PONDEROMOTIVE INSTABILITIES, MICROPHONICS, and RF CONTROL

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Frequency Control

Energy gain $$W = qV\cos\phi$$

Energy gain error $$\frac{\delta W}{W} = \frac{\delta V}{V} - \delta\phi \tan\phi$$

The fluctuations in cavity field amplitude and phase come mostly from the fluctuations in cavity frequency

Need for fast frequency control

Minimization of rf power requires matching of average cavity frequency to reference frequency

Need for slow frequency tuners

Some Definitions

- **Ponderomotive effects: changes in frequency caused by the electromagnetic field (radiation pressure)**
  - **Static Lorentz detuning (cw operation)**
  - **Dynamic Lorentz detuning (pulsed operation)**

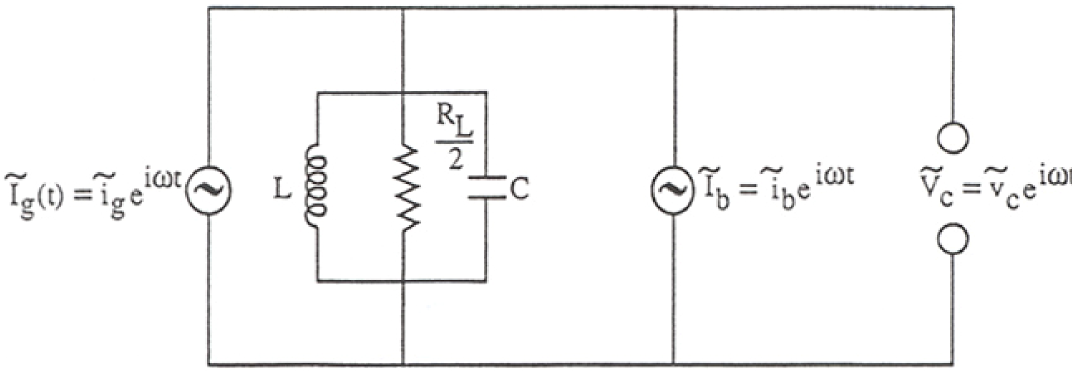
- **Microphonics: changes in frequency caused by connections to the external world**
  - **Vibrations**
  - **Pressure fluctuations**

Note: The two are not completely independent.

When phase and amplitude feedbacks are active, ponderomotive effects can change the response to external disturbances

Equivalent Circuit for a Cavity with Beam

- **Beam in the rf cavity is represented by a current generator.**
- **Equivalent circuit:**



$$R_L = \frac{R_{sh}}{(1+\beta)}$$

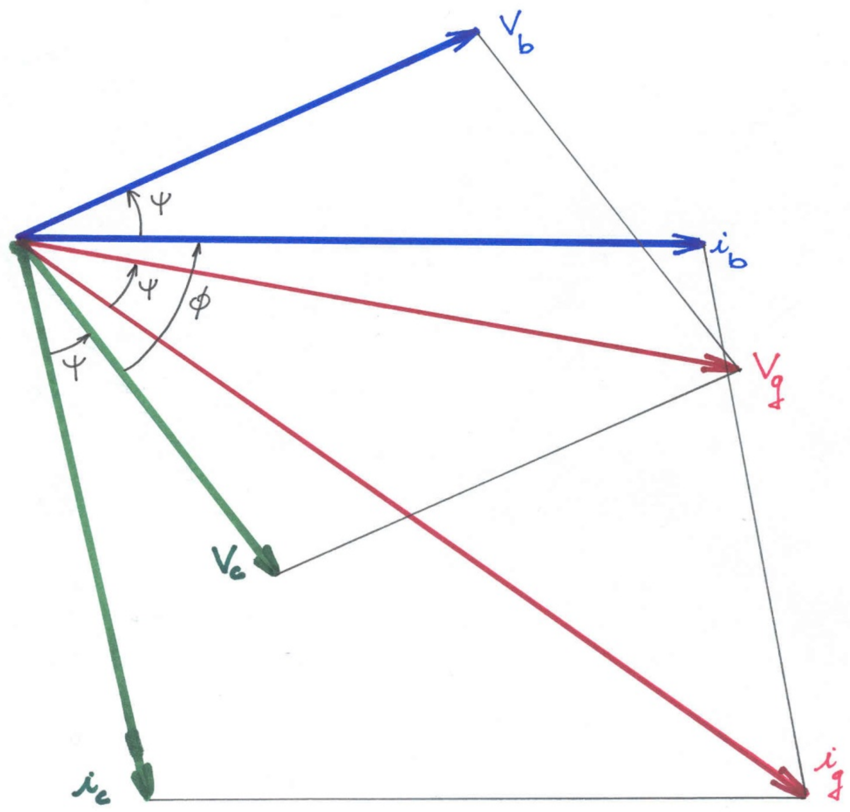
$\tilde{i}_b$ produces $\tilde{V}_b$ with phase ψ (detuning angle)

$\tilde{i}_g$ produces $\tilde{V}_g$ with phase ψ

$$\tilde{V}_c = \tilde{V}_g - \tilde{V}_b$$

$$\tan \psi = -2\frac{Q_0}{1+\beta}\frac{\Delta\omega}{\omega_0}$$

Equivalent Circuit for a Cavity with Beam



$$V_g = (P_g R_{sh})^{1/2} \frac{2\beta^{1/2}}{1+\beta} \cos\psi$$

$$V_b = \frac{i_b R_{sh}}{2(1+\beta)} \cos\psi$$

$$i_b = 2 i_0 \frac{\sin\frac{\theta_b}{2}}{\frac{\theta_b}{2}}$$

i_b: beam rf current

i_0: beam dc current

θ_b: beam bunch length

Equivalent Circuit for a Cavity with Beam

$$P_g = \frac{V_c^2}{R_{sh}} \frac{1}{4\beta} \left\{ (1+\beta+b)^2 + \left[(1+\beta)\tan\psi - b\tan\phi \right]^2 \right\}$$

$$b = \frac{\text{Power absorbed by the beam}}{\text{Power dissipated in the cavity}} = \frac{R_{sh} i_0 \cos\phi}{V_c}$$

Minimize P_g:

$$(1+\beta_{opt})\tan\psi_{opt} = b\ \tan\phi$$

$$\beta_{opt} = |1+b|$$

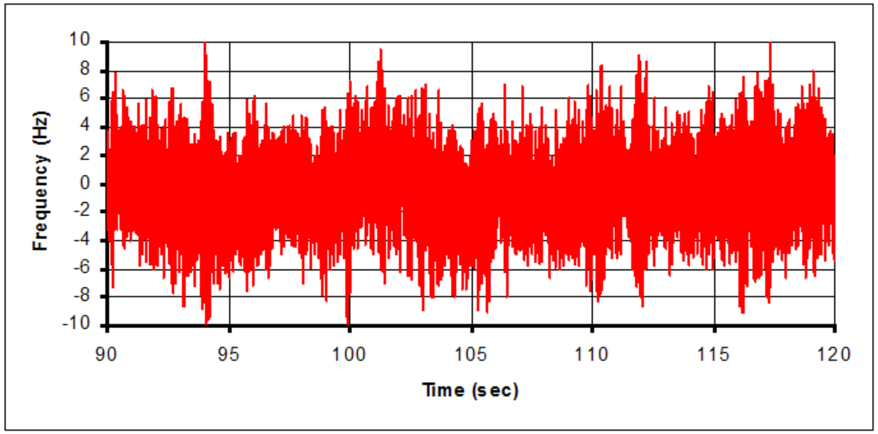
$$P_g^{opt} = \frac{V_c^2}{R_{sh}} \frac{|1+b| + (1+b)}{2}$$

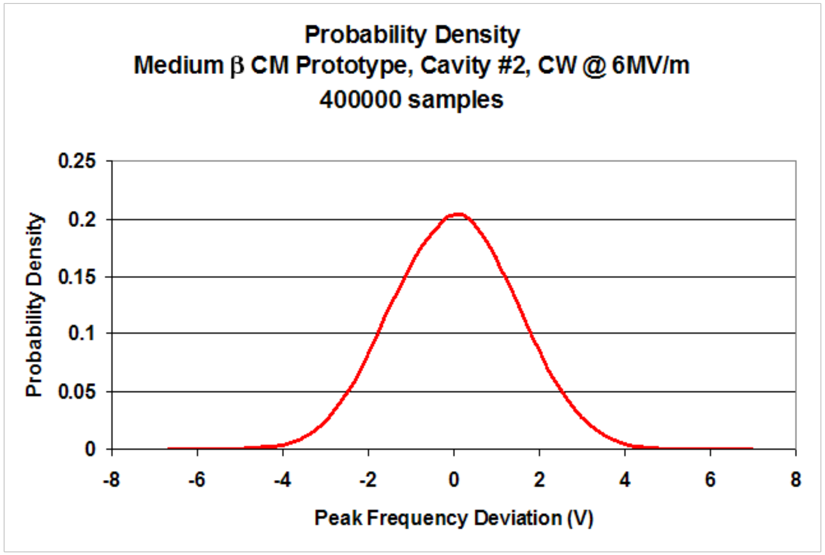
Cavity with Beam and Microphonics

- The detuning is now $\tan\psi = -2Q_L \dfrac{\delta\omega_0 \pm \delta\omega_m}{\omega_0}$ $\tan\psi_0 = -2Q_L \dfrac{\delta\omega_0}{\omega_0}$

  where $\delta\omega_0$ is the static detuning (controllable)

  and $\delta\omega_m$ is the random dynamic detuning (uncontrollable)





Q_{ext} Optimization with Microphonics

Condition for optimum coupling:

$$\beta_{opt} = \sqrt{(b+1)^2 + \left(2Q_0 \frac{\delta\omega_m}{\omega_0}\right)^2}$$

and

$$P_g^{opt} = \frac{V_c^2}{2R_{sh}}\left[(b+1) + \sqrt{(b+1)^2 + \left(2Q_0 \frac{\delta\omega_m}{\omega_0}\right)^2}\right]$$

In the absence of beam (b=0):

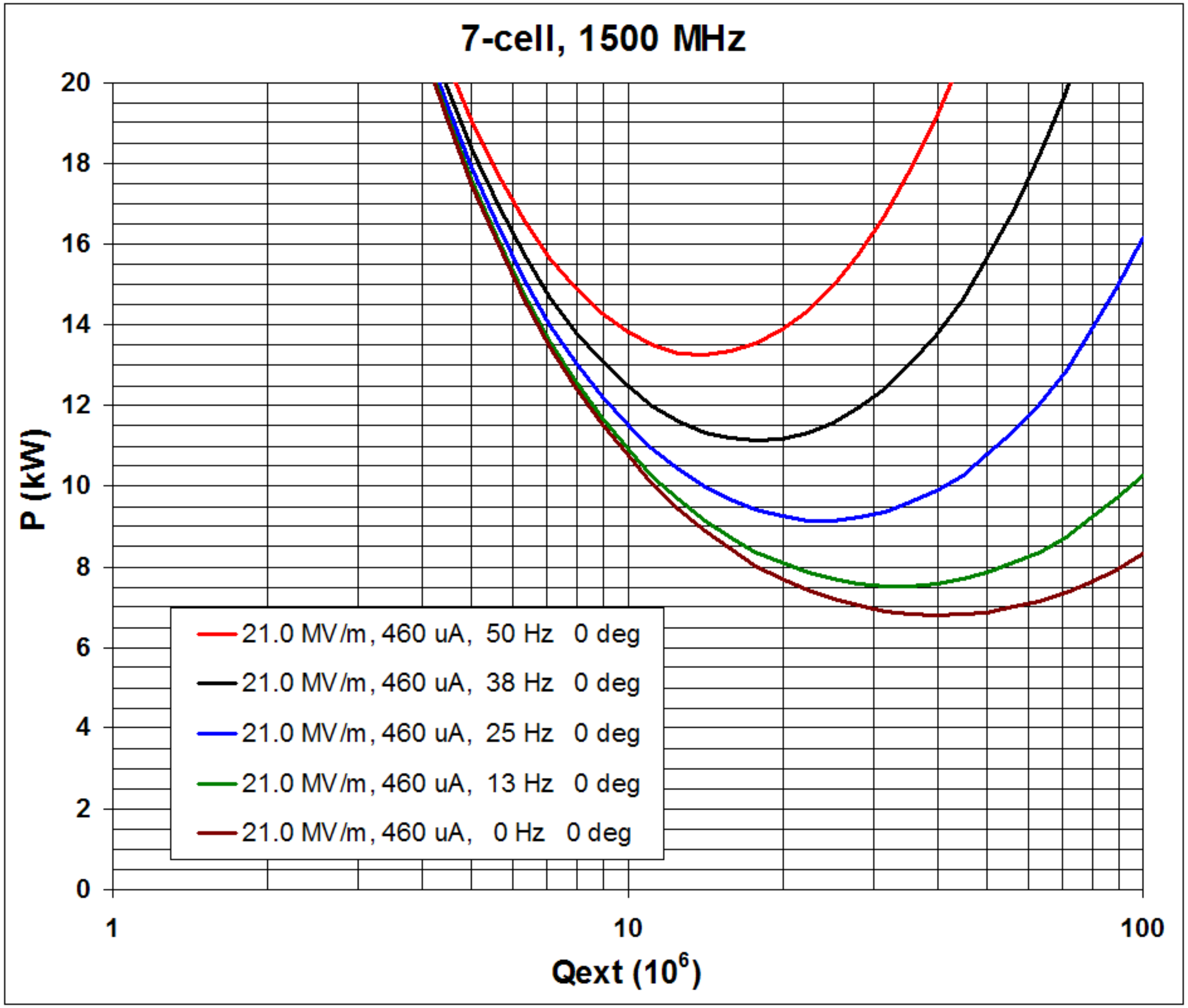
$$\beta_{opt} = \sqrt{1 + \left(2Q_0 \frac{\delta\omega_m}{\omega_0}\right)^2}$$

and

$$P_g^{opt} = \frac{V_c^2}{2R_{sh}}\left[1 + \sqrt{1 + \left(2Q_0 \frac{\delta\omega_m}{\omega_0}\right)^2}\right]$$

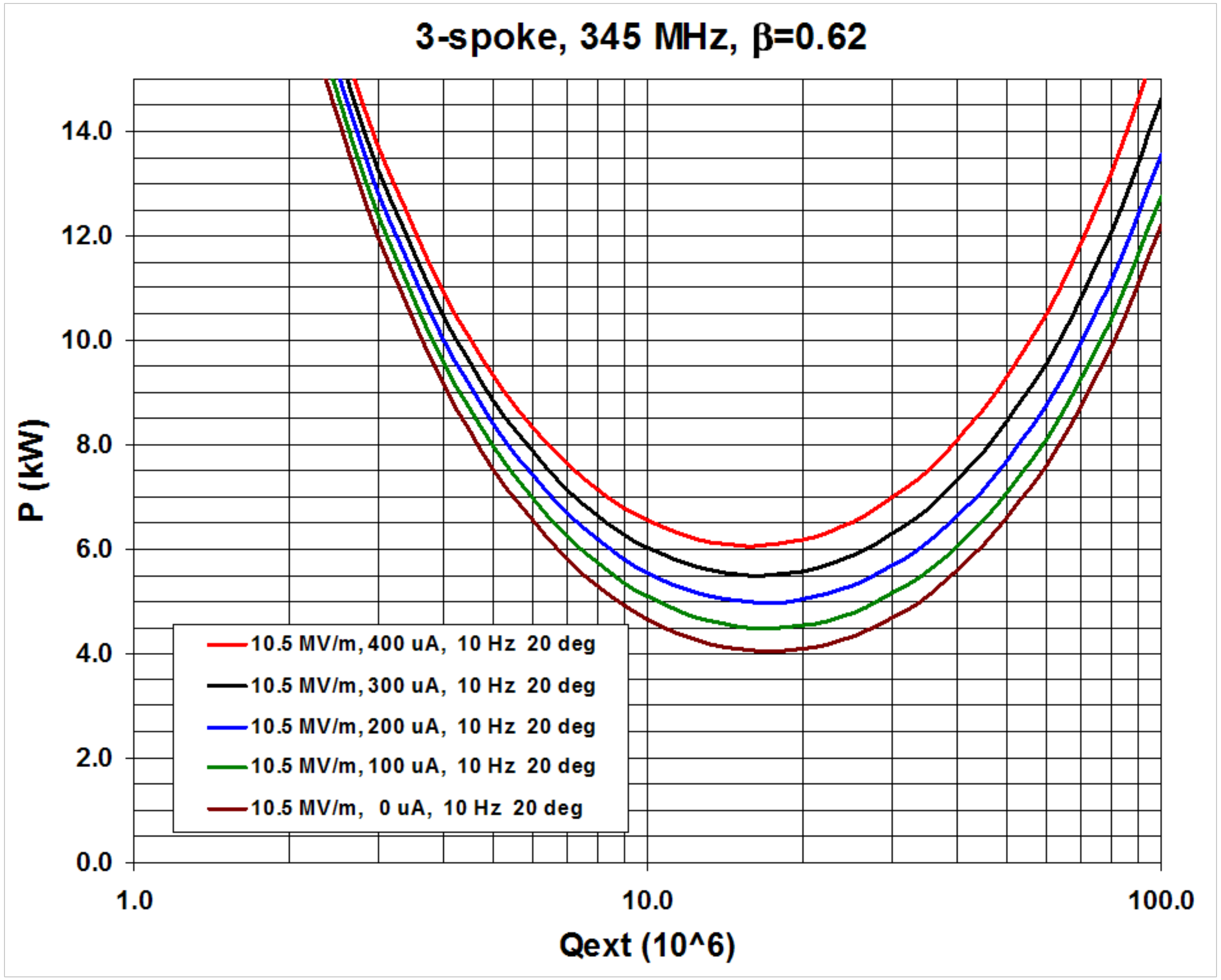
$\simeq U\,\delta\omega_m$ If $\delta\omega_m$ is very large

Example



7-cell, 1500 MHz
20
18
16
14
12
10
8
6
4
2
0
P (kW)
1
10
100
Qext (10^6)
21.0 MV/m, 460 uA, 50 Hz 0 deg
21.0 MV/m, 460 uA, 38 Hz 0 deg
21.0 MV/m, 460 uA, 25 Hz 0 deg
21.0 MV/m, 460 uA, 13 Hz 0 deg
21.0 MV/m, 460 uA, 0 Hz 0 deg

Example

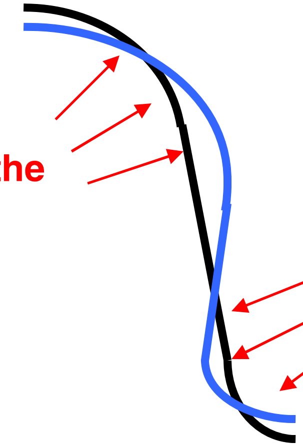

3-spoke, 345 MHz, β=0.62
14.0
12.0
10.0
8.0
6.0
4.0
2.0
0.0
P (kW)
1.0
10.0
100.0
Qext (10^6)
10.5 MV/m, 400 uA, 10 Hz 20 deg
10.5 MV/m, 300 uA, 10 Hz 20 deg
10.5 MV/m, 200 uA, 10 Hz 20 deg
10.5 MV/m, 100 uA, 10 Hz 20 deg
10.5 MV/m, 0 uA, 10 Hz 20 deg

Lorentz Detuning

Pressure deforms the cavity wall:

RF power produces radiation pressure:

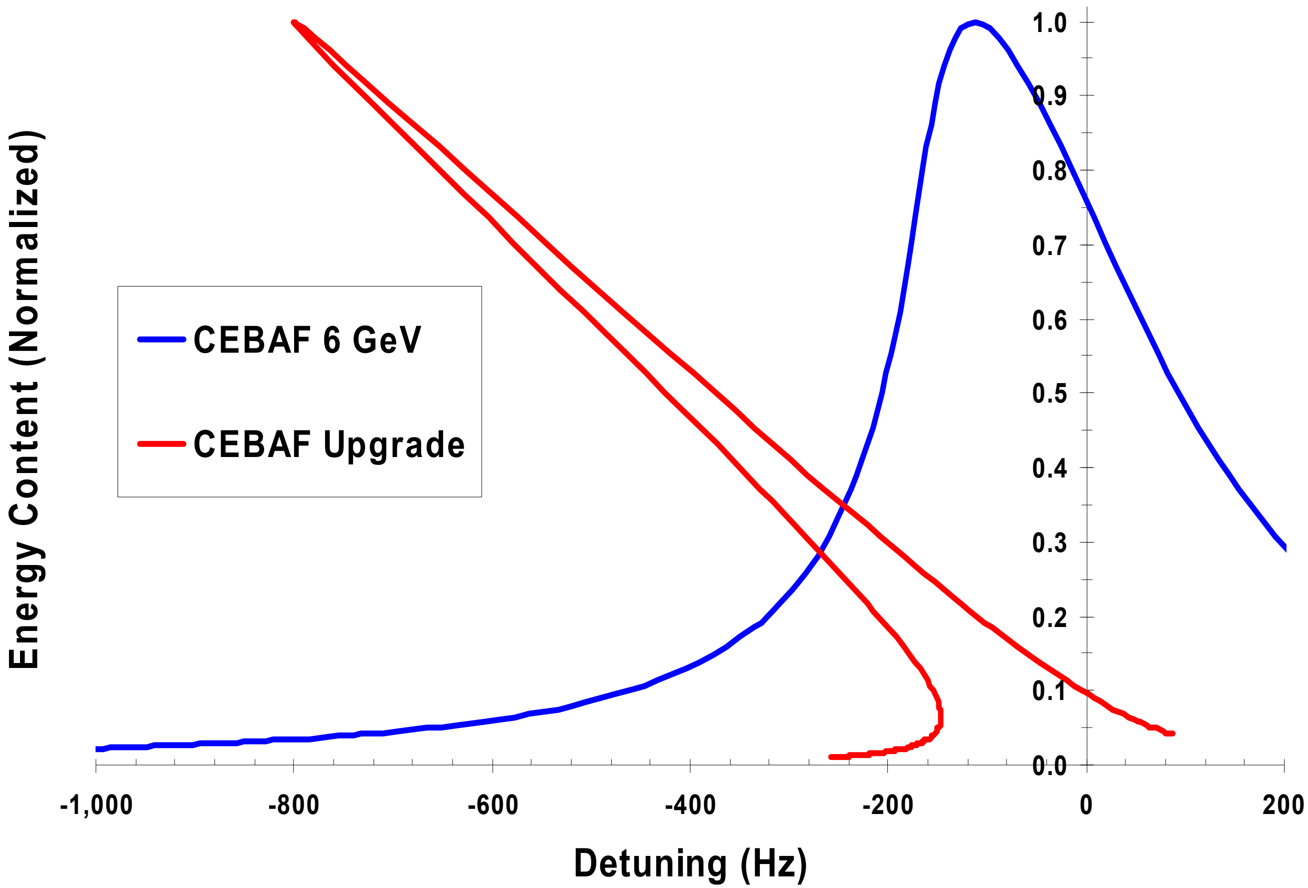
$$P = \frac{\mu_0 H^2 - \varepsilon_0 E^2}{4}$$



Deformation produces a frequency shift:

$$\Delta f = -k_L E_{acc}^2$$

Lorentz Detuning

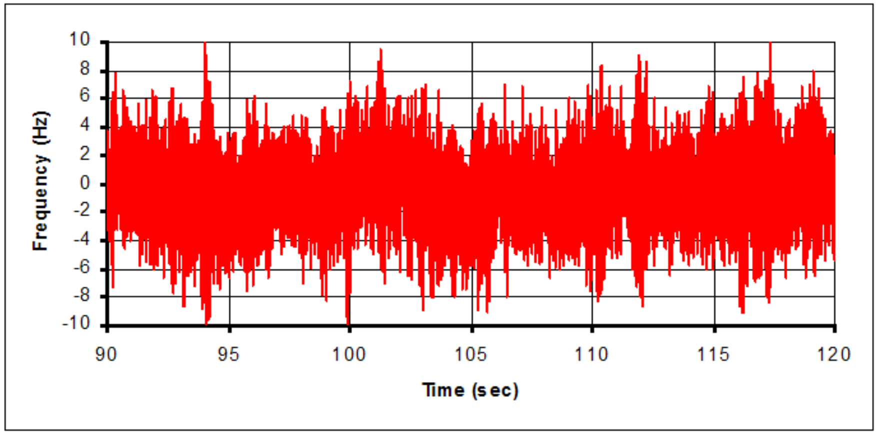


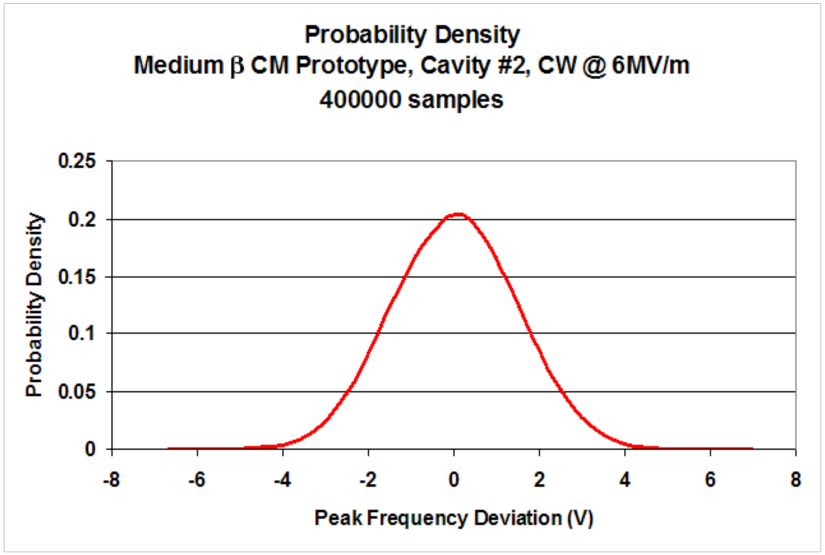
Microphonics

- **Total detuning** $\delta\omega_0 + \delta\omega_m$

  where $\delta\omega_0$ is the static detuning (controllable)

  and $\delta\omega_m$ is the random dynamic detuning (uncontrollable)





Ponderomotive Effects

- **Adiabatic theorem applied to harmonic oscillators (Boltzmann-Ehrenfest)**

If $\varepsilon = \dfrac{1}{\omega^2}\dfrac{d\omega}{dt} \ll 1$, then $\dfrac{U}{\omega}$ is an adiabatic invariant to all orders

$$\Delta\left(\frac{U}{\omega}\right)\Big/\left(\frac{U}{\omega}\right) \sim o(e^{-d/\varepsilon}) \quad \Rightarrow \quad \boxed{\frac{\Delta\omega}{\omega} = \frac{\Delta U}{U}}$$ (Slater)

Quantum mechanical picture: the number of photons is constant: $U = N\hbar\omega$

$$U = \int_V dv \left[\frac{\mu_0}{4} H^2(\vec{r}) + \frac{\varepsilon_0}{4} E^2(\vec{r})\right]$$ (energy content)

$$\Delta U = -\int_S dS\, \vec{n}(\vec{r}) \cdot \vec{\xi}(\vec{r}) \left[\frac{\mu_0}{4} H^2(\vec{r}) - \frac{\varepsilon_0}{4} E^2(\vec{r})\right]$$ (work done by radiation pressure)

Ponderomotive Effects

$$\frac{\Delta\omega}{\omega} = -\frac{\int_S dS\, \vec{n}(\vec{r})\cdot\vec{\xi}(\vec{r}) \left[\frac{\mu_0}{4}H^2(\vec{r}) - \frac{\varepsilon_0}{4}E^2(\vec{r})\right]}{\int_V dv \left[\frac{\mu_0}{4}H^2(\vec{r}) + \frac{\varepsilon_0}{4}E^2(\vec{r})\right]}$$

Expand wall displacements and forces in normal modes of vibration $\phi_\mu(\vec{r})$ of the resonator

$$\int_S dS\, \phi_\mu(\vec{r})\, \phi_\nu(\vec{r}) = \delta_{\mu\nu}$$

$$\xi(\vec{r}) = \sum_\mu q_\mu \phi_\mu(\vec{r}) \qquad q_\mu = \int_S \xi(\vec{r})\, \phi_\mu(\vec{r})\, dS$$

$$F(\vec{r}) = \sum_\mu F_\mu \phi_\mu(\vec{r}) \qquad F_\mu = \int_S F(\vec{r})\, \phi_\mu(\vec{r})\, dS$$

Ponderomotive Effects

Equation of motion of mechanical mode μ

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{q}_\mu} - \frac{\partial L}{\partial q_\mu} + \frac{\partial \Phi}{\partial \dot{q}_\mu} = F_\mu \qquad L = T - U \qquad \text{(Euler-Lagrange)}$$

$$U = \frac{1}{2}\sum_\mu c_\mu q_\mu^2 \quad \text{(elastic potential energy)}$$

c_μ: elastic constant

$$T = \frac{1}{2}\sum_\mu c_\mu \frac{\dot{q}_\mu^2}{\Omega_\mu^2} \quad \text{(kinetic energy)}$$

Ω_μ: frequency

$$\Phi = \sum_\mu \frac{c_\mu}{\tau_\mu}\frac{\dot{q}_\mu^2}{\Omega_\mu^2} \quad \text{(power loss)}$$

τ_μ: decay time

$$\boxed{\ddot{q}_\mu + \frac{2}{\tau_\mu}\dot{q}_\mu + \Omega_\mu^2 q_\mu = \frac{\Omega_\mu^2}{c_\mu}F_\mu}$$

Ponderomotive Effects

The frequency shift $\Delta\omega_\mu$ caused by the mechanical mode μ is proportional to q_μ

$$\Delta\ddot{\omega}_\mu + \frac{2}{\tau_\mu}\Delta\dot{\omega}_\mu + \Omega_\mu^2\Delta\omega_\mu = -\frac{\omega_0}{c_\mu}\left(\frac{F_\mu}{U}\right)^2 \Omega_\mu^2\, U = -k_\mu \Omega_\mu^2 V^2$$

Total frequency shift: $\Delta\omega(t) = \sum_\mu \Delta\omega_\mu(t)$

Static frequency shift: $\Delta\omega_0 = \sum_\mu \Delta\omega_{\mu 0} = -V^2 \sum_\mu k_\mu$

Static Lorentz coefficient: $k = \sum_\mu k_\mu$

Ponderomotive Effects – Mechanical Modes

$$\Delta\ddot{\omega}_\mu + \frac{2}{\tau_\mu}\Delta\dot{\omega}_\mu + \Omega_\mu{}^2\Delta\omega_\mu = -\Omega_\mu^2 k_\mu V_0{}^2 + \cancel{n(t)}$$

Fluctuations around steady state:

$$\Delta\omega_\mu = \Delta\omega_{\mu o} + \delta\omega_\mu$$

$$V = V_0(1+\delta v)$$

Linearized equation of motion for mechanical mode:

$$\delta\ddot{\omega}_\mu + \frac{2}{\tau_\mu}\delta\dot{\omega}_\mu + \Omega_\mu^2\delta\omega_\mu = -2\Omega_\mu^2 k_\mu V_0{}^2\delta v$$

The mechanical mode is driven by fluctuations in the electromagnetic mode amplitude.

Variations in the mechanical mode amplitude causes a variation of the electromagnetic mode frequency, which can cause a variation of its amplitude.

→Closed feedback system between electromagnetic and mechanical modes, that can lead to instabilities.

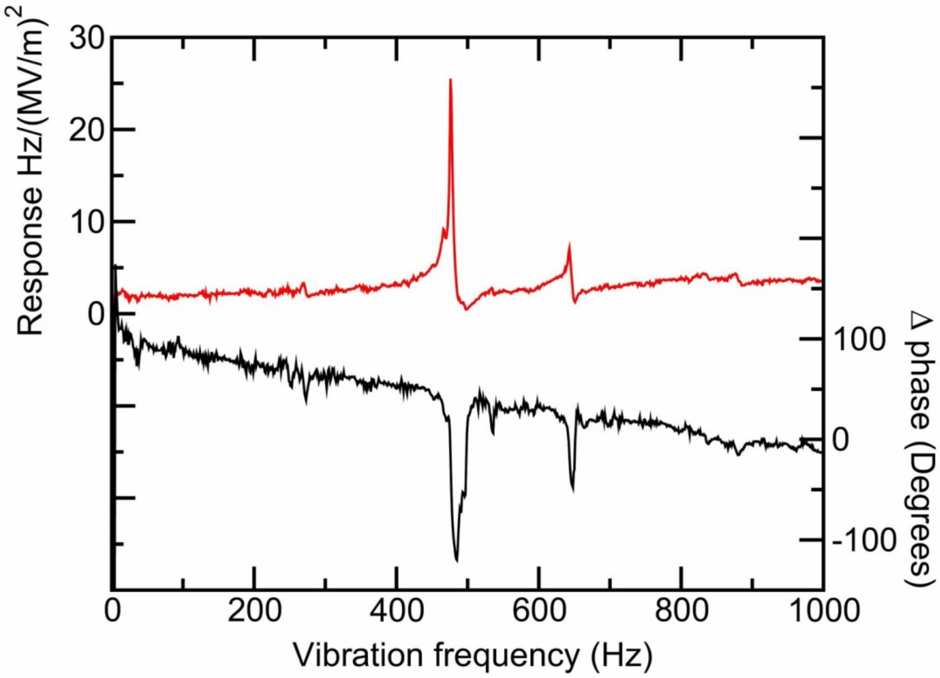
Lorentz Transfer Function

$$\delta\ddot{\omega}_\mu + \frac{2}{\tau_\mu}\delta\dot{\omega}_\mu + \Omega_\mu^2\delta\omega_\mu = -2\Omega_\mu^2 k_\mu V_0{}^2\delta v$$

$$\delta\omega_\mu(\omega) = \frac{-2\Omega_\mu^2 k_\mu V_0{}^2}{\left(\Omega_\mu^2 - \omega^2\right) + \frac{2}{\tau_\mu} i\omega}\delta v(\omega)$$

TEM-class cavities
ANL, single-spoke,
354 MHz, β=0.4

simple spectrum with few modes

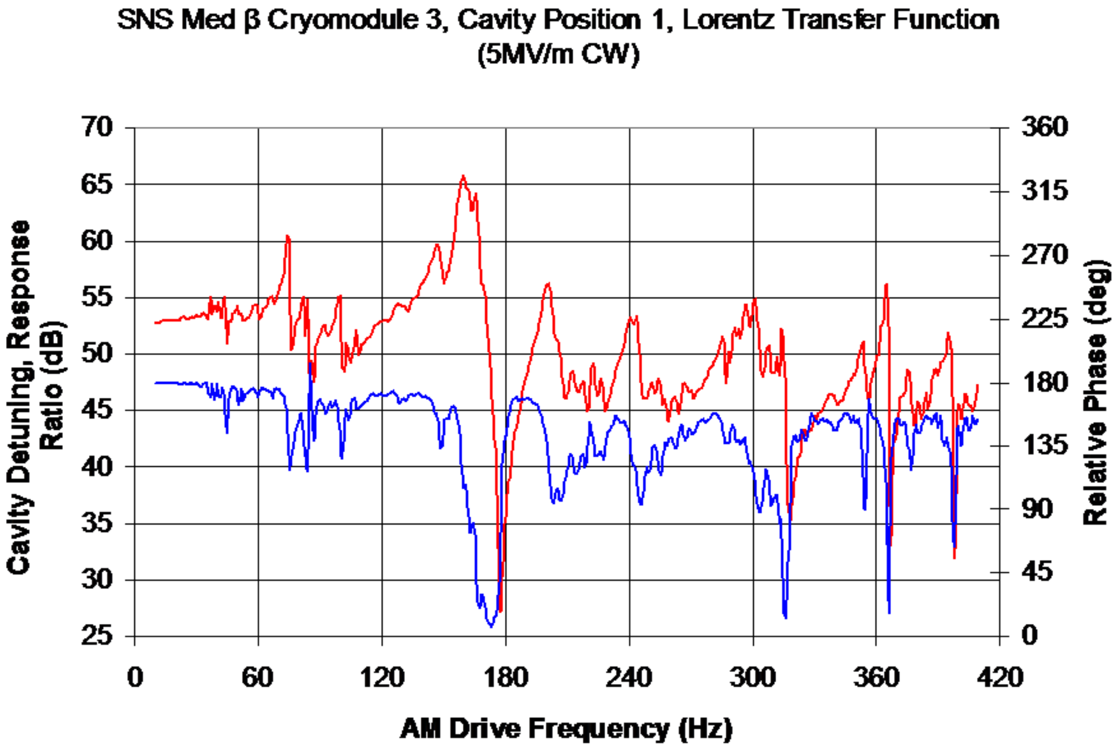


Lorentz Transfer Function

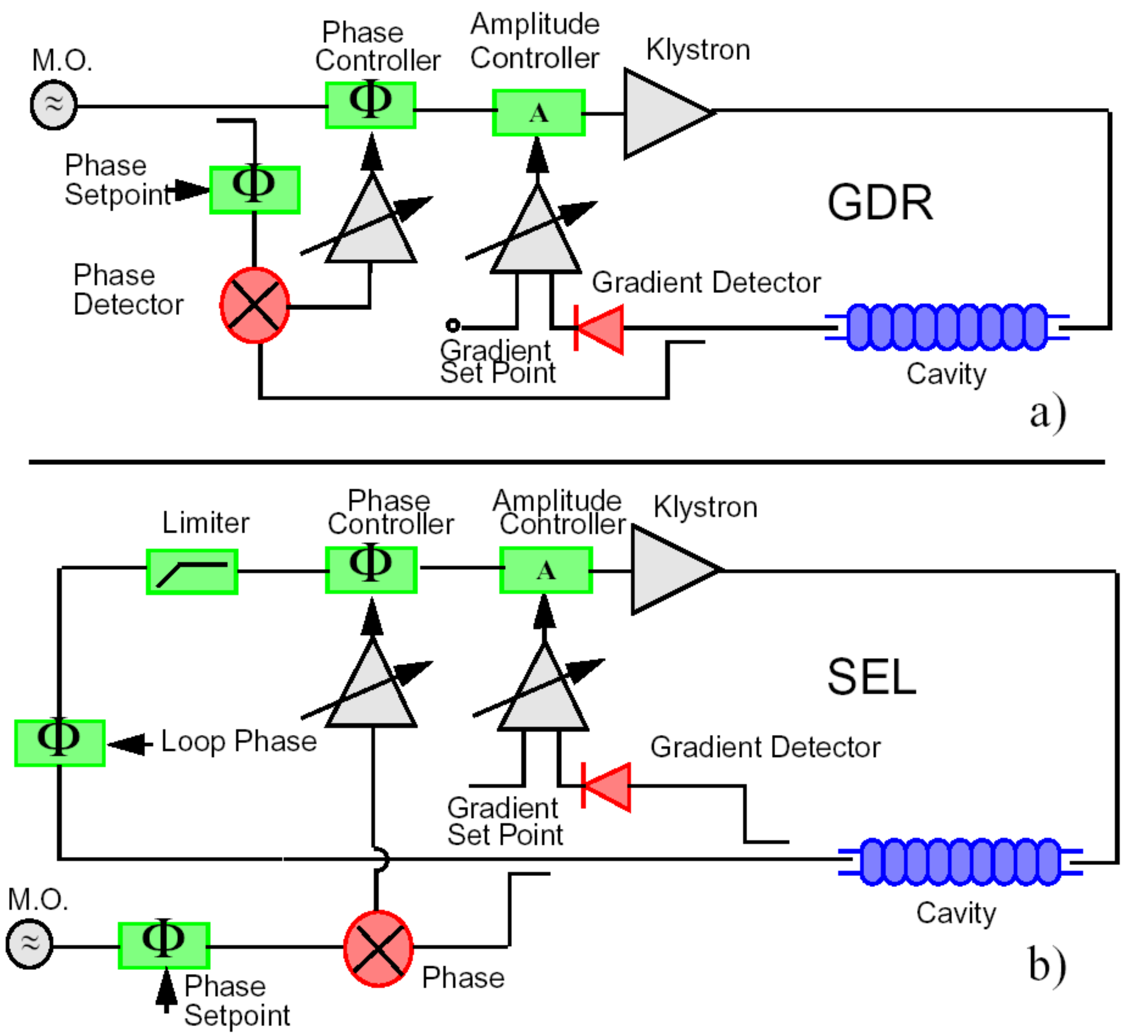
TM-class cavities (Jlab, 6-cell elliptical, 805 MHz, β=0.61)

Rich frequency spectrum from low to high frequencies

Large variations between cavities



GDR and SEL


M.O.
Phase Controller
Amplitude Controller
Klystron
Phase Setpoint
GDR
Phase Detector
Gradient Detector
Gradient Set Point
Cavity
a)
Limiter
Phase Controller
Amplitude Controller
Klystron
SEL
Loop Phase
Gradient Detector
Gradient Set Point
M.O.
Cavity
Phase
Phase Setpoint
b)

Generator-Driven Resonator

- **In a generator-driven resonator the coupling between the electromagnetic and mechanical modes can lead to two ponderomotive instabilities**

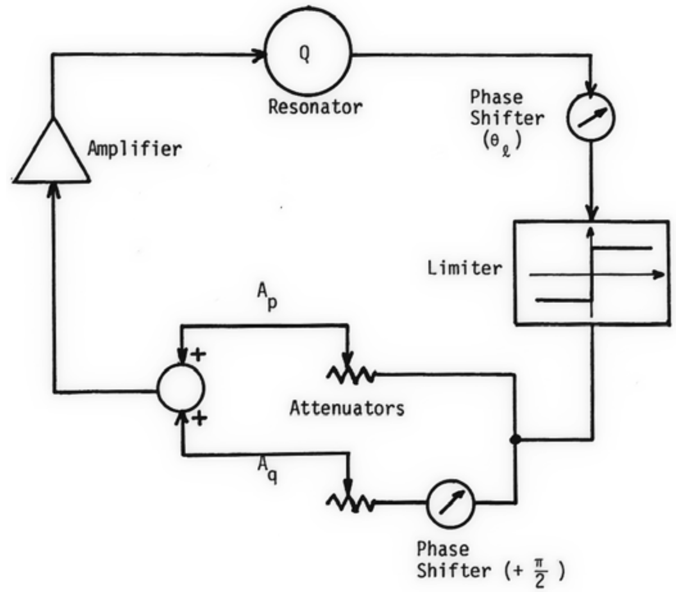
- **Monotonic instability : Jump phenomenon where the amplitudes of the electromagnetic and mechanical modes increase or decrease exponentially until limited by non-linear effects**

- **Oscillatory instability : The amplitudes of both modes oscillate and increase at an exponential rate until limited by non-linear effects**

Self-Excited Loop-Principle of Stabilization

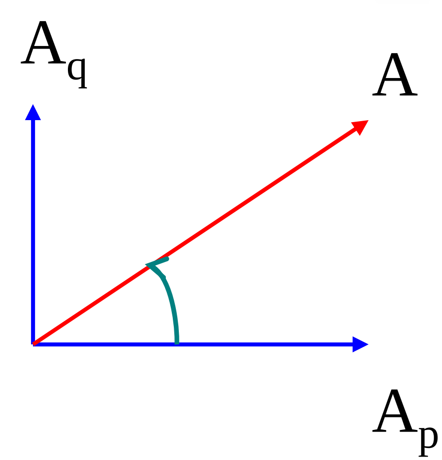
Controlling the external phase shift $_l$ can compensate for the fluctuations in the cavity frequency ω_c so the loop is phase locked to an external frequency reference ω_r.

$$\omega = \omega_c + \frac{\omega_c}{2Q} \tan\theta_l$$

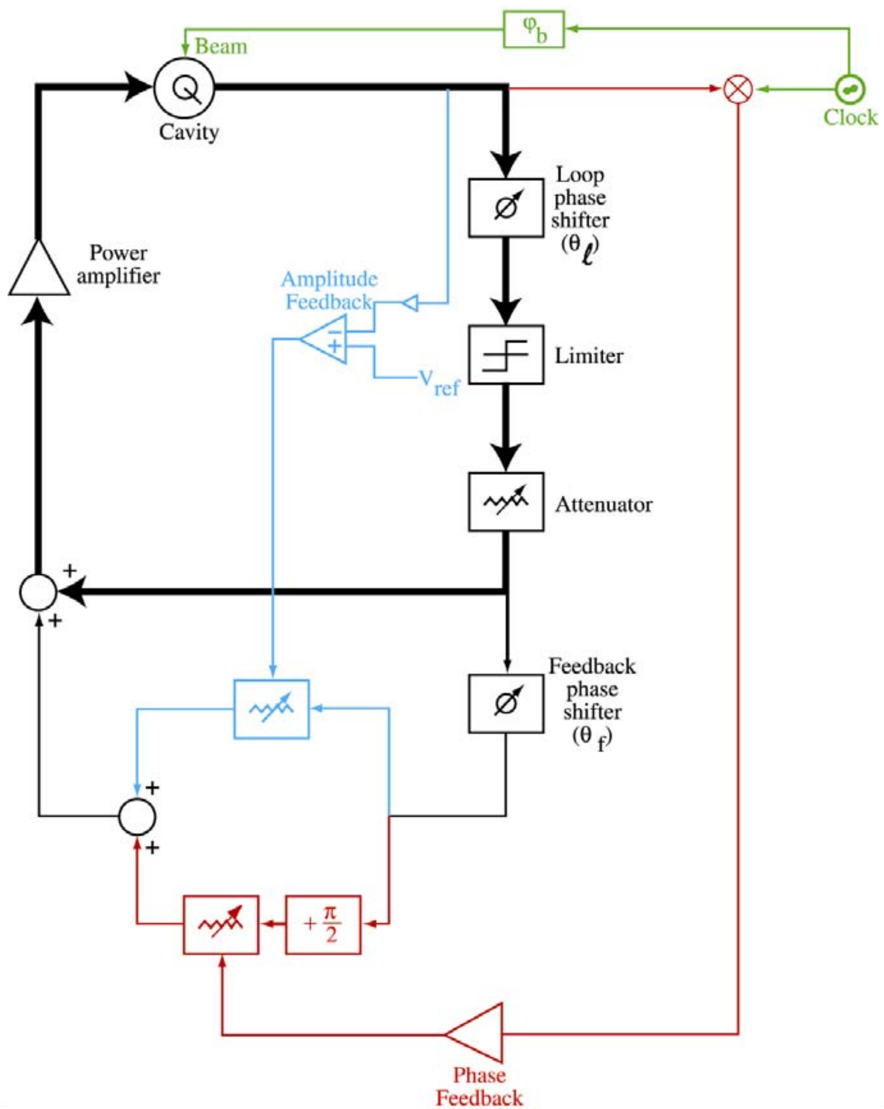


Instead of introducing an additional external controllable phase shifter, this is usually done by adding a signal in quadrature

→ The cavity field amplitude is unaffected by the phase stabilization even in the absence of amplitude feedback.



Self-Excited Loop – Block Diagram



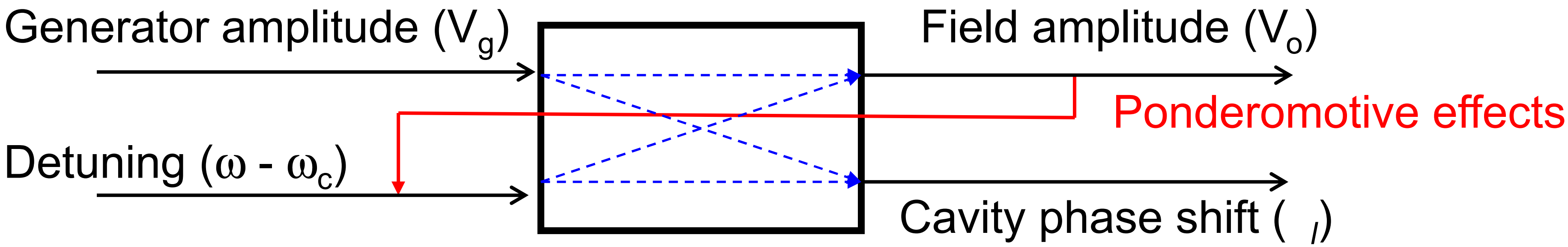
Self-Excited Loop

- Resonators operated in self-excited loops in the absence of feedback are free of ponderomotive instabilities. An SEL is equivalent to the ideal VCO.
  - Amplitude is stable
  - Frequency of the loop tracks the frequency of the cavity

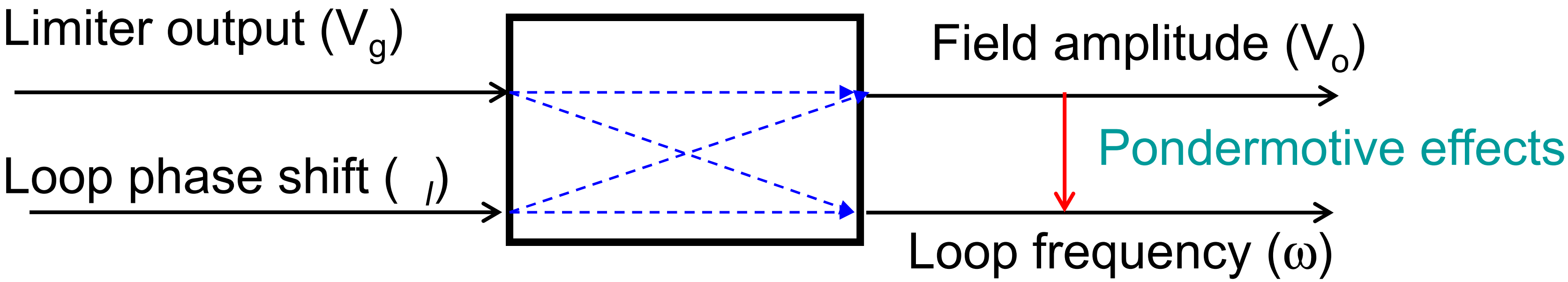
- Phase stabilization can reintroduce instabilities, but they are easily controlled with small amount of amplitude feedback

Input-Output Variables

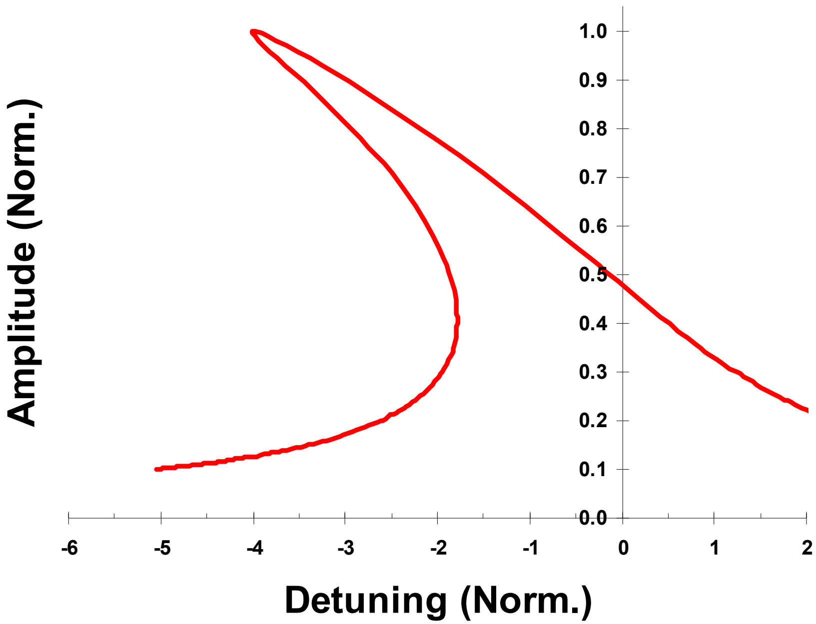
- **Generator - driven cavity**

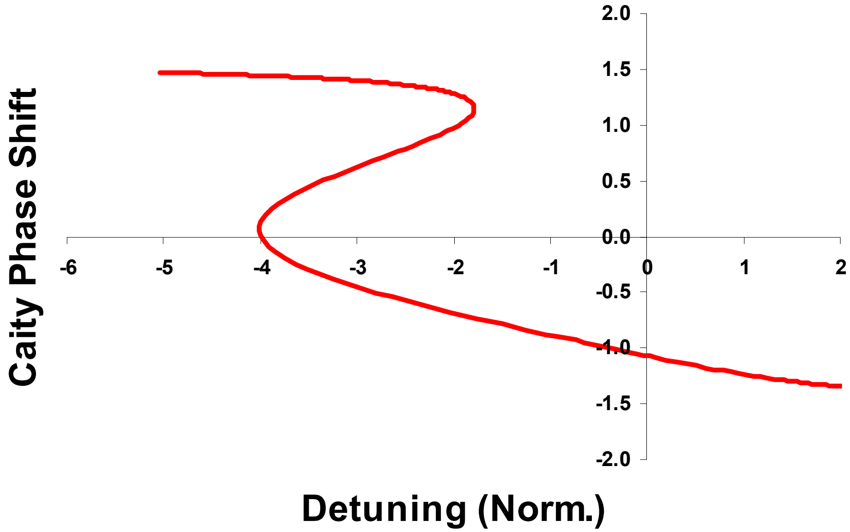


- **Cavity in a self-excited loop**

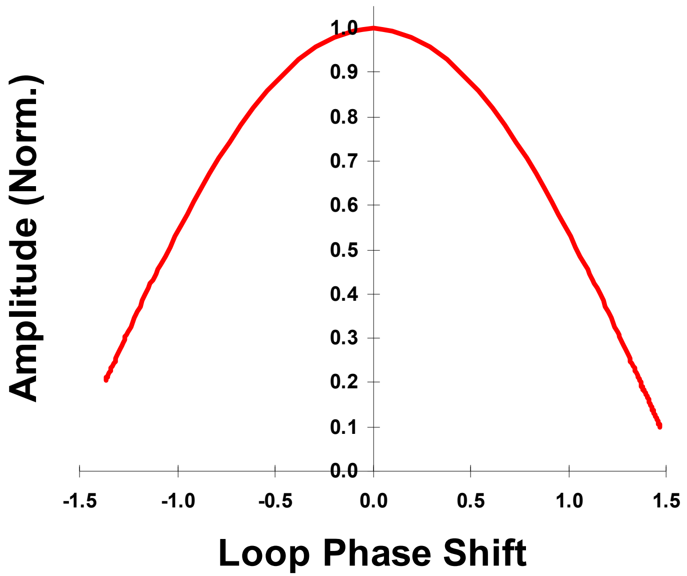


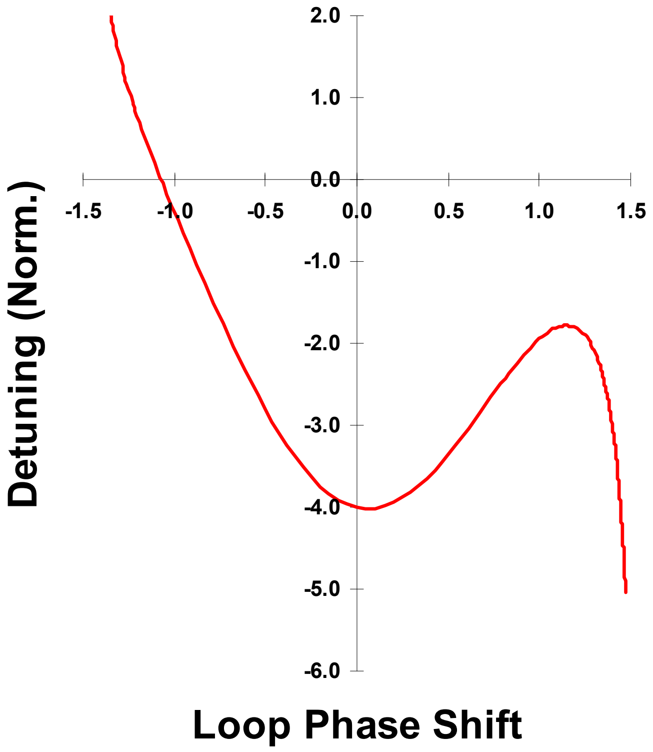
Input-Output Variables
Generator-Driven Resonator





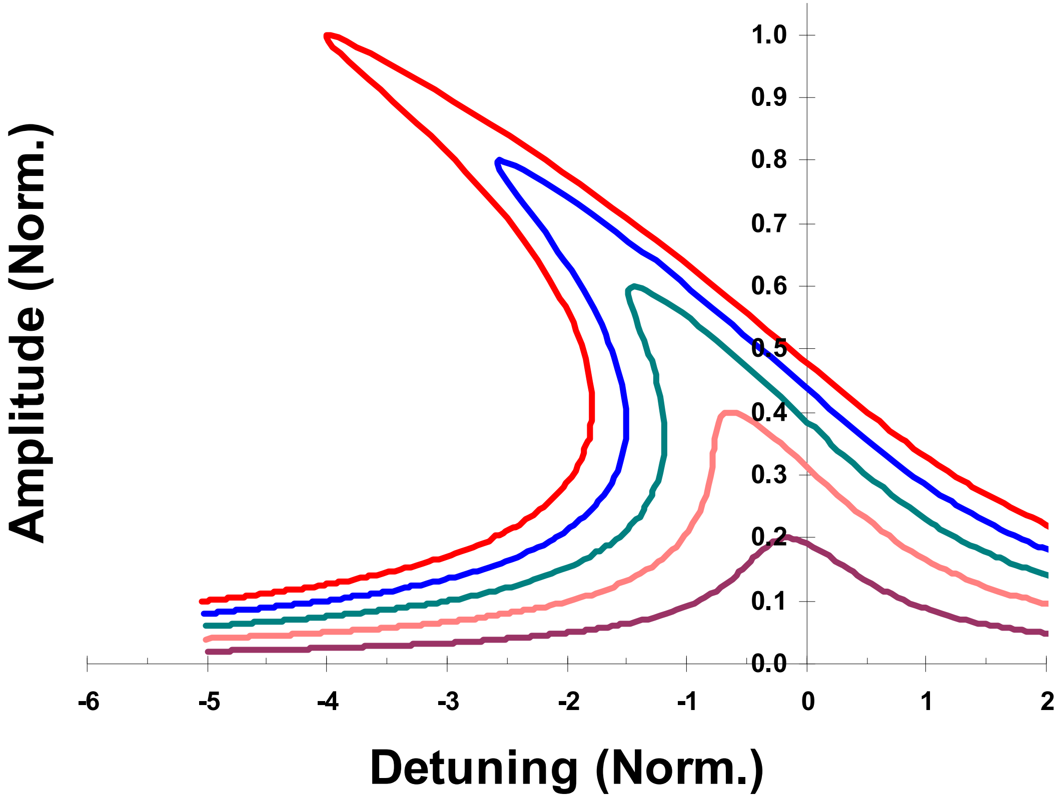
Input-Output Variables
Self-Excited Loop





Lorentz Detuning

During transient operation (rise time and decay time) the loop frequency automatically tracts the resonator frequency. Lorentz detuning has no effect and is automatically compensated



Microphonics

- Microphonics: changes in frequency caused by connections to the external world
  - Vibrations
  - Pressure fluctuations

When phase and amplitude feedbacks are active, ponderomotive effects can change the response to external disturbances

$$\delta\ddot{\omega}_\mu + \frac{2}{\tau_\mu}\delta\dot{\omega}_\mu + \Omega_\mu{}^2\delta\omega_\mu = -2\Omega_\mu^2 k_\mu V_0{}^2\delta v + n(t)$$

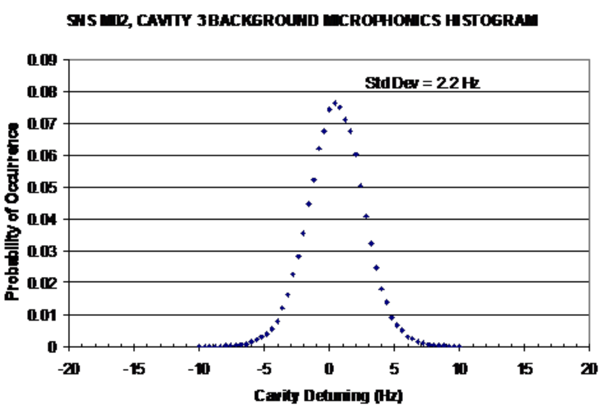
Microphonics

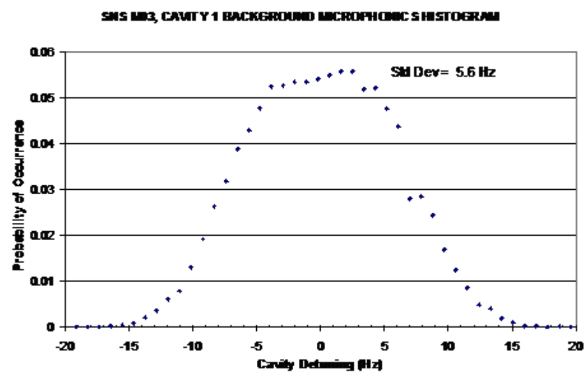
Two extreme classes of driving terms:

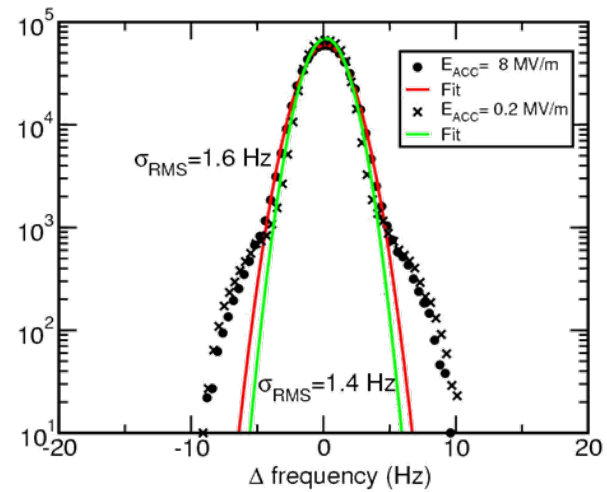
- Deterministic, monochromatic
  - Constant, well defined frequency
  - Constant amplitude
- Stochastic
  - Broadband (compared to bandwidth of mechanical mode)
  - Will be modeled by gaussian stationary white noise process

Microphonics (probability density)

Single gaussian	Bimodal	Multi-gaussian
Noise driven	Single-frequency driven	Non-stationary noise







805 MHz TM	805 MHz TM	172 MHz TEM

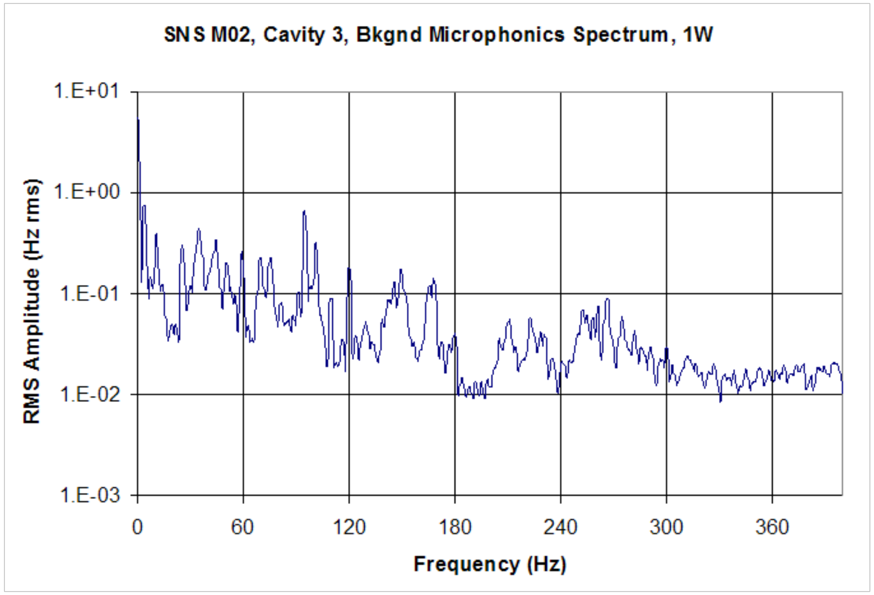
Microphonics (frequency spectrum)

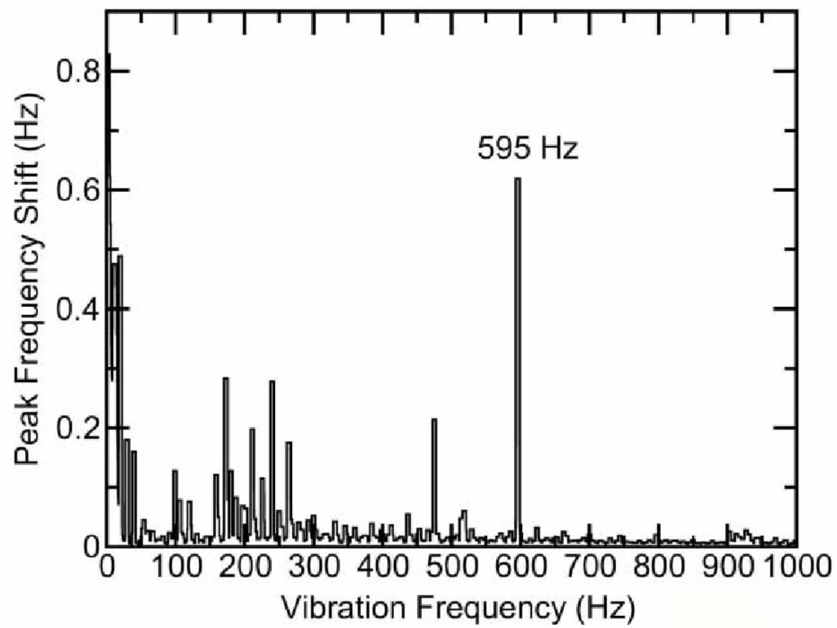
TM-class cavities (JLab, 6-cell elliptical, 805 MHz, β=0.61)

- Rich frequency spectrum from low to high frequencies
- Large variations between cavities

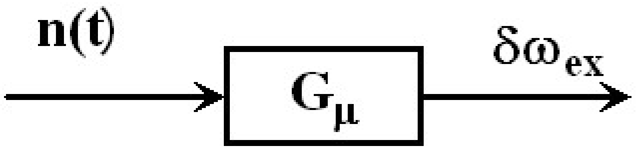
TEM-class cavities (ANL, single-spoke, 354 MHz, β=0.4)

- Dominated by low frequency (<10 Hz) from pressure fluctuations
- Few high frequency mechanical modes that contribute little to microphonics level.





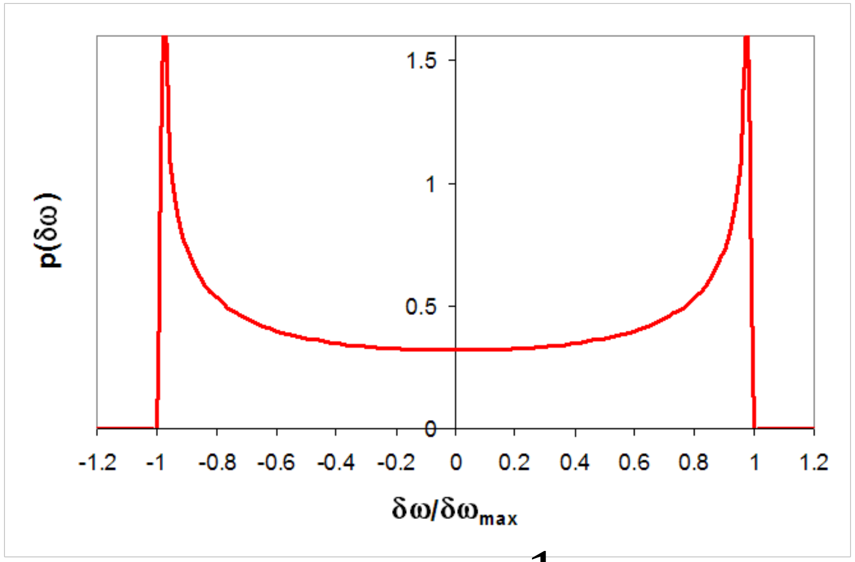
Probability Density (histogram)

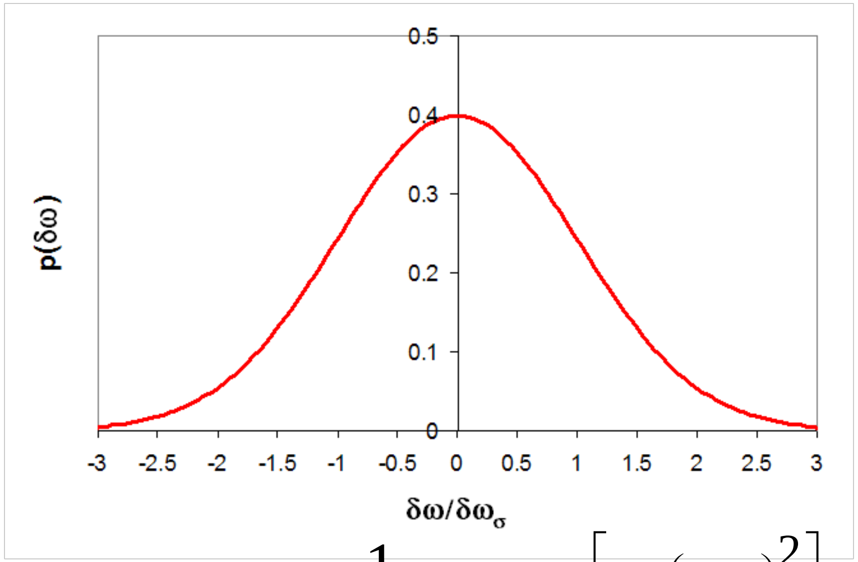


Harmonic oscillator (Ω_μ,τ_μ) driven by:

Single frequency, constant amplitude

White noise, gaussian





$$p(\delta\omega) = \frac{1}{\pi\sqrt{\delta\omega_{\max}^2 - \delta\omega^2}}$$

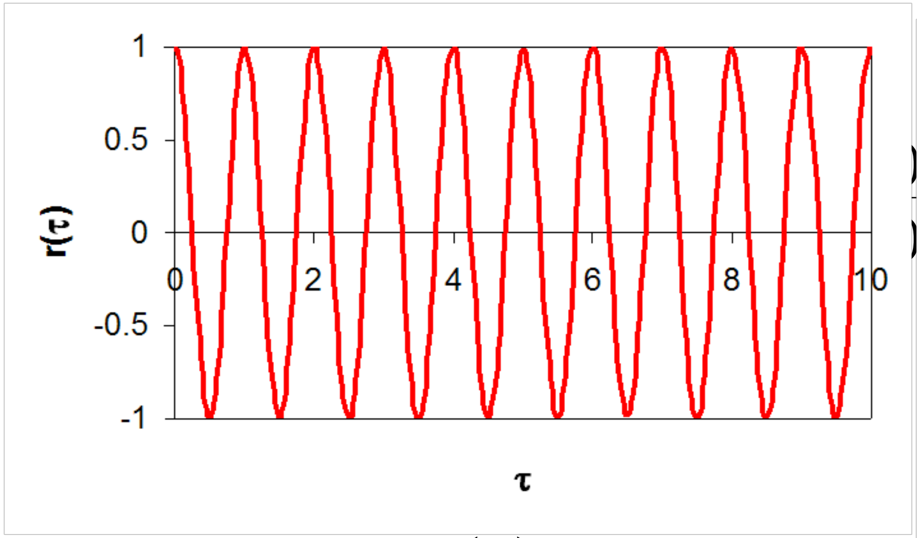
$$p(\delta\omega) = \frac{1}{\sigma_\omega\sqrt{2\pi}}\exp\left[-\frac{1}{2}\left(\frac{\delta\omega}{\sigma_\omega}\right)^2\right]$$

Autocorrelation Function

$$R_x(\tau) = \langle x(t)\,x(t+\tau)\rangle = \lim_{T\to\infty}\frac{1}{T}\int_0^T x(t)\,x(t+\tau)\,dt$$

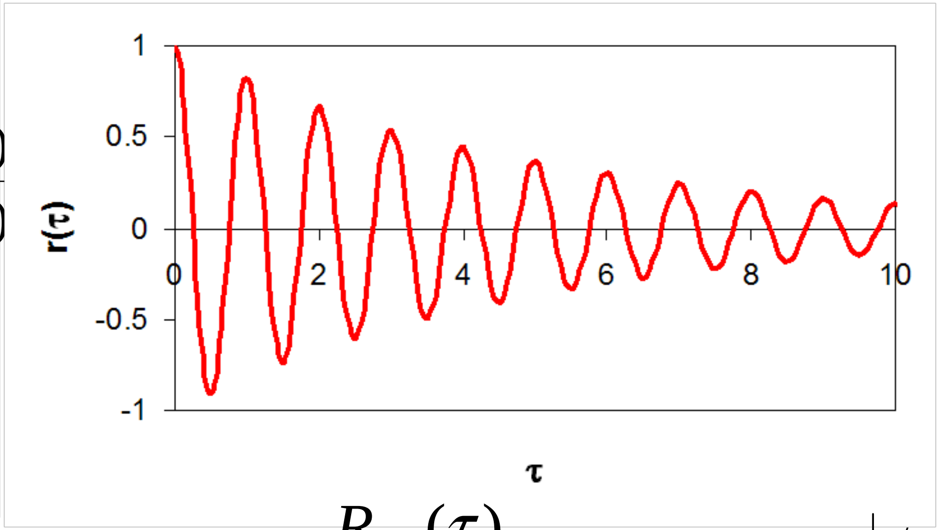
Harmonic oscillator (Ω_μ,τ_μ) driven by:

Single frequency, constant amplitude



$$r_{\delta\omega}(\tau) = \frac{R_{\delta\omega}(\tau)}{R_{\delta\omega}(0)} = \cos(\omega_d \tau)$$

White noise, gaussian



$$r_{\delta\omega}(\tau) = \frac{R_{\delta\omega}(\tau)}{R_{\delta\omega}(0)} = \cos(\Omega_\mu \tau)e^{-|\tau/\tau_\mu|}$$

Stationary Stochastic Processes

$x(t)$: stationary random variable

Autocorrelation function: $$R_x(\tau) = \langle x(t)\, x(t+\tau) \rangle = \lim_{T\to\infty} \frac{1}{T} \int_0^T x(t)\, x(t+\tau)\, dt$$

Spectral Density $S_x(\omega)$: Amount of power between ω and $d\omega$

$S_x(\omega)$ and $R_x(\tau)$ are related through the Fourier Transform (Wiener-Khintchine)

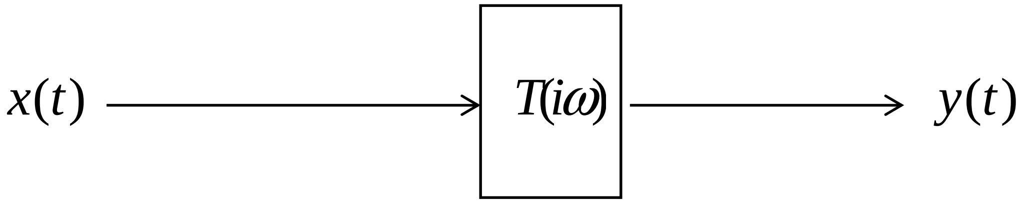
$$S_x(\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} R_x(\tau)\, e^{-i\omega\tau}\, d\tau \qquad R_x(\tau) = \int_{-\infty}^{\infty} S_x(\omega)\, e^{i\omega\tau}\, d\omega$$

Mean square value:

$$\langle x^2 \rangle = R_x(0) = \int_{-\infty}^{\infty} S_x(\omega)\, d\omega$$

Stationary Stochastic Processes

For a stationary random process driving a linear system



$$\left\langle y^2 \right\rangle = R_y(0) = \int_{-\infty}^{+\infty} S_y(\omega)\, d\omega \qquad \left\langle x^2 \right\rangle = R_x(0) = \int_{-\infty}^{+\infty} S_x(\omega)\, d\omega$$

$R_y(\tau)$ $\left[R_x(\tau)\right]$: auto correlation function of $y(t)$ $\left[x(t)\right]$

$S_y(\omega)$ $\left[S_x(\omega)\right]$: spectral density of $y(t)$ $\left[x(t)\right]$

$$\boxed{S_y(\omega) = S_x(\omega)\left|T(i\omega)\right|^2}$$

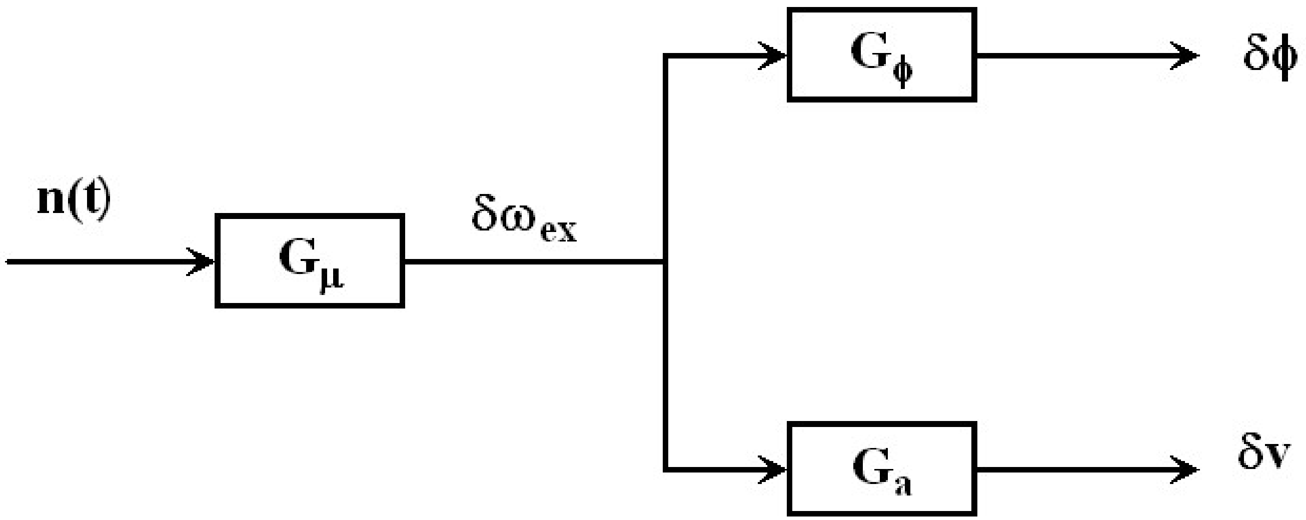
$$\boxed{\left\langle y^2 \right\rangle = \int_{-\infty}^{+\infty} S_x(\omega)\left|T(i\omega)\right|^2 d\omega}$$

Performance of Control System

Residual phase and amplitude errors caused by microphonics

Can also be done for beam current amplitude and phase fluctuations

Assume a single mechanical oscillator of frequency Ω_μ and decay time τ_μ excited by white noise of spectral density A^2



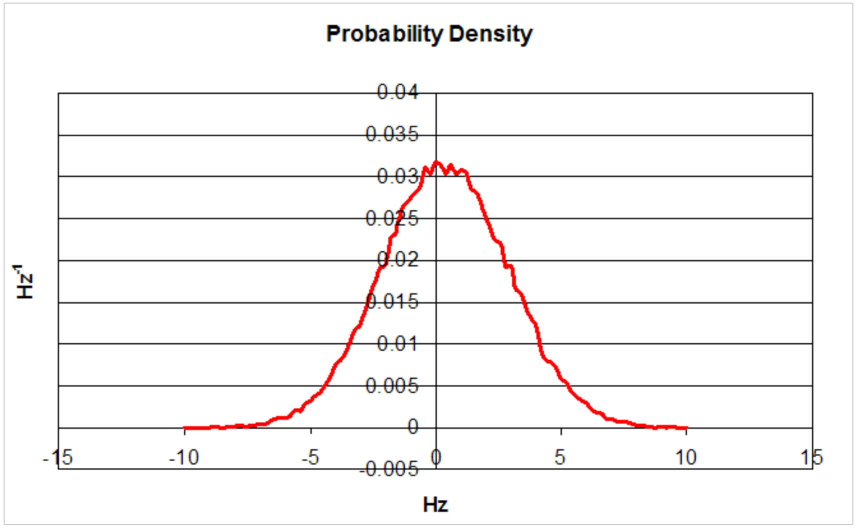
Performance of Control System

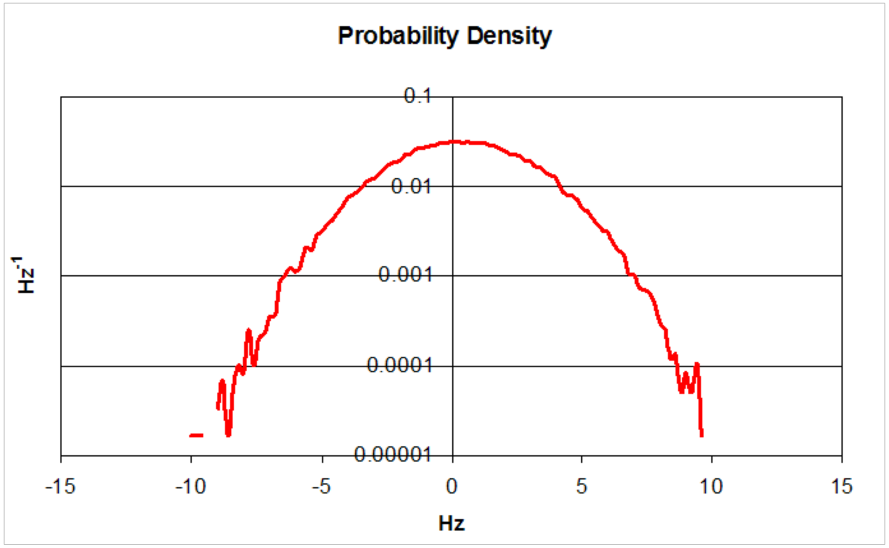
$$<\delta\omega_{ex}^2>=A^2\int_{-\infty}^{+\infty}\left|G_\mu(i\omega)\right|^2 d\omega = A^2\int_{-\infty}^{+\infty}\frac{d\omega}{\left|-\omega^2+\frac{2}{\tau_\mu}i\omega+\Omega_\mu^2\right|^2} = A^2\frac{\pi\tau_\mu}{2\Omega_\mu^2}$$

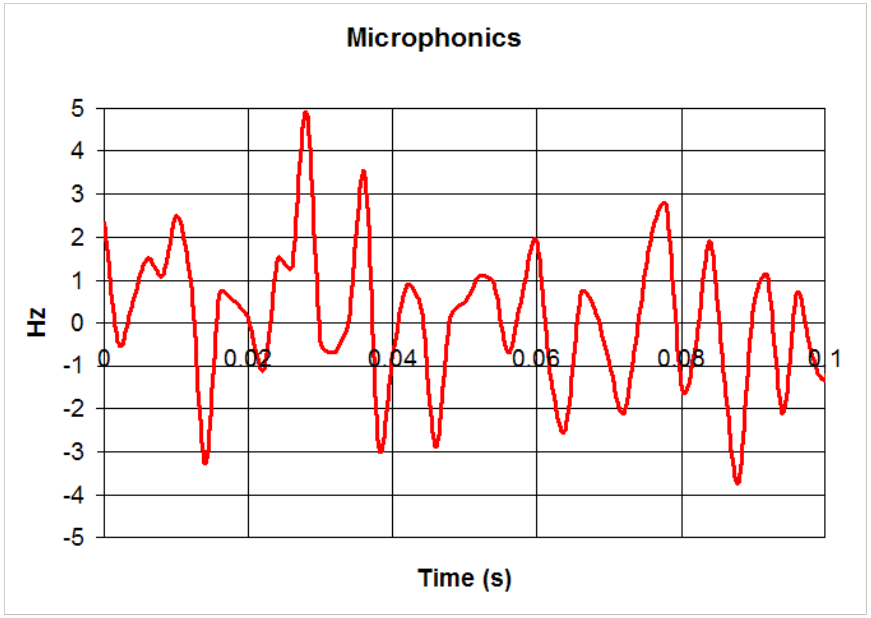
$$<\delta v^2>=A^2\int_{-\infty}^{+\infty}\left|G_\mu(i\omega)G_a(i\omega)\right|^2 d\omega = <\delta\omega_{ex}^2>\frac{2\Omega_\mu^2}{\pi\tau_\mu}\int_{-\infty}^{+\infty}\left|\frac{G_a(i\omega)}{-\omega^2+\frac{2}{\tau_\mu}i\omega+\Omega_\mu^2}\right|^2 d\omega$$

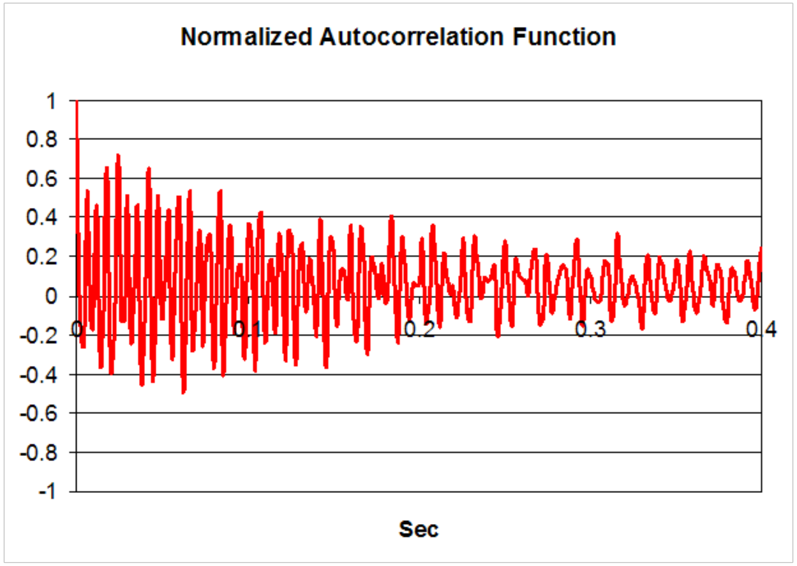
$$<\delta\varphi^2>=A^2\int_{-\infty}^{+\infty}\left|G_\mu(i\omega)G_\varphi(i\omega)\right|^2 d\omega = <\delta\omega_{ex}^2>\frac{2\Omega_\mu^2}{\pi\tau_\mu}\int_{-\infty}^{+\infty}\left|\frac{G_\varphi(i\omega)}{-\omega^2+\frac{2}{\tau_\mu}i\omega+\Omega_\mu^2}\right|^2 d\omega$$

The Real World

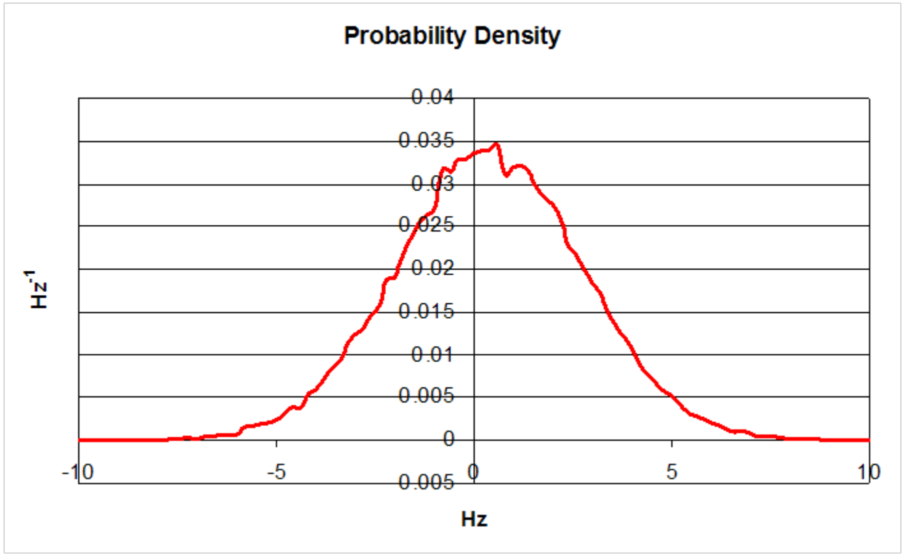


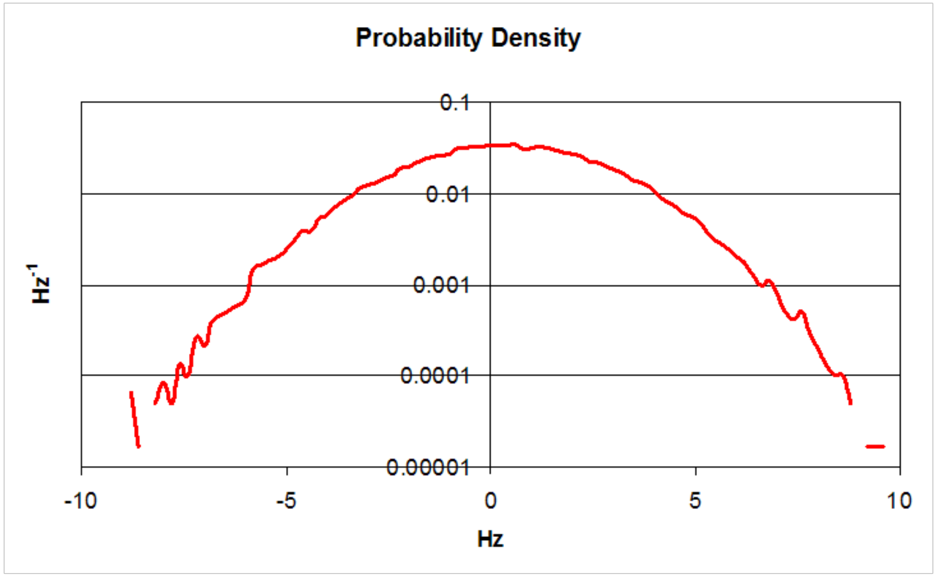


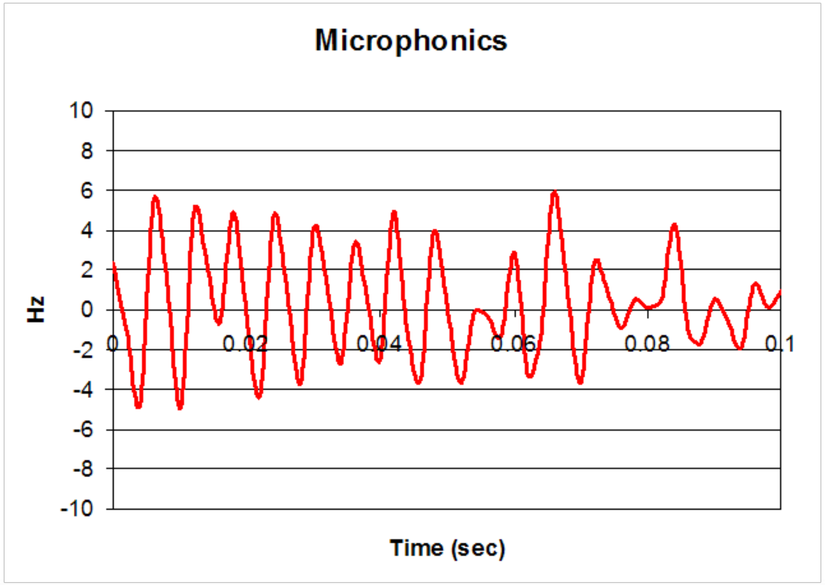


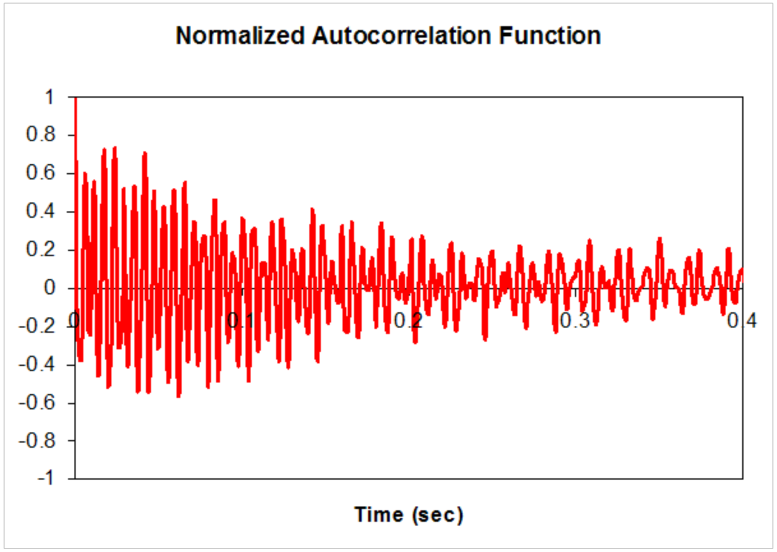


The Real World

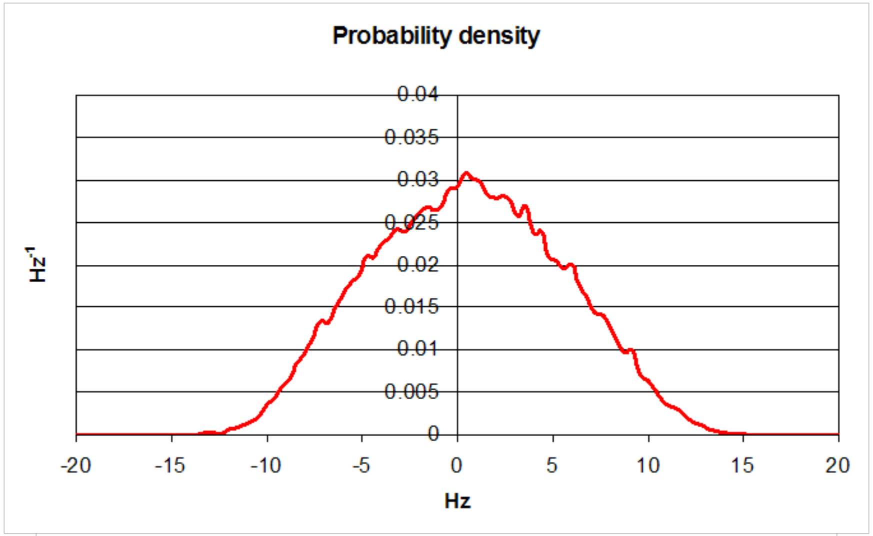


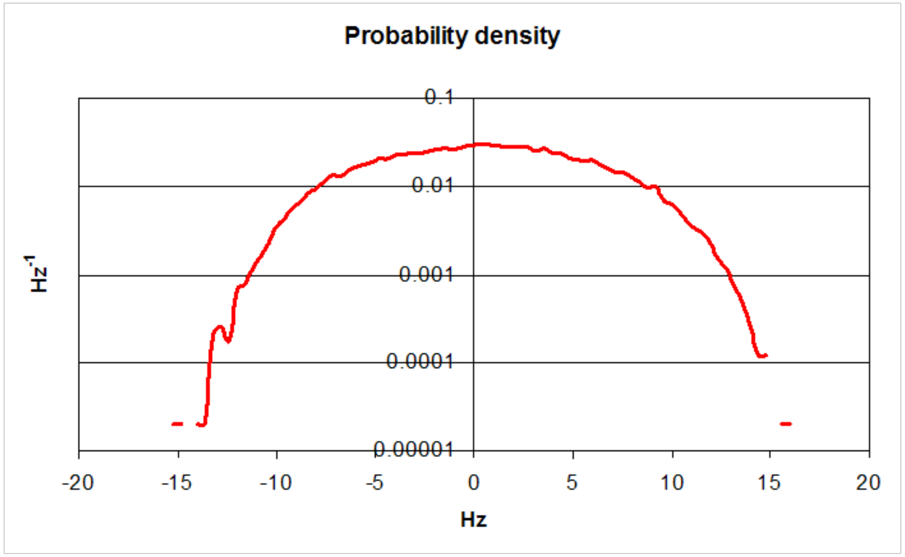


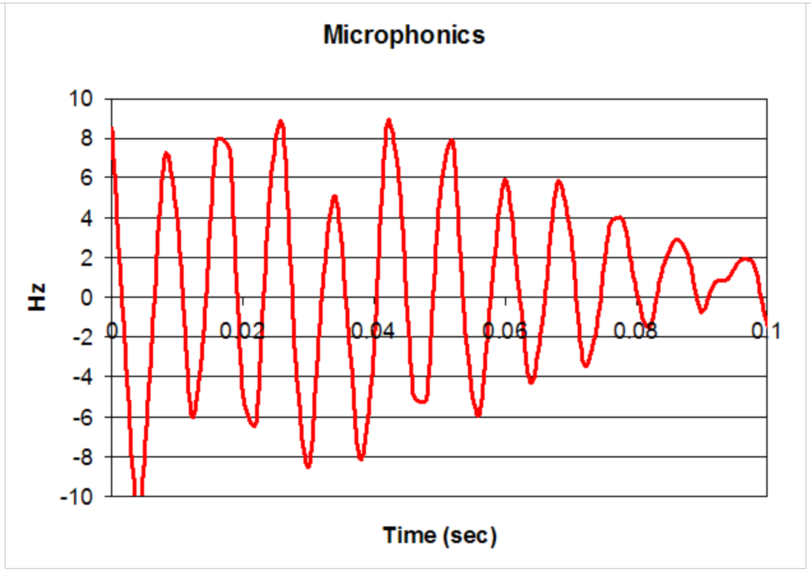


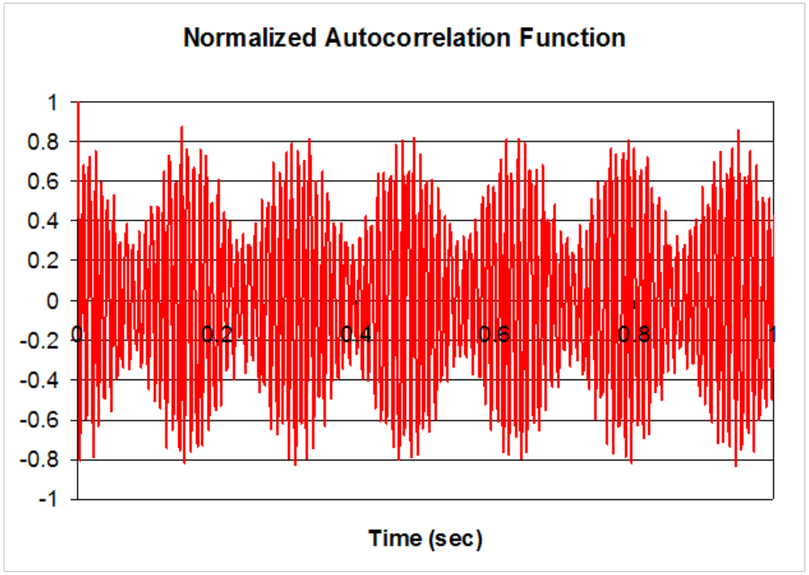


The Real World









Piezo control of microphonics

MSU, 6-cell elliptical 805 MHz, β=0.49

Adaptive feedforward compensation

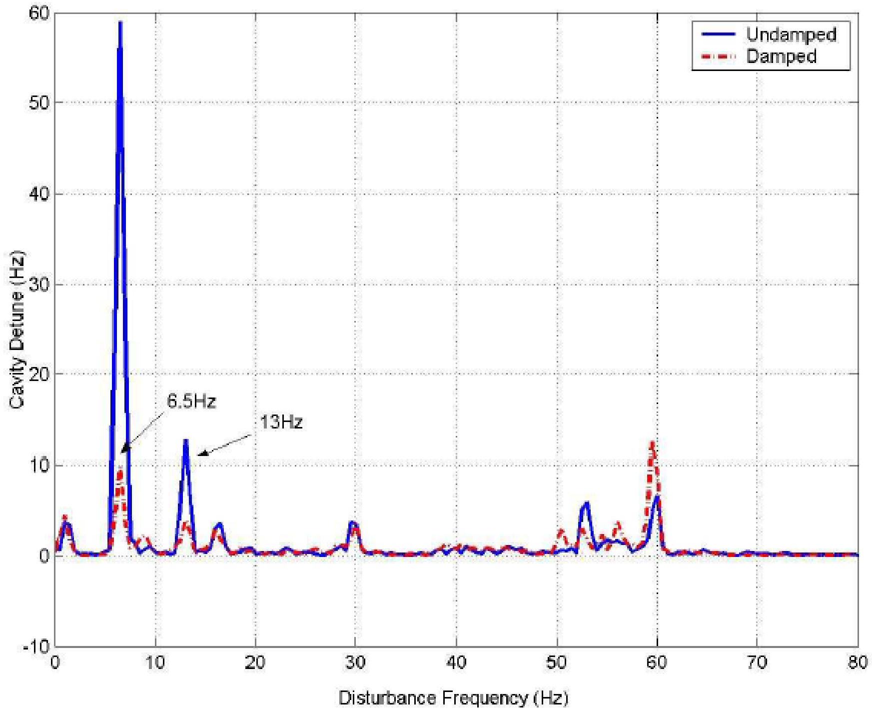


Figure 2. Active damping of helium oscillations at 2K.

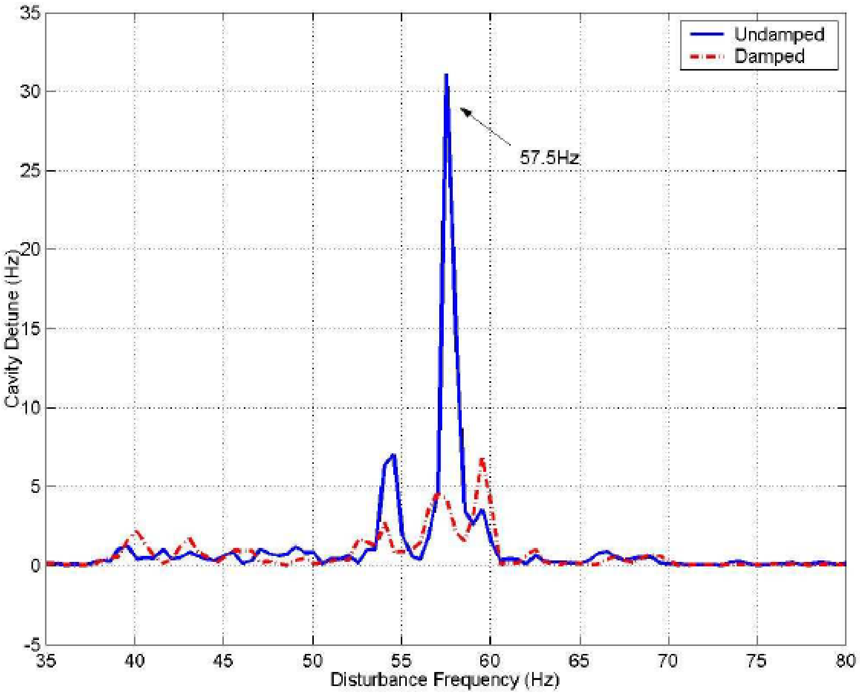
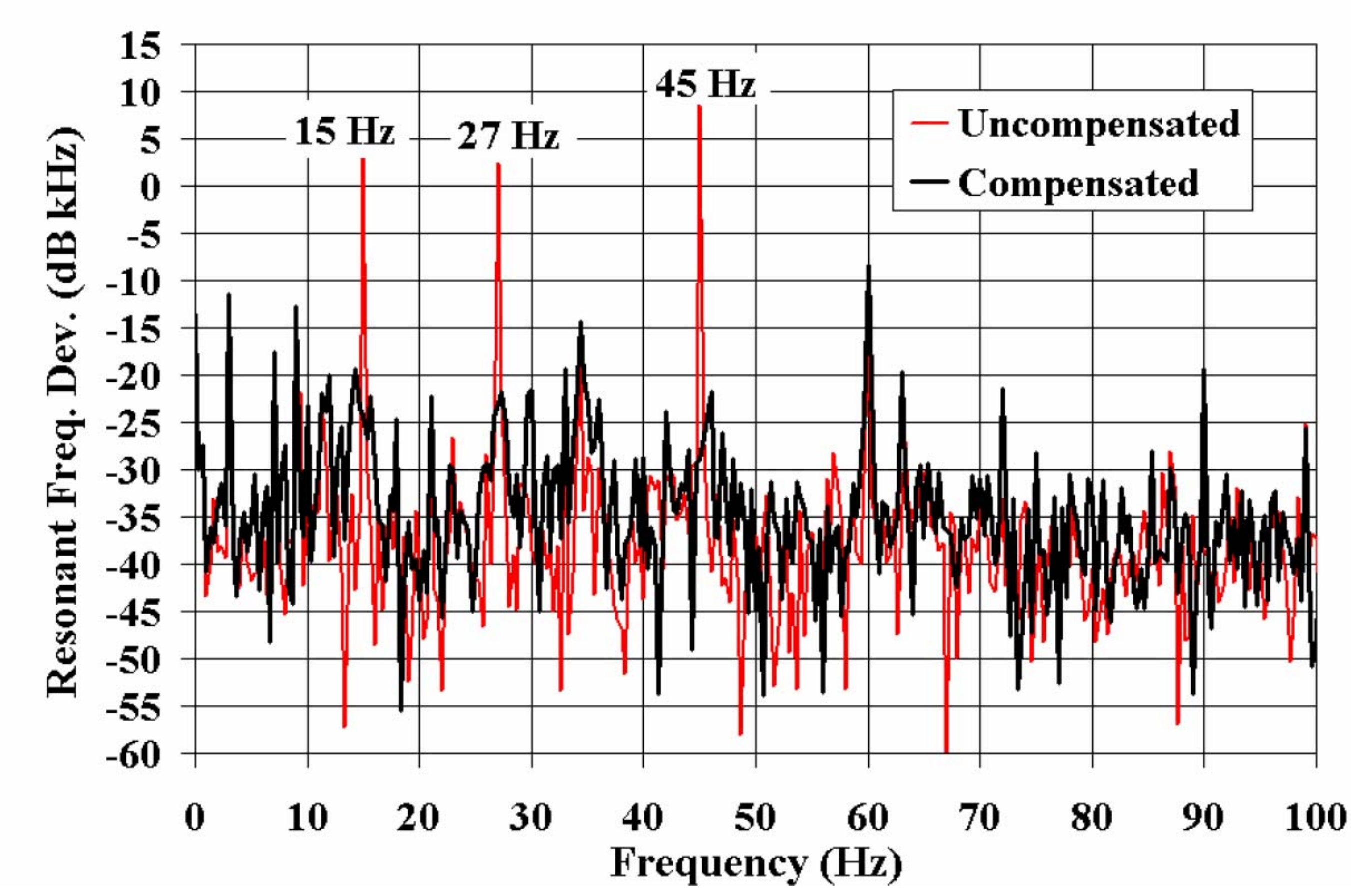


Figure 3. Active damping of external vibration at 2K.

Piezo Control of Microphonics

FNAL, 3-cell 3.9 GHz



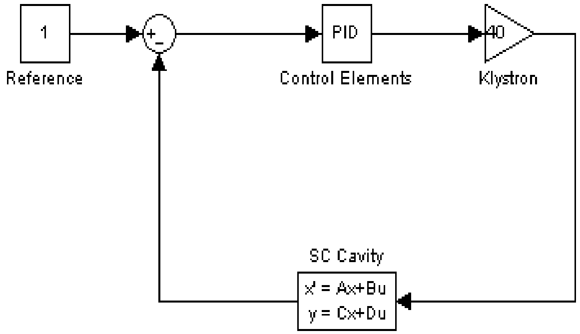
SEL and GDR

- **SEL are best suited for high gradient, high-loaded Q cavities operated cw.**
  - Well behaved with respect to ponderomotive instabilities
  - Unaffected by Lorentz detuning at power up
  - Able to run independently of external rf source
  - Rise time can be random and slow (starts from noise)

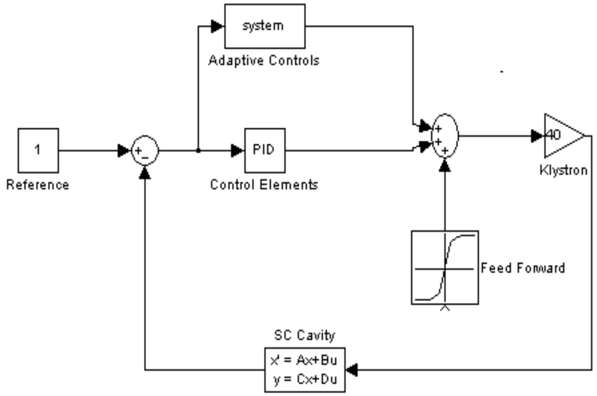
- **GDR are best suited for low-Q cavities operated for short pulse length.**
  - Fast predictable rise time
  - Power up can be hampered by Lorentz detuning

SC Control Systems

- CW accelerators (Atlas, CEBAF) use simple proportional negative feedback.



- Pulsed accelerators (TESLA, SNS) need more complex control methods, adaptive control, and feed forward techniques.



Control System Example

At CEBAF, Nuclear experiments require an energy spread of ~ 10^{-4}

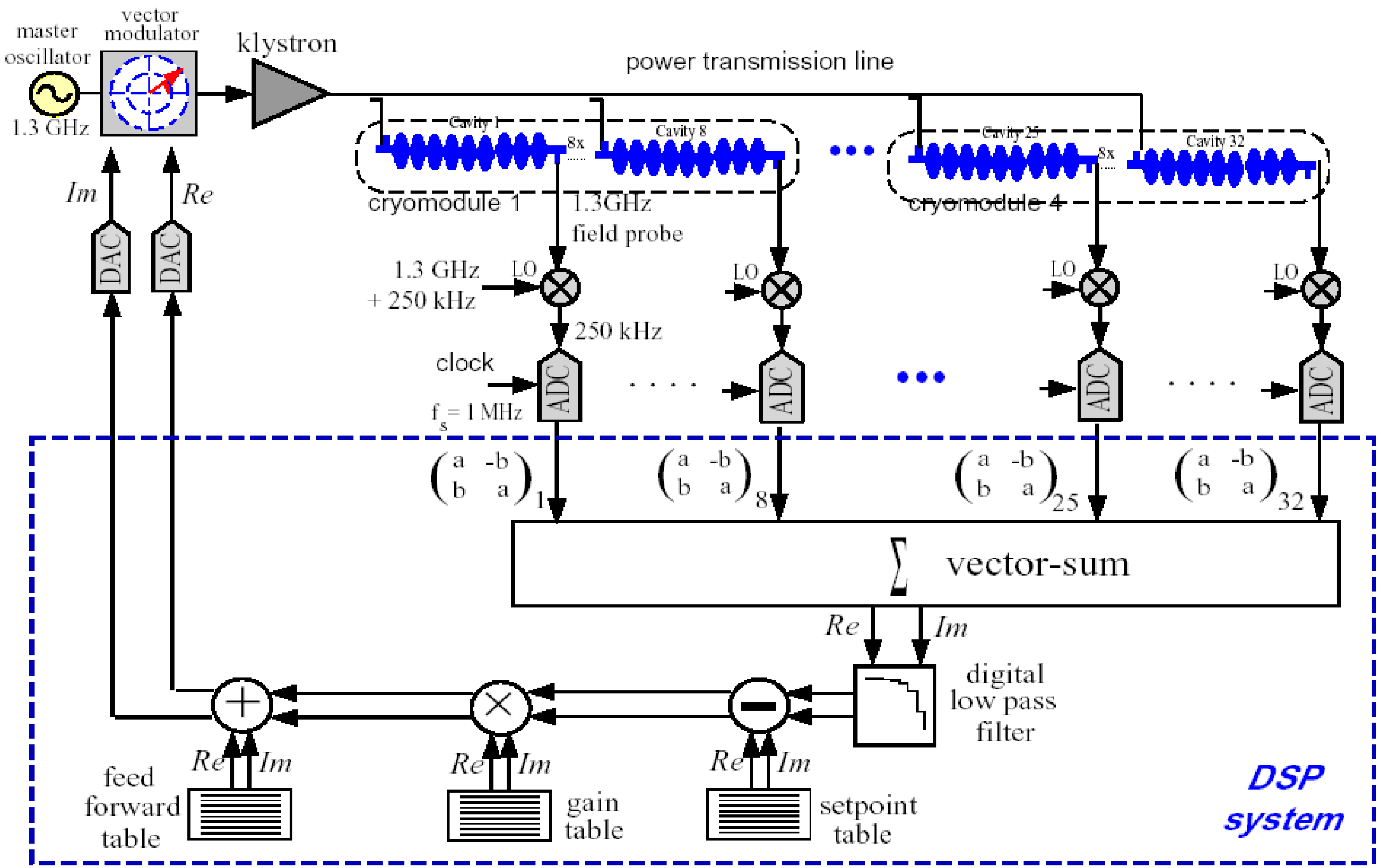
To meet this each individual cavity must have no more than ~ 10^{-5} amplitude variation.

[$\Delta E/E \sim 1/N^{1/2}$ where N is the number of cavities]

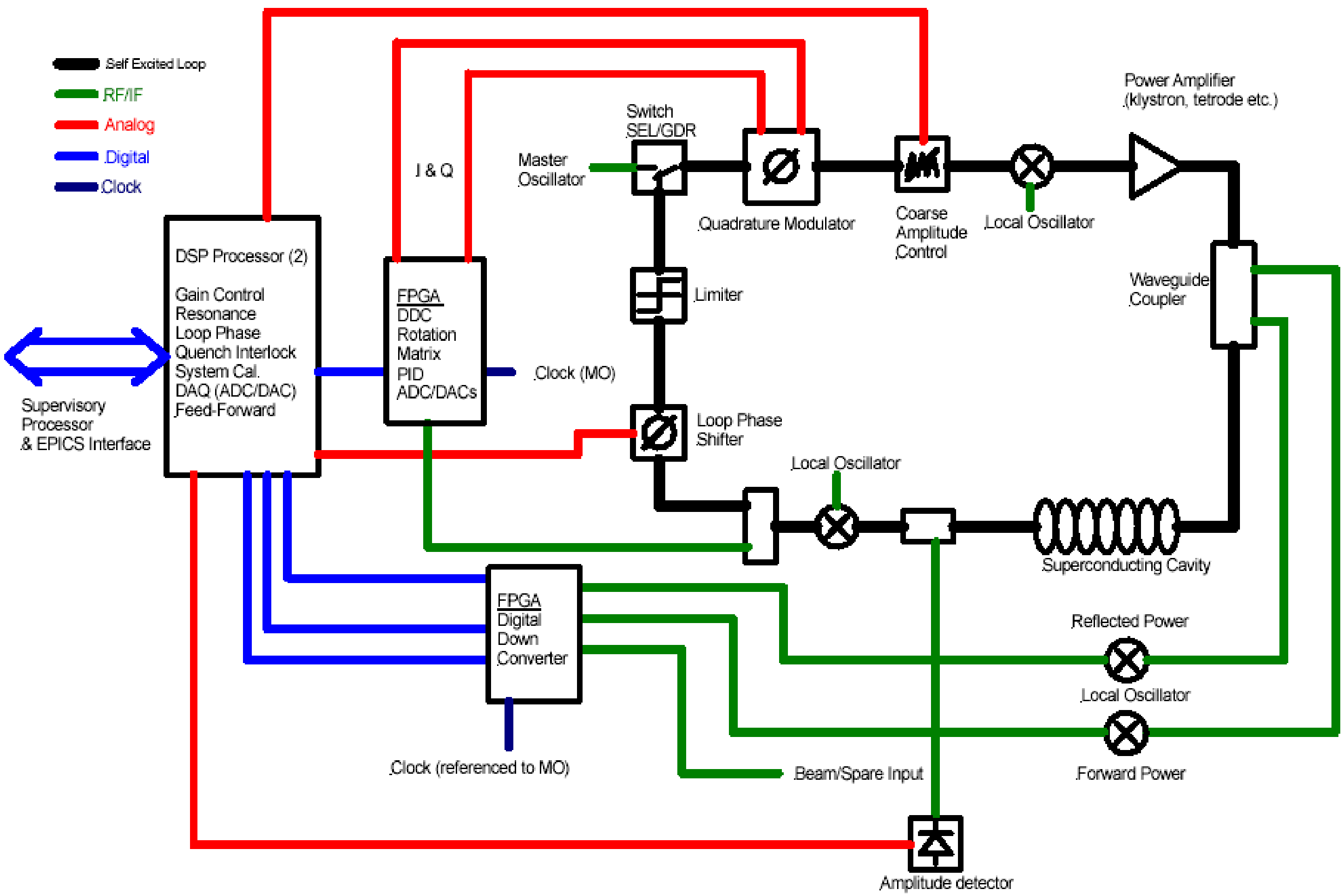
Background microphonics are 5% (peak) do to $Q_L = 10^7$

Therefore gain required to control the cavity field is 500 or ~ 53 dB in gain.

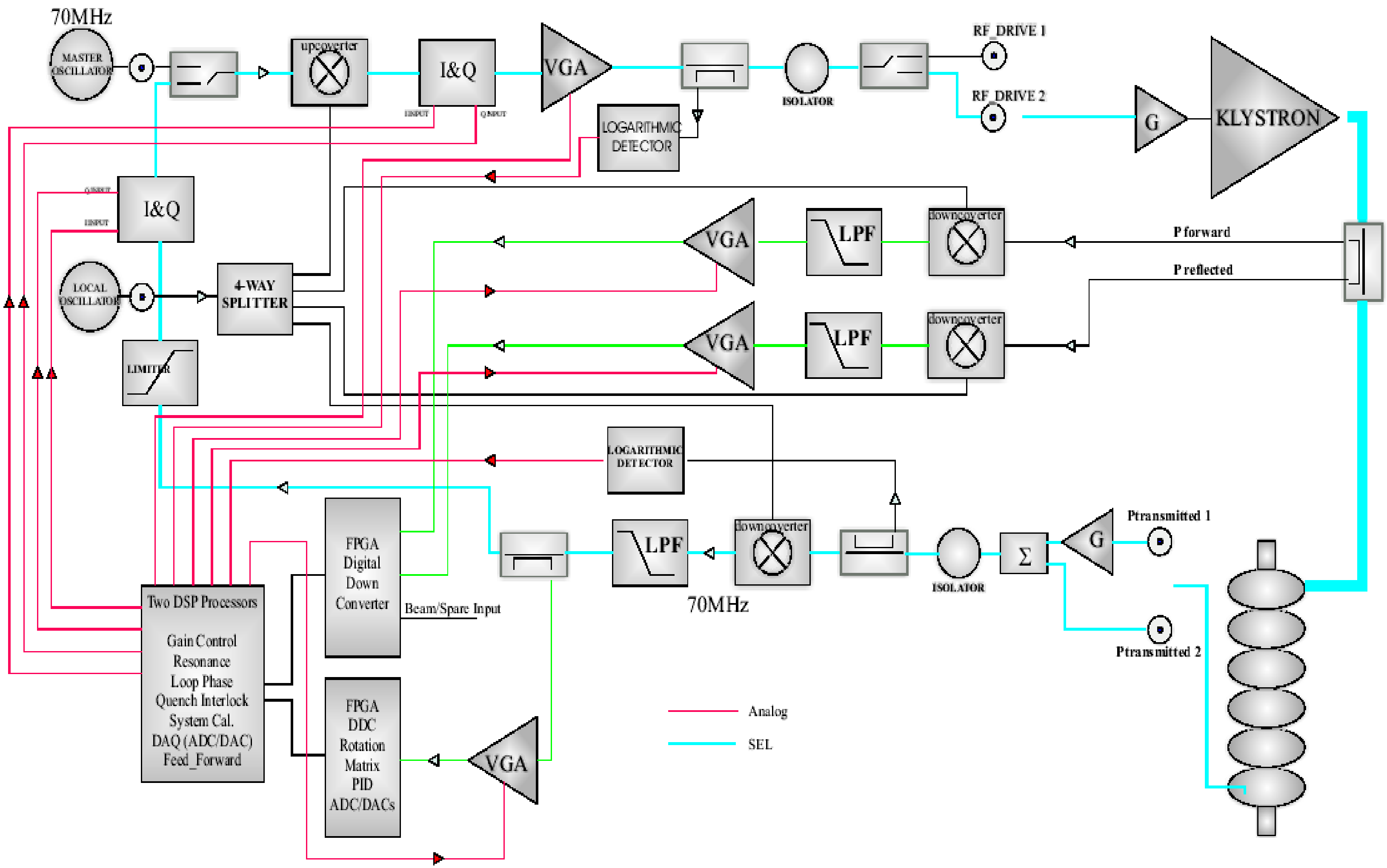
TESLA Control System



Basic LLRF Block Diagram



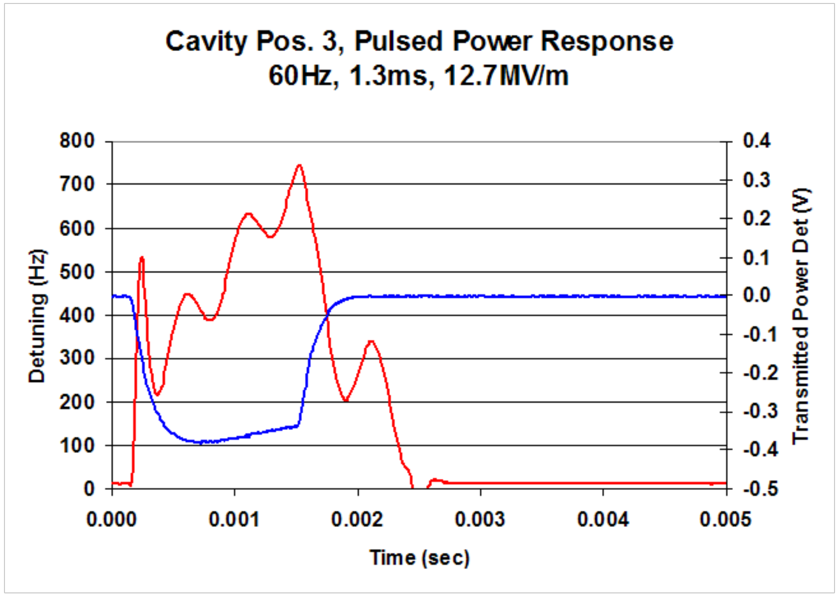
Low level rf control development



Concept for a LLRF control system

Pulsed Operation

- **Under pulsed operation Lorentz detuning can have a complicated dynamic behavior**



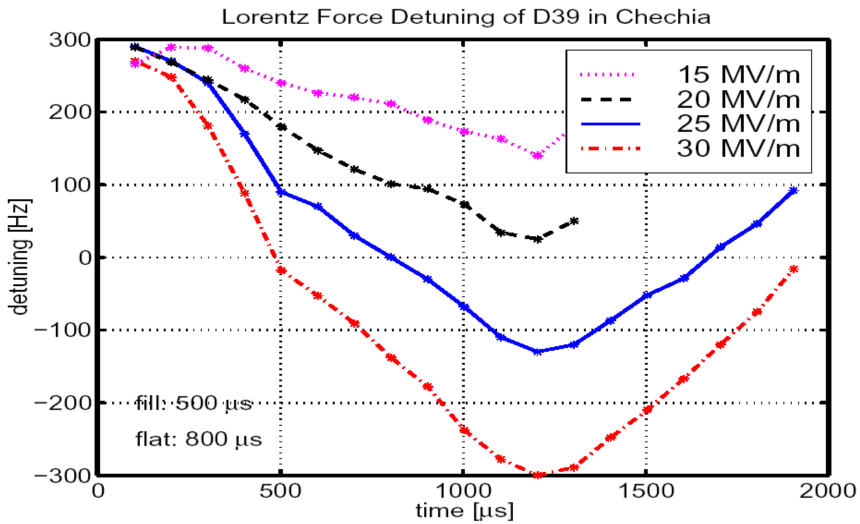
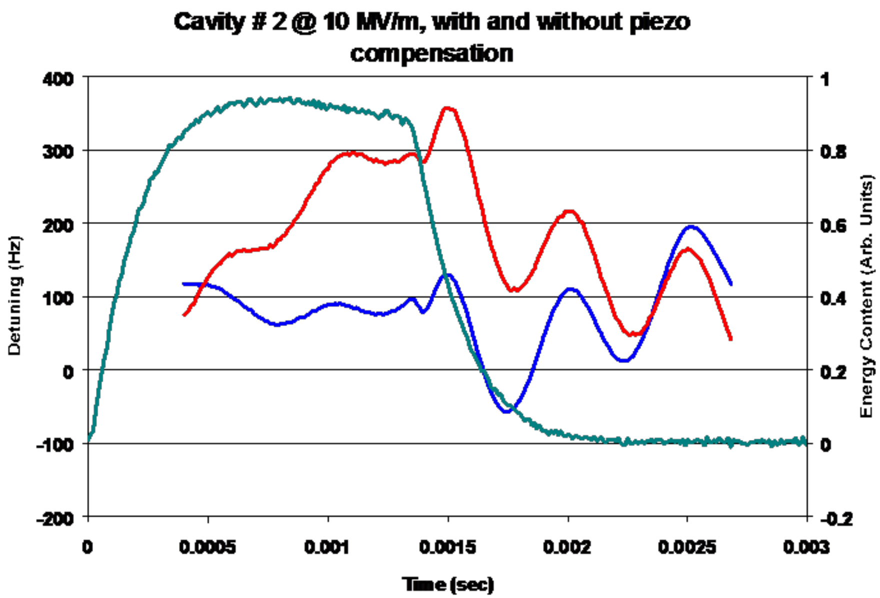


Fig. 2: Lorentz force detuning measured for a TESLA cavity at different gradients.

Pulsed Operation

- **Fast piezoelectric tuners can be used to compensate the dynamic Lorentz detuning**



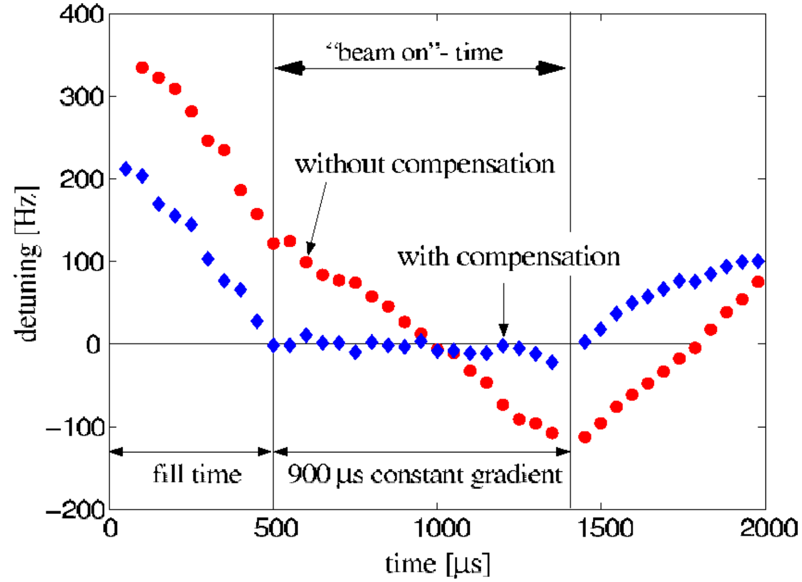


Figure 2. Lorentz force compensation at the TTF

Status of Microphonics Control

- Microphonics and ponderomotive instabilities issues in high-Q SRF cavities were "hot topics" in the early days (~70s), especially in low-β applications
- They were solved and are well understood
- They are being rediscovered in medium- to high-β applications
- Today's challenges:
  - Large scale (cavities and accelerators): need for optimization
  - Finite beam loading
    - Small but non-negligible current (e.g. RIA)
    - Low current resulting from the not quite perfect cancellation of 2 large currents (ERLs)