RF FUNDAMENTALS and BEAM LOADING

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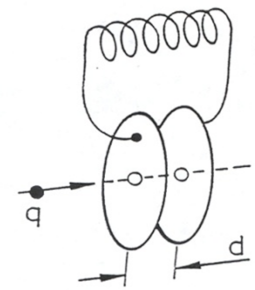
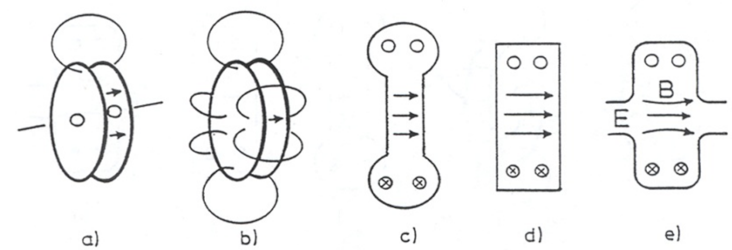
Equivalent Circuit for an rf Cavity

Simple LC circuit representing an accelerating resonator



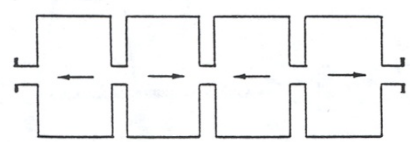
Simple lumped L-C circuit repesenting an accelerating resonator.
$\omega_o^2 = 1/LC$

Metamorphosis of the LC circuit into an accelerating cavity



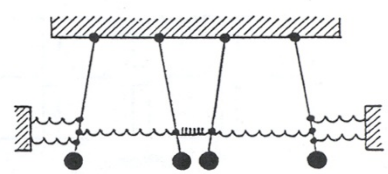
Metamorphosis of the L-C circuit of Fig. 1 into an accelerating cavity (after R.P.Feynman[33]). Fig. 5d shows the cylindrical "pillbox cavity" and Fig. 5e a slightly modified pillbox cavity with beam holes (typical β between 0.5 and 1.0). Fig. 5c resembles a low β version of the pillbox variety (0.2<β<0.5).

Chain of weakly coupled pillbox cavities representing an accelerating cavity



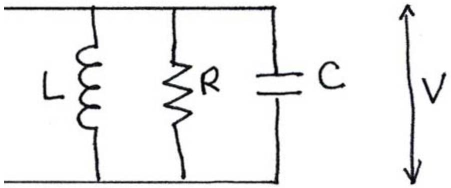
Chain of weakly-coupled pillbox cavities representing an accelerating module

Chain of coupled pendula as its mechanical analogue



Chain of coupled pendula as a mechanical analogue to Fig. 6a

Parallel Circuit Model of an Electromagnetic Mode



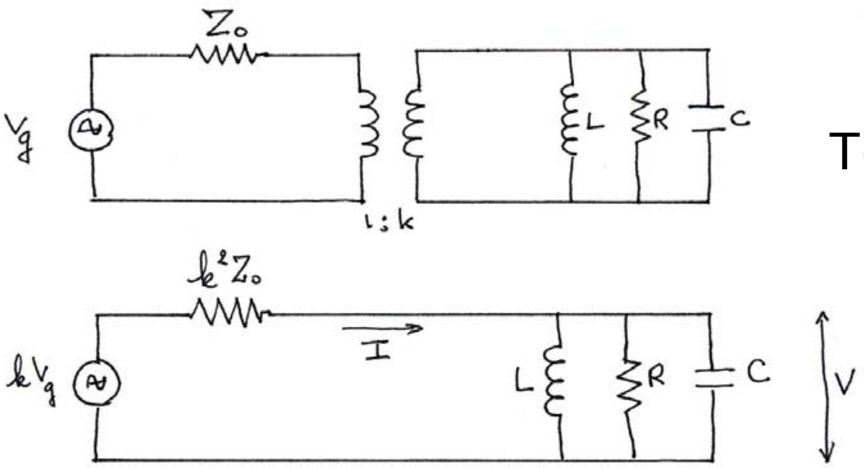
- Power dissipated in resistor R: $P_{diss} = \frac{1}{2}\frac{V_c^2}{R}$
- Shunt impedance: $R_{sh} \equiv \frac{V_c^2}{P_{diss}}$ $\Rightarrow \; R_{sh} = 2R$
- Quality factor of resonator:

$$Q_0 \equiv \frac{\omega_0 U}{P_{diss}} = \omega_0 CR = \frac{R}{L\omega_c} = R\left(\frac{C}{L}\right)^{1/2}$$

$$\tilde{Z} = R\left[1 + iQ_0\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right)\right]^{-1}$$

$$\omega \approx \omega_0 \;, \qquad \tilde{Z} \approx R\left[1 + 2iQ_0\left(\frac{\omega - \omega_0}{\omega_0}\right)\right]^{-1}$$

1-Port System



Total impedance: $k^2 Z_0 + \dfrac{R}{1 + 2i\dfrac{Q_0}{\omega_0}\Delta\omega}$

$$I = \frac{kV_g}{k^2 Z_0 + \dfrac{R}{1 + 2i\dfrac{Q_0}{\omega_0}\Delta\omega}} \qquad V = kV_g \frac{R}{R + k^2 Z_0\left(1 + 2i\dfrac{Q_0}{\omega_0}\Delta\omega\right)}$$

1-Port System

Energy content $U = \frac{1}{2} CV^2 = \frac{1}{2}\frac{Q_0}{\omega R} V^2$

$$= \frac{1}{2}\frac{Q_0}{\omega R} \; k^2 V_g^2 \; \frac{R^2}{\left(R + k^2 Z_0\right)^2 + 4k^4 Z_0^2 Q_0^2 \left(\frac{\Delta\omega}{\omega_0}\right)^2}$$

Incident power: $P_{inc} = \frac{V_g^2}{8Z_0}$

Define coupling coefficient: $\beta = \frac{R}{k_0^2 Z_0}$

$$\frac{U}{P_{inc}} = \frac{Q_0}{\omega_0} \frac{4\beta}{(1+\beta)^2} \frac{1}{1 + \left(\frac{2Q_0}{1+\beta}\right)^2 \left(\frac{\Delta\omega}{\omega_0}\right)^2}$$

1-Port System

Power dissipated

$$P_{diss} = \frac{\omega U}{Q_0} = P_{inc} \, \frac{4\beta}{(1+\beta)^2} \, \frac{1}{1+\left(\frac{2Q_0}{1+\beta}\right)^2 \left(\frac{\Delta\omega}{\omega_0}\right)^2}$$

Optimal coupling: $\frac{U}{P_{inc}}$ maximum or $P_{diss} = P_{inc}$

$\Rightarrow \Delta\omega = 0, \quad \beta = 1$: critical coupling

Reflected power

$$P_{ref} = P_{inc} - P_{diss} = P_{mc} \left[1 - \frac{4\beta}{(1+\beta)^2} \, \frac{1}{1+\left(\frac{2Q_0}{1+\beta} \, \frac{\Delta\omega}{\omega_0}\right)^2} \right]$$

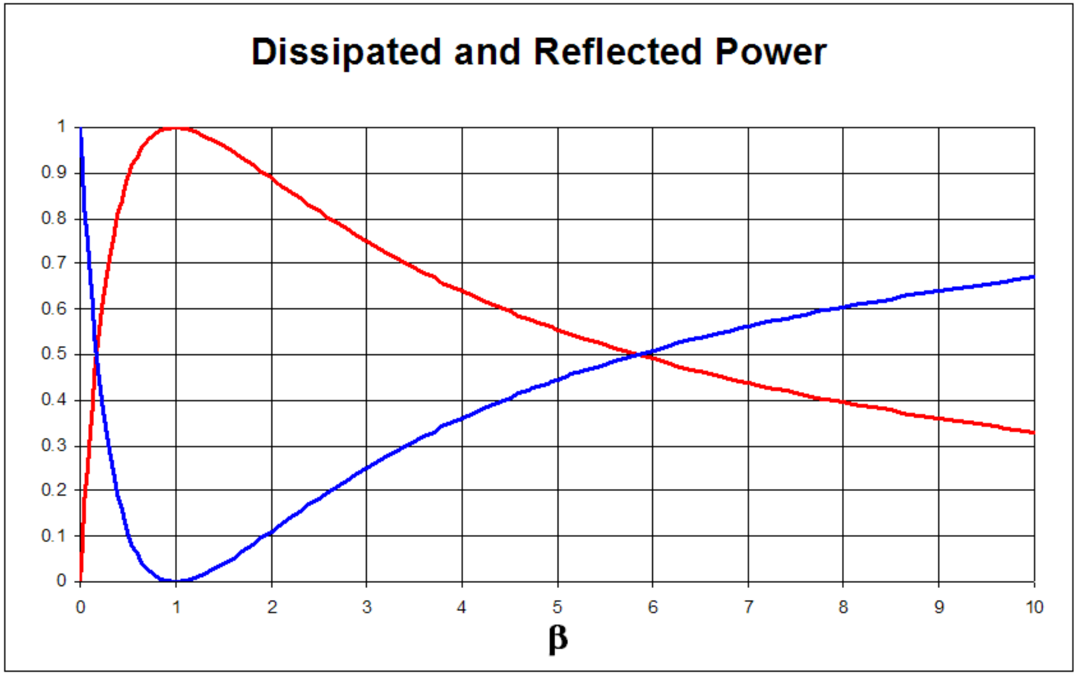
1-Port System

At resonance

$$U = \frac{Q_0}{\omega_0} \frac{4\beta}{(1+\beta)^2} P_{inc}$$

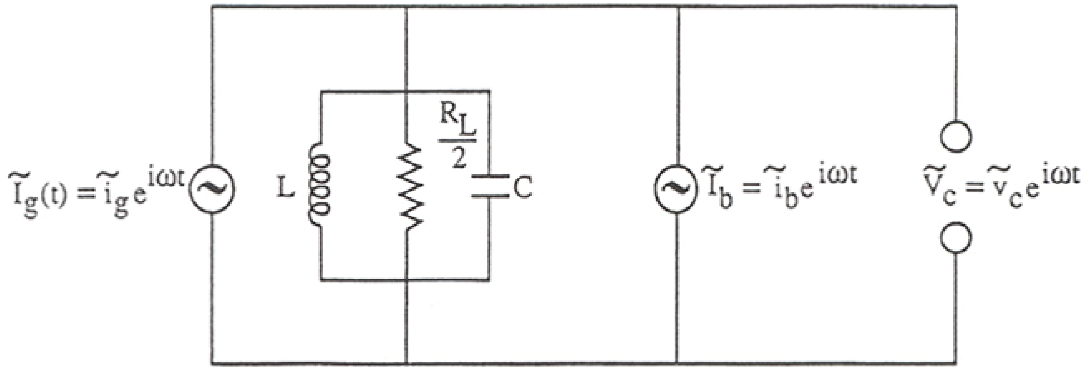
$$P_{diss} = \frac{4\beta}{(1+\beta)^2} P_{inc}$$

$$P_{ref} = \left(\frac{1-\beta}{1+\beta}\right)^2 P_{inc}$$



Equivalent Circuit for a Cavity with Beam

- Beam in the rf cavity is represented by a current generator.
- Equivalent circuit:



$$R_L = \frac{R_{sh}}{(1+\beta)}$$

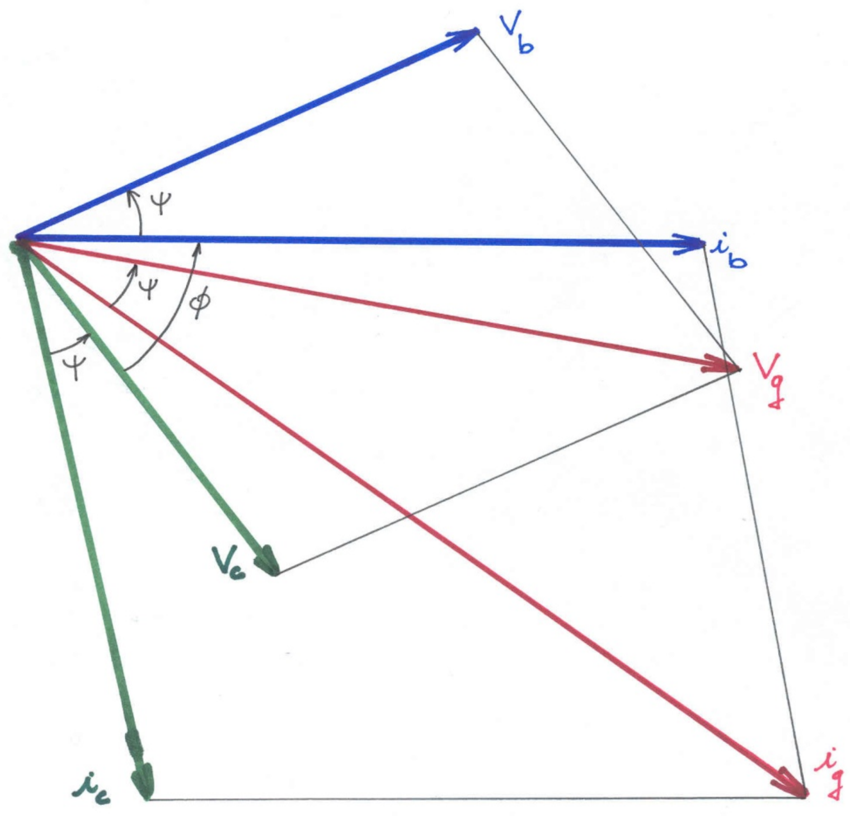
$\tilde{i}_b$ produces $\tilde{V}_b$ with phase ψ (detuning angle)

$\tilde{i}_g$ produces $\tilde{V}_g$ with phase ψ

$$\tilde{V}_c = \tilde{V}_g - \tilde{V}_b$$

$$\tan\ \psi = -2\frac{Q_0}{1+\beta}\frac{\Delta\omega}{\omega_0}$$

Equivalent Circuit for a Cavity with Beam



$$V_g = (P_g R_{sh})^{1/2} \frac{2\beta^{1/2}}{1+\beta} \cos\psi$$

$$V_b = \frac{i_b R_{sh}}{2(1+\beta)} \cos\psi$$

$$i_b = 2i_0 \frac{\sin\frac{\theta_b}{2}}{\frac{\theta_b}{2}}$$

i_b: beam rf current

i_0: beam dc current

θ_b: beam bunch length

Equivalent Circuit for a Cavity with Beam

$$P_g = \frac{V_c^2}{R_{sh}} \frac{1}{4\beta} \left\{ (1+\beta+b)^2 + \left[(1+\beta)\tan\psi - b\tan\phi \right]^2 \right\}$$

$$b = \frac{\text{Power absorbed by the beam}}{\text{Power dissipated in the cavity}} = \frac{R_{sh} i_0 \cos\phi}{V_c}$$

Minimize P_g:

$$(1+\beta_{opt})\tan\psi_{opt} = b\ \tan\phi$$

$$\beta_{opt} = |1+b|$$

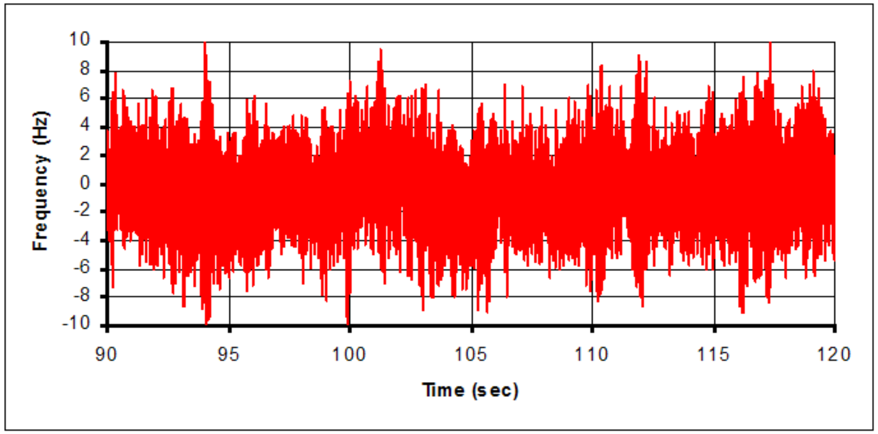
$$P_g^{opt} = \frac{V_c^2}{R_{sh}} \frac{|1+b| + (1+b)}{2}$$

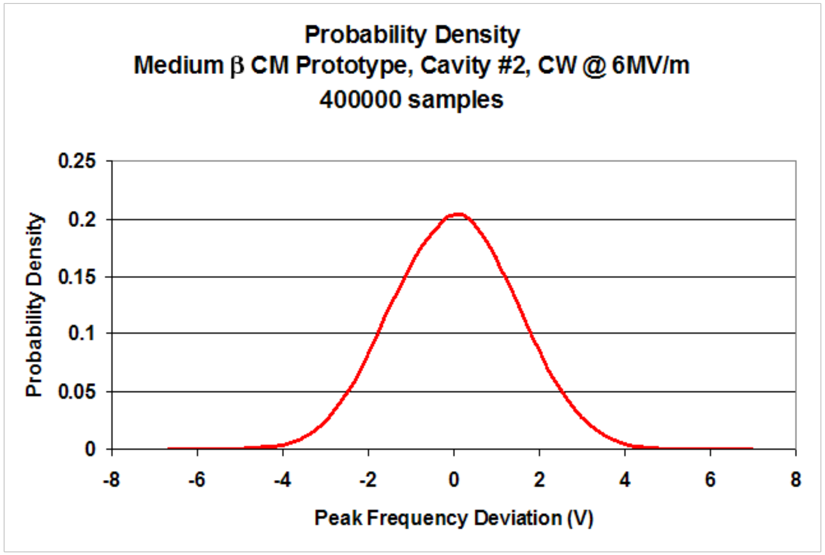
Cavity with Beam and Microphonics

- The detuning is now $\tan\psi = -2Q_L \frac{\delta\omega_0 \pm \delta\omega_m}{\omega_0}$ $\tan\psi_0 = -2Q_L \frac{\delta\omega_0}{\omega_0}$

where $\delta\omega_0$ is the static detuning (controllable)

and $\delta\omega_m$ is the random dynamic detuning (uncontrollable)





Q_{ext} Optimization with Microphonics

Condition for optimum coupling:

$$\beta_{opt} = \sqrt{(b+1)^2 + \left(2Q_0 \frac{\delta\omega_m}{\omega_0}\right)^2}$$

and

$$P_g^{opt} = \frac{V_c^2}{2R_{sh}} \left[(b+1) + \sqrt{(b+1)^2 + \left(2Q_0 \frac{\delta\omega_m}{\omega_0}\right)^2} \right]$$

In the absence of beam (b=0):

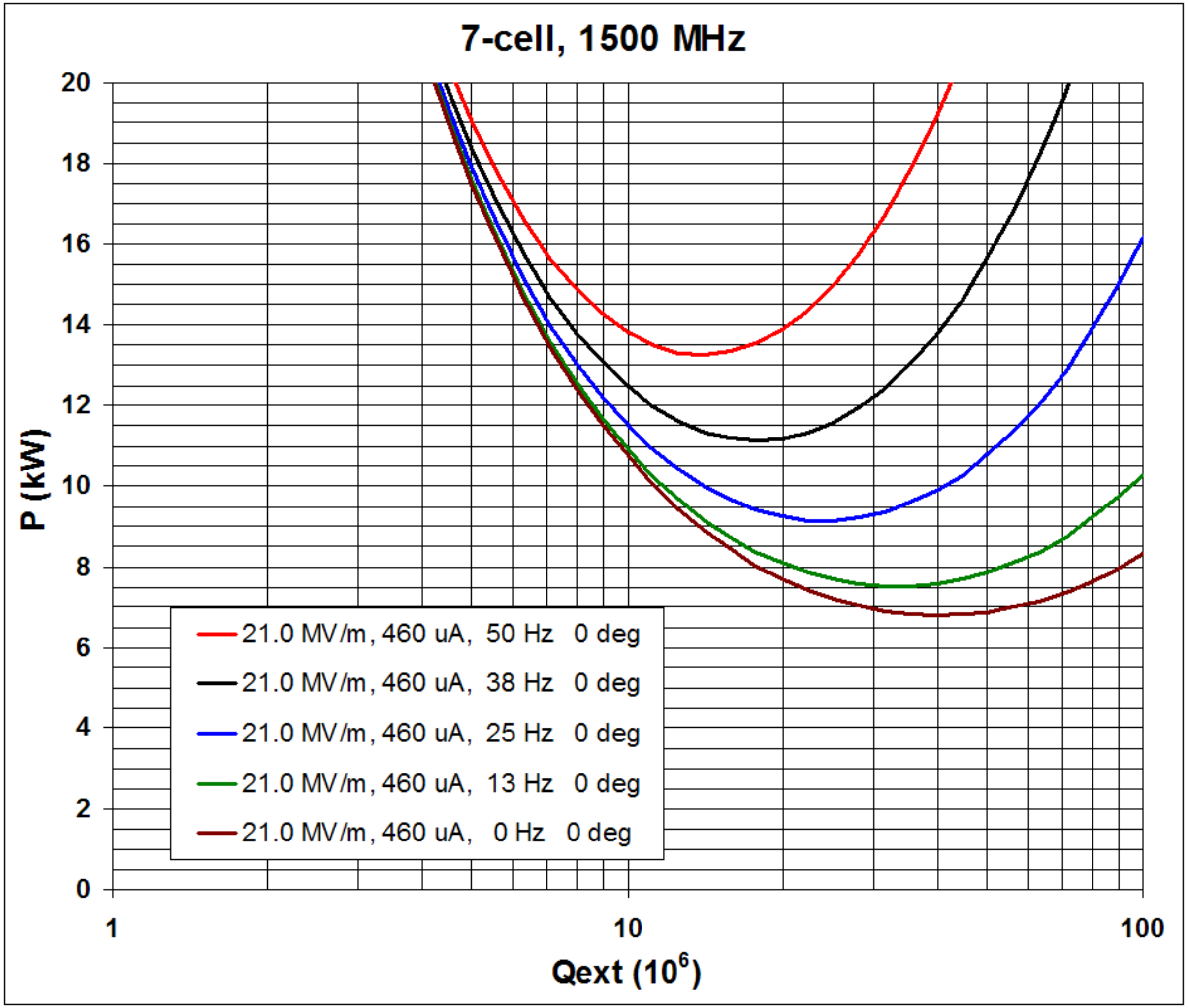
$$\beta_{opt} = \sqrt{1 + \left(2Q_0 \frac{\delta\omega_m}{\omega_0}\right)^2}$$

and

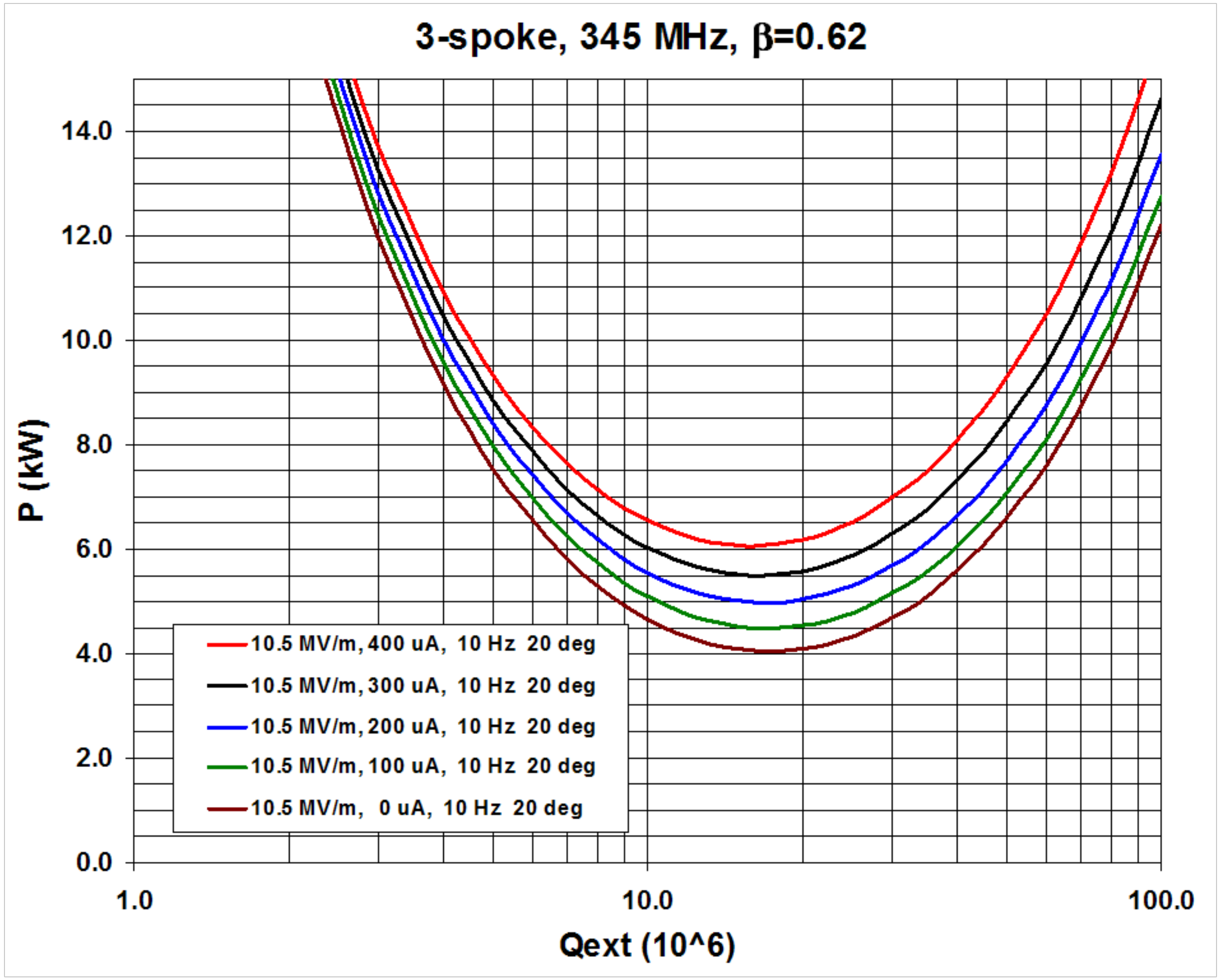
$$P_g^{opt} = \frac{V_c^2}{2R_{sh}} \left[1 + \sqrt{1 + \left(2Q_0 \frac{\delta\omega_m}{\omega_0}\right)^2} \right]$$

$\simeq U\ \delta\omega_m$ If $\delta\omega_m$ is very large

Example


7-cell, 1500 MHz
P (kW)
20
18
16
14
12
10
8
6
4
2
0
1
10
100
Qext (10^6)
21.0 MV/m, 460 uA, 50 Hz 0 deg
21.0 MV/m, 460 uA, 38 Hz 0 deg
21.0 MV/m, 460 uA, 25 Hz 0 deg
21.0 MV/m, 460 uA, 13 Hz 0 deg
21.0 MV/m, 460 uA, 0 Hz 0 deg

Example


3-spoke, 345 MHz, β=0.62
P (kW)
14.0
12.0
10.0
8.0
6.0
4.0
2.0
0.0
1.0
10.0
100.0
Qext (10^6)
10.5 MV/m, 400 uA, 10 Hz 20 deg
10.5 MV/m, 300 uA, 10 Hz 20 deg
10.5 MV/m, 200 uA, 10 Hz 20 deg
10.5 MV/m, 100 uA, 10 Hz 20 deg
10.5 MV/m, 0 uA, 10 Hz 20 deg

Example

- **ERL Injector and Linac:**

  f_m=25 Hz, Q_0=1x10^{10} , f_0=1300 MHz, I_0=100 mA, V_c=20 MV/m, L=1.04 m, R_a/Q_0=1036 ohms per cavity

- **ERL linac: Resultant beam current, I_{tot} = 0 mA (energy recovery)**

  and $_{opt}$=385 Q_L=2.6x10^7 P_g = 4 kW per cavity.

- **ERL Injector: I_0=100 mA and $_{opt}$= 5x10^4 ! Q_L= 2x10^5 P_g = 2.08 MW per cavity!**

Note: I_0V_a = 2.08 MW optimization is entirely dominated by beam loading.

RF System Modeling

- To include amplitude and phase feedback, nonlinear effects from the klystron and be able to analyze transient response of the system, response to large parameter variations or beam current fluctuations
  - We developed a model of the cavity and low level controls using SIMULINK, a MATLAB-based program for simulating dynamic systems.
- Model describes the beam-cavity interaction, includes a realistic representation of low level controls, klystron characteristics, microphonic noise, Lorentz force detuning and coupling and excitation of mechanical resonances

RF System Model