANSYS is then generated



Cavity Geometrical Parameters

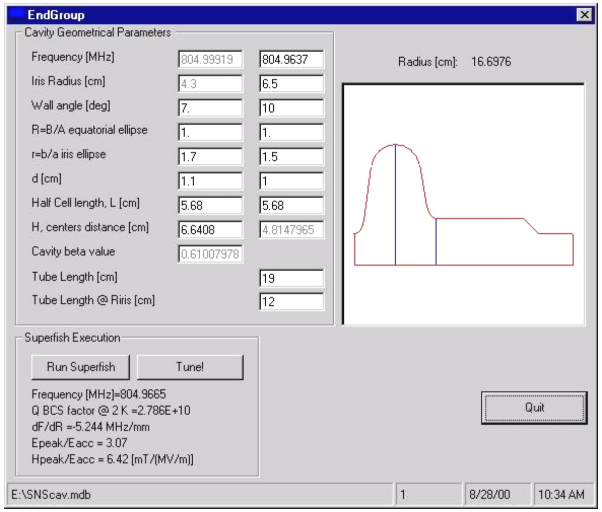
Frequency [MHz] 804.99919
Iris Radius [cm] 4.3
Wall angle [deg] 7.
R=B/A equatorial ellipse 1.
r=b/a iris ellipse 1.7
d [cm] 1.1
Half Cell length, L [cm] 5.68
H, centers distance [cm] 6.27500121
Cavity beta value 0.61007978

Radius [cm]: 16.3764

Superfish Execution

Frequency [MHz]=804.9992
Epeak/Eacc=2.63
Hpeak/Eacc [mT/(MV/m)]=5.44
Q BCS factor @ 2 K =2.736E+10
r/Q [Ohm]=24.56
dF/dR at equator [MHz/mm]=0.00
K [coupling, %]=1.53%

E:\SNScav.mdb | Tuned | Cav #: 1 | 8/28/00 | 10:23 AM



Cavity Geometrical Parameters

Frequency [MHz] 804.99919 804.9637
Iris Radius [cm] 4.3 6.5
Wall angle [deg] 7. 10
R=B/A equatorial ellipse 1. 1.
r=b/a iris ellipse 1.7 1.5
d [cm] 1.1 1
Half Cell length, L [cm] 5.68 5.68
H, centers distance [cm] 6.6408 4.8147965
Cavity beta value 0.61007978
Tube Length [cm] 19
Tube Length @ Riris [cm] 12

Radius [cm]: 16.6976

Superfish Execution

Frequency [MHz]=804.9665
Q BCS factor @ 2 K =2.786E+10
dF/dR =-5.244 MHz/mm
Epeak/Eacc = 3.07
Hpeak/Eacc = 6.42 [mT/(MV/m)]

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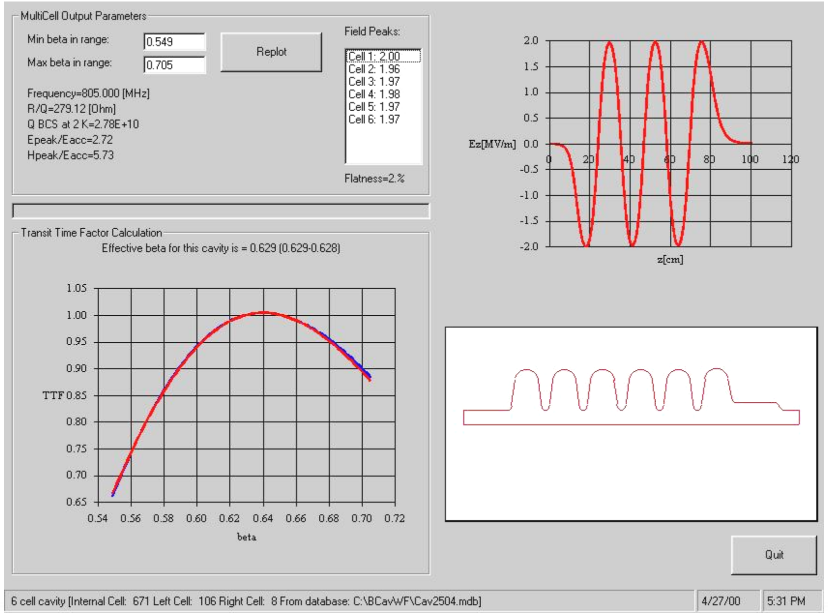


MultiCell Output Parameters

Min beta in range: 0.549
Max beta in range: 0.705

Frequency=805.000 [MHz]
R/Q=279.12 [Ohm]
Q BCS at 2 K=2.78E+10
Epeak/Eacc=2.72
Hpeak/Eacc=5.73

Field Peaks:
Cell 1: 2.00
Cell 2: 1.96
Cell 3: 1.97
Cell 4: 1.98
Cell 5: 1.97
Cell 6: 1.97

Flatness=2.%

Transit Time Factor Calculation

Effective beta for this cavity is = 0.629 (0.629-0.628)

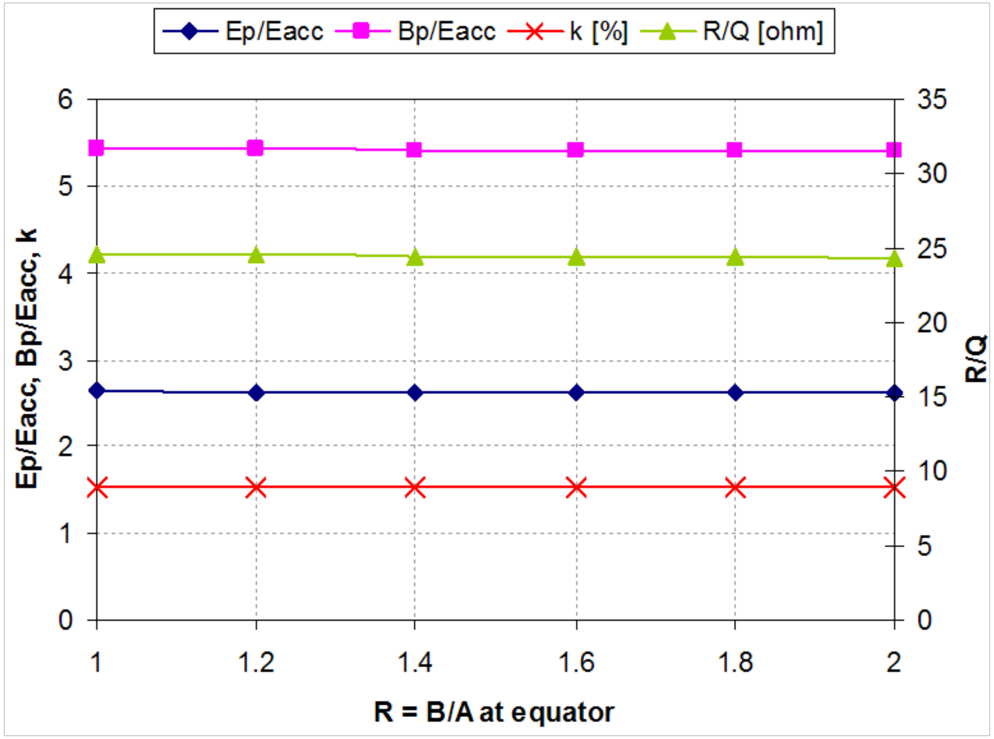
6 cell cavity [Internal Cell: 671 Left Cell: 106 Right Cell: 8 From database: C:\BCavWF\Cav2504.mdb] | 4/27/00 | 5:31 PM

Inner cell data

L = 56.8 mm
R = 1
r = 1.7
α = 7°
d = 11 mm
R_{iris} = 43 mm

R: “mechanical” parameter

- **The equator aspect ratio (R) is a free parameter for what concerns the -mode e.m. design but the cavity mechanical parameters are greatly affected by the equator shape: R>1 allows better stress distribution in the unstiffened cavity but a bigger Lorentz force coefficient**
- **The cell tuning strategy doesn’t affect the cell’s performances (e.m. and mechanical)**



Reference data

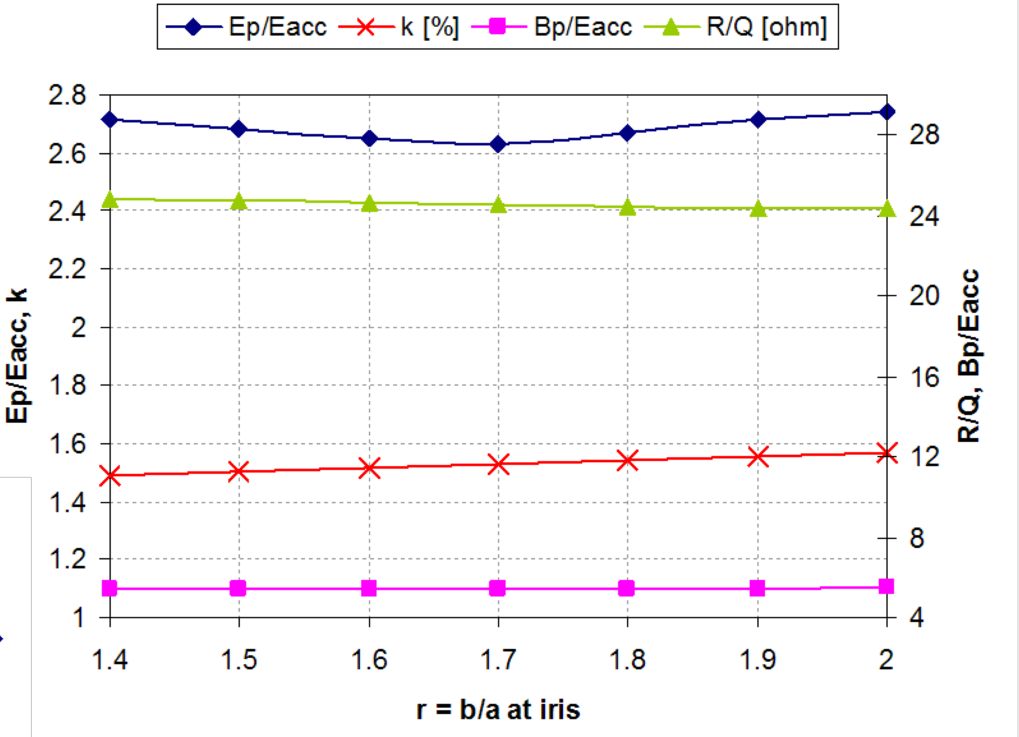
L = 56.8 mm   r = 1.7 $\alpha = 7°$

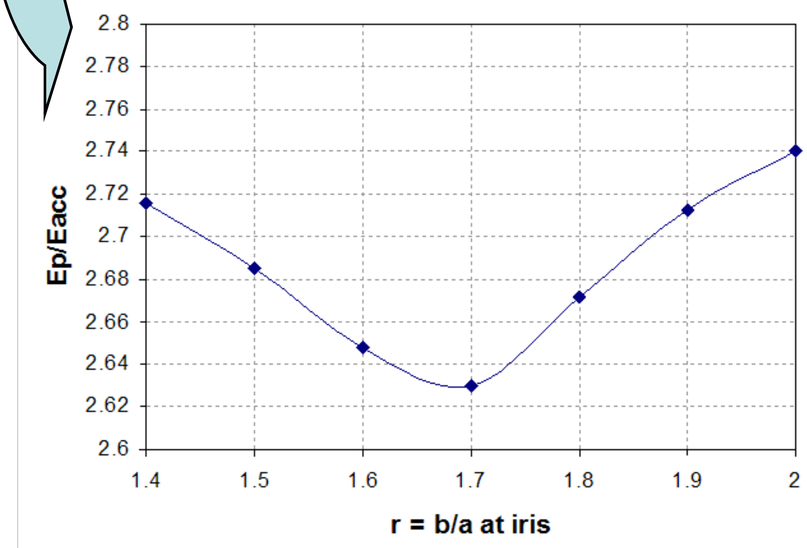
d = 11 mm R_{iris} = 43 mm

Chosen R = 1

Optimal value for r

- The iris ellipse aspect ratio has always an optimal value that minimize the peak surface E field
- All the other cavity parameters (e.m. and mechanical) are unchanged

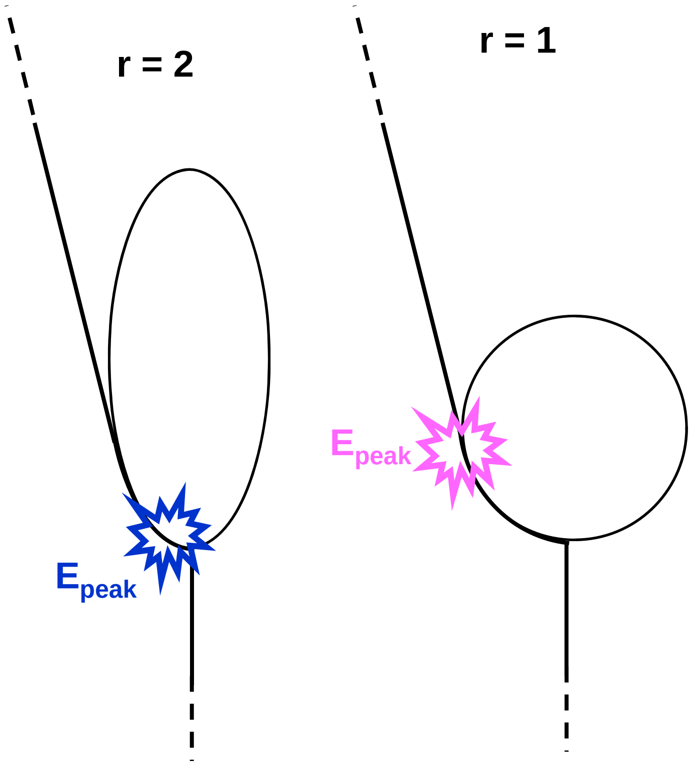


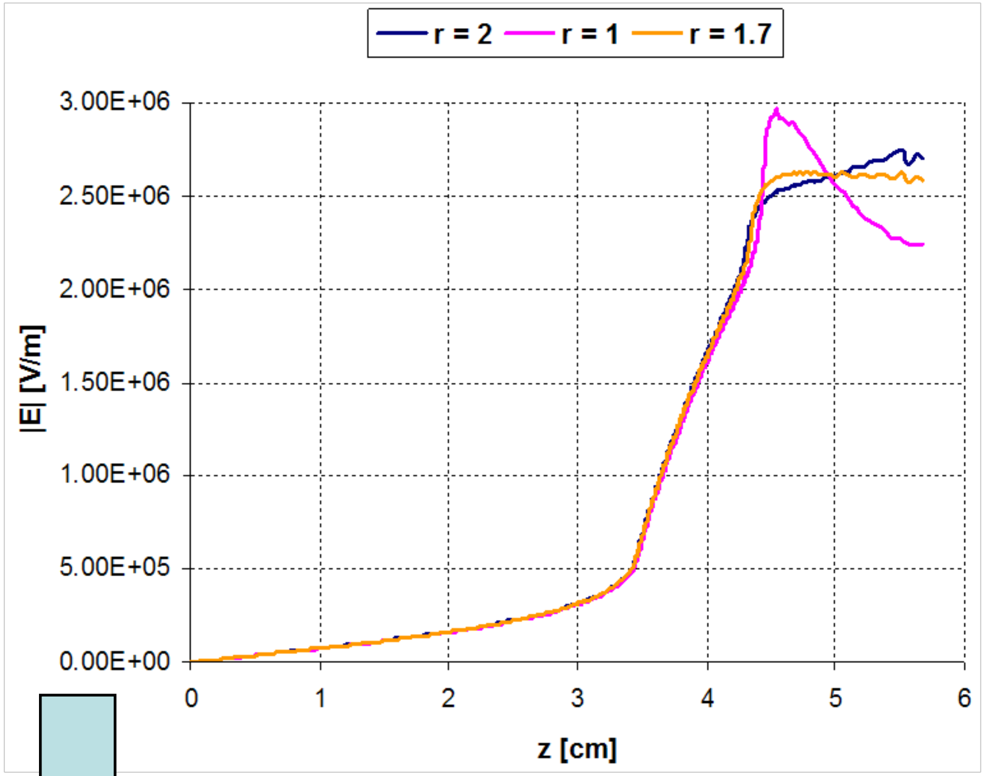


Reference data

$L = 56.8$ mm $R = 1$ $\alpha = 7°$
$d = 11$ mm $R_{iris} = 43$ mm

Optimal value for r

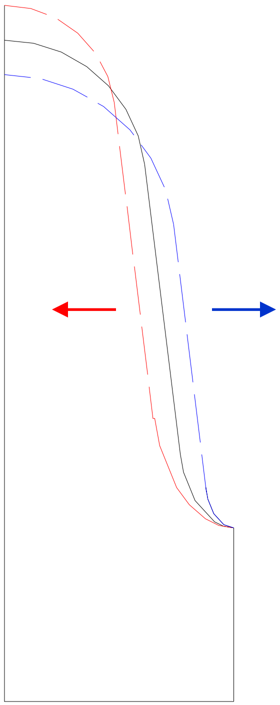




Optimization of r means best field distribution on the cell’s nose

Chosen r = 1.7

Influence of d @ constant R_{iris}

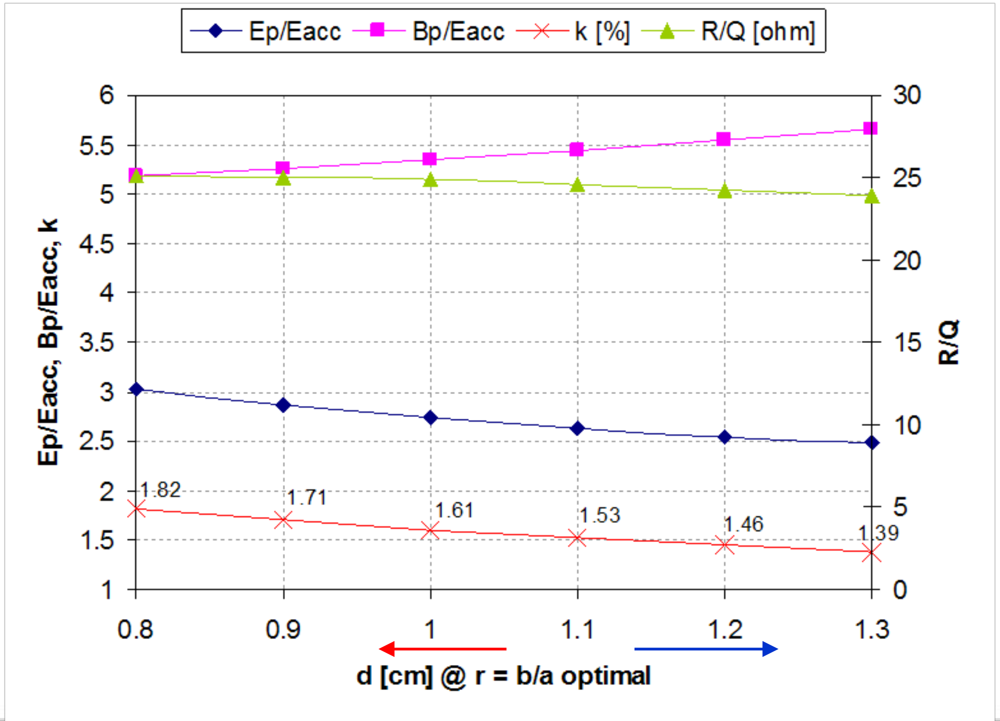


- d can be used to balance the electric and magnetic volumes of the cavity
  - The value of r is always optimal.
  - The cell to cell coupling changes, if R_{iris} is kept constant

Reference data

L = 56.8 mm
R = 1 α = 7°
r = optimal
R_{iris} = 43 mm

- Better mechanical performances are reached with decreasing d



Influence of d @ constant k

- If we want to keep a constant cell to cell coupling we have to adjust Riris

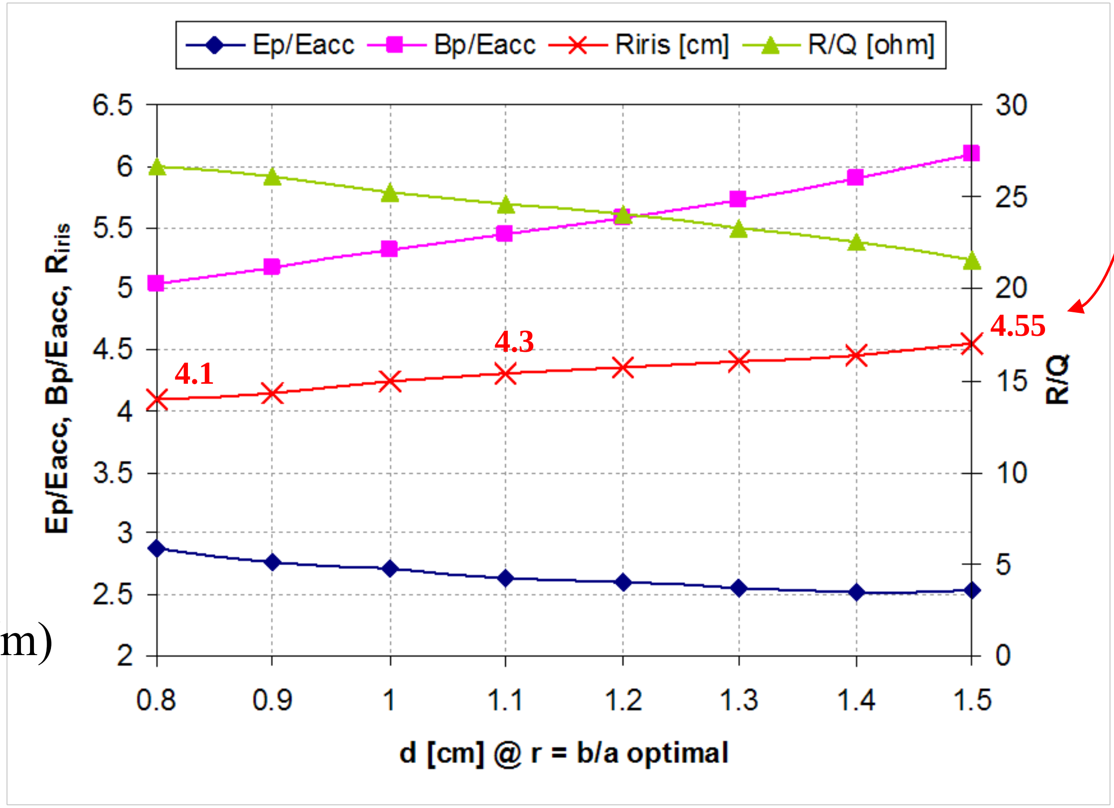
Reference data

L = 56.8 mm
R = 1

α = 7°
r = optimal
k = 1.5 %

Chosen d=11 mm

B_{peak}/E_{peak} = 2.07 mT/(MV/m)

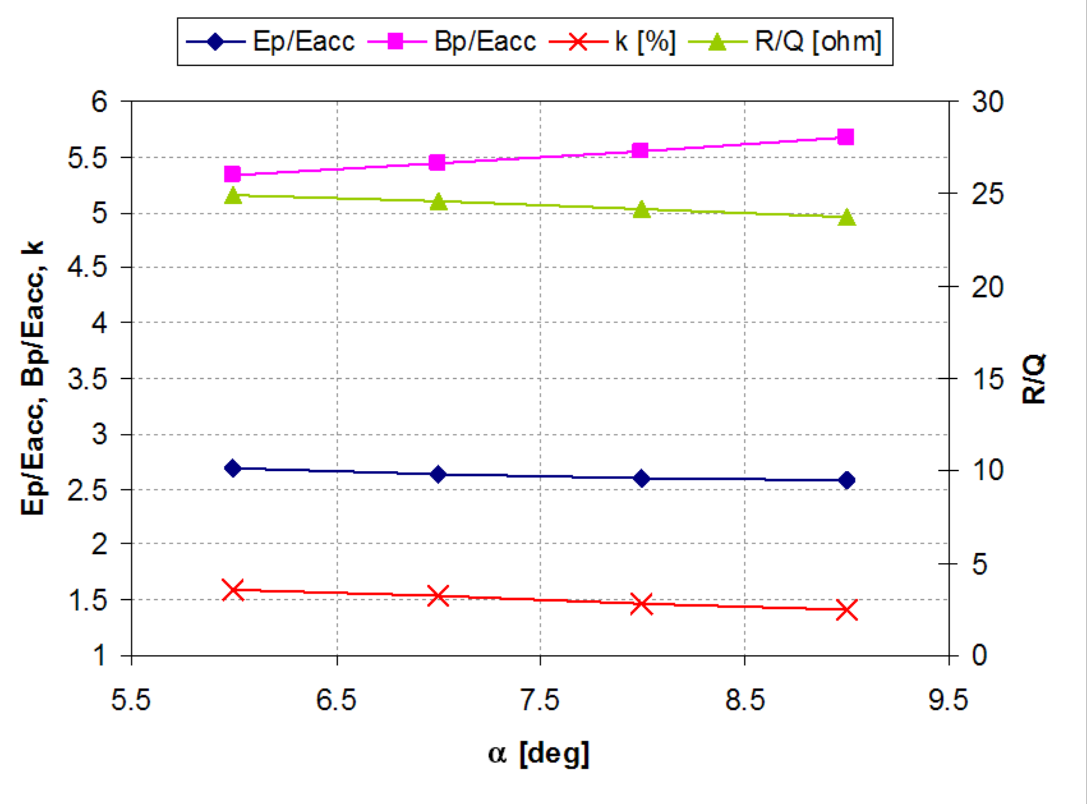


Dependence on α

The wall angle α slightly affects all the e.m. parameters, but has a strong effect on the mechanical performances:

- **Lower values are preferred for Lorentz force detuning**
- **Too small α could be critical for chemistry and cleaning**

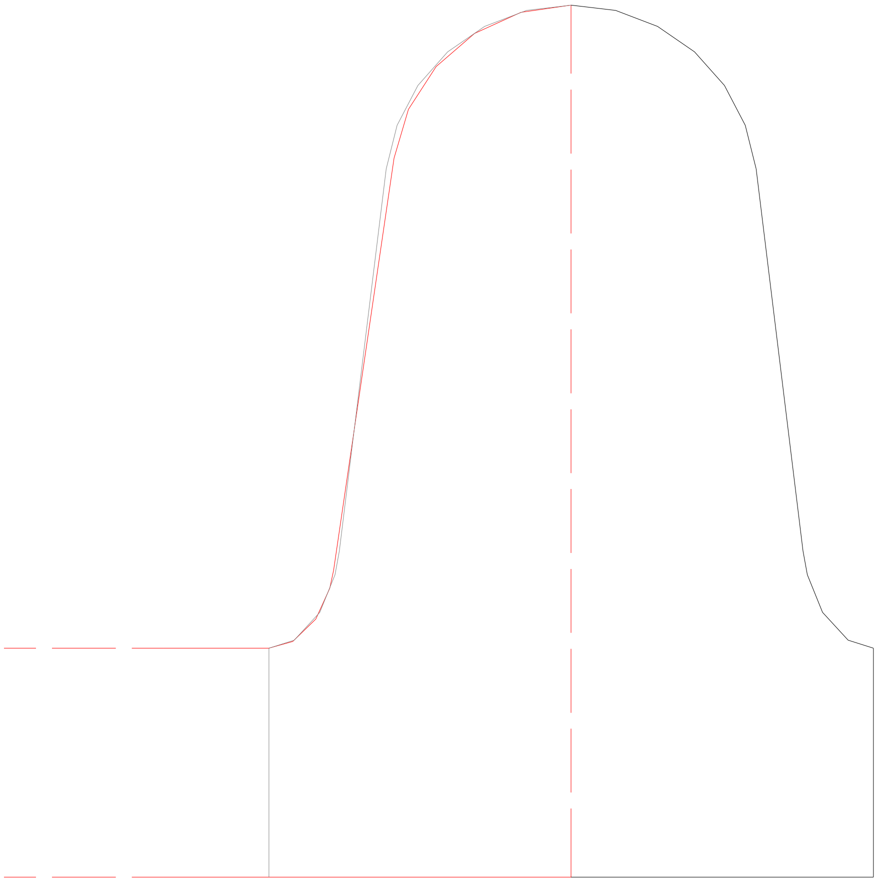
Chosen α = 7 deg



Reference data

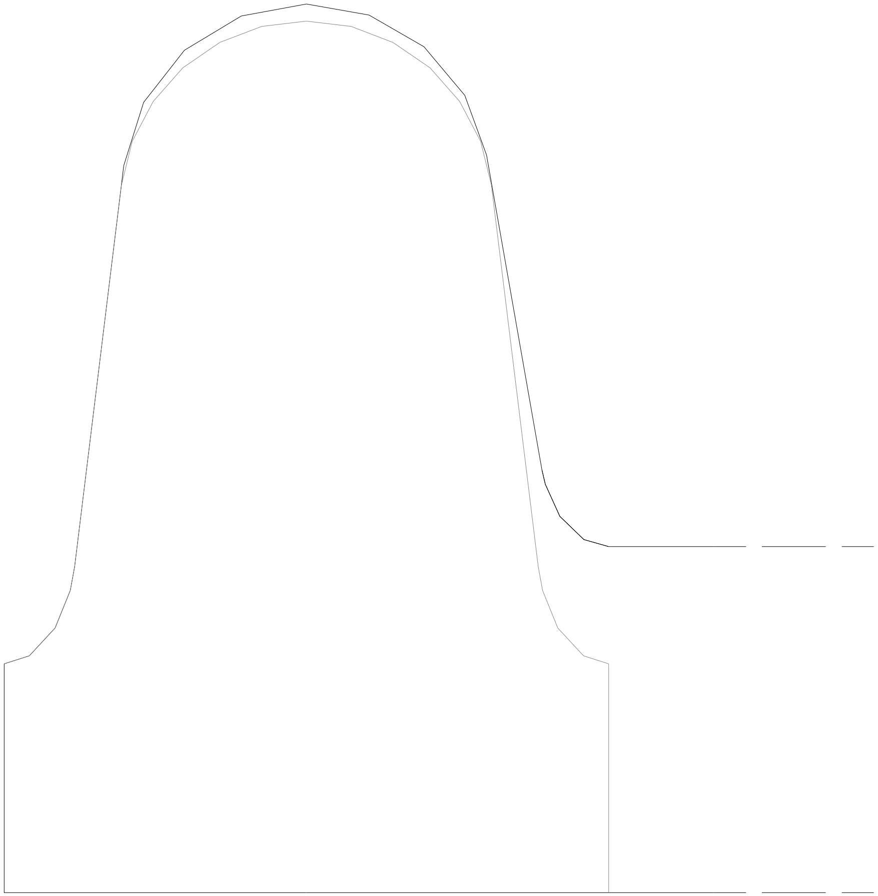
L = 56.8 mm
R = 1
d =11 mm
r = 1.7
R_{iris} = 43 mm

End cell tune



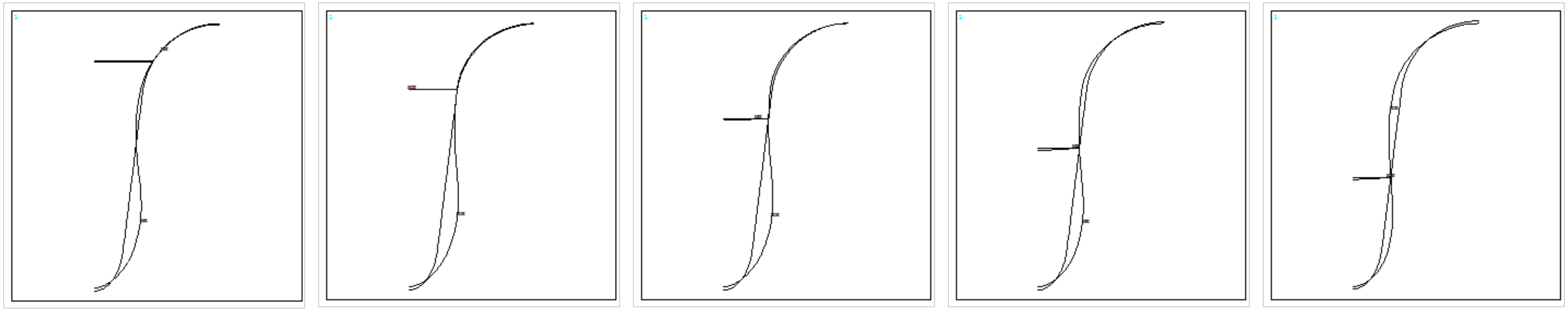
- d set 1 mm lower than the in-cell
- optimization of r = b/a at iris
- Slater compensation (decrease of the magnetic volume) of the cut-off tube and d reduction (↓f), increasing the wall angle α. This gives also the necessary stiffening to the end cell
- the frequency of end cell + tube is about 50 kHz lower than the in-cell's due to the asimmetry

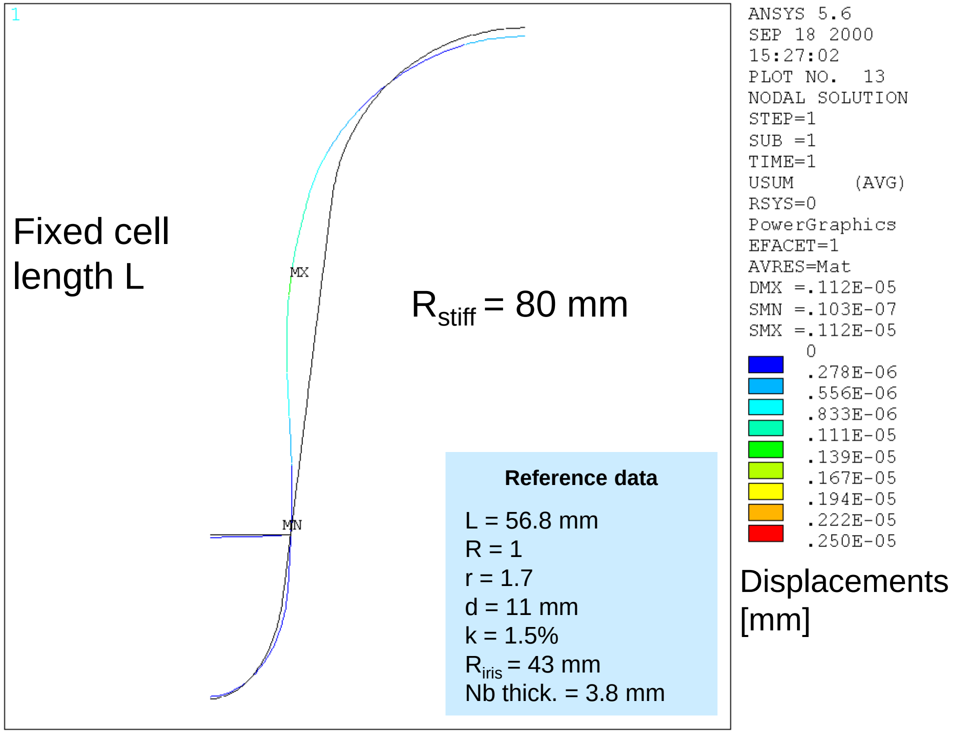
End cell @ FPC side tune



- R_{iris} set to 65 mm to have enough field at the power coupler antenna
- d set 1 mm lower than the in-cell
- optimization of r = b/a at iris
- α set to 10 deg to have the necessary stiffening
- Slater compensation (increase of the magnetic volume) of the cut-off tube (↓f), d reduction (↓f), α and R_{iris} increase (↑f) by increasing the equator radius ⇨ 4 dies
- the frequency of end cell + tube is about 40 kHz lower than the in-cell's due to the asimmetry

Optimal stiffening ring position





Fixed cell length L

R_{stiff} = 80 mm

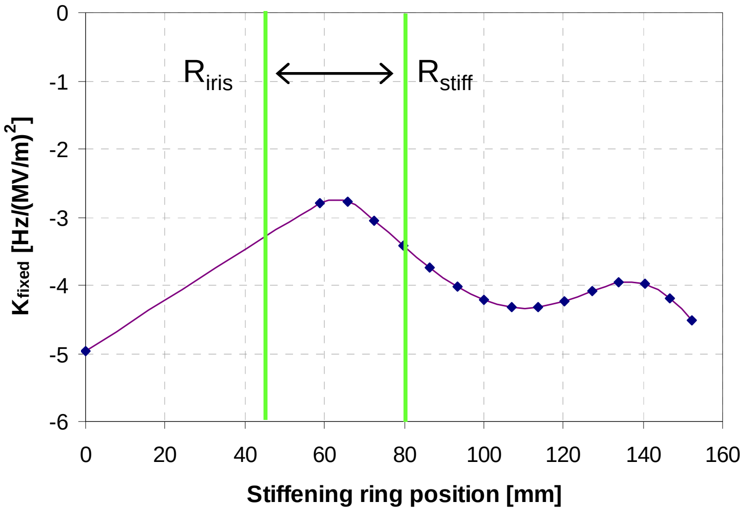
Reference data

L = 56.8 mm
R = 1
r = 1.7
d = 11 mm
k = 1.5%
R_{iris} = 43 mm
Nb thick. = 3.8 mm

```
ANSYS 5.6
SEP 18 2000
15:27:02
PLOT NO.   13
NODAL SOLUTION
STEP=1
SUB =1
TIME=1
USUM       (AVG)
RSYS=0
PowerGraphics
EFACET=1
AVRES=Mat
DMX =.112E-05
SMN =.103E-07
SMX =.112E-05
      0
      .278E-06
      .556E-06
      .833E-06
      .111E-05
      .139E-05
      .167E-05
      .194E-05
      .222E-05
      .250E-05
```

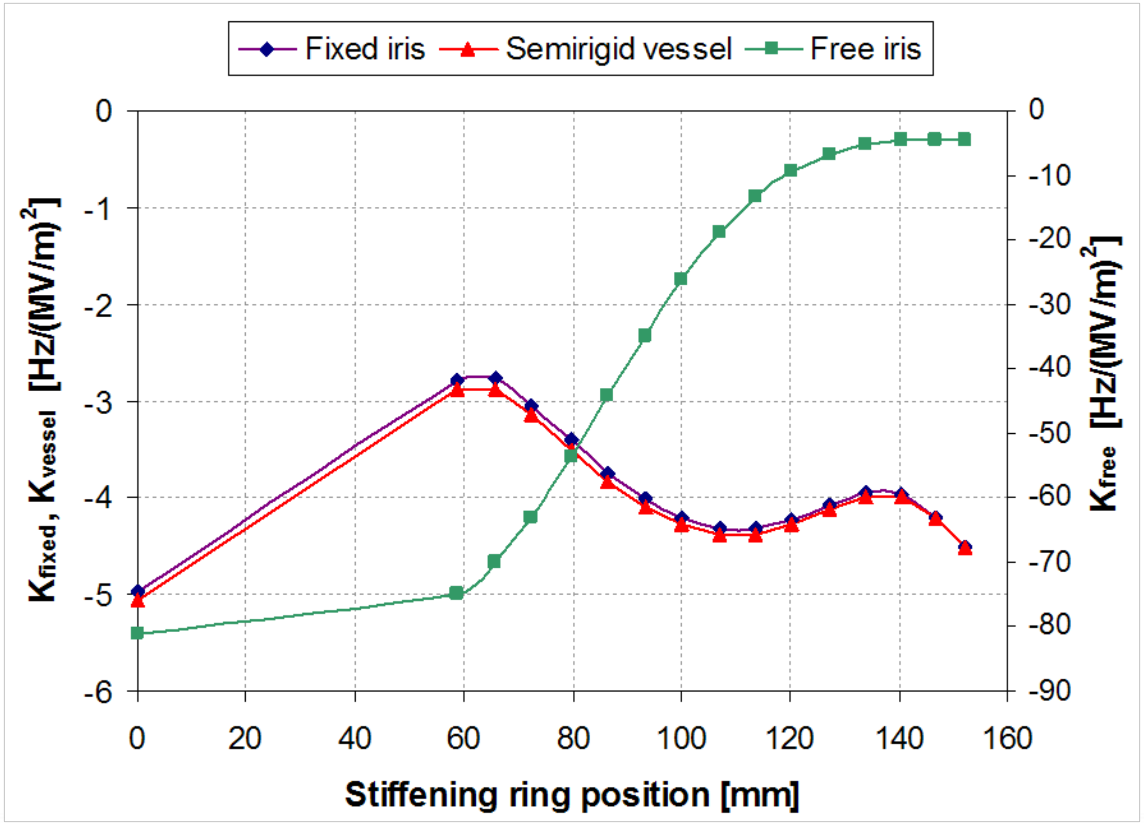
Displacements [mm]

The Lorentz forces coefficients for 15 different stiffening ring positions are evaluated automatically with ANSYS, preparing the geometry and reading the fields from the SFO output from SUPERFISH



KL for different boundary conditions

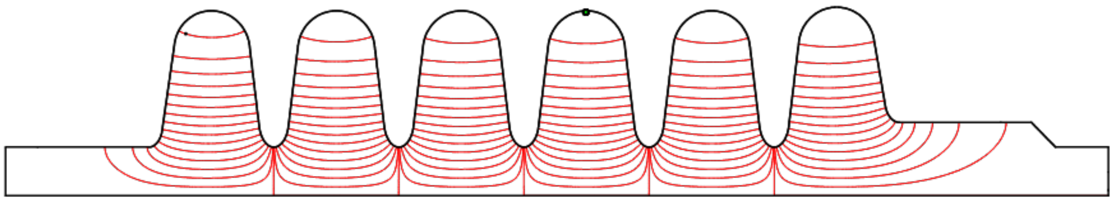
- The estimate for KL strongly depends on the cell boundaries. We compute it for 3 different cases:
  - Fixed cell length
  - Free cell length
  - Helium Vessel/Tuning System (= 3 tubes with diameter 30 mm and thickness 2 mm)



β_g = 0.61 Cavity for SNS

Effective β that matches the TTF curve = 0.630

E_p/E_{acc}	2.72 (2.63 inner cell)
B_p/E_{acc} [mT/(MV/m)]	5.73 (5.44 inner cell)
R/Q [Ω]	279
G [Ω]	214
k [%]	1.53
Q_{BCS} @ 2 K [10^9]	27.8
Frequency [MHz]	805.000
Field Flatness [%]	2



KL70 = -2.9 [Hz/(MV/m)^2] KL80 = -3.4 [Hz/(MV/m)^2]

Nb thickness = 3.8 mm-

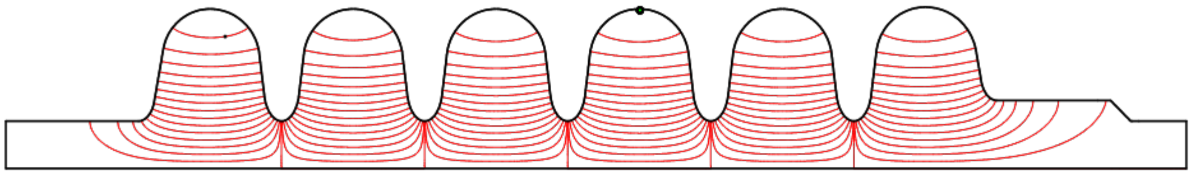
Geometrical Parameters

	Inner cell	**End Cell Left**	**End Group (coupler)**	
			Left	**Right**
L [cm]	**5.68**	**5.68**	**5.68**	
R_{iris} [cm]	**4.3**	**4.3**	**4.3**	**6.5**
D [cm]	**16.376**	**16.376**	**16.698**	
d [cm]	**1.1**	**1.0**	**1.1**	**1.0**
r	**1.7**	**1.5**	**1.7**	**1.5**
R	**1.0**	**1.0**	**1.0**	
α [deg]	**7.0**	**8.36**	**7.0**	**10.0**

β_g = 0.81 Cavity for SNS

Effective β that matches the TTF curve = 0.832

E_p/E_{acc}	2.19 (2.14 inner cell)
B_p/E_{acc} [mT/(MV/m)]	4.72 (4.58 inner cell)
R/Q [Ω]	484.8
G [Ω]	233
k [%]	1.52
Q_{BCS} @ 2 K [10^9]	36.2
Frequency [MHz]	805.004
Field Flatness [%]	1.1



KL70 = -0.7 [Hz/(MV/m)^2]

KL80 = -0.8 [Hz/(MV/m)^2]

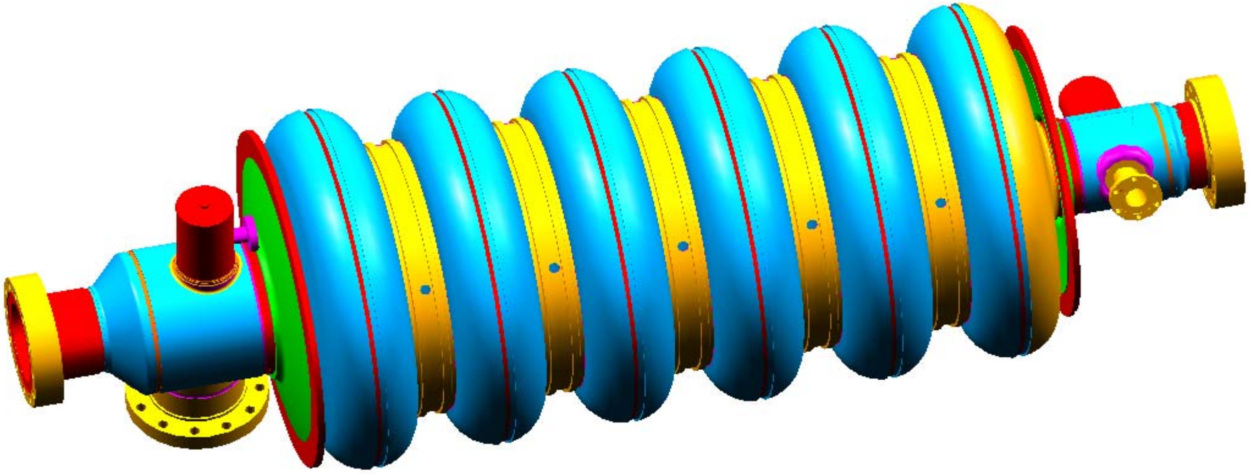
Nb thickness = 3.8 mm-

Geometrical Parameters

	Inner cell	End Cell Left	End Group (coupler)	
			Left	Right
L [cm]	7.55	7.55	7.55	
R_{iris} [cm]	4.88	4.88	4.88	7.0
D [cm]	16.415	16.415	16.611	
d [cm]	1.5	1.3	1.5	1.3
r	1.8	1.6	1.8	1.6
R	1.0	1.0	1.0	
α [deg]	7.0	10.072	7.0	10.0

Stress and Modal Analysis

- Nominal Medium Beta Cavity



SNS Cavity Modal Analysis

Medium Beta Cavity

End Condition	Load (atm)	127 mm Rings (Hz)	70 mm Rings (Hz)	No Rings (Hz)
Fixed-Guided	-	85	48	38
Fixed-Fixed	-	126 (*204)	57 (*59)	48 (*42)
Fixed-Fixed Mid Supt	-	149 (*220)	95 (~*108)	88
Compressed 0.4mm	1.65	125	-	46
Compressed 1.25 mm	1.65	124	-	46

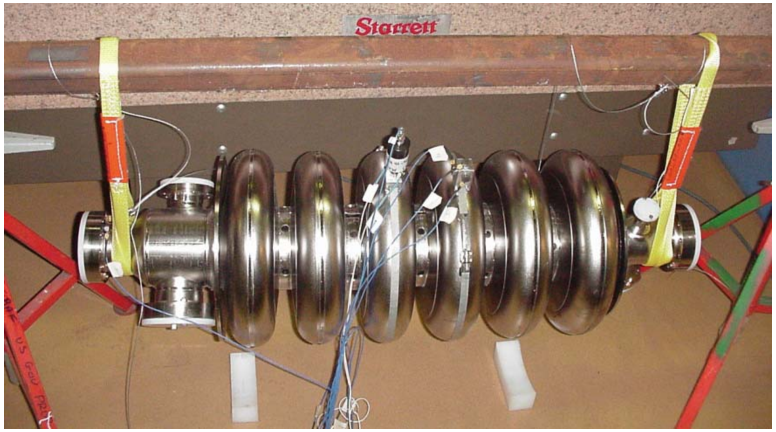
(*D. Schrage, LANL)
(~ Beta = 0.76)

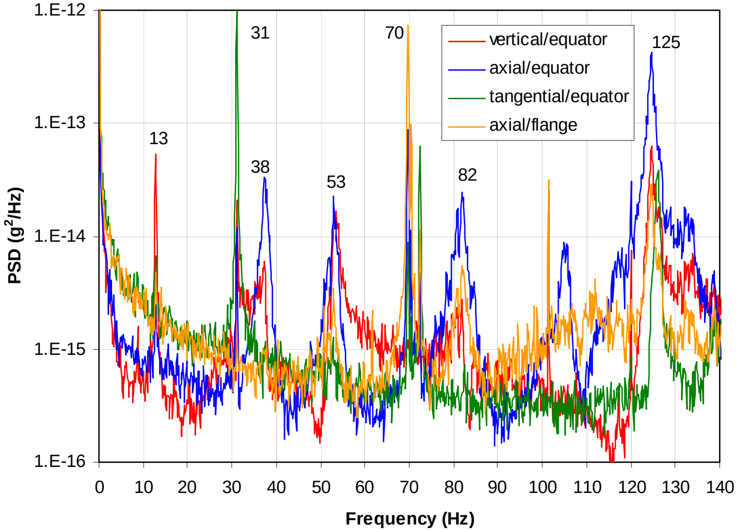
SNS Cavity Modal Analysis

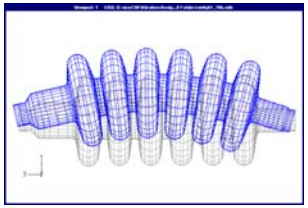
High Beta Cavity

End Condition	Load (atm)	127 mm Rings (Hz)	70 mm Rings (Hz)	No Rings (Hz)
Fixed-Fixed	-	120	-	46
Fixed-Guided	-	107	-	34
Compressed 0.4mm	1.65	120	-	44
Compressed 1.25 mm	1.65	119	-	44

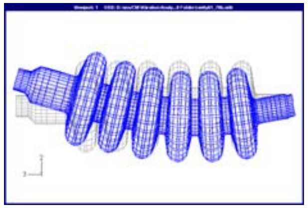
Mode Analysis, Beta = 0.81



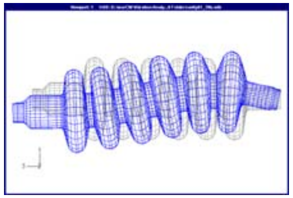




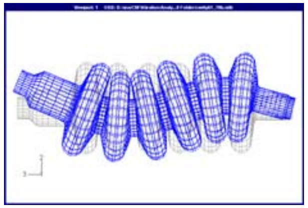
Mode 1 – 14 Hz



Mode 2 – 26 Hz



Mode 3 – 40 Hz



Mode 5 – 72 Hz

	Natural Frequency (Hz)	
Mode	**Test Data**	**FE Analysis**
1	13	14
2	31	26
3	38	40
4	53	48
5	70	72
6	82	83
7	125	124

SNS Cavity Mechanical Design Requirements

- Minimize/prevent microphonics
- Withstand loss of vacuum accident up to 5 atm
- Withstand cool down at 1.65 atm
- Adhere to intent of ASME B&P Code
  - – Allowable Stress (Sm) = 2/3 Yield Stress
  - – Primary Membrane Stress (Pm) <= (Sm)
  - – Pm + Bending <= 1.5*Sm
  - – Pm + Bending + Secondary Stress <= 3*Sm
    - Allowable Stresses
      - »**Warm Niobium = 4,667 psi**
      - »**Cold Niobium = 53,333 psi**

Medium Beta Stress Analysis

SNS Medium Beta Cavity Wall Stresses

Compression (mm/end)	Loads (atm)	127 mm Stiffening Ring Max Stress (psi)	70 mm Stiffening Ring Max Stress (psi)	No Stiffening Ring Max Stress (psi)
0.2	1.65	-	-	-
0.4	1.65	3,960	-	4,310
0.5	1.65	4,610	-	4,550
0.75	1.65	7,500	-	4,670
1.25	1.65	17,500	5,730 (1.8 atm)	5,000
0.75	5	11,200	-	12,900
1.25	5	14,300	10,100	47,100

High Beta Stress Analysis

SNS High Beta Cavity Wall Stresses				
Compression (mm/end)	Loads (atm)	127 mm Stiffening Ring Max Stress (psi)	70 mm Stiffening Ring Max Stress (psi)	No Stiffening Ring Max Stress (psi)
0.2	1.65	3,040	-	-
0.4	1.65	6,350	-	3,140
0.5	1.65	8,070	-	3,350
0.75	1.65	12,500	-	3,940
1.25	1.65	21,400	-	5,830
0.75	5	11,500	-	9,130
1.25	5	14,300	-	9,590